Ultralight Metallic/Composite Materials with Architected Cellular Structures

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Abstract. Due to the emergence of technologies enabling the fabrication of complex cellular materials, new materials with higher mechanical efficiency than the constituent material have been introduced. Combination of optimized cellular architectures with high-performance metals and composites can result in lightweight materials with mechanical properties previously unattainable at low densities. Consequently, new materials can be designed to maximally fit the target application. There is a wide range of important applications including energy absorption, metamaterial, thermal management, and bioscaffold for ultralight architected cellular metals as well as airframes and shape morphing for ultralight architected cellular composites. Therefore, it is of importance to propose a nearly exact and highly efficient methodology to study low-mass metal and composite systems with architected cellular structures. We present the simplest initial computational framework for the analysis, design, and topology optimization of such cellular materials. In the present methodology, the repetitive Representative Volume Element (RVE) approach is employed to model the actual cellular metallic/composite micro-lattices. Each member of the cellular material is modeled using only one finite beam element with 12 degrees of freedom (DOF), and the nonlinear coupling of axial, bidirectional-bending, and torsional deformations is studied for each spatial three-dimensional (3D) beam element. The large deformation analysis of the cellular strut members is performed utilizing mixed variational principle in the updated Lagrangian co-rotational reference frame. The explicit form of the stiffness matrix is calculated under the effect of plasticity for the case of the cellular metals and under the effect of nonlinear flexible connections for the case of the cellular composites. Then, we use newly proposed homotopy methods to solve the algebraic equations.

Keywords: Architected cellular metals/composites · Plastic hinge mechanism · Nonlinear flexible connections · Homotopy methods

1 Introduction

Nature benefits from high stiffness and strength low-weight materials by evolving architected cellular structures. For example, trabecular bone, beaks and bones of birds, plant

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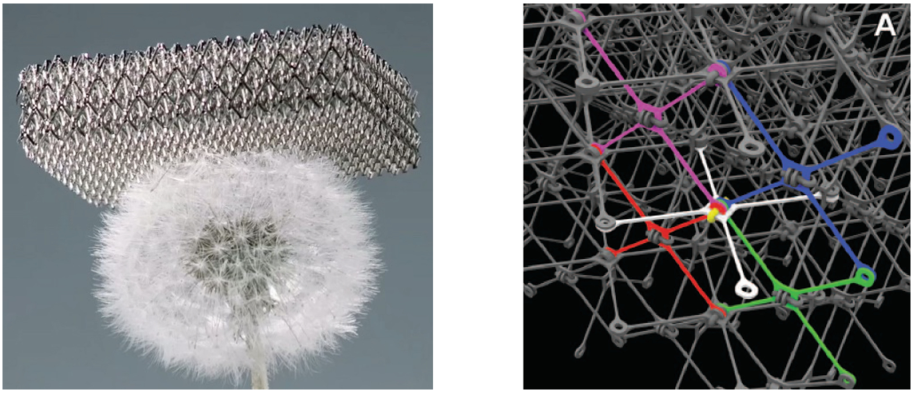
parenchyma, and sponge optimize superior mechanical properties at low density by implementing a highly porous, complex architected cellular core [1]. The same engineering and architectural principles at the material scale have been used by humankind to develop materials with higher mechanical efficiency and lower mass in many weight-critical applications. The emergence of advanced manufacturing technologies such as additive manufacturing and three-dimensional (3D) laser lithography offer the opportunity to fabricate ultralight metallic and composite materials with intricate cellular architecture to location-specific requirements. For example, the world's lightest metal [2, 3], Fig. 1(a), and reversibly assembled ultralight carbon-fiber-reinforced composite materials [4], Fig. 1(b), with micro-architected cellular structures have been recently fabricated at HRL Laboratories and MIT Media Lab-Center for Bits and Atoms, respectively.



(a) (b)

Fig. 1. (a) Metallic cellular microarchitecture [2, 3] and (b) assembly of a cuboct carbon-fiber-reinforced composite microlattice [4].

Architected cellular materials enable to include the effect of the structural hierarchy in the determination of the bulk material properties [5]. For example, fiber-reinforced cellular composites recently fabricated at MIT Media Lab-Center for Bits and Atoms exhibit structure on more than one length scale [4], and metallic microlattice materials recently fabricated at HRL Laboratories span three different length scales (nm, μm, mm) [2, 3]. Cheung and Gershenfeld [4] fabricated a cubic lattice of vertex-connected octahedrons, called cuboct, consisting of solid bars with square cross section $t \times t$ and length l. They examined only ultralight cellular composites with thickness-to-length ratio, $= t/l$ smaller than 0.1 and obtained stretch-dominated lattice-based composites. To form volume-filling lattice structure, they [4] produced many small cross-shaped building blocks assembled by mechanical interlocking connections. Each building block constitutes four conjoined strut members to one locally central node where a shear clip is inserted after the assemblage of all blocks. Strut members are composed of unidirectional fiber composite beams and looped fiber load-bearing holes [4]. Schaedler et al. [2] and Torrents et al. [3] fabricated nickel-based micro-architected cellular materials composed of hollow tube members with $L = 1$ to 4 mm node-to-node spacing, $D = 100$

to 500 μm strut diameter, $t = 100$ to 500 nm wall thickness, and $\theta = 60^{\circ}$ inclination angle. The unit cell of micro-architecture was an octahedron without any basal members. Such architected cellular materials have offered specific mechanical properties (stiffness, strength, toughness and energy absorption) at low-density regions, which makes them appropriate for a variety of engineering applications. For example, efforts are under way to employ the functionality of the cellular metallic/composite materials in energy absorption [6], mechanical metamaterials [7], bioscaffolds [8], and in adaptive structures [9].

In the current work, the fiber-reinforced cellular composites recently fabricated at MIT Media Lab-Center [4] and nickel-based micro-architected cellular materials recently fabricated at HRL Laboratories [2, 3] are modeled using repetitive Representative Volume Element (RVE) approach consisting of nodes and strut members which mimic the topology of the cellular material. Each member of the ultralight cellular metallic/composite materials is considered as single finite three-dimensional (3D) beam element, and member generalized strains and stresses are calculated under the nonlinear coupling of axial, bidirectional-bending, and torsional deformations. The plastic hinge method [10, 11] is employed to study the effect of plasticity on the mechanical response of the micro-architected cellular metal. Using this method, plastic hinge can be formed along the cellular member everywhere the plasticity condition in terms of generalized stress resultants is satisfied. The effect of nonlinear flexible connections of the micro-architected cellular composite is studied using the standardized Ramberg-Osgood function [12] for the moment-rotation relation of flexible connections. The explicit form of the tangent stiffness matrix is derived utilizing the mixed variational principle [13] in the co-rotational updated Lagrangian reference frame. Then, we employ the Newton homotopy algorithm [14] to solve the algebraic equation $\boldsymbol{F}(\boldsymbol{X}) = 0$, in which $\boldsymbol{X}$ is the solution vector for the equilibrated nodal generalized coordinates. In contrast to the Newton-type algorithms which require to invert the Jacobian matrix, homotopy methods avoid inverting the Jacobian matrix, which makes them simpler to use when the Jacobian is nearly singular.

The outline of the paper is as follows. The fundamental concepts of the present methodology are given in Sect. 2. Section 3 is devoted to model micro-architected cellular metallic materials fabricated at HRL Laboratories [2, 3] and ultralight cellular composites fabricated at MIT Media Lab-Center [4] using the current methodology and compare the calculated mechanical properties with the corresponding experimental measurements. Finally, a conclusion is presented in Sect. 4.

2 Computational Approach

Due to the consideration of the nonlinear coupling of the axial, bidirectional-bending, and torsional deformations for the large-deformation analysis, the following displacement field is considered for each 3D spatial beam element in the co-rotational updated Lagrangian reference

$$u_1(x_1, x_2, x_3) = u_{1T}(x_2, x_3) + u_{10}(x_1) - x_2 \frac{\partial u_{20}(x_1)}{\partial x_1} - x_3 \frac{\partial u_{30}(x_1)}{\partial x_1},$$

$$u_2(x_1, x_2, x_3) = u_{20}(x_1) - \hat{\theta}x_3,$$
$$u_3(x_1, x_2, x_3) = u_{30}(x_1) + \hat{\theta}x_2, \tag{1}$$

where, $u_{1T}(x_2, x_3)$ is the warping displacement due to the torsion T, $u_{10}(x_1)$ is the axial displacement at the centroid ($x_2 = x_3 = 0$), $u_{20}(x_1)$ and $u_{30}(x_1)$ are the transverse bending displacements at the centroid along x_2- and x_3-axes, respectively, and $\hat{\theta}$ is the twist angle around x_1-axis. Using the Green-Lagrange strain components in the updated Lagrangian co-rotational frame, the member generalized strain, $\boldsymbol{E}$ is determined as below

$$\boldsymbol{E} = \begin{bmatrix} u_{10,1} + \frac{1}{2}\left(u_{20,1}\right)^2 + \frac{1}{2}\left(u_{30,1}\right)^2 \\ -u_{20,11} \\ -u_{30,11} \\ \hat{\theta}_{,1} \end{bmatrix}, \tag{2}$$

resulting in the following member generalized stresses, $\boldsymbol{\sigma}$

$$\boldsymbol{\sigma} = \boldsymbol{DE}. \tag{3}$$

The matrix $\boldsymbol{D}$ is determined upon the mechanical properties of the constituent material; for the cellular composite, it includes the anisotropic properties of the base material, and for the cellular metal, it includes the linear elastic properties of the parent material.

The functional of the mixed variational principle, $\mathcal{H}$ for an RVE consisting of N members can be expressed in terms of the incremental components of the second Piola-Kirchhoff stress tensor, S_{ij}^1. and the displacement field, u_i, as follows

$$\mathcal{H} = \sum_{m=1}^{N} \left(\int_{V_m} \left\{ -B\left[S_{ij}^1\right] + \frac{1}{2}\tau_{ij}^0 u_{k,i}u_{k,j} + \frac{1}{2}S_{ij}\left(u_{i,j} + u_{j,i}\right) - \rho b_i u_i \right\} dV - \int_{S_{\sigma m}} \bar{T}_i u_i dS \right), \tag{4}$$

where, $V_m(m = 1, 2, \cdots, N)$ is the volume of the mth member, S_{σ_m} is the m member surface with the prescribed traction, and $\bar{T}_i$ and b_i $(i = 1, 2, 3)$ are, respectively, the components of the boundary tractions and the body forces per unit volume in the current configuration. Invoking the variation of $\mathcal{H}$ leads to the following relation

$$\delta\mathcal{H} = \sum_{m=1}^{N} \delta\boldsymbol{\beta}^T(-\boldsymbol{H\beta} + \boldsymbol{Ga}) + \sum_{m=1}^{N} \delta\boldsymbol{a}^T\left(\boldsymbol{G}^T\boldsymbol{\beta} + \boldsymbol{K}_N\boldsymbol{a} - \boldsymbol{F} + \boldsymbol{F}^0\right) = 0. \tag{5}$$

To study the effect of plasticity for the case of the micro-architected cellular metal, the plastic hinge mechanism is employed, which considers the formation of plastic hinge everywhere along the beam element when the plasticity condition is satisfied. The increment of the plastic work at the ith plastic hinge, dw_i^P is determined on the basis of the incremental plastic nodal displacement, $d\boldsymbol{a}_i^P$ as

$$dW_i^p = d\boldsymbol{a}_i^{pT}\boldsymbol{\sigma}. \tag{6}$$

The incremental plastic nodal displacement can be expressed using the potential function $\phi_k = \left[\frac{\partial f_k(\boldsymbol{\sigma}, \sigma_Y)}{\partial \boldsymbol{\sigma}}\right]$ as

$$d\boldsymbol{a}_i^p = d\lambda_k\boldsymbol{\phi}_k, \tag{7}$$

here, $d\lambda_k$ is a positive scalar, and $f(\boldsymbol{\sigma}, \sigma_Y) = 0$ is the plastic potential explaining the plasticity condition in terms of the stress components at the location of the plastic hinge and the Yield stress, σ_Y.

To study the effect of nonlinear flexible connections for the case of the micro-architected cellular composite, nonlinear rotational springs are modeled at the ends of the adjacent members. The increment of the spring rotation can be expressed as

$$\Delta^\alpha \phi_i = (-1)^\alpha \frac{\Delta^\alpha M_i}{{}^\alpha S_i^t} \qquad i = 2, 3, \alpha = 1, 2, \tag{8}$$

in which, $\Delta^\alpha M_i$ is the incremental momentum along $x_i - axis$ at node α, and ${}^\alpha S_i^t$ is the instantaneous rotational rigidity of the spring along $x_i - axis$ at node α. Employing the moment-rotation relation based on the standardized Ramberg-Osgood function, S^t which is the slope of the moment-rotation curve, $\frac{dM}{d\phi}$ is obtained as

$$S^t = \frac{dM}{d\phi} = \frac{(\text{Ꝁ} M)_0}{\phi_0 \text{Ꝁ}\left[1 + n\left(\frac{\text{Ꝁ} M}{(\text{Ꝁ} M)_0}\right)^{n-1}\right]} \qquad M > 0. \tag{9}$$

Including the plastic works done by plastic hinges for the case of the cellular metal and including the incremental energy spent at nonlinear rotational springs for the case of the cellular composite into the mixed variational principle and then invoking its variation modifies Eq. (5) to the following equation

$$\sum\nolimits_{m=1}^{N} \delta\hat{\boldsymbol{\beta}}^T\left(-\hat{\boldsymbol{H}}\hat{\boldsymbol{\beta}} + \hat{\boldsymbol{G}}\boldsymbol{a}\right) + \sum\nolimits_{m=1}^{N} \delta\boldsymbol{a}^T\left(\hat{\boldsymbol{G}}^T\hat{\boldsymbol{\beta}} + \boldsymbol{K}_N\boldsymbol{a} - \boldsymbol{F} + \boldsymbol{F}^0\right) = 0. \tag{10}$$

Therefore, the stiffness matrix, $\hat{\boldsymbol{K}}$, for the large deformation analysis of ultralight cellular metal/composite is derived explicitly as

$$\hat{\boldsymbol{K}} = \hat{\boldsymbol{G}}^T\hat{\boldsymbol{H}}^{-1}\hat{\boldsymbol{G}} + \boldsymbol{K}_N. \tag{11}$$

Then, to solve the incremental tangent stiffness equations ($\boldsymbol{F}(\boldsymbol{X}) = 0$) using homotopy method, we consider the following scalar Newton homotopy function,

$$h_n(\boldsymbol{X}, t) = \frac{1}{2}\|\boldsymbol{F}(\boldsymbol{X})\|^2 + \frac{1}{2Q(t)}\boldsymbol{F}(\boldsymbol{X}_0)^2, \qquad t \geq 0, \tag{12}$$

resulting in,

$$\dot{\boldsymbol{X}} = -\frac{1}{2}\frac{\dot{Q}\|\boldsymbol{F}\|^2}{Q\left\|\boldsymbol{B}^T\boldsymbol{F}\right\|^2}\boldsymbol{B}^T\boldsymbol{F}, \qquad t \geq 0, \tag{13}$$

where, $\boldsymbol{B}$ is the Jacobian (tangent stiffness) matrix evaluated with $\boldsymbol{B} = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{X}}$, and $Q(t)$ is a positive and monotonically increasing function to enhance the convergence speed.

3 Ultralight Architected Cellular Metallic/Composite Materials

3.1 Metallic Octahedral Micro-architecture

Metallic cellular materials fabricated at HRL Laboratories [2, 3] are nickel-based materials with octahedral configuration without any basal members. Micro-architected metals consist of hollow tube strut members with the length of $L = 1 - 4$ mm, the strut diameter of $D = 100 - 500\,\mu$m, the wall thickness of $t = 100 - 500$ nm, and the inclination angle of $\theta = 60^{\circ}$. Using the RVE approach in conjunction with plastic hinge method, we model a nickel-based cellular material with $L = 1200\,\mu$m, $D = 175\,\mu$m, and $t = 26\,\mu$m and compare the calculated mechanical results with the corresponding experimental measurements. For this study, an RVE consisting of 36 nodes and 64 strut members is considered under compressive loading, Fig. 2(a). The structural geometry of the strut member is also shown in Fig. 2(b).

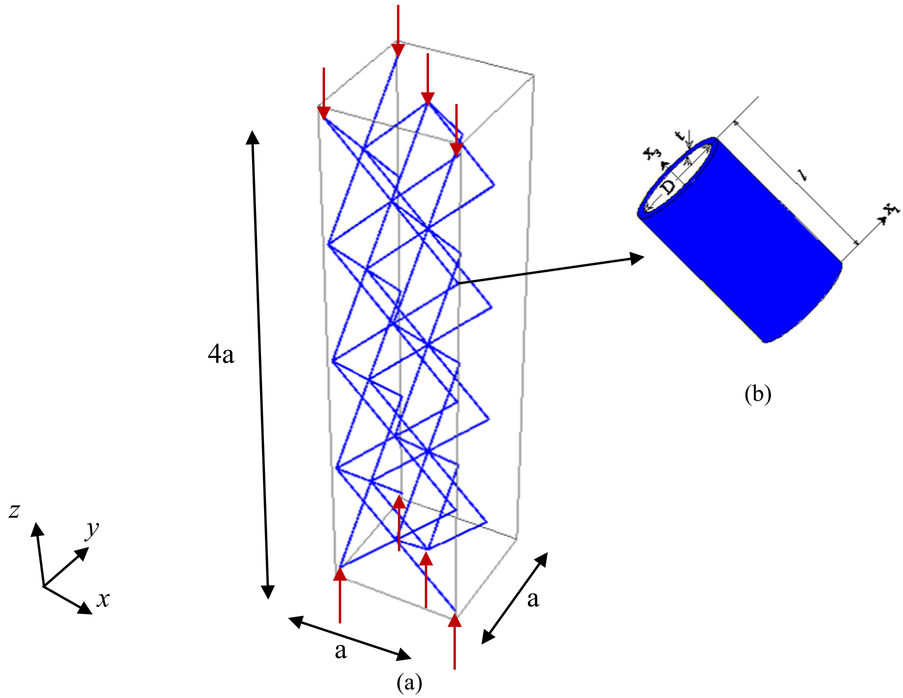


Fig. 2. (a) An RVE consisting of 36 nodes and 64 strut members and (b) the structural geometry of the strut member.

Using the present methodology, the engineering stress-engineering strain curve is calculated and plotted in Fig. 3. The Young's modulus and the yield stress are calculated as 0.7841 GPa and 7.3704 MPa, respectively. The experimental measurements reported by Torrents et al. [3] for the tested micro-lattice with the strut diameter $D = 175 \pm 26\,\mu$m, strut length $L = 1200 \pm 36\,\mu$m, and wall thickness $t = 26.00 \pm 2.6\,\mu$m exhibit the Young's modulus of 0.58 ± 0.003 GPa and the yield stress of 8.510 ± 0.025 MPa. As it is found, there is a good agreement between our calculated mechanical properties and those measured experimentally by Torrents et al. [3].

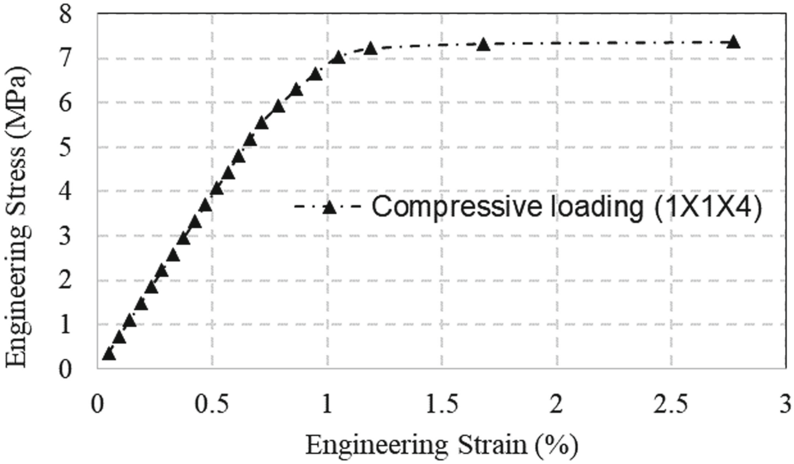


Fig. 3. Stress-strain curve of the cellular metallic micro-lattice subjected to compression.

3.2 Flexible Cuboct Carbon Fiber-Reinforced Polymer Composite

The cuboct cellular lattice of carbon fiber-reinforced polymer composite at MIT Media Lab-Center [4] is fabricated by vertex-connected octahedrons, Fig. 4(a). The lattice is composed of slender members with $\phi = t/l < 0.1$ in which t is the width of the square cross section and l is the length of the strut member, Fig. 4(b). Since the cellular structure is formed by assembling identical building blocks, we consider that connections are flexible and introduce nonlinear rotational springs at flexible connections. Using the repetitive RVE approach and the standardized Ramberg-Osgood functions for the moment-rotation relation of the nonlinear rotational springs at flexible connections, we model a cuboct sample with $l = 0.9$ cm including 21 nodes and 48 elements. By changing the width of the cross section, t, various samples with different values of ϕ are modeled and, then, loaded under compression. The Young's moduli for the ultralight cellular composite materials with $t/l < 0.1$ are calculated and compared with the corresponding experimental results given by Cheung and Gershenfeld [4] in Table 1. As it is seen from Table 1, there is an excellent agreement between computational and experimental results.

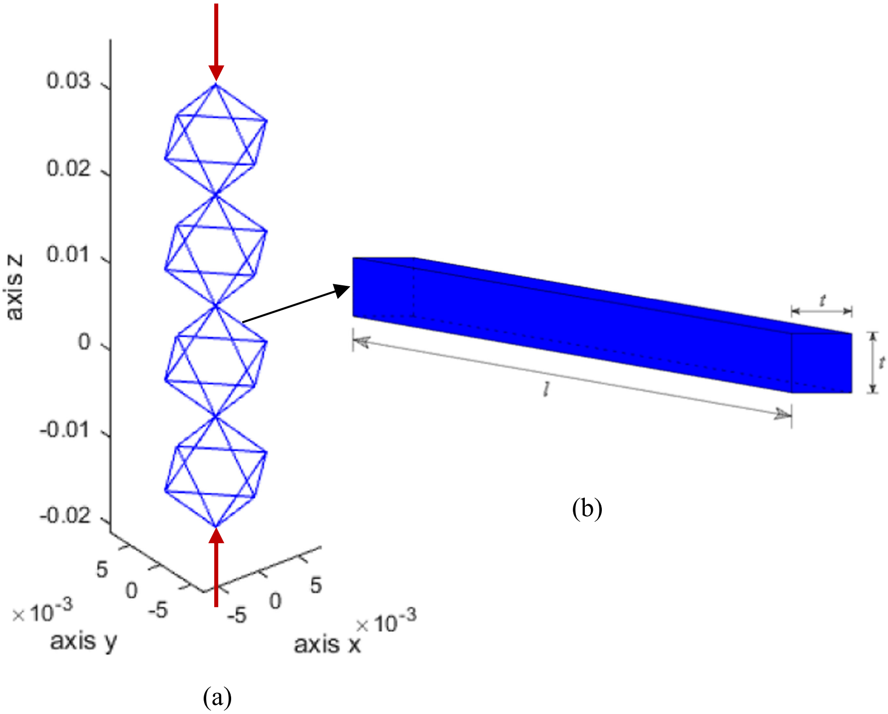


Fig. 4. (a) An RVEs including 21 nodes and 48 elements and (b) strut member geometry.

Table 1. Comparison between results calculated from the current methodology and corresponding experimental results reported by Cheung and Gershenfeld [4].

t/l	Young's Modulus (GPa)		
	Present Study		Experiment (Ref. [4])
	Flexible Connections	Rigid Connections	
4.6995×10^{-2}	1.8729×10^{-2}	1.8731×10^{-2}	1.4190×10^{-2}
4.7127×10^{-2}	1.8839×10^{-2}	1.8842×10^{-2}	1.4080×10^{-2}
4.7226×10^{-2}	1.8919×10^{-2}	1.8921×10^{-2}	1.4760×10^{-2}
4.7456×10^{-2}	1.9116×10^{-2}	1.9118×10^{-2}	1.4430×10^{-2}
4.7522×10^{-2}	1.9171×10^{-2}	1.9173×10^{-2}	1.4300×10^{-2}

4 Conclusion

Throughout this study, we present a computational methodology for the analysis, design, and topology optimization of ultralight metals or composites with architected cellular structures. The repetitive RVE approach is employed to mimic the fabricated cellular material. Each member of the architecture is modeled using only one finite 3D beam element with 12 DOF, and the nonlinear coupling of axial, bidirectional-bending, and torsional deformations is studied for each spatial beam element. For the large elastic-plastic deformation analysis of micro-architected cellular metallic materials, the plastic hinge method is utilized in such a way that plastic hinges can be formed everywhere along the beam when the plasticity condition is satisfied. For the large deformation analysis of the cuboct cellular composite with flexible connections, the standardized Ramberg-Osgood function is employed to introduce the moment-rotation relation at flexible nodes. One of the fabricated nickel-based cellular microstructures is modeled using an RVE with 36 nodes and 64 elements, and its mechanical properties under compression are calculated and compared with the corresponding experimental results. Moreover, a series of fabricated cuboct cellular composites with different thickness-to-length ratio are simulated using an RVE with 21 nodes and 48 elements, and the calculated Young's moduli are compared with the corresponding experimental measurements. We find a very good agreement between our computational results and experimental reports available in the literature.

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