Mathematical Modeling of Water Infiltration in Unsaturated Latosol Samples

Patricia Figuereido de Sousa[1](✉), João de Mendonça Naime[2], Silvio Crestana[2], and André Luís Brasil Cavalcante[1]

[1] University of Brasilia, Campus Universitário Darcy Ribeiro, Asa Norte, Brasília, Brazil
patriciafiguereidodesousa@gmail.com
[2] Embrapa Instrumentação Agropecuária, Rua XV de Novembro, 1452, São Carlos, Brazil

Abstract. Understanding the soil in the unsaturated state means, among other things, understanding how the presence of water transiently affects the physical and hydraulic properties of the soil environment. This is extremely important in solving soil resistance problems involving water, such as the stability of land slopes. Among the hydraulic properties that are essential in understanding water flow in porous media, flow velocity and soil water retention capacity are the most important defining characteristics of the hydraulic behavior of the medium. In the physical model used in this study, these characteristics were represented by the advective velocity and hydraulic diffusivity parameters that were obtained from the infiltration data by conducting tests on the latosol columns. The main results obtained can be used for the estimation of the wetting front advance given the uncertainties of the variables obtained from the confidence envelope curves.

Keywords: Unsaturated soils · Richards equation · Infiltration tests · Wetting front

1 Unsaturated Transient Vertical Flow

The Richards equation, a nonlinear partial differential equation, is used to describe the unsaturated flow phenomenon in porous media. This equation governs soil fluid migration assuming the validity of Darcy's law and the continuity equation. For the vertical flow condition of an incompressible and homogeneous fluid, the Richards equation can be expressed by Eq. (1), as can be seen in the literature.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(\frac{k_z(\theta)}{\gamma_w}\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z}\right) - \frac{\partial k_z(\theta)}{\partial z}\frac{\partial \theta}{\partial z}. \quad (1)$$

where θ is the volumetric moisture content, k_z is permeability coefficient, ψ is suction and z is flow direction. The two important terms defined in Eq. (1), namely, hydraulic diffusivity (D_z) and advective velocity (a_s) are presented in Eqs. (2) and (3).

M. Barla et al. (Eds.): IACMAG 2021, LNCE 125, pp. 616–623, 2021.
https://doi.org/10.1007/978-3-030-64514-4_63

$$D_z = \frac{k_z(\theta)}{\gamma_w}\frac{\partial\psi}{\partial\theta}, \tag{2}$$

$$a_s = \frac{\partial k_z(\theta)}{\partial z}. \tag{3}$$

Substituting the values of D_z and a_s in Eqs. (2) and (3) to that in Eq. (1), the Richards equation can be expressed as a function of these values as shown in Eq. (4).

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D_z\frac{\partial\theta}{\partial z}\right) - a_s\frac{\partial\theta}{\partial z}. \tag{4}$$

Due to the complexity of the differential equation that governs water migration, there is not yet a closed form analytical solution for all cases and still requires some considerations for its solution. The analytical solution used in this research, which was presented by Cavalcante and Zornberg (2017), assumes that hydraulic diffusivity and advective velocity do not vary as the water moves through the soil or that their variations are small enough to be considered negligible. This means they can be considered as constant values $(\bar{D}, \bar{a})$ for a given soil sample. Hence, Eq. (4) can be rewritten as

$$\frac{\partial\theta}{\partial t} = \overline{D}\frac{\partial^2\theta}{\partial z^2} - \overline{a}\frac{\partial\theta}{\partial z}. \tag{5}$$

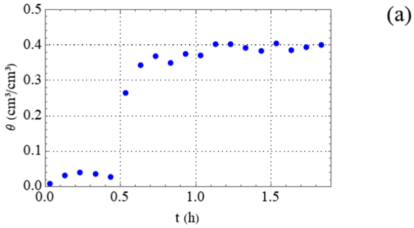
In the literature, equations with a similar structure to Eq. (5), present analytical solutions for different initial and boundary conditions, as described in Ogata and Banks (1961), Brenner (1962), Lindstrom and Boersma (1971), Cleary and Adrian (1973), among many others. Cavalcante and Zornberg (2017) presented solutions to some problems, which can be divided into four cases. In this study, we focused on one of these solutions, which corresponded to a constant volumetric moisture content model at the top of a semi-infinite soil column. This analytical solution is presented in Eq. (6), where $\theta(0,t) = \theta_o$ (upper boundary condition), $\theta(z,0) = \theta_i$ (initial condition) and erfc is the complete error function, whose expression can be seen in the article by Cavalcante and Zornberg (2017).

$$\theta(z,t) = \theta_i + (\theta_o - \theta_i)\operatorname{erfc}\left(\frac{z-\overline{a}}{2\sqrt{\overline{D}t}}\right) + \exp\left(\frac{\overline{a}z}{\overline{D}}\right)\operatorname{erfc}\left(\frac{z+\overline{a}}{2\sqrt{\overline{D}t}}\right). \tag{6}$$

2 Materials and Methods

The samples used in the infiltration study were obtained from the Embrapa Farm, in São Carlos, São Paulo State, Brazil. The soil was composed of 45% clay, 13% silt, and 42% sand (Naime 2001). The clay fraction of the soil was made of kaolinite (1:1), which was responsible for the low retention capacity observed in the soil samples.

The data underlying this study are volumetric moisture content values obtained by Naime (2001) during infiltration into soil columns monitored by the use of gamma-tomography. More details on specimen test methodology can be found in Naime (2001). Figure 1a and 1b show the data that were analyzed in this study.



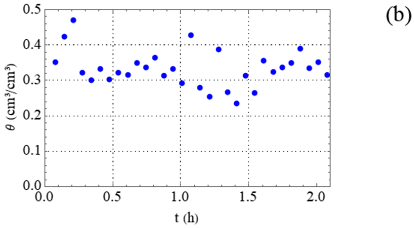


Fig. 1. Infiltration data (a) 0.0935 m from the top of the column, mean of two samples (b) measured in a variable section of a sample (z ranging from 0 to 0.15 m, with an increment of 0.005 m between each reading)

To back analyze the infiltration data presented in Figs. 1a and 1b, adjustments were made to the temporal variation curve of volumetric moisture content using the function described in Eq. (6) for constant z (data from Fig. 1a only) and variable z (combined data from Figs. 1a and 1b). These adjustments were performed with the aid of Wolfram Mathematica software using the nonlinear fit model tuning command to find the best hydraulic diffusivity and advective velocity values according to the experimental data analysis. The input data for model calibration were: (a) evaluated position (z), which was equal to the section of readings taken by the tomograph; (b) initial column volumetric moisture content (θ_i), which was assumed to be the lowest volumetric moisture content measured during the tests (equal to 1.0% for the samples evaluated in this study); (c) top column volumetric moisture content (θ_o), which was assumed to be the characteristic volumetric moisture content value of upper threshold points [40% based on the analysis of the readings and consistent with the one presented in Varandas (2011)]. At the end of the computational analysis, the fit was considered good if the function obtained could describe the experimental points well ($R^2 > 0.75$) and if the values obtained for $\bar{D}$ and $\bar{a}$ agree with the expected values for this type of soil.

3 Results and Discussion

3.1 Case 1: Variation of Volumetric Moisture Content with Time at Constant Z

From the infiltration data presented in Fig. 1a, the volumetric moisture content moisture function was adjusted following the methodology presented in Sect. 2. The parameters obtained from the adjustment are presented in Table 1. A comparison between the curve obtained from the adjustment and the one from the experimental points is shown in Fig. 2a. The estimation of the advance of the wetting front for the other sections, through the function $\theta(z, t)$ defined by the adjustment is presented in Fig. 2b. It should be noted that this extrapolation is valid only if, along the tested soil column, the hydraulic properties have been kept constant, as assumed in this study.

Table 1. Parameters obtained from the adjustment of temporal variation of volumetric moisture content

Parameters	Value	R^2
Hydraulic Diffusivity (m^2/h)	4.00×10^{-2}	~1.00
Advective velocity (m/h)	1.82×10^{-1}	

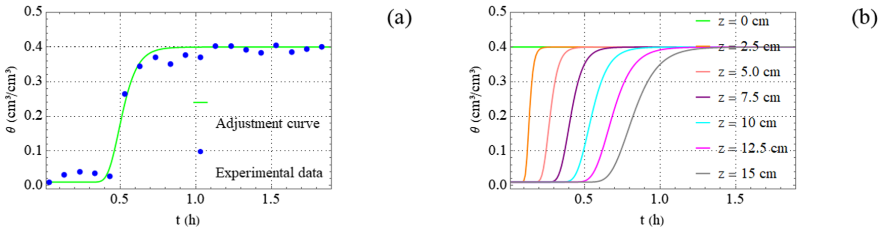


Fig. 2. (a) Curve fitted to experimental data (b) wetting front estimate

3.2 Analysis of Confidence Intervals– Case 1 Adjustment

Another way to visualize the parameters obtained from the adjustments is through confidence envelopes as they help in the determination of probable ranges for the hydraulic behavior of the soil. Confidence envelopes also provide a more conservative and realistic evaluation of the environmental behavior given the uncertainties due to the natural heterogeneity of the material. In this study, the ranges were established for the 95%, 99%, and 99.9% confidence levels. Table 2 shows the lower and upper limit values of these ranges for the adjusted constants.

Table 2. Confidence intervals for the adjusted parameters – case 1

Parameters	Hydraulic diffusivity (m^2/h)		Advective velocity (m/h)	
Trust level	Inferior limit	Upper limit	Inferior limit	Upper limit
95%	2.64×10^{-2}	5.30×10^{-2}	1.75×10^{-1}	1.89×10^{-1}
99%	2.14×10^{-2}	5.80×10^{-2}	1.72×10^{-1}	1.91×10^{-1}
99.99%	1.47×10^{-2}	6.47×10^{-2}	1.68×10^{-1}	1.95×10^{-1}

One way to draw reliable envelopes from the values shown in Table 2 is to keep one parameter constant while varying the others, so the influence of each parameter can be evaluated separately (Figs. 3a and 3b). It is noted that the use of higher hydraulic diffusivity values (upper limits) (Fig. 3a) could lead to a faster advancement of the wetting front to the point where this behavior reverses. When the volumetric moisture content was about 20% (an average between the values of θ_i and θ_o) at about 0.5 h, the $\partial\theta/\partial t$ was at maximum; at this point the advective component dominated the water movement as can be seen in Fig. 4. Figure 3b shows that higher values of advective velocity led to a faster advancement of the wetting front or a greater ease of the propagation of the wetting front. In practice, for example, the use of larger advective velocity values in slope stability analysis could lead to faster loss of strength during soil saturation due to higher flow velocity. Thus, lower safety factors are used when scaling slope cut or some containment structure. At the thresholds, the confidence envelope area is almost negligible.

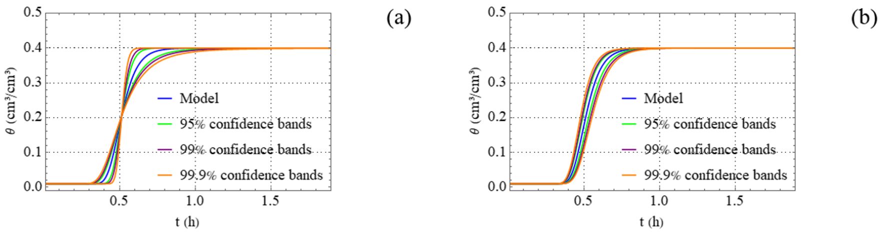


Fig. 3. Confidence envelopes for case 1 based on lower and upper limit values for the (a) hydraulic diffusivity (b) advective velocity

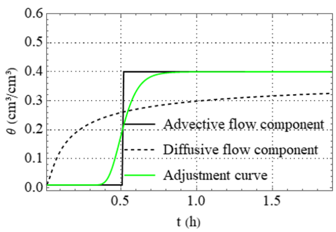


Fig. 4. Influence of diffusive and advective flow plots at z = 0.0935 m

3.3 Case 2: Variation of Volumetric Moisture Content with Time at Variable Z

The volumetric moisture content function was fitted to the experimental data by joining the readings data of a single section with various sections during infiltration, as shown in Figs. 1a and 1b. The surface obtained from the adjusted function is shown in Fig. 5. The estimated parameters obtained by fitting the volumetric moisture content function to the experimental data is presented in Table 3.

Comparing the data presented in Table 3 with that presented in Table 2, it is evident that the only difference is the hydraulic diffusivity, which was higher for case 2. This caused a lower rate of the wetting front advance for this case. Figure 6a shows the comparison between the experimental data presented in Fig. 1a with the prediction of the fit-defined model for case 2. On comparing Fig. 2a with Fig. 6a, it is observed that the section fit was more effective to estimate the infiltration data presented in Fig. 1. However, this does not mean that it is more effective to extrapolate the other sections as can be seen in the difference of the residuals shown in Fig. 7 between case 1 and 2 fitting solutions with the experimental points presented in Fig. 2. In the residual plot, the smaller the area, the better the solution.

Through the volumetric moisture content function obtained in this type of adjustment, it was possible to estimate the advance of the wetting front to the other sections more coherently than those presented previously. The curves for the other sections are shown in Fig. 6b. It should be noted that when comparing Figs. 2b and 6b, the advance of the wetting front occurs at a lower rate by increasing the value of the diffusive flow portion only. This may be explained by the greater importance of the diffusive flow component than the advective component in dissipating the wetting front (see Figs. 8a and 8b).

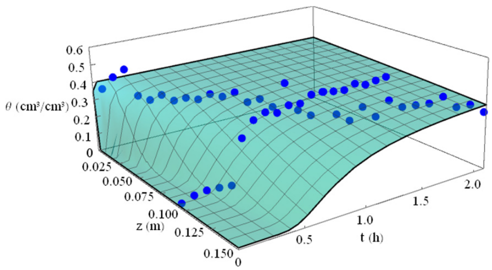


Fig. 5. Surface obtained from fitting experimental data

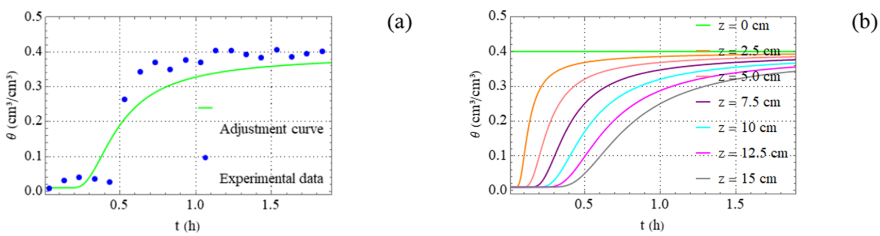


Fig. 6. For case 2 (a) Comparison between fixed position experimental data and the model obtained from the adjustment (b) wetting front estimate

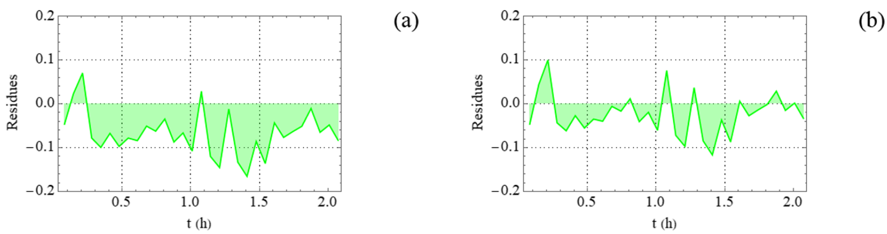


Fig. 7. Difference between experimental values of volumetric moisture content measured in different sections with the function defined in (a) case 1 (b) case 2

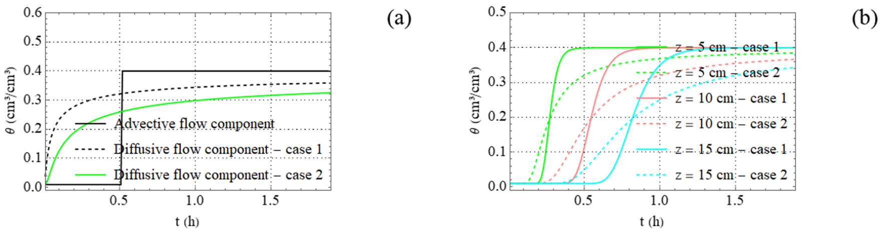


Fig. 8. Influence of increased hydraulic diffusivity on water movement (a) comparison between flow portions (b) comparison between estimates for some sections

Table 3. Parameters obtained in the adjustment of the points of temporal variation of the volumetric moisture content with variable z

Parameters	Value	R^2
Hydraulic Diffusivity (m^2/h)	13.4×10^{-2}	~0.97
Advective velocity (m/h)	1.77×10^{-1}	

3.4 Analysis of Confidence Intervals –Case 2 Adjustment

Similar to case 1, the confidence intervals were defined for the parameters adjusted in case 2 (Table 4). Figure 9a and 9b show, the confidence envelopes for the adjusted function based on hydraulic diffusivity and advective velocity variations, respectively, for section z = 0.0935 m. The confidence envelopes for this case were wider than those for the previous case. This may be due to the greater dispersion of the experimental data used in the case 2 back-analysis, which resulted in a greater difference between the lower and upper limits in each confidence level range.

Table 4. Confidence intervals for adjusted parameters – case 2

Parameters	Hydraulic Diffusivity (m^2/h)		Advective velocity (m/h)	
Trust level	Inferior limit	Upper limit	Inferior limit	Upper limit
95%	8.46×10^{-2}	18.36×10^{-2}	1.47×10^{-1}	2.08×10^{-1}
99%	6.80×10^{-2}	20.01×10^{-2}	1.36×10^{-1}	2.19×10^{-1}
99,99%	4.78×10^{-2}	22.04×10^{-2}	1.24×10^{-1}	2.31×10^{-1}

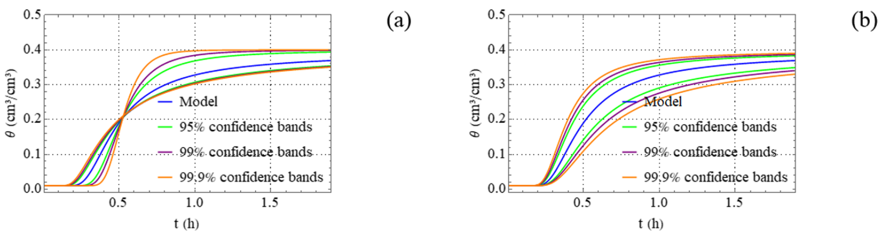


Fig. 9. Confidence envelopes for case 2 based on lower and upper limit values for the (a) hydraulic diffusivity (b) advective velocity

4 Conclusions

One of the objectives of modeling the hydraulic behavior of unsaturated soil is to estimate the advance of the wetting front imposed by a given hydraulic gradient. In this study, through the back analysis of infiltration data obtained by tomography, the hydraulic diffusivity and advective velocity values of the samples were predicted. In addition, these values for the soil under study were extrapolated by defining ranges for different confidence levels. This helped to visualize these properties in a practical way considering their dispersion, something still uncommon in geotechnical practice. By using the function $\theta(z, t)$ obtained from the back analysis of the experimental data, the water movement along the sections of the soil samples was estimated, which allowed visualization of the wetting fronts through a surface in space. It also allowed evaluation of the spatial and temporal variation of volumetric moisture content in this type of soil. It helped to understand and complete the analysis of the infiltration process in unsaturated soils.

Acknowledgements. The authors acknowledge the support provided by the following institutions for funding this research: the National Council for Scientific and Technological Development (CNPq Grant 304721/2017-4), Support Research of the Federal District Foundation (FAP-DF Grants 0193.001563/2017 and 0193.002014/2017-68), Coordination for the Improvement of Higher Level Personnel (CAPES), and University of Brasilia.

References

Brenner, H.: The diffusion model of longitudinal mixing in beds of finite length. Numer. Values. Chem. Eng. Sci. **17**(4), 229–243 (1962)

Cavalcante, A.L.B., Zornberg, J.G.: Efficient approach to solving transient unsaturated flow problems. I: Analytical solutions. Int. J. Geomech. **17**(7), 04017013 (2017). https://doi.org/10.1061/(asce)gm.1943-5622.0000875

Cleary, R.W., Adrian, D.D.: Analytical solution of the convective dispersive equation for cation adsorption in soils. Soil Sci. Soc. Am. J. **37**(2), 197–199 (1973)

Lindstrom, F.T., Boersma, L.: A theory on the mass transport of previously distributed chemicals in a water satured sorbing porous medium. Soil Sci. **111**, 192–199 (1971)

Naime, J.M.: A new method for in situ dynamic studies of water infiltration in unsaturated soil. Doctoral Thesis, University of São Carlos, São Carlos School of Engineering, p. 145 (2001)

Ogata, A., Banks, R.B.: A solution of the differential equation of longitudinal dispersion in porous media. Professional Paper 411-A, A1-A9, USGS, Reston, VA (1961)

Varandas, J.M.M.: Evaluation of soil physical quality on a microbasin scale. Doctoral Thesis, University of São Paulo, Center for Nuclear Energy in Agriculture, p. 88 (2011)