Chapter 20
Collaboration in Transport and Logistics Networks

Behzad Hezarkhani, Marco Slikker, and Tom Van Woensel

1 Introduction

Transport and logistics companies invested substantially to increase the efficiency of their individual operations. Research has also been fruitful in finding ways to optimize problems of planning routes, scheduling deliveries, designing networks, and deploying resources. It is generally well-understood that economy of scale in transportation and logistics plays a crucial role in increasing efficiency. Yet, efforts towards internal optimization cannot always increase the economies of scale for organizations beyond their operational scope. This is problematic as the logistics and transportation sector is fragmented and many operators of different sizes are present. It is no wonder then that the logistics sector suffers from low overall efficiency—for example, more than 20% for all truck movements in Europe is completely empty and the remainder is hardly ever full.

The success of new network design approaches, building on concepts, models and methodologies such as the Physical Internet, City Logistics, synchromodal networks, etc., is also to a large part depending upon the ability to successfully collaborate and agree on these cost-and-benefit sharing mechanisms. Collaboration is a way to open possibilities for achieving these important economies of scale needed for a successful implementation any Physical Internet or City Logistics solution. In fact, collaboration may positively affect many aspects. For example, by consolidating their loads, carriers can increase their service level and reduce their

---

B. Hezarkhani
Brunel Business School, Brunel University, London, UK
e-mail: Behzad.Hezarkhani@brunel.ac.uk

M. Slikker · T. Van Woensel (✉)
School of Industrial Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands
e-mail: M.Slikker@tue.nl

T. G. Crainic et al. (eds.), *Network Design with Applications to Transportation and Logistics*, https://doi.org/10.1007/978-3-030-64018-7_20

total costs. Carriers could increase the utilization rate of their assets when combining their delivery demands. Finally, as a result of consolidated cargo and combined trips, the socio-environmental problems of transport and logistics can also be mitigated.

Despite the clear advantages of collaborative logistics, in practice, cooperation and collaboration among organizations are exceptions. Collaboration among carriers is often hampered by their competitive positions and by the risks of divulging information and losing customers. Shippers, on the other hand, may hesitate to collaborate as they might not have a clear understanding of collaborative mechanisms employed and whether or not they receive a fair share out of collaborative operations. Finally, designing a fair cost sharing scheme is a major impediment for collaboration.

This type of collaboration problems falls in the area of cooperative game theory, where coalitions and their respective cost sharing issues are researched. The intention of this chapter is not to give an exhaustive review of cost sharing problems, but to provide an overview of relevant approaches in dealing with cost sharing problems for collaboration in the setting of logistics network design problems.

By abstracting a cooperative situation into a cooperative game, consisting of a player set and a function that determines the cost of different groups of players, cooperative game theory studies solutions that satisfy collections of logically desirable properties expressed in relation to such an abstraction. The players in these situations require or provide transportation-based logistics services. The cost of groups of players is obtained via a network design optimization problem. The specific features of cooperative situations under study provide grounds for refining well-known solutions in cooperative game theory or develop new ones that are appropriate for special situations.

Important to note is that cooperative games are build on stylized situations. A situation is a description of the real-life problem to handle (e.g., network optimization or service network design). However, for these situations, we need to obtain the exact value of possible coalitions (e.g., players working together). From an Operations Research perspective, many of these underlying situations are combinatorial problems, leading to significant calculation times to obtain the relevant (optimal) values. That is why a large body of cooperative game theory literature is build around stylized models. Clearly, solutions in cooperative game context can prove to be unsatisfactory in more complex situations.

This chapter is build around three parts. In the first part, we discuss the most important components around cooperation within a transport and logistics network setting. In the second part, we discuss cost-sharing problems in some basic and stylized network design models. The simplicity of underlying situations in this category allows for adoption of well-known game theoretic solutions such as the core (Shapley 1955) and the Shapley value (Shapley 1953). The search for the core of cooperative games in network situations has motivated a large body of literature, and implementation of the Shapley value is suggested by a host of research in collaborative logistics. In the third part, we look at more operational problems in collaborative logistics and overview the cooperative truckload delivery situations where logistics providers jointly devise plans for their daily pickup/transport/deliver operations. We discuss desirable properties for allocations rules in these situations and introduce an appropriate one for these situations.

This chapter is organized as follows. Section 2 discusses the key concepts revolving around collaboration in transport and logistics networks. Section 3 provides some background and preliminaries on cooperative game theory and the relevant main concepts. In Section 4, we discuss the cost sharing problem in stylized cooperative network design problems, in particular, minimum cost spanning tree, facility location, and hub location. In Section 5, we turn our attention to designing logistics service networks and focus on cooperative truckload delivery situations. Section 6 concludes the chapter.

2 Key Collaboration Concepts in Transport and Logistics Networks

Transport and Logistics networks collaboration involves different aspects: Communication, Coordination, and Consolidation. Many different actors are involved in Transport and Logistics activities. One way of reducing costs is to consolidate activities, e.g. freight consolidation or capacity consolidation, as such reducing empty mileage or under-filled resources. But, these stakeholders hardly communicate with each other, let alone that there is a form of coordination.

Over the past years, more and more different types of collaboration emerged. **Vertical collaboration**, getting popular in the 90s, involves collaboration within the supply chain, i.e. connecting the upstream and downstream partners. This lead to concepts like Vendor Managed Inventory (VMI), factory gate pricing, Collaborative Planning Forecasting and Replenishment (CPFR), and Efficient Consumer Response (ECR). At this moment, these concepts were mainly focused on costs efficiency in the different key supply chain decision areas like inventory, transportation, forecasting, etc. Early 200, next to costs efficiency, companies also started to consider other drivers like sustainability and greenhouse gas emissions. Also in transport and logistics, continued observations on low vehicle utilisations, and a large number of empty running vehicles, lead to strong understanding that collaboration could be a solution towards costs reductions but also to significant reductions in the environmental pressure.

Next to vertical collaboration, **horizontal collaboration** started to gain momentum over the past 10 years. Here, collaboration in distribution and coordination among similar stakeholders, e.g. logistics service providers or shippers, is the focus. The rationale is that bundling of physical good flows into (urban) areas, results in fewer negative impacts (decongestion, less negative externalities in cities). Clearly, Transport and Logistics networks are large constructs of multiple many-to-many interconnected stakeholders, active in both horizontal and vertical relations.

Cruijssen et al. (2007) investigated the opportunities and obstacles carriers face in horizontal collaborations. They organized a spectrum of collaboration types from basically no collaboration (i.e. "arms length") to a full integration, which is similar to a merger of companies. In between these two extremes, three different levels (denoted as Type I, II and III) are distinguished. **Type I** consists of partners who know and trust each other. They coordinate their activities and planning on a limited basis. The collaboration partnership may be short-term and a single division of each company may focus on one single activity. **Type II** collaboration maintains a longer collaborative relationship. The scope of collaboration for the participants is not only to coordinate, but also to integrate part of their business planning. The horizon is of a long though finite length and multiple divisions or functions of the companies are involved. **Type III** collaboration refers to those organizations which have a significant level of integration, and each company treat others as an extension of its own business unit. There is no end date for this kind of collaboration.

Other collaboration (Communication, Coordination, and Consolidation) concepts also arise in other Transport and Logistics networks fields. Again aiming to reduce vehicle movements and/or increase utilization, crowd logistics is a sharing economy concept. Unorganized individuals (the *crowd*) offer their services (e.g. movement or capacity) to the platform. In this setup, transportation is outsourced to the crowd or *crowdsourced*. Efficient use of different transportation modes, enabled by the use of standardized containers, presents a challenge. Synchro-modality as structured, efficient and synchronic combination of two or more transportation modes also brings interesting collaboration issues, as it also involves multiple stakeholders (i.e. modalities). In these concepts, issues around pricing, revenue and cost sharing are abundantly around.

These logistics processes can also be transformed to the *Physical Internet (PI)* paradigm. This PI acts as an autonomously managed network with nodes (locations where freight is collected, transferred or delivered) and flows (transport movements). For each request, a specific path from the origin to the destination through the network is determined, using standardized transport unit (e.g. containers). A number of prerequisites for successful Physical Internet implementations are real-time monitoring within dispatching systems, integrated in an information-sharing platform, high-level advanced predictions of the future supply of transport movements and advanced collaborative decision support systems, including pain-and-gain sharing mechanisms.

3 Cost Sharing: Preliminaries

Consider a situation wherein a set of players (partners) collaborate among themselves to improve upon their joint costs. The cost sharing problem in a situation entails finding ways to allocate the joint costs among the players. A solution to a cost sharing problem indicates appropriate ways to do the latter.

We distinguish between two alternative approaches to solve cost sharing problems. The first approach (α) defines a cooperative game associated with the situation and uses cooperative game theory to come up with allocations and/or cost-shares. A cooperative game among a set of players is defined by the joint costs of collaboration among the grand coalition as well as all sub-coalitions. The second approach (β) deals directly with the situation at hand and obtains cost-shares using the information contained in the situation. In this approach, the solution often relies on the underlying optimization problems.

Situations can be either more succinct or more expressive than their associated games. Cooperative games explicitly describe the costs of every sub-coalition, while these costs do not appear explicitly in the underlying situation. In this regard, a situation may present relevant justifications for a certain solution that cannot be devised just by focusing on costs of sub-coalitions. However, as the game theoretical solutions abstract away the details of underlying situations, they provide a generic framework to tackle cost sharing problems. In the remainder of this section, we introduce some 5 notions from cooperative game theory. Figure 20.1 illustrates the two approaches possible to cost sharing problems.

3.1 Cooperative Cost Games

A **cooperative game** is a pair (N, c) consisting of a player set $N = \{1, \ldots, n\}$ and a **characteristic cost function** c which assigns to every group of players $S \subseteq N$, hereafter a **coalition**, the cost $c(S) \in \mathbb{R}$. For the empty set we fix $c(\emptyset) = 0$.

The cooperative game (N, c) is **subadditive** if for every two disjoint coalitions S and T, i.e., $S, T \subseteq N$ with $S \cap T = \emptyset$, we have

$$c(S \cup T) \leq c(S) + c(T).$$

If a game is subadditive, then the cost of a combination of disjoint coalitions are always at most as much as the sum of their stand-alone costs so cooperation among players could be beneficial. We focus our attention in this chapter on subadditive games.

The cooperative game (N, c) is **concave** if for every S and T with $S \subset T \subset N$ and every $i \in N \setminus T$ we have

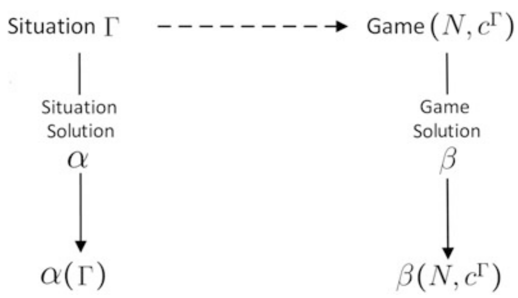

Fig. 20.1 A situation Γ, its associated game (N, c^{Γ}), and two approaches to cost sharing

$$c(S \cup \{i\}) - c(S) \geq c(T \cup \{i\}) - c(T).$$

Concavity of a game implies that the marginal cost of adding a new player to a larger coalition is non-increasing.

Example 1 Consider a cooperative game among three players, $N = \{1, 2, 3\}$. The costs for various coalitions are as follows. For $S \subseteq N$ we have: $c(S) = 10$ if $|S| = 1$, $c(S) = 19$ if $|S| = 2$, and $c(S) = 24$ if $|S| = 3$. Compared to the sum of their stand-alone costs, two-player coalitions save one and the grand coalition saves a total of 6. The game is sub-additive. It is also concave—for instance, the marginal cost of adding player 1 to player 2 is 9 units and the marginal cost of adding player 1 to the coalition of players 2 and 3 is 5 units.

The example above motivates an alternative approach in defining cooperative games. For every cooperative cost game (N, c) there exists a dual **cost-savings game** (N, v) where for every $S \subseteq N$: $v(S) = \sum_{i \in S} c(\{i\}) - c(S)$. The characteristic function in a savings game gives the amount of savings that can be made in coalitions compared to the stand-alone costs of the players involved.

Let $a_i \in \mathbb{R}$ be the cost-share of player $i \in N$. An **allocation** $a = (a_i)_{i \in N}$ is a vector of cost-shares for all players. A basic set of properties can be defined to reflect appropriate conditions that allocations should satisfy. Let (N, c) be an arbitrary but fixed game for the rest of this section.

An allocation a satisfies the **Efficiency property** if $\sum_{i \in N} a_i = c(N)$. With an efficient allocation, the entire cost of the grand coalition is shared among the players so that no excess or shortage occurs.

An allocation a satisfies the **Individual Rationality property** if for every $i \in N$ we have $a_i \leq c(\{i\})$. If an allocation fails to satisfy the individual rationality property, then some players would be better off not collaborating.

Two players $i, j \in N$ are substitutable if $c(S \cup \{i\}) = c(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. An allocation a satisfies the **Symmetry property** if for every pair of substitutable players $i, j \in N$ it holds that $a_i = a_j$. This property reflects a basic fairness feature, that is, for two players that are identical in contributions to costs, their cost-shares must be equal as well.

Example 2 In Example 1, both allocations $a = (8, 8, 8)$ and $(6, 9, 10)$ satisfy efficiency, and individually rationality. Only the former allocation satisfies the symmetry property.

3.2 Solutions for Cooperative Cost Games

Let $\mathscr{G}$ be the set of all cooperative cost games. Let $\mathscr{G}' \subseteq \mathscr{G}$ be a subset of all cooperative cost games. A (game) **solution** on $\mathscr{G}'$ is a set-valued function β that determines a set of allocations for every cooperative cost game in $\mathscr{G}'$. A solution

β on $\mathscr{G}'$ is called **single-valued** if $|\beta(N,c)| = 1$ for every $(N,c) \in \mathscr{G}'$. For any single-valued solution β on $\mathscr{G}'$ we refer to the function that assigns to any game $(N,c) \in \mathscr{G}'$ the unique element in $\beta(N,c)$ as an **allocation rule**.

We introduce some of the well-known solutions for cooperative games.

3.2.1 Core

The individual rationality property can be extended over all coalitions of players by requiring that the sum of cost-shares of players in every coalition be at most as much as the characteristic cost of that coalition. An allocation a is **stable** for the game $(N,c) \in \mathscr{G}$ if for every $S \subseteq N$ we have $\sum_{i \in S} a_i \leq c(S)$. The **core** of game $(N,c) \in \mathscr{G}$ is the set of all efficient and stable allocations. That is,

$$\mathscr{C}(N,c) = \left\{ a \in \mathbb{R}^n \,\middle|\, \sum_{i \in N} a_i = c(N) \text{ and } \sum_{i \in S} a_i \leq c(S), \forall S \subseteq N \right\}.$$

Given a game (N,c), consider the following linear program:

$$\begin{aligned} \max \quad & \sum_{i \in N} a_i \\ \text{s.t.} \quad & \sum_{i \in S} a_i \leq c(S) & \forall S \subseteq N \end{aligned}$$

The core of (N,c) is non-empty if and only if at optimality the objective function of the above program is $c(N)$, that is, an optimal solution to the above program a^* satisfies $\sum_{i \in N} a_i^* = c(N)$. If the latter holds, then every optimal solution to the program above is an allocation in the core and vice versa. Consider the dual to the program above:

$$\begin{aligned} \min \quad & \sum_{S \subseteq N} \delta_S c(S) \\ \text{s.t.} \quad & \sum_{S \subseteq N, S \ni i} \delta_S = 1 & \forall i \in N \end{aligned}$$

By the strong duality theorem, the core of the game (N,c) is non-empty if and only if the optimal value of the objective function in the dual formulation is also $c(N)$. Bondareva (1963) and Shapley (1967) provide a related condition for non-emptiness of the core of a game. A map $\kappa : 2^N \setminus \{\emptyset\} \rightarrow [0,1]$ is a **balanced map** if for all $i \in N$ we have

$$\sum_{S \subseteq N, S \ni i} \kappa(S) = 1.$$

The game (N, c) is a **balanced game** if for every balanced map κ it holds that

$$\sum_{S \in 2^N \setminus \{\emptyset\}} \kappa(S) c(S) \geq c(N).$$

Bondareva (1963) and Shapley (1967) show independently that the core of a game is non-empty if and only if it is a balanced game.

Example 3 Let $N = \{1, 2, 3\}$ and consider the game (N, c). An example of a balanced map in this case is $\kappa(S) = 0.5$ if $S \subset N$ and $|S| = 2$ and $\kappa(S) = 0$ for all other $S \subseteq N$. A necessary, but not sufficient, condition for the game to have a non-empty core is to have $0.5c(\{1, 2\}) + 0.5c(\{1, 3\}) + 0.5c(\{2, 3\}) \geq c(N)$, i.e., $c(\{1, 2\}) + c(\{1, 3\}) + c(\{2, 3\}) \geq 2c(N)$. Hence whenever the latter condition is violated the core of (N, c) would be empty.

The following example shows that the core of a game can be empty.

Example 4 Consider the game (N, c) with $N = \{1, 2, 3\}$. The costs for various coalitions of players are as follows. For $S \subseteq N$ we have: $c(S) = 11$ if $|S| = 1$, $c(S) = 17$ if $|S| = 2$, $c(S) = 28$ if $|S| = 3$. Note that $c(\{1, 2\}) + c(\{1, 3\}) + c(\{2, 3\}) = 17 + 17 + 17 = 51 < 56 = 2c(N)$. By the condition established in Example 3 we conclude that $\mathscr{C}(N, c) = \emptyset$.

3.2.2 Shapley Value

The Shapley value is a single-valued solution, i.e. for every game it results in a set with a single element (a singleton). To describe the allocation rule leading to this element, to which we refer as the Shapley value as well, let $\sigma : N \to N$ be a bijection of players in N. σ can represent the order in which players join in. Denote the set of all such permutations with $\Pi(N)$. For a given permutation σ, let $\sigma(i)$ be the position of player i in the order and $P_i^\sigma = \{j \in N | \sigma(j) \leq \sigma(i)\}$ be the set of players that come before i, including i itself, in σ. We define the marginal contribution of a player in an order as the cost that the player adds to the coalition of players joining before him. Given the game $(N, c) \in \mathscr{G}$, the marginal contribution of player i in σ is

$$m_i^\sigma(N, c) = c(P_i^\sigma) - c(P_i^\sigma \setminus \{i\})$$

Let $m^\sigma(N, c) = \left(m_i^\sigma(N, c)\right)_{i \in N}$ be the vector of marginal contributions of all players in σ. The Shapley value of a game (N, c) is defined as

$$\Phi(N,c) = \frac{1}{n!} \sum_{\sigma \in \Pi(N)} m^{\sigma}(N,c).$$

The Shapley value divides the total cost of the grand coalition according to the average marginal contributions of players in all different orders that they can join the cooperative game. Note that there are exactly $n!$ of such orders. An alternative formulation of the Shapley value is

$$\Phi(N,c) = \left(\sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!} [c(S) - c(S \setminus \{i\})] \right)_{i \in N}.$$

Example 5 In Examples 1 and 3 the corresponding Shapley values are (8, 8, 8) and (9, 9, 9) respectively.

Shapley (1971) shows that if (N, c) is a concave game then we have $\Phi(N,c) \in \mathscr{C}(N,c)$, i.e., the Shapley value is in the core. For ease of comparison we refer to the set containing $\Phi(N,c)$ as $SH(N,c)$, that is, $SH(N,c) = \{\Phi(N,c)\}$ for every $(N,c) \in \mathscr{G}$.

3.2.3 Least-Core

The intuitive appeal of the stability concept and the possibility of having empty cores motivates alternative solutions that address the stability-related issues. An allocation a for the game $(N,c) \in \mathscr{G}$ is ϵ-stable if $\sum_{i \in S} a_i - \epsilon \leq c(S)$ for all $S \subseteq N$. The set of all ϵ-stable allocations of the game associated with a situation comprises the *ϵ-core* (Shapley and Shubik 1966).

The **least-core** of a game (Maschler et al. 1979) is the intersection of all non-empty ϵ-cores of it. Accordingly, the least-core of a game $(N,c) \in \mathscr{G}$ is defined as:

$$\mathscr{LC}(N,c) = \left\{ a \in \mathbb{R}^N \;\middle|\; \sum_{i \in N} a_i = c(N) \text{ and } \sum_{i \in S} a_i - \epsilon^{\min} \leq c(S), \forall S \subset N \right\}.$$

where

$$\epsilon^{\min}(N,c) = \min \left\{ \epsilon \in \mathbb{R} \;\middle|\; \sum_{i \in N} a_i = c(N) \text{ and } \sum_{i \in S} a_i - \epsilon \leq c(S), \forall S \subset N \right\}.$$

Considering the definition of ϵ-core, it can be observed that when the core is not empty, then the least-core is a subset of the core. Also, for every game one can always find values of ϵ such that the corresponding ϵ-core is non-empty.

3.2.4 Nucleolus

The nucleolus (Schmeidler 1969) is another well-studied solution for cooperative games. Let $(N, c) \in \mathscr{G}$ be a given cooperative cost game. Define the *imputation set* of (N, c) as

$$I(N, c) = \left\{ a \in \mathbb{R}^N \middle| \sum_{i \in N} a_i = c(N) \text{ and } a_i \leq c(\{i\}), \forall i \in N \right\}.$$

Observe that if (N, c) is a subadditive game, then the imputation set of the game is non-empty. Consider an allocation $a \in \mathbb{R}^N$. Define the *coalitional unhappiness* of every coalition $S \subseteq N$ as

$$\bar{\theta}_S(a) = \sum_{i \in S} a_i - c(S).$$

Let $\theta(a)$ contain the elements $(\bar{\theta}_S(a))_{S \subset N}$ in a non-increasing order. For two vectors $\theta, \theta' \in \mathbb{R}^m$, the *lexicographical order* $\theta \leq_L \theta'$ implies that either $\theta = \theta'$, or there is $1 \leq t \leq m$ such that $\theta_i = \theta'_i$ for $1 \leq j < t$ and $\theta_t < \theta'_t$. The **nucleolus** of the game (N, c), i.e. $\eta(N, c)$, is the set of imputations whose associated vectors of unhappiness are lexicographically minimal:

$$\eta(N, c) = \left\{ a \in I(N, c) \,\middle|\, \theta(a) \leq_L \theta(a'), \forall a' \in I(N, c) \right\}.$$

The nucleolus of a game has the least maximum unhappiness over all coalitions in a lexicographical manner. For every subadditive cooperative game, the nucleolus is always non-empty, unique, and is contained in the least-core (Schmeidler 1969). We remark that the allocation rule leading to the unique element of the nucleolus is often times referred to as the nucleolus as well.

3.2.5 Comparing Solutions

We present some desirable properties for solutions and compare the aforementioned ones across these properties.

A solution β on $\mathscr{G}'$ satisfies the **non-emptiness property** if for every $(N, c) \in \mathscr{G}'$ it holds that $\beta(N, c) \neq \emptyset$. The non-emptiness of a solution assures that it can suggest ways for cost sharing in all games.

As we saw in Examples 1 and 2, the core can include many allocations or no allocation at all. The **least-unstability property** is the next best thing to maintain if stability is not achievable. A solution β on $\mathscr{G}'$ satisfies the **least-unstability property** if for every $(N, c) \in \mathscr{G}'$ and every $a \in \beta(N, c)$ we have $\sum_{i \in S} a_i - \epsilon^* \leq c(S)$ for every $S \subset N$ where

Table 20.1 Comparing solutions on subadditive games; NE: non-emptiness, SV: single-value, LU: least-unstability, S: stability

Allocation rule		NE	SV	LU	S
Core	$\mathscr{C}$	×	×	✓	✓
Shapley value	SH	✓	✓	×	×
Least-core	$\mathscr{LC}$	✓	×	✓	×
Nucleolus	η	✓	✓	✓	×

$$\epsilon^*(N, c) = \min \left\{ \epsilon \in \mathbb{R}_+ \middle| \sum_{i \in N} a_i = c(N) \text{ and } \sum_{i \in S} a_i - \epsilon \leq c(S), \forall S \subset N \right\}.$$

If the latter holds while $\epsilon^* = 0$, we say that the solution is **stable**.

Table 20.1 compares core, Shapley value, least-core and nucleolus on the class of subadditive games along these properties. As can be seen from this table, there is no perfect solution that can satisfy all these properties. The core is the only solution that guarantees stability. However, the core can be empty. The Shapley value is a single-valued solution but it may fail to be stable—or least-unstable when the core is empty. The least-core and nucleolus are both least-unstable (they are stable if the core is not empty). Furthermore, the nucleolus is a single-valued solution, that is, it always obtains a unique allocation.

3.3 Solutions for Situations

As mentioned earlier, a collaborative situation is a succinct description of relevant information necessary to analyze the context. Formally, we denote a collaborative situation with Γ. The set of all situations with player set N is also denoted with $\mathscr{T}$. The joint cost of collaboration among all players in N in situation $\Gamma \in \mathscr{T}$ is $c^{\Gamma}(N)$.

Let $\mathscr{T}'$ be a subset of all situations. A (situation) **solution** on $\mathscr{T}'$ is a set-valued function α that determines a set of allocations for every situation $\Gamma \in \mathscr{T}'$. In line with solutions for cooperative cost games we can consider single-valued solutions and allocation rules for situations as well.

If a situation allows for explicit calculation of joint costs for all sub-coalitions, then one can construct a cooperative cost game associated with the situation. Given such a situation Γ the associated game is denoted with (N, c^{Γ}). In this case, a situation solution α on $\mathscr{T}'$ can be defined by drawing upon a game solution β on $\mathscr{G}'$, that is, $\alpha(\Gamma) = \beta(N, c^{\Gamma})$, if for every $\Gamma \in \mathscr{T}'$ we have $(N, c^{\Gamma}) \in \mathscr{G}'$. Accordingly, one can redefine the properties defined for game solutions in the previous sub-section to situation solutions by requiring the properties to hold in the associated games. The advantage of using situation solutions over game solutions is their ability to incorporate more details from situations that allows for formalizing properties which cannot otherwise be defined over associated games. We elaborate further on this issue in next sections.

4 Cost Sharing in Logistics Network Situations

In this section, we discuss cost sharing in some of the stylized logistics network design situations. We particularly focus on possibilities for having a non-empty core in the games associated with these situations.

4.1 Minimum Cost Spanning Tree (mcst) Games

The minimum cost spanning tree (*mcst*) problem is a well-studied problem in operations research. An *mcst* problem consists of a set of nodes including a special node called "source". The costs of establishing links among all nodes are known. Subsequently, a minimum cost spanning tree is a set of links between the nodes that connects all nodes to the source and has the lowest total cost of establishing links among all possibilities to do so.

The cooperative version of an *mcst* problem represents the situation where each node, except the source, corresponds to a player and the players collaborate to establish a network of paths to reach the source at the lowest total cost. In the context of logistics, players on nodes can represent a set of suppliers who want to establish transportation channels to a customer. The issue of sharing the cost of an *mcst* among the players is critical in such contexts.

Formally, let $N = \{1, \dots, n\}$ be a set of players each corresponding to a node and denote the source node with 0. The set of all nodes is denoted with $N^+ = \{0, 1, \dots, n\}$. The set of links that can be established in the network is denoted with $L^+ = \{\{i, j\} | i, j \in N^+, i \neq j\}$. The connection cost function $w : L^+ \to \mathbb{R}_+$ gives the cost that needs to be incurred in order to establish a link between any pair of nodes in the network. For convenience we refer to $w(\{i, j\})$ as w_{ij} for every $\{i, j\} \in L^+$. A minimum cost spanning tree (*mcst*) situation can be represented with the tuple:

$$\Gamma = (N^+, w).$$

For every coalition $S \subseteq N$, let E_S be a set of links constituting a minimum cost spanning tree for players in S using the nodes in S^+ only. The **cooperative *mcst* game** associated with situation Γ is the pair (N, c^Γ) where for every $S \subseteq N$ we have $c^\Gamma(S) = \sum_{ij \in E_S} w_{ij}$.

Example 6 Figure 20.2 illustrates a network with four nodes that corresponds to an *mcst* situation Γ with three players. The connection costs along all links are presented in the figure. The *mcst* for the grand coalition is indicated with bold lines. Observe that $c^\Gamma(N) = 27$, while $c^\Gamma(\{1\}) = 6$, $c^\Gamma(\{2\}) = 17$, $c^\Gamma(\{3\}) = 18$, $c^\Gamma(\{1\}) = 6$, $c^\Gamma(\{1, 2\}) = 23$, $c^\Gamma(\{1, 3\}) = 19$, and $c^\Gamma(\{2, 3\}) = 25$. Note that $c(\{1, 3\}) - c(\{3\}) = 1$, that is, the contribution of player 1 when he joins player 3 is 1. However, we have $c(\{1, 2, 3\}) - c(\{2, 3\}) = 2$. Therefore, player 1's contribution

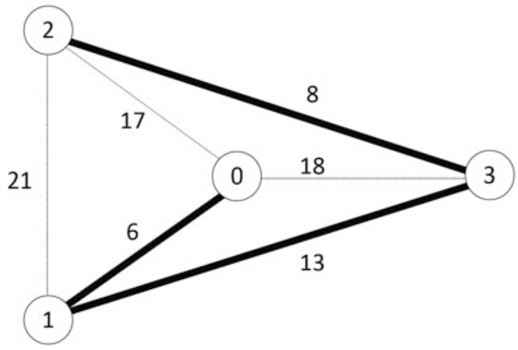

Fig. 20.2 An *mcst* example (Norde et al. 2004)

to costs increases when joining coalition of players 2 and 3. Thus, the game is not concave.

The fundamental result regarding cores of *mcst* games is as follows.

Theorem 1 *The core of an mcst game is non-empty.*

The first proof for non-emptiness of the core of an *mcst* game is given by Bird (1976). Tamir (1991) shows that the characteristic function of an *mcst* game can be represented with a mixed-integer linear program and that allocations in the core can be obtained via solutions to the dual of the integer relaxation of such program. Nevertheless, an interesting feature of *mcst* games is that one can obtain allocations in the core without solving a linear program and directly from the situation. This was shown by Bird (1976) using the Prim (1957) algorithm for solving an *mcst* problem.

The **Prim's algorithm** for finding an *mcst* over a given network starts by establishing a link between the source and the node such that the cost of this link is the lowest among all. It continues by establishing another link between a connected node and an unconnected node with the lowest connection cost. By repeating the last step the algorithm connects all nodes to the source. The solution for *mcst* situations that obtains by requiring newly connected players to pay their connection costs is called Bird's solution. The literature often referred to this solution as Bird's rule.

Bird's Solution Given situation $\Gamma = (N^+, w)$, let E_N^P be an *mcst* obtained from Prim's algorithm. For every player $i \in N$, find $j \in N^+$ such that i is directly connected to j in E_N^P on the path toward the source. Let $a_i^B(\Gamma) = w_{ij}$. Bird's solution $\alpha^B(\Gamma)$ is the set of all allocations that are obtained in this manner.

Let σ^* be an ordering of nodes as they are connected to the source using Prim's algorithm. The ordering is such that if $\sigma^*(i) < \sigma^*(j)$ for any $i, j \in N$, then i is on the path from j to the source. Then the allocation to player $i \in N$ obtained by Bird's solution with respect to σ^* is exactly his marginal contribution, that is

$$a_i^B(\Gamma) = c(P_i^{\sigma^*}) - c(P_i^{\sigma^*} \setminus \{i\}).$$

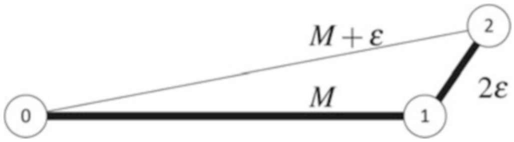

Fig. 20.3 The *mcst* situation in Example 7

Bird's solution always obtains allocations in the core, as implied by the theorem below.

Theorem 2 *For every mcst situation* Γ *we have* $\alpha^B(\Gamma) \subseteq \mathscr{C}(N, c^\Gamma)$.

It should be noted that Prim's algorithm does not necessarily produce unique *mcst*s thus the allocations obtained from Bird's solution need not be unique. We remark that convex combinations of allocations obtained via Bird's solution also generate allocations in the core (Curiel 1997).

Bird's solution provides a straightforward approach to obtain allocations in the core of these games. This solution directly builds upon the situation and thus one does not need to obtain the costs of all sub-coalitions or solve a linear program for finding core allocations. But is this solution always satisfactory? Consider the following example.

Example 7 Consider an *mcst* situation Γ with two players (see Fig. 20.3). Let M be a large number and ε a small number. Bird's solution obtains the unique allocation $a^B(\Gamma) = (M, 2\varepsilon)$ which is in the core. In this example, both players are at a long distance from the source although player 2 is slightly further away, i.e. $c^\Gamma(\{2\}) = c^\Gamma(\{1\}) + \varepsilon$. Still, Bird's solution requires player 1 to pay the entire cost of its connection while player 2 pays almost nothing. One would argue that this is not fair—especially when there are other allocations in the core. The core of the game is $\mathscr{C}(N, c^\Gamma) = \{(x, M+2\varepsilon-x) | \varepsilon \leq x \leq M\}$. Notice that in the allocation $(M, 2\varepsilon)$ players 1 and 2 are paying respectively the maximum and minimum amounts that they could pay in any core allocation.

The issue observed in Example 7 concerning Bird's solution is not coincidental. In fact, Bird's solution always gives extreme points in the cores of *mcst* games (Granot and Huberman 1981). Subsequently, Bird's solution always makes every group of players who are directly connected to the source collectively pay their stand-alone costs. The players that join such coalitions only pay their marginal cost of connection and thus enjoy the benefits of collaboration the most. A closer look at Bird's solution reveals some other shortcomings. We present an example.

Example 8 Consider again the situation Γ in Fig. 20.2. The highlighted *mcst* is indeed the one obtained by Prim's algorithm. We have $a^B(\Gamma) = (6, 8, 13)$ which is in the core of the game. It can be verified that in coalition $\{2, 3\}$ player 3 is allocated with 8 according to Bird's solution which is less than what that player pays in the grand coalition, i.e. 13.

As seen in Example 8, Bird's solution may result in some players being allocated with higher costs in larger coalitions. If this is the case, then such players might object to the inclusion of more players to the game despite the fact that the grand coalition can benefit from having more players (due to subaditivity). Accordingly, Bird's solution does not satisfy the population monotonicity property (Sprumont 1990).

An alternative approach for obtaining the allocations in the core of *mcst* games without recourse to the characteristic function is proposed by Norde et al. (2004) which is closely related to the Kruskal (1956) algorithm for obtaining *mcst*s. This solution is slightly less straightforward to obtain than Bird's solution. However, it has the additional advantage of producing allocations that ensure players in smaller coalitions would never be worse off by the addition of new players to the coalition (and thus satisfies the population monotonicity property).

There are several extensions of *mcst* games in the literature. We discuss two of such extensions briefly in this section.

Extension 1 Recall that in the original *mcst* game the cost of sub-coalitions are defined with regard to the *mcst*s that connect their members to the source drawing upon the nodes in their corresponding network only. That is, members of $S \subset N$ cannot use nodes involving players not in S for connecting to the source. The first extension of *mcst* games relaxes this assumption, that is, coalitions of players can construct their connection to the source using the nodes corresponding to other players. For instance, suppose the players in a coalition correspond to factories in different cities who would like to construct a network of pipelines to a supplier of water. Then the factories can indeed construct the network through the cities where other factories are located at. Given the *mcst* situation Γ, in the associated **monotone *mcst* game** (Granot and Huberman 1981), $(N, \bar{c}^{\Gamma})$, for every $S \subseteq N$ we have

$$\bar{c}^{\Gamma}(S) = \min_{S \subseteq T \subseteq N} c^{\Gamma}(T).$$

Theorem 3 *The core of a monotone mcst game is non-empty. Bird's solution gives extreme points of the cores of monotone mcst games.*

Extension 2 In the previous extension, we allowed sub-coalitions to use outsiders' nodes to construct their path to the source. Still, the grand coalition of players constituted all available nodes except the source node. Another extension of *mcst* games allows for additional nodes in the network, i.e. nodes that correspond to no players in N. Let

$$\Gamma = (V, N, w)$$

be a situation with a set of nodes V that includes the source node, the set of players $N \subset V$, and the connection cost function w defined over pairs of nodes in V. In the **Steiner Tree game** (Megiddo 1978; Sharkey 1995) associated with Γ, $(N, \bar{\bar{c}}^{\Gamma})$, the

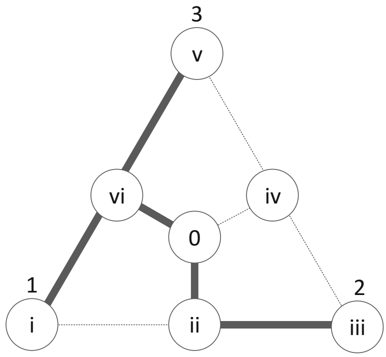

Fig. 20.4 An extended *mcst* example (Sharkey 1995)

cost of each coalition $S \subseteq N$ is the cost of *mcst* that connects players in S to the source while using any nodes in V. The following example shows that the core of these games can be empty.

Example 9 An extended *mcst* situation is depicted in Fig. 20.4. The node set includes six locations in addition to the source. There are three players in the grand coalition. The cost of connection on all links are 1. Observe that $\bar{\bar{c}}^{\Gamma}(S) = 2$ whenever $|S| = 1$, $\bar{\bar{c}}^{\Gamma}(S) = 3$ whenever $|S| = 2$, and $\bar{\bar{c}}^{\Gamma}(N) = 5$. Note that $\bar{\bar{c}}^{\Gamma}(\{1, 2\}) + \bar{\bar{c}}^{\Gamma}(\{1, 3\}) + \bar{\bar{c}}^{\Gamma}(\{2, 3\}) = 9 < 2\bar{\bar{c}}^{\Gamma}(N) = 10$. By the condition established in Example 3 we conclude that the core of the game is empty.

4.2 Facility Location Games

In facility location games, players collaborate to jointly open facilities as well as to establish connections to their locations. The basic facility location situation can be formulated as follows. Let V be a set of nodes. The player set is a subset of the nodes, that is $N \subseteq V$. A flow function $f : N \to \mathbb{R}_+$ gives the requirement of demand for each player. Let $E \subseteq \{\{i, j\}|i, j \in V\}$ be the link set representing feasible connections between the nodes. A connection cost function $w : E \to \mathbb{R}_+$ gives the cost of providing one unit of service across each link. We let $w_{ii} = 0$ for all $i \in N$. The function $t : V \to \mathbb{R}_+$ gives the total investment needed to establish facilities at different nodes (fixed costs). A facility location situation is thus

$$\Gamma = (V, N, f, E, w, t).$$

Given the facility location situation Γ, the associated cooperative cost games is the pair (N, c^{Γ}) where for every $S \subseteq N$ we have:

$$c^{\Gamma}(S) = \min \sum_{i \in S, k \in V : \{i,k\} \in E} f_i w_{ik} x_{ik} + \sum_{k \in V} t_k y_k \tag{20.1}$$

$$\text{s.t.} \quad \sum_{k \in V : \{i,k\} \in E} x_{ik} = 1 \qquad \forall i \in S \tag{20.2}$$

$$y_k - x_{ik} \geq 0 \qquad \forall i \in S, \forall k \in V : \{i,k\} \in E \tag{20.3}$$

$$x_{ik}, y_k \in \{0, 1\} \qquad \forall i \in S, \forall k \in V : \{i,k\} \in E \tag{20.4}$$

The program above minimizes the total cost of flow as well as opening facilities. The optimal solution satisfies the following constraints. First, all players in a coalition must be connected to a facility. Second, a facility should be established if there is a link to a player. Finally, integrality constraints ensure the feasibility of solution. The dual program associated with the relaxation of program (20.1)–(20.4) for N is

$$\bar{c}^{\Gamma}(N) = \max \sum_{i \in N} a_i \tag{20.5}$$

$$\text{s.t.} \quad a_i - \mu_{ik} \leq f_i w_{ik} \qquad \forall k \in V, \forall i \in N : \{i,k\} \in E \tag{20.6}$$

$$\sum_{i \in N : \{i,k\} \in E} \mu_{ik} \leq t_k \qquad \forall k \in V \tag{20.7}$$

$$\mu_{ik} \geq 0 \qquad \forall i \in N, \forall k \in V : \{i,k\} \in E \tag{20.8}$$

The solutions of the primal and the dual programs can be used to provide insights regarding non-emptiness of the core. Kolen (1983), Chardaire (1998), and Goemans and Skutella (2004) show that the dual program above is exactly the same as the program for obtaining the core of the game. Therefore, non-emptiness of the core can be guaranteed when the optimal objective function of the dual equals that of the original (un-relaxed) program. In other words, the core is non-empty if the duality gap is zero.

Theorem 4 *Let $\Gamma = (V, N, f, E, w, t)$ be a facility location situation. The core of the associated game (N, c^{Γ}) is non-empty if and only if $c(N) = \bar{c}(N)$, that is, there is no integrality gap between the primal and dual (of the relaxation) programs for the grand coalition.*

As implied by the result above, an integrality gap renders the core of a facility location game empty. The example below shows that facility location games can have empty cores.

Example 10 The network of a facility location situation is depicted in Fig. 20.5. All nodes are situated on a circle and require a unit flow. The distance between every

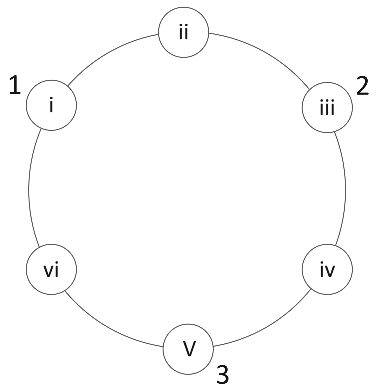

Fig. 20.5 A facility location situation on a circle (Goemans and Skutella 2004)

pair of adjacent nodes, which constitute the link set, is one and the cost of flow equals the distance. There are three players, $N = \{1, 2, 3\}$, located at nodes i, iii, and v respectively. The cost of opening a facility on nodes ii, iv, and vi is two and for the other nodes the cost is a large number. In the associated cooperative game individual players each need one facility adjacent to them thus $c^\Gamma(S) = 2 + 1 = 3$ for $|S| = 1$. In two-player coalitions one facility can serve both players so $c^\Gamma(S) = 2+1+1 = 4$ for $|S| = 2$. Finally, in the grand coalition the best option is to open two facilities thus $c^\Gamma(N) = 2+2+1+1+1 = 7$. Note that $c^\Gamma(\{1, 2\})+c^\Gamma(\{1, 3\})+c^\Gamma(\{2, 3\}) = 12 < 14 = 2c^\Gamma(N)$. By the condition established in Example 3 we conclude that the core of the game is empty.

There are several special situations where the zero duality gap between the primal and dual programs, and subsequently non-emptiness of the core, can be proven to always hold. For instance, suppose that the underlying graph of the situation (V, E) is a tree—i.e. there is exactly one path between any two nodes—and that the costs of connection between any pair of nodes correspond to the metric distance between those nodes on the corresponding planar graph. In this case, the original program can be re-written in the following way. For each player $i \in N$, let $0 = r_{i1} \leq r_{i2} \leq \ldots \leq r_{i|V|}$ be the ordered sequence of distances between player i's node and all other nodes. Also let $r_{iV+1} = M$ where M is a sufficiently large number. Define the variables z_{ij} such that $z_{ij} = 1$ if player i is not connected to an open facility which is situated at the distance less than or equal to r_{ij}, and $z_{ij} = 0$ otherwise. Also, define u^j_{ik} such that $u^j_{ik} = 1$ if $c_{ik} \leq r_{ij}$ and $u^k_{ij} = 0$ otherwise. Then we have (see Kolen 1983):

$$c^\Gamma(S) = \min \sum_{i\in S, j\in V} f_i(r_{ij+1} - r_{ij})z_{ij} + \sum_{k\in V} t_k y_k \tag{20.9}$$

$$\text{s.t.} \quad \sum_{k\in V:\{i,k\}\in E} u^j_{ik} y_k + z_{ij} \geq 1 \qquad \forall i \in S, \forall j \in V \tag{20.10}$$

$$z_{ij}, y_k \in \{0, 1\} \qquad \forall i \in S, \forall k \in V \tag{20.11}$$

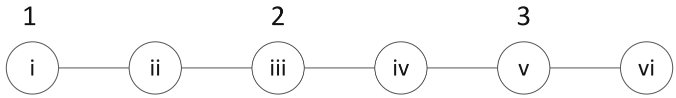

Fig. 20.6 A facility location situation on a line

Constraint (20.10) ensures that whenever there is no open facility within the range r_{ij} from i, then $z_{ij} = 1$. Kolen (1983) show that the constraint coefficient matrix in the program above has a special feature which guarantees a zero duality gap. Thus, in this class of situations the core is always non-empty.

Example 11 A facility location situation Γ' is depicted in Fig. 20.6. The only difference between this situation and the one in Example 10 is that the nodes are now situated on a line. In the cooperative game associated with this situation individual players each need one facility adjacent to them thus $c^{\Gamma'}(S) = 2 + 1 = 3$ for $|S| = 1$. In two-player coalitions $\{1, 2\}$ and $\{2, 3\}$ one facility can serve both players so $c^{\Gamma'}(S) = 2 + 1 + 1 = 4$ for $S = \{\{1, 2\}, \{1, 3\}\}$. However, for coalition $\{1, 3\}$ we have $c^{\Gamma'}(\{1, 3\}) = 2 + 2 + 1 + 1 = 6$. Finally, in the grand coalition the best option is again to open two facilities thus $c^{\Gamma'}(N) = 2 + 2 + 1 + 1 + 1 = 7$. Notice that allocation $a = (2, 2, 3)$ is in the core.

4.3 Hub Location Games

Another class of collaborative situations related to logistical problems pertains to finding the locations of logistical hubs, i.e., points of consolidation in a network, which allows for more efficient dispatching of vehicles. The basic hub location situation encompass hub-spoke structures where the transport costs in between hubs are cheaper due to the use of more efficient means of movement. Accordingly, in these collaborative situations players jointly establish hubs and connections to reduce the cost of their aggregated network.

Let V be a set of nodes in a network and let the player set N be situated amongst the nodes, i.e., $N \subseteq V$. Each player is positioned on a node and has transportation requirements from his node to other nodes. Let the requirement function $f : N \times V \to \mathbb{R}_+$ represent the latter. Define the link cost function $w : V \times V \to \mathbb{R}_+$ and hub cost function $t : V \to \mathbb{R}_+$. The cost of direct movements between nodes are sufficiently high so that it is beneficial that flows of goods between two nodes always pass through hubs. The link costs satisfy the triangular inequalities which ensure that transports between any two nodes does not need more than two hubs involved. Finally, let the coefficient $\lambda \in [0, 1]$ be the discount factor for movements between hubs. This means, if there are two hubs established at nodes i and j, then the unit cost of transportation from i to j drops from w_{ij} to λw_{ij}. A hub location situation is a tuple

$$\Gamma = (V, N, f, w, t, \lambda).$$

The players collectively decide to open hubs at some nodes in order to satisfy the flow requirements with the minimum cost. Skorin-Kapov (1998) gives a formulation of the optimal cost for the grand coalition as

$$c(N) = \min \sum_{i \in N; j,k,m \in V} f_{ij}(w_{ik} + \lambda w_{km} + w_{mj}) x_{ijkm} + \sum_{k \in V} t_k y_{kk} \quad (20.12)$$

$$\text{s.t.} \sum_{k \in V} y_{ik} = 1 \qquad \forall i \in N \quad (20.13)$$

$$y_{kk} - y_{ik} \geq 0 \qquad \forall i, k \in V \quad (20.14)$$

$$\sum_{m \in V} x_{ijkm} = y_{ik} \qquad \forall i \in N, \forall j, k \in V \quad (20.15)$$

$$\sum_{k \in V} x_{ijkm} = y_{jm} \qquad \forall i \in N, \forall j, m \in V \quad (20.16)$$

$$x_{ijkm} \geq 0 \qquad \forall i \in N, \forall k, j, m \in V \quad (20.17)$$

$$y_{ik} \in \{0, 1\} \qquad \forall i, k \in V \quad (20.18)$$

The program above minimizes the total cost of movements between nodes and through hubs, plus the cost of establishing the hubs. Each player must be connected to a hub. The variable x_{ijkm} represents the fraction of flow from i's node to j that passes through hubs k and m. The zero-one variable $y = (y_{ij})_{i,j \in V}$ also indicates the connections between nodes and hubs with y_{ii} indicating the establishment of a hub at node i. The constraint (20.15) (respectively constraint (20.16)) indicates that the entire flow from a player i's node to destination j will be routed via link ik (link mj) if and only if i is allocated to hub k (j is allocated to hub m) independently of the destination (source). Let (x^*, y^*) be an optimal solution to the above problem.

There are different possibilities for defining cooperative games associated with hub locations situations based on how the cost of sub-coalitions are defined. In the basic hub location game (Skorin-Kapov 1998) the cost of sub-coalitions are calculated on the network which is optimal for the grand coalition. Let $y_k^{*S} = 1$ whenever there is $i \in S$ such that $x^*_{ijkm} > 0$ or $x^*_{ijmk} > 0$, that is, $y_k^{*S} = 1$ if a member of S uses the hub k. Then, the basic hub-location game associated with situation Γ is (N, c^Γ) where $c(N)$ is defined above and for $S \subset N$ we have

$$c(S) = \sum_{\substack{i \in S \\ j,k,m \in V}} f_{ij}(w_{ik} + \lambda w_{km} + w_{mj}) x^*_{ijkm} + \sum_{k \in V} t_k y_{kk}^{*S} \quad (20.19)$$

Therefore, the cost of a sub-coalition S is the total transportation cost of movements on the optimal network plus the cost of establishing the hubs that S uses on the optimal network.

Theorem 5 *Let $\Gamma = (V, N, f, w, t, \lambda)$ be a hub location situation. The core of the basic hub location game associated with Γ is non-empty.*

4.4 Delivery Consolidation Games

With the increasing attention to reducing the negative side effects of transportation such as congestion and pollution, Urban Consolidation Centers (UCC) became an important new logistical initiative. Through a UCC, logistics providers can combine their LTL cargo and collaboratively dispatch FTL trucks to urban areas. However, the cost of joint dispatches must be shared among the users. In this section we overview a cost sharing game associated with UCCs introduced in Hezarkhani et al. (2019). The carriers (players) have deliveries that are destined for the same area. Instead of individually driving to their destinations, the players can arrive at the consolidation center and bundle their cargo into full-truck loads. The deliveries are time-sensitive and the amounts of savings that the carriers obtain are dependent on their dispatch times.

The network V consists of only two nodes: a consolidation center and a common destination and the players in N can drive the distance between the two nodes either individually or jointly. We call r_i the arrival time of delivery i to UCC and assume that deliveries have non-identical arrival times and that N is arranged by increasing order of arrival times, i.e., $r_1 < r_2 < \ldots < r_n$. Let $p_i \geq 0$ be the waiting penalty rate for player i, that is the cost that he incurs when his cargo sits in the consolidation center for a unit of time. Thus, the cost to player i if dispatched from the consolidation center at time $d_i \geq r_i$ is $p_i(d_i - r_i)$. The cost of dispatching a truck from the consolidation center to the common destination is $W \geq 0$. We assume players have small yet time sensitive cargo and the capacity of a truck is not a restriction. Accordingly, a Dispatch Consolidation (DC) situation can be defined by the tuple $\Gamma = (V, N, \boldsymbol{r}, \boldsymbol{p}, W)$.

The consolidation center decides a collection of dispatches, representing consolidated subsets of players, and their associated dispatch times. The objective of the UCC is to minimize the sum of waiting and dispatching costs for all players. One can verify that the optimal time for the dispatch of a fixed group of players in $T \subseteq N$, is the arrival time of the last player in T. Denote the first and last arriving delivery in T with $b(T)$ and $e(T)$, respectively. Since the players are ordered by their arrival times, $b(T)$ and $e(T)$ also represent respectively the smallest and largest elements in T. The cost function f for a group of players $T \subseteq N$ is

$$f_T = \sum_{i \in T} \left[p_i \left(r_{e(T)} - r_i \right) \right] + W.$$

We can construct the optimization problem as a set packing formulation and define the associated dispatch consolidation (DC) game by letting $c(S)$ to be

$$c^{\Gamma}(S) = \min \sum_{T \subseteq S} x_T f_T$$

$$s.t. \quad \sum_{T \subseteq S : S \ni i} x_T \geq 1 \qquad \forall i \in S$$

$$x_T \in \{0, 1\} \qquad \forall T \subseteq S$$

DC games are special instances of the class of set packing games (Deng et al. 1999). The general characterization of the conditions for non-emptiness of the cores of set packing games gives us the following result; The core of a DC game is non-empty if and only if the integer relaxation of the program above for N does not affect optimality.

Using the results of Barany et al. (1986) regarding zero duality gap of set packing problems on trees via their sub-trees, Hezarkhani et al. (2019) show that integer relaxation of the program above in DC games does not affect optimality. Therefore, the core of any DC games is non-empty.

The extension of DC games to incorporate restrictive capacities of the trucks is also considered by Hezarkhani et al. (2019). With restrictive capacities, DC games might have an empty core. In this case, Hezarkhani et al. (2019) introduce the notion of component-wise core as an alternative notion of stability and prove that DC games with restrictive capacities have non-empty component-wise cores.

5 Cost Sharing in Cooperative Truck-Load Delivery Situations

The logistical situations studied in the previous section were all concerned with establishing the physical network which is comprised of links, facilities, and hubs. The corresponding decisions are at the strategic level and as such necessitate a long term cooperation time line. However, there are other opportunities for cooperative logistics which deal with day-to-day activities of participating players and target operational decisions. In these *service logistics* situations, the nature of cost sharing problems can be different. In this section, we discuss the Cooperative Truck-Load Delivery (CTLD) situations, introduced by Hezarkhani et al. (2016), that arise in service network design and explain how an appropriate allocation rule for these situations can be devised.

CTLD situations are comprised of a number of logistics providers and their individual resources—e.g. depots, trucks, drivers, equipment, etc. Players have delivery requirements. A delivery requirement is simplified as an order for picking up cargo at some location and transporting it to another location. In practice, delivery requirements can be more complex and involve time windows, special equipment and personnel, and other constraints. The delivery requirements must be fulfilled by vehicles in feasible trips. In this context, the players seek to collaboratively design their service network at the tactical as well as operational levels.

Formally, let V be a set of nodes corresponding to spatial locations, and $w : V \times V \rightarrow \mathbb{R}^+$ be a distance function which satisfies the triangular inequalities. We assume hereafter that cost and distance are equivalent. A set of delivery requirements $\{d^1, \ldots, d^m\}$ is given. A *delivery requirement* d^k corresponds to an arc (i^k, j^k), consisting of the corresponding pickup location $i^k \in V$ and delivery location $j^k \in V, i^k \neq j^k$. The *fulfillment* of the delivery requirement d^k corresponds to a single traverse of the arc (i^k, j^k) for requirement k. A non-empty set of depots $\{o^1, \ldots, o^h\} \subseteq V$ is available. The depots station vehicles that fulfill the delivery requirements. Delivery requirements must be fulfilled in trips. A trip is a sequence of deliveries that start and ends at a particular depot. Thus a trip l can be defined as a tuple (o^l, D^l, σ^l) where o^l is the origin/destination, D^l is a subset of deliveries that are fulfilled in l, and σ^l is an ordering of deliveries in D^l which represents the sequence of fulfillments in trip l. Let $\mathscr{L}$ be the set of all such trips. Let $L \subseteq \mathscr{L}$ be the feasible trip set. The feasibility of a trip can depend on the number and type of deliveries it fulfills, specific depots and equipment that must be employed, and other details.

The cost of a feasible trip l, w^l, is the sum of costs of the arcs traversed in trip l. The *full kilometers cost* of a trip is independent of both the choice of the trip's depot and the sequence of fulfillments:

$$w_F^l = \sum_{k:d^k \in D^l} w\left(i^k, j^k\right).$$

The second part of a trip's cost, i.e. *empty kilometers cost*, is the cost associated with the distance travelled from/to the depot and among different fulfillments:

$$w_E^l = w\left(o^l, i^{\sigma_1^l}\right) + \sum_{k=1}^{|D^l|-1} w\left(j^{\sigma_k^l}, i^{\sigma_{k+1}^l}\right) + w\left(j^{\sigma_{|D^l|}^l}, o^l\right).$$

where the shorthand notation σ_k^l represents the index of the delivery requirement that is fulfilled after all the $k-1$ deliveries preceding it in σ^l are fulfilled. By $|D^l|$ we denote the number of deliveries in D^l. The *cost of trip l* is defined by $w^l = w_F^l + w_E^l$.

A fulfillment plan P from O to D is a collection of feasible trips in $L(O, D)$ that fulfills all deliveries in D exactly once. The deliveries fulfilled in the trips of the plan P partition the corresponding set of delivery requirements, i.e. $\bigcup_{l \in P} D^l = D$ and $D^l \cap D^k = \emptyset$ for all $k, l \in P$ with $l \neq k$. The cost of the fulfillment plan P is the total cost of its trips, i.e. $w(P) = \sum_{l \in P} w^l$. Accordingly, $w(P)$ is decomposable into full and empty movements:

$$w(P) = w_F(P) + w_E(P),$$

where $w_F(P) = \sum_{l \in P} w_F^l$ and $w_E(P) = \sum_{l \in P} w_E^l$ are the total costs of full and empty kilometers of P respectively. Let $\mathscr{P}(O, D)$ be the set of feasible plans from O to D. The cost of optimal plan from O to D is

$$w^*(O, D) = \min_{P \in \mathscr{P}(O,D)} w(P).$$

Consider a non-empty set $N = \{1, \ldots, n\}$ of players. Each player $i \in N$ possesses a set of delivery requirements $D_i = \{d_i^1, \ldots, d_i^{m_i}\}$ and a non-empty set of depots $O_i = \{o_i^1, \ldots, o_i^{h_i}\}$ such that $\cup_{i \in N} D_i = \{d^1, \ldots, d^m\}$ and $\cup_{i \in N} O_i = \{o^1, \ldots, o^h\}$. Let $O_S = \cup_{i \in S} O_i$ and $D_S = \cup_{i \in S} D_i$ denote the combined set of depots and delivery requirements of players in coalition $S \subseteq N$. The set $L(O_S, D_S)$ contains all feasible trips that coalition $S \subseteq N$ can use to fulfill its combined delivery requirements. Combining all this, a CTLD situation is a tuple:

$$\Gamma = (N, V, w, (D_i)_{i \in N}, (O_i)_{i \in N}, L).$$

Let $\mathscr{T}'$ be the set of all CTLD situations. By joint planning of fulfillments, a coalition in a CTLD situation could reduce the cost of its empty kilometers. The cost saving generated by a coalition can be due to utilization of a larger pool of depots for constructing trips or combining fulfillments together more efficiently in trips, or both. It can be verified that shrinking the set of delivery requirements cannot increase the minimum cost of delivery, and augmenting the set of depots cannot increase the minimum cost of delivery. Also, there is a subadditive effect with regard to the minimum costs of fulfillment that results from aggregated planning of delivery requirements (see Hezarkhani et al. 2016).

We refer to the cost games associated with CTLD situations as the CTLD games. The characteristic function in CTLD game (N, c^Γ) associated with situation Γ assigns to coalition $S \subseteq N$ the cost

$$c^\Gamma(S) = w^*(O_S, D_S).$$

Although there are special CTLD situations where the core is always non-empty (see Özener and Ergun 2008 and Hezarkhani et al. 2014), in general, CTLD games can have empty cores, as shown in the example below.

Example 12 Consider the CTLD situation Γ depicted in Fig. 20.7. There are three players $N = \{1, 2, 3\}$ each having a depot and a delivery requirement. The distance between the pickup and delivery locations for all delivery requirements is two and the distance from the depots to any pickup/delivery point is one. The set of feasible

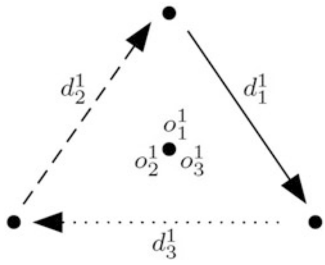

Fig. 20.7 A CTLD situation where players have different competitive positions

trips includes all trips which fulfill no more than two delivery requirements, i.e. $L = \{l \in \mathscr{L} \| D^l| \leq 2\}$ (only two deliveries can be fulfilled sequentially during a day). For $S \subseteq N$ we have $c^\Gamma(S) = 4$ if $|S| = 1$, $c^\Gamma(S) = 6$ if $|S| = 2$, and $c^\Gamma(N) = 10$. Applying the condition in Example 3, we obtain that the core of this game is empty.

5.1 Desirable Properties for CTLD Solutions

In order to find solutions for CTLD situations, i.e., solutions defined over the set of all CTLD situations $\mathscr{T}'$, we define a set of properties that could be considered as desirable in these situations.

The notion of stability is a critical concept in many cooperative situations, including CTLD situations. Given the possibility of having empty cores, we seek for the best possible outcomes in terms of instability of allocations. Thus the first desirable property for CTLD solutions is that of least-unstability. A solution α on $\mathscr{T}'$ satisfies the least-unstability property if for every $\Gamma \in \mathscr{T}'$ and every $a \in \alpha(\Gamma)$ we have $\sum_{i \in S} a_i - \epsilon^* \leq c^\Gamma(S)$ for every $S \subset N$ where ϵ^* is defined in the same way as in Sect. 3.2.5 for the associated game (N, c^Γ).

The highly competitive nature of logistics markets as well as the limited number of potential participants necessitate solutions that are capable of incorporating the notion of competitiveness among the logistics providers. The two properties discussed in the remainder of this section are specific to CTLD situations and address issues concerning the competitive positions of the players and the scope beyond which the network of deliveries of a player should be ignored by the solution. We start by introducing two special classes of delivery requirements in CTLD situations. Let $\Gamma \in \mathscr{T}'$ be a CTLD situation with player set N. $D \subseteq D_i$ is a separable delivery set (SDS) of player i if

$$w^*(O_i, D) + w^*(O_N, D_N \setminus D) = w^*(O_N, D_N). \tag{20.20}$$

Let $SDS_i(\Gamma)$ be the set of separable delivery sets of i. The stand-alone cost of fulfilling a separable delivery set of a player is additive to the cost of fulfilling

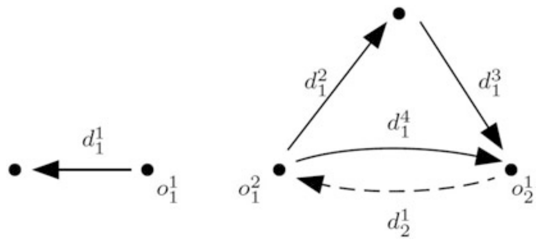

Fig. 20.8 Separable and irrelevant deliveries

the remaining deliveries in the grand coalition. Therefore, a player can individually fulfill a separable delivery set of itself without disrupting the optimality of delivery plans in the grand coalition. Let $\Gamma \in \mathcal{T}'$ be a CTLD situation with player set N. $D \subseteq D_i$ is an **irrelevant delivery set** (IDS) of i if for all $D' \subseteq D$, all $S \subseteq N$ with $i \in S$, and all $D'' \subseteq D_S \setminus D$ it holds that

$$w^*(O_i, D') + w^*(O_S, D'') = w^*(O_S, D' \cup D''). \tag{20.21}$$

Let $IDS_i(\Gamma)$ be the set of irrelevant delivery sets of i. The cost of fulfilling any subsets of irrelevant deliveries of a player is additive to any subset of the set of remaining deliveries in any coalition that includes that player, so the player can fulfill such deliveries separately in any possible combination with other deliveries. The following example elaborates on the notion of separable and irrelevant deliveries.

Example 13 Figure 20.8 depicts a CTLD situation Γ with two players $N = \{1, 2\}$. It is easy to see that player 1 can individually fulfill the delivery requirement $\{d_1^1\}$. Also, player 1 can take out either $\{d_1^2, d_1^3\}$ or $\{d_1^4\}$ (but not both sets!) from the grand coalition's delivery requirements and fulfill them separately such that the total cost of fulfillment does not increase. Thus, we have $SDS_1(\Gamma) = \big\{\{d_1^1\}, \{d_1^2, d_1^3\}, \{d_1^4\}, \{d_1^1, d_1^2, d_1^3\}, \{d_1^1, d_1^4\}\big\}$ and $IDS_1 = \big\{\{d_1^1\}\big\}$.

Given $D_i' \subseteq D_i$, let $\Gamma \setminus D_i'$ be a CTLD situation that coincides with Γ except for the delivery set of i which is replaced by $D_i \setminus D_i'$. Define the *independence of irrelevant deliveries* property as the insensitivity of a solution to the exclusion of irrelevant deliveries of the players. A solution for CTLD situations α satisfies the **independence of irrelevant deliveries property** if for every $\Gamma \in \mathcal{T}'$, every $a \in \alpha(\Gamma)$, and every $a' \in \alpha(\Gamma \setminus D)$ it holds for every $i \in N$ and every $D \in IDS_i(\Gamma)$ that $a_i = a_i' + w^*(O_i, D)$ and $a_j = a_j'$ for every $j \in N \setminus \{i\}$.

The last property addresses the competitive aspect of solutions in CTLD situations.

We define the *average cost of fulfillment* from O to $D \neq \emptyset$ as

$$z(O, D) = \frac{w^*(O, D)}{w_F(D)} \tag{20.22}$$

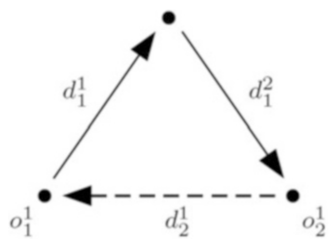

Fig. 20.9 A CTLD situation where players have different competitive positions

where $w_F(D)$ is the cost of full kilometers needed to be traversed to fulfill D. The average cost of fulfillment $z(O, D)$ represents the average distance (cost) that need to be traveled (incurred) in D in order to fulfill a unit distance of delivery requirement.

The average cost of fulfillment provides a basis for calculating unit delivery prices in logistics markets. However, it can also be utilized as a measure of comparison among the players. This idea is motivated by the observation that a lower average cost of fulfillment of a logistics player compared to that of another logistics player allows the former to charge a lower unit price for its delivery services while remaining profitable. Therefore, if for two players i and j it holds that $z(O_i, D_i) < z(O_j, D_j)$, it can be stated that prior to cooperation, i is in a better competitive position than j. The definition of average cost of fulfillment can be naturally extended to incorporate the savings allocated to the players after the cooperation. Given an allocation a and player $i \in N$, $D_i \neq \emptyset$, define the average cost of fulfillment of a player i under a as

$$z_i^a(O_i, D_i) = \frac{a_i}{w_F(D_i)} \tag{20.23}$$

We are now ready to present a competitiveness property defined over a restricted set of CTLD situations. Let $\hat{\mathscr{T}}'$ be the set of all CTLD situations $\Gamma \in \mathscr{T}'$ with player set N such that $SDS_i(\Gamma) = \{\emptyset\}$ for all $i \in N$. A CTLD solution satisfies the **restricted competitiveness property** if for every situation Γ with player set $N = \{1, 2\}$ and any $a \in \alpha(\Gamma)$ it holds that

$$z_1^a(O_1, D_1)z(O_2, D_2) = z_2^a(O_2, D_2)z(O_1, D_1). \tag{20.24}$$

Example 14 Figure 20.9 represents a CTLD situation with two players $N = \{1, 2\}$. Assuming that the distance between any two locations is 1, we get $z(O_1, D_1) = 1.5$ and $z(O_2, D_2) = 2$. The cooperation in this case results in $c^\Gamma(N) = 3$, i.e., 2 units of saving compared to individual fulfillments $c^\Gamma(\{1\}) = 3$ and $c^\Gamma(\{2\}) = 2$. Observe that the allocation $a = (1.8, 1.2)$ preserves the competitive positions of players 1 and 2 before and after the cooperation, resulting in $z_1^a(O_1, D_1) = 0.9$ and $z_2^a(O_2, D_2) = 1.2$.

5.2 A Solution for CTLD Situations

The proposed CTLD solution is constructed in two steps. In the first step, we introduce a proportional allocation, a^P, which incorporates the notions of competitiveness and scope defined in the previous section. In the second step, we use the latter proportional allocation to construct a least-unstable solution, α^P, for CTLD situations.

Let $\Gamma \in \mathscr{T}'$ be a CTLD situation with player set N. $D \subseteq D_i$ is a **minimal essential delivery set** (MEDS) of player i if

$$w^*(O_i, D_i \setminus D) + w^*(O_N, D_{N\setminus i} \cup D) = w^*(O_N, D_N). \tag{20.25}$$

and for every $D' \subset D$, $D \neq \emptyset$:

$$w^*(O_i, D_i \setminus D') + w^*(O_N, D_{N\setminus i} \cup D') > w^*(O_N, D_N) \tag{20.26}$$

and $w^*(O_i, D) \leq w^*(O_i, D')$ for any D' that satisfy the above two conditions.

Fix Γ, let $D_i^m \in MEDS_i(\Gamma)$, and define

$$a_i^p(\Gamma) = w^*(O_i, D_i) - \frac{w^*(O_i, D_i^m)}{\sum_{j\in N} w^*(O_j, D_j^m)} \left(\sum_{j\in N} w^*(O_j, D_j) - w^*(O_N, D_N) \right) \tag{20.27}$$

The allocation a^P obtains a unique efficient allocation that divides the savings obtained in the grand coalition of CTLD situation Γ among players with non-empty essential delivery sets proportional to the stand-alone cost of their minimal essential deliveries. The above formulation assumes that the essential delivery set of all players are non-empty. See Hezarkhani et al. (2016) for the treatment of the other case. The allocation a^P completely preserves the competitive positions of the players with regard to their minimal essential delivery sets. This means that for every pair of players $i, j \in N$ with non-empty essential delivery sets we have

$$\frac{z_i^{a^P(\Gamma)}(O_i, D_i^m)}{z_i(O_i, D_i^m)} = \frac{z_j^{a^P(\Gamma)}(O_j, D_j^m)}{z_j(O_j, D_j^m)}.$$

The allocation a^P, however, does not necessarily obtain a least-unstable allocation. In order to achieve this, we present our CTLD solution α^P:

$$\alpha^P(\Gamma) = \arg\min_{a \in \mathbb{R}^N} \sum_{i \in N} (a_i^P(\Gamma) - a_i)^2 \tag{20.28}$$

$$s.t. \quad \sum_{i \in S} a_i - \epsilon^* \leq w^*(O_S, D_S) \qquad \forall S \subset N \tag{20.29}$$

$$\sum_{i \in N} a_i = w^*(O_N, D_N) \tag{20.30}$$

where ϵ^* is defined in Sect. 3.2.3. Given the situation Γ, $\alpha^P(\Gamma)$ gives the set of all ϵ^*-stable allocations that have the shortest distance from the proportional allocation $a^P(\Gamma)$. The following result is proven by Hezarkhani et al. (2016).

Theorem 6 α^P *satisfies the nonemptiness, uniqueness, least-unstability, independence of irrelevant deliveries, and restricted competitiveness properties for all CTLD situations.*

6 Bibliographical Notes

We split this section in two parts: literature on collaborations and literature on the relevant game theorical background.

6.1 Collaborations

Quak and Tavasszy (2011) report that among more than 100 initiatives in urban logistics collaborations, more than half of them fail during implementation. There are several underlying reasons for this (Vanovermeire et al. 2014), e.g. collaboration among carriers is often hampered by their competitive positions and by the risks of divulging information and losing customers. Shippers, on the other hand, may hesitate to collaborate as they might not have a clear understanding of collaborative mechanisms employed and whether or not they receive a fair share out of collaborative operations. In a survey based on a large number of logistics service providers (LSPs) in Belgium, Cruijssen et al. (2007) observe that despite the obvious benefits of cooperation, designing a fair cost sharing scheme is a major impediment for collaboration among LSPs. For more information on the fill rates of vehicles refer to (Eurostat 2018).

Good examples of such cost sharing reviews already exist in the literature (see e.g. Deng and Fang 2008; Marinakis et al. 2008). Although the literature often associates the definition of the core to Gillies (1959), it was Shapley who first defined the core in its current form (Zhao 2018).

In their review paper, Gansterer and Hartl (2018) distinguish between centralized versus decentralized planning in cooperation. Having perfect information with regards to all requests (central planning) leads to profit sharing approaches, usually based on game-theoretical principles. In decentralized planning, imperfect information to no request information is assumed. Most research is circulating around horizontal collaboration and cost sharing concepts. Early research on horizontal collaboration considering independent freight carriers is discussed in Kopfer and Pankratz (1998), researching a groupage system, and coining the term Collaborative Transport Planning (CTP). One fair allocation of the savings can be done via the Shapley value introduced by Shapley (1953) that uniquely distributes the savings among the participants.

Cruijssen, and Salomon (2004) showed that order sharing potentially leads to remarkable savings up to 15%. In a follow up paper, Cruijssen et al. (2007) investigated the opportunities and obstacles carriers face in horizontal collaborations. Topics such as a fair allocation of the savings, carrier differentiation, trust and the extent of cooperation are important drivers for success or failure (see also Pomponi et al. (2015)).

Krajewska and Kopfer (2006) introduced an exchange mechanism build around three phases: preprocessing, exchange mechanism, and profit sharing. These cooperation mechanisms are applied to the pickup and delivery problem with time windows (PDPTW) in Krajewska and Kopfer (2006) and Krajewska et al. (2008). This problem is extended with transshipment points for the collaborating carriers by Vornhusen et al. (2014). Wang et al. (2017) investigated the capacitated VRP. Cuervo et al. (2016) did simulations on the effects of partner characteristics. Larger order portfolios lead to larger gains through collaborative coalitions.

Berger and Bierwirth (2010) focused on the exchange mechanism in cooperation for the traveling salesman problem with pickup and delivery. The auctioning of request bundles is an NP–hard combinatorial auctioning problem (CAP). Wang and Kopfer (2014) showed potential cost savings of on average 18.2% up to 64.8%. Wang et al. (2017) applied a route—based bidding mechanism to the PDPTW. Li et al. (2015) formulated a single request exchange approach. Jacob and Buer (2018) investigated the effects of non-truthful bidding and showed that is individually rational but not collectively rational, resulting into a variant of the famous prisoner's dilemma.

Gansterer and Hartl (2016) investigated several request evaluation strategies building on Berger and Bierwirth (2010). Using heuristics, they solve larger instances for the TSP with precedence constraints. Gansterer and Hartl (2018) showed that attractive subsets of predefined bundles can be effectively identified, reducing the computation complexity. More recently, Gansterer et al. (2020) showed the advantage to bundle requests rather than individual requests. Karels et al. (2020) investigate an auction mechanism to facilitate collaboration amongst carriers while maintaining autonomy for the individual carriers, based on a traditional vehicle routing problem.

6.2 Game Theoretical Concepts

Lloyd Shapley introduced two of the most well-known game theoretic solutions, i.e., the core (Shapley 1955), and the Shapley value (Shapley 1953). Although the literature often associates the definition of the core to Gillies (1959), it was Shapley who first defined the core in its current form (Zhao 2018). The search for the core of cooperative games in network situations has motivated a large body of literature (e.g. Borm et al. 2001; Curiel 2008), and implementation of the Shapley value has been suggested by a host of research in collaborative logistics (e.g. Krajewska et al. 2008).

The Nucleolus was first developed by Schmeidler (1969). The unhappiness function used in the definition of the nucleolus can be defined in other ways as well. See Tijs and Driessen (1986) for a review of alternative definitions. Alternative approaches for proving non-emptiness of the cores of *mcst* games have been proposed in the literature, (e.g., Bird 1976, Granot 1986, Granot and Huberman 1981, and Tamir 1991). Although the basic *mcst* situation presented here deals with undirected graphs, similar results also hold for the more general situations with directed graphs. The proof in Tamir (1991) is for directed situations. The proof of Theorem 2 is given in Granot and Huberman (1981). Other solutions for *mcst* situations have been discussed, among others, by Aarts and Driessen (1993) and Bogomolnaia and Moulin (2010) via the concept of the irreducible core, which gives subsets of core allocations. It is worth mentioning that the Shapley value in *mcst* games is also studied in Kar (2002) who provides an axiomatization of this allocation rule for the class of *mcst* games. Interested readers can refer to Granot and Huberman (1981) for the proof of Theorem 3.

Further extensions of the facility location game are studied in the literature, see for instance Mallozzi (2011) and Xu and Du (2006).

The proof of Theorem 5 is given by Skorin-Kapov (1998) where he also considers other variations of hub location games. Further extensions of hub locations games are discussed in Matsubayashi et al. (2005) and Skorin-Kapov (2001).

6.3 Other Classes of Stylized Situations Related to Cooperative Network Design Problems

There are several other classes of stylized situations related to cooperative network design problems for which the cost sharing problems have been studied in the literature. In traveling salesman situations, the goal is to construct cycles with minimum total cost from a source through a set of given nodes representing the players. Accordingly, in traveling salesman games players in a coalition cooperate to establish such cycles among themselves and the source. The main difference between the traveling salesman and *mcst* situations is that of cycles versus trees in constructing solutions respectively. It has been proven that all traveling salesman games with five or less players have non-empty cores (Potters et al. 1992; Tamir 1989; Kuipers 1993). However, for games with six players and above the core can be empty (Tamir 1989; Faigle et al. 1998). In vehicle routing situations, the players would have demands with specific sizes that must be satisfied with vehicles with limited capacity via tours from an origin node. As the class of vehicle routing situations contains the traveling salesman situations as a special instance, the negative results regarding the emptiness of the cores of associated games holds as well. However, Göthe-Lundgren et al. (1996) casts the vehicle routing situations as set partitioning problems and show that non-emptiness of the core can be guaranteed whenever the duality gap for the corresponding linear relaxation is zero. Interested readers are also referred to Chinese-Postman Games (Hamers et al. 1999; Platz and Hamers 2013), Delivery Scheduling Games (Hezarkhani 2016), and Delivery Consolidation Games (Hezarkhani et al. 2019).

7 Conclusions and Perspectives

In this chapter, we looked into the role of cooperation within Transport and Logistics networks. The success concepts like the Physical Internet, urban hubs, or crowd-sourcing, depends heavily on managing the pain-and-gain sharing mechanisms. Clearly, having multiple stakeholders involved in the transportation processes, leads to important cooperation issues. The drivers for cooperation are mainly related to resource utilization optimizations, leading to e.g. less empty mileage or increase truckloads. Game theory helps us to model, understand and optimize these collaborations from a cost sharing perspective. Cooperative game theory provides a set of tools and techniques to address such problems.

Most discussed Transport and Logistics applications (including the network design models) involve very complex situations, as their underlying models are not easy to solve to optimality in a tractable way. This poses a serious problem in adoption of available solutions originating from cooperative game theory. Hence, finding appropriate cost shares is challenging for the Operations Research-based network design models, and we have to revert to the more basic and stylized network design models.

In these highly stylized situations, it might be possible to directly use well-known solutions. Accordingly, one might be able to devise solutions that obtain appropriate cost shares, e.g. allocations in the core, directly from the underlying optimization problems. Specifically, in a collaborative network design situation, there might be a straightforward connection between the optimization program and the appropriate cost shares.

However, classical approaches in cooperative game theory alone are not able to satisfactorily solve cost-sharing problems in the more complex network design situations. On the one hand, the core of games associated with these simulations might be empty—even in relatively simple situations. On the other hand, inherent difficulties in solving the underlying optimization problem can render these solutions too complex (or time consuming).

Despite the theoretical appeal of basic problems discussed in the previous sections, collaborative situations in practice are often complicated by many factors and constraints. Solutions might need to satisfy properties that are specific to a collaborative situations and cannot be captured by standard game-theoretic solutions.

All this motivates research on situation-specific solutions for more advanced network design models. In developing reasonable solutions for these situations, one can formulate practical requirements in terms of desirable properties. We argue that on exactly on this interface of cooperative game theory and network design models, investigating the desirable properties of these solutions and their formal definition ex-ante is needed to obtain more meaningful results, rather than using standard game theoretical solutions.

References

Aarts, H., & Driessen, T. (1993). The irreducible core of a minimum cost spanning tree game. *Mathematical Methods of Operations Research, 38*(2), 163–174.

Barany, I., Edmonds, J., & Wolsey, L. A. (1986). Packing and covering a tree by subtrees. *Combinatorica, 6*(3), 221–233.

Berger, S., & Bierwirth, C. (2010). Solutions to the request reassignment problem in collaborative carrier networks. *Transportation Research Part E: Logistics and Transportation Review, 46*, 627–638.

Bird, C. G. (1976). On cost allocation for a spanning tree: a game theoretic approach. *Networks, 6*(4), 335–350.

Bogomolnaia, A., & Moulin, H. (2010). Sharing a minimal cost spanning tree: Beyond the folk solution. *Games and Economic Behavior, 69*(2), 238–248.

Bondareva, O. N. (1963). Some applications of linear programming methods to the theory of cooperative games. *Problemy kibernetiki, 10*, 119–139 (In Russian).

Borm, P., Hamers, H., & Hendrickx, R. (2001). Operations research games: A survey. Discussion Paper 45, Tilburg University, Center for Economic Research.

Chardaire, P. (1998). *Facility location optimization and cooperative games.* (PhD thesis, University of East Anglia, 1998).

Cruijssen, F., & Salomon, M. (2004). Empirical study: Order sharing between transportation companies may result in cost reductions between 5 to 15 percent. *CentER Discussion Paper*, (2004-80).

Cruijssen, F., Cools, M., & Dullaert, W. (2007). Horizontal cooperation in logistics: Opportunities and impediments. *Transportation Research Part E, 43*(2), 129–142.

Cuervo, D. P., Vanovermeire, C., & Sörensen, K. (2016). Determining collaborative profits in coalitions formed by two partners with varying characteristics. *Transportation Research Part C: Emerging Technologies, 70*, 171–184.

Curiel, I. (1997). Minimum cost spanning tree games. In *Cooperative game theory and applications* (pp. 129–148). Berlin: Springer.

Curiel, I. (2008). Cooperative combinatorial games. In *Pareto optimality, game theory and equilibria* (pp. 131–157). Berlin: Springer.

Deng, X., & Fang, Q. (2008). Algorithmic cooperative game theory. *Pareto Optimality, Game Theory And Equilibria, 17*, 159–185.

Deng, X., Ibaraki, T., & Nagamochi, H. (1999). Algorithmic aspects of the core of combinatorial optimization games. *Mathematics of Operations Research, 24*(3), 751–766.

Eurostat. (2018). Annual road freight transport vehicle movements, loaded and empty, by reporting country. http://epp.eurostat.ec.europa.eu/portal/page/portal/transport/data/database

Faigle, U., Fekete, S. P., Hochstättler, W., & Kern, W. (1998). On approximately fair cost allocation in Euclidean TSP games. *OR Spectrum, 20*(1), 29–37.

Gansterer, M., & Hartl, R. F. (2016). Request evaluation strategies for carriers in auction-based collaborations. *OR Spectrum, 38*, 3–23.

Gansterer, M., & Hartl, R. (2018). Centralized bundle generation in auction-based collaborative transportation *OR Spectrum, 40*, 613–635.

Gansterer, M., Hartl, R. F., & Sörensen, K. (2020). Pushing frontiers in auction-based transport collaborations. *Omega, 94*, 102042.

Gillies, D. B. (1959). Solutions to general non-zero-sum games. *Contributions to the Theory of Games, 4*, 47–85.

Goemans, M. X., & Skutella, M. (2004). Cooperative facility location games. *Journal of Algorithms, 50*(2), 194–214.

Göthe-Lundgren, M., Jörnsten, K., Värbrand, P. (1996). On the nucleolus of the basic vehicle routing game. *Mathematical Programming, 72*(1), 83–100.

Granot, D. (1986). A generalized linear production model: A unifying model. *Mathematical Programming, 34*(2), 212–222.

Granot, D., & Huberman, G. (1981). Minimum cost spanning tree games. *Mathematical Programming, 21*(1), 1–18.

Hamers, H., Borm, P., van de Leensel, R., & Tijs, S. (1999). Cost allocation in the Chinese postman problem. *European Journal of Operational Research, 118*(1), 153–163.

Hezarkhani, B. (2016). Pairwise mergers in bipartite matching games with an application in collaborative logistics. *Operations Research Letters, 44*(6), 818–822.

Hezarkhani, B., Slikker, M., & Van Woensel, T. (2014). On characterization of the core of lane covering games via dual solutions. *Operations Research Letters, 42*(8), 505–508.

Hezarkhani, B., Slikker, M., & Van Woensel, T. (2016). A competitive solution for cooperative truckload delivery. *OR Spectrum, 38*(1), 51–80.

Hezarkhani, B., Slikker, M., & Van Woensel, T. (2019). Gain-sharing in urban consolidation centers. *European Journal of Operational Research, 279*(2), 380–392.

Jacob, J., & Buer, T. (2018). Impact of non-truthful bidding on transport coalition profits. *Operations Research Proceedings 2016, 84*, 203–208.

Kar, A. (2002). Axiomatization of the Shapley value on minimum cost spanning tree games. *Games and Economic Behavior, 38*(2), 265–277.

Karels, V. C., Veelenturf, L. P., & Van Woensel, T. (2020). An auction for collaborative vehicle routing: Models and algorithms. *EURO Journal on Transportation and Logistics, 9*(2), 100009.

Kolen, A. (1983). Solving covering problems and the uncapacitated plant location problem on trees. *European Journal of Operational Research, 12*(3), 266–278.

Kopfer, H., & Pankratz, G. (1998). Das Groupage-Problem kooperierender Verkehrsträger. In *Operations Research Proceedings* (pp. 453–462). Berlin: Springer.

Krajewska, M.A., & Kopfer, H. (2006). Collaborating freight forwarding enterprises. *OR Spectrum, 28*(3), 301–317 (2006)

Krajewska, M. A., Kopfer, H., Laporte, G., Ropke, S., & Zaccour, G. (2008). Horizontal cooperation among freight carriers: request allocation and profit sharing. *Journal of Operational Research Society, 59*(11), 1483–1491.

Kruskal, J. B. (1956). On the shortest spanning subtree of a graph and the traveling salesman problem. *Proceedings of the American Mathematical Society, 7*(1), 48–50.

Kuipers, J. (1993). A note on the 5-person traveling salesman game. *Mathematical Methods of Operations Research, 38*(2), 131–139.

Li, J., Rong, G., & Feng, Y. (2015). Request selection and exchange approach for carrier collaboration based on auction of a single request. *Transportation Research Part E: Logistics and Transportation Review, 84*, 23–39.

Mallozzi, L. (2011). Cooperative games in facility location situations with regional fixed costs. *Optimization Letters, 5*(1), 173–181.

Marinakis, Y., Migdalas, A., & Pardalos, P. M. (2008). Cost allocation in combinatorial optimization games. In *Pareto optimality, game theory and equilibria* (pp. 217–247). Berlin: Springer

Maschler, M., Peleg, B., & Shapley, L. S. (1979). Geometric properties of the kernel, nucleolus, and related solution concepts. *Mathematics of Operations Research, 4*(4), 303–338.

Matsubayashi, N., Umezawa, M., Masuda, Y., & Nishino, H. (2005). A cost allocation problem arising in hub–spoke network systems. *European Journal of Operational Research, 160*(3), 821–838.

Megiddo, N. (1978). Cost allocation for Steiner trees. *Networks, 8*(1), 1–6.

Norde, H., Moretti, S., & Tijs, S. (2004). Minimum cost spanning tree games and population monotonic allocation schemes. *European Journal of Operational Research, 154*(1), 84–97. ISSN 0377-2217.

Özener, O. Ö., Ergun, Ö. (2008). Allocating costs in a collaborative transportation procurement network. *Transportation Science, 42*(2), 146–165.

Platz, T., & Hamers, H. (2013). On games arising from multi-depot Chinese postman problems. Technical report, Tilburg University, Center for Economic Research.

Pomponi, F., Fratocchi, L., Tafuri, S. R. (2015). Trust development and horizontal collaboration in logistics: A theory based evolutionary framework. *Supply Chain Management: An International Journal*, 20(1):83–97.

Potters, J. A. M., Curiel, I. J., & Tijs, S. H. (1992). Traveling salesman games. *Mathematical Programming, 53*(1), 199–211.

Prim, R. C. (1957). Shortest connection networks and some generalizations. *Bell Labs Technical Journal, 36*(6), 1389–1401.

Quak, H., Tavasszy, L. (2011). Customized solutions for sustainable city logistics: The viability of urban freight consolidation centres. In *Transitions towards sustainable mobility* (pp. 213–233). Berlin: Springer.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics, 17*(6), 1163–1170.

Shapley, L. (1953). *A value for n-person games*, volume II, chapter Contributions to the Theory of Games (pp. 307–317). Princeton, NJ: Princeton University Press.

Shapley, L. (1971). Cores of convex games. *International Journal of Game Theory, 1*(1), 11–26.

Shapley, L., & Shubik, M. (1966). Quasi-cores in a monetary economy with nonconvex preferences. *Econometrica: Journal of the Econometric Society, 34*(4), 805–827.

Shapley, L. S. (1955). Markets as cooperative games. Technical report, Rand Corporation Paper P-629.

Shapley, L. S. (1967). On balanced sets and cores. *Naval Research Logistics, 14*(4), 453–460.

Sharkey, W. W. (1995). *Handbooks in Operations Research and Management Science*, (Vol. 8, pp. 713–765), chapter Network models in economics. Network routing. Amsterdam: Elsevier.

Skorin-Kapov, D. (1998). Hub network games. *Networks, 31*(4), 293–302.

Skorin-Kapov, D. (2001). On cost allocation in hub-like networks. *Annals of Operations Research, 106*(1), 63–78.

Sprumont, Y. (1990). Population monotonic allocation schemes for cooperative games with transferable utility. *Games and Economic Behavior, 2*(4), 378–394.

A. Tamir. (1989). On the core of a traveling salesman cost allocation game. *Operations Research Letters, 8*(1), 31–34.

Tamir, A. (1991). On the core of network synthesis games. *Mathematical Programming, 50*(1), 123–135.

Tijs, S. H., & Driessen, T. S. H. (1986). Game theory and cost allocation problems. *Management Science, 32*(8), 1015–1028.

Vanovermeire, C., Sörensen, K., Van Breedam, A., Vannieuwenhuyse, B., & Verstrepen, S. (2014). Horizontal logistics collaboration: Decreasing costs through flexibility and an adequate cost allocation strategy. *International Journal of Logistics Research and Applications, 17*(4), 339–355.

Vornhusen, B., Wang, X., & Kopfer, H. (2014). Vehicle Routing under Consideration of Transhipment in Horizontal Coalitions of Freight Carriers. *Procedia CIRP, 19*(1), 117–122.

Wang, X., & Kopfer, H. (2014). Collaborative transportation planning of less-than-truckload freight. *OR Spectrum, 36*, 357–380.

Wang, Y., Ma, X., Li, Z., Liu, Y., Xu, M., & Wang, Y. (2017). Profit distribution in collaborative multiple centers vehicle routing problem. *Journal of Cleaner Production, 144*, 203–219.

Xu, D., & Du, D. (2006). The k-level facility location game. *Operations Research Letters, 34*(4), 421–426.

Zhao, J. (2018). Three little-known and yet still significant contributions of Lloyd Shapley. *Games and Economic Behavior, 108*, 592–599.