

# Chapter 19

## Logistics Network Design



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### 1 Introduction

The design of logistics networks is one of the most important areas of application for multicommodity network design models. Logistics networks (or supply chains) connect suppliers, manufacturing plants, warehouses, distribution centers and customers to coordinate the acquisition of raw materials and components, their transformation into finished products and the delivery of these products to the customers. The design of these networks is complex and involves making a large number of interdependent decisions concerning the selection of suppliers, the location of production and distribution facilities, the assignment of products to the facilities, the selection of transportation modes, and the determination of the flows of raw materials, components and finished products in the network. Figure 19.1 illustrates the structure of a classical logistics network with suppliers providing raw materials to production plants which, in turn, deliver finished products to warehouses. Finally, these warehouses are responsible for serving the demand of the customers. Nodes that are not connected to others represent potential facilities that are not currently part of the network.

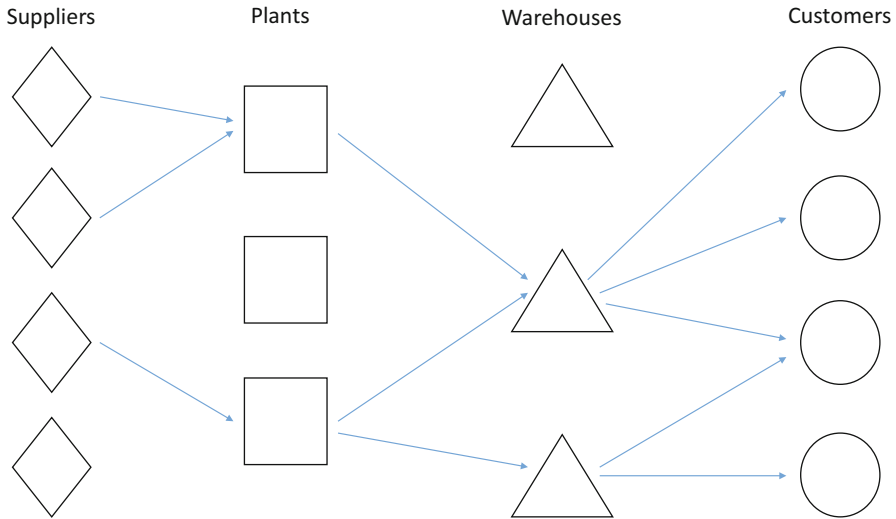
Logistics network design is of course strategic in nature and concerns the long term. However, it is essential when designing a network to take into account the

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**Fig. 19.1** Example of a three-echelon logistics network

tactical and operational repercussions of the strategic decisions. For instance, the selection of proper locations and capacity levels for production plants depends to a large extent on the selection of suppliers, on the choice of which products will be made in each plant, on the transportation modes used to connect the various nodes in the network, and even on the amount of flow circulating on the arcs. Of course, tactical and operational decisions will be revised more frequently as demand and other parameters change in the environment of the firm.

In addition to combining multiple types of intertwined decisions, logistics network design problems usually involve several categories of costs: fixed costs associated with facility locations, acquisition costs for raw materials and components, production costs, transportation costs, inventory holding costs, etc. Not surprisingly, several studies have shown that very important savings can be achieved by taking the design decisions in an integrated way (see, e.g. Arntzen et al. 1995; Fleischmann et al. 2006; Ulstein et al. 2006). Companies often benefit from the re-optimization of their logistics networks following changes in their business environment: new market opportunities, changes in production or transportation costs, new technologies, changes in trade regulations, etc. Mergers and acquisitions also often create the need for a firm to revise the structure of its logistics network. Accordingly, logistics networks are rarely designed from scratch and optimization models usually aim at finding the best way to adapt an existing network to new market conditions.

Over the last 40 years, the realism of logistics network design models has greatly improved and efficient solution methods have been developed to solve these models. There is now a vast literature on the topic with a very large number of models addressing the many problem variants encountered in practice (see, e.g. Melo et al.

2009; Martel and Klibi 2016). The purpose of this chapter is to provide a general modeling framework that can be used to express many of these variants and to give a brief overview of the main solution methodologies. We also devote attention to two important and recent trends: the treatment of risk and uncertainty in the design of logistics networks and the incorporation of environmental, sustainability and reverse logistics aspects.

The rest of the chapter is organized as follows. Section 2 introduces the modeling framework and discusses various extensions. This is followed by a discussion of risk and uncertainty concepts in Sect. 3 and of reverse logistics, environmental and sustainability aspects in Sect. 4. Section 5 is devoted to solution methods. Finally, Sect. 6 provides bibliographic notes and Sect. 7 concludes the chapter.

## 2 A General Modeling Framework for Logistics Network Design

The purpose of this section is to introduce a general formulation that captures the fundamental aspects of logistics network design and can also serve as a basis to incorporate various extensions that are often required in practical applications. This formulation is itself based on the model proposed by Cordeau et al. (2006) but it has been generalized to consider multiple time periods as well as arbitrary network structures and bills of materials.

### 2.1 Notation

We denote by  $\mathcal{K}$  be the set of all item types circulating in the logistics network. This set can be partitioned into a subset  $\mathcal{R}$  of raw materials, a subset  $\mathcal{A}$  of assemblies and a subset  $\mathcal{F}$  of finished products such that  $\mathcal{K} = \mathcal{R} \cup \mathcal{A} \cup \mathcal{F}$ . We assume here that raw materials are acquired from external suppliers whereas finished products are delivered to customers. Assemblies are intermediate components that are made either from raw materials or from other assemblies. For every item  $k \in \mathcal{A} \cup \mathcal{F}$ , let  $\mathcal{B}^k \subseteq \mathcal{R} \cup \mathcal{A}$  denote the subset of items that are needed to make item  $k$ . Similarly, for every raw material or assembly  $\ell \in \mathcal{R} \cup \mathcal{A}$ , let  $\mathcal{K}^\ell \subseteq \mathcal{A} \cup \mathcal{F}$  denote the assemblies and finished products that require item  $\ell$ . For every  $k \in \mathcal{K}$  and  $\ell \in \mathcal{B}^k$ , we denote by  $b^{k\ell}$  the amount of item  $\ell$  required for the production of one unit of item  $k$ . The set  $\mathcal{B}^k$  and values  $b^{k\ell}$  define the bill of materials for item  $k$ .

The set of potential suppliers is denoted by  $\mathcal{S}$  and, for every raw material  $r \in \mathcal{R}$ , we let  $\mathcal{S}^r \subseteq \mathcal{S}$  represent the subset of suppliers that may provide  $r$ . We also let  $\mathcal{P}$  and  $\mathcal{W}$  denote the sets of potential locations for plants and warehouses, respectively. For every item  $k \in \mathcal{K}$ , we let  $\mathcal{P}^k$  denote the subset of plants where item  $k$  can be produced or used in the production of other items. Similarly,  $\mathcal{W}^k$  represents the

warehouses where item  $k$  can be stored. Finally, we denote by  $\mathcal{C}$  the set of customer locations. In most applications, a customer  $c$  would not represent an individual customer or a specific company but rather an aggregation of the demand of a given region.

To simplify the writing of the model, we let  $\mathcal{O} = \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$  and  $\mathcal{D} = \mathcal{P} \cup \mathcal{W} \cup \mathcal{C}$  denote the sets of possible origins and destinations for the items circulating in the network. For every  $k \in \mathcal{K}$ , we also define  $\mathcal{O}^k \subseteq \mathcal{O}$  and  $\mathcal{D}^k \subseteq \mathcal{D}$  as the sets of possible origins and destinations for item  $k$ , respectively. For any node  $i \in \mathcal{O}$ , let also  $\mathcal{K}_i = \{k \in \mathcal{K} \mid i \in \mathcal{O}^k \cup \mathcal{D}^k\}$  be the set of items which may originate from or be destined to node  $i$ .

The planning horizon is divided into a set of consecutive time periods denoted by  $\mathcal{T}$ . For every  $c \in \mathcal{C}$ ,  $f \in \mathcal{F}$  and  $t \in \mathcal{T}$ , let  $d_c^{ft}$  be the demand of customer  $c$  for finished product  $f$  in period  $t$ .

The model uses three types of binary variables to represent decisions related to the selection of nodes in the network and the assignment of items to these nodes. For every node  $i \in \mathcal{O}$ , we define a binary variable  $y_i$  equal to 1 if and only if the node is selected, and we let  $c_i$  be the fixed cost of selecting this node. For every item  $k \in \mathcal{K}$  and every node  $i \in \mathcal{O}^k$ , let also  $v_i^k$  be a binary variable, with cost  $c_i^k$ , taking value 1 if and only if item  $k$  is assigned to node  $i$ . These variables represent the decisions to acquire raw materials from certain suppliers or to make and store products in certain plants and warehouses. Finally, for every  $k \in \mathcal{K}$ ,  $i \in \mathcal{O}^k$  and  $j \in \mathcal{D}^k$ , let  $w_{ij}^k$  be a binary variable, with cost  $c_{ij}^k$ , equal to 1 if and only if origin  $i$  provides item  $k$  to destination  $j$ .

For every node  $i \in \mathcal{O}$ , let  $q_i$  be the output capacity of this node, and for every  $k \in \mathcal{K}_i$ , let  $u_i^k$  be the amount of capacity required by one unit of item  $k$  at node  $i$ . For every  $k \in \mathcal{K}$  and  $i \in \mathcal{O}^k$ , let also  $q_i^k$  be the capacity of node  $i$  for item  $k$  and  $q_{ij}^k$  be the maximum that can be provided to destination  $j \in \mathcal{D}^k$ .

We assume that a set  $\mathcal{M}_{ij}$  of potential transportation modes is associated to every origin-destination pair  $(i, j) \in \mathcal{O} \times \mathcal{D}$ . For every  $m \in \mathcal{M}_{ij}$ , we then define a binary variable  $z_{ij}^m$  equal to 1 if and only if transportation mode  $m$  is used between origin  $i$  and destination  $j$ . We use the notation  $c_{ij}^m$  to represent the fixed cost of using mode  $m$  and let  $q_{ij}^m$  be its capacity in each time period. For every  $k \in \mathcal{K}$ ,  $i \in \mathcal{O}^k$  and  $j \in \mathcal{D}^k$ ,  $\mathcal{M}_{ij}^k \subseteq \mathcal{M}_{ij}$  is the subset of feasible transportation modes between  $i$  and  $j$  for item  $k$ , and  $u^{km}$  is the unit capacity consumption for item  $k$  in mode  $m$ .

Finally, the model uses two types of continuous variables to represent acquisition, production, storage and transportation decisions. For every  $m \in \mathcal{M}_{ij}^k$  and  $t \in \mathcal{T}$ , we define a non-negative variable  $x_{ij}^{kmt}$ , with cost  $c_{ij}^{kmt}$ , representing the number of units of item  $k$  transported from  $i$  to  $j$  using mode  $m$  in period  $t$ . Unit costs  $c_{ij}^{kmt}$  should include not only transportation expenses but also relevant acquisition, production and handling costs at the origin node  $i$ . This does not cause any loss of generality since the total flow going through a node can be computed as the sum of the flows on the arcs leaving that node. For every item  $k \in \mathcal{K}$  and every node  $w \in \mathcal{W}^k$ , we also let  $I_w^{kt}$  denote the inventory of item  $k$  in node  $w$  at the end of period  $t$  and we let

**Table 19.1** Summary of notation for sets

$\mathcal{A}$	Set of assemblies
$\mathcal{B}^k$	Set of items needed to make item $k$
$\mathcal{C}$	Set of customers
$\mathcal{C}^f$	Set of customers that require product $f$
$\mathcal{D}$	Set of destinations
$\mathcal{D}^k$	Set of potential destinations for item $k$
$\mathcal{F}$	Set of finished products
$\mathcal{H}$	Set of all items
$\mathcal{H}^\ell$	Set of items that require item $\ell$
$\mathcal{H}_i$	Set of items that may originate or be destined to $i$
$\mathcal{M}_{ij}$	Set of transportation modes between $i$ and $j$
$\mathcal{M}_{ij}^k$	Set of transportation modes between $i$ and $j$ for item $k$
$\mathcal{O}$	Set of origins
$\mathcal{O}^k$	Set of potential origins for item $k$
$\mathcal{P}$	Set of potential plant locations
$\mathcal{P}^k$	Set of plant locations where item $k$ can be produced or used
$\mathcal{R}$	Set of raw materials
$\mathcal{S}$	Set of potential suppliers
$\mathcal{S}^\ell$	Set of potential suppliers providing raw material $\ell$
$\mathcal{T}$	Set of time periods
$\mathcal{W}$	Set of potential warehouse locations
$\mathcal{W}^k$	Set of warehouse locations where item $k$ can be stored

$g_w^{kt}$  be the cost of holding one unit of item  $k$  at node  $w$  in period  $t$ . We assume here that inventory is held only at warehouse nodes. If a plant possesses storage areas for assemblies or finished products, it can be modeled by multiple nodes connected among themselves and representing the various functions of the plant.

Tables 19.1, 19.2 and 19.3 provide a summary of the notation.

## 2.2 Formulation

The *logistics network design problem* (LNDP) consists in minimizing the following objective function, which comprises all the fixed costs associated with the binary design variables and unit costs associated with the flow and inventory variables:

$$\text{Minimize } \sum_{i \in \mathcal{O}} \left[ c_i y_i + \sum_{j \in \mathcal{D}} \sum_{m \in \mathcal{M}_{ij}} c_{ij}^m z_{ij}^m \right] +$$

**Table 19.2** Summary of notation for parameters

$b^{k\ell}$	Amount of item $\ell$ needed in one unit of item $k$
$c_i$	Fixed cost of selecting origin $i$
$c_i^k$	Fixed cost of assigning item $k$ to origin $i$
$c_{ij}^k$	Fixed cost of providing item $k$ to destination $j$ from origin $i$
$c_{ij}^m$	Fixed cost of using transportation mode $m$ between $i$ and $j$
$c_{ij}^{kmt}$	Unit cost for providing item $k$ to $j$ from $i$ with mode $m$ in period $t$
$d_c^{ft}$	Demand of customer $c$ for product $f$ in period $t$
$g_w^{kt}$	Cost of holding one unit of item $k$ at node $w$ during period $t$
$q_i$	Capacity of node $i$ in equivalent units
$q_{ij}^m$	Capacity of mode $m$ between $i$ and $j$ in equivalent units
$q_i^k$	Upper limit on the amount of item $k$ shipped from node $i$
$q_{ij}^k$	Upper limit on the amount of item $k$ shipped from $i$ to $j$
$u_i^k$	Amount of capacity required by one unit of item $k$ at node $i$
$u^{km}$	Amount of capacity required by one unit of item $k$ in mode $m$

**Table 19.3** Summary of notation for variables

$I_w^{kt}$	Inventory of item $k$ in location $w$ at the end of period $t$
$x_{ij}^{kmt}$	Amount of item $k$ shipped from $i$ to $j$ with mode $m$ in period $t$
$y_i$	= 1 if node $i$ is selected
$v_i^k$	= 1 if item $k$ is assigned to origin $i$
$w_{ij}^k$	= 1 if node $i$ provides item $k$ to destination $j$
$z_{ij}^m$	= 1 if mode $m$ is selected between $i$ and $j$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{O}^k} \left[ c_i^k v_i^k + \sum_{j \in \mathcal{D}^k} \left[ c_{ij}^k w_{ij}^k + \sum_{m \in \mathcal{M}_{ij}^k} \sum_{t \in \mathcal{T}} c_{ij}^{kmt} x_{ij}^{kmt} \right] \right] + \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}^k} \sum_{t \in \mathcal{T}} g_w^{kt} I_w^{kt}. \quad (19.1)$$

The first group of constraints comprises equations related to the flow of items in the network:

$$\sum_{i \in \mathcal{O}^\ell} \sum_{m \in \mathcal{M}_{ip}^\ell} x_{ip}^{\ell mt} - \sum_{k \in \mathcal{K}^\ell} \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{pj}^k} b^{k\ell} x_{pj}^{kmt} = 0 \quad \ell \in \mathcal{R} \cup \mathcal{A}; p \in \mathcal{P}^\ell; t \in \mathcal{T} \quad (19.2)$$

$$\sum_{i \in \mathcal{O}^k} \sum_{m \in \mathcal{M}_{iw}^k} x_{iw}^{kmt} - \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{wj}^k} x_{wj}^{kmt} + I_w^{k,t-1} - I_w^{kt} = 0 \quad k \in \mathcal{K}; w \in \mathcal{W}^k; t \in \mathcal{T} \quad (19.3)$$

$$\sum_{i \in \mathcal{O}^f} \sum_{m \in \mathcal{M}_{ic}^f} x_{ic}^{f mt} = d_c^{ft} \quad f \in \mathcal{F}; c \in \mathcal{C}^f; t \in \mathcal{T}. \quad (19.4)$$

Constraints (19.2) force the amount of raw material or assembly  $\ell$  shipped to plant  $p$  in period  $t$  to be equal to the amount required by all assemblies and finished products made at this plant during the same period. Constraints (19.3) ensure that the amount of item  $k$  entering warehouse  $w$  during period  $t$  plus the inventory available at the beginning of period  $t$  is equal to the amount leaving the warehouse during that period plus the amount available at the end of the period. To ensure consistent inventory levels, the last period in the horizon can be connected to the first one, which has the effect of forcing the inventory level at the end of the last period to be equal to the inventory level at the beginning of the first one. Demand constraints are imposed by Eqs. (19.4).

The second group of constraints comprises inequalities related to the capacity of the nodes and arcs in the network:

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{ij}^k} u_i^k x_{ij}^{k mt} - q_i y_i \leq 0 \quad i \in \mathcal{O}; t \in \mathcal{T} \quad (19.5)$$

$$\sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{ij}^k} x_{ij}^{k mt} - q_i^k v_i^k \leq 0 \quad k \in \mathcal{K}; i \in \mathcal{O}^k; t \in \mathcal{T} \quad (19.6)$$

$$\sum_{m \in \mathcal{M}_{ij}^k} x_{ij}^{k mt} - q_{ij}^k w_{ij}^k \leq 0 \quad k \in \mathcal{K}; i \in \mathcal{O}^k; j \in \mathcal{D}^k; t \in \mathcal{T} \quad (19.7)$$

$$\sum_{k \in \mathcal{K}} u^{km} x_{ij}^{k mt} - q_{ij}^m z_{ij}^m \leq 0 \quad i \in \mathcal{O}; j \in \mathcal{D}; m \in \mathcal{M}_{ij}; t \in \mathcal{T}. \quad (19.8)$$

Constraints (19.5) impose aggregate capacity limits on suppliers, plants and warehouses, whereas limits for individual items are enforced through (19.6). Constraints (19.7) force node  $i$  to be selected if units of item  $k$  are transported from  $i$  to  $j$ . Finally, capacity constraints for individual transportation modes are represented by (19.8).

It should be noted that single-sourcing for item  $k$  at destination  $j$  can be imposed with the constraint

$$\sum_{i \in \mathcal{O}^k} w_{ij}^k \leq 1. \quad (19.9)$$

Furthermore, the following constraint can impose a single-assignment rule to ensure that all units of an item  $k$  come from the same origin:

$$\sum_{i \in \mathcal{O}^k} v_i^k \leq 1. \quad (19.10)$$

## 2.3 Extensions

The above formulation captures the essential aspects of the LNPD but it can also be generalized to address a variety of practical situations. For the sake of clarity, each extension is presented separately although the different extensions can obviously be combined.

### 2.3.1 Lower Bounds and Capacity Alternatives

Lower limits on acquisition, production, storage and transportation activities can be imposed by using constraints similar to (19.5)–(19.8) but reversing the inequality sign. In particular, lower bounds can be used to model quantity discounts or any other situation where a minimal volume is necessary for a given unit cost to be applicable. The combined use of lower and upper bounds can also serve to model situations where several capacity alternatives, with different operating costs, exist for the configuration of a node in the network. This also applies to different layouts or configurations of facilities.

### 2.3.2 Multi-Period Design Decisions

The above model assumes that all binary decisions are made at the beginning of the planning horizon and that the network structure thus remains the same throughout this horizon. However, facility location decisions are usually highly dependent on the value of some parameters such as the fixed costs of the facilities and the customer demand. If these parameters are expected to vary over time, it may be desirable to plan in advance for future adjustments in the number and location of facilities and in other related decisions. In this case, locating a set of facilities becomes a question not only of “where” but also of “when”. This can be achieved by associating a time index to the binary design decisions and by adapting constraints (19.5)–(19.8) to reflect the fact that network configuration and facility capacity evolve over time. In particular, let  $y_{it}$  indicate whether the facility at node  $i$  is open and operating in period  $t$ . Simply adding the time index would allow for frequent opening and closing of facilities, which does not adequately reflect an implementable evolution of a real-world supply chain network. To depict the network evolution of a growing enterprise, constraints (19.11) ensure that facilities that have once been opened remain open throughout the planning horizon:



$$y_{i,t-1} - y_{it} \leq 0 \quad i \in \mathcal{I}; t \in \mathcal{T} \setminus \{1\}. \quad (19.11)$$

When planning the optimal evolution of an existing supply chain network, the successive closing of existing facilities becomes important. In such phase-in/phase-out models the set of nodes  $\mathcal{I}$  can be partitioned into two subsets:  $\mathcal{I}^C$ , the set of locations where existing facilities can be removed, and  $\mathcal{I}^O$ , the set of locations where new facilities can be installed. To ensure consistency in the resulting network configuration constraints (19.11) are replaced with constraints (19.12) and (19.13):

$$y_{i,t-1} - y_{it} \leq 0 \quad i \in \mathcal{I}^O; t \in \mathcal{T} \setminus \{1\} \quad (19.12)$$

$$y_{it} - y_{i,t-1} \leq 0 \quad i \in \mathcal{I}^C; t \in \mathcal{T} \setminus \{1\}. \quad (19.13)$$

The explicit consideration of opening and closing costs for facilities in this context was first introduced by Wesolowsky and Truscott (1975).

Multi-period models are sometimes called dynamic (location) models in the literature. Note that this is somewhat misleading because design decisions can only be made at specific moments, namely at the beginning of each indexed period. Some models go even further and, mostly in an attempt to reduce complexity, divide the set of periods  $\mathcal{T}$  into strategic and tactical periods, whereby changes in the network configuration are restricted to the strategic periods. A multi-period modeling framework involves one extra dimension in the decision space: the timing. Hence, the resulting models tend to be large and harder to solve, even for instances of moderate size. Accordingly, one may ask whether it is worth considering this extra dimension instead of making static decisions even though costs, demands and other parameters may vary over time. An answer to this question can be given by the *value of the multi-period solution*, a concept first introduced by Alumur et al. (2012) in the context of a multi-period reverse logistics network design problem. The value of the multi-period solution compares the optimal value of the multi-period problem and the value of a solution found by solving a static counterpart. We refer the interested reader to Nickel and Saldanha-da-Gama (2015) for an in-depth discussion of multi-period facility location models and for details on the value of the multi-period solution.

### 2.3.3 Inventory Level Constraints

Lower and upper bounds on inventory levels in warehouses can be added to the model by introducing constraints on the  $I_w^{kt}$  variables. In particular, an upper bound  $\hat{q}_w^{kt}$  on the amount of item  $k$  held in inventory in warehouse  $w$  at the end of period  $t$  can be imposed with the following constraint:

$$I_w^{kt} \leq \hat{q}_w^{kt} v_w^k. \quad (19.14)$$

Similarly, an upper bound  $\hat{q}_w^t$  on the total amount of inventory held in warehouse  $w$  at the end of period  $t$  can be imposed with the following constraint:

$$\sum_{k \in \mathcal{K}} u_w^k I_w^{kt} \leq \hat{q}_w^t y_w. \tag{19.15}$$

Because the LNDP is a strategic planning problem defined over a long planning horizon, the length of the time periods usually does not allow for a detailed representation of operational inventory decisions. Nevertheless, constraints can be imposed on expected safety stocks and cyclic (replenishment) inventory levels by relying on turnover ratios. Let  $\rho_w^{kt}$  denote the expected turnover ratio for item  $k$  at warehouse  $w$  in period  $t$ . A lower bound on the total inventory level at the end of period  $t$  can be imposed as follows:

$$I_w^{kt} \geq \sum_{d \in D^k} \sum_{m \in \mathcal{M}_{wd}^k} \frac{1}{\rho_w^{kt}} x_{wd}^{kmt}. \tag{19.16}$$

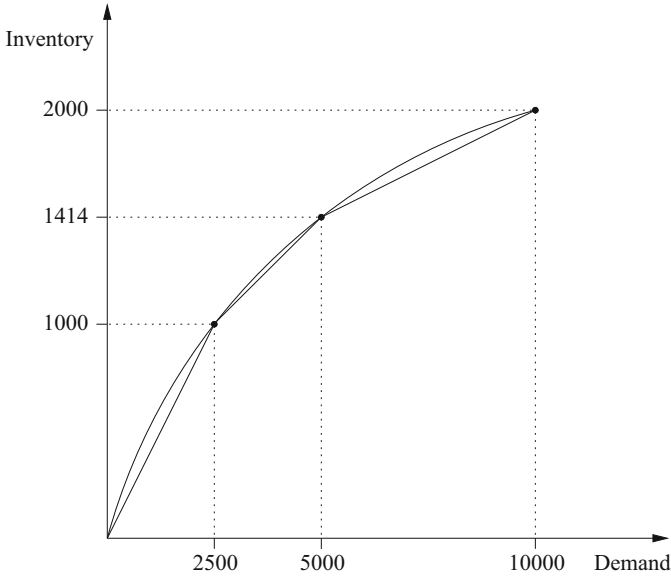
The actual inventory level can of course be larger than the lower bound when fluctuations in demand make it beneficial to accumulate inventory in some periods to be used in later ones. It is well known in inventory control that, because of pooling effects, the amount of safety stock needed increases less than linearly with the amount of demand served by a warehouse. To capture this non-linear relationship between demand volume and operational inventory levels, one may define several copies of the same warehouse with different turnover ratios for the given product. Better approximations of the concave relationship between throughput and inventory levels can be obtained by using a continuous piece-wise linear function specified as a set of base levels (the equivalent of a fixed cost) and unit rates of increase (the equivalent of a unit cost). Mathematically, this relationship can be imposed by using the constraint

$$I_w^{kt} \geq \alpha_w^{kt} + \sum_{d \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{wd}^k} \beta_w^{kt} x_{wd}^{kmt}, \tag{19.17}$$

where  $\alpha_w^{kt}$  represents the base level and  $\beta_w^{kt}$  is the unit increase rate. This will lead to the type of approximation represented in Fig. 19.2.

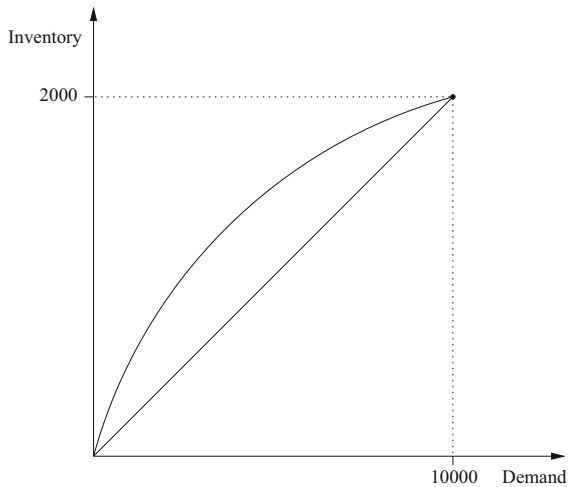
It should be observed that by setting  $\alpha_w^{kt} = 0$  for every segment, one obtains the first approximation (19.16) based only on turnover ratios. Furthermore, if a single segment is used, the value of  $\beta_w^{kt}$  should correspond to the inverse of the expected turnover ratio. If this value is estimated with respect to the maximum possible throughput, one will obtain the approximation illustrated in Fig. 19.3.

We refer to Martel (2005) for a detailed treatment of inventory representation in LNDPs.



**Fig. 19.2** Inventory levels with piece-wise linear segments

**Fig. 19.3** Inventory levels with a single segment



**2.3.4 Profit Maximization**

The traditional focus in logistics network design is to minimize costs while satisfying an exogenous demand. In a value-creation paradigm, however, it may be more appropriate to consider a profit maximization objective function that captures both costs and revenues. Model (1)–(8) can be modified in different ways to account for profit maximization. A common approach is to consider the demand as a decision

variable. This can be accomplished by associating a unit revenue  $r_c^f$  with product  $f$  and customer  $c$  and by replacing equality constraints (19.4) with two inequalities imposing minimum and maximum demand levels to be served in each market as follows:

$$\sum_{i \in \mathcal{O}^f} \sum_{m \in \mathcal{M}_{ic}^f} x_{ic}^{f mt} \geq \bar{d}_c^{ft} \quad f \in \mathcal{F}; c \in \mathcal{C}^f; t \in \mathcal{T} \quad (19.18)$$

$$\sum_{i \in \mathcal{O}^f} \sum_{m \in \mathcal{M}_{ic}^f} x_{ic}^{f mt} \leq \hat{d}_c^{ft} \quad f \in \mathcal{F}; c \in \mathcal{C}^f; t \in \mathcal{T}. \quad (19.19)$$

Here,  $\bar{d}_c^{ft}$  and  $\hat{d}_c^{ft}$  represent, respectively, the minimum amount of demand to be served and the maximum potential demand of customer  $c$  for product  $f$  in period  $t$ . Setting both parameters to the same value corresponds to imposing the equality constraints (19.4). The following term should be added to the objective function to measure total revenue:

$$\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{O}^f} \sum_{m \in \mathcal{M}_{ic}^f} r_c^f x_{ic}^{f mt}.$$

The latter expression assumes that unit revenue is constant, which makes the objective function linear and preserves model tractability. However, if demand is assumed to depend on price and price is itself treated as a decision variable, then the objective function becomes non-linear and makes the problem harder to solve. The above approach also assumes that demand is independent of the network design. In practice, sales are often affected not only by price but also by the design of the logistics network itself because it has an impact on various aspects of service such as response time, i.e., the time it takes to deliver a product to the customer. Figure 19.4 illustrates how, for a given price, the revenues ( $R^S$ ) and costs ( $C^S$ ) may depend on the response time  $S$  provided by the logistics network and, consequently, how the economic value added by the network can be expressed as the difference between revenue and cost. It also shows that beyond a given response time  $S^{max}$  revenues can decline abruptly.

This suggests another approach to address profit maximization: one can perform a sensitivity analysis based on the price and response time variations. For given values of price and response time, the demand can be estimated and the revenues calculated a priori. Then, the cost minimization model can be solved after removing from the model the variables that correspond to customer assignments that would violate the target response time  $S$ . Varying the price and response time allows one to approximate the two curves shown in Fig. 19.4 and to identify the value of  $S$  for which the difference  $P^S$  between  $R^S$  and  $C^S$  is maximized.

Finally, a more sophisticated approach to treat profit maximization is to model the set of potential market policies offered by a company through binary policy selection variables. A market policy specifies the price, desired response time and other

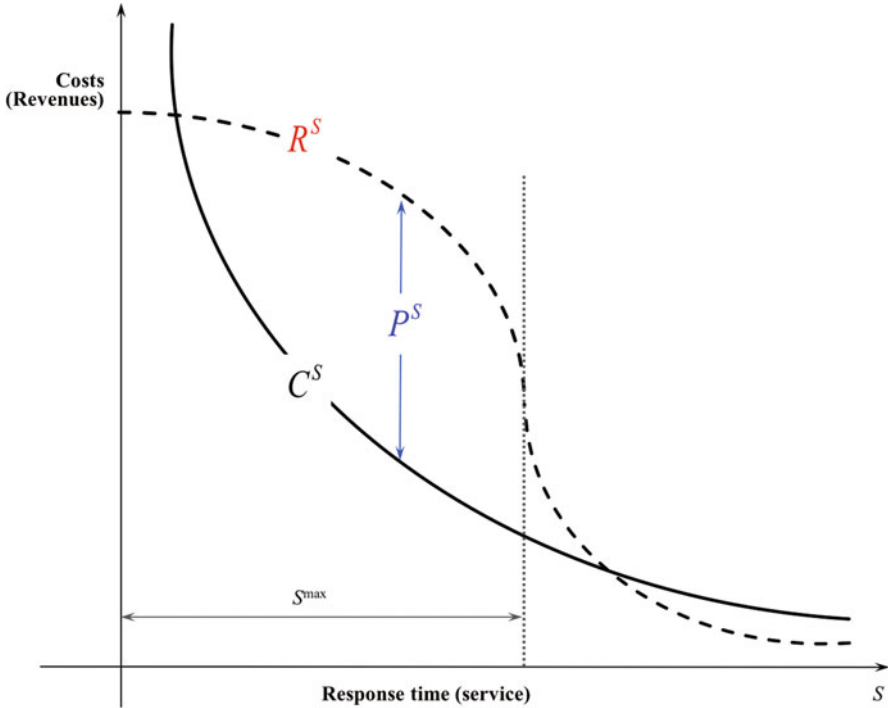


Fig. 19.4 Economic value added for a given logistics network design

attributes, and is characterised by a fixed implementation cost, service constraints and demand bounds. This approach is explained in more detail by Martel and Klibi (2016).

### 2.3.5 International Aspects

Some aspects related to international operations can easily be taken into consideration with this model. For example, exchange rates should be used to convert monetary values into a unique, common currency. In addition, tariffs and duties for products that cross a border can be added directly to the cost of the corresponding arcs. Local content rules can often be enforced in the form of lower bounds on a sum of arc flow variables. However, more complex questions such as transfer pricing and taxation require the introduction of additional variables and constraints. In particular, transfer pricing usually leads to non-linear formulations because of the need to determine both costs and flows simultaneously on some arcs of the networks. A detailed treatment of international aspects is beyond the scope of this chapter and we instead refer to Arntzen et al. (1995), Martel (2005) and to Martel and Klibi (2016).

### 3 Risk and Uncertainty

Because logistics network design decisions concern the long term, several of the input parameters are often subject to risk and uncertainty. Several modeling approaches can be considered to take this uncertainty into account. In this section, we explain how two of these approaches, stochastic programming and robust optimization, can be applied to the LNDP.

#### 3.1 Stochastic Programming

Stochastic programming is a well-known optimization technique to address mathematical programs in which some of the data are random variables. It assumes that the probability distributions of the random parameters are known a priori. One can distinguish between two main types of models: (1) stochastic programs with recourse that explicitly model recourse decisions to hedge against uncertainty, and (2) chance-constrained programs that impose restrictions on the probability that a constraint is violated due to stochasticity. Here, we limit ourselves to the case of stochastic programming with recourse, which is the most common approach in the context of logistics network design.

Referring to formulation (1)–(8) of the LNDP, the aim is to incorporate notions of cost, demand and capacity uncertainty in the model. When all design decisions are made at once, the problem can be formulated as a two-stage stochastic program with recourse. When several design periods are considered, one obtains a multi-stage program, which is more difficult to solve. Under a two-stage setting, some decisions are made in the first stage before the uncertain information is known. In the second stage, the values of the random variables become known and the recourse actions are taken. In our context, the first stage could, for example, correspond to fixing all of the binary design variables. The second stage would consist in choosing the flows and resulting inventory levels in the network and also in choosing the short-term actions required to match supply and demand (i.e., use of additional capacity, subcontracting, or overtime). Uncertainty should thus be restricted to parameters that affect only the flow variables in connection with demand, unit costs and capacities. However, fixed costs associated with the binary variables should be deterministic. Structural information such as bills of materials should also be known with certainty. As a basic rule of thumb, one may consider that information that varies from period to period in the deterministic model can be considered random in the stochastic program, while information that is not period-dependent should be deterministic.

Let  $\mathcal{H}$  denote the set of possible scenarios and, for each scenario  $h \in \mathcal{H}$ , denote by  $p_h$  the probability of scenario  $h$  being realized. Each scenario represents a realization of the vector of uncertain parameters. When continuous probability distributions are considered, the set  $\mathcal{H}$  is implicitly infinite. However, a subset of

scenarios can be generated by a sampling method. The main modification required to the demand, cost, and capacity parameters is the addition of a scenario index  $h$ . For every scenario  $h$ , let  $x_{ij}^{kmth}$  and  $I_w^{kth}$  denote the flows and inventory levels under scenario  $h$ . Using this notation, the generic LNDP model (19.1)–(19.8) can be transformed into the following two-stage stochastic program with recourse:

$$\begin{aligned} \text{Minimize } & \sum_{i \in \mathcal{O}} \left[ c_i y_i + \sum_{j \in \mathcal{D}} \sum_{m \in \mathcal{M}_{ij}} c_{ij}^m z_{ij}^m \right] + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{O}^k} \left[ c_i^k v_i^k + \sum_{j \in \mathcal{D}^k} c_{ij}^k w_{ij}^k \right] + \\ & \mathbb{E}_{\mathcal{H}} [Q(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)], \end{aligned} \quad (19.20)$$

where  $Q(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)$  is the optimal value of the second-stage program and  $\mathbb{E}_{\mathcal{H}}[\cdot]$  denotes the expectation with respect to the scenario set  $\mathcal{H}$ . For a given scenario  $h$  and given values  $\bar{\mathbf{y}}, \bar{\mathbf{z}}, \bar{\mathbf{v}}, \bar{\mathbf{w}}$  of the first-stage decisions, the second-stage problem can be expressed as follows:

$$\text{Minimize } \sum_{i \in \mathcal{T}} \sum_{k \in \mathcal{K}} \left[ \sum_{i \in \mathcal{O}^k} \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{ij}^k} c_{ij}^{kmth} x_{ij}^{kmth} + \sum_{w \in \mathcal{W}^k} g_w^{kth} I_w^{kth} \right] \quad (19.21)$$

subject to

$$\sum_{i \in \mathcal{O}^\ell} \sum_{m \in \mathcal{M}_{ip}^\ell} x_{ip}^{\ell mth} - \sum_{k \in \mathcal{K}^\ell} \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{pj}^k} b^{k\ell} x_{pj}^{kmth} = 0 \quad \ell \in \mathcal{R} \cup \mathcal{A}; p \in \mathcal{P}^\ell; t \in \mathcal{T} \quad (19.22)$$

$$\sum_{i \in \mathcal{O}^k} \sum_{m \in \mathcal{M}_{iw}^k} x_{iw}^{kmth} - \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{wj}^k} x_{wj}^{kmth} + I_w^{k,t-1,h} - I_w^{kth} = 0 \quad k \in \mathcal{K}; w \in \mathcal{W}^k; t \in \mathcal{T} \quad (19.23)$$

$$\sum_{i \in \mathcal{O}^f} \sum_{m \in \mathcal{M}_{ic}^f} x_{ic}^{fmth} = d_c^{fmth} \quad f \in \mathcal{F}; c \in \mathcal{C}^f; t \in \mathcal{T} \quad (19.24)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{ij}^k} u_i^k x_{ij}^{kmth} - q_i^h \bar{y}_i \leq 0 \quad i \in \mathcal{O}; t \in \mathcal{T} \quad (19.25)$$

$$\sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{ij}^k} x_{ij}^{kmth} - q_i^{kh} \bar{v}_i^k \leq 0 \quad k \in \mathcal{K}; i \in \mathcal{O}^k; t \in \mathcal{T} \quad (19.26)$$

$$\sum_{m \in \mathcal{M}_{ij}^k} x_{ij}^{kmth} - q_{ij}^{kh} \bar{w}_{ij}^k \leq 0 \quad k \in \mathcal{K}; i \in \mathcal{O}^k; j \in \mathcal{D}^k; t \in \mathcal{T} \quad (19.27)$$

$$\sum_{k \in \mathcal{K}} u^{km} x_{ij}^{kmth} - q_{ij}^{mh} z_{ij}^m \leq 0 \quad i \in \mathcal{O}; j \in \mathcal{D}; m \in \mathcal{M}_{ij}; t \in \mathcal{T}. \quad (19.28)$$

In the presence of uncertainty, it may not be possible to fully satisfy the customer demand under every scenario. To ensure that the second stage problem is always feasible, one may introduce in the demand constraints (19.24) recourse variables  $a_c^{fth}$  representing the amount of demand not satisfied for customer  $c$  and product  $f$  in period  $t$  under scenario  $h$ . These variables can be appended to the objective function with a recourse cost of  $e_c^{fth}$  per unit to represent the cost of the recourse needed when demand cannot be fully satisfied. Alternatively, one may introduce recourse variables in the capacity constraints if the demand should always be satisfied in full but possible recourse actions consist in acquiring extra capacity through subcontracting, overtime or any other means.

### 3.2 Robust Optimization

The above approach can be used to deal with general forms of uncertainty concerning demand, cost and capacity parameters. It assumes, however, that information is available on the likelihood of each scenario and that planners are risk-neutral. Under these assumptions, minimizing the expected cost is perfectly reasonable. In practice, however, decision-makers are often risk-averse and they may care more about the worst-case cost than the expected cost. In addition, rare events such as natural disasters or terrorist attacks do not have well-defined probabilities of occurrence. Even if they do, the probabilities are usually very small and will not have any real impact on the optimal solution to the problem although the corresponding events may have dramatic consequences.

One way to overcome these limitations of stochastic programming is to use robust optimization. In classical robust optimization, one is interested in finding a solution that minimizes the cost under the worst possible scenario. This leads to a min-max objective function, which is often seen as too pessimistic because it assumes that all uncertain parameters can take their worst possible value at the same time. An interesting alternative to worst-case optimization is the “budget-of-uncertainty” approach of Bertsimas and Sim (2004). This approach assumes that the number of uncertain parameters that can deviate from their nominal value or the sum of these deviations is bounded from above by a value known as the budget of uncertainty. This approach generally leads to more balanced solutions that are less sensitive to extreme scenarios. However, it still puts the focus on the worst-case at the expense of average performance.

A possibly more appropriate approach in practice is to combine the idea of average cost optimization with some notion of robust optimization in the same model. For example, risk aversion can be taken into account by adding to the



objective function of the two-stage stochastic program an extra term measuring the worst-case cost with respect to the different possible scenarios. The problem then becomes parametric because one must define the relative weight of this extra term in the objective function.

Let  $\mathbb{D}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)]$  denote a risk measure that the decision-makers want to consider in the selection process of robust solutions. Shapiro et al. (2009) identified a set of coherent risk measures that satisfy a number of convexity, monotonicity, translation equivalence, and positive homogeneity properties in stochastic programming. For instance the mean absolute upper semi-deviation from the mean is a common downside risk measure which can be written as follows:

$$\mathbb{D}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)] = \mathbb{E}_{\mathcal{H}} [(C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h) - \mathbb{E}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)])^+], \quad (19.29)$$

where  $(x)^+ = \max\{0, x\}$ . Alternatively, a worst case measure is given by the maximum upper semi-deviation:

$$\mathbb{D}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)] = \max_{\mathcal{H}} [(C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h) - \mathbb{E}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)])^+]. \quad (19.30)$$

With either of these measures, one can transform the objective function (19.21) into the following weighted sum:

$$\text{Minimize } \mathbb{E}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)] + \omega \mathbb{D}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)], \quad (19.31)$$

where  $\omega \geq 0$  is a scaling parameter.

Finally, one can also use the risk measures (19.29) or (19.30) to impose a constraint on risk inside a stochastic program. This is achieved by adding the constraint

$$\mathbb{D}_{\mathcal{H}} [C(\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{w}, h)] \leq \omega^0, \quad (19.32)$$

where  $\omega^0$  corresponds to the upper bound on risk tolerance.

## 4 Reverse Logistics, Environmental Aspects and Sustainability

While traditional logistics network design has focused on forward logistics, i.e., the movement of goods from suppliers to end customers, there is a growing interest in both the scientific literature and the industry for the design of reverse logistics networks to manage upstream flows from end customers back to the plants and even to the suppliers. The logistics network for recovery and revalorization of used products can take several forms, depending on the industrial context. This could be a network composed by a second-hand market independent of the original equipment

manufacturer (OEM) or an internal network based on demand for parts or used products by the OEM's manufacturing or remanufacturing activities. We note that when forward and reverse flows are coupled the resulting problem is referred to as a closed-loop logistics network design problem (Akçalı et al. 2009; Easwaran and Uster 2010).

The decisions related to the design of a reverse logistics network may involve the determination of the optimal locations and capacities of collection centers, inspection centers, remanufacturing facilities, and recycling plants in addition to the optimal shipment strategies between these facilities. There usually are various options available including repair, refurbishing, and recycling of products as well as alternatives such as inspection, disassembly, disposal, or selling to suppliers, to the secondary market or to external remanufacturing facilities. Different actors and facilities are also involved in reverse logistics networks, e.g., disposers, remanufacturers, and the secondary market. Moreover, unlike forward networks, which are mostly driven by economic considerations, there are further factors motivating the establishment of reverse logistics networks such as environmental laws and regulations (Mota et al. 2014). Finally, uncertainty is also prevalent because the supply of returned products is often highly unpredictable. Hence, reverse LNDPs are usually quite complex.

Items that are shipped in a reverse logistics network include used, repaired, or refurbished products, as well as components or raw materials of such products. The set of items  $\mathcal{K}$  must thus account for different states (used, repaired, refurbished) of the same product. The transitions between the stages of products at nodes of the network as well as the reverse bills of materials need to be considered when modeling these problems. The most important type of constraints in reverse logistics network design models is the flow balance constraints. Flow balance needs to account for the total amount of products recovered at a location as well as the transition between different states of the product through various recovery options. For example, a used product may turn into a refurbished product at a remanufacturing facility. Another important issue to consider within the flow balance constraints of a reverse logistics network design model is the reverse bills of materials. A product may be decomposed into its components at a disassembly facility. Let  $\delta^{\ell k}$  denote the amount of item  $k$  obtained by recovering one unit of item  $\ell$ . One needs to define a new decision variable  $r_p^{kt}$  representing the amount of item  $k$  recovered at location  $p$  in period  $t$ . Assuming that recovery takes one period of time, the flow conservation constraint for item  $k$  at recovery plant  $p$  in period  $t$  can then be formulated as follows:

$$\sum_{i \in \mathcal{O}^k} \sum_{m \in \mathcal{M}_{ip}^k} x_{ip}^{kmt} - \sum_{j \in \mathcal{D}^k} \sum_{m \in \mathcal{M}_{pj}^k} x_{pj}^{kmt} + \sum_{\ell \in \mathcal{K}} \delta^{\ell k} r_p^{\ell, t-1} - r_p^{kt} = 0. \quad (19.33)$$

As noted above, the major driving forces in reverse logistics networks include not only economic factors, but also legislation and environmental consciousness.

Hence, there can be constraints associated with recovery targets. An example of such a constraint would be:

$$\sum_{p \in \mathcal{P}} \sum_{\tau=1}^t r_p^{k\tau} \geq RT^{kt} \quad k \in \mathcal{K}; t \in \mathcal{T}, \quad (19.34)$$

where  $RT^{kt}$  denotes the recovery target of item  $k$  up until the end of period  $t$ . This constraint forces the recovery target of each item to be met by the end of each period.

In terms of the objective function, it is common to consider profit maximization in reverse LNDPs rather than cost minimization. As noted in Alumur et al. (2012), a company could fully outsource its reverse logistics operations if the only motivation is to satisfy legislation or regulations. Moreover, there are usually multiple actors involved in the design and operation of a reverse logistics network in addition to those involved in a forward network. These multiple stakeholders include producers, distributors, third-party logistics providers, disposers, and municipalities. Multiple actors may obviously lead to multi-objective decision problems.

In practice, the estimation of greenhouse gas (GHG) emissions may be difficult because they cover upstream and downstream activities that are not under the control of a single company. For GHG, the common measure is the CO<sub>2</sub>-equivalent emissions (in kgCO<sub>2</sub>e/unit load) that could apply to inbound flows associated to suppliers and to outbound flows to include transportation emissions to customers. Accordingly, there is a growing preoccupation for the incorporation of environmental constraints in the design of forward and reverse logistics networks (Mota et al. 2014). Certain types of constraints can be imposed easily in the LNDP. For example, a limit on CO<sub>2</sub> emissions can be treated by imposing aggregate constraints on the sum of flows in the network. If  $e_i^k$  and  $e_{ij}^{km}$  denote the emissions produced by the flow (e.g., the production or storage) of one unit of item  $k$  at node  $i$  and the transportation of one unit of item  $k$  from node  $i$  to node  $j$  with mode  $m$ , then an upper bound  $E_t$  on total emissions in period  $t$  can be imposed with the following constraint:

$$\sum_{i \in \mathcal{O}} \sum_{j \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}_{ij}^k} (e_i^k + e_{ij}^{km}) x_{ij}^{kmt} \leq E_t. \quad (19.35)$$

In addition to environmental efficiency, there is a growing focus on social sustainability (Tang and Zhou 2012; Giusti et al. 2019). However, characterizing and measuring social well-being in logistics is still in its infancy. Employment is often cited as the main social indicator in connection with regional development (Mota et al. 2014). Other social objectives can take the form of an equity concern related to space, returns, or production factors and can be expressed in the LNDP as equity constraints in terms of lower and upper bounds.

## 5 Solution Methods

Because the LNDP is usually formulated as a mixed-integer program (MIP), it is often solved by general-purpose branch-and-bound or branch-and-cut solvers. However, the formulations are sometimes very large and solving them to near-optimality can be a challenge, even for state-of-the-art solvers. Hence, decomposition methods are often used to separate the problem into smaller and more tractable components. Several heuristic algorithms have also been developed to identify good solutions in reasonable time. This section provides an overview of the main solution methods.

### 5.1 Exact Algorithms

MIP formulations such as the one provided in Sect. 2.2 can be solved successfully by branch-and-cut for moderate size instances. However, like many network design problems, these formulations tend to have large integrality gaps caused by the presence of fixed costs associated with the binary design decisions. When facilities or arcs in the network are capacitated, linear programming (LP) solutions tend to be very fractional and a significant amount of branching is required to reach an optimal integer solution. The performance of branch-and-bound algorithms can be improved by strengthening the LP relaxations through the addition of valid inequalities, either directly in the formulation, or in the form of cuts in a branch-and-cut algorithm. General families of inequalities, such as cutset inequalities, can often be used directly or can be adapted to the special network structure considered. For example, the simple inequalities  $v_i^k \leq y_i, \forall i \in \mathcal{O}$ , have been shown to considerably strengthen the LP relaxation of formulation (1)–(8). Cordeau et al. (2006) provide several other families of valid inequalities. Even with the addition of valid inequalities, large-scale instances of the LNDP can remain formidably difficult to solve. Hence, several authors have turned to decomposition methods.

#### 5.1.1 Lagrangian Relaxation

If one relaxes demand and flow conservations (2)–(4), the resulting subproblem becomes separable by origin node. This relaxation has been successfully exploited by some authors. For example, Pirkul and Jayaraman (1998) observed that by relaxing the demand constraints and the flow conservation constraints at the warehouses, their formulation decomposed into a set of independent continuous knapsack problems, one for each warehouse and each plant. In the same way, Hinojosa et al. (2008) relaxed the demand and flow conservations constraints connecting the distribution levels in a two-echelon warehouse location model with multiple commodities. The resulting subproblem decomposes by echelon, by facility and by time period. More recently, Pimentel et al. (2013) used a Lagrangian

heuristic procedure to solve a stochastic LNDP, whereas Badri et al. (2013) used Lagrangian relaxation for a deterministic variant. In general, the weakness of these approaches is that they rarely provide feasible integer solutions. Hence, Lagrangian heuristics are usually necessary to produce good feasible solutions.

### 5.1.2 Benders Decomposition

The most popular decomposition approach for the LNDP is Benders decomposition. By keeping all binary design decisions in the master problem, one obtains a linear and continuous subproblem which usually takes the form of a capacitated multicommodity network flow problem. Although this problem is not separable by commodity, it is nevertheless much easier to solve than the original formulation. When the number of binary design decisions is small but the number of arcs in the network and the number of commodities is large, this decomposition can be very beneficial. The use of Benders decomposition to solve variants of the LNDP was investigated, among others, by Dogan and Goetschalckx (1999) and by Cordeau et al. (2006). Santoso et al. (2005) considered several acceleration techniques to solve a stochastic variant of the LNDP within a sample average approximation (SAA) framework. In particular, the use of a trust region for the master problem together with knapsack inequalities, cut strengthening and cut disaggregation was shown to improve performance. More recently, Mariel and Minner (2017) also used Benders decomposition in a heuristic way to solve a problem with a bilinear objective function arising in the context of supply chain design under NAFTA local content requirements. It is worth noting that in a scenario-based formulation of the LNDP, the decomposition scheme can decompose the subproblem by scenario and allow the use of parallelism.

## 5.2 Heuristic Algorithms

Several authors have observed that the LP relaxation solution sometimes contains many location variables that naturally take value 0 or 1. This has led to the idea of gradually rounding the LP solution into an integer one. For example, Thanh et al. (2010) solve a sequence of LPs by fixing some binary decisions at each step until all binary variables are integer or the remaining MIP can be solved to optimality. The idea of rounding the LP solution was also used by Melo et al. (2014), who proposed four rounding strategies followed by a local search algorithm to repair infeasibility or improve the resulting integer solution.

An interesting idea when designing heuristics for LNDPs is to explore the space of the integer design variables while relying on a general-purpose LP solver to set the values of the continuous variables to their optimal values. This idea was exploited, for example, by Melo et al. (2012) who use tabu search for a dynamic facility location problem. Their heuristic explores the space of the binary facility

location variables while the remaining (continuous) variables are set by solving an LP. A rounding procedure is used to compute an initial solution and infeasible solutions that violate the budget constraints are allowed during the search. The neighborhood considered consists of solutions obtained by changing the status of a single facility at a time. The idea was also used by Carle et al. (2012) who described an agent-based metaheuristic for multi-period LNDPs with an explicit calculation of inventory levels. This metaheuristic combines tabu search procedures with iterated local search and mixed-integer programming to separately and iteratively optimize different components of the problem. Cordeau et al. (2008) devised an iterated local search heuristic for a special case of the problem with single assignment. Under single assignment constraints, the problem becomes purely combinatorial in nature and it can be stated directly with just the binary variables. Hence, the impact of opening or closing a facility or of changing the product assignment can be easily computed, which allows a fast exploration of a solution's neighborhood. Finally, an interesting approach is to take advantage of the strategic-tactical structure of the LNDP model to devise a hierarchical or nested heuristic approach. This allows decoupling the location and allocation decisions from the product flows and inventory decisions, and solving the corresponding parts of the problem with an appropriate exact or heuristic algorithm (see, for instance, You and Grossmann 2008 and Klibi et al. 2010a).

## 6 Bibliographical Notes

The design of logistics networks is rooted in discrete facility location (Daskin 2011; Laporte et al. 2016) and most early models were direct extensions of capacitated or uncapacitated facility location problems with fixed costs (Aikens 1985; ReVelle and Eiselt 2005). One of the first papers addressing the design of multicommodity distribution networks is that of Geoffrion and Graves (1974). Over the years, many models have been introduced with a focus on improving the realism and comprehensiveness of the problem setting. In particular, several models have been introduced to combine production and distribution decisions (Cohen and Lee 1989; Vidal and Goetschalckx 1997; Elhedhli and Goffin 2005).

An important area of research concerns the incorporation of international aspects in supply chain design. Arntzen et al. (1995) introduced a formulation capturing duties, duty drawback, local content rules and offset requirements. This was followed by Vidal and Goetschalckx (2001) and Goetschalckx et al. (2002) who took into account issues of transfer pricing. More recently, Mariel and Minner (2017) have modeled the problem of locating plants and planning production and distribution under the North American Free Trade Agreement local content rules.

The most comprehensive and realistic modeling framework described to date in the scientific literature is probably that of Martel (2005). The objective function aims to maximize after tax net revenue by taking into account the impact of delivery times on demand. Complex product structures with arbitrary bills of materials are

considered along with facility layout and capacity options. A detailed representation of inventory levels is embedded in the formulation as well as a rather rich cost structure. Finally, many aspects of transfer prices, taxes, tariffs and duties are taken into account. The resulting formulation contains nonlinear terms in the objective function but it can be solved iteratively by using piecewise linear approximations of these terms that are updated each time the problem is solved.

Another important stream of literature concerns the dynamic location of facilities over a multiple-period planning horizon. Multi-period models were proposed, among others, by Martel (2005), Melo et al. (2006), Thanh et al. (2008), Hinojosa et al. (2008), and Jena et al. (2015). Melo et al. (2006) have introduced a general framework that not only supports facility relocation or shutdown but also capacity expansion and reduction under a budget constraint in each time period. This framework is mostly targeted at distribution network design as it does not consider supplier selection or the transformation of raw materials into finished products. Another multi-period model for supply chain design was introduced by Thanh et al. (2008). Facilities can be opened and closed during the planning horizon and modular capacities are considered for each facility. They also consider both seasonal and cyclic stocks. Finally, the work of Klibi and Martel (2013) provides a modeling framework for logistics network design under uncertainty. The problem is cast as a two-level organizational decision (strategic–operational) and is characterized by multiple design cycles and multiple planning periods.

Beside time aspects, the treatment of risk and uncertainty related to logistics networks is of inherent importance for realistic models (Dunke et al. 2018). Surprisingly the literature lacks a clear distinction between the notion of risk and uncertainty. An in-depth study of the notion of supply chain risk can be found in Heckmann et al. (2015). An approach for modelling risk and for the generation of scenarios in the LNDP can be found in Klibi and Martel (2012). Finally, in Heckmann and Nickel (2017) a more general discussion of common flaws in supply chain risk analysis is presented.

The application of robust optimization techniques to LNDPs is still a relatively new area of research. An interesting comparison and application of different formulations to a case study can be found in Govindan and Fattahi (2017). Robust optimization approaches were proposed to deal with simple location problems (Snyder and Daskin 2006). We refer to Klibi et al. (2010b) for a broader discussion of robustness issues in the design of logistics networks. With respect to uncertainty, Santoso et al. (2005) and Schutz et al. (2009) proposed to apply the sample average approximation (SAA) method coupled with Benders decomposition and dual decomposition, respectively, to solve stochastic LNDPs. For more details and background on stochastic programming in the context of logistics network design the reader is referred to Correia and Saldanha-da-Gama (2015), Fan et al. (2017), and to Govindan et al. (2017).

There is a quickly growing literature on green logistics network design and the incorporation of sustainability objectives and constraints. Two classes of problems can be distinguished: reverse logistics network and closed-loop logistics network design problems. The recent work of Mota et al. (2014) is linked to the former class

with the establishment of reverse logistics networks including environmental laws and regulations. The work of Chaabane et al. (2012) considers life cycle assessment principles and emission trading schemes to design sustainable logistics network. The work of Akçalı et al. (2009) and Easwaran and Uster (2010) is oriented towards closed-loop network design problems. We also refer to Tang and Zhou (2012) for a discussion of social sustainability and to Garcia and You (2015) for the inclusion of sustainability issues in multi-objective optimization of LNDPs.

## 7 Conclusions and Perspectives

Logistics network design problems are challenging combinatorial optimization problems with widespread applications and a high potential for impact in terms of cost reduction and performance improvement for companies involved in the manufacturing and distribution of goods. The field has attracted a lot of attention in the operations research community and the models and algorithms currently available can solve instances of reasonable size with a sufficient degree of realism. Nevertheless, there still exist many opportunities for improvement.

One of the main challenges is the treatment of uncertainty. Although it is easy to formulate two-stage or multi-stage versions of the LNDP, solving these models remains extremely difficult. Because the problems involve a large number of uncertain parameters, one must consider large sets of scenarios to obtain a sufficient degree of precision in the representation of uncertainty. Decomposition methods such as Benders decomposition can be used to obtain some form of separability but convergence to an optimal solution is usually very slow.

Another area that could largely benefit from additional research is the integration of forward and reverse logistics network design decisions into so-called “closed-loop” supply chains. With environmental regulations becoming more stringent in most countries, there is a growing need to design forward networks that can also handle reverse flows in an effective way from the start. The resulting problems are again highly complex and difficult to solve.

Finally, we would like to point out that the growing importance of on-line commerce is slowly changing the way logistics networks are designed. The focus is now placed less on cost minimization through economies of scale and more on revenue maximization through improved quality of service to the end customers. The reorganization of distribution, in the retail sector in particular, gives rise to multi-channel distribution network structures and to a mix between fulfillment and storage strategies.



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