2

Competition for Market Share and for Market Size

In this chapter, we build on the ideas developed in Chap. 1 to formulate a more general model, although portraying the simplified economy imagined by Dixit and Stiglitz (1977), an economy reduced to two sectors, one oligopolistic, the other competitive, with firms selling each a single good to a representative consumer. In the following, we will first present the canonical model and then explore several extensions. Assuming either weak or homothetic separability, we define a general concept of oligopolistic equilibrium. The first-order conditions are used to derive a simple formula where the relative markup is a function of the intra- and intersectoral elasticities of substitution. This leads to a parameterisation of equilibria in terms of firms' competitive toughnesses defining possible regimes of oligopolistic competition. It is then shown that such a formula is robust to supposing that firms take into account the income feedback effects of distributed income (the so-called Ford effects). In the homogeneous good case, we compare our approach to alternative ones, such as the conjectural variation and the supply function approaches. Finally, the methodology is applied to two policy questions: Is tougher competition price decreasing? Does it foster innovation?

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Before providing the technical details, we want to emphasise that the two-sector economy is the essential basis for the canonical model in which we introduce our oligopolistic equilibrium concept. In this economy the fundamental feature is that the representative consumer preferences are assumed weakly separable relative to the two sectors. The differentiated goods produced in the oligopolistic sector can thus be aggregated into a composite good (through the sub-utility function defined on these goods) and the aggregate good produced in the competitive sector, representing the rest of the economy, is taken as numeraire. Applying two-stage budgeting, one obtains the main ingredients to be used in our own setting: (1) within the oligopolistic sector, the Hicksian demand for each differentiated good and the corresponding intrasectoral elasticity of substitution of each such good for the composite good and (2) across sectors, the Marshallian demand for the composite good and the corresponding intersectoral elasticity of substitution of each good for aggregate consumption.

An oligopolistic equilibrium supposes that each firm in the oligopolistic sector maximises profit (allowing for no-production) in both price and quantity, under two constraints, on market share (depending upon Hicksian demand) and on market size (depending upon Marshallian demand). It is from the first-order conditions that we get the basic equations that should be used for estimation purposes. These equations determine the equilibrium markup of each firm as a weighted mean of the reciprocals of the two elasticities of substitution. The crucial advantage is that the weights explicitly involve the firm's "competitive toughness" as a continuous parameter (varying between 0 and 1) derived from the Lagrange multipliers associated with the two constraints. By varying the competitive toughness parameters we hence get a continuum of regimes of competition, including standard ones such as tacit collusion, pure price and pure quantity equilibria.

Assuming symmetry and negligible market shares, we retrieve the Dixit-Stiglitz monopolistic competition equilibrium, with the equilibrium markup just equal to the reciprocal of the intrasectoral elasticity of substitution, simplicity being however counterbalanced by the loss of any intersectoral effects. As we will see, the same reduced form can be obtained, even with large firms, via cutthroat competition (maximal competitive toughness), an illustration of the so-called Bertrand paradox. Competitive toughness is thus an important factor to explain firm heterogeneity. In a recent empirical study, Hottman et al. (2016) observe that firms with the largest market shares have "substantially higher markups" and that this effect is much greater under quantity competition than under price competition.[1] Firm heterogeneity is thus reinforced by softer competition. However, from our point of view, this statement should not be reduced to the Cournot-Bertrand dichotomy, but apply to the continuum of possible competition regimes between the two extremes of cutthroat competition and pure collusion.

The existence of large firms implies that these firms influence the size of their own market through the income they distribute. That such income feedback effects can be taken into account by large firms was already well illustrated by the industrialist Henry Ford in the 1920s when advocating a high wage policy: "Our own sales depend in a measure upon the wages we pay" (Ford 1922, p. 124). This kind of effects can be integrated into our canonical model. In particular, in the case where the firms internalise the income feedback effect within the oligopolistic sector, taking as given the expenditure in the competitive sector, we will show that the equilibrium markup formula will keep the same structure. These "Ford effects" essentially modify the relevant elasticity of intersectoral substitution.

Coming back to the possible continuum of equilibria in oligopolistic markets, we shall mention three previous approaches (the conjectural variation, the supply function and the pricing scheme approaches) explaining this indeterminateness in the special case where all firms produce the same homogeneous good. These alternative approaches have been useful in different contexts, for example in the studies of the New Empirical Industrial Organisation (NEIO) for the conjectural variations approach, in the analysis of electricity markets for the supply function equilibrium approach, and to model facilitating practices for the pricing schemes

[1] To quote: "In most sectors, the largest firm has a market share above 20%, which enables it to charge a markup that is 24% higher than that of the median firm under price competition and double that under quantity competition." (Hottman et al., 2016, p.5).

approach. We will show that by restricting properly the admissible class of the corresponding instruments (conjectural variations, supply functions or pricing schemes), we can recover exactly the set of oligopolistic outcomes.

To end this chapter, we will discuss two important applications. First, to enhance the role of the two elasticities of substitution, we shall examine the price effects of intensifying competition. Both under price and quantity competition, an average markup can be written as a function of the Herfindhal index of concentration. This function is increasing (resp. decreasing) if the intersectoral elasticity of substitution is smaller (resp. larger) than the intrasectoral elasticity of substitution, thus implying a pro-competitive (resp. anti-competitive) effect of abating concentration. Second, we will use our continuous measure of competitive toughness to re-examine the classical debate on the role of competition for firm innovative activity, opposing the Darwinian view for which a competitive firm is forced (or has more incentives, according to Arrow 1962) to innovate than a monopolist, and the Schumpeterian view for which innovation requires some monopoly rent.

1 The Canonical Model

The main idea underlying our model is that competition has essentially two dimensions: a dimension of conflicting interests of firms fighting against each other for their market shares and a dimension of convergent interests of firms implicitly competing together, against the other sector, for their market size.

A Representative Consumer with General Separable Preferences

Assuming existence of a representative consumer with preferences separable with respect to the two composite goods supplied by the two sectors allows to use the standard analytical framework for consumption decisions and in particular to exploit duality, a very convenient property for the study of price-quantity competition. We suppose that the representative consumer supplies inelastically L units of labour at a wage equal to 1 (the labour productivity in the competitive sector) and receives a profit Π from the imperfectly competitive sector (the equilibrium profit of the other sector being necessarily zero). He chooses a basket $\mathbf{x} \in [0, \infty)^N$ of N differentiated goods (sold at prices $\mathbf{p} \in (0, \infty]^N$) and a quantity $z \in [0, \infty)$ of the numeraire good (an *implicitly* composite good resulting from the aggregation of the rest of the economy). This choice is made so as to maximise, with an income $Y = L + \Pi$ and under the budget constraint $\mathbf{px} + z \leq Y$, a separable utility function $U(X(\mathbf{x}), z)$. The utility function U and the sub-utility function X, which aggregates the quantities of the differentiated goods into the volume of a composite good, are assumed increasing and strongly quasi-concave (except, for X, in the linear and Leontief limit cases and, for U, in the case of quasilinearity[2] in z). Notice that, apart from standard properties of the utility function and from separability, essential for a Dixit-Stiglitz economy, we are not imposing homotheticity, additivity or symmetry to the aggregator function X.

The maximisation can be performed in two stages, to which correspond, as we will see, the two mentioned dimensions of competition. At the first stage, the consumer chooses the quantity x_i of each differentiated good i given some quantity $\underline{X}$ of the composite good (some level of sub-utility X), by solving the programme

$$\min_{x \in \mathbb{R}_+^N} \left\{ \mathbf{px} \,\middle|\, X(\mathbf{x}) \geq \underline{X} \right\} \equiv e\left(\mathbf{p}, \underline{X}\right), \tag{1.1}$$

which defines the expenditure function e. We obtain:

$$p_i = \partial_X e\left(\mathbf{p}, \underline{X}\right) \partial_i X(\mathbf{x}) \qquad \text{(first-order condition)} \tag{1.2}$$

$$x_i = \partial_{p_i} e\left(\mathbf{p}, \underline{X}\right) \equiv H_i\left(\mathbf{p}, \underline{X}\right) \qquad \text{(Shephard's lemma)}, \tag{1.3}$$

[2]This is the special case where $U(X(\mathbf{x}), z) = \hat{U}(X(\mathbf{x})) + z$. It is the one considered by Spence (1976) seminal contribution.

where H_i is the *Hicksian demand* function for good i (associated with sub-utility X).

Using these two equations, we can compute the *intrasectoral elasticity of substitution* s_i of good i for the composite good, that is, the absolute value of the elasticity of x_i/X with respect to the relative price p_i/P (where X and P are the quantity and price of the composite good).[3] This computation may be alternatively performed in terms of quantities or in terms of prices, by taking respectively $X = X(\mathbf{x})$ and $p_i/P = \partial_i X(\mathbf{x})$, the marginal rate of substitution of x_i for X, or by taking $x_i = H_i\left(\mathbf{p}, \underline{X}\right)$ and $P = \partial_X e\left(\mathbf{p}, \underline{X}\right)$, the shadow price of $\underline{X}$. We thus obtain two equivalent formulas:

$$s_i = \frac{1 - \epsilon_i X(\mathbf{x})}{-\epsilon_i\left(\partial_i X(\mathbf{x})\right)} = \frac{-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)}{1 - \left[\epsilon_i X(\mathbf{x})\right]\left[\epsilon_X H_i\left(\mathbf{p}, \underline{X}\right)\right]}. \tag{1.4}$$

At the second stage, the consumer chooses the quantities $\underline{X}$ of the composite good and z of the numeraire good by solving the programme

$$\max_{\left(\underline{X}, z\right) \in \mathbb{R}_+^2} \left\{U\left(\underline{X}, z\right) \middle| e\left(\mathbf{p}, \underline{X}\right) + z \leq Y\right\}. \tag{1.5}$$

The solution to this programme determines the *Marshallian demand* $\underline{X} = D\left(\mathbf{p}, Y\right)$ for the composite good and the demand $z = Y - e\left(\mathbf{p}, D\left(\mathbf{p}, Y\right)\right)$ for the numeraire good. We can then define the *inter-sectoral elasticity of substitution* of good i as the elasticity of substitution

[3] In the standard use of the concept and from the point of view of good i, it is its substitutability with respect to some other good j rather than to the composite good that is considered. When the substitutability differs among pairs of goods, conventional elasticities must be averaged (see, e.g. Bertoletti and Etro 2018, who use averages of the Morishima elasticticities of substitution and complementarity). We avoid introducing arbitrary averages by directly referring to substitutability with respect to the composite good. The relation between our concept and the conventional one is examined in the appendix of d'Aspremont and Dos Santos Ferreira (2016).

σ_i of good i for the aggregate consumption Y in the whole economy:

$$\sigma_i \equiv -\left.\frac{d\left(x_i/Y\right)}{d\left(p_i/1\right)}\right|_{X(\mathbf{x})=D(\mathbf{p},Y)} \frac{p_i/1}{x_i/Y}$$
$$= -\frac{(1/Y)\,\partial_{p_i} D\left(\mathbf{p}, Y\right)}{\partial_i X\left(\mathbf{x}\right)} \frac{p_i}{x_i/Y} = \frac{-\epsilon_{p_i} D\left(\mathbf{p}, Y\right)}{\epsilon_i X\left(\mathbf{x}\right)}. \quad (1.6)$$

In the computation of σ_i we are taking into account the variation of the Marshallian demand $D\left(\mathbf{p}, Y\right)$ rather than the mere share adjustment expressed in the elasticity of the Hicksian demand $-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)$.

A stronger form of separability of the utility function, *homothetic separability*, applying when the aggregator X is homothetic (more specifically, homogeneous of degree one, without loss of generality), simplifies computations and allows to further exploit duality. In this case, the expenditure function and, obviously, the Hicksian demand function are linear in $\underline{X}$: $e\left(\mathbf{p}, \underline{X}\right) = P\left(\mathbf{p}\right)\underline{X}$ and $H_i\left(\mathbf{p}, \underline{X}\right) = \partial_i P\left(\mathbf{p}\right)\underline{X}$, where P is a *price aggregator* function, and the Marshallian demand function becomes homothetically separable: $D\left(\mathbf{p}, Y\right) = \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)$. As a consequence, we obtain dual, perfectly symmetric, expressions for the first-order condition (1.2) and for Shephard's lemma (1.3):

$$p_i = P\left(\mathbf{p}\right)\partial_i X\left(\mathbf{x}\right) \text{ and } x_i = X\left(\mathbf{x}\right)\partial_i P\left(\mathbf{p}\right), \quad (1.7)$$

respectively. Also, for cost minimising consumption bundles, the budget share of good i is equal to the elasticity of anyone of the two aggregator functions:

$$\frac{p_i x_i}{P\left(\mathbf{p}\right) X\left(\mathbf{x}\right)} = \epsilon_i P\left(\mathbf{p}\right) = \epsilon_i X\left(\mathbf{x}\right) \equiv \alpha_i. \quad (1.8)$$

Finally, the price formula for the intrasectoral elasticity of substitution is then symmetric with respect to the quantity formula:

$$s_i = \frac{1 - \epsilon_i X\left(\mathbf{x}\right)}{-\epsilon_i\left(\partial_i X\left(\mathbf{x}\right)\right)} = \frac{-\epsilon_i\left(\partial_i P\left(\mathbf{p}\right)\right)}{1 - \epsilon_i P\left(\mathbf{p}\right)}, \quad (1.9)$$

and the intersectoral elasticity of substitution is just equal to the demand elasticity, now the same for any differentiated good:

$$\sigma_i \equiv -\epsilon_i D\left(\mathbf{p}, Y\right)/\alpha_i = -\epsilon_P \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)\epsilon_i P\left(\mathbf{p}\right)/\alpha_i = \sigma.$$

Firms' Competitive Behaviour and Oligopolistic Equilibria

We consider competition among N firms, each firm i producing a single component of the composite good with a constant positive unit cost c_i and a non-negative fixed cost ϕ_i incurred only when production is positive.[4] Firms behave strategically in price-quantity pairs: $(p_i, x_i) \in \mathbb{R}_+^2$ for each firm $i = 1, \ldots, N$. These pairs have to satisfy two admissibility constraints, generalising the two constraints as specified in Chap. 1.

The first is a *constraint on market share*, focusing on competition within the sector which produces the differentiated goods and referring to the first stage of the consumer's utility maximisation. It bounds the quantity of good i by the corresponding Hicksian demand:

$$x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}\right), X\left(x_i, \mathbf{x}_{-i}\right)\right). \tag{1.10}$$

The second is a *constraint on market size*, focusing on competition of the whole set of producers of the differentiated goods with the sector which produces the numeraire good. It refers to the second stage of the consumer's utility maximisation, and bounds the size of the market for the differentiated goods by the Marshallian demand:

$$X\left(x_i, \mathbf{x}_{-i}\right) \leq D\left(\left(p_i, \mathbf{p}_{-i}\right), Y\right). \tag{1.11}$$

[4] We assume positivity of unit costs for *all* firms to keep the exposition simple. The case of zero unit costs has already been examined in Chap. 1. The concept of oligopolistic equilibrium has been introduced in d'Aspremont et al. (2007), and further explored in d'Aspremont and Dos Santos Ferreira (2009, 2010, 2016).

We recall that the constraint on market share emphasises the conflictual side of competition between the oligopolists, whereas the constraint on market size translates their common interest as a sector.

We define the concept of oligopolistic equilibrium.

Definition 1 An *oligopolistic equilibrium* is a $2N$-tuple $\left(p_i^*, x_i^*\right)_{i=1,\ldots,N} \in \overline{\mathbb{R}}_+^{2N}$ such that, for any i,

$$\begin{aligned} \left(p_i^*, x_i^*\right) \in \arg & \max_{(p_i, x_i) \in \mathbb{R}_+^2} \left(p_i - c_i\right) x_i \\ \text{s.t.} \qquad & x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and } X\left(x_i, \mathbf{x}_{-i}^*\right) & \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right), \end{aligned} \tag{1.12}$$

and such that $Y^* = L + \sum_{i=1}^N \left(\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right) \phi_i\right)$. In addition, we require the profits to be non-negative, namely $\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right) \phi_i \geq 0$ for each i, and the consumer to be non-rationed.

Non-rationing of the consumer implies that both constraints are satisfied as equalities for each firm i at equilibrium. It makes the equilibrium compatible with the consumer's programme and the resulting demand functions. Notice that, according to this definition, all firms are not necessarily active in an oligopolistic equilibrium.[5] We shall in general assume that n firms are active, each one i choosing a positive strategy $\left(p_i^*, x_i^*\right)$, and that $N - n$ firms are inactive, choosing each a strategy $(\infty, 0)$. Of course, inactive firms are also maximising profits at equilibrium: no admissible strategy would allow them to obtain a positive profit. As the fixed cost is incurred only at a positive output, choosing a zero output is a way to ensure that the profit is at least non-negative. The price strategy is then arbitrary. We suppose it to be infinite in order to avoid consumer rationing. As already discussed in Chap. 1, existence

[5] Inactive firms do play a role though. Shubik (1959) suggests to call such firms "firms in being" by analogy to the famous term "fleet in being," introduced by Lord Torrington in 1690 and used by Kipling.

of inactive firms at equilibrium allows to qualify that equilibrium as a free entry equilibrium, but only if the inactive firms have the same opportunities as the active ones, which imposes the oligopoly game to be symmetric, a restrictive assumption that we are not making in general.

We next show that an oligopolistic equilibrium can be characterised by a simple expression for individual (relative) *markups* (or *Lerner indices* of the degree of monopoly power) at that equilibrium, that is, $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ for each active firm i. This markup is derived from the first-order conditions of producer i's programme in Definition 1.[6] To obtain that simple expression, we refer to the intra- and intersectoral elasticities of substitution of good i, s_i^* and σ_i^* respectively, again at the considered equilibrium, and we introduce in addition simplifying notations for two additional elasticities. The elasticity $\alpha_i \equiv \epsilon_i X(\mathbf{x})$ measures the impact of a variation in the quantity of good i on the volume of the composite good. The elasticity $\beta_i \equiv \epsilon_X H_i\left(\mathbf{p}, \underline{X}\right)$ measures the reverse impact of a variation of the quantity of the composite good on the demand for its component i, at given prices $\mathbf{p}$. The product of these two elasticities, which appears in the multiplier $1/\left(1 - \alpha_i \beta_i\right)$ applied to the elasticity $-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)$ of the Hicksian demand in the price formula for s_i, Eq. (1.4), measures the intensity of the feedback originating in a variation of the quantity of good i and going through the volume of the composite good.

For easier reference, we recall the expressions for these four elasticities in Table 2.1:

Table 2.1 Elasticities appearing in the markup formula

Intrasectoral substitution:	$s_i = \frac{1-\alpha_i}{-\epsilon_i \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, X(\mathbf{x}))}{1-\alpha_i \beta_i}$
Intersectoral substitution:	$\sigma_i = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\alpha_i}$
Impact of x_i on X	$\alpha_i \equiv \epsilon_i X(\mathbf{x})$
Impact of X on x_i	$\beta_i \equiv \epsilon_X H_i(\mathbf{p}, X(\mathbf{x}))$

[6] Reference to the markup μ_i^* replaces the direct reference to the price p_i^* used in Chap. 1 for the case of zero unit costs (a case in which the Lerner index is always equal to 1).

Take an oligopolistic equilibrium $\left(\left(p_i^*, x_i^*\right)_{i=1,\ldots,n}, (\infty, 0)^{N-n}\right)$ (with positive prices and quantities for the first n firms), henceforth denoted $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} = \left(\mathbf{p}^*, \mathbf{x}^*\right)$ for simplicity. In the rest of our book, we shall often resort to this abusive simplifying notation referring to the sole active firms. The markup of active firm i at this equilibrium will be expressed, according to the following proposition, as a weighted mean of the reciprocals of the two elasticities of substitution s_i^* and σ_i^* at that equilibrium. The corresponding weights will involve, for each firm i, the elasticities α_i^* and β_i^* measuring the two reciprocal effects of quantity variations of good i and of the composite good, as well as a conduct parameter[7] $\theta_i^* \in [0, 1]$, stemming from the first-order conditions and interpreted as the *competitive toughness* displayed by firm i towards its rival oligopolists at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$.

Proposition 4 *Let $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ be an oligopolistic equilibrium. Then the markup $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ of each firm i is given by*

$$\mu_i^* = \frac{\theta_i^*\left(1 - \alpha_i^*\beta_i^*\right) + \left(1 - \theta_i^*\right)\alpha_i^*}{\theta_i^*\left(1 - \alpha_i^*\beta_i^*\right)s_i^* + \left(1 - \theta_i^*\right)\alpha_i^*\sigma_i^*}, \tag{1.13}$$

for some $\theta_i^* \in [0, 1]$.

Proof We start by making dimensionally homogeneous the two constraints in the programme of firm i, rewriting them in terms of the two ratios:

$$\frac{x_i}{H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right)} \leq 1 \text{ and } \frac{X\left(x_i, \mathbf{x}_{-i}^*\right)}{D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right)} \leq 1. \tag{1.14}$$

[7] To use the terminology of the New Empirical Industrial Organization (see Bresnahan, 1989).

The first-order necessary conditions for profit maximisation at $\left(p_i^*, x_i^*\right)$ under these two constraints (holding as equalities at equilibrium) can then be expressed, for non-negative Lagrange multipliers λ_i^* and ν_i^*, as

$$\begin{aligned} x_i^* &= \lambda_i^* \frac{-\partial_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} + \nu_i^* \frac{-\partial_{p_i} D\left(\mathbf{p}^*, Y^*\right)}{D\left(\mathbf{p}^*, Y^*\right)} \\ &= \frac{\lambda_i^*}{p_i^*}\left[-\epsilon_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] + \frac{\nu_i^*}{p_i^*}\left[-\epsilon_{p_i} D\left(\mathbf{p}^*, Y^*\right)\right], \end{aligned} \tag{1.15}$$

and

$$\begin{aligned} p_i^* - c_i &= \lambda_i^* \frac{1 - \left[\partial_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\left[\partial_i X\left(\mathbf{x}^*\right)\right]}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} + \nu_i^* \frac{\partial_i X\left(\mathbf{x}^*\right)}{X\left(\mathbf{x}^*\right)} \\ &= \frac{\lambda_i^*}{x_i^*}\left(1 - \left[\epsilon_i X\left(\mathbf{x}^*\right)\right]\left[\epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\right) + \frac{\nu_i^*}{x_i^*}\epsilon_i X\left(\mathbf{x}^*\right). \end{aligned} \tag{1.16}$$

We can use these two conditions and the notations of Table 2.1 to obtain the markup formula for firm i at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, as given by (1.13), with $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^*\right)$. □

Since λ_i^* is the Lagrange multiplier associated with the constraint on market share, which emphasises the conflictual side of competition between firm i and its rivals in the sector, whereas ν_i^* is the Lagrange multiplier associated with the constraint on market size, which reflects converging interests of the competitors in the sector, the normalised multiplier θ_i^* can be interpreted, as suggested in Chap. 1, as the competitive toughness experienced by firm i at the particular equilibrium $\left(p_j^*, x_j^*\right)_{j=1,\ldots,n}$. Inspection of Eq. (1.13) shows that the weight put on the reciprocal of *intra*sectoral elasticity of substitution s_i^* (relative to the weight put on the reciprocal of its *inter*sectoral homologue σ_i^*) naturally increases with the competitive toughness θ_i^* experienced by firm i at equilibrium.

The equilibrium markup of firm i is a weighted *harmonic* mean of the reciprocals of the two elasticities of substitution s_i^* and σ_i^*. If we assume homothetic separability of the consumer's utility function (not necessarily the CES specification of the aggregator X), we can use, as we did in Sect. 3 of Chap. 1 (Proposition 3), the dual constraints on market share and market size in order to obtain

$$\mu_i^* = \frac{\theta_i'^* \left(1-\alpha_i^*\right)\left(1/s_i^*\right) + \left(1-\theta_i'^*\right)\alpha_i^* \left(1/\sigma^*\right)}{\theta_i'^* \left(1-\alpha_i^*\right) + \left(1-\theta_i'^*\right)\alpha_i^*}, \tag{1.17}$$

where μ_i^* appears as an *arithmetic* mean of the reciprocals of the two elasticities s_i^* and σ^* (the latter uniform for the whole oligopolistic sector because of the homotheticity assumption).[8] The equilibrium is the same (s_i^*, σ^* and the budget share α_i^* are identical, with $\beta_i^* = 1$ by linearity of the Hicksian demand), but the conduct parameters are specific to this dual form, the equilibrium parameterisation differing between the dual forms of the two constraints. By identifying the formula of μ_i^* given by (1.17) and that given by (1.13) (with $\sigma_i^* = \sigma^*$ and $\beta_i^* = 1$), we can easily establish the relation between the two parameters: for $s_i^* \notin \{0, \sigma^*, \infty\}$,

$$\frac{1/\theta_i'^* - 1}{\sigma^*} = \frac{1/\theta_i^* - 1}{s_i^*} \tag{1.18}$$

Regimes of Competition

The vector $\boldsymbol{\theta}^* = \left(\theta_1^*, \ldots, \theta_n^*\right)$ of the competitive toughnesses of the different active firms (or its dual counterpart $\boldsymbol{\theta}'^*$ in the homothetic case) specifies a *regime of competition*, which can be continuously modified by varying this vector in $[0, 1]^n$. Tracing the set of potential equilibria

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[8] Recall that

$$\sigma_i = \frac{-\epsilon_{p_i} D\left(\mathbf{p}, Y\right)}{\alpha_i} = \frac{-\epsilon_{p_i} \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)}{\alpha_i} = \frac{-\epsilon_P \widehat{D} \cdot \epsilon_i P}{\alpha_i} = -\epsilon_P \widehat{D} = \sigma,$$

with σ denoting the elasticity of the Marshallian demand function $\widehat{D}\left(\cdot, Y\right)$, as in Chap. 1.

by varying $\boldsymbol{\theta}^*$ allows us in particular to retrieve standard regimes like price and quantity equilbria ($\theta_i^* = 1/2$ and $\theta_i'^* = 1/2$, respectively, for any active firm i), or the collusive solution ($\boldsymbol{\theta}^* \equiv \mathbf{0}$), although existence of the whole spectrum of potential equilibria (for all values of $\boldsymbol{\theta} \in [0,1]^n$) is generally not satisfied, as already shown in Chap. 1. The markup formula is also useful in limit cases. We shall examine in Sect. 3 the perfect substitutability case ($s_i = \infty$) referring to the dual version (1.17) of the formula. Now, if we assume that preferences are symmetric, that the unit costs are identical for all firms ($c_i = c$ for all i) and that the number of active firms go to infinity, say for a sequence of symmetric oligopolistic equilibria, then the oligopolistic sector becomes "large" in the sense of Chamberlin, meaning that every individual firm becomes negligible ($\alpha_i^* \simeq 0$), and we get what we may call the *Dixit-Stiglitz monopolistic competition equilibrium*, with $\mu_i^* = 1/s_i^*$. The standard case is when the markup remains positive, that is $\lim_{n\to\infty}\left(1/s_i^*\right) > 0$, as it is when the aggregator X is CES with $s_i^* = s > 0$. Then there is another way to obtain the outcome of monopolistic competition, which is to assume that firms' conduct is sufficiently tough ($\theta_i^* \simeq 1$). This holds, even in the "small" group case.[9] However, in the large group case, we get the competitive (Walrasian) equilibrium when $\lim_{n\to\infty}\left(1/s_i^*\right) = 0$.[10]

To summarise, the main competition regimes under homothetic separability can be characterised by the competitive toughness and markup values (applying to all firms) displayed in Table 2.2.

[9] Shimomura and Thisse (2012) introduce a *mixed market* structure. They consider a Dixit-Stiglitz economy where U is Cobb-Douglas and X CES, defined over the union of a discrete set of goods produced by large firms and a continuum of goods produced by small firms. This continuum is a monopolistically competitive fringe, with mass determined by the zero-profit condition, under free entry restricted to the fringe. Quantity competition is also assumed. The resulting mixed market quantity equilibrium outcome can of course be (approximately) obtained within our canonical model, by letting $\alpha_i^* \simeq 0$ iff firm i is small or, alternatively, by letting $\theta_i'^* \simeq 1$ if firm i is small (and $\theta_i'^* = 1/2$ if firm i is large).

[10] As well emphasised in Thisse and Ushchev (2018), this depends on the preferences. Other examples will be given below.

Table 2.2 Competition regimes under homothetic separability

Competition regime	Competitive toughness	Markup
Collusion	$\theta_i = \theta'_i = 0$	$\mu^*_i = 1/\sigma^*$
Quantity competition	$\theta'_i = 1/2$; $\theta_i = 1/\left(1 + s^*_i/\sigma^*\right)$	$\mu^*_i = \left(1 - \alpha^*_i\right)/s^*_i + \alpha^*_i/\sigma^*$
Price competition	$\theta_i = 1/2$; $\theta'_i = 1/\left(1 + \sigma^*/s^*_i\right)$	$\mu^*_i = 1/\left(\left(1 - \alpha^*_i\right)s^*_i + \alpha^*_i\sigma\right)$
Monopolistic competition	$\theta_i = \theta'_i = 1$	$\mu^*_i = 1/s^*_i$
	$\theta_i = \theta'_i > 0$ and $\alpha^*_i \simeq 0$	$\mu^*_i = 1/s^*_i$
Perfect competition	$\theta_i = \theta'_i > 0$ and $\alpha^*_i \simeq 0$	–
	with $\lim_{n\to\infty}\left(1/s^*_i\right) = 0$	$\mu^*_i = 0$

To illustrate, take for instance the case, represented in Fig. 2.1, of a symmetric differentiated duopoly with the CES specification for the aggregator X, the isoelastic demand function $\widehat{D}(P, Y) = YP^{-5/2}$ and the constant unit cost $c = 1$. We represent the degree of complementarity $1/(1+s)$ on the horizontal axis and the competitive toughness θ (the same for both firms) on the vertical axis. The set of values which parameterise existent oligopolistic equilibria is represented by the region inside the thick curves. We see that as the two goods become more and more complementary, potential equilibria cease to be enforceable as competitive toughness becomes too high or too low. The same is true if substitutability is very high and competitive toughness very low. The thin horizontal line ($\theta = 1/2$) represents the competitive toughness displayed in the price equilibrium (existent at any degree of complementarity) and the thin increasing curve represents the competitive toughness associated with the quantity equilibrium (existent only if complementarity is not too large).

The set of existent oligopolistic equilibria is also represented for the same example in Fig. 2.2 as the region of the space $1/(1+s) \times \mu$ between the thick curves. Relative to Fig. 1.2 in Chap. 1, the main difference is the presence of a positive unit cost, allowing to replace on the vertical axis the price index $P = 2^{1/(1-s)}p$ by the markup $\mu = (p-1)/p = \left(P - 2^{1/(1-s)}\right)/P$.

The thick curve switching from concave to convex is quite similar to the corresponding one in Fig. 1.2 of Chap. 1. It is the *soft competition*

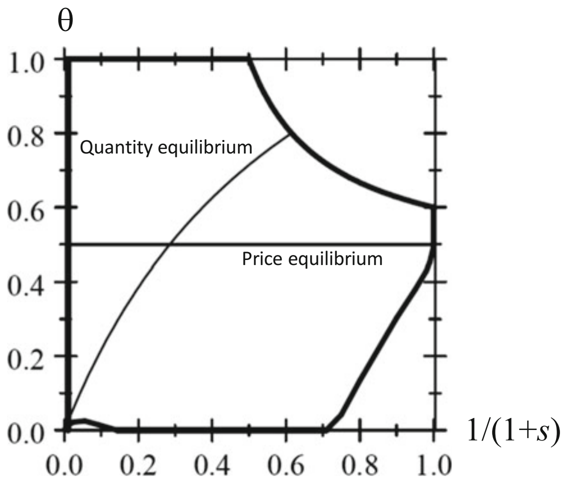

Fig. 2.1 Competitive toughness compatible with equilibrium existence

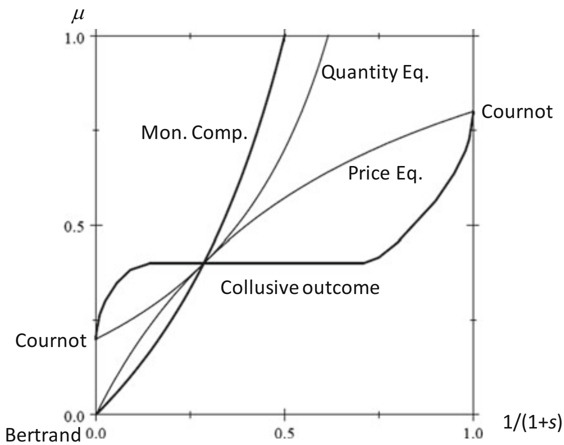

Fig. 2.2 Equilibrium regimes

frontier representing the markup that is closest to the collusive one ($1/\sigma = 0.4$ in this example) and linking the two Cournot solutions (the one for the homogeneous duopoly at its left end and the one for complementary monopoly at its right end). The thick convex curve starting at the origin from the Bertrand solution ($\mu = 0$) is the *tough competition frontier*, representing the markup $1/s$ resulting from maximal competitive toughness ($\theta = 1$). By the Bertrand paradox, this maximal competitive toughness entails the monopolistic competition outcome usually associated with a continuum of producers. Because of the positive unit cost, $\theta = 1$ is not linked to corner solutions, so that this frontier essentially differs from the graph of the step function in Fig. 1.2 in Chap. 1. The thin concave curve, linking the Bertrand and the Cournot complementary monopoly solutions, represents the price equilibrium markup and the thin convex curve starting from the Cournot homogeneous duopoly solution represents the quantity equilibrium markup, always above the price equilibrium markup.

2 Introducing Ford Effects

In the preceding section, when formulating the constraint on market share, we have treated income Y parametrically, its value being of course adjusted at equilibrium. Large firms may however influence the size of their own market through the income they distribute, in a way that is far from negligible. And they may well have a good perception of that influence, taking it into account in their decisions. A well-known example of that perception is the high wage policy advocated from an industrialist point of view by Henry Ford:

> I believe in the first place that, all other considerations aside, our own sales depend in a measure upon the wages we pay. If we can distribute high wages, then that money is going to be spent and it will serve to make storekeepers and distributors and manufacturers and workers in other lines more prosperous and their prosperity will be reflected in our sales. Country-wide high wages spell country-wide prosperity, provided, however, the higher wages are paid for higher production (Ford 1922, p.124).

What makes this idea quite remarkable is that it is formulated in "general equilibrium" terms. The income feedback effect of higher distributed income—that may be called the *Ford effect*[11] —works through the variations it induces outside the sector in which it originates.

The consequences for the equilibrium markup of introducing Ford effects working through the different income components (wages, profits and their sum) can be evaluated (see d'Aspremont and Dos Santos Ferreira, 2017). Let us here consider Ford effects extended to the whole income of the oligopolistic sector, when firms in this sector take as given the (wage) income z generated (or spent) in the competitive sector. We redefine accordingly the concept of oligopolistic equilibrium.

Definition 2 An *oligopolistic equilibrium with Ford effects* is a tuple of pairs $\left(p_i^*, x_i^*\right)_{i=1,\ldots,N} \in \overline{\mathbb{R}}_+^{2N}$ such that, for any i,

$$\begin{aligned} \left(p_i^*, x_i^*\right) \in \arg & \max_{(p_i, x_i) \in \mathbb{R}_+^2} \left(p_i - c_i\right) x_i \\ \text{s.t.} \quad & x_i \leq H_i \left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and } X\left(x_i, \mathbf{x}_{-i}^*\right) & \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), z^* + \textstyle\sum_{j \neq i} p_j^* x_j^* + p_i x_i\right), \end{aligned} \tag{2.1}$$

and such that the profits are non-negative, namely that $\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right) \phi_i \geq 0$ for each i, and also such that the consumer is non-rationed (implying $z^* = L - \sum_{i=1}^N \left(c_i x_i^* + \operatorname{sgn}\left(x_i^*\right) \phi_i\right)$).

The general formula obtained for the equilibrium markup is modified, while remaining easy to interpret.

Proposition 5 *Let* $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ *be an oligopolistic equilibrium with Ford effects. Then the equilibrium markup* $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ *of each firm* i *is given by*

$$\mu_i^* = \frac{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) + \left(1 - \theta_i^*\right) \left[\alpha_i^* - \eta_i^* \epsilon_Y D^*\right]}{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) s_i^* + \left(1 - \theta_i^*\right) \left[\alpha_i^* \sigma_i^* - \eta_i^* \epsilon_Y D^*\right]}, \tag{2.2}$$

[11]As in d'Aspremont et al. 1989a and 1989b.

where $\eta_i^* \equiv p_i^* x_i^* / Y^*$ *is the budget share of good* i *in the* whole *expenditure, for some* $\theta_i^* \in [0, 1]$.

Proof The only modification in the programme of firm i concerns the income as an argument of the Marshallian demand function in the constraint for market size. Thus, by referring to the programme (2.1) and building on first-order conditions (1.15) and (1.16) in the proof of Proposition 4, we easily modify these conditions to obtain:

$$
\begin{aligned}
x_i^* &= \frac{\lambda_i^*}{p_i^*}\left[-\epsilon_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] \\
&\quad + \frac{\nu_i^*}{p_i^*}\left[-\epsilon_{p_i} D\left(\mathbf{p}^*, Y^*\right) - \epsilon_Y D\left(\mathbf{p}^*, Y^*\right)\epsilon_{p_i} Y^*\right] \\
p_i^* - c_i &= \frac{\lambda_i^*}{x_i^*}\left(1 - \left[\epsilon_i X\left(\mathbf{x}^*\right)\right]\left[\epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\right) \\
&\quad + \frac{\nu_i^*}{x_i^*}\left[\epsilon_i X\left(\mathbf{x}^*\right) - \epsilon_Y D\left(\mathbf{p}^*, Y^*\right)\epsilon_{x_i} Y^*\right].
\end{aligned}
\tag{2.3}
$$

By dividing the two handsides of the second equation by the corresponding handsides of the first and then using Table 2.1 and $\epsilon_{p_i} Y^* = \epsilon_{x_i} Y^* = p_i^* x_i^* / Y^* \equiv \eta_i^*$ plus $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^*\right)$ to make the appropriate simplifications, we obtain indeed the markup formula (2.2). □

This expression for the equilibrium markup, similar to formula (1.13) in Proposition 4, is again a harmonic mean of the elasticities (in absolute value) of the two frontiers at the equilibrium point (in the space $x_i \times p_i$), $1/s_i^*$ for the market share frontier and $\left(\alpha_i^* - \eta_i^* \epsilon_Y D^*\right) / \left(\alpha_i^* \sigma_i^* - \eta_i^* \epsilon_Y D^*\right) \equiv 1/\widehat{\sigma}_i^*$, rather than $1/\sigma_i^*$, for the market size frontier. The redefined elasticity of intersectoral substitution $\widehat{\sigma}_i^*$ is larger (resp. smaller) than the original σ_i^* if $\sigma_i^* > 1$ (resp. $\sigma_i^* < 1$). In other words, the Ford effect increases (resp. decreases) the relevant elasticity of intersectoral substitution and accordingly exerts *ceteris paribus* a depressing (resp. enhancing) effect on the equilibrium markup when good i and the numeraire good are substitutes (resp. complements).

The equilibrium markup formula (2.2) becomes simpler in the particular case of homotheticity of the utility function U and homogeneity of degree 1 of the aggregator function X. In this case, $\epsilon_X e^* = \epsilon_X H_i^* = \epsilon_Y D^* = 1$ and $\eta_i^* = \alpha_i^* \left(P^* X^* / Y^*\right) \equiv \alpha_i^* \gamma^*$, where γ^* is the budget share of the composite product of the oligopolistic sector in the whole expenditure, so that $\widehat{\sigma}^* = (\sigma^* - \gamma^*) / (1 - \gamma^*)$ and

$$\mu_i^* = \frac{\theta_i^* \left(1 - \alpha_i^*\right) + \left(1 - \theta_i^*\right) \alpha_i^* \left(1 - \gamma^*\right)}{\theta_i^* \left(1 - \alpha_i^*\right) s_i^* + \left(1 - \theta_i^*\right) \alpha_i^* \left(1 - \gamma^*\right) \widehat{\sigma}^*}. \tag{2.4}$$

The markup μ_i^* is a weighted harmonic mean of the reciprocals of two elasticities of substitution, s_i^* and $\widehat{\sigma}^*$, where the intersectoral elasticity of substitution $\widehat{\sigma}^*$ has been implicitly redefined to refer to the substitution of X/z with respect to $P/1$ rather than that of x_i/Y with respect to $p_i/1$. We have indeed, using quantity and price indices of the composite good produced in the oligopolistic sector thanks to homotheticity:

$$\begin{aligned}
\widehat{\sigma} &\equiv -\epsilon_P \left[\frac{\widehat{D}(P,Y)}{Y - P\widehat{D}(P,Y)}\right] \\
&= \underbrace{-\epsilon_P \widehat{D}(P,1)}_{\sigma} - \underbrace{\frac{P\widehat{D}(P,1)}{1 - P\widehat{D}(P,1)}}_{\gamma/(1-\gamma)} \underbrace{\left(1 + \epsilon_P \widehat{D}(P,1)\right)}_{1-\sigma} \\
&= \frac{\sigma - \gamma}{1 - \gamma}.
\end{aligned} \tag{2.5}$$

Let us compare the markup formula (2.4) and the formula (1.13) prevailing in the absence of Ford effects (but under homotheticity, leading to $\beta_i^* = 1$ and $\sigma_i^* = \sigma^*$ as the price elasticity of demand for the composite good). The weight on the reciprocal of the intrasectoral elasticity of substitution is not modified: this elasticity completely determines the markup of firm i in the limit cases of a negligible market share ($\alpha_i^* \to 0$) or of maximum competitive toughness ($\theta_i^* \to 1$). Correspondingly, the intersectoral elasticity of substitution completely determines the markup of firm i in the opposite limit cases of monopoly ($\alpha_i^* \to 1$) or collusion

($\theta_i^* \to 0$). An increasing weight put on the reciprocal of the intersectoral elasticity of substitution may however also result from a decreasing budget share γ^* of the composite product of the oligopolistic sector. But the most significant consequence of the Ford effect is the transformation of the intersectoral elasticity of substitution itself: with $\widehat{\sigma}^*$ increasing from $\sigma^* > 1$ to infinity as the budget share γ^* increases from 0 to 1 (when the two composite goods are substitutable) and with $\widehat{\sigma}^*$ decreasing from $\sigma^* < 1$ to zero as the budget share γ^* increases from 0 to 1 (when the two composite goods are complementary).

This concludes our analysis of the consequences of introducing Ford effects extended to the whole income of the oligopolistic sector. Some more limited form could be considered, but in the present model, restricting Ford effects to wages does not make sense. As the economy has a single labour market which is perfectly competitive, as labour productivity in the numeraire sector is assumed constant and as labour supply is rigid, economy-wide wage income is insensitive to oligopolistic firms' decisions, at least when expressed in terms of the numeraire. It is just equal to L.

Restricting Ford effects to profits, so that the economy income, as conjectured by firm i at some equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, is

$$Y = L + \sum_{j \neq i} \left(\left(p_j^* - c_j \right) x_j^* - \phi_j \right) + \left(\left(p_i - c_i \right) x_i - \phi_i \right), \qquad (2.6)$$

will not have any consequence either. Consider the programme (1.12) of firm i, expressed as the maximisation of the Lagrangian

$$\max_{(p_i, x_i)} f_i\left(p_i, x_i\right) - \lambda_i g_i\left(p_i, x_i\right) - \nu_i h\left(p_i, x_i, Y\left(f_i\left(p_i, x_i\right)\right)\right), \qquad (2.7)$$

where f_i is the objective function, and where $g_i\left(p_i, x_i\right) \leq 0$ and $h(p_i, x_i, Y\left(f_i\left(p_i, x_i\right)\right)) \leq 0$ are the two constraints, on market share and on market size respectively, and λ_i and ν_i the corresponding Lagrange multipliers. The strategies of other firms, implicit arguments of functions g_i and h, are omitted for simplicity of notation. The crucial point is that Y depends upon the strategy pair (p_i, x_i) only through the objective function f_i. As a consequence, the first-order condition for an interior solution is

$$\left[1 - \nu_i \partial_Y h \cdot Y' \left(f_i \left(p_i, x_i\right)\right)\right] \boldsymbol{\partial}_{(p_i, x_i)} f_i \left(p_i, x_i\right)$$
$$= \lambda_i \boldsymbol{\partial}_{(p_i, x_i)} g_i \left(p_i, x_i\right) + \nu_i \boldsymbol{\partial}_{(p_i, x_i)} h \qquad (2.8)$$

where the gradient $\boldsymbol{\partial}_{(p_i, x_i)} f_i \left(p_i, x_i\right)$ is multiplied, not by 1 as when Ford effects are ignored, but by a positive factor which depends upon the strategy pair (p_i, x_i). Thus, taking into account Ford effects restricted to profits only changes proportionately the two Lagrange multipliers without modifying the equilibrium markup, which depends only on the ratio of those multipliers.

3 Back to the Homogeneous Good Case: Comparison with Alternative Approaches

In the first section of Chap. 1, we have considered the case of a duopoly producing a homogeneous good at zero cost under perfectly symmetric conditions. Let us now suppose N firms, each firm i producing the same good with a technology described by an increasing cost function C_i, which is continuously differentiable on $(0, \infty)$ and such that $C_i(0) = 0$.[12] The demand D for the good is a function of market price P, with a finite continuous derivative $D'(P) < 0$ over all the domain where it is positive and such that $\lim_{P \to \overline{P}} D(P) = 0$, for some $\overline{P} \in (0, \infty]$. Our purpose is to review different approaches to oligopolistic competition in the homogeneous good case, used in different contexts, also leading to the same kind of indeterminacy. We start by our own approach.

[12] Following d'Aspremont and Dos Santos Ferreira (2009), and for the sake of comparing our oligopolistic equilibrium concept with alternative concepts, the technology assumption is weakened with respect to the one in section 2.1, where $C_i\left(x_i\right) = \phi_i + c_i x_i$ for $x_i > 0$ (with $c_i > 0$).

Our Market Share and Market Size Approach

Using the dual form of the market share and market size constraints given in Eq. (3.7) of Chap. 1, the definition of oligopolistic equilibrium is straightforwardly adapted.

Definition 3 An *oligopolistic equilibrium* is a $2N$-tuple $(\mathbf{p}^*, \mathbf{x}^*)$ such that, for each firm $i = 1, \dots, N$, (p_i^*, x_i^*) is solution to the programme

$$\max_{(p_i, x_i) \in \mathbb{R}_+^2} \left\{ p_i x_i - C_i(x_i) | \ p_i \leq \min_{j \neq i} \{p_j^*\} \text{ and } p_i \leq D^{-1}\left(x_i + \sum_{j \neq i} x_j^*\right) \right\}, \tag{3.1}$$

and satisfies

$$\sum_j x_j^* = D\left(\min_j \{p_j^*\}\right). \tag{3.2}$$

Since both constraints are binding for any active firm at an oligopolistic equilibrium and since we are in the homogeneous good case, an *equilibrium outcome* is simply given by the pair $(P^*, \mathbf{x}^*)$ with $P^* = \min_j \{p_j^*\}$.

It is easy to see that both the Cournot outcome $(P^{\mathrm{C}}, \mathbf{x}^{\mathrm{C}})$ satisfying

$$\begin{aligned} x_i^{\mathrm{C}} &\in \arg \max_{x_i \in [0, \infty)} \left\{ D^{-1}\left(x_i + \sum_{j \neq i} x_j^{\mathrm{C}}\right) x_i - C_i(x_i) \right\} \text{ for } i = 1, \dots, N, \\ P^{\mathrm{C}} &= D^{-1}\left(\sum_j x_j^{\mathrm{C}}\right), \end{aligned} \tag{3.3}$$

and the competitive (Walrasian) outcome $(P^{\mathrm{W}}, \mathbf{x}^{\mathrm{W}})$ satisfying

$$x_i^{\mathrm{W}} \in \arg\max_{x_i \in [0,\infty)} \{P^{\mathrm{W}} x_i - C_i(x_i)\} \text{ for } i = 1, \ldots, N,$$

$$P^{\mathrm{W}} = D^{-1}\left(\sum_j x_j^{\mathrm{W}}\right) \tag{3.4}$$

are oligopolistic equilibria. If, indeed, there were, for some i, a deviation $(p_i, x_i) \in \mathbb{R}_+^2$ satisfying the two constraints in (3.1) such that the profit $p_i x_i - C_i(x_i)$ were strictly larger than the Cournot profit $P^{\mathrm{C}} x_i^{\mathrm{C}} - C_i(x_i^{\mathrm{C}})$ (resp. the Walrasian profit $P^{\mathrm{W}} x_i^{\mathrm{W}} - C_i(x_i^{\mathrm{W}})$), then we would get the contradiction $P^{\mathrm{C}} x_i^{\mathrm{C}} - C_i(x_i^{\mathrm{C}}) < D^{-1}(x_i + \sum_{j \neq i} x_j^{\mathrm{C}}) x_i - C_i(x_i)$ (resp. $P^{\mathrm{W}} x_i^{\mathrm{W}} - C_i(x_i^{\mathrm{W}}) < P^{\mathrm{W}} x_i - C_i(x_i)$).

As to the Bertrand outcome $(P^{\mathrm{B}}, \mathbf{x}^{\mathrm{B}})$ with $P^{\mathrm{B}} = \min_i\{p_i^{\mathrm{B}}\}$, it is now characterised by

$$p_i^{\mathrm{B}} \in \arg\max_{p_i \in [0,\infty)} \left\{p_i d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) - C_i\left(d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right)\right)\right\} \tag{3.5}$$

where the demand to firm i is $d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) = D\left(p_i\right) / (\#\arg\min\left\{p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right\})$ if $p_i = \min\left\{p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right\}$ and $d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) = 0$ otherwise. It is also an oligopolistic equilibrium, since a profitable deviation (p_i, x_i) for some i in the extended Cournot-Bertrand game would have $p_i \leq P^{\mathrm{B}}$ and hence would be also feasible and profitable in the Bertrand game, again a contradiction. Finally, as already noticed in the symmetric duopoly case, and in contrast with the differentiated good case, the collusive outcome $(P^{\mathrm{m}}, x^{\mathrm{m}})$ corresponding to

$$(P^{\mathrm{m}}, \mathbf{x}^{\mathrm{m}}) \in \arg\max_{(P,\mathbf{x}) \in \mathbb{R}_+^{n+1}} \left\{ P \sum_i x_i - \sum_i C_i(x_i) \,\middle|\, \sum_i x_i \leq D(P) \right\}, \tag{3.6}$$

cannot be an oligopolistic equilibrium in the homogeneous good case unless it coincides with the Cournot outcome. Indeed, if $(P^{\mathrm{m}}, \mathbf{x}^{\mathrm{m}})$ is not a Cournot outcome, we have, for some i, some $x_i \in \mathbb{R}_+$ and $P = D^{-1}(x_i + \sum_{j \neq i} x_j^{\mathrm{m}})$,

$$\begin{aligned} &P x_i - C_i(x_i) + P^{\mathrm{m}} \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}}) \\ &> P^{\mathrm{m}} x_i^{\mathrm{m}} - C_i(x_i^{\mathrm{m}}) + P^{\mathrm{m}} \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}}), \end{aligned} \tag{3.7}$$

and, since it is collusive,

$$P^{\mathrm{m}} \sum_j x_j^{\mathrm{m}} - \sum_j C_j(x_j^{\mathrm{m}}) \geq P x_i - C_i(x_i) + P \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}}) \tag{3.8}$$

implying $P < P^{\mathrm{m}}$. Therefore, (P, x_i) is an admissible deviation for firm i in the oligopoly game.

Looking now at the first-order conditions of firm i at an oligopolistic equilibrium (with multipliers $(\lambda_i', \nu_i') \in \mathbb{R}_+^2 \setminus \{\mathbf{0}\}$ associated with the first and second constraints in (3.1)), they require, by the positivity of p_i^* and of x_i^* (if firm i is active) that $x_i^* - \lambda_i'^* - \nu_i'^* = 0$, and $p_i^* - C_i'(x_i^*) + \nu_i'^*/D'(P^*) = 0$. If firm i is inactive, both constraints cease to bind, so that we let $\lambda_i'^* = \nu_i'^* = 0$. Using the normalised parameter $\theta_i'^* \equiv \lambda_i'^* / \left(\lambda_i'^* + \nu_i'^*\right) \in [0, 1]$, we can rewrite the first-order conditions to characterise the markup of each firm i in the set I^* of active firms as a function of $\theta_i'^*$, with $P^* = \min_j\{p_j^*\}$:

$$\mu_i^* = \frac{P^* - C_i'(x_i^*)}{P^*} = \left(1 - \theta_i'^*\right) \frac{x_i^* / \sum_j x_j^*}{-\epsilon D(P^*)} \equiv (1 - \theta_i'^*) \frac{\alpha_i^*}{\sigma(P^*)},\ i \in I^*. \tag{3.9}$$

This formula generalises formula (1.5) in Chap. 1 for the duopoly case with zero marginal cost. As above, $\theta_i'^*$ may be interpreted as measuring the competitive toughness of firm i at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$. When competitive toughness is maximal ($\boldsymbol{\theta}'^* = (1, \ldots, 1)$), each active firm equalising marginal cost to price, we get the competitive equilibrium (or the Bertrand equilibrium for constant marginal costs). At the other extreme, when competitive toughness is minimal ($\boldsymbol{\theta}'^* = (0, \ldots, 0)$), we get the standard markup formula for the Cournot equilibrium. All other oligopolistic equilibria correspond to intermediate values of $\boldsymbol{\theta}'^*$.

Notice that μ_i^* in formula (3.9) is equal to the numerator of μ_i^* in formula (1.17) in the limit case $s_i^* = \infty$. But, looking at the denominator of μ_i^* in formula (1.17), we see that the parameterisation of competitive toughness is different in the two formulas (although we have kept the same notation $\theta_i'^*$ in both formulas). Indeed, if we apply formula (1.17), it entails $\mu_i^* = 0$ for $\theta_i'^* = 1$ (Bertrand) and $\mu_i^* = \alpha_i^*/\sigma^*$ for $\theta_i'^* = 1/2$ (Cournot), whereas by applying formula (3.9) we still obtain $\mu_i^* = 0$ for $\theta_i'^* = 1$ (Bertrand) but now $\mu_i^* = \alpha_i^*/\sigma^*$ for $\theta_i'^* = 0$ (Cournot).[13]

For the sake of comparison, let us now look at other approaches to oligopolistic competition in the homogeneous good case.

The Conjectural Variation Approach

The parameterisation we have obtained in (3.9) is equivalent to the one used in the empirical literature, building econometric models that incorporate general equations where each firm conduct in setting price or quantity is represented by a parameter, itself viewed as an index of competitiveness. This is the so-called "conduct parameter method" which has been at the basis of the New Empirical Industrial Organisation (NEIO) and has generated a large number of empirical studies (for a synthesis see Bresnahan 1989, Einav and Levin 2010). It is related to

[13] In the homogeneous good case (a limit case of the homothetic case), differentiability of P is lost. The left-hand elasticity $\epsilon_i^- P\left(\mathbf{p}^*\right)$, the one that must be applied when considering a deviation along the market size frontier, is equal to 1, so that we get

$$\frac{1}{\sigma_i^*} = \frac{\alpha_i^*}{-\epsilon_{p_i}^- D\left(\mathbf{p}^*, Y\right)} = \frac{\alpha_i^*}{-\epsilon_P \widehat{D} \cdot \epsilon_i^- P\left(\mathbf{p}^*\right)} = \frac{\alpha_i^*}{\sigma^*}.$$

Compare to footnote 8 in Sect. 1.

the conjectural variation approach but, as stressed by Bresnahan, "the phrase "conjectural variations" has to be understood in two ways: it means something different in the theoretical literature than the object which has been estimated in the empirical papers."

In the theoretical approach to conjectural variations, each firm i when choosing its quantity is also supposed to make a specified type of conjecture concerning the reaction of the other firms to any of its deviations. These conjectures, though, are not game-theoretically founded. They are introduced directly into the first-order conditions. Following the presentation in Dixit (1986), a sufficient specification[14] consists in introducing conjectural derivatives $r_i = \sum_{j \neq i} \partial x_j / \partial x_i$ for each i. These are called compensating (or non-collusive) variations if each r_i is restricted to be in the interval $[-1, 0]$, for every i. The corresponding first-order conditions are:

$$\frac{P^* - C_i'(x_i^*)}{P^*} = (1 + r_i) \frac{x_i^* / \sum_j x_j^*}{-\epsilon D(P^*)}. \tag{3.10}$$

If matching variations ($r_i > 0$) are excluded, and in particular those leading to the collusive solution, this gives the same characterisation as (3.9) with $r_i = -\theta_i'$. In other words, comparing first-order conditions, the set of oligopolistic equilibrium outcomes appears as the selected subset of outcomes obtained by non-collusive conjectural variations. The concept of oligopolistic equilibrium thus provides some game-theoretic foundation to the concept of conjectural variations, since the conjectural variation terms (within the relevant class) can be identified with the parameterisation of the equilibria of a fully specified game.

[14]Dixit considers the more general case where r_i is a function of both x_i and $\sum_{j \neq i} x_j$. More generally, in the empirical approach to conjectural variations with differentiated products, there are as many conjectural variation parameters as pairs of products (Nevo, 1998). As we have noticed in the case of demand estimation (see footnote 3 in Sect. 1), for estimation on the supply side our approach is also more parcimonious, with one parameter per product.

The Supply Function Approach

Another approach, initiated by Grossman (1981) and Hart (1982), assumes that firms strategies are supply functions. A firm i strategy is a *supply function* S_i associating with every price p_i in $[0, \infty)$ a quantity $x_i = S_i(p_i)$. In order to compare this concept with our own, we shall restrict strategies to the set $\mathbb{S}_+$ of *non-decreasing supply functions*.[15] To define the payoffs of the corresponding game, we have to solve in P the following equation for any N-tuple $\mathbf{S}$ of supply functions in $\mathbb{S}_+^N$

$$\sum_{j=1}^{N} S_j(P) = D(P). \tag{3.11}$$

Since the market demand is strictly decreasing and the supply function of each firm is non-decreasing, if a solution $\hat{P}(\mathbf{S})$ clearing the market exists, then it is unique.

The payoffs are defined as follows. We let

$$\Pi_i(S_i, \mathbf{S}_{-i}) = \hat{P}(S_i, \mathbf{S}_{-i}) S_i\big(\hat{P}(S_i, \mathbf{S}_{-i})\big) - C_i\big(S_i(\hat{P}\big(S_i, \mathbf{S}_{-i}))\big),$$

if the market clearing price $\hat{P}(\mathbf{S})$ exists, and

$$\Pi_i(S_i, \mathbf{S}_{-i}) = 0, \text{ otherwise.} \tag{3.12}$$

A *supply function equilibrium* is a Nash equilibrium $\mathbf{S}^*$ of the resulting game.

We can define the residual demand function of firm i at an equilibrium $\mathbf{S}^*$:

$$D_i^*(P, \mathbf{S}_{-i}^*) = \max\left\{ D(P) - \sum_{j \neq i} S_j^*(P), 0 \right\},$$

[15] As Delgado and Moreno (2004) do. However, they assume in addition that firms are identical.

Then for any firm i, maximising in S_i the profit $\Pi_i(S_i, \mathbf{S}^*_{-i})$ amounts to select P^* in

$$\arg\max_{P \in \mathbb{R}_+} \{P\, D_i^*(P, \mathbf{S}^*_{-i}) - C_i(D_i^*(P, \mathbf{S}^*_{-i}))\}. \tag{3.13}$$

or, equivalently, to choose any supply function S_i for which $S_i(P) = D_i^*(P, \mathbf{S}^*_{-i})$ has the unique solution P^*. The multiplicity of supply function equilibria is well known, but we have the following characterisation.

Proposition 6 *If strategies are restricted to non-decreasing supply functions, the set of outcomes of the supply function game,*

$$\left\{ \left(P^*, \mathbf{x}^*\right) \in \mathbb{R}_+^{N+1} \middle| \mathbf{x}^* = \mathbf{S}^* \left(P^*\right) \text{ with } \mathbf{S}^* \text{ a supply function equilibrium} \right\},$$

coincides with the set of oligopolistic equilibrium outcomes.

Proof Let $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ be an oligopolistic equilibrium. We then construct a supply function equilibrium giving the same outcome, each firm i choosing a supply function $S_i^* \in \mathbb{S}_+$ simply characterised by the price-quantity pair (p_i^*, x_i^*), that is such that $S_i^*(P) = x_i^*$ if $P \leq p_i^*$, and $S_i^*(P) = \infty$ otherwise. Clearly, the solution to (3.13) cannot be larger than $\min_{j \neq i}\{p_j^*\}$, hence any profitable deviation by some firm i from S_i^* must involve a price below $\min_{j \neq i}\{p_j^*\}$ and a quantity below $D(p_i) - \sum_{j \neq i} x_j^*$, and thus constitute a deviation with respect to the oligopolistic equilibrium.

To prove the converse, let $\mathbf{S}^* \in \mathbb{S}_+^N$ be a supply function equilibrium. Observe that, for any i, the residual demand $D_i^*(P, \mathbf{S}^*_{-i})$ is decreasing in P and that the profit $p_i x_i - C_i(x_i)$ is increasing in p_i for $x_i > 0$. Hence, by (3.13), (p_i^*, x_i^*) maximises $p_i x_i - C_i(x_i)$ on

$$A_i \equiv \left\{ (p_i, x_i) \in \mathbb{R}_+^2 \middle| x_i \leq D_i^*(p_i, \mathbf{S}^*_{-i}) \right\}. \tag{3.14}$$

For $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, with $\mathbf{x}^* = \mathbf{S}^*(\hat{P}(\mathbf{S}^*))$ and $p_j^* = \hat{P}(\mathbf{S}^*)$ for any j, to be an oligopolistic equilibrium, (p_i^*, x_i^*) should maximise $p_i x_i - C_i(x_i)$ on

$$\widehat{A}_i \equiv \Bigg\{ (p_i, x_i) \in \mathbb{R}_+^2 \Big| \, p_i \leq \min_{j \neq i} \left(p_j^*\right), \\ x_i \leq \max \left\{ D\left(p_i\right) - \sum_{j \neq i} x_j^*, 0 \right\} \Bigg\}, \tag{3.15}$$

for every i. Since, $\left(p_i^*, x_i^*\right) \in \widehat{A}_i$ and $\widehat{A}_i \subset A_i$, the result follows. □

This shows that, for any oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, there is a supply function equilibrium $\mathbf{S}^* \in \mathbb{S}_+^N$ such that $\mathbf{S}^*\left(\min_j\{p_j^*\}\right) = \mathbf{x}^*$ and, conversely, for any supply function equilibrium $\mathbf{S}^* \in \mathbb{S}_+^N$, there is an oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ such that $p_j^* = \hat{P}(\mathbf{S}^*)$, for any j, and $\mathbf{x}^* = \mathbf{S}^*(\hat{P}(\mathbf{S}^*))$.

If we consider the differentiable case (restricting $\mathbf{S}_{-i}^*$ to differentiable supply functions in $\mathbb{S}_+^{N-1}$), we may get back formula (3.9) if we derive the first-order condition to firm i programme (3.13) at equilibrium:

$$x_i^* + \left(D'(P^*) - \sum_{j \neq i} S_j^{*\prime}(P^*) \right) (P^* - C_i'(x_i^*)) = 0, \text{ with} \\ x_i^* = D_i^*(P^*, \mathbf{S}_{-i}^*), \tag{3.16}$$

or, equivalently,

$$\frac{P^* - C_i'(x_i^*)}{P^*} = \frac{x_i^* / \sum_k x_k^*}{-\epsilon D(P^*) + \sum_{j \neq i}(x_j^* / \sum_k x_k^*) \epsilon S_j^*(P^*)}. \tag{3.17}$$

Taking

$$1 - \theta_i' = \frac{-\epsilon D(P^*)}{-\epsilon D(P^*) + \sum_{j \neq i} (x_j^* / \sum_k x_k^*) \epsilon S_j^*(P^*)}. \tag{3.18}$$

we obtain formula (3.9).

In this formula, the term $\sum_{j \neq i} (x_j^* / \sum_k x_k^*) \epsilon S_j^*(P^*)$ may be interpreted as measuring the "reactivity of the other firms" (with respect to prices) as anticipated by firm i at the supply function equilibrium. It has a positive impact on the competitive toughness θ_i' of firm i as measured at the oligopolistic equilibrium. The elasticity of the supply function chosen by firm i is indifferent from the point of view of the firm itself since only the price-quantity pair (p_i, x_i) matters. However varying the elasticities of the other firms'supply functions allows to cover the whole range of admissible values of θ_i'. In particular the Cournot solution corresponds to an elasticity $\epsilon S_j^*(P^*)$ of the supply functions equal to 0 for all j, and the competitive solution to $\epsilon S_j^*(P^*) = \infty$ for at least two j's.

The Pricing Scheme Approach

In Chap. 1, we have introduced in the duopoly case a concept of "pricing scheme" associating with a vector of price announcements the resulting market price. It was mentioned that, if the pricing scheme (which is nothing else than a coordination device) is sufficiently responsive to individual price signals, then we get the Cournot equilibrium. This leads to the interpretation of a Cournot equilibrium as the coordinated optimal decisions of a set of monopolists, each facing some (imperfectly elastic) residual demand. In the original Cournot model, the same coordination is ensured by the use of the inverse demand function. Formally, pricing schemes have the same status as auctions or bidding mechanisms. They could be assimilated to the "facilitating practices" already discussed in Sect. 2 of Chap. 1 (see also d'Aspremont et al., 1991a,b). In this subsection, we shall first come back to the Cournot case where the pricing scheme is supposed to be sufficiently responsive and then examine the case of facilitating practices implying *de facto* that the pricing-scheme is the min-pricing scheme.

In the pricing scheme approach, the market price is supposed to be determined by a *pricing scheme* Ψ, a continuous non-decreasing function from $\mathbb{R}_+^N$ to $\mathbb{R}_+$, associating with each vector of price signals $\boldsymbol{\psi} = (\psi_1, \ldots, \psi_i, \ldots, \psi_N)$ a single price $\Psi(\boldsymbol{\psi})$. For a given pricing scheme Ψ, we thus obtain a game involving the N firms, the strategies of firm i being the set of nonnegative price-quantity pairs (ψ_i, x_i). For any vector $(\boldsymbol{\psi}, \mathbf{x})$ of such strategies, the payoff of firm i is given by the profit function

$$\Pi_i(\boldsymbol{\psi}, \mathbf{x}) \equiv \Psi(\boldsymbol{\psi})\, x_i - C_i(x_i), \tag{3.19}$$

with $(\boldsymbol{\psi}, \mathbf{x})$ satisfying

$$\sum_{i=1}^{N} x_i \leq D(\Psi(\boldsymbol{\psi})). \tag{3.20}$$

A Ψ*-equilibrium* is a vector $(\boldsymbol{\psi}^*, \mathbf{x}^*)$ in $\mathbb{R}_+^{2N}$, such that $\sum_{i=1}^{N} x_i^* = D(\Psi(\boldsymbol{\psi}^*))$ and, for every $i \in N$, (ψ_i^*, x_i^*) is a solution to

$$\max_{(\psi_i, x_i) \in \mathbb{R}_+^2} \Psi(\psi_i, \boldsymbol{\psi}_{-i}^*)\, x_i - C_i(x_i), \text{ s.t. } x_i \leq D(\Psi(\psi_i, \boldsymbol{\psi}_{-i}^*)) - \sum_{j \neq i}^{N} x_j^*. \tag{3.21}$$

If we now consider the Cournot model with, say, the function $D^{-1}(x_i, \mathbf{x}_{-i})\, x_i - C_i(x_i)$ differentiable and strictly quasi-concave in x_i, and assume that the pricing scheme Ψ is differentiable, onto and strictly increasing in each variable ψ_i (and hence strongly responsive), then the Ψ-equilibrium outcome coincides with the Cournot outcome, that is, $(\Psi(\boldsymbol{\psi}^*), \mathbf{x}^*) = (P^C, \mathbf{x}^C)$. Looking at the first-order conditions for an active firm $i \in I^*$ at a Ψ-equilibrium, we obtain

$$\partial_i \Psi(\boldsymbol{\psi}^*) \left[x_i^* + (\Psi(\boldsymbol{\psi}^*) - C_i'(x_i^*))\, D'(\Psi(\boldsymbol{\psi}^*))\right] = 0, \tag{3.22}$$

leading to the same conditions as (3.9) for the Cournot case ($\boldsymbol{\theta}' = \mathbf{0}$):

$$\frac{\Psi\left(\boldsymbol{\psi}^*\right) - C_i'\left(x_i^*\right)}{\Psi\left(\boldsymbol{\psi}^*\right)} = \frac{x_i^* / \sum_j x_j^*}{-\epsilon D\left(\Psi\left(\boldsymbol{\psi}^*\right)\right)}, i \in I^*. \tag{3.23}$$

The essential property to get this result is the manipulability (upwards and downwards) of the market price by each individual producer. This means that the pricing scheme can be eventually obliterated. The equilibrium can simply be described as having each firm i choosing its monopoly solution (p_i, x_i) on its residual demand, that is, by maximising its profit $p_i x_i - C_i(x_i)$ in price and quantity under the residual demand constraint $x_i \leq D(p_i) - \sum_{j \neq i} x_j^*$. Each firm i will thus end up choosing the Cournot solution x_i^{C} and the same price $p_i^{\mathrm{C}} = P^{\mathrm{C}}$, clearing the market $\sum_j x_j^{\mathrm{C}} = D\left(P^{\mathrm{C}}\right)$.

But, of course, firms can also adopt a different conduct, based on other forms of price coordination, such as facilitating practices. For instance, each firm can include a *meeting competition clause* (or *price-match guarantee*) in its sales contracts, guaranteeing its customers that they are not paying more than what they would to a competitor, so that each customer acts as if facing the single market price $\Psi^{\min}(\mathbf{p}) = \min_j\{p_j\}$, where $\Psi^{\min}$ is called *the min-pricing scheme*. Combining this guarantee with the assumption that each firm i brings x_i to the market, we infer that it should be willing to sell this output at the discount price $P = \min\{\Psi^{\min}(\mathbf{p}), D^{-1}(\sum_j x_j)\}$. We thus get the following payoff function for firm i:

$$\Pi_i\left(p_i, \mathbf{p}_{-i}, x_i, \mathbf{x}_{-i}\right) \equiv \min\left\{\Psi^{\min}(\mathbf{p}), D^{-1}\left(\sum_j x_j\right)\right\} x_i - C_i(x_i). \tag{3.24}$$

This defines a *price-matching oligopoly game* in prices and quantities. The corresponding oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, called a $\Psi^{\min}$-*equilibrium*, is a Nash equilibrium satisfying in addition the *no-rationing* *restriction*

$$\sum_j x_j^* = D\left(\Psi^{\min}\left(\mathbf{p}^*\right)\right) \tag{3.25}$$

to eliminate equilibria where customers would be willing to buy more at the equilibrium price $\Psi^{\min}\left(\mathbf{p}^*\right)$. The following proposition states that the equilibria of the price-matching oligopoly game coincide, when the output is homogeneous, with the oligopolistic equilibria.

Proposition 7 *A 2n-tuple* $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ *is a* $\Psi^{\min}$*-equilibrium if and only if it is an oligopolistic equilibrium.*

Proof Suppose first that $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ is a $\Psi^{\min}$-equilibrium (so that, for every i, $p_i^* = \Psi^{\min}\left(\mathbf{p}^*\right) = D^{-1}\left(\sum_j x_j^*\right)$), but that, for some i, and some $(p_i, x_i) \in \mathbb{R}_+^2$, $p_i x_i - C_i(x_i) > p_i^* x_i^* - C_i\left(x_i^*\right)$, with $p_i \le \min\left\{\mathbf{p}_{-i}^*, D^{-1}\left(x_i + \sum_{j\neq i} x_j^*\right)\right\}$. Then,

$$\begin{aligned} &\min\left\{\Psi^{\min}\left(p_i, \mathbf{p}_{-i}^*\right), D^{-1}\left(x_i + \textstyle\sum_{j\neq i} x_j^*\right)\right\} x_i - C_i(x_i) \\ &= p_i x_i - C_i(x_i) \\ &> \min\left\{\Psi^{\min}\left(\mathbf{p}^*\right), D^{-1}\left(\textstyle\sum_j x_j^*\right)\right\} x_i^* - C_i(x_i^*), \end{aligned}$$

and (p_i, x_i) is a profitable deviation to the $\Psi^{\min}$-equilibrium, a contradiction.

To prove the other direction, suppose now (p^*, q^*) is an oligopolistic equilibrium (so that again $\sum_j x_j^* = D\left(\min_j\left\{p_j^*\right\}\right)$), but that, for some i, some $(p_i, x_i) \in \mathbb{R}_+^2$, and $p_i' \equiv \min\left\{p_i, \mathbf{p}_{-i}^*, D^{-1}\left(x_i + \sum_{j\neq i} x_j^*\right)\right\}$, we have $p_i' x_i - C_i(x_i) > p_i^* x_i^* - C_i\left(x_i^*\right) \ge 0$. Then $\left(p_i', x_i\right)$ satisfies the two constraints in (3.1) and gives higher profit to firm i, again a contradiction. □

Hence, the min-pricing scheme approach is another way to get oligopolistic equilibria and a relevant one to investigate the large number of markets where the price-match guarantee is offered.

4 The Effects of Intensifying Competition: Two Applications of the Model

The conventional view of the consequences of intensifying competition, through abatement of concentration or restriction of collusive practices, is that it increases welfare by reducing prices and spurring innovation. This view has however been challenged, as more intense competition can be price increasing (Chen and Riordan, 2008; Thisse and Ushchev, 2018; Zhelobodko et al., 2012) and its influence on R&D investment non-monotone (for a synthesis, see Aghion et al., 2005). These two questions can be easily addressed using our framework.

Stiffer Competition: Is It Price Decreasing or Price Increasing?

The basis of the conventional view that an increase in competitive intensity has a price decreasing effect can be traced back to Cournot (1838), where the symmetric equilibrium condition $\sigma(p) = 1/n$ in the case of nil costs, with an increasing function σ, has the consequence that "the resulting value of p would diminish indefinitely with the indefinite increase of the number n" (p. 94). The same result can be obtained without entry, through tougher competitive conduct, as implied by Bertrand's objection to Cournot. Does this view hold when we proceed from the homogeneous to the differentiated oligopoly and from partial to general equilibrium?

A simple way of answering this question is to recall our equilibrium markup formula given in Proposition 4. The markup appears in this formula as a weighted mean of the reciprocals of intra- and intersectoral elasticities of substitution. The weight on the former, $\theta_i\left(1-\alpha_i\beta_{ii}\right)$ for firm i, is increasing in the competitive toughness displayed by firm i and decreasing in the impact α_i of firm i's production on the aggregate output. Hence, more intense competition translates into a higher relative weight put on the reciprocal of intrasectoral elasticity of substitution, so that it decreases (resp. increases) the price of good i if the differentiated goods are more (resp. less) substitutable among themselves than for the numeraire good.

To make this analysis sharper, let us (1) take the CES case, (2) consider the two standard regimes of price and quantity competition (continuously increasing θ_i is anyway equivalent to continuously decreasing α_i) and (3) refer to the average markup in the oligopolistic sector. By (1.13) and taking $\theta_i = 1/2$ for a price equilibrium $\mathbf{p}^{\mathrm{B}}$, we obtain the following harmonic mean of the markups of all firms, weighted by their budget shares:

$$\overline{\mu}^{\mathrm{B}} = \left(\sum_i \frac{\alpha_i^{\mathrm{B}}}{\mu_i^{\mathrm{B}}}\right)^{-1} = \frac{1}{s + \left(\sigma^{\mathrm{B}} - s\right)\sum_i \left(\alpha_i^{\mathrm{B}}\right)^2}. \tag{4.1}$$

The most sensible assumption to make concerning the difference $\sigma^{\mathrm{B}} - s$ is that it is negative, differentiated goods being more substitutable among themselves than for the numeraire good (see Dixit and Stiglitz, 1977). Then, the average markup $\overline{\mu}^{\mathrm{B}}$ is an increasing function of the *Herfindahl index of concentration* $\sum_i \left(\alpha_i^{\mathrm{B}}\right)^2$, so that abating concentration has a price decreasing effect and entry has a pro-competitive effect. The opposite case can however not be excluded, leading to opposite effects: more intense competition is then welfare degrading.

We naturally obtain the same kind of results under quantity competition. By (1.17) and taking $\theta_i' = 1/2$ for a quantity equilibrium $\mathbf{x}^{\mathrm{C}}$, the arithmetic mean of the markups of all firms, weighted by their budget shares, is

$$\overline{\mu}^{\mathrm{C}} = 1/s + \left(1/\sigma^{\mathrm{C}} - 1/s\right)\sum_i \left(\alpha_i^{\mathrm{C}}\right)^2. \tag{4.2}$$

Hence, abating concentration has a price decreasing (resp. increasing) effect and entry has a pro-competitive (resp. anti-competitive) effect if $s > \sigma^{\mathrm{C}}$ (resp. $s < \sigma^{\mathrm{C}}$).

The relation between intra- and intersectoral elasticities of substitution thus appears crucial to settle the sense of price effects of higher competitive intensity. Take however the case of a continuum $[0, N]$ of differentiated goods, a case where budget shares and the ensuing index of concentration are equal to zero (Chamberlin's case of a large group of producers). We then lose the general equilibrium dimension and the markup is simply equal to $1/s$, a constant in the CES case. We can however take instead the case of symmetric *variable elasticity of substitution* (Thisse and Ushchev, 2018). With $\alpha_i = 0$, the expression (1.4) becomes, at a symmetric profile where $x_i = x$ if $i \in [0, n]$ and $x_i = 0$ if $i \in [n, N]$,

$$s_i = \frac{1}{-\partial^2_{ii} X(\mathbf{x})\, x / \partial_i X(\mathbf{x})}. \tag{4.3}$$

If we stick to the homotheticity assumption, the denominator on the RHS of this equation is homogeneous of degree 0 in $\mathbf{x}$ and only depends on the mass n of produced goods, so that the elasticity of substitution at a symmetric profile is a function of n. In this case, we can consequently have pro- or anti-competitive effects of entry if s is an increasing or a decreasing function, respectively. If the aggregator function is additive ($X(\mathbf{x}) = \int_0^n \xi(x) + \int_n^N \xi(0)$), we obtain for any active firm the elasticity of substitution $s(x) = 1/\left(-\epsilon_{\xi'}(x)\right)$, the reciprocal of the relative love for variety (an index of the local curvature of the aggregator function). As the equilibrium value of x is itself a function of n, we obtain again the two possible pro- and anti-competitive effects according to the sense of variation of s as a composite function of n. So, even under monopolistic competition, the possibility of anti-competitive effects of entry undermines the conventional view against collusive practices and barriers to entry.

Tougher Competition: Does It Foster Innovation?

Two opposite views contend on this question: the Darwinian view for which competition is needed to force firms to innovate in order to survive[16] and the Schumpeterian view for which monopoly rent is required to support innovative activity, tougher competition having a negative impact on innovation (Schumpeter, 1942). These two views refer to two contrary effects, the presence of which may lead to the observed non-monotonicity of the relation between competitive toughness and innovative activity. In recent theoretical and empirical work, Aghion et al. (2005) suggest an explanation for this observation. More intense competition enhances R&D investment when firms are at the same technological level (the Darwinian view), but discourages it when technological leaders and laggards coexist (the Schumpeterian view). By averaging R&D intensities across all industries, an inverted U-relationship between the average innovation rate and product market competition obtains through a composition effect. Non-monotonicity has however deeper roots, within each industry, and does not necessarily appear only at the aggregate level (d'Aspremont et al., 2010).

Let us examine the question in the context of a homogeneous oligopoly where process innovation reduces constant unit production costs. Consider a two-stage game played by N firms, which decide at the first stage whether or not to make, at a fixed cost ϕ, a R&D investment allowing to reduce the unit cost from $\overline{c}$ to $\underline{c}$ (with $\overline{c} > \underline{c} > 0$). At the second stage, n innovators produce at unit cost $\underline{c}$ and $N - n$ laggards at unit cost $\overline{c}$ and compete for demand $1/P$ with toughness θ. The competitive toughness is taken as uniform across all firms and exogenously given, characterising a specific regime of competition (between the two extremes of $\theta = 0$ for Cournot and $\theta = 1$ for Bertrand). Firm i's equilibrium markup $1 - c_i/P^*$ is consequently, by (3.9), equal to $(1 - \theta)\,\alpha_i^*$, with α_i^* the equilibrium market share of firm i. Firm i's profit is consequently $(1 - c_i/P^*)\,\alpha_i^* = (1 - \theta)\,\alpha_i^{*2}$. By aggregating the markup formula over

[16] Or, according to Arrow (1962), a monopolist has less incentive to invent than a competitive firm. This is due to a "replacement effect": the profits resulting from innovation replace profits that are smaller for a competitive firm than for a monopolist (Tirole, 1988). See also the discussion in Dasgupta and Stiglitz (1980).

all firms, we can easily compute the equilibrium price

$$P^* = \frac{n\underline{c} + (N-n)\,\overline{c}}{N-(1-\theta)}, \tag{4.4}$$

and then, introducing the notation $\kappa \equiv \left(\overline{c} - \underline{c}\right)/\overline{c} \in (0,1)$ for the *relative cost advantage* of the innovators, the equilibrium market shares $\overline{\alpha}$ and $\underline{\alpha}$ of the innovator and the laggard, respectively

$$\overline{\alpha}\,(\theta, n, N, \kappa) = \min\left\{\frac{1-\kappa+(N-n)\,\kappa/(1-\theta)}{N-n\kappa}, \frac{1}{n}\right\} \text{ and}$$

$$\underline{\alpha}\,(\theta, n, N, \kappa) = \max\left\{\frac{1-n\kappa/(1-\theta)}{N-n\kappa}, 0\right\}. \tag{4.5}$$

We can now refer to the *gain of innovating* for a firm confronted with n rival innovators ($0 \le n < N$), namely

$$G\,(\theta, n, N, \kappa) = (1-\theta)\left[\left(\overline{\alpha}\,(\theta, n+1, N, \kappa)\right)^2 - \left(\underline{\alpha}\,(\theta, n, N, \kappa)\right)^2\right]. \tag{4.6}$$

First notice that laggards are eliminated if $n\kappa \ge 1-\theta$, the case of drastic innovations, with many innovators benefitting from a high relative cost advantage. The gain of innovating is then $(1-\theta)/(n+1)^2$, a decreasing function of θ, leaving us with the markup squeezing effect of higher competitive toughness, discouraging innovation in the Schumpeterian mood. By contrast, if $0 < n\kappa < 1-\theta$, the case of non-drastic innovations, the innovator's market share $\overline{\alpha}\,(\theta, n+1, N, \kappa)$ is increasing and the laggard's market share $\underline{\alpha}\,(\theta, n, N, \kappa)$ decreasing in θ. Although the markup squeezing effect is still working, tougher competition may spur innovation through some sort of Darwinian selection pressure, eroding the territory of the least apt to the benefit of the fittest. It is however interesting to notice that the possible stimulating influence of tougher competition on innovation works here through higher asymmetry of

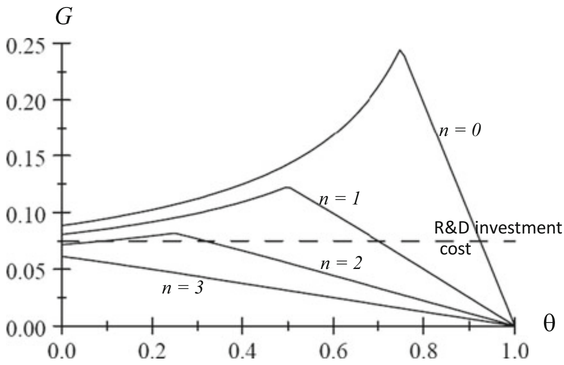

Fig. 2.3 Gain of innovating as a function of competitive toughness

market shares, hence through a concentration effect, thus preserving in some sense the Schumpeterian view.

The function $G(\cdot, n, N, \kappa)$ is strictly quasi-concave, either always decreasing or, if $n\kappa$ is small enough, inverse V-shaped. We illustrate its behaviour in Fig. 2.3 for $\kappa = 0.2$, $N = 8$ and $n \in \{0, 1, 2, 3\}$ (for a full treatment, see Lemma 1 in d'Aspremont et al. 2010).

The function $G(\theta, \cdot, N, \kappa)$ is decreasing, the higher curve in Fig. 2.3 corresponding to $n = 0$ and the lower to $n = 3$. For $\phi = 0.075$, represented by the dashed horizontal line, the subgame perfect equilibrium number n^* of innovators depends upon the competitive toughness: $n^* = 0$ if $\theta \in [0.92, 1]$, $n^* = 1$ if $\theta \in [0.70, 0.92]$, $n^* = 2$ if $\theta \in [0, 0.095] \cup [0.32, 0.70]$ and $n^* = 3$ if $\theta \in [0.095, 0.32]$. In this example, high competitive toughness discourages innovation, but the largest number of innovators is associated with low, but not too low levels of competitive toughness. The relation between competitive toughness and the number of innovators is again non-monotone.

In our deterministic model, a particular equilibrium has to be selected by randomly choosing n^* R&D investors, hence innovators, among N identical firms. It would be more realistic to go farther and assume that R&D investment at the first stage does not necessarily succeed, ensuring only a higher probability of innovating at the second. The decision to invest is then made by comparing the R&D investment cost and the expected gain of innovating. However, the results of such stochastic extension of the model mainly reproduce the preceding analysis and will accordingly be omitted (see d'Aspremont et al., 2010 for the stochastic and general equilibrium extension).

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