Claude d'Aspremont · Rodolphe Dos Santos Ferreira

The Economics of Competition, Collusion and In-between

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"This is an excellent and concise study of the field of imperfect competition in both a general and partial equilibrium setting by two of the world's experts in this field. Starting from the basics of Edgeworth and Cournot, the authors gradually build up and extend the framework to include within and between group competition, the intensity of competitive behaviour and general equilibrium Ford effects. Throughout the micro-foundations are rigorous and clearly explained, with simple examples and diagrams to illustrate the general insights. I recommend this book to any economist who wants to understand imperfect competition from a theoretical perspective."

—Huw Dixon, *Professor of Economics, Cardiff Business School (Cardiff University)*

"Empirical evidence shows that many markets are characterized by a few large firms that behave strategically, while cohabiting with a competitive fringe of small firms. Thus, there is a need to consider general equilibrium settings that account explicitly for strategic interactions among big firms. In this short but deep book, d'Aspremont and Dos Santos Ferreira provide several solutions that can reconcile "old" and "new" approaches to market competition through a series of nested frameworks. What makes this book unique is that the authors recognize explicitly the key role played by the labor market for the product market outcome. Their work is, therefore, a fundamental contribution that will allow us to understand better how markets work. Readers will also find a wide range of tools that can be used in different applications."

—Jacques-François Thisse, *CORE, Université Catholique de Louvain*

"Oligopoly in general equilibrium has proved an elusive goal in theory for decades, with Claude d'Aspremont and Rodolphe Dos Santos Ferreira among the small group of scholars that have taken it seriously. In recent years, the big data revolution has revealed just how important very large firms are, especially in international trade, and has encouraged more work breaking away from the perfectly and monopolistically competitive paradigms. This book could not be more timely therefore, and should be required reading for anyone working in this important field."

—Peter Neary, *Georgetown University Qatar and Merton College, University of Oxford*

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Claude d'Aspremont
Université Catholique de Louvain
Belgium

Rodolphe Dos Santos Ferreira
Université de Strasbourg
France

ISBN 978-3-030-63601-2 ISBN 978-3-030-63602-9 (eBook)
https://doi.org/10.1007/978-3-030-63602-9

This Palgrave Macmillan imprint is published by the registered company Springer Nature Switzerland AG.

The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Introduction

This book is devoted to oligopolistic competition. In our global world, it is increasingly recognised that more and more markets are dominated by large firms acting strategically, a typical industry configuration being a dominant group of large firms and a competitive fringe of small firms. This is of course at the heart of the various models in industrial organisation studying firm behaviour in particular markets, and aiming at providing adequate tools for improving competition policy. But the behaviour of large firms is certainly important at the aggregate level too (the so-called *granularity hypothesis*), having in particular significant effects on international exchanges, employment and growth. Yet, international trade theory and macroeconomics are still dominated by models of either perfect or monopolistic competition, assuming, for analytical convenience, symmetric preferences and a large number of negligible firms with vanishing strategic power. The main ambition of this book is to propose an alternative tractable model where such unrealistic assumptions can be avoided, preserving both strategic interactions among a few heterogeneous firms and a strong general equilibrium flavour, and also improving empirical applicability.

In addition to the difficulty of integrating strategic interactions in a general equilibrium model, one can add another obstacle to modelling oligopolistic competition. This is the choice of the strategic variable.

In most models of oligopolistic competition it is assumed that the strategic firms privilege one strategic variable, the quantity produced or the selling price. This of course corresponds to the two main traditions in modelling firms' behaviour. The first tradition, dating back to Cournot's fundamental contribution (1838), finds convenient to use the inverse demand function and assumes that firms behaving strategically affect the competitive price mechanism through quantity-setting. In the second tradition, that of Bertrand (1883), Edgeworth (1897), and the monopolistic competition of Robinson (1933) and Chamberlin (1933), as well as the spatial competition of Launhardt (1885) and Hotelling (1929), again only firms are supposed to behave strategically, but the strategic variables are prices.

This obstacle is still present when defining a general equilibrium. If the privileged strategic variable is quantity, as in the Cournot-Walras general equilibrium approach of Gabszewicz and Vial (1972), the firms are given the ability to anticipate correctly, and for every move, the result of the market mechanism, not only in the market in which they act strategically, as in the Cournot partial equilibrium approach, but in all markets simultaneously. Hence the existence of a unique Walrasian equilibrium is assumed to be associated with every choice of quantities made by the firms and the equilibrium of the resulting game is the Cournot-Walras equilibrium. When the privileged strategic variable is price, the difficulty[1] is to model how the quantities adjust to any system of chosen prices and to construct an "effective demand function" in the sense of Nikaido (1975), taking into account both direct and indirect effects (through dividends or wage income) of a change in prices.

The difficulties facing such "objective" approaches to general oligopolistic equilibrium seemed unsolvable, but fortunately a *deus ex machina* occurred. Following Spence (1976), Dixit and Stiglitz (1977) introduced a simple and tractable general equilibrium model of monopolistic competition. It is a two-sector model with an imperfectly competitive sector producing differentiated goods under increasing returns and a perfectly competitive sector producing a homogeneous

[1]Well discussed in Marschak and Selten (1974).

good under constant returns. The key idea in their model is to assume separability of the representative consumer preferences, defining a sub-utility function on the bundles of differentiated goods and implying the analysis "to depend on the intra- and intersector elasticities of substitution" (1977, p.297). But what makes this model so successful not only in industrial organisation (the first proposed issue was in this field) but in various other fields such as economic geography, international trade and macroeconomics is an additional set of assumptions, that is, the firm negligibility assumption combined with further restrictions on the preferences (mainly symmetry and additivity of the sub-utility) and free entry in the sense of a zero-profit condition. This brings in several convenient features. Each individual firm acting as a monopoly in its own niche, there is no need any more to choose between price and quantity as firms' decision variable and, by the zero-profit condition, there is no income feedback effect on demand. However, the negligibility assumption obliterates part of the benefit of assuming separable preferences: intersectoral effects are indeed negligible for negligible firms.[2]

Our purpose is to take a middle-road approach. Keeping the basic Dixit-Stiglitz framework with preference separability, and even extending this separability to several "groups" within the oligopolistic sector (to use the terminology of Dixit and Stiglitz 1977, III), we propose to put aside the additional set of assumptions so as to recover the strategic influence of heterogeneous firms and the role of the relationship between intra- and intersectoral substitution. We also propose to replace the conventional dichotomy price versus quantity competition by a single form of price-quantity competition.[3]

Discarding the conventional opposition between quantity and price competition in favour of a single form of competition with varying conduct is also an opportunity to revisit the Cournot-Bertrand debate, which should not be reduced to the choice of the relevant strategic

[2]As emphasised by Parenti et al. (2017), even without additive sub-utility, the "primitive" of the model remains the elasticity of intrasectoral substitution and the competitive effects are mainly determined by the properties of the sub-utility.

[3]This approach was pioneered by Shubik (1959).

variable. The opposition between Bertrand and Cournot is rather the result of the emphasis put on each one of the two sides of competition: for market share, expressing the divergent interests of the competitors, and for market size, reflecting their convergent interests against outsiders. For Bertrand, a lower price means undercutting the competitors' prices to get a higher market share, even to appropriate the whole market independently of its size. For Cournot, the market price "is necessarily the same" for all firms and, given competitors' supplies, a lower price means higher sales through the resulting increase in market size.

In the two-sector Dixit-Stiglitz framework (assuming weak separability but no symmetry), our basic concept of oligopolistic equilibrium will exploit these two sides of competition. Each firm in the oligopolistic sector maximises profit both in price and in quantity under two constraints, a market share constraint (involving the Hicksian demand) and a market size constraint (involving the Marshallian demand). The solution is indeterminate and typically involves a continuum of competition regimes. Analytical tractability is obtained through the first-order conditions that can be written as equations involving a manageable number of parameters both on the demand side and on the supply side, allowing for the estimation of each firm's competitive toughness as a continuous parameter. Standard regimes of competition, such as tacit collusion, pure price and pure quantity equilibria, correspond to particular values of the competitive toughness parameters.

In the limit case of perfectly substitutable goods we have two extreme regimes. When only the market share constraint is binding (corresponding to maximal competitive toughness) the equilibrium coincides with the competitive equilibrium (or the Bertrand solution), and at the other extreme when only the market size constraint is binding (corresponding to minimal competitive toughness) one gets the Cournot solution. When goods are differentiated, price and quantity competition are in-between competition regimes and for intervals of values of intra- and intersectoral substitution, even the collusive regime may become enforceable as the softest oligopolistic equilibrium.

In this general approach, firms are non-negligible players choosing price-quantity strategies. Moreover, if they are large, their choices may have a direct effect on macro variables such as aggregate income and factor prices.[4] As regards aggregate income, we show that the testable equations are not significantly modified when firms are assumed to perceive feedback effects through wages or profits (the so-called Ford effects). As regards factor prices, the wage in particular, we introduce an enlarged variant of the model, as the enlarged Dixit-Stiglitz model used in the macroeconomic and trade literatures.[5] This enlarged model includes labour as an additional good and it does not lead to a general equilibrium analysis until the wage rate, taken as given in a first step, is adjusted competitively or strategically. This is important when studying unemployment and we will see that the degree of competitive toughness matters in this context.

To introduce several (or many) groups of commodities produced in the oligopolistic sector, and to assume separability between groups, is an important extension for the applicability and the tractability of the model. If there are several groups and several firms in each group, this is a considerable simplification of the conjectures each firm has to form about others' strategies when maximising its profit. Indeed, the market share constraint only involves prices and quantities of firms in the same group. Price strategies of firms in other groups do appear in the market size constraint, but adding homothetic separability, they are summarised by group price indices. So, the focus is put on the direct rivalry among firms in the same group. A particular case is when each group produces a single homogeneous good and firms play Cournot within their group, implying only one price per group. Assuming a large number of groups implies that firms are big in their own group but small in the economy.

In Chap. 1, we start from the Cournot-Bertrand debate, contrasting the opposition between quantities and prices as the relevant strategic variables and the opposition between market size and market share as

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[4] As well stated by Neary, "if firms are large in their own market, and if that market constitutes a significant segment of the economy, then the firms have direct influence on economy-wide variables. Assuming they behave rationally, they should exploit this influence, taking account of their effect on both aggregate income and economy-wide factor prices when choosing their output or price" (2003, p.485).

[5] See, for example, Weitzman (1985), Blanchard and Kiyotaki (1987), d'Aspremont et al. (1990, 1996) and Neary (2016).

the relevant targets and then see how the "law of one price" introduced by Cournot can be implemented through what antitrust case law calls "facilitating practices." Two other factors that could restrict the intensity of competition are then formally introduced: product differentiation and concentration.

In Chap. 2, adopting the two-sector model of Dixit and Stiglitz with either weak or homothetic separability, we define our basic concept of oligopolistic equilibrium. We then derive from first-order conditions the simple relative markup formula to be used for empirical estimation. It is shown that such a formula is robust to taking into account Ford effects and, for the homogeneous case, our approach is compared to alternative ones, such as the conjectural variation and the supply function approaches. Finally, we illustrate the potential of our methodology by applying it to two policy issues, the effect of increasing competition on prices and on innovative activity.

In Chap. 3, we introduce the subdivision of the oligopolistic sector into several groups in order to simplify firms' conjectures and reinforce tractability. In the limit case where perfect substitutability holds within each group, using this approach, Cournotian competition in general equilibrium can be introduced more simply than in the Cournot-Walras approach. This approach is also useful to study empirically the interaction of groups within an industry. As an example, we review the method to estimate empirically the dominant firms' competitive toughness in a particuler industry (as proposed by Sakamoto and Stiegert 2018).

In the fourth and last chapter, we extend our basic model in several directions to get new perspectives. One extension is to introduce a labour market and examine the possibility of Keynesian involuntary unemployment resulting from oligopolistic behaviour of firms. We then extend the model to an overlapping generations economy with two representative consumers, one young the other old, and the possibility for firms to invest in capital in a first period, preceding the production period. Assuming a continuum of groups, we compare the dynamic properties of the model near the stationary solution in two cases, with or without strategic investment. In the last extension, competition is localised in a product space *à la* Hotelling and, as in the delegation literature, we re-interpret competitive toughness as a managerial attitude that can be strategically chosen.

A last comment is in order. This book offers a synthesis of our own work on oligopolistic competition, spread over a long period and published in various places. Although references will be made to the work of many authors, our purpose is not to provide a survey: there will be no attempt to review systematically other available methodologies and their applications. Our goal has been, and still is, to offer a specific methodology allowing to analyse oligopolistic markets in a comprehensive way and from a general equilibrium viewpoint and, in particular, to propose a well-founded measure of competitive toughness ready to use in empirical relevant applications.

Acknowledgements Most of the articles, or the preliminary papers, that are at the basis of this book were jointly written and the main ideas discussed with Louis-André Gérard-Varet before, sadly, he passed away in 2001. The work presented here is part of the long-term project we have developed together on general equilibrium and imperfect competition. Other co-authors have also inspired some important elements of this book, Massimo Motta in Chap. 1 and Jacques Thépot in Chap. 4. We also owe an intellectual debt to other co-authors of related papers quoted in this book, Camille Cornand, Frédéric Dufourt, David Encaoua, Jean Gabszewicz, Teresa Lloyd-Braga, Jean-Pierre Ponssard and Jacques Thisse.

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1

Modelling the Intensity of Competition

In this chapter we argue, using a simple model, that Cournot's founding contribution to oligopoly theory should not be reduced to quantity competition but viewed instead as introducing a particular competitive conduct of the producers, between the two extreme conducts that will be singled out much later by Bertrand's critique: collusion and pure price competition. We then analyse how producers can coordinate their conduct through "facilitating practices" of which the best-price guarantee is an example, and so implement Cournot's regime of competition. As a second factor shaping the intensity of competition, we let producers differentiate their products either to attract different types of consumers or to respond to their taste for variety. When products are neither too-close substitutes nor too-close complements, the collusive regime may become enforceable, which was excluded in Cournot's case of non-cooperative producers of a homogeneous good. The last factor we examine as a source of producers' market power is concentration, looking at the way it varies with the number of firms in a symmetric context, and then how it varies with the dispersion of market shares.

Before developing these four topics, let us point out the main ideas we want to put forward in this chapter. The first one is that we should reject

C. d'Aspremont, R. Dos Santos Ferreira, *The Economics of Competition, Collusion and In-between*, https://doi.org/10.1007/978-3-030-63602-9_1

the ubiquitous opposition between quantity and price competition which results from arbitrarily imposing a single strategy variable when modelling oligopolistic competition. As a reading, turned legendary, of the Cournot-Bertrand debate, this ubiquitous opposition is historically unfounded (Magnan de Bornier, 1992). It is also theoretically impoverishing in hiding the essential distinction between competition for market size, which involves convergent interests of otherwise non-cooperative producers and was highlighted by Cournot, and competition for market shares, which displays conflicting interests of the producers and was emphasised by Bertrand.

The second main idea is that combining these two dimensions of competition is a fruitful way to design an oligopoly model of price-quantity competition among more or less aggressive producers. It naturally entails the existence of a large set of equilibrium outcomes, characterised by variable degrees of producers' competitive toughness, between collusive conduct and fierce price undercutting. We thus retrieve Edgeworth's equivalence of competitive imperfection and equilibrium indeterminateness: "Contract with more or less perfect competition is less or more indeterminate" (Edgeworth, 1881, p. 20). Of course, we are here taking indeterminate in the sense that "contract is indeterminate when there are an indefinite number of final settlements" (*op. cit.*, p. 19), not in the alternative sense that "there [is] an indeterminate tract through which the index of value [oscillates ...] for an indefinite length of time," without ever reaching "that determinate position of equilibrium which is characteristic of perfect competition" (Edgeworth, 1897, [1925], p. 118). In other words, indeterminateness should here be understood as existence of a continuum of equilibria, certainly not as equilibrium inexistence.

Cournot showed that producers' non-cooperative behaviour made the collusive solution unenforceable when the products were either perfect substitutes or perfect complements. Our third main idea is that this unenforceability is not the consequence of non-cooperation alone, since tacit collusion is a possible non-cooperative equilibrium when goods are neither too-close substitutes (making undercutting profitable) nor too-close complements (making overbidding profitable). More generally, under product differentiation, our price-quantity modelling of oligopolistic competition enlarges the set of equilibrium outcomes beyond the frontiers set by pure price competition and pure quantity competition.

The last main idea concerns the deeply rooted conviction, at the basis of much of both macroeconomic modelling and antitrust policy design, that the absence of barriers to entry intensifies competition by fostering firm creation, so that profits are eventually dissipated and welfare is necessarily enhanced as entry becomes free. The question is in fact trickier than it seems. Ever since Cournot, we know that this conviction does not apply to the case of complementary products: "An association of monopolists, working for their own interest, in this instance will also work for the interest of consumers" (Cournot 1838, [1897] p. 103). Furthermore, given product substitutability, the intensity of competition is jointly determined by structure (concentration) and conduct (competitive toughness), which should not be considered separately. Last, free entry does not necessarily translate into zero profits, since some potential producers, taking into account their conjectures of others' actions, may rationally decide not to produce. In other words, free entry equilibria—typically multiple—are not necessarily characterised by the zero-profit condition.

1 Cournot and Bertrand: Competition Regimes and Competitive Conduct

Confronting Cournot (1838) and Bertrand (1883) views of competition between producers of the same good is a nice way to introduce the central theme of the book. Both authors start from the collusive solution (the only one for Bertrand). This solution is optimal from the producers' point of view, but it is unenforceable through competition. Unenforceability is the consequence of the non-cooperative behaviour of the producers, "each one acting on his own" in Cournot's words. Now, Bertrand wrongly interprets Cournot by supposing that non-cooperation would mean price undercutting by the producers, competing for higher market shares. In Bertrand's words,

> Cournot supposes that one of the competitors will lower his prices to attract the buyers to himself and that, the other lowering in turn his prices, even more, to get the buyers back, they will not stop doing so until each one of them would not gain anything more by lowering his prices further, even if his competitor would give up the fight. One decisive objection occurs: under this assumption no solution is possible, the lowering of prices would have no limit (Bertrand 1883, p. 503; our translation).

Thus, Bertrand objects that this competition regime is incompatible with the existence of an equilibrium (implicitly at a positive price). Cournot, by assuming that the market is permanently ruled by the law of one price, which excludes price undercutting, proposes in fact a halfway solution between collusion and a price war. The two authors attribute different competitive conducts to the producers, tougher for Bertrand, softer for Cournot, and characterise accordingly two different competition regimes. Unfortunately, the opposition between these two competition regimes has been reduced by the posterity to the difference between the strategic variables supposedly chosen by the competitors: the price in the case of Bertrand and the quantity in that of Cournot.

Let us take a closer look at the opposition between Bertrand and Cournot. In order to make the argument as clear as possible, we shall provisionally stick to the context found in the historical debate, namely that of a homogeneous duopoly with zero production costs. Concerning the aggregate variables, price P and quantity X, the collusive solution is the monopoly solution, stemming from the maximisation of the revenue PX under the constraint imposed by demand: $X \leq D(P)$. We will assume that the demand function D is decreasing, twice continuously differentiable and such that the Marshallian elasticity $\sigma(P) \equiv -D'(P)P/D(P)$ is increasing, with values below and above 1. This assumption ensures that $P(D(P) - x)$ is unimodal for $x \geq 0$. Thus, the monopoly price $P^{\mathrm{m}} = \arg\max_P PD(P)$ is unique and the corresponding aggregate quantity $D(P^{\mathrm{m}})$ will be taken as equally divided between the two producers, since the situation is fully symmetric.

Let us now move from cooperation between the producers to non-cooperation. In Bertrand's view, each producer i maximises his own revenue $p_i x_i$ under a constraint bounding the quantity x_i and depending on the conjectured price p_j of the competitor:

$$x_i \leq \begin{cases} X & \text{if } p_i < p_j \\ X/2 & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}. \tag{1.1}$$

As Bertrand notes, "whatever the adopted common price, if one of the competitors cuts his price alone, he attracts [...] the whole sale, and he will double his revenue if his competitor lets him do so" (Bertrand, 1883, p. 503; our translation). As a consequence, "no solution is possible" since any potential equilibrium will always trigger an undercutting deviation by one of the two duopolists. Of course, if we extend the price strategy spaces to zero prices, we will obtain an equilibrium at $p_i = p_j = 0$, with $\max\{x_i, x_j\} \leq X/2$.

We must emphasise that this result is not only independent from the quantities conjectured by the duopolists, but completely independent from the precise value X of aggregate demand and from its virtual dependence upon prices. The constraint on x_i that generates the Bertrand outcome is actually a *constraint on the market share* x_i/X of each producer i. In the competition regime imagined by Bertrand (and wrongly attributed to Cournot), there is no reference to the market size, as determined by the "law of demand" $D(P)$.

Cournot concentrates on the contrary on the *constraint on the market size* inherited from the monopoly market, namely $X \leq D(P)$ or, since $X = x_i + x_j$,

$$x_i \leq D(P) - x_j. \tag{1.2}$$

Hence, each producer acts as a monopolist maximising Px_i relative to his residual demand $[D(P) - x_j]$. Practically however, Cournot finds "convenient" to reformulate this constraint by referring to the inverse demand $P \leq D^{-1}(x_i + x_j)$, which leads to the maximisation of $x_i D^{-1}(x_i + x_j)$—a practice that has certainly reinforced the interpretation of Cournot competition as pure quantity competition.[1] The Cournot equilibrium price can then be obtained as the solution to the equation $\sigma(P) = 1/2$, where $\sigma(P)$ is the demand elasticity and $1/2$ stands for the symmetric duopoly market share $x_i/\left(x_i + x_j\right)$.

Two features should be emphasised in Cournot's approach. The first one concerns the law of one price ruling perfect markets, even out of equilibrium. Cournot insists: "the price is necessarily the same for each proprietor" supplying competitively the same market (Cournot, 1838, p. 88). Consequently, there is no point in distinguishing p_i and p_j, and price undercutting as the means to increase market shares is excluded. The second feature concerns the way market shares are conjectured by each duopolist. It cannot be $D(P)/2$, since this would make us go back to monopoly (maximising $PD(P)$ or $PD(P)/2$ leads to the same solution), so that the collusive solution would then be enforceable as a Cournot equilibrium. Since we are referring to residual demand, producer i's conjectured market share is $1 - x_j/D(P)$, which means that he is accepting the realisation of his rival's sale target. The two features, making the duopolists refer to the same price and making them accomodate their competitors' sale targets, characterise a soft competitive conduct, in fact some sort of semi-cooperative conduct, to be contrasted with Bertrand's cutthroat competition.

A problem remains in this approach: who sets the common price P? The problem is somehow hidden by the use of the inverse demand, which implicitly leaves that task to the impersonal market. However, the choice of x_i by producer i, given his rival's choice x_j and the law of demand $D(P)$, is effected, according to Cournot, "by properly modifying the

[1]Cournot adopts a dual approach in his *complementary monopoly* model (Sonnenschein, 1968), with which he studies producers' *concurrence* (Cournot, 1838, chap.IX), by contrast with producers' *competition* (*ibid*., chap.VII). Two producers sell goods that can only be used if combined, say one unit of each to get one unit of a composite good, so that there is no point in distinguishing x_i and x_j. Instead of using the inverse demand function $D^{-1}\left(x_i + x_j\right)$, one can now use the demand function $D\left(p_i + p_j\right)$ for the quantity of the composite good demanded at the total price $p_i + p_j$ where p_i and p_j are the prices to be paid to the two producers (marginal costs being again assumed nil). By duality, the Cournot solution in quantities for the homogeneous duopoly can be translated into a Cournot solution in prices for the complementary monopoly.

price" (Cournot, 1838, p. 89).[2] The way to perform the market price adjustment is however left implicit. We may, for instance, think of some coordinating device allowing the duopolists to manipulate the market price, by making it respond through a monotonic *pricing scheme* to price signals (ψ_1, ψ_2) emitted by the duopolists: $P = \Psi(\psi_1, \psi_2)$ (see d'Aspremont et al., 1991). The essential property for such pricing scheme is the resulting manipulability (upwards and downwards) of the market price by each individual producer. We hence recover the Cournot equilibrium, each producer i choosing the same price P^* and the quantity x_i^* in order to maximise the revenue $P\left(D(P) - x_j^*\right)$. In the Cournot model this is done through the inverse demand function, the producer choosing adequately the price and a corresponding quantity, under the conjectured quantity of the other producer.

We shall come back to this question in the next section, but will presently exploit another way to enforce at equilibrium the law of one price while letting the duopolists set their own prices. This way is a combination of Cournot and Bertrand consisting in the assumption that each producer i maximises his revenue in *both* variables and under *both* market share and market size constraints:[3]

$$\max_{(p_i, x_i)} p_i x_i$$

$$\text{s.t. } x_i \leq \begin{cases} x_i + x_j & \text{if } p_i < p_j \\ (x_i + x_j)/2 & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases} \text{ and } x_i \leq D\left(\min\{p_i, p_j\}\right) - x_j. \tag{1.3}$$

We can then formulate the following proposition characterising the equilibria of this *symmetric Cournot-Bertrand game*.

[2]We read "by properly adjusting *his* price" in the English translation of Cournot's *Researches* (by N. Bacon 1897), but this translation is incorrect and has confused prominent readers who did not have access to the original French text (see Magnan de Bornier, 1992).

[3]The minimum of the two prices in the market size constraint may be viewed as a particular "pricing scheme" that does not allow upward price manipulability.

Proposition 1 *Any equilibrium of the symmetric Cournot-Bertrand game at positive prices is symmetric, such that the demand is fully served and such that the common price is at most equal to the Cournot price. Any profile* $((p_1, x_1), (p_2, x_2))$ *satisfying these conditions is an equilibrium.*

Proof Take any feasible profile with $0 < p_j < p_i$. Producer i who, by the market share constraint, has zero sales can then ensure a positive revenue by deviating[4] to $p_i' \in \left(0, p_j\right)$ and $x_i' \in \left(0, D\left(p_i\right) - x_j\right)$. So, an equilibrium at positive prices is always symmetric in prices and, again by the market share constraint, also symmetric in quantities. It is also such that demand is fully served, otherwise any duopolist would be able to increase his revenue by capturing the rationed demand through a small downward price deviation. If the common price is higher than the Cournot price, any producer will want to make a downward price deviation and an upward quantity deviation, compatible with both constraints and leading to a higher revenue. By contrast, if the common price were lower than the Cournot price, any producer would want to make an upward price deviation, but such deviation would violate the market share constraint. □

Thus, price-quantity competition between duopolists constrained on their market shares as well as on market size—a combination of Bertrand and Cournot—leads to existence of a continuum of symmetric equilibria with prices between zero (the so-called Bertrand equilibrium price) and the Cournot equilibrium price. Intermediate prices correspond to equilibria such that a duopolist setting a higher price would hit the constraint on market share and be thrown out of the market, and such that a duopolist setting a lower price in order to increase his market share would hit the constraint on market size and undergo a revenue decrease.

Duopolist i's revenue maximisation under the two constraints can be reformulated in terms of the maximisation in (p_i, x_i) of the Lagrangian

[4] To define an interval in the real line, we use parentheses (.) or brackets [.] to indicate whether the endpoints are excluded or included, respectively.

function

$$\mathcal{L}(p_i, x_i, \lambda_i, \nu_i) = p_i x_i - \lambda_i \left(p_i - p_j\right) - \nu_i \left(p_i - D^{-1}\left(x_i + x_j\right)\right) \tag{1.4}$$

(with non-negative Lagrange multipliers λ_i and ν_i), where the constraints have been rewritten so as to become dimensionally homogeneous. At equilibrium, ignoring subscripts and using $x = D(p)/2$ because of symmetry, the first-order conditions when both constraints are binding are $x - (\lambda + \nu) = 0$ and $p + \nu / D'(p) = 0$, leading to $\nu = -D'(p)\,p$ and $D/2 = \lambda + \left(-D'p\right)$ and to the equation:

$$\theta \equiv \frac{\lambda}{\lambda + \nu} = 1 - 2\frac{-D'(p)\,p}{D(p)} = 1 - 2\sigma(p). \tag{1.5}$$

In the limit case where $\theta = 0$, the constraint on market share ceases to bite and we recover the Cournot equilibrium condition, with $\sigma(p) = 1/2$. Observe that any price P larger than the Cournot price cannot be enforced since it would imply $\sigma(P) > 1/2$ and $\theta < 0$. This is in particular true of the monopoly price P^{m} since $\sigma(P^{\mathrm{m}}) = 1$. As the weight λ of the constraint on market share increases relative to the weight ν on market size, the elasticity $\sigma(p)$ and hence the price p both decrease. Lower and lower equilibrium prices are the expression of higher and higher relative weight put on the constraint on market share, which we can view as a higher and higher *competitive toughness* θ displayed by the duopolists.

2 Implementing Cournot Through *Facilitating Practices*

As we have stressed, a weakness of Cournot's approach to competition is that it leaves implicit the way the law of one price is practically enforced, and in particular how the competitors manage to coordinate on a common market price. Our suggested combination of Cournot and

Bertrand partially corrects this weakness by explicitly considering price making competitors, but remains at the purely conceptual level and leads to equilibrium indeterminacy, without ensuring the prevalence of the Cournot outcome. In practice however, there are "specific actions taken by firms to make coordination easier or more effective *without the need for an explicit agreement*" (Hay, 2008, p. 1203). These are *facilitating practices* in the terminology of anti-trust case law (see Salop, 1986). Examples are clauses in sales contracts guaranteeing to the buyer that no lower price will be offered to another customer, that is, the *most-favoured-customer* clause (MFC), nor obtained from another seller (the *best-price* guarantee). These clauses contribute to the enforcement of the law of one price, the MFC by forbidding price discrimination, the best-price guarantee by automatising price matching in the case of rivals' undercutting. The main result of the economic literature on facilitating practices is to show how such clauses can implement prices above the competitive price.[5]

Three quotations illustrate each of these two clauses, the latter under the two forms of *price-match* and *meet-or-release* (allowing the seller to avoid the price match by releasing the buyer from the contract):

> The Bidder certifies that the price proposed is not in excess of the lowest price charged anyone else, including the Bidder's *most favoured customer*, for the like quality and quantity of the goods, services or both (Standard Acquisition Clauses and Conditions Manual, C0001T, 2007-05-25, Public Works and Government Services, Canada; our emphasis).
>
> If you find a lower-priced advertised on the internet for an identical electronic product or an equivalent (as determined by Dell) Dell, HP, Apple or Lenovo computer, Dell will *match that price* (Dell's Price Match Guarantee; our emphasis).

[5] For example, Kalai and Satterthwaite (1994) and Doyle (1988) get the implementation of the collusive price (supposing that firms use price strategies only, ignoring their rivals' market shares). By introducing discount possibilities below list prices in a second stage, though, Holt and Scheffman (1987) get the Cournot price as the maximal implementable price. In a specific model, d'Aspremont and Dos Santos Ferreira (2005) obtain by combining the "most-favoured-customer" and the "meet-or-release" clauses, the Cournot solution as the subgame perfect equilibrium of a two-stage duopoly game. An experimental approach to the effects of facilitating practices has been initiated by Grether and Plott (1984).

> If Buyer is offered material of equal quality at a price lower than stated herein before this order is filled and furnishes satisfactory evidence of such lower-price offer, Seller will either *meet such price* with respect to the quantity so offered or *allow Buyer to purchase said material so offered*, the amount so purchased to be deducted from the quantity specified herein (Solvay Advanced Polymers L.L.C., Standard Procurement Terms and Conditions; our emphasis).

In order to illustrate how these clauses may modify the equilibrium outcome, let us introduce two versions of a simple two-stage symmetric duopoly game, one corresponding to the straight price-match clause, the other to the meet-or-release clause (both clauses together with the most-favoured-customer clause). Interestingly it will appear that the possible outcomes are quite disjoint except for the Cournot outcome. In both versions, the two firms address a continuum $[0, 1]$ of consumers, each one wanting to buy one unit of the homogeneous good if its price does not exceed his reservation price. The demand function D, assumed to have the same properties as in Sect. 1, indicates for each price P the mass of consumers whose reservation prices are at least equal to P. At the first stage, each firm i chooses a price-output pair (p_i, x_i) and, by symmetry, is supposed to be randomly contacted by half of the consumers. As the probability for a randomly selected consumer to buy a good priced P is equal to $D(P)$, firm i sells at price p_i the quantity $\min\{x_i, D(p_i)/2\}$. Although, for simplicity, we assume zero production costs, we suppose that each duopolist incurs a cost (which is not explicit) if its output exceeds the quantity it can eventually sell.[6]

At the second stage, listed prices become publicly known and the firm quoting the higher price, if any, has to conform to the clause corresponding to the chosen version of the game: (i) under the price-match guarantee, it just has to meet its rival's listed price applying the appropriate discount to *all* its customers (by MFC); (ii) under the

[6] In the whole of this chapter, we keep Cournot's (provisional) assumption of zero production costs. An equivalent assumption would involve identical linear production costs cx, allowing to use the transformations $p - c = \widetilde{p}$ and $\widetilde{D}(\widetilde{p}) = D(\widetilde{p} + c)$. Firm i's profit is then $\widetilde{p}_i x_i$ if $x_i \leq \widetilde{D}(\widetilde{p}_i)/2$ and $\widetilde{p}_i \widetilde{D}(\widetilde{p}_i)/2 - c\left(x_i - \widetilde{D}(\widetilde{p}_i)/2\right)$, with a cost proportional to the excess of output over demand.

meet-or-release clause, it has to decide whether to meet the lower price (respecting MFC) or to release its customers. If both listed prices are equal or if the firm having quoted the higher price meets its rival's listed price, the profit of each firm i is then

$$\pi\left(P, x_i, x_j\right) = P \min\left\{x_i, \max\left\{D\left(P\right)/2, D\left(P\right) - x_j\right\}\right\} \text{ with}$$
$$P = \min\left\{p_i, p_j\right\}, \tag{2.1}$$

with the residual demand $D(P) - x_j$ becoming effective if $x_j < D(P)/2$. Now, under the meet-or-release clause, if the firm i announcing the higher price decides instead to release its customers of their contract, its sales will be restrained to those of its customers rationed by firm j and wanting to buy even at its higher listed price, thus obtaining the profit

$$\overline{\pi}\left(p_i, x_i, x_j\right) = p_i \min\left\{x_i, \max\left\{D\left(p_i\right) - x_j, 0\right\}\right\} \tag{2.2}$$

while letting its competitor j get

$$\underline{\pi}\left(p_j, x_j\right) = p_j \min\left\{x_j, D\left(p_j\right)\right\}. \tag{2.3}$$

Now, suppose that firm j conjectures the price p_i (any positive price) to be set by its rival. In the absence of any best-price guarantee, in a one-stage game, we would naturally retrieve Bertrand: by (slightly) undercutting its rival's price and choosing $x_j = D\left(p_j\right)$, firm j is able to (almost) double its profit. Under the price-match guarantee, this will not be possible, because price undercutting is automatically neutralised. Setting a price *below* p_i can at most allow firm j to catch the additional residual demand, as in Cournot. However, under the meet-or-release clause, firm i can decide not to match p_j, obtaining $\overline{\pi}\left(p_i, x_i, x_j\right) > \pi\left(P, x_i\right)$ if $p_i\left(D\left(p_i\right) - x_j\right) > p_j D\left(p_j\right)/2$. When choosing a price *above* p_j, the natural objective of firm i is then to maximise its profit relative to its residual demand $D(p_i) - x_j$ as, again, in Cournot but now with the price varying in the opposite sense. The following proposition states the consequences of adopting each one of the two versions of the best-price guarantee, with or without the release possibility (together with the most-favoured-customer clause).

Proposition 2 *Any subgame perfect equilibrium* $\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right)$ *of the two-stage duopoly game is such that* $x_1^* = x_2^* = D\left(\min\left\{p_1^*, p_2^*\right\}\right)/2 \equiv D\left(P^*\right)/2$ *and such that: (i) under the price-match guarantee and MFC,* $P^* \in \left[0, P^C\right]$*, with* P^C *the Cournot equilibrium price; (ii) under the meet-or-release clause and MFC,* $P^* \in \left[P^C, P^m\right]$*, with* P^m *the monopoly equilibrium price, firm* i *always deciding to match* p_j^* *at the second stage if* $p_i^* > p_j^*$*.*

Proof Take a subgame perfect equilibrium $\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right)$. If $p_j^* < p_i^*$, under the price-match guarantee or, under the meet-or-release clause, if firm i decides to meet at the second stage, $x_i^* = D\left(P^*\right)/2$ at the first stage. Indeed, $x_i^* = D\left(p_i^*\right)/2 < D\left(p_j^*\right)/2$ would lead to an unexploited excess demand for its output at the second stage. Also, an excess of output over demand would be costly, so that $x_1^* = x_2^* = D\left(\min\left\{p_1^*, p_2^*\right\}\right)/2$. Now, let us consider price deviations.

(i) Under the price-match guarantee any price increase is ineffective. As to a price decrease triggered by firm i, since $P < P^*$ entails $D(P) - x_j^* = D(P) - D\left(P^*\right)/2 > D(P)/2$, it is profitable, by (2.1), if and only if $P\left(D(P) - D\left(P^*\right)/2\right)$ is decreasing in P at P^*, hence if and only if

$$1 - \frac{D\left(P^*\right)/2}{D(P)} + \frac{P D'(P)}{D(P)} = \frac{1}{2} - \sigma\left(P^*\right) < 0,$$

which, as $\sigma\left(P^{\mathrm{C}}\right) = 1/2$ and σ is increasing, translates into $P^* > P^{\mathrm{C}}$. The condition for the downward price deviation to be unprofitable is consequently $P^* \in \left[0, P^{\mathrm{C}}\right]$.

(ii) Under the meet-or-release clause, we first show that the decision to release at the second stage is excluded. It would imply that the output of the firm having set the lowest price, say j, satisfies $x_j^* < D\left(p_j^*\right)$, otherwise $\overline{\pi}\left(p_i^*, x_i^*, x_j^*\right) = 0$, leading firm i to match p_j^*. However, if $x_j^* < D\left(p_j^*\right)$, firm j can increase its profit by choosing $p_j \in \left(p_j^*, p_i^*\right)$. Consider next an upward price deviation by firm i: $p_i > P^*$. Of course, for such deviation to be operative at the second stage, firm i would have to release its customers, getting a higher profit if $p_i\left(D\left(p_i\right) - D\left(P^*\right)/2\right) > P^* D\left(P^*\right)/2$. Since we get an equality for $p_i = P^*$, the preceding inequality stands if the left-hand side is increasing in p_i at P^*, that is, if

$$1 - \frac{D\left(P^*\right)/2}{D\left(p_i\right)} + \frac{p_i D'\left(p_i\right)}{D\left(p_i\right)} \equiv \frac{1}{2} - \sigma\left(P^*\right) > 0.$$

As $\sigma\left(P^{\mathrm{C}}\right) = 1/2$ and σ is increasing, this condition translates into $P^* < P^{\mathrm{C}}$. Thus, an upward price deviation is unprofitable if and only if the equilibrium price is at least equal to the Cournot price. What about a downward price deviation? Clearly, such deviation will be profitable if and only if $PD\left(P\right)$ is decreasing at P^*, that is, if and only if the equlibrium price is higher than the monopoly price. We thus obtain $P^* \in \left[P^{\mathrm{C}}, P^{\mathrm{m}}\right]$ and the proof is complete.

□

Thus, the price-match guarantee (together with the most-favoured-customer clause) leads in our two-stage game to the same equilibrium outcomes as the one-stage Cournot-Bertrand game of the last section. The meet-or-release clause allows to obtain higher equilibrium prices, the Cournot price being the sole that can result at equilibrium from both versions of the best-price guarantee.

3 Product Differentiation

Let us now introduce product differentiation, which is the most common case in practice. Indeed, from the producers' point of view, by supplying specific varieties of their products, firms strive to generate the market power that is associated with some niche. In other words, a way for producers to soften competition is to differentiate their products, and thus attract different types of consumers or else exploit their taste for variety. From the consumers' point of view, product differentiation has indeed two particular functions. One is to adapt different products to different types of consumers, but another is to respond to the taste any consumer may have for variety.

There are accordingly two main approaches in the literature. The first is in the Launhardt (1885) and Hotelling (1929) tradition, assuming a continuum of consumers each having an ideal variety represented by a location in the product space (a subset of the real line in the simplest case). We shall use this approach in Sect. 3 of Chap. 4, sticking now to the other one, which goes back to Chamberlin (1933) and emphasises the taste for variety. Models in this line often use the "representative consumer" device, assuming a single consumer who summarises the behaviour of the market, consuming all varieties that are offered.[7]

Equilibria of the Extended Cournot-Bertrand Game

Following Dixit and Stiglitz (1977), we assume that the representative consumer's preferences are symmetric in the different goods produced in the oligopolistic sector and can be represented by a utility function $X(\mathbf{x})$, aggregating the vector[8] $\mathbf{x}$ of quantities of those goods into a composite good. In this section, we keep our simplifying assumption of nil costs and adopt the most popular version of the Dixit and Stiglitz model where X is the CES *quantity aggregator*:

$$X(\mathbf{x}) = \left(\sum_{i=1}^{n} x_i^{(s-1)/s} \right)^{s/(s-1)}, \tag{3.1}$$

[7] In Anderson et al. (1992, chap. 3) conditions are given to show how a representative consumer's utility function and the corresponding demand function can be derived from an underlying population of consumers making discrete choices (and conversely). The special case where this representative utility is CES is also considered.

[8] Vectors are denoted in bold.

with s (positive and different from 1) the constant elasticity of substitution between goods. Minimisation at given prices of the consumer's expenditure $\mathbf{px}$ under the constraint $X(\mathbf{x}) \geq \underline{X}$ leads to the first-order condition:[9]

$$p_i = P \partial_i X(\mathbf{x}) \text{ for } i = 1, \ldots, n. \tag{3.2}$$

It is easy to check by a simple computation that the Lagrange multiplier P is a function of the price vector:

$$P(\mathbf{p}) = \left(\sum_{i=1}^{n} p_i^{1-s} \right)^{1/(1-s)}, \tag{3.3}$$

which can be seen as a *price aggregator*. Observe that, when s tends to infinity (the perfect substitutability case of the preceding sections), the quantity and price aggregators tend respectively to $X(\mathbf{x}) = \sum_i x_i$ and $P(\mathbf{p}) = \min(\mathbf{p})$.[10] By computing the derivative $\partial_i X(\mathbf{x})$, we see that the first-order condition expresses a relation between the *market share* and the relative price, namely

$$\frac{x_i}{X(\mathbf{x})} = \left(\frac{p_i}{P(\mathbf{p})} \right)^{-s}. \tag{3.4}$$

If we maintain the partial equilibrium viewpoint of the preceding section, the *market size* of the composite good is simply assumed to be

---

[9] We use the simplifying notations:

$$\partial_i F(\mathbf{x}) = \frac{\partial F(\mathbf{x})}{\partial x_i} \text{ and } \epsilon_i F(\mathbf{x}) = \frac{\partial F(\mathbf{x})}{\partial x_i} \frac{x_i}{F(\mathbf{x})}.$$

[10] A second limit case is the Leontief case of perfect complementarity ($s = 0$), where the quantity and price aggregators are respectively given by $X(\mathbf{x}) = \min(\mathbf{x})$ and $P(\mathbf{p}) = \sum_i p_i$. A third limit case is the Cobb-Douglas case ($s = 1$), where the quantity and price aggregators are respectively given by $X(\mathbf{x}) = \prod_i x_i$ and $P(\mathbf{p}) = \prod_i p_i$.

determined by the demand

$$\underline{X} = D(P), \tag{3.5}$$

where the function D has the same properties as before, namely is decreasing, twice continuously differentiable and has an increasing Marshallian elasticity $\sigma(P) \equiv -D'(P)P/D(P)$ with values below and above 1. We can then extend our concept of Cournot-Bertrand game to the present context of product differentiation by adapting the market share and market size constraints, each producer being now assumed to solve the following programme:

$$\begin{aligned} &\max_{(p_i,x_i)} p_i x_i \\ &\text{s.t. } x_i \leq \left(p_i/P\left(p_i, \mathbf{p}_{-i}\right)\right)^{-s} X\left(x_i, \mathbf{x}_{-i}\right) \\ &\text{and } X\left(x_i, \mathbf{x}_{-i}\right) \leq D\left(P\left(p_i, \mathbf{p}_{-i}\right)\right). \end{aligned} \tag{3.6}$$

In the duopoly case of perfect substitutable goods ($n = 2$ and $s = \infty$), this programme coincides with (1.3), except for the symmetry condition applying to the case of equal prices set by the two duopolists. Notice also that the two constraints can be equivalently formulated in the dual form:

$$\begin{aligned} &p_i \leq \left(x_i/X\left(x_i, \mathbf{x}_{-i}\right)\right)^{-1/s} P\left(p_i, \mathbf{p}_{-i}\right) \text{ and} \\ &P\left(p_i, \mathbf{p}_{-i}\right) \leq D^{-1}\left(X\left(x_i, \mathbf{x}_{-i}\right)\right). \end{aligned} \tag{3.7}$$

We can state the following proposition establishing the main equilibrium conditions.

Proposition 3 *If $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ is an equilibrium of the extended Cournot-Bertrand game characterised by the programme (3.6), the following conditions*

must be satisfied for $i = 1, \ldots, n$*:*
for some vector $\boldsymbol{\theta} \in [0,1]^n$,

$$\frac{\theta_i \left(1 - \alpha_i^*\right) + (1 - \theta_i)\, \alpha_i^*}{\theta_i \left(1 - \alpha_i^*\right) s + (1 - \theta_i)\, \alpha_i^* \sigma \left(P\left(\mathbf{p}^*\right)\right)} = 1 \tag{3.8}$$

and, alternatively, for some vector $\boldsymbol{\theta}' \in [0,1]^n$,

$$\frac{\theta_i' \left(1 - \alpha_i^*\right)(1/s) + \left(1 - \theta_i'\right) \alpha_i^* \left(1/\sigma \left(P\left(\mathbf{p}^*\right)\right)\right)}{\theta_i' \left(1 - \alpha_i^*\right) + \left(1 - \theta_i'\right) \alpha_i^*} = 1, \tag{3.9}$$

with $\alpha_i^* \equiv p_i^* x_i^* / \mathbf{p}^* \mathbf{x}^*$ *denoting the budget share of good* i.

Proof Oligopolist i's revenue maximisation under the two constraints of (3.6) can be reformulated, as above for the perfect substitutability case and also by preserving dimensional homogeneity, in terms of the maximisation in (p_i, x_i) of the Lagrangian function

$$\begin{aligned} \mathcal{L}\left(p_i, x_i, \lambda_i, \nu_i\right) = p_i x_i - \lambda_i \left(x_i \left(\frac{p_i}{P\left(p_i, \mathbf{p}_{-i}\right)} \right)^s - X\left(x_i, \mathbf{x}_{-i}\right) \right) \\ - \nu_i \left(X\left(x_i, \mathbf{x}_{-i}\right) - D\left(P\left(p_i, \mathbf{p}_{-i}\right)\right)\right). \end{aligned} \tag{3.10}$$

We can easily compute the first-order conditions at an equilibrium where both constraints are binding (denoting $P^* = P\left(\mathbf{p}^*\right)$ and $X^* = X\left(\mathbf{x}^*\right)$):

$$x_i^* = \lambda_i \frac{1 - p_i^* \partial_i P^* / P^*}{p_i^*} X^* s - \nu_i D'\left(P^*\right) \partial_i P^* \tag{3.11}$$

and

$$p_i^* = \lambda_i \left(\frac{X^*}{x_i^*} - \partial_i X^* \right) + \nu_i \partial_i X^*. \tag{3.12}$$

From (3.4), it is easy to establish:

$$\frac{p_i^* \partial_i P^*}{P^*} = \left(\frac{p_i^*}{P^*}\right)^{1-s} = \frac{p_i^* x_i^*}{P^* X^*} = \left(\frac{x_i^*}{X^*}\right)^{(s-1)/s} = \frac{x_i^* \partial_i X^*}{X^*} \text{ and}$$
$$P^* X^* = \mathbf{p}^* \mathbf{x}^*, \tag{3.13}$$

so that $\epsilon_i P^* = \epsilon_i X^* = \alpha_i^*$. By multiplying both sides of (3.11) and (3.12) by p_i^* and x_i^*, respectively, and equalising the two resulting expressions of $p_i^* x_i^*$, we then obtain

$$\lambda_i \left(1 - \alpha_i^*\right) s + \nu_i \alpha_i^* \sigma \left(P^*\right) = \lambda_i \left(1 - \alpha_i^*\right) + \nu_i \alpha_i^*$$

and, by denoting $\theta_i \equiv \lambda_i / (\lambda_i + \nu_i)$, formula (3.8).

If we replace the two constraints of (3.6) by their dual (3.7), we can use the Lagrangian function

$$\mathcal{L}' \left(p_i, x_i, \lambda_i, \nu_i\right) = p_i x_i - \lambda_i' \left(p_i \left(\frac{x_i}{X \left(x_i, \mathbf{x}_{-i}\right)}\right)^{1/s} - P \left(p_i, \mathbf{p}_{-i}\right)\right)$$
$$- \nu_i' \left(P \left(p_i, \mathbf{p}_{-i}\right) - D^{-1} \left(X \left(x_i, \mathbf{x}_{-i}\right)\right)\right). \tag{3.14}$$

First-order conditions at an equilibrium where both constraints are binding are then

$$x_i^* = \lambda_i' \left(\frac{P^*}{p_i^*} - \partial_i P^*\right) + \nu_i' \partial_i P^* \tag{3.15}$$

and

$$p_i^* = \lambda_i' \left(1 - x_i^* \partial_i X^* / X^*\right) \frac{P^*}{x_i^*} \frac{1}{s} - \nu_i' \frac{\partial_i X^* / X^*}{D' \left(P^*\right) / D \left(P^*\right)}. \tag{3.16}$$

By multiplying both sides of these equations by p_i^* and x_i^*, respectively, and equalising the two resulting expressions of $p_i^* x_i^*$, we obtain

$$\lambda_i' \left(1 - \alpha_i^*\right) + \nu_i' \alpha_i^* = \lambda_i' \left(1 - \alpha_i^*\right) \frac{1}{s} + \nu_i' \alpha_i^* \frac{1}{\sigma\left(P^*\right)} \tag{3.17}$$

and, by denoting $\theta_i' \equiv \lambda_i' / \left(\lambda_i' + \nu_i'\right)$, formula (3.9). □

The left-hand sides of Eqs. (3.8) and (3.9) are weighted means (harmonic and arithmetic, respectively) of the reciprocals of the elasticity of substitution s and of the elasticity of demand $\sigma\left(P^*\right)$. These reciprocals are, in absolute value, the elasticities of the two frontiers defined by the constraint on market share (for $1/s$) and by the constraint on market size (for $1/\sigma\left(P^*\right)$) in the space $x_i \times p_i$, at their point of intersection. The parameters θ_i and θ_i', the relative weights on the market share constraint for firm i and at a specific equilibrium, can be interpreted as measuring the *competitive toughness* displayed at that equilibrium by that firm.

To compare the present analysis with that of the Cournot-Bertrand game of Sect. 1, we must take into account the non-differentiability of the price aggregator $P\left(\mathbf{p}\right) = \min\left(\mathbf{p}\right)$ when $s = \infty$, making us distinguish the two cases $\partial_i^+ P^* = 0$ and $\partial_i^- P^* = 1$ for upward and downward price deviations, respectively. An equilibrium in which the constraint on market share is the only binding constraint is necessarily associated with the so-called Bertrand solution $P^* = 0$, since $\nu_i' = 1/s = 0$ in Eq. (3.16). In the opposite case, an equilibrium in which the constraint on market size is the only binding constraint, hence with $\theta_i' = \lambda_i' = 0$, will not be associated with the collusive solution $\sigma\left(P^*\right) = 1$, as a simple inspection of condition (3.9) might suggest. The reason is that $\alpha_i^* = \epsilon_i X^*$ in the numerator of the LHS of (3.9), whereas $\alpha_i^* = \epsilon_i^- P^* = 1$ in the corresponding denominator, leading to the Cournot solution $\sigma\left(P^*\right) = 1/n$ under symmetry.

Regimes of Competition

Assuming $s < \infty$, it is easy to show that the outcomes of standard price and quantity equilibria can be obtained as equilibrium outcomes of the extended Cournot-Bertrand game. Consider indeed the case where $\theta_i = 1/2$ or $\lambda_i = \nu_i$ for any firm i, a case in which the Lagrangian function (3.10) becomes

$$\mathcal{L}(p_i, x_i, \lambda_i, \lambda_i) = p_i x_i - \lambda_i \left(x_i \left(\frac{p_i}{P(p_i, \mathbf{p}_{-i})} \right)^s - D\left(P\left(p_i, \mathbf{p}_{-i}\right)\right) \right), \tag{3.18}$$

coinciding with the Lagrangian of the programme

$$\max_{(p_i, x_i)} p_i x_i \text{ s.t. } x_i \leq \left(\frac{p_i}{P(p_i, \mathbf{p}_{-i})} \right)^{-s} D\left(P\left(p_i, \mathbf{p}_{-i}\right)\right). \tag{3.19}$$

This programme is equivalent to

$$\max_{p_i} p_i \left(\frac{p_i}{P(p_i, \mathbf{p}_{-i})} \right)^{-s} D\left(P\left(p_i, \mathbf{p}_{-i}\right)\right), \tag{3.20}$$

the programme characterising a price equilibrium.

Similarly, if we take $\theta_i' = 1/2$, or $\lambda_i' = \nu_i'$ for any firm i, we obtain the Lagrangian

$$\mathcal{L}'(p_i, x_i, \lambda_i, \lambda_i) = p_i x_i - \lambda_i' \left(p_i \left(\frac{x_i}{X(x_i, \mathbf{x}_{-i})} \right)^{1/s} - D^{-1}\left(X\left(x_i, \mathbf{x}_{-i}\right)\right) \right),$$

the maximisation of which in (p_i, x_i) is equivalent to the programme

$$\max_{x_i} x_i \left(\frac{x_i}{X(x_i, \mathbf{x}_{-i})} \right)^{-1/s} D^{-1}\left(X\left(x_i, \mathbf{x}_{-i}\right)\right) \tag{3.21}$$

characterising a quantity equilibrium.

Other standard regimes of competition involve identical extreme competitive conducts with a sole binding constraint at equilibrium. If goods are independent ($s = 1$), the oligopolists play in fact as monopolists, each one in his own niche, so that the equilibrium is the monopoly one, with $\sigma(P^{\mathrm{m}}) = 1$. If $s \neq 1$, the toughest conduct, with the constraint on market share being the sole binding ($\theta = 1$), leads necessarily in the present context of nil costs, by (3.8) or (3.9), to corner solutions, both resulting in zero profits. Hence, if goods are substitutable ($s > 1$), so that the oligopolists face an elastic demand, the equilibrium price must be zero. If goods are complementary ($s < 1$), so that the demand is inelastic, the equilibrium quantity must be zero.

At the other extreme, we have the softest competition regime, with the constraint on market size as the sole binding ($\theta = 0$). This is of course tacit collusion, provided this regime is enforceable as a non-cooperative equilibrium. Indeed, as goods become highly substitutable (resp. highly complementary), there is an incentive for oligopolists to make a downward (resp. upward) price deviation and the softest enforceable competitition regime is one with positive competitive toughness, becoming larger as the elasticity of substitution increases to infinity (resp. decreases to zero).

Figure 1.1 presents an illustration of such unenforceability of the collusive solution when goods are substitutes.[11] In the quantity-price space of firm i, the two frontiers are represented by continuous curves (the market share frontier for higher prices, the market size frontier for lower prices, in this example with substitutable goods)[12] intersecting at

[11]The curves are computed for $n = 2$, $s = 10$ and linear demand $D(P) = 2 - P$. The collusive solution is given by the monopoly price and quantity $P^{\mathrm{m}} = X^{\mathrm{m}} = 1$ and, for each duopolist, $p^{\mathrm{m}} = 0.5^{1/(1-s)} \simeq 1.08$ and $x^{\mathrm{m}} = 0.5^{s/(s-1)} \simeq 0.46$.

[12]The relative position of the two frontiers would be inverted in the case of complementary goods. The two frontiers are reminiscent of the two demand curves considered by Chamberlin: the curve DD', showing "the falling off in sales which would attend an increase in price, *provided* other prices did not also increase," and the curve dd', showing "the demand for the product of any one seller at various prices on the assumption that his competitors' prices are always identical with his," the latter "evidently [] much less elastic than the former" (Chamberlin, 1933, p. 90). Chamberlin's curves are explicitly the branches of the *kinked demand curve* of Hall and Hitch (1939, pp. 23–24 and 29n) and, implicitly, of Sweezy (1939, p. 569).

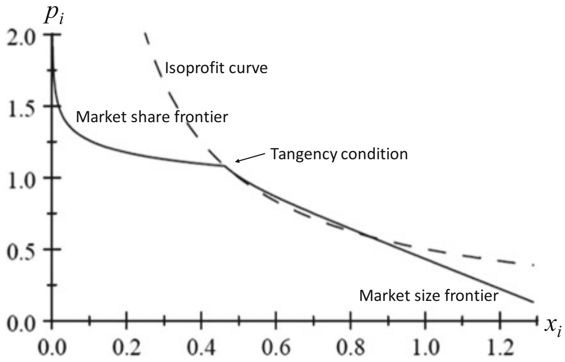

Fig. 1.1 Unenforceable collusion

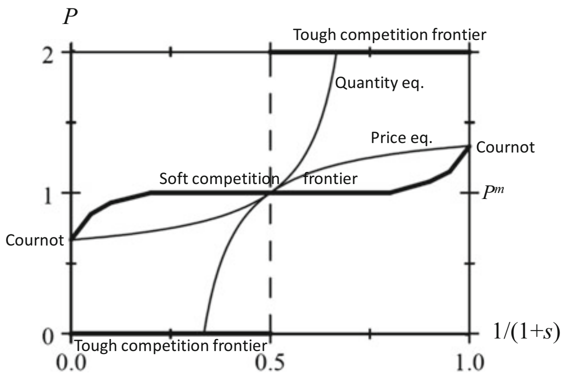

Fig. 1.2 Equilibrium regimes

the collusive values $\left(x_i^{\mathrm{m}}, p_i^{\mathrm{m}}\right)$. The dashed isoprofit curve going through this point and tangent to the market size frontier is partly below this frontier, so that a downward price deviation would ensure a higher profit without violating the constraint on market size.

Let us now represent the partition of the parameter space into different regimes of competition. For this representation, we will take $n = 2$ and use linear demand $D(P) = 2 - P$. In Fig. 1.2, we display on the horizontal axis a normalised index of complementarity $1/(1+s)$ (0 corresponding to perfect substitutability and 1 to perfect complementarity) and on the vertical axis the price aggregator P. Equilibrium prices correspond to the two regions between the thick curves, for substitutable goods ($s > 1$) on the left and for complementary goods ($s < 1$) on the right. The horizontal thick segments at $P = 0$ for $s > 1$ and at $P = 2$ for $s < 1$ form the *tough competition frontier*, with the sole binding market share constraint ($\theta = 1$). The thick curve switching from concave to convex is the *soft competition frontier*, that is, the frontier representing the softest competition regimes that are enforceable. It has an intermediate horizontal piece at $P = 1$, the collusive solution, with the sole binding market size constraint ($\theta = 0$), and links the two Cournot solutions: the one for homogeneous duopoly ($s = \infty$) at its left extremity $(0, 2/3)$ and the one for complementary monopoly ($s = 0$) at its right extremity $(1, 4/3)$.

Observe that the tough competition frontier lies below (resp. above) the soft competition frontier for $s > 1$ (resp. $s < 1$), so that the usual idea that intensifying competition allows to lower prices and improve welfare works only for substitutable goods, as already established by Cournot (for the two extreme cases $s = \infty$ and $s = 0$).[13] The thin concave (resp. convex) curve is the locus of *price equilibria* (resp. *quantity equilibria*).[14]

Notice that the Cournot homogeneous duopoly solution coincides with a quantity equilibrium and the Cournot complementary monopoly solution with a price equilibrium, reflecting the conventional view of dual quantity vs. price equilibrium concepts in Cournot. However, the common characteristic of the two Cournot solutions is that they are the outcome of the softest competitive conduct among those that are non-cooperatively enforceable. Here lies, we believe, the essential characteristic opposing Cournot to Bertrand.

[13] In the complementary monopoly regime, where producers are *concurring*, "an association of monopolists, working for their own interest, in this instance will also work for the interest of consumers, which is exactly the opposite of what happens with *competing* producers" (Cournot 1838 [1897], p. 103). See also Ellet (1839, pp. 79–80) for a similar observation in a spatial model of complementary monopoly.

[14] Equilibrium prices are always lower at price equilibria than at quantity equilibria, independently of goods being substitutable or complementary. This is consistent with Vives (1985).

4 Concentration and Market Power

After conduct and product differentiation, we consider a third important factor influencing market power, namely concentration. Concentration is the expression of an increase in market shares, either as the consequence of a decrease in the number of competitors or of quantity asymmetry at equilibrium. We shall first look at the way concentration varies with the number of firms in a symmetric context, and then consider how concentration varies with the dispersion of market shares.

Varying the Number of Competitors

That concentration varies with the number n of firms is the phenomenon emphasised by Cournot. For symmetric equilibria, the market share of the individual firm is $\alpha^* = 1/n$ and, as n indefinitely increases, this market share correspondingly decreases indefinitely. Market shares (and, with them, market power) eventually become negligible and competition *indefinite*, perhaps a more appropriate term than *perfect*, nevertheless preferred by Cournot's posterity.

Product differentiation is however a way of resisting to the complete vanishing of market power as n tends to infinity and, consequently, α_i^* to zero. In Eq. (3.9), which expresses first-order conditions, the left-hand side—an index of market power—tends indeed to $1/s$, a positive value which increases with the degree of product differentiation. This is the founding feature of Chamberlin's monopolistic competition within a large group, where firms are infinitesimal but retain some monopoly power. Now, it is true that the present model specification, with constant elasticity of substitution and nil production costs, implies a corner solution in the limit. So, the residual market power cannot keep on being exploited as n tends to infinity, but we will see in the next chapter that this feature is specific to the zero-cost case.

Looking at Eq. (3.8) and referring to the symmetric case, we observe that the weights put on the reciprocals of the two elasticities, of substitution s and of demand $\sigma(P^*)$, are proportional to $\theta(1 - 1/n)$ and $(1 - \theta)(1/n)$, respectively. This means that variations in conduct may be seen as substitutes to variations in structure: increasing n to obtain a large group in the sense of Chamberlin (the usual regime of monopolistic competition) is equivalent to moving θ towards its ceiling. This is the essence of *Bertrand's paradox* (in the homogeneous oligopoly case): two competitors are enough to obtain the perfectly competitive outcome, provided they compete with maximum toughness.

As market power is jointly determined by structure and conduct or, more explicitly, by concentration and competitive toughness, there is a trade-off for antitrust authorities. A price decrease will increase welfare, if welfare is evaluated as the sum of profit and consumer surplus. But how to ensure this price decrease? By diminishing concentration or by augmenting competitive toughness? The erosion of market power due to the entry of new firms suggests the idea that the absence of barriers to entry should be enough to ensure the vanishing of market power or at least, in the Chamberlinian tradition, the dilution of profits. According to this idea, *free entry* would be fully characterised by the zero-profit condition applied to a specific regime of competition: as long as profits are positive, there is an incentive for new competitors to enter the market. This idea supposes symmetry, or more precisely *equal treatment* of all firms having decided to enter. Although there is no imperative reason to assume post-entry equal treatment, we shall now look at the trade-off between the toughness of the competition regime and the number of competitors entering the market under free entry (following d'Aspremont and Motta 2000a and 2000b).[15] This will first be done by comparing the maximum number of viable firms, able to cover a positive fixed cost ϕ, under the two alternative regimes of price and quantity competition. Then we will compare the total welfare obtained in the oligopolistic sector under these two regimes, when applying the zero-profit condition.

In our simple framework, given n active producers, total welfare W^n is the sum of consumer surplus $X(\mathbf{x}) - \mathbf{px}$ and producers' surplus $\mathbf{px} - n\phi$, hence $W^n = X(\mathbf{x}) - n\phi$. Trivially, given n, W^n increases with X or, equivalently, decreases with P. The price equilibrium entails consequently a higher total welfare than the quantity equilibrium. How-

[15] See also d'Aspremont et al. (2000b).

ever, the number of viable firms is not necessarily the same under the two regimes, so that the trade-off between that number and competitive toughness has to be taken seriously. When varying the number of firms two important scale effects intervene. The first is on the production side: increasing the number of producers represents an efficiency loss due to the multiplication of individual fixed costs. The second is on the consumption side: increasing the number of products better satisfies the consumer's taste for variety.

A simple example will illustrate this issue. Assume the linear demand $D(P) = A - P$, so that $\sigma(P) = P/(A - P)$. Following the arguments in the last subsection, the equilibrium price for n firms can be computed to be

$$P^{\mathrm{B}n} = \left(1 - \frac{1}{n + 1 - (n-1)s}\right) A \text{ and } P^{\mathrm{C}n} = \frac{1}{n + 1 - (n-1)/s} A \tag{4.1}$$

for a price game and for a quantity game, respectively. Enforceability as a free entry equilibrium of the quantity equilibrium with 3 firms and of the price equilibrium with 2 firms results from the following condition:[16]

$$\max\left\{\frac{P^{\mathrm{B}3}\left(A - P^{\mathrm{B}3}\right)}{3}, \frac{P^{\mathrm{C}4}\left(A - P^{\mathrm{C}4}\right)}{4}\right\} < \phi \leq \min\left\{\frac{P^{\mathrm{B}2}\left(A - P^{\mathrm{B}2}\right)}{2}, \frac{P^{\mathrm{C}3}\left(A - P^{\mathrm{C}3}\right)}{3}\right\}. \tag{4.2}$$

Taking $s = 1.25$, this condition translates into

$$\max\{0.0741, 0.0617\} < \phi/A^2 \leq \min\{0.122, 0.081\}.$$

[16] A similar example has been developed for the linear-quadratic utility case by d'Aspremont and Motta (2000b).

We next proceed to welfare comparisons. With the adopted demand specification the consumer surplus becomes $(A - P)(1 - P)$ which, if positive, is a decreasing function of P. Using (4.1), we compute:

$$P^{\mathrm{B2}} = 0.429A > 0.417A = P^{\mathrm{C3}}, \tag{4.3}$$

so that the price is lower and the consumer surplus higher at the free entry quantity equilibrium in spite of this regime displaying a lower competitive toughness. As to total welfare, the quantity equilibrium still dominates the price equilibrium even though total fixed costs are equal to 3ϕ for the former and only 2ϕ for the latter provided $\phi/A < 0.012$ (compatible with $0.074 < \phi/A^2 \leq 0.081$ if $A < 0.162$).

Empirical evidence seems to reinforce such conclusions. In the United States, a consequence of the antitrust policy introduced by the Sherman Act (1890) was a sharp increase in the number of mergers (Bittlingmayer 1985), hence reducing the number of competitors in many industries. A similar consequence was observed in Europe, after the Rome Treaty (1957) and its articles against collusive behaviour, so that a new regulation on mergers and acquisitions was introduced in 1989.

Dispersion of Market Shares

Up to now, we have limited our analysis to symmetric games and focused on symmetric equilibria. With a fixed number of competitors, concentration may result from advantages in product characteristics or in production costs leading to the dominance of some firms or even to the elimination of their rivals. It can however arise even in a symmetric game. In the homogeneous duopoly examined in Sect. 1, equilibria are necessarily symmetric by construction, as a result of the constraint on market share as formulated in (1.3). Even disregarding this constraint, the standard Cournot solution of a symmetric game is necessarily symmetric: in the case of nil costs, for instance, the equilibrium condition imposes that the market share of the two duopolists be equal to the *same* Marshallian demand elasticity.

This is however no longer the case as soon as we refer to two constraints and suppress the symmetry requirement in the constraint on market share following its formulation in (3.6). We can then use Eq. (3.9) in Proposition 3. This equation is compatible with the corresponding equation for firm j with $\alpha_i^* < \alpha_j^*$, provided $\theta'_i < \theta'_j$: tougher competitors obtain higher market shares. And since concentration increases with dispersion of market shares (because the Herfindahl index of concentration—the sum of squared market shares—is strictly convex), it increases with the dispersion of competitive conducts.

An extreme case of asymmetry is provided by equilibria in which only part of the firms are active, the best response of the others being to remain inactive. Existence of equilibria of this type commands a redefinition of the concept of free entry equilibrium once we abandon the symmetry assumption and require *equal opportunities*, not equal treatment at equilibrium, for all the firms (see d'Aspremont et al. 2000a, and Dos Santos Ferreira and Dufourt 2007).

As an example, let us consider the simple case of the homogeneous oligopoly ($s = \infty$). Take n as the number of *active* firms (having a positive production). For a $2n$-tuple $\left(\left(p_1^*, x_1^*\right), \ldots, \left(p_n^*, x_n^*\right)\right)$ to be a free entry equilibrium of the extended Cournot-Bertrand game, a first condition is that the pair $\left(p_i^*, x_i^*\right)$ solve for any active firm i the programme (3.6), being in addition *profitable*, that is, ensuring non-negative profits ($p_i^* x_i^* \geq \phi$). A second condition is that any admissible pair $\left(p_j, x_j\right)$, satisfying both constraints of programme (3.6), lead to a non-positive profit ($p_j x_j \leq \phi$) for any inactive firm j, with $x_j^* = 0$. If there is at least one inactive firm, we will then say that the equilibrium is *sustainable* under free entry.

To give an illustration, take the linear demand function $D(P) = 2a - P$. We know, by Proposition 1, that the highest possible equilibrium price, symmetric with respect to n active firms, is the *Cournot price* $P_n^{\mathrm{C}} = 2a/(n+1)$, satisfying $\sigma\left(P_n^{\mathrm{C}}\right) = 1/n$. The aggregate profitability condition for n active firms is $PD(P) \geq n\phi$, so that an equilibrium price must belong to the interval $\left[P_n, P_n^{\mathrm{C}}\right] = \left[a - \sqrt{a^2 - n\phi}, 2a/(n+1)\right]$, P_n being the *break-even price*. This interval is non-empty if $n \leq 2a/\sqrt{\phi} - 1 < a^2/\phi$. For an inactive firm, setting a price p smaller than the equilibrium price P^* is a necessary condition to have a positive (residual) demand $2a - p - (2a - P^*) = P^* - p$. Sustainability imposes that $\max_p p (P^* - p) = (P^*/2)^2 \leq \phi$, an inequality which defines the *limit price* $\overline{P} = 2\sqrt{\phi}$, that is, the maximum sustainable equilibrium price. Hence, a free entry equilibrium price P^* must belong to the interval $\left[P_n, \min\left(P_n^{\mathrm{C}}, \overline{P}\right)\right]$, which is non-empty for $n \leq \min\left(4a/\sqrt{\phi} - 3, 2a/\sqrt{\phi}\right) - 1$.

As an extreme case of a free entry equilibrium, we have the monopoly equilibrium in which both the single active firm and the inactive firm(s) set the price $P^* = \min\left(P^{\mathrm{m}}, \overline{P}\right)$, with $P^{\mathrm{m}} = P_1^{\mathrm{C}} = a$, the former choosing the quantity $D(P^*)$. The condition for existence of this equilibrium is $2 \leq \min\left(4a/\sqrt{\phi} - 3, 2a/\sqrt{\phi}\right)$, implying $a/\sqrt{\phi} \geq 5/4$. Thus, a free entry monopoly equilibrium exists even if, say, a symmetric Cournot oligopoly equilibrium with n active firms remains profitable (if $a/\sqrt{\phi} \geq (n+1)/2$), contrary to the conventional view of a free entry equilibrium, making it depend upon negative profits for all firms once a new entry takes place.

Notice further that the condition $P_n \leq P_n^{\mathrm{C}} \leq \overline{P}$, ensuring existence of a free entry symmetric Cournot oligopoly equilibrium, is $(n-1)/2 \leq a/\sqrt{\phi} - 1 \leq n$. This condition leaves place to (a maximum of) $\underline{n} + 2$ integers, where $\underline{n}$ is the least integer that is at least equal to $a/\sqrt{\phi} - 1$.[17] Hence, the oligopolistic equilibrium indeterminacy introduced by more or less aggressive competitive conduct results in addition from the actual participation of more or less potential competitors. From a macroeconomic point of view, such indeterminacy is significant as a possible source of stochastic endogenous fluctuations (see Dos Santos Ferreira and Dufourt 2006, Dos Santos Ferreira and Lloyd-Braga 2008).

[17] The condition is $(n-1)/2 \leq a/\sqrt{\phi} - 1 \leq n$. The least integer satisfying this condition is $\underline{n} = \left\lceil a/\sqrt{\phi} - 1\right\rceil$, and the greatest integer $\overline{n} = \left\lfloor 2a/\sqrt{\phi} - 1\right\rfloor$. Hence, $\overline{n} - \underline{n} \leq \left(2a/\sqrt{\phi} - 1\right) - \left(a/\sqrt{\phi} - 1\right) = a/\sqrt{\phi} \leq \underline{n} + 1$, or $\overline{n} = \underline{n} + k$, with $k = 0, 1, \ldots, \underline{n} + 1$.

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2

Competition for Market Share and for Market Size

In this chapter, we build on the ideas developed in Chap. 1 to formulate a more general model, although portraying the simplified economy imagined by Dixit and Stiglitz (1977), an economy reduced to two sectors, one oligopolistic, the other competitive, with firms selling each a single good to a representative consumer. In the following, we will first present the canonical model and then explore several extensions. Assuming either weak or homothetic separability, we define a general concept of oligopolistic equilibrium. The first-order conditions are used to derive a simple formula where the relative markup is a function of the intra- and intersectoral elasticities of substitution. This leads to a parameterisation of equilibria in terms of firms' competitive toughnesses defining possible regimes of oligopolistic competition. It is then shown that such a formula is robust to supposing that firms take into account the income feedback effects of distributed income (the so-called Ford effects). In the homogeneous good case, we compare our approach to alternative ones, such as the conjectural variation and the supply function approaches. Finally, the methodology is applied to two policy questions: Is tougher competition price decreasing? Does it foster innovation?

C. d'Aspremont, R. Dos Santos Ferreira, *The Economics of Competition, Collusion and In-between*, https://doi.org/10.1007/978-3-030-63602-9_2

Before providing the technical details, we want to emphasise that the two-sector economy is the essential basis for the canonical model in which we introduce our oligopolistic equilibrium concept. In this economy the fundamental feature is that the representative consumer preferences are assumed weakly separable relative to the two sectors. The differentiated goods produced in the oligopolistic sector can thus be aggregated into a composite good (through the sub-utility function defined on these goods) and the aggregate good produced in the competitive sector, representing the rest of the economy, is taken as numeraire. Applying two-stage budgeting, one obtains the main ingredients to be used in our own setting: (1) within the oligopolistic sector, the Hicksian demand for each differentiated good and the corresponding intrasectoral elasticity of substitution of each such good for the composite good and (2) across sectors, the Marshallian demand for the composite good and the corresponding intersectoral elasticity of substitution of each good for aggregate consumption.

An oligopolistic equilibrium supposes that each firm in the oligopolistic sector maximises profit (allowing for no-production) in both price and quantity, under two constraints, on market share (depending upon Hicksian demand) and on market size (depending upon Marshallian demand). It is from the first-order conditions that we get the basic equations that should be used for estimation purposes. These equations determine the equilibrium markup of each firm as a weighted mean of the reciprocals of the two elasticities of substitution. The crucial advantage is that the weights explicitly involve the firm's "competitive toughness" as a continuous parameter (varying between 0 and 1) derived from the Lagrange multipliers associated with the two constraints. By varying the competitive toughness parameters we hence get a continuum of regimes of competition, including standard ones such as tacit collusion, pure price and pure quantity equilibria.

Assuming symmetry and negligible market shares, we retrieve the Dixit-Stiglitz monopolistic competition equilibrium, with the equilibrium markup just equal to the reciprocal of the intrasectoral elasticity of substitution, simplicity being however counterbalanced by the loss of any intersectoral effects. As we will see, the same reduced form can be obtained, even with large firms, via cutthroat competition (maximal competitive toughness), an illustration of the so-called Bertrand paradox. Competitive toughness is thus an important factor to explain firm heterogeneity. In a recent empirical study, Hottman et al. (2016) observe that firms with the largest market shares have "substantially higher markups" and that this effect is much greater under quantity competition than under price competition.[1] Firm heterogeneity is thus reinforced by softer competition. However, from our point of view, this statement should not be reduced to the Cournot-Bertrand dichotomy, but apply to the continuum of possible competition regimes between the two extremes of cutthroat competition and pure collusion.

The existence of large firms implies that these firms influence the size of their own market through the income they distribute. That such income feedback effects can be taken into account by large firms was already well illustrated by the industrialist Henry Ford in the 1920s when advocating a high wage policy: "Our own sales depend in a measure upon the wages we pay" (Ford 1922, p. 124). This kind of effects can be integrated into our canonical model. In particular, in the case where the firms internalise the income feedback effect within the oligopolistic sector, taking as given the expenditure in the competitive sector, we will show that the equilibrium markup formula will keep the same structure. These "Ford effects" essentially modify the relevant elasticity of intersectoral substitution.

Coming back to the possible continuum of equilibria in oligopolistic markets, we shall mention three previous approaches (the conjectural variation, the supply function and the pricing scheme approaches) explaining this indeterminateness in the special case where all firms produce the same homogeneous good. These alternative approaches have been useful in different contexts, for example in the studies of the New Empirical Industrial Organisation (NEIO) for the conjectural variations approach, in the analysis of electricity markets for the supply function equilibrium approach, and to model facilitating practices for the pricing schemes

[1]To quote: "In most sectors, the largest firm has a market share above 20%, which enables it to charge a markup that is 24% higher than that of the median firm under price competition and double that under quantity competition." (Hottman et al., 2016, p.5).

approach. We will show that by restricting properly the admissible class of the corresponding instruments (conjectural variations, supply functions or pricing schemes), we can recover exactly the set of oligopolistic outcomes.

To end this chapter, we will discuss two important applications. First, to enhance the role of the two elasticities of substitution, we shall examine the price effects of intensifying competition. Both under price and quantity competition, an average markup can be written as a function of the Herfindhal index of concentration. This function is increasing (resp. decreasing) if the intersectoral elasticity of substitution is smaller (resp. larger) than the intrasectoral elasticity of substitution, thus implying a pro-competitive (resp. anti-competitive) effect of abating concentration. Second, we will use our continuous measure of competitive toughness to re-examine the classical debate on the role of competition for firm innovative activity, opposing the Darwinian view for which a competitive firm is forced (or has more incentives, according to Arrow 1962) to innovate than a monopolist, and the Schumpeterian view for which innovation requires some monopoly rent.

1 The Canonical Model

The main idea underlying our model is that competition has essentially two dimensions: a dimension of conflicting interests of firms fighting against each other for their market shares and a dimension of convergent interests of firms implicitly competing together, against the other sector, for their market size.

A Representative Consumer with General Separable Preferences

Assuming existence of a representative consumer with preferences separable with respect to the two composite goods supplied by the two sectors allows to use the standard analytical framework for consumption decisions and in particular to exploit duality, a very convenient property for the study of price-quantity competition. We suppose that the representative consumer supplies inelastically L units of labour at a wage equal to 1 (the labour productivity in the competitive sector) and receives a profit Π from the imperfectly competitive sector (the equilibrium profit of the other sector being necessarily zero). He chooses a basket $\mathbf{x} \in [0, \infty)^N$ of N differentiated goods (sold at prices $\mathbf{p} \in (0, \infty]^N$) and a quantity $z \in [0, \infty)$ of the numeraire good (an *implicitly* composite good resulting from the aggregation of the rest of the economy). This choice is made so as to maximise, with an income $Y = L + \Pi$ and under the budget constraint $\mathbf{px} + z \leq Y$, a separable utility function $U(X(\mathbf{x}), z)$. The utility function U and the sub-utility function X, which aggregates the quantities of the differentiated goods into the volume of a composite good, are assumed increasing and strongly quasi-concave (except, for X, in the linear and Leontief limit cases and, for U, in the case of quasilinearity[2] in z). Notice that, apart from standard properties of the utility function and from separability, essential for a Dixit-Stiglitz economy, we are not imposing homotheticity, additivity or symmetry to the aggregator function X.

The maximisation can be performed in two stages, to which correspond, as we will see, the two mentioned dimensions of competition. At the first stage, the consumer chooses the quantity x_i of each differentiated good i given some quantity $\underline{X}$ of the composite good (some level of sub-utility X), by solving the programme

$$\min_{x \in \mathbb{R}_+^N} \left\{ \mathbf{px} \,\middle|\, X(\mathbf{x}) \geq \underline{X} \right\} \equiv e\left(\mathbf{p}, \underline{X}\right), \tag{1.1}$$

which defines the expenditure function e. We obtain:

$$p_i = \partial_X e\left(\mathbf{p}, \underline{X}\right) \partial_i X(\mathbf{x}) \qquad \text{(first-order condition)} \tag{1.2}$$

$$x_i = \partial_{p_i} e\left(\mathbf{p}, \underline{X}\right) \equiv H_i\left(\mathbf{p}, \underline{X}\right) \qquad \text{(Shephard's lemma)}, \tag{1.3}$$

[2]This is the special case where $U(X(\mathbf{x}), z) = \hat{U}(X(\mathbf{x})) + z$. It is the one considered by Spence (1976) seminal contribution.

where H_i is the *Hicksian demand* function for good i (associated with sub-utility X).

Using these two equations, we can compute the *intrasectoral elasticity of substitution* s_i of good i for the composite good, that is, the absolute value of the elasticity of x_i/X with respect to the relative price p_i/P (where X and P are the quantity and price of the composite good).[3] This computation may be alternatively performed in terms of quantities or in terms of prices, by taking respectively $X = X(\mathbf{x})$ and $p_i/P = \partial_i X(\mathbf{x})$, the marginal rate of substitution of x_i for X, or by taking $x_i = H_i(\mathbf{p}, \underline{X})$ and $P = \partial_X e(\mathbf{p}, \underline{X})$, the shadow price of $\underline{X}$. We thus obtain two equivalent formulas:

$$s_i = \frac{1 - \epsilon_i X(\mathbf{x})}{-\epsilon_i(\partial_i X(\mathbf{x}))} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})}{1 - [\epsilon_i X(\mathbf{x})]\left[\epsilon_X H_i(\mathbf{p}, \underline{X})\right]}. \tag{1.4}$$

At the second stage, the consumer chooses the quantities $\underline{X}$ of the composite good and z of the numeraire good by solving the programme

$$\max_{(\underline{X}, z) \in \mathbb{R}_+^2} \left\{ U(\underline{X}, z) \,\middle|\, e(\mathbf{p}, \underline{X}) + z \leq Y \right\}. \tag{1.5}$$

The solution to this programme determines the *Marshallian demand* $\underline{X} = D(\mathbf{p}, Y)$ for the composite good and the demand $z = Y - e(\mathbf{p}, D(\mathbf{p}, Y))$ for the numeraire good. We can then define the *inter-sectoral elasticity of substitution* of good i as the elasticity of substitution

[3]In the standard use of the concept and from the point of view of good i, it is its substitutability with respect to some other good j rather than to the composite good that is considered. When the substitutability differs among pairs of goods, conventional elasticities must be averaged (see, e.g. Bertoletti and Etro 2018, who use averages of the Morishima elasticticities of substitution and complementarity). We avoid introducing arbitrary averages by directly referring to substitutability with respect to the composite good. The relation between our concept and the conventional one is examined in the appendix of d'Aspremont and Dos Santos Ferreira (2016).

σ_i of good i for the aggregate consumption Y in the whole economy:

$$\sigma_i \equiv - \left.\frac{d\,(x_i/Y)}{d\,(p_i/1)}\right|_{X(\mathbf{x})=D(\mathbf{p},Y)} \frac{p_i/1}{x_i/Y}$$
$$= -\frac{(1/Y)\,\partial_{p_i} D\left(\mathbf{p}, Y\right)}{\partial_i X\,(\mathbf{x})}\frac{p_i}{x_i/Y} = \frac{-\epsilon_{p_i} D\left(\mathbf{p}, Y\right)}{\epsilon_i X\,(\mathbf{x})}. \tag{1.6}$$

In the computation of σ_i we are taking into account the variation of the Marshallian demand $D\left(\mathbf{p}, Y\right)$ rather than the mere share adjustment expressed in the elasticity of the Hicksian demand $-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)$.

A stronger form of separability of the utility function, *homothetic separability*, applying when the aggregator X is homothetic (more specifically, homogeneous of degree one, without loss of generality), simplifies computations and allows to further exploit duality. In this case, the expenditure function and, obviously, the Hicksian demand function are linear in $\underline{X}$: $e\left(\mathbf{p}, \underline{X}\right) = P\left(\mathbf{p}\right)\underline{X}$ and $H_i\left(\mathbf{p}, \underline{X}\right) = \partial_i P\left(\mathbf{p}\right)\underline{X}$, where P is a *price aggregator* function, and the Marshallian demand function becomes homothetically separable: $D\left(\mathbf{p}, Y\right) = \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)$. As a consequence, we obtain dual, perfectly symmetric, expressions for the first-order condition (1.2) and for Shephard's lemma (1.3):

$$p_i = P\left(\mathbf{p}\right)\partial_i X\,(\mathbf{x}) \text{ and } x_i = X\,(\mathbf{x})\,\partial_i P\left(\mathbf{p}\right), \tag{1.7}$$

respectively. Also, for cost minimising consumption bundles, the budget share of good i is equal to the elasticity of anyone of the two aggregator functions:

$$\frac{p_i x_i}{P\left(\mathbf{p}\right) X\,(\mathbf{x})} = \epsilon_i P\left(\mathbf{p}\right) = \epsilon_i X\,(\mathbf{x}) \equiv \alpha_i. \tag{1.8}$$

Finally, the price formula for the intrasectoral elasticity of substitution is then symmetric with respect to the quantity formula:

$$s_i = \frac{1 - \epsilon_i X\,(\mathbf{x})}{-\epsilon_i\,(\partial_i X\,(\mathbf{x}))} = \frac{-\epsilon_i\left(\partial_i P\left(\mathbf{p}\right)\right)}{1 - \epsilon_i P\left(\mathbf{p}\right)}, \tag{1.9}$$

and the intersectoral elasticity of substitution is just equal to the demand elasticity, now the same for any differentiated good:

$$\sigma_i \equiv -\epsilon_i D\left(\mathbf{p}, Y\right)/\alpha_i = -\epsilon_P \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)\epsilon_i P\left(\mathbf{p}\right)/\alpha_i = \sigma.$$

Firms' Competitive Behaviour and Oligopolistic Equilibria

We consider competition among N firms, each firm i producing a single component of the composite good with a constant positive unit cost c_i and a non-negative fixed cost ϕ_i incurred only when production is positive.[4] Firms behave strategically in price-quantity pairs: $(p_i, x_i) \in \mathbb{R}_+^2$ for each firm $i = 1, \ldots, N$. These pairs have to satisfy two admissibility constraints, generalising the two constraints as specified in Chap. 1.

The first is a *constraint on market share*, focusing on competition within the sector which produces the differentiated goods and referring to the first stage of the consumer's utility maximisation. It bounds the quantity of good i by the corresponding Hicksian demand:

$$x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}\right), X\left(x_i, \mathbf{x}_{-i}\right)\right). \tag{1.10}$$

The second is a *constraint on market size*, focusing on competition of the whole set of producers of the differentiated goods with the sector which produces the numeraire good. It refers to the second stage of the consumer's utility maximisation, and bounds the size of the market for the differentiated goods by the Marshallian demand:

$$X\left(x_i, \mathbf{x}_{-i}\right) \leq D\left(\left(p_i, \mathbf{p}_{-i}\right), Y\right). \tag{1.11}$$

[4]We assume positivity of unit costs for *all* firms to keep the exposition simple. The case of zero unit costs has already been examined in Chap. 1. The concept of oligopolistic equilibrium has been introduced in d'Aspremont et al. (2007), and further explored in d'Aspremont and Dos Santos Ferreira (2009, 2010, 2016).

We recall that the constraint on market share emphasises the conflictual side of competition between the oligopolists, whereas the constraint on market size translates their common interest as a sector.

We define the concept of oligopolistic equilibrium.

Definition 1 An *oligopolistic equilibrium* is a $2N$-tuple $\left(p_i^*, x_i^*\right)_{i=1,\ldots,N} \in \overline{\mathbb{R}}_+^{2N}$ such that, for any i,

$$\begin{aligned} \left(p_i^*, x_i^*\right) \in \arg & \max_{(p_i,x_i)\in\mathbb{R}_+^2} \left(p_i - c_i\right) x_i \\ \text{s.t.} \qquad & x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and } X\left(x_i, \mathbf{x}_{-i}^*\right) & \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right), \end{aligned} \tag{1.12}$$

and such that $Y^* = L + \sum_{i=1}^{N}\left(\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right)\phi_i\right)$. In addition, we require the profits to be non-negative, namely $\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right)\phi_i \geq 0$ for each i, and the consumer to be non-rationed.

Non-rationing of the consumer implies that both constraints are satisfied as equalities for each firm i at equilibrium. It makes the equilibrium compatible with the consumer's programme and the resulting demand functions. Notice that, according to this definition, all firms are not necessarily active in an oligopolistic equilibrium.[5] We shall in general assume that n firms are active, each one i choosing a positive strategy $\left(p_i^*, x_i^*\right)$, and that $N - n$ firms are inactive, choosing each a strategy $(\infty, 0)$. Of course, inactive firms are also maximising profits at equilibrium: no admissible strategy would allow them to obtain a positive profit. As the fixed cost is incurred only at a positive output, choosing a zero output is a way to ensure that the profit is at least non-negative. The price strategy is then arbitrary. We suppose it to be infinite in order to avoid consumer rationing. As already discussed in Chap. 1, existence

[5] Inactive firms do play a role though. Shubik (1959) suggests to call such firms "firms in being" by analogy to the famous term "fleet in being," introduced by Lord Torrington in 1690 and used by Kipling.

of inactive firms at equilibrium allows to qualify that equilibrium as a free entry equilibrium, but only if the inactive firms have the same opportunities as the active ones, which imposes the oligopoly game to be symmetric, a restrictive assumption that we are not making in general.

We next show that an oligopolistic equilibrium can be characterised by a simple expression for individual (relative) *markups* (or *Lerner indices* of the degree of monopoly power) at that equilibrium, that is, $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ for each active firm i. This markup is derived from the first-order conditions of producer i's programme in Definition 1.[6] To obtain that simple expression, we refer to the intra- and intersectoral elasticities of substitution of good i, s_i^* and σ_i^* respectively, again at the considered equilibrium, and we introduce in addition simplifying notations for two additional elasticities. The elasticity $\alpha_i \equiv \epsilon_i X(\mathbf{x})$ measures the impact of a variation in the quantity of good i on the volume of the composite good. The elasticity $\beta_i \equiv \epsilon_X H_i\left(\mathbf{p}, \underline{X}\right)$ measures the reverse impact of a variation of the quantity of the composite good on the demand for its component i, at given prices $\mathbf{p}$. The product of these two elasticities, which appears in the multiplier $1/\left(1-\alpha_i\beta_i\right)$ applied to the elasticity $-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)$ of the Hicksian demand in the price formula for s_i, Eq. (1.4), measures the intensity of the feedback originating in a variation of the quantity of good i and going through the volume of the composite good.

For easier reference, we recall the expressions for these four elasticities in Table 2.1:

Table 2.1 Elasticities appearing in the markup formula

Intrasectoral substitution:	$s_i = \frac{1-\alpha_i}{-\epsilon_i \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, X(\mathbf{x}))}{1-\alpha_i\beta_i}$
Intersectoral substitution:	$\sigma_i = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\alpha_i}$
Impact of x_i on X	$\alpha_i \equiv \epsilon_i X(\mathbf{x})$
Impact of X on x_i	$\beta_i \equiv \epsilon_X H_i(\mathbf{p}, X(\mathbf{x}))$

[6] Reference to the markup μ_i^* replaces the direct reference to the price p_i^* used in Chap. 1 for the case of zero unit costs (a case in which the Lerner index is always equal to 1).

Take an oligopolistic equilibrium $\left(\left(p_i^*, x_i^*\right)_{i=1,\ldots,n}, (\infty, 0)^{N-n}\right)$ (with positive prices and quantities for the first n firms), henceforth denoted $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} = \left(\mathbf{p}^*, \mathbf{x}^*\right)$ for simplicity. In the rest of our book, we shall often resort to this abusive simplifying notation referring to the sole active firms. The markup of active firm i at this equilibrium will be expressed, according to the following proposition, as a weighted mean of the reciprocals of the two elasticities of substitution s_i^* and σ_i^* at that equilibrium. The corresponding weights will involve, for each firm i, the elasticities α_i^* and β_i^* measuring the two reciprocal effects of quantity variations of good i and of the composite good, as well as a conduct parameter[7] $\theta_i^* \in [0, 1]$, stemming from the first-order conditions and interpreted as the *competitive toughness* displayed by firm i towards its rival oligopolists at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$.

Proposition 4 *Let* $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ *be an oligopolistic equilibrium. Then the markup* $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ *of each firm* i *is given by*

$$\mu_i^* = \frac{\theta_i^*\left(1 - \alpha_i^*\beta_i^*\right) + \left(1 - \theta_i^*\right)\alpha_i^*}{\theta_i^*\left(1 - \alpha_i^*\beta_i^*\right)s_i^* + \left(1 - \theta_i^*\right)\alpha_i^*\sigma_i^*}, \tag{1.13}$$

for some $\theta_i^* \in [0, 1]$.

Proof We start by making dimensionally homogeneous the two constraints in the programme of firm i, rewriting them in terms of the two ratios:

$$\frac{x_i}{H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right)} \leq 1 \text{ and } \frac{X\left(x_i, \mathbf{x}_{-i}^*\right)}{D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right)} \leq 1. \tag{1.14}$$

[7] To use the terminology of the New Empirical Industrial Organization (see Bresnahan, 1989).

The first-order necessary conditions for profit maximisation at $\left(p_i^*, x_i^*\right)$ under these two constraints (holding as equalities at equilibrium) can then be expressed, for non-negative Lagrange multipliers λ_i^* and ν_i^*, as

$$\begin{aligned} x_i^* &= \lambda_i^* \frac{-\partial_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} + \nu_i^* \frac{-\partial_{p_i} D\left(\mathbf{p}^*, Y^*\right)}{D\left(\mathbf{p}^*, Y^*\right)} \\ &= \frac{\lambda_i^*}{p_i^*}\left[-\epsilon_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] + \frac{\nu_i^*}{p_i^*}\left[-\epsilon_{p_i} D\left(\mathbf{p}^*, Y^*\right)\right], \end{aligned} \tag{1.15}$$

and

$$\begin{aligned} p_i^* - c_i &= \lambda_i^* \frac{1 - \left[\partial_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\left[\partial_i X\left(\mathbf{x}^*\right)\right]}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} + \nu_i^* \frac{\partial_i X\left(\mathbf{x}^*\right)}{X\left(\mathbf{x}^*\right)} \\ &= \frac{\lambda_i^*}{x_i^*}\left(1 - \left[\epsilon_i X\left(\mathbf{x}^*\right)\right]\left[\epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\right) + \frac{\nu_i^*}{x_i^*}\epsilon_i X\left(\mathbf{x}^*\right). \end{aligned} \tag{1.16}$$

We can use these two conditions and the notations of Table 2.1 to obtain the markup formula for firm i at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, as given by (1.13), with $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^*\right)$. □

Since λ_i^* is the Lagrange multiplier associated with the constraint on market share, which emphasises the conflictual side of competition between firm i and its rivals in the sector, whereas ν_i^* is the Lagrange multiplier associated with the constraint on market size, which reflects converging interests of the competitors in the sector, the normalised multiplier θ_i^* can be interpreted, as suggested in Chap. 1, as the competitive toughness experienced by firm i at the particular equilibrium $\left(p_j^*, x_j^*\right)_{j=1,\ldots,n}$. Inspection of Eq. (1.13) shows that the weight put on the reciprocal of *intra*sectoral elasticity of substitution s_i^* (relative to the weight put on the reciprocal of its *inter*sectoral homologue σ_i^*) naturally increases with the competitive toughness θ_i^* experienced by firm i at equilibrium.

The equilibrium markup of firm i is a weighted *harmonic* mean of the reciprocals of the two elasticities of substitution s_i^* and σ_i^*. If we assume homothetic separability of the consumer's utility function (not necessarily the CES specification of the aggregator X), we can use, as we did in Sect. 3 of Chap. 1 (Proposition 3), the dual constraints on market share and market size in order to obtain

$$\mu_i^* = \frac{\theta_i'^* \left(1-\alpha_i^*\right)\left(1/s_i^*\right) + \left(1-\theta_i'^*\right)\alpha_i^* \left(1/\sigma^*\right)}{\theta_i'^* \left(1-\alpha_i^*\right) + \left(1-\theta_i'^*\right)\alpha_i^*}, \tag{1.17}$$

where μ_i^* appears as an *arithmetic* mean of the reciprocals of the two elasticities s_i^* and σ^* (the latter uniform for the whole oligopolistic sector because of the homotheticity assumption).[8] The equilibrium is the same (s_i^*, σ^* and the budget share α_i^* are identical, with $\beta_i^* = 1$ by linearity of the Hicksian demand), but the conduct parameters are specific to this dual form, the equilibrium parameterisation differing between the dual forms of the two constraints. By identifying the formula of μ_i^* given by (1.17) and that given by (1.13) (with $\sigma_i^* = \sigma^*$ and $\beta_i^* = 1$), we can easily establish the relation between the two parameters: for $s_i^* \notin \{0, \sigma^*, \infty\}$,

$$\frac{1/\theta_i'^* - 1}{\sigma^*} = \frac{1/\theta_i^* - 1}{s_i^*} \tag{1.18}$$

Regimes of Competition

The vector $\boldsymbol{\theta}^* = \left(\theta_1^*, \ldots, \theta_n^*\right)$ of the competitive toughnesses of the different active firms (or its dual counterpart $\boldsymbol{\theta}'^*$ in the homothetic case) specifies a *regime of competition*, which can be continuously modified by varying this vector in $[0,1]^n$. Tracing the set of potential equilibria

[8] Recall that

$$\sigma_i = \frac{-\epsilon_{p_i} D\left(\mathbf{p}, Y\right)}{\alpha_i} = \frac{-\epsilon_{p_i} \widehat{D}\left(P\left(\mathbf{p}\right), Y\right)}{\alpha_i} = \frac{-\epsilon_P \widehat{D} \cdot \epsilon_i P}{\alpha_i} = -\epsilon_P \widehat{D} = \sigma,$$

with σ denoting the elasticity of the Marshallian demand function $\widehat{D}\left(\cdot, Y\right)$, as in Chap. 1.

by varying $\boldsymbol{\theta}^*$ allows us in particular to retrieve standard regimes like price and quantity equilbria ($\theta_i^* = 1/2$ and $\theta_i'^* = 1/2$, respectively, for any active firm i), or the collusive solution ($\boldsymbol{\theta}^* \equiv \mathbf{0}$), although existence of the whole spectrum of potential equilibria (for all values of $\boldsymbol{\theta} \in [0, 1]^n$) is generally not satisfied, as already shown in Chap. 1. The markup formula is also useful in limit cases. We shall examine in Sect. 3 the perfect substitutability case ($s_i = \infty$) referring to the dual version (1.17) of the formula. Now, if we assume that preferences are symmetric, that the unit costs are identical for all firms ($c_i = c$ for all i) and that the number of active firms go to infinity, say for a sequence of symmetric oligopolistic equilibria, then the oligopolistic sector becomes "large" in the sense of Chamberlin, meaning that every individual firm becomes negligible ($\alpha_i^* \simeq 0$), and we get what we may call the *Dixit-Stiglitz monopolistic competition equilibrium*, with $\mu_i^* = 1/s_i^*$. The standard case is when the markup remains positive, that is $\lim_{n\to\infty}\left(1/s_i^*\right) > 0$, as it is when the aggregator X is CES with $s_i^* = s > 0$. Then there is another way to obtain the outcome of monopolistic competition, which is to assume that firms' conduct is sufficiently tough ($\theta_i^* \simeq 1$). This holds, even in the "small" group case.[9] However, in the large group case, we get the competitive (Walrasian) equilibrium when $\lim_{n\to\infty}\left(1/s_i^*\right) = 0$.[10]

To summarise, the main competition regimes under homothetic separability can be characterised by the competitive toughness and markup values (applying to all firms) displayed in Table 2.2.

[9] Shimomura and Thisse (2012) introduce a *mixed market* structure. They consider a Dixit-Stiglitz economy where U is Cobb-Douglas and X CES, defined over the union of a discrete set of goods produced by large firms and a continuum of goods produced by small firms. This continuum is a monopolistically competitive fringe, with mass determined by the zero-profit condition, under free entry restricted to the fringe. Quantity competition is also assumed. The resulting mixed market quantity equilibrium outcome can of course be (approximately) obtained within our canonical model, by letting $\alpha_i^* \simeq 0$ iff firm i is small or, alternatively, by letting $\theta_i'^* \simeq 1$ if firm i is small (and $\theta_i'^* = 1/2$ if firm i is large).

[10] As well emphasised in Thisse and Ushchev (2018), this depends on the preferences. Other examples will be given below.

Table 2.2 Competition regimes under homothetic separability

Competition regime	Competitive toughness	Markup
Collusion	$\theta_i = \theta'_i = 0$	$\mu^*_i = 1/\sigma^*$
Quantity competition	$\theta'_i = 1/2$; $\theta_i = 1/\left(1 + s^*_i/\sigma^*\right)$	$\mu^*_i = \left(1 - \alpha^*_i\right)/s^*_i + \alpha^*_i/\sigma^*$
Price competition	$\theta_i = 1/2$; $\theta'_i = 1/\left(1 + \sigma^*/s^*_i\right)$	$\mu^*_i = 1/\left(\left(1 - \alpha^*_i\right) s^*_i + \alpha^*_i \sigma\right)$
Monopolistic competition	$\theta_i = \theta'_i = 1$	$\mu^*_i = 1/s^*_i$
	$\theta_i = \theta'_i > 0$ and $\alpha^*_i \simeq 0$	$\mu^*_i = 1/s^*_i$
Perfect competition	$\theta_i = \theta'_i > 0$ and $\alpha^*_i \simeq 0$	–
	with $\lim_{n\to\infty}\left(1/s^*_i\right) = 0$	$\mu^*_i = 0$

To illustrate, take for instance the case, represented in Fig. 2.1, of a symmetric differentiated duopoly with the CES specification for the aggregator X, the isoelastic demand function $\widehat{D}(P, Y) = YP^{-5/2}$ and the constant unit cost $c = 1$. We represent the degree of complementarity $1/(1+s)$ on the horizontal axis and the competitive toughness θ (the same for both firms) on the vertical axis. The set of values which parameterise existent oligopolistic equilibria is represented by the region inside the thick curves. We see that as the two goods become more and more complementary, potential equilibria cease to be enforceable as competitive toughness becomes too high or too low. The same is true if substitutability is very high and competitive toughness very low. The thin horizontal line ($\theta = 1/2$) represents the competitive toughness displayed in the price equilibrium (existent at any degree of complementarity) and the thin increasing curve represents the competitive toughness associated with the quantity equilibrium (existent only if complementarity is not too large).

The set of existent oligopolistic equilibria is also represented for the same example in Fig. 2.2 as the region of the space $1/(1+s) \times \mu$ between the thick curves. Relative to Fig. 1.2 in Chap. 1, the main difference is the presence of a positive unit cost, allowing to replace on the vertical axis the price index $P = 2^{1/(1-s)}p$ by the markup $\mu = (p-1)/p = \left(P - 2^{1/(1-s)}\right)/P$.

The thick curve switching from concave to convex is quite similar to the corresponding one in Fig. 1.2 of Chap. 1. It is the *soft competition*

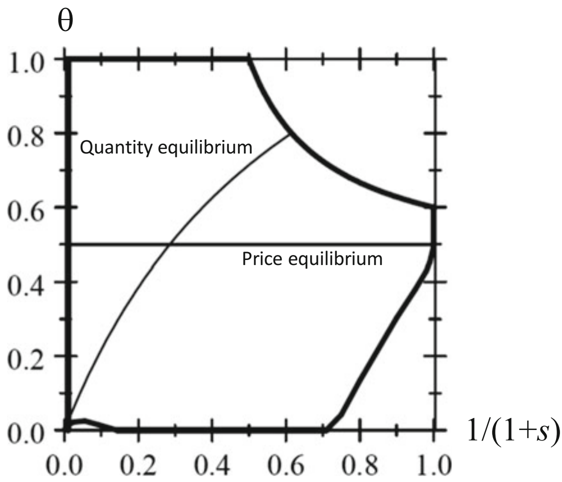

Fig. 2.1 Competitive toughness compatible with equilibrium existence

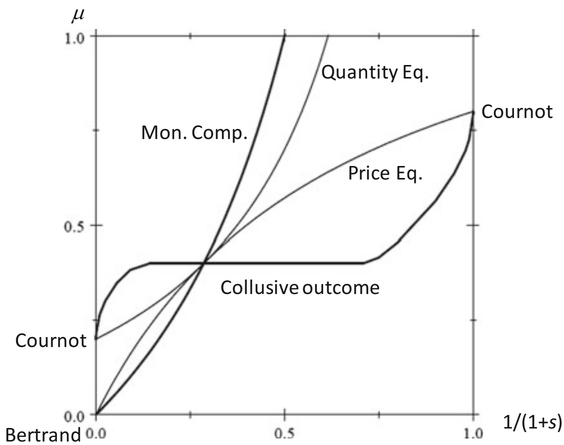

Fig. 2.2 Equilibrium regimes

frontier representing the markup that is closest to the collusive one ($1/\sigma = 0.4$ in this example) and linking the two Cournot solutions (the one for the homogeneous duopoly at its left end and the one for complementary monopoly at its right end). The thick convex curve starting at the origin from the Bertrand solution ($\mu = 0$) is the *tough competition frontier*, representing the markup $1/s$ resulting from maximal competitive toughness ($\theta = 1$). By the Bertrand paradox, this maximal competitive toughness entails the monopolistic competition outcome usually associated with a continuum of producers. Because of the positive unit cost, $\theta = 1$ is not linked to corner solutions, so that this frontier essentially differs from the graph of the step function in Fig. 1.2 in Chap. 1. The thin concave curve, linking the Bertrand and the Cournot complementary monopoly solutions, represents the price equilibrium markup and the thin convex curve starting from the Cournot homogeneous duopoly solution represents the quantity equilibrium markup, always above the price equilibrium markup.

2 Introducing Ford Effects

In the preceding section, when formulating the constraint on market share, we have treated income Y parametrically, its value being of course adjusted at equilibrium. Large firms may however influence the size of their own market through the income they distribute, in a way that is far from negligible. And they may well have a good perception of that influence, taking it into account in their decisions. A well-known example of that perception is the high wage policy advocated from an industrialist point of view by Henry Ford:

> I believe in the first place that, all other considerations aside, our own sales depend in a measure upon the wages we pay. If we can distribute high wages, then that money is going to be spent and it will serve to make storekeepers and distributors and manufacturers and workers in other lines more prosperous and their prosperity will be reflected in our sales. Country-wide high wages spell country-wide prosperity, provided, however, the higher wages are paid for higher production (Ford 1922, p.124).

What makes this idea quite remarkable is that it is formulated in "general equilibrium" terms. The income feedback effect of higher distributed income—that may be called the *Ford effect*[11] —works through the variations it induces outside the sector in which it originates.

The consequences for the equilibrium markup of introducing Ford effects working through the different income components (wages, profits and their sum) can be evaluated (see d'Aspremont and Dos Santos Ferreira, 2017). Let us here consider Ford effects extended to the whole income of the oligopolistic sector, when firms in this sector take as given the (wage) income z generated (or spent) in the competitive sector. We redefine accordingly the concept of oligopolistic equilibrium.

Definition 2 An *oligopolistic equilibrium with Ford effects* is a tuple of pairs $\left(p_i^*, x_i^*\right)_{i=1,\ldots,N} \in \overline{\mathbb{R}}_+^{2N}$ such that, for any i,

$$\begin{aligned} \left(p_i^*, x_i^*\right) \in \arg & \max_{(p_i, x_i) \in \mathbb{R}_+^2} \left(p_i - c_i\right) x_i \\ \text{s.t.} \quad & x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and } X\left(x_i, \mathbf{x}_{-i}^*\right) & \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), z^* + \textstyle\sum_{j \neq i} p_j^* x_j^* + p_i x_i\right), \end{aligned} \tag{2.1}$$

and such that the profits are non-negative, namely that $\left(p_i^* - c_i\right) x_i^* - \operatorname{sgn}\left(x_i^*\right) \phi_i \geq 0$ for each i, and also such that the consumer is non-rationed (implying $z^* = L - \sum_{i=1}^N \left(c_i x_i^* + \operatorname{sgn}\left(x_i^*\right) \phi_i\right)$).

The general formula obtained for the equilibrium markup is modified, while remaining easy to interpret.

Proposition 5 *Let* $\left(p_i^*, x_i^*\right)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ *be an oligopolistic equilibrium with Ford effects. Then the equilibrium markup* $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ *of each firm i is given by*

$$\mu_i^* = \frac{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) + \left(1 - \theta_i^*\right)\left[\alpha_i^* - \eta_i^* \epsilon_Y D^*\right]}{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) s_i^* + \left(1 - \theta_i^*\right)\left[\alpha_i^* \sigma_i^* - \eta_i^* \epsilon_Y D^*\right]}, \tag{2.2}$$

[11] As in d'Aspremont et al. 1989a and 1989b.

where $\eta_i^* \equiv p_i^* x_i^* / Y^*$ *is the budget share of good* i *in the* whole *expenditure, for some* $\theta_i^* \in [0, 1]$.

Proof The only modification in the programme of firm i concerns the income as an argument of the Marshallian demand function in the constraint for market size. Thus, by referring to the programme (2.1) and building on first-order conditions (1.15) and (1.16) in the proof of Proposition 4, we easily modify these conditions to obtain:

$$\begin{aligned} x_i^* &= \frac{\lambda_i^*}{p_i^*} \left[-\epsilon_{p_i} H_i \left(\mathbf{p}^*, D \left(\mathbf{p}^*, Y^* \right) \right) \right] \\ &\quad + \frac{\nu_i^*}{p_i^*} \left[-\epsilon_{p_i} D \left(\mathbf{p}^*, Y^* \right) - \epsilon_Y D \left(\mathbf{p}^*, Y^* \right) \epsilon_{p_i} Y^* \right] \\ p_i^* - c_i &= \frac{\lambda_i^*}{x_i^*} \left(1 - \left[\epsilon_i X \left(\mathbf{x}^* \right) \right] \left[\epsilon_X H_i \left(\mathbf{p}^*, D \left(\mathbf{p}^*, Y^* \right) \right) \right] \right) \\ &\quad + \frac{\nu_i^*}{x_i^*} \left[\epsilon_i X \left(\mathbf{x}^* \right) - \epsilon_Y D \left(\mathbf{p}^*, Y^* \right) \epsilon_{x_i} Y^* \right]. \end{aligned} \tag{2.3}$$

By dividing the two handsides of the second equation by the corresponding handsides of the first and then using Table 2.1 and $\epsilon_{p_i} Y^* = \epsilon_{x_i} Y^* = p_i^* x_i^* / Y^* \equiv \eta_i^*$ plus $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^* \right)$ to make the appropriate simplifications, we obtain indeed the markup formula (2.2). □

This expression for the equilibrium markup, similar to formula (1.13) in Proposition 4, is again a harmonic mean of the elasticities (in absolute value) of the two frontiers at the equilibrium point (in the space $x_i \times p_i$), $1/s_i^*$ for the market share frontier and $\left(\alpha_i^* - \eta_i^* \epsilon_Y D^* \right) / \left(\alpha_i^* \sigma_i^* - \eta_i^* \epsilon_Y D^* \right) \equiv 1/\widehat{\sigma}_i^*$, rather than $1/\sigma_i^*$, for the market size frontier. The redefined elasticity of intersectoral substitution $\widehat{\sigma}_i^*$ is larger (resp. smaller) than the original σ_i^* if $\sigma_i^* > 1$ (resp. $\sigma_i^* < 1$). In other words, the Ford effect increases (resp. decreases) the relevant elasticity of intersectoral substitution and accordingly exerts *ceteris paribus* a depressing (resp. enhancing) effect on the equilibrium markup when good i and the numeraire good are substitutes (resp. complements).

The equilibrium markup formula (2.2) becomes simpler in the particular case of homotheticity of the utility function U and homogeneity of degree 1 of the aggregator function X. In this case, $\epsilon_X e^* = \epsilon_X H_i^* = \epsilon_Y D^* = 1$ and $\eta_i^* = \alpha_i^* (P^* X^* / Y^*) \equiv \alpha_i^* \gamma^*$, where γ^* is the budget share of the composite product of the oligopolistic sector in the whole expenditure, so that $\widehat{\sigma}^* = (\sigma^* - \gamma^*) / (1 - \gamma^*)$ and

$$\mu_i^* = \frac{\theta_i^* \left(1 - \alpha_i^*\right) + \left(1 - \theta_i^*\right) \alpha_i^* \left(1 - \gamma^*\right)}{\theta_i^* \left(1 - \alpha_i^*\right) s_i^* + \left(1 - \theta_i^*\right) \alpha_i^* \left(1 - \gamma^*\right) \widehat{\sigma}^*}. \tag{2.4}$$

The markup μ_i^* is a weighted harmonic mean of the reciprocals of two elasticities of substitution, s_i^* and $\widehat{\sigma}^*$, where the intersectoral elasticity of substitution $\widehat{\sigma}^*$ has been implicitly redefined to refer to the substitution of X/z with respect to $P/1$ rather than that of x_i/Y with respect to $p_i/1$. We have indeed, using quantity and price indices of the composite good produced in the oligopolistic sector thanks to homotheticity:

$$\begin{aligned}
\widehat{\sigma} &\equiv -\epsilon_P \left[\frac{\widehat{D}(P, Y)}{Y - P\widehat{D}(P, Y)}\right] \\
&= \underbrace{-\epsilon_P \widehat{D}(P, 1)}_{\sigma} - \underbrace{\frac{P\widehat{D}(P, 1)}{1 - P\widehat{D}(P, 1)}}_{\gamma/(1-\gamma)} \underbrace{\left(1 + \epsilon_P \widehat{D}(P, 1)\right)}_{1-\sigma} \\
&= \frac{\sigma - \gamma}{1 - \gamma}.
\end{aligned} \tag{2.5}$$

Let us compare the markup formula (2.4) and the formula (1.13) prevailing in the absence of Ford effects (but under homotheticity, leading to $\beta_i^* = 1$ and $\sigma_i^* = \sigma^*$ as the price elasticity of demand for the composite good). The weight on the reciprocal of the intrasectoral elasticity of substitution is not modified: this elasticity completely determines the markup of firm i in the limit cases of a negligible market share ($\alpha_i^* \to 0$) or of maximum competitive toughness ($\theta_i^* \to 1$). Correspondingly, the intersectoral elasticity of substitution completely determines the markup of firm i in the opposite limit cases of monopoly ($\alpha_i^* \to 1$) or collusion

($\theta_i^* \to 0$). An increasing weight put on the reciprocal of the intersectoral elasticity of substitution may however also result from a decreasing budget share γ^* of the composite product of the oligopolistic sector. But the most significant consequence of the Ford effect is the transformation of the intersectoral elasticity of substitution itself: with $\widehat{\sigma}^*$ increasing from $\sigma^* > 1$ to infinity as the budget share γ^* increases from 0 to 1 (when the two composite goods are substitutable) and with $\widehat{\sigma}^*$ decreasing from $\sigma^* < 1$ to zero as the budget share γ^* increases from 0 to 1 (when the two composite goods are complementary).

This concludes our analysis of the consequences of introducing Ford effects extended to the whole income of the oligopolistic sector. Some more limited form could be considered, but in the present model, restricting Ford effects to wages does not make sense. As the economy has a single labour market which is perfectly competitive, as labour productivity in the numeraire sector is assumed constant and as labour supply is rigid, economy-wide wage income is insensitive to oligopolistic firms' decisions, at least when expressed in terms of the numeraire. It is just equal to L.

Restricting Ford effects to profits, so that the economy income, as conjectured by firm i at some equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, is

$$Y = L + \sum_{j \neq i} \left(\left(p_j^* - c_j\right) x_j^* - \phi_j\right) + \left(\left(p_i - c_i\right) x_i - \phi_i\right), \qquad (2.6)$$

will not have any consequence either. Consider the programme (1.12) of firm i, expressed as the maximisation of the Lagrangian

$$\max_{(p_i, x_i)} f_i\left(p_i, x_i\right) - \lambda_i g_i\left(p_i, x_i\right) - \nu_i h\left(p_i, x_i, Y\left(f_i\left(p_i, x_i\right)\right)\right), \qquad (2.7)$$

where f_i is the objective function, and where $g_i\left(p_i, x_i\right) \leq 0$ and $h(p_i, x_i, Y\left(f_i\left(p_i, x_i\right)\right)) \leq 0$ are the two constraints, on market share and on market size respectively, and λ_i and ν_i the corresponding Lagrange multipliers. The strategies of other firms, implicit arguments of functions g_i and h, are omitted for simplicity of notation. The crucial point is that Y depends upon the strategy pair $\left(p_i, x_i\right)$ only through the objective function f_i. As a consequence, the first-order condition for an interior solution is

$$\begin{aligned}&\left[1-\nu_i\partial_Y h\cdot Y'\left(f_i\left(p_i,x_i\right)\right)\right]\boldsymbol{\partial}_{(p_i,x_i)}f_i\left(p_i,x_i\right)\\&=\lambda_i\boldsymbol{\partial}_{(p_i,x_i)}g_i\left(p_i,x_i\right)+\nu_i\boldsymbol{\partial}_{(p_i,x_i)}h\end{aligned}\tag{2.8}$$

where the gradient $\boldsymbol{\partial}_{(p_i,x_i)}f_i\left(p_i,x_i\right)$ is multiplied, not by 1 as when Ford effects are ignored, but by a positive factor which depends upon the strategy pair (p_i, x_i). Thus, taking into account Ford effects restricted to profits only changes proportionately the two Lagrange multipliers without modifying the equilibrium markup, which depends only on the ratio of those multipliers.

3 Back to the Homogeneous Good Case: Comparison with Alternative Approaches

In the first section of Chap. 1, we have considered the case of a duopoly producing a homogeneous good at zero cost under perfectly symmetric conditions. Let us now suppose N firms, each firm i producing the same good with a technology described by an increasing cost function C_i, which is continuously differentiable on $(0,\infty)$ and such that $C_i(0)=0$.[12] The demand D for the good is a function of market price P, with a finite continuous derivative $D'(P)<0$ over all the domain where it is positive and such that $\lim_{P\to\overline{P}}D(P)=0$, for some $\overline{P}\in(0,\infty]$. Our purpose is to review different approaches to oligopolistic competition in the homogeneous good case, used in different contexts, also leading to the same kind of indeterminacy. We start by our own approach.

[12]Following d'Aspremont and Dos Santos Ferreira (2009), and for the sake of comparing our oligopolistic equilibrium concept with alternative concepts, the technology assumption is weakened with respect to the one in section 2.1, where $C_i\left(x_i\right)=\phi_i+c_ix_i$ for $x_i>0$ (with $c_i>0$).

Our Market Share and Market Size Approach

Using the dual form of the market share and market size constraints given in Eq. (3.7) of Chap. 1, the definition of oligopolistic equilibrium is straightforwardly adapted.

Definition 3 An *oligopolistic equilibrium* is a $2N$-tuple $(\mathbf{p}^*, \mathbf{x}^*)$ such that, for each firm $i = 1, \ldots, N$, (p_i^*, x_i^*) is solution to the programme

$$\max_{(p_i,x_i)\in\mathbb{R}_+^2} \left\{ p_i x_i - C_i(x_i) \middle| \; p_i \leq \min_{j\neq i}\{p_j^*\} \text{ and } p_i \leq D^{-1}\left(x_i + \sum_{j\neq i} x_j^*\right)\right\}, \tag{3.1}$$

and satisfies

$$\sum_j x_j^* = D\left(\min_j \left\{p_j^*\right\}\right). \tag{3.2}$$

Since both constraints are binding for any active firm at an oligopolistic equilibrium and since we are in the homogeneous good case, an *equilibrium outcome* is simply given by the pair $(P^*, \mathbf{x}^*)$ with $P^* = \min_j\{p_j^*\}$.

It is easy to see that both the Cournot outcome $(P^{\mathrm{C}}, \mathbf{x}^{\mathrm{C}})$ satisfying

$$\begin{aligned} x_i^{\mathrm{C}} &\in \arg\max_{x_i\in[0,\infty)} \left\{ D^{-1}\left(x_i + \sum_{j\neq i} x_j^{\mathrm{C}}\right) x_i - C_i(x_i)\right\} \text{ for } i = 1, \ldots, N, \\ P^{\mathrm{C}} &= D^{-1}\left(\sum_j x_j^{\mathrm{C}}\right), \end{aligned} \tag{3.3}$$

and the competitive (Walrasian) outcome $(P^{\mathrm{W}}, \mathbf{x}^{\mathrm{W}})$ satisfying

$$x_i^{\mathrm{W}} \in \arg \max_{x_i \in [0,\infty)} \{P^{\mathrm{W}} x_i - C_i(x_i)\} \text{ for } i = 1, \ldots, N,$$

$$P^{\mathrm{W}} = D^{-1}\left(\sum_j x_j^{\mathrm{W}}\right) \tag{3.4}$$

are oligopolistic equilibria. If, indeed, there were, for some i, a deviation $(p_i, x_i) \in \mathbb{R}_+^2$ satisfying the two constraints in (3.1) such that the profit $p_i x_i - C_i(x_i)$ were strictly larger than the Cournot profit $P^{\mathrm{C}} x_i^{\mathrm{C}} - C_i(x_i^{\mathrm{C}})$ (resp. the Walrasian profit $P^{\mathrm{W}} x_i^{\mathrm{W}} - C_i(x_i^{\mathrm{W}})$), then we would get the contradiction $P^{\mathrm{C}} x_i^{\mathrm{C}} - C_i(x_i^{\mathrm{C}}) < D^{-1}(x_i + \sum_{j \neq i} x_j^{\mathrm{C}}) x_i - C_i(x_i)$ (resp. $P^{\mathrm{W}} x_i^{\mathrm{W}} - C_i(x_i^{\mathrm{W}}) < P^{\mathrm{W}} x_i - C_i(x_i)$).

As to the Bertrand outcome $(P^{\mathrm{B}}, \mathbf{x}^{\mathrm{B}})$ with $P^{\mathrm{B}} = \min_i \{p_i^{\mathrm{B}}\}$, it is now characterised by

$$p_i^{\mathrm{B}} \in \arg \max_{p_i \in [0,\infty)} \left\{p_i d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) - C_i\left(d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right)\right)\right\} \tag{3.5}$$

where the demand to firm i is $d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) = D\left(p_i\right) / (\# \arg\min \left\{p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right\})$ if $p_i = \min\left\{p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right\}$ and $d_i\left(p_i, \mathbf{p}_{-i}^{\mathrm{B}}\right) = 0$ otherwise. It is also an oligopolistic equilibrium, since a profitable deviation (p_i, x_i) for some i in the extended Cournot-Bertrand game would have $p_i \leq P^{\mathrm{B}}$ and hence would be also feasible and profitable in the Bertrand game, again a contradiction. Finally, as already noticed in the symmetric duopoly case, and in contrast with the differentiated good case, the collusive outcome $(P^{\mathrm{m}}, x^{\mathrm{m}})$ corresponding to

$$(P^{\mathrm{m}}, \mathbf{x}^{\mathrm{m}}) \in \arg \max_{(P,\mathbf{x}) \in \mathbb{R}_+^{n+1}} \left\{ P \sum_i x_i - \sum_i C_i(x_i) \,\middle|\, \sum_i x_i \leq D(P) \right\}, \tag{3.6}$$

cannot be an oligopolistic equilibrium in the homogeneous good case unless it coincides with the Cournot outcome. Indeed, if $(P^{\mathrm{m}}, \mathbf{x}^{\mathrm{m}})$ is not a Cournot outcome, we have, for some i, some $x_i \in \mathbb{R}_+$ and $P = D^{-1}(x_i + \sum_{j \neq i} x_j^{\mathrm{m}})$,

$$Px_i - C_i(x_i) + P^{\mathrm{m}} \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}})$$
$$> P^{\mathrm{m}} x_i^{\mathrm{m}} - C_i(x_i^{\mathrm{m}}) + P^{\mathrm{m}} \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}}), \tag{3.7}$$

and, since it is collusive,

$$P^{\mathrm{m}} \sum_j x_j^{\mathrm{m}} - \sum_j C_j(x_j^{\mathrm{m}}) \geq Px_i - C_i(x_i) + P \sum_{j \neq i} x_j^{\mathrm{m}} - \sum_{j \neq i} C_j(x_j^{\mathrm{m}}) \tag{3.8}$$

implying $P < P^{\mathrm{m}}$. Therefore, (P, x_i) is an admissible deviation for firm i in the oligopoly game.

Looking now at the first-order conditions of firm i at an oligopolistic equilibrium (with multipliers $(\lambda_i', \nu_i') \in \mathbb{R}_+^2 \setminus \{\mathbf{0}\}$ associated with the first and second constraints in (3.1)), they require, by the positivity of p_i^* and of x_i^* (if firm i is active) that $x_i^* - \lambda_i'^* - \nu_i'^* = 0$, and $p_i^* - C_i'(x_i^*) + \nu_i'^*/D'(P^*) = 0$. If firm i is inactive, both constraints cease to bind, so that we let $\lambda_i'^* = \nu_i'^* = 0$. Using the normalised parameter $\theta_i'^* \equiv \lambda_i'^* / \left(\lambda_i'^* + \nu_i'^*\right) \in [0, 1]$, we can rewrite the first-order conditions to characterise the markup of each firm i in the set I^* of active firms as a function of $\theta_i'^*$, with $P^* = \min_j\{p_j^*\}$:

$$\mu_i^* = \frac{P^* - C_i'(x_i^*)}{P^*} = \left(1 - \theta_i'^*\right) \frac{x_i^* / \sum_j x_j^*}{-\epsilon D(P^*)} \equiv (1 - \theta_i'^*) \frac{\alpha_i^*}{\sigma(P^*)}, \; i \in I^*. \tag{3.9}$$

This formula generalises formula (1.5) in Chap. 1 for the duopoly case with zero marginal cost. As above, $\theta_i'^*$ may be interpreted as measuring the competitive toughness of firm i at the equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$. When competitive toughness is maximal ($\boldsymbol{\theta}'^* = (1, \ldots, 1)$), each active firm equalising marginal cost to price, we get the competitive equilibrium (or the Bertrand equilibrium for constant marginal costs). At the other extreme, when competitive toughness is minimal ($\boldsymbol{\theta}'^* = (0, \ldots, 0)$), we get the standard markup formula for the Cournot equilibrium. All other oligopolistic equilibria correspond to intermediate values of $\boldsymbol{\theta}'^*$.

Notice that μ_i^* in formula (3.9) is equal to the numerator of μ_i^* in formula (1.17) in the limit case $s_i^* = \infty$. But, looking at the denominator of μ_i^* in formula (1.17), we see that the parameterisation of competitive toughness is different in the two formulas (although we have kept the same notation $\theta_i'^*$ in both formulas). Indeed, if we apply formula (1.17), it entails $\mu_i^* = 0$ for $\theta_i'^* = 1$ (Bertrand) and $\mu_i^* = \alpha_i^*/\sigma^*$ for $\theta_i'^* = 1/2$ (Cournot), whereas by applying formula (3.9) we still obtain $\mu_i^* = 0$ for $\theta_i'^* = 1$ (Bertrand) but now $\mu_i^* = \alpha_i^*/\sigma^*$ for $\theta_i'^* = 0$ (Cournot).[13]

For the sake of comparison, let us now look at other approaches to oligopolistic competition in the homogeneous good case.

The Conjectural Variation Approach

The parameterisation we have obtained in (3.9) is equivalent to the one used in the empirical literature, building econometric models that incorporate general equations where each firm conduct in setting price or quantity is represented by a parameter, itself viewed as an index of competitiveness. This is the so-called "conduct parameter method" which has been at the basis of the New Empirical Industrial Organisation (NEIO) and has generated a large number of empirical studies (for a synthesis see Bresnahan 1989, Einav and Levin 2010). It is related to

[13] In the homogeneous good case (a limit case of the homothetic case), differentiability of P is lost. The left-hand elasticity $\epsilon_i^- P\left(\mathbf{p}^*\right)$, the one that must be applied when considering a deviation along the market size frontier, is equal to 1, so that we get

$$\frac{1}{\sigma_i^*} = \frac{\alpha_i^*}{-\epsilon_{p_i}^- D\left(\mathbf{p}^*, Y\right)} = \frac{\alpha_i^*}{-\epsilon_P \widehat{D} \cdot \epsilon_i^- P\left(\mathbf{p}^*\right)} = \frac{\alpha_i^*}{\sigma^*}.$$

Compare to footnote 8 in Sect. 1.

the conjectural variation approach but, as stressed by Bresnahan, "the phrase "conjectural variations" has to be understood in two ways: it means something different in the theoretical literature than the object which has been estimated in the empirical papers."

In the theoretical approach to conjectural variations, each firm i when choosing its quantity is also supposed to make a specified type of conjecture concerning the reaction of the other firms to any of its deviations. These conjectures, though, are not game-theoretically founded. They are introduced directly into the first-order conditions. Following the presentation in Dixit (1986), a sufficient specification[14] consists in introducing conjectural derivatives $r_i = \sum_{j \neq i} \partial x_j / \partial x_i$ for each i. These are called compensating (or non-collusive) variations if each r_i is restricted to be in the interval $[-1, 0]$, for every i. The corresponding first-order conditions are:

$$\frac{P^* - C_i'(x_i^*)}{P^*} = (1 + r_i) \frac{x_i^* / \sum_j x_j^*}{-\epsilon D(P^*)}. \tag{3.10}$$

If matching variations ($r_i > 0$) are excluded, and in particular those leading to the collusive solution, this gives the same characterisation as (3.9) with $r_i = -\theta_i'$. In other words, comparing first-order conditions, the set of oligopolistic equilibrium outcomes appears as the selected subset of outcomes obtained by non-collusive conjectural variations. The concept of oligopolistic equilibrium thus provides some game-theoretic foundation to the concept of conjectural variations, since the conjectural variation terms (within the relevant class) can be identified with the parameterisation of the equilibria of a fully specified game.

[14] Dixit considers the more general case where r_i is a function of both x_i and $\sum_{j \neq i} x_j$. More generally, in the empirical approach to conjectural variations with differentiated products, there are as many conjectural variation parameters as pairs of products (Nevo, 1998). As we have noticed in the case of demand estimation (see footnote 3 in Sect. 1), for estimation on the supply side our approach is also more parcimonious, with one parameter per product.

The Supply Function Approach

Another approach, initiated by Grossman (1981) and Hart (1982), assumes that firms strategies are supply functions. A firm i strategy is a *supply function* S_i associating with every price p_i in $[0, \infty)$ a quantity $x_i = S_i(p_i)$. In order to compare this concept with our own, we shall restrict strategies to the set $\mathbb{S}_+$ of *non-decreasing supply functions*.[15] To define the payoffs of the corresponding game, we have to solve in P the following equation for any N-tuple $\mathbf{S}$ of supply functions in $\mathbb{S}_+^N$

$$\sum_{j=1}^{N} S_j(P) = D(P). \tag{3.11}$$

Since the market demand is strictly decreasing and the supply function of each firm is non-decreasing, if a solution $\hat{P}(\mathbf{S})$ clearing the market exists, then it is unique.

The payoffs are defined as follows. We let

$$\Pi_i(S_i, \mathbf{S}_{-i}) = \hat{P}(S_i, \mathbf{S}_{-i}) S_i\big(\hat{P}(S_i, \mathbf{S}_{-i})\big) - C_i\Big(S_i(\hat{P}\big(S_i, \mathbf{S}_{-i}))\Big),$$

if the market clearing price $\hat{P}(\mathbf{S})$ exists, and

$$\Pi_i(S_i, \mathbf{S}_{-i}) = 0, \text{ otherwise.} \tag{3.12}$$

A *supply function equilibrium* is a Nash equilibrium $\mathbf{S}^*$ of the resulting game.

We can define the residual demand function of firm i at an equilibrium $\mathbf{S}^*$:

$$D_i^*(P, \mathbf{S}_{-i}^*) = \max\left\{ D(P) - \sum_{j \neq i} S_j^*(P), 0 \right\},$$

[15] As Delgado and Moreno (2004) do. However, they assume in addition that firms are identical.

Then for any firm i, maximising in S_i the profit $\Pi_i(S_i, \mathbf{S}^*_{-i})$ amounts to select P^* in

$$\arg\max_{P \in \mathbb{R}_+} \{P\, D_i^*(P, \mathbf{S}^*_{-i}) - C_i(D_i^*(P, \mathbf{S}^*_{-i}))\}. \tag{3.13}$$

or, equivalently, to choose any supply function S_i for which $S_i(P) = D_i^*(P, \mathbf{S}^*_{-i})$ has the unique solution P^*. The multiplicity of supply function equilibria is well known, but we have the following characterisation.

Proposition 6 *If strategies are restricted to non-decreasing supply functions, the set of outcomes of the supply function game,*

$$\left\{ \left(P^*, \mathbf{x}^*\right) \in \mathbb{R}_+^{N+1} \middle| \mathbf{x}^* = \mathbf{S}^* \left(P^*\right) \text{ with } \mathbf{S}^* \text{ a supply function equilibrium} \right\},$$

coincides with the set of oligopolistic equilibrium outcomes.

Proof Let $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ be an oligopolistic equilibrium. We then construct a supply function equilibrium giving the same outcome, each firm i choosing a supply function $S_i^* \in \mathbb{S}_+$ simply characterised by the price-quantity pair (p_i^*, x_i^*), that is such that $S_i^*(P) = x_i^*$ if $P \leq p_i^*$, and $S_i^*(P) = \infty$ otherwise. Clearly, the solution to (3.13) cannot be larger than $\min_{j \neq i}\{p_j^*\}$, hence any profitable deviation by some firm i from S_i^* must involve a price below $\min_{j \neq i}\{p_j^*\}$ and a quantity below $D(p_i) - \sum_{j \neq i} x_j^*$, and thus constitute a deviation with respect to the oligopolistic equilibrium.

To prove the converse, let $\mathbf{S}^* \in \mathbb{S}_+^N$ be a supply function equilibrium. Observe that, for any i, the residual demand $D_i^*(P, \mathbf{S}^*_{-i})$ is decreasing in P and that the profit $p_i x_i - C_i(x_i)$ is increasing in p_i for $x_i > 0$. Hence, by (3.13), (p_i^*, x_i^*) maximises $p_i x_i - C_i(x_i)$ on

$$A_i \equiv \left\{ (p_i, x_i) \in \mathbb{R}_+^2 \middle| x_i \leq D_i^*(p_i, \mathbf{S}^*_{-i}) \right\}. \tag{3.14}$$

For $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, with $\mathbf{x}^* = \mathbf{S}^*(\hat{P}(\mathbf{S}^*))$ and $p_j^* = \hat{P}(\mathbf{S}^*)$ for any j, to be an oligopolistic equilibrium, (p_i^*, x_i^*) should maximise $p_i x_i - C_i(x_i)$ on

$$\widehat{A}_i \equiv \left\{ (p_i, x_i) \in \mathbb{R}_+^2 \middle| \, p_i \leq \min_{j \neq i} \left(p_j^*\right), \right.$$
$$\left. x_i \leq \max \left\{ D\left(p_i\right) - \sum_{j \neq i} x_j^*, 0 \right\} \right\}, \tag{3.15}$$

for every i. Since, $\left(p_i^*, x_i^*\right) \in \widehat{A}_i$ and $\widehat{A}_i \subset A_i$, the result follows. □

This shows that, for any oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, there is a supply function equilibrium $\mathbf{S}^* \in \mathbb{S}_+^N$ such that $\mathbf{S}^*\left(\min_j\{p_j^*\}\right) = \mathbf{x}^*$ and, conversely, for any supply function equilibrium $\mathbf{S}^* \in \mathbb{S}_+^N$, there is an oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ such that $p_j^* = \hat{P}(\mathbf{S}^*)$, for any j, and $\mathbf{x}^* = \mathbf{S}^*(\hat{P}(\mathbf{S}^*))$.

If we consider the differentiable case (restricting $\mathbf{S}_{-i}^*$ to differentiable supply functions in $\mathbb{S}_+^{N-1}$), we may get back formula (3.9) if we derive the first-order condition to firm i programme (3.13) at equilibrium:

$$x_i^* + \left(D'(P^*) - \sum_{j \neq i} S_j^{*\prime}(P^*) \right) (P^* - C_i'(x_i^*)) = 0, \text{ with}$$
$$x_i^* = D_i^*(P^*, \mathbf{S}_{-i}^*), \tag{3.16}$$

or, equivalently,

$$\frac{P^* - C_i'(x_i^*)}{P^*} = \frac{x_i^* / \sum_k x_k^*}{-\epsilon D(P^*) + \sum_{j \neq i} (x_j^* / \sum_k x_k^*) \epsilon S_j^*(P^*)}. \tag{3.17}$$

Taking

$$1-\theta_i' = \frac{-\epsilon D(P^*)}{-\epsilon D(P^*) + \sum_{j\neq i}(x_j^*/\sum_k x_k^*)\epsilon S_j^*(P^*)}. \tag{3.18}$$

we obtain formula (3.9).

In this formula, the term $\sum_{j\neq i}(x_j^*/\sum_k x_k^*)\epsilon S_j^*(P^*)$ may be interpreted as measuring the "reactivity of the other firms" (with respect to prices) as anticipated by firm i at the supply function equilibrium. It has a positive impact on the competitive toughness θ_i' of firm i as measured at the oligopolistic equilibrium. The elasticity of the supply function chosen by firm i is indifferent from the point of view of the firm itself since only the price-quantity pair (p_i, x_i) matters. However varying the elasticities of the other firms'supply functions allows to cover the whole range of admissible values of θ_i'. In particular the Cournot solution corresponds to an elasticity $\epsilon S_j^*(P^*)$ of the supply functions equal to 0 for all j, and the competitive solution to $\epsilon S_j^*(P^*) = \infty$ for at least two j's.

The Pricing Scheme Approach

In Chap. 1, we have introduced in the duopoly case a concept of "pricing scheme" associating with a vector of price announcements the resulting market price. It was mentioned that, if the pricing scheme (which is nothing else than a coordination device) is sufficiently responsive to individual price signals, then we get the Cournot equilibrium. This leads to the interpretation of a Cournot equilibrium as the coordinated optimal decisions of a set of monopolists, each facing some (imperfectly elastic) residual demand. In the original Cournot model, the same coordination is ensured by the use of the inverse demand function. Formally, pricing schemes have the same status as auctions or bidding mechanisms. They could be assimilated to the "facilitating practices" already discussed in Sect. 2 of Chap. 1 (see also d'Aspremont et al., 1991a,b). In this subsection, we shall first come back to the Cournot case where the pricing scheme is supposed to be sufficiently responsive and then examine the case of facilitating practices implying *de facto* that the pricing-scheme is the min-pricing scheme.

In the pricing scheme approach, the market price is supposed to be determined by a *pricing scheme* Ψ, a continuous non-decreasing function from $\mathbb{R}_+^N$ to $\mathbb{R}_+$, associating with each vector of price signals $\boldsymbol{\psi} = (\psi_1, \ldots, \psi_i, \ldots, \psi_N)$ a single price $\Psi(\boldsymbol{\psi})$. For a given pricing scheme Ψ, we thus obtain a game involving the N firms, the strategies of firm i being the set of nonnegative price-quantity pairs $\left(\psi_i, x_i\right)$. For any vector $(\boldsymbol{\psi}, \mathbf{x})$ of such strategies, the payoff of firm i is given by the profit function

$$\Pi_i\left(\boldsymbol{\psi}, \mathbf{x}\right) \equiv \Psi\left(\boldsymbol{\psi}\right) x_i - C_i\left(x_i\right), \tag{3.19}$$

with $(\boldsymbol{\psi}, \mathbf{x})$ satisfying

$$\sum_{i=1}^{N} x_i \leq D\left(\Psi\left(\boldsymbol{\psi}\right)\right). \tag{3.20}$$

A Ψ*-equilibrium* is a vector $\left(\boldsymbol{\psi}^*, \mathbf{x}^*\right)$ in $\mathbb{R}_+^{2N}$, such that $\sum_{i=1}^N x_i^* = D\left(\Psi\left(\boldsymbol{\psi}^*\right)\right)$ and, for every $i \in N$, $\left(\psi_i^*, x_i^*\right)$ is a solution to

$$\max_{(\psi_i, x_i) \in \mathbb{R}_+^2} \Psi\left(\psi_i, \boldsymbol{\psi}_{-i}^*\right) x_i - C_i\left(x_i\right), \text{ s.t. } x_i \leq D\left(\Psi\left(\psi_i, \boldsymbol{\psi}_{-i}^*\right)\right) - \sum_{j \neq i}^{N} x_j^*. \tag{3.21}$$

If we now consider the Cournot model with, say, the function $D^{-1}\left(x_i, \mathbf{x}_{-i}\right) x_i - C_i\left(x_i\right)$ differentiable and strictly quasi-concave in x_i, and assume that the pricing scheme Ψ is differentiable, onto and strictly increasing in each variable ψ_i (and hence strongly responsive), then the Ψ-equilibrium outcome coincides with the Cournot outcome, that is, $\left(\Psi\left(\boldsymbol{\psi}^*\right), \mathbf{x}^*\right) = \left(P^{\mathrm{C}}, \mathbf{x}^{\mathrm{C}}\right)$. Looking at the first-order conditions for an active firm $i \in I^*$ at a Ψ-equilibrium, we obtain

$$\partial_i \Psi\left(\boldsymbol{\psi}^*\right)\left[x_i^* + \left(\Psi\left(\boldsymbol{\psi}^*\right) - C_i'\left(x_i^*\right)\right) D'\left(\Psi(\boldsymbol{\psi}^*)\right)\right] = 0, \tag{3.22}$$

leading to the same conditions as (3.9) for the Cournot case ($\boldsymbol{\theta}' = \mathbf{0}$):

$$\frac{\Psi\left(\boldsymbol{\psi}^*\right) - C_i'\left(x_i^*\right)}{\Psi\left(\boldsymbol{\psi}^*\right)} = \frac{x_i^* / \sum_j x_j^*}{-\epsilon D\left(\Psi\left(\boldsymbol{\psi}^*\right)\right)}, \; i \in I^*. \tag{3.23}$$

The essential property to get this result is the manipulability (upwards and downwards) of the market price by each individual producer. This means that the pricing scheme can be eventually obliterated. The equilibrium can simply be described as having each firm i choosing its monopoly solution (p_i, x_i) on its residual demand, that is, by maximising its profit $p_i x_i - C_i(x_i)$ in price and quantity under the residual demand constraint $x_i \leq D(p_i) - \sum_{j \neq i} x_j^*$. Each firm i will thus end up choosing the Cournot solution x_i^{C} and the same price $p_i^{\mathrm{C}} = P^{\mathrm{C}}$, clearing the market $\sum_j x_j^{\mathrm{C}} = D\left(P^{\mathrm{C}}\right)$.

But, of course, firms can also adopt a different conduct, based on other forms of price coordination, such as facilitating practices. For instance, each firm can include a *meeting competition clause* (or *price-match guarantee*) in its sales contracts, guaranteeing its customers that they are not paying more than what they would to a competitor, so that each customer acts as if facing the single market price $\Psi^{\min}(\mathbf{p}) = \min_j \{p_j\}$, where $\Psi^{\min}$ is called *the min-pricing scheme*. Combining this guarantee with the assumption that each firm i brings x_i to the market, we infer that it should be willing to sell this output at the discount price $P = \min\{\Psi^{\min}(\mathbf{p}), D^{-1}(\sum_j x_j)\}$. We thus get the following payoff function for firm i:

$$\Pi_i\left(p_i, \mathbf{p}_{-i}, x_i, \mathbf{x}_{-i}\right) \equiv \min\left\{\Psi^{\min}(\mathbf{p}), D^{-1}\left(\sum_j x_j\right)\right\} x_i - C_i(x_i). \tag{3.24}$$

This defines a *price-matching oligopoly game* in prices and quantities. The corresponding oligopolistic equilibrium $\left(\mathbf{p}^*, \mathbf{x}^*\right)$, called a $\Psi^{\min}$-*equilibrium*, is a Nash equilibrium satisfying in addition the *no-rationing* *restriction*

$$\sum_j x_j^* = D\left(\Psi^{\min}\left(\mathbf{p}^*\right)\right) \tag{3.25}$$

to eliminate equilibria where customers would be willing to buy more at the equilibrium price $\Psi^{\min}\left(\mathbf{p}^*\right)$. The following proposition states that the equilibria of the price-matching oligopoly game coincide, when the output is homogeneous, with the oligopolistic equilibria.

Proposition 7 *A 2n-tuple $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ is a $\Psi^{\min}$-equilibrium if and only if it is an oligopolistic equilibrium.*

Proof Suppose first that $\left(\mathbf{p}^*, \mathbf{x}^*\right)$ is a $\Psi^{\min}$-equilibrium (so that, for every i, $p_i^* = \Psi^{\min}\left(\mathbf{p}^*\right) = D^{-1}\left(\sum_j x_j^*\right)$), but that, for some i, and some $(p_i, x_i) \in \mathbb{R}_+^2$, $p_i x_i - C_i(x_i) > p_i^* x_i^* - C_i\left(x_i^*\right)$, with $p_i \leq \min\left\{\mathbf{p}_{-i}^*, D^{-1}\left(x_i + \sum_{j \neq i} x_j^*\right)\right\}$. Then,

$$\begin{aligned} &\min\left\{\Psi^{\min}\left(p_i, \mathbf{p}_{-i}^*\right), D^{-1}\left(x_i + \sum_{j \neq i} x_j^*\right)\right\} x_i - C_i(x_i) \\ &= p_i x_i - C_i(x_i) \\ &> \min\left\{\Psi^{\min}\left(\mathbf{p}^*\right), D^{-1}\left(\sum_j x_j^*\right)\right\} x_i^* - C_i(x_i^*), \end{aligned}$$

and (p_i, x_i) is a profitable deviation to the $\Psi^{\min}$-equilibrium, a contradiction.

To prove the other direction, suppose now (p^*, q^*) is an oligopolistic equilibrium (so that again $\sum_j x_j^* = D\left(\min_j\left\{p_j^*\right\}\right)$), but that, for some i, some $(p_i, x_i) \in \mathbb{R}_+^2$, and $p_i' \equiv \min\left\{p_i, \mathbf{p}_{-i}^*, D^{-1}\left(x_i + \sum_{j \neq i} x_j^*\right)\right\}$, we have $p_i' x_i - C_i(x_i) > p_i^* x_i^* - C_i\left(x_i^*\right) \geq 0$. Then $\left(p_i', x_i\right)$ satisfies the two constraints in (3.1) and gives higher profit to firm i, again a contradiction. □

Hence, the min-pricing scheme approach is another way to get oligopolistic equilibria and a relevant one to investigate the large number of markets where the price-match guarantee is offered.

4 The Effects of Intensifying Competition: Two Applications of the Model

The conventional view of the consequences of intensifying competition, through abatement of concentration or restriction of collusive practices, is that it increases welfare by reducing prices and spurring innovation. This view has however been challenged, as more intense competition can be price increasing (Chen and Riordan, 2008; Thisse and Ushchev, 2018; Zhelobodko et al., 2012) and its influence on R&D investment non-monotone (for a synthesis, see Aghion et al., 2005). These two questions can be easily addressed using our framework.

Stiffer Competition: Is It Price Decreasing or Price Increasing?

The basis of the conventional view that an increase in competitive intensity has a price decreasing effect can be traced back to Cournot (1838), where the symmetric equilibrium condition $\sigma(p) = 1/n$ in the case of nil costs, with an increasing function σ, has the consequence that "the resulting value of p would diminish indefinitely with the indefinite increase of the number n" (p. 94). The same result can be obtained without entry, through tougher competitive conduct, as implied by Bertrand's objection to Cournot. Does this view hold when we proceed from the homogeneous to the differentiated oligopoly and from partial to general equilibrium?

A simple way of answering this question is to recall our equilibrium markup formula given in Proposition 4. The markup appears in this formula as a weighted mean of the reciprocals of intra- and intersectoral elasticities of substitution. The weight on the former, $\theta_i\left(1-\alpha_i\beta_{ii}\right)$ for firm i, is increasing in the competitive toughness displayed by firm i and decreasing in the impact α_i of firm i's production on the aggregate output. Hence, more intense competition translates into a higher relative weight put on the reciprocal of intrasectoral elasticity of substitution, so that it decreases (resp. increases) the price of good i if the differentiated goods are more (resp. less) substitutable among themselves than for the numeraire good.

To make this analysis sharper, let us (1) take the CES case, (2) consider the two standard regimes of price and quantity competition (continuously increasing θ_i is anyway equivalent to continuously decreasing α_i) and (3) refer to the average markup in the oligopolistic sector. By (1.13) and taking $\theta_i = 1/2$ for a price equilibrium $\mathbf{p}^{\mathrm{B}}$, we obtain the following harmonic mean of the markups of all firms, weighted by their budget shares:

$$\overline{\mu}^{\mathrm{B}} = \left(\sum_i \frac{\alpha_i^{\mathrm{B}}}{\mu_i^{\mathrm{B}}}\right)^{-1} = \frac{1}{s + \left(\sigma^{\mathrm{B}} - s\right) \sum_i \left(\alpha_i^{\mathrm{B}}\right)^2}. \tag{4.1}$$

The most sensible assumption to make concerning the difference $\sigma^{\mathrm{B}} - s$ is that it is negative, differentiated goods being more substitutable among themselves than for the numeraire good (see Dixit and Stiglitz, 1977). Then, the average markup $\overline{\mu}^{\mathrm{B}}$ is an increasing function of the *Herfindahl index of concentration* $\sum_i \left(\alpha_i^{\mathrm{B}}\right)^2$, so that abating concentration has a price decreasing effect and entry has a pro-competitive effect. The opposite case can however not be excluded, leading to opposite effects: more intense competition is then welfare degrading.

We naturally obtain the same kind of results under quantity competition. By (1.17) and taking $\theta_i' = 1/2$ for a quantity equilibrium $\mathbf{x}^{\mathrm{C}}$, the arithmetic mean of the markups of all firms, weighted by their budget shares, is

$$\overline{\mu}^{\mathrm{C}} = 1/s + \left(1/\sigma^{\mathrm{C}} - 1/s\right) \sum_i \left(\alpha_i^{\mathrm{C}}\right)^2. \tag{4.2}$$

Hence, abating concentration has a price decreasing (resp. increasing) effect and entry has a pro-competitive (resp. anti-competitive) effect if $s > \sigma^{\mathrm{C}}$ (resp. $s < \sigma^{\mathrm{C}}$).

The relation between intra- and intersectoral elasticities of substitution thus appears crucial to settle the sense of price effects of higher competitive intensity. Take however the case of a continuum $[0, N]$ of differentiated goods, a case where budget shares and the ensuing index of concentration are equal to zero (Chamberlin's case of a large group of producers). We then lose the general equilibrium dimension and the markup is simply equal to $1/s$, a constant in the CES case. We can however take instead the case of symmetric *variable elasticity of substitution* (Thisse and Ushchev, 2018). With $\alpha_i = 0$, the expression (1.4) becomes, at a symmetric profile where $x_i = x$ if $i \in [0, n]$ and $x_i = 0$ if $i \in [n, N]$,

$$s_i = \frac{1}{-\partial_{ii}^2 X(\mathbf{x})\, x / \partial_i X(\mathbf{x})}. \tag{4.3}$$

If we stick to the homotheticity assumption, the denominator on the RHS of this equation is homogeneous of degree 0 in $\mathbf{x}$ and only depends on the mass n of produced goods, so that the elasticity of substitution at a symmetric profile is a function of n. In this case, we can consequently have pro- or anti-competitive effects of entry if s is an increasing or a decreasing function, respectively. If the aggregator function is additive ($X(\mathbf{x}) = \int_0^n \xi(x) + \int_n^N \xi(0)$), we obtain for any active firm the elasticity of substitution $s(x) = 1/\left(-\epsilon\xi'(x)\right)$, the reciprocal of the relative love for variety (an index of the local curvature of the aggregator function). As the equilibrium value of x is itself a function of n, we obtain again the two possible pro- and anti-competitive effects according to the sense of variation of s as a composite function of n. So, even under monopolistic competition, the possibility of anti-competitive effects of entry undermines the conventional view against collusive practices and barriers to entry.

Tougher Competition: Does It Foster Innovation?

Two opposite views contend on this question: the Darwinian view for which competition is needed to force firms to innovate in order to survive[16] and the Schumpeterian view for which monopoly rent is required to support innovative activity, tougher competition having a negative impact on innovation (Schumpeter, 1942). These two views refer to two contrary effects, the presence of which may lead to the observed non-monotonicity of the relation between competitive toughness and innovative activity. In recent theoretical and empirical work, Aghion et al. (2005) suggest an explanation for this observation. More intense competition enhances R&D investment when firms are at the same technological level (the Darwinian view), but discourages it when technological leaders and laggards coexist (the Schumpeterian view). By averaging R&D intensities across all industries, an inverted U-relationship between the average innovation rate and product market competition obtains through a composition effect. Non-monotonicity has however deeper roots, within each industry, and does not necessarily appear only at the aggregate level (d'Aspremont et al., 2010).

Let us examine the question in the context of a homogeneous oligopoly where process innovation reduces constant unit production costs. Consider a two-stage game played by N firms, which decide at the first stage whether or not to make, at a fixed cost ϕ, a R&D investment allowing to reduce the unit cost from $\overline{c}$ to $\underline{c}$ (with $\overline{c} > \underline{c} > 0$). At the second stage, n innovators produce at unit cost $\underline{c}$ and $N - n$ laggards at unit cost $\overline{c}$ and compete for demand $1/P$ with toughness θ. The competitive toughness is taken as uniform across all firms and exogenously given, characterising a specific regime of competition (between the two extremes of $\theta = 0$ for Cournot and $\theta = 1$ for Bertrand). Firm i's equilibrium markup $1 - c_i/P^*$ is consequently, by (3.9), equal to $(1 - \theta)\,\alpha_i^*$, with α_i^* the equilibrium market share of firm i. Firm i's profit is consequently $(1 - c_i/P^*)\,\alpha_i^* = (1 - \theta)\,\alpha_i^{*2}$. By aggregating the markup formula over

[16] Or, according to Arrow (1962), a monopolist has less incentive to invent than a competitive firm. This is due to a "replacement effect": the profits resulting from innovation replace profits that are smaller for a competitive firm than for a monopolist (Tirole, 1988). See also the discussion in Dasgupta and Stiglitz (1980).

all firms, we can easily compute the equilibrium price

$$P^* = \frac{n\underline{c} + (N - n)\,\overline{c}}{N - (1 - \theta)}, \tag{4.4}$$

and then, introducing the notation $\kappa \equiv \left(\overline{c} - \underline{c}\right)/\overline{c} \in (0, 1)$ for the *relative cost advantage* of the innovators, the equilibrium market shares $\overline{\alpha}$ and $\underline{\alpha}$ of the innovator and the laggard, respectively

$$\overline{\alpha}\,(\theta, n, N, \kappa) = \min\left\{\frac{1 - \kappa + (N - n)\,\kappa/\,(1 - \theta)}{N - n\kappa}, \frac{1}{n}\right\} \text{ and}$$
$$\underline{\alpha}\,(\theta, n, N, \kappa) = \max\left\{\frac{1 - n\kappa/\,(1 - \theta)}{N - n\kappa}, 0\right\}. \tag{4.5}$$

We can now refer to the *gain of innovating* for a firm confronted with n rival innovators ($0 \leq n < N$), namely

$$G\,(\theta, n, N, \kappa) = (1 - \theta)\left[(\overline{\alpha}\,(\theta, n + 1, N, \kappa))^2 - \left(\underline{\alpha}\,(\theta, n, N, \kappa)\right)^2\right]. \tag{4.6}$$

First notice that laggards are eliminated if $n\kappa \geq 1 - \theta$, the case of drastic innovations, with many innovators benefitting from a high relative cost advantage. The gain of innovating is then $(1 - \theta)\,/\,(n + 1)^2$, a decreasing function of θ, leaving us with the markup squeezing effect of higher competitive toughness, discouraging innovation in the Schumpeterian mood. By contrast, if $0 < n\kappa < 1 - \theta$, the case of non-drastic innovations, the innovator's market share $\overline{\alpha}\,(\theta, n + 1, N, \kappa)$ is increasing and the laggard's market share $\underline{\alpha}\,(\theta, n, N, \kappa)$ decreasing in θ. Although the markup squeezing effect is still working, tougher competition may spur innovation through some sort of Darwinian selection pressure, eroding the territory of the least apt to the benefit of the fittest. It is however interesting to notice that the possible stimulating influence of tougher competition on innovation works here through higher asymmetry of

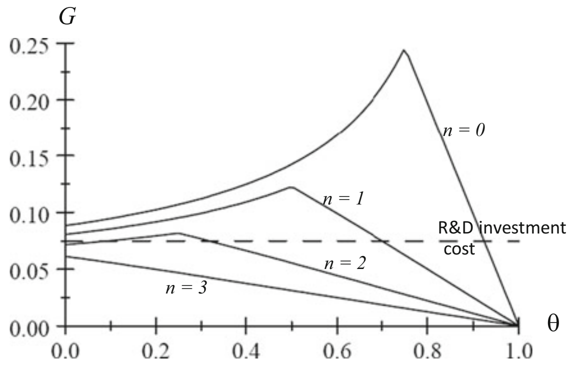

Fig. 2.3 Gain of innovating as a function of competitive toughness

market shares, hence through a concentration effect, thus preserving in some sense the Schumpeterian view.

The function $G(\cdot, n, N, \kappa)$ is strictly quasi-concave, either always decreasing or, if $n\kappa$ is small enough, inverse V-shaped. We illustrate its behaviour in Fig. 2.3 for $\kappa = 0.2$, $N = 8$ and $n \in \{0, 1, 2, 3\}$ (for a full treatment, see Lemma 1 in d'Aspremont et al. 2010).

The function $G(\theta, \cdot, N, \kappa)$ is decreasing, the higher curve in Fig. 2.3 corresponding to $n = 0$ and the lower to $n = 3$. For $\phi = 0.075$, represented by the dashed horizontal line, the subgame perfect equilibrium number n^* of innovators depends upon the competitive toughness: $n^* = 0$ if $\theta \in [0.92, 1]$, $n^* = 1$ if $\theta \in [0.70, 0.92]$, $n^* = 2$ if $\theta \in [0, 0.095] \cup [0.32, 0.70]$ and $n^* = 3$ if $\theta \in [0.095, 0.32]$. In this example, high competitive toughness discourages innovation, but the largest number of innovators is associated with low, but not too low levels of competitive toughness. The relation between competitive toughness and the number of innovators is again non-monotone.

In our deterministic model, a particular equilibrium has to be selected by randomly choosing n^* R&D investors, hence innovators, among N identical firms. It would be more realistic to go farther and assume that R&D investment at the first stage does not necessarily succeed, ensuring only a higher probability of innovating at the second. The decision to invest is then made by comparing the R&D investment cost and the expected gain of innovating. However, the results of such stochastic extension of the model mainly reproduce the preceding analysis and will accordingly be omitted (see d'Aspremont et al., 2010 for the stochastic and general equilibrium extension).

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3

Competition within and Between Groups of Firms

In this chapter, we subdivide the oligopolistic sector into several separable groups and adapt our concept of oligopolistic equilibrium by simplifying accordingly both the market share and the market size constraints. Next we introduce another path to further simplify firms' conjectures: each firm anticipates the income to be spent in its group as if all groups were independent. Then, we further exploit the subdivision into groups by restricting our analysis to the limit case where perfect substitutability holds within each group. Assuming Cournot competition within each group leads to the concept of Cournotian monopolistic competition equilibrium. Through an example and a simple existence proposition, we show that this concept is less demanding than the Cournot-Walras equilibrium concept. In the last section, we present an empirical application to an industry divided into two groups, a dominant group and a competitive fringe, and, finally, examine in a theoretical example the limit of collusion.

The objective of this chapter is to reinforce the potential applicability of our methodology to various fields (e.g. applied industrial organisation, economic geography, international trade, macroeconomics and macrodynamics), as well as its empirical tractability. As already mentioned, the key

C. d'Aspremont, R. Dos Santos Ferreira, *The Economics of Competition, Collusion and In-between*, https://doi.org/10.1007/978-3-030-63602-9_3

factor in Dixit and Stiglitz (1977) seminal contribution is to assume weak separability of preferences between sectors, typically two, the oligopolistic sector and the competitive sector. This is enough in our framework, as we have seen previously, to get an equilibrium markup formula for each firm expressed as the weighted mean of the reciprocals of the intra- and intersectoral elasticities of substitution, the weights being functions of the firm's competitive toughness parameter. But what has made the Dixit-Stiglitz approach so popular for applications are two additional extremely convenient assumptions, a symmetric CES sub-utility function and a large number of firms allowing for the introduction of price and quantity indices to shape firms' conjectures. This has however the drawback of neglecting intersectoral effects and of fixing the intrasectoral effects.

To these two assumptions we want to substitute another type of model simplification, obtained by extending separability within the oligopolistic sector, now subdivided into several groups. We thus obtain a corresponding number of quantity aggregators (or sub-utility functions), not necessarily symmetric. By assuming in addition that the separability is homothetic, we can derive a price aggregator (or price index) for each group. This is the key to adapt the concept of oligopolistic equilibrium to simplified conjectures, the producers of each group competing among themselves and taking as given the prices of the other groups, as summarised by the price indices. The equilibrium markup formula for each firm in each group is now a weighted mean of the reciprocals of the intra- and intergroup elasticities of substitution, the former being group specific and the latter uniform within the group. The weights are still functions of the competitive toughness parameters to be estimated, one for each firm in each group, thus preserving a parsimonious parameter space. For the sake of comparison, we will mention another type of simplified conjectures, not requiring homotheticity, by which firms simply anticipate the income to be spent in their group. This can be viewed as an approximation which becomes exact when groups are independent, that is, when preferences are Cobb-Douglas, clearly a very strong assumption. This assumption, though, has been conveniently adopted in the recent trade literature on granularity, acknowledging the dominant role of large firms in international trade.

An important development in this multiproduct approach allowed by homothetic separability has been to suppose in addition, within each group, perfect substitutability, entailing the law of one price. Firms are accordingly assumed to behave *à la* Cournot, taking as given the price indices of other groups. The oligopolistic equilibrium coincides then with the so-called Cournotian monopolistic competition equilibrium whereby a firm is considered to be large in its own group but small economy-wide. A limit case in that respect is to assume a continuum of groups, thus restricting firm market power to their own group. In that limit case, Neary (2003) general oligopolistic equilibrium (GOLE) is obtained if we assume continuum-quadratic preferences and Dixit-Stiglitz monopolistic competition is obtained if all groups are singletons. While keeping a strong general equilibrium flavour, the Cournotian monopolistic competition equilibrium concept (or more generally the oligopolistic equilibrium concept with different types of firm conduct), appears to be more advantageous for applications, and less demanding for existence, than the Cournot-Walras general equilibrium concept.

Also, the possibility to concentrate on some group, and to measure firms' competitive toughness within that group, opens the way to different types of empirical studies. For example, adopting this methodology, Sakamoto and Stiegert (2018) exploit weak separability and use a multistage demand system to study an industry divided into a dominant group and a competitive fringe, a division reflected in the consumer preferences.[1] After some pre-testing to determine which firms belong to the dominant group, and using the elasticities estimated from a three-stage demand system, they estimate the competitive toughness parameters of the firms in the dominant group. The estimated values of the competitive toughness parameters are close to zero, indicating a collusive conduct against the competitive fringe. However, the fact that full collusion is not observed cannot be surprising. Avoiding the risk of detection might be a

[1]One can refer here to the empirical study of Hottman et al. (2016) who observe (i) "that the typical sector is characterized by a few large firms with substantial markets shares and a competitive fringe of firms with trivial markets shares, and (ii) that "appeal" (quality or taste) explains in large part the success of the firms in the dominant group. We should add that the conduct, in particular a collusive conduct, of the dominant firms reinforces this success.

sufficient reason. As we will show, even without this risk, enforcing full collusion may become impossible as the elasticity of substitution within the collusive group is large enough, making a downward price deviation sufficiently attractive.

1 Simplifying Firms' Conjectures

In the preceding chapter we have kept the main simplification introduced by Dixit and Stiglitz (1977), that is, to assume weak separability of the representative consumer's preferences determining only two sectors, one competitive the other oligopolistic. In this section, we first reinforce this separability in two ways, first by subdividing the oligopolistic sector into several groups,[2] and also (following d'Aspremont and Dos Santos Ferreira, 2020) by exploiting two kinds of separability: weak and homothetic. We show that these properties simplify the problem of a profit-maximising firm, weak separability simplifying quantity conjectures and homothetic separability simplifying price conjectures. The benefit is to reinforce the general equilibrium flavour of our model while keeping tractability. We then introduce another kind of simplified conjectures not requiring homothetic separability.

Simplifying Price and Quantity Conjectures

In the Dixit-Stiglitz approach, the two sectors correspond to the two arguments of the representative consumer's separable utility function $U(X(\mathbf{x}), z)$, where X is a function aggregating N differentiated goods into a single composite good and z is the quantity of a numeraire good, the

[2] As Dixit and Stiglitz (1977), we prefer to use the more general term "group" than the more specific term "industry." As well emphasised by Neary (2004, p. 161), "previous writers had debated the appropriate definition of an "industry," or, in Chamberlin's preferred term, a "group." Typically, definitions were given in terms of cross-elasticities of demand, sometimes of *both* direct and inverse demand functions. […] DS cut through all this fog: instead of restricting the demand functions by imposing arbitrary limits on inter- and intra-industry substitutability, they made a single restriction on the utility function, which implies that (in symmetric equilibria) all products within an industry should have the *same* degree of substitutability with other goods."

composition of which is left implicit. Here, we want to go a step further, by assuming separability of X itself, meaning that the set of differentiated goods can be partitioned into K groups of size N_k ($k = 1, \ldots, K$), each group being aggregated into a composite good X^k so that[3]

$$X(\mathbf{x}) \equiv \widetilde{X}\left(X^1\left(\mathbf{x}^1\right), \ldots, X^K\left(\mathbf{x}^K\right)\right).$$

Assuming only *weak separability* of the representative consumer's utility, with the oligopolistic sector divided into K groups of goods, the utility function of the representative consumer can be simply written as

$$\begin{aligned} U\left(X(\mathbf{x}), z\right) &= U\left(\widetilde{X}\left(X^1\left(\mathbf{x}^1\right), \ldots, X^K\left(\mathbf{x}^K\right)\right), z\right) \\ &= \widetilde{U}\left(X^1\left(\mathbf{x}^1\right), \ldots, X^K\left(\mathbf{x}^K\right), z\right), \end{aligned} \tag{1.1}$$

where $X^1, \ldots, X^K$ are increasing functions.

As in the canonical model presented in Chap. 2, thanks to separability, we may consider two stages when solving the utility maximisation programme. Following the same procedure, but now for each group k, we get at the first stage the *Hicksian demand function* $H_i^k\left(\mathbf{p}^k, X_k\right)$, for good i in group k, and, at the second stage, the *Marshallian demand function* $D^k\left(\mathbf{p}^1, \ldots, \mathbf{p}^K, Y\right)$ for the composite good k, and finally the numeraire good consumption $z = Y - \sum_{k=1}^{K} e^k\left(\mathbf{p}^k, D^k\left(\mathbf{p}^1, \ldots, \mathbf{p}^K, Y\right)\right)$, with $e^k\left(\mathbf{p}^k, D^k\left(\mathbf{p}^1, \ldots, \mathbf{p}^K, Y\right)\right)$ denoting the expenditure in the composite good k.

On the producers' side, we assume (as in the canonical model) an inelastic supply of L units of labour and a wage equal to 1, the constant unit cost in the competitive sector. For notational simplicity, we suppose, for each producer i in group k, a constant marginal cost c_i^k. Income is accordingly equal to the sum of wages and profits, namely $Y = L + \Pi$, where Π is the profit of the imperfectly competitive sector (the profit of the other sector being necessarily zero). Weak separability of the K groups allows to limit the quantity conjectures of each firm to the quantities in

[3] We denote $\mathbf{x}^k \equiv \left(x_i^k\right)_{i=1}^{N_k} \in \mathbb{R}_+^{N_k}$ and $X^k : \mathbf{x}^k \mapsto X^k\left(\mathbf{x}^k\right) \in \mathbb{R}_+$.

its own group. When there is more than a single group ($K > 1$), this is an important simplification of the concept of oligopolistic equilibrium defined in the canonical model. As before, all firms are not necessarily active in an oligopolistic equilibrium and we assume that n_k firms are active in group k and that $N_k - n_k$ firms are inactive. Using the simplifying notations $\mathbf{p} = \left(\mathbf{p}^1, \ldots, \mathbf{p}^K\right) = \left(\mathbf{p}^k, \mathbf{p}^{-k}\right) = \left(p_i^k, \mathbf{p}_{-i}^k, \mathbf{p}^{-k}\right)$ in order to point to the (price) strategy of firm i in group k, and letting $N \equiv \sum_k N_k$, we have the following

Definition 4 An *oligopolistic equilibrium* is a K-tuple of $2N_k$-tuples $\left(\mathbf{p}^{k*}, \mathbf{x}^{k*}\right)_k$ in $\mathbb{R}_+^{2N}$ such that, for any i in group k,

$$\begin{aligned} \left(p_i^{k*}, x_i^{k*}\right) \in \arg & \max_{\left(p_i^k, x_i^k\right) \in \mathbb{R}_+^2} \left(p_i^k - c_i^k\right) x_i^k \\ \text{s.t.} \qquad & x_i^k \leq H_i^k\left(p_i^k, \mathbf{p}_{-i}^{k*}, X^k\left(x_i^k, \mathbf{x}_{-i}^{k*}\right)\right) \\ \text{and } X^k\left(x_i^k, \mathbf{x}_{-i}^{k*}\right) & \leq D^k\left(p_i^k, \mathbf{p}_{-i}^{k*}, \mathbf{p}^{-k*}, Y^*\right), \end{aligned} \tag{1.2}$$

with $Y^* = L + \sum_{k=1}^K \sum_{i=1}^{n_k} \left(p_i^{k*} - c_i^k\right) x_i^{k*}$ and no rationing of the consumer.

With a single group ($K = 1$), we retrieve the oligopolistic equilibrium concept of the canonical model. But it should be stressed that here, with weak separability and more than one group, the market share constraint of producer i in group k involves only strategic variables (prices and quantities) of producers in group k. Moreover, in the market size constraint of i, the only strategic variables of producers outside group k are price strategies appearing through the Marshallian demand D^k.

Also, as in the canonical model, we can derive s_i^k the *intra-group elasticity of substitution* of good i in group k (for the composite good produced by the group)[4] Accordingly, $1/s_i^k$ is the *elasticity of the market*

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[4] s_i^k is the elasticity (in absolute value) of $x_i^k/X_k = x_i^k/H_i^k\left(\mathbf{p}^k, X_k\right)$ with respect to $p_i^k/P_k = p_i^k/\partial_X e^k\left(\mathbf{p}^k, X_k\right)$, where P_k denotes the shadow price $\partial_X e^k\left(\mathbf{p}^k, X_k\right)$ of the composite good k (see d'Aspremont and Dos Santos Ferreira, 2016, Appendix).

share frontier for each good i at the equilibrium point, and relative to the group. We can also derive σ_i^k the *inter-group elasticity of substitution* of good i that measures the intensity of the response of the consumption x_i^k to a change in the price p_i^k taking into account the variation of the Marshallian demand D^k, so that $1/\sigma_i^k$ expresses (in absolute value) the *elasticity of the market size frontier*.[5]

Under *homothetic separability*, all the aggregator functions X^k $(k = 1, \ldots, K)$ are assumed to be homogeneous of degree one and the expenditure function, defined by the consumer's first-stage programme, is linear at the utility level: $e^k\left(\mathbf{p}^k, X_k\right) = P^k\left(\mathbf{p}^k\right) X_k$, with $P^k\left(\mathbf{p}^k\right)$ viewed as a price index for group k. As a consequence, the Marshallian demand derived from the solution to the consumer's second-stage programme is separable: $D^k\left(\mathbf{p}, Y\right) = \widehat{D}^k\left(P^1\left(\mathbf{p}^1\right), \ldots, P^K\left(\mathbf{p}^K\right), Y\right)$. Thus, producers in each group may be assumed to conjecture price index values for the other groups, rather than having to form dispensable conjectures on the corresponding price vectors.

Another significant difference introduced by homothetic separability is the linearity in X_k of the Hicksian demand function for the group k: $H_i^k\left(\mathbf{p}^k, X_k\right) = \partial_i P^k\left(\mathbf{p}^k\right) X_k$ (by Shephard's lemma). We consequently have that β_i^k, the elasticity of x_i^k with respect to X_k, is equal to 1. Also, by referring in addition to the first-order condition of the consumer's first-stage programme, we know (by analogy with formula (1.8) in Chap. 2) that $\alpha_i^k = \epsilon_i X^k\left(\mathbf{x}^k\right)$ can now be identified with the budget share of good i in group k. Finally, the intersectoral elasticity of substitution is just equal to the demand elasticity: $\sigma^k = -\epsilon_{P^k}\widehat{D}^k\left(P^1\left(\mathbf{p}^1\right), \ldots, P^K\left(\mathbf{p}^K\right), Y\right)$, now the same for any differentiated good in group k.

Adapting the notation given in Table 2.1 in Chap. 2, we thus obtain the following table (Table 3.1).

[5]Notice that the elasticities of the two frontiers in the space $x_i^k \times p_i^k$ (with quantity and price in the horizontal and vertical axes, respectively, according to the Marshallian tradition) are here taken as the two corresponding expressions for $-\left(dp_i^k/dx_i^k\right)\left(x_i^k/p_i^k\right)$. In d'Aspremont and Dos Santos Ferreira (2020, equations (8) and (9)) we have adopted the opposite convention, using $-\left(dx_i^k/dp_i^k\right)\left(p_i^k/x_i^k\right)$ for the elasticities of the two frontiers.

Table 3.1 Elasticities under weak and homothetic separability

Elasticities	Weak separability	Homothetic separability
of X^k wrt x_i^k	$\alpha_i^k \equiv \epsilon_i X^k\left(\mathbf{x}^k\right)$	$\alpha_i^k = \frac{p_i^k x_i^k}{P^k\left(\mathbf{p}^k\right)X^k\left(\mathbf{x}^k\right)}$
of x_i^k wrt X^k (via H_i^k)	$\beta_i^k \equiv \epsilon_X H_i^k\left(\mathbf{p}^k, X^k\left(\mathbf{x}^k\right)\right)$	$\beta_i^k = 1$
of intra-group substitution	$s_i^k \equiv \frac{-\epsilon_i H_i^k\left(\mathbf{p}^k, X^k\left(\mathbf{x}^k\right)\right)}{1-\alpha_i^k \beta_i^k}$	$s_i^k = \frac{-\epsilon_i \partial_i P^k\left(\mathbf{p}^k\right)X^k\left(\mathbf{x}^k\right)}{1-\alpha_i^k}$
of inter-group substitution	$\sigma_i^k \equiv \frac{-\epsilon_i D^k(\mathbf{p}, Y)}{\alpha_i^k}$	$\sigma_i^k = \sigma^k$

The following proposition derives the corresponding oligopolistic equilibrium markup formula. The proof of this proposition follows the same line of argument as the one of Proposition 2 in Chap. 2. So we shall not repeat it.

Proposition 8 *Assume weak separability of the representative consumer's utility function U into K groups of goods produced in the oligopolistic sector. Let $\left(\mathbf{p}^{k*}, \mathbf{x}^{k*}\right)_{k=1,\ldots,K}$ be an oligopolistic equilibrium. Then the markup of each firm i in each group k is given by*

$$\frac{p_i^{k*} - c_i^k}{p_i^{k*}} = \frac{\theta_i^{k*}\left(1 - \alpha_i^{k*}\beta_i^{k*}\right) + \left(1 - \theta_i^{k*}\right)\alpha_i^{k*}}{\theta_i^{k*}\left(1 - \alpha_i^{k*}\beta_i^{k*}\right)s_i^{k*} + \left(1 - \theta_i^{k*}\right)\alpha_i^{k*}\sigma_i^{k*}} \equiv \mu_i^{k*}, \tag{1.3}$$

for some $\theta_i^{k*} \in [0, 1]$.

For each firm i in group k, the parameter θ_i^{k*} measures the relative weight put, at the equilibrium $\left(\mathbf{p}^{k*}, \mathbf{x}^{k*}\right)_{k=1,\ldots,K}$, on the market share constraint and hence reflects the intra-group rivalry. It is interpreted as the *competitive toughness* displayed by firm i in group k on its rivals of the same group, as evaluated at the reference equilibrium. Looking at the isoprofit curve of firm i in the space $x_i^k \times p_i^k$ through the intersection of the two frontiers, that is, the equilibrium point, its elasticity (in absolute value) is equal to the markup μ_i^{k*} and must take an intermediate value between the elasticities (in absolute value) of those frontiers, namely $1/s_i^{k*}$ for market share and $1/\sigma_i^{k*}$ for market size.

When $\alpha_i^{k*} \simeq 0$, meaning that firm i is "small" relative to the size of the group, its equilibrium markup coincides with the elasticity of the market share frontier: $\mu_i^{k*} \simeq 1/s_i^{k*}$. There are neither inter-group nor intersectoral feedback effects. But, as seen before for the one group case, the same conclusion holds for firm i when its competitive toughness is high whatever its relative size.

Under homothetic separability, the markup formula (1.3) simplifies to

$$\frac{p_i^{k*}-c_i^k}{p_i^{k*}}=\frac{\theta_i^{k*}\left(1-\alpha_i^{k*}\right)+\left(1-\theta_i^{k*}\right)\alpha_i^{k*}}{\theta_i^{k*}\left(1-\alpha_i^{k*}\right)s_i^{k*}+\left(1-\theta_i^{k*}\right)\alpha_i^{k*}\sigma^{k*}}\equiv\mu_i^{k*}. \tag{1.4}$$

Moreover, in the limit case (i) of perfect substitutability within some group k (with $n_k > 1$), $X^k\left(\mathbf{x}^k\right) = \sum_i x_i^k$ and $P^k\left(\mathbf{p}^k\right) = \min\left(p_1^k, \ldots, p_{n_k}^k\right)$, so that the market share frontier is an horizontal line and differentiability is lost for P^k. Similarly, in the limit case (ii) of perfect complementarity, $X^k\left(\mathbf{x}^k\right) = \min\left(x_1^k, \ldots, x_{n_k}^k\right)$ and $P^k\left(\mathbf{p}^k\right) = \sum_i p_i^k$, so that the market share frontier is a vertical line and differentiability is lost for X^k. In these two cases, we must argue in terms of left- and right-hand derivatives.[6] Now consider, in case (i), a strategy profile $\left(\mathbf{p}^k, \mathbf{x}^k\right) \in \mathbb{R}_{++}^{2n^k}$ with all prices equal. Any tentative upward price deviation would result in a single binding constraint, the market share one, leading to the Bertrand zero markup, since $s_i^k = \infty$. By contrast, any tentative downward price deviation would result in a single binding constraint, the market size one, with x_i^k bounded by the residual demand $D^k\left(p_i^k, \mathbf{p}_{-i}^k, \mathbf{P}_{-k}, Y\right) - \sum_{j\neq i} x_j^k$, leading to the Cournot markup. This is equal to $1/\sigma_i^k = \alpha_i^k/\left(-\epsilon_k \hat{D}^k\right)$, the reciprocal of the Marshallian elasticity of the residual demand. In the limit case of perfect substitutability, the markup of an oligopolistic equilibrium may thus take any intermediate value between the Bertrand and Cournot markups, corresponding to upward and downward price deviations,

[6]So, $\partial_i^- P^k(p, ..., p) = 1$ and $\partial_i^+ P^k(p, ..., p) = 0$ in case (i) and $\partial_i^- X^k(x, ..., x) = 1$ and $\partial_i^+ X^k(x, ..., x) = 0$ in case (ii). As a consequence, $\sigma_i^k \equiv -\epsilon_k \widehat{D}^k\left(P^k(p, ..., p), \mathbf{P}^{-k}\left(\mathbf{p}^{-k}\right), Y\right)/\alpha_i^k$ for a downward price deviation in case (i) and $\sigma_i^k \equiv -\epsilon_k \widehat{D}^k\left(P^k(p, ..., p), \mathbf{P}^{-k}\left(\mathbf{p}^{-k}\right), Y\right)\epsilon_i P^k(p, ..., p)$ for a downward quantity deviation in case (ii).

respectively. This is given by a formula similar to formula (3.9) in Chap. 2:

$$\frac{p_i^{k*} - c_i^k}{p_i^{k*}} = \left(1 - \theta_i'^{k*}\right) \frac{\alpha_i^{k*}}{-\epsilon_k \widehat{D}^{k*}} = \frac{1 - \theta_i'^{k*}}{\sigma_i^{k*}}. \tag{1.5}$$

Cobb-Douglas Utility and Independence Between Groups

We have seen how homothetic separability allows to drastically simplify producers' conjectures by introducing price aggregators for each group. We also have mentioned how introducing economy-wide variables into the perceived demand curve and assuming infinitesimal firms could be a useful approximation, of course at the cost of neglecting inter-group and intersectoral feedback effects. We now consider another kind of approximation, also reinforcing the partial equilibrium dimension, which consists in reducing firms conjectures to conjectures about the income to be spent in their group, as if their group were independent.

Of course such independence is fully valid if we assume that the consumer's utility function is Cobb-Douglas:

$$\widetilde{U}(\mathbf{X}, z) = \prod_k X_k^{\alpha_k} z^{1-\alpha},$$

with $\alpha = \sum_k \alpha_k$. In this case, whatever the prices chosen by the producers, the representative consumer wants to spend $Y^k = \alpha_k Y$ in the goods produced by group k. Groups of oligopolistic firms become independent from each other and firms in group k have only to conjecture Y, knowing α_k. They do not have to make conjectures on prices and quantities prevailing outside group k. For firm i in group k, the market size constraint simply becomes

$$e^k\left(p_i^k, \mathbf{p}_{-i}^k, X^k\left(x_i^k, \mathbf{x}_{-i}^k\right)\right) \leq \alpha_k Y. \tag{1.6}$$

The assumption that the representative consumer's preferences are represented by a Cobb-Douglas utility function (or, equivalently, by a log-linear utility function), defined over a set of groups with CES aggregators, is used in recent papers exploiting the *granularity hypothesis*[7] in international trade theory, recognising that large ("granular") firms play a dominant role in world trade (Breinlich et al., 2020; Gaubert and Itskhoki, 2020; di Giovanni and Levchenko, 2012). Gaubert and Itskhoki (2020) propose a model to analyse the role of granular firms in determining the comparative advantage of countries and estimate this model using French firm-level data on domestic and export sales across manufacturing industries. By assuming pure price competition, Gaubert and Itskhoki (2020, equation (9)) obtain as the equilibrium markup for firm i in group k the weighted harmonic mean of 1 (with relative weight α_i^{k*}) and $1/s$, the reciprocal of the elasticity of substitution, assumed uniform over a continuum of groups of unit size. This is of course a particular case of our Eq. (1.3) with $\theta_i^{k*} = 1/2$, $\sigma_i^{k*} = 1$ and $s_i^{k*} = s$.

Breinlich et al. (2020) analyse how gravity equations[8] for trade flows have to be modified to account for oligopolistic competition. Using combined French and Chinese firm-level export data and a sample of product-level imports by European countries, they show that introducing oligopolistic competition leads to important changes. In their model, they consider a multi-country world with a continuum of groups. The representative consumer's utility function is assumed to be log-linear and, for each group k, the sub-utility X^k is CES with intragroup elasticity of substitution s^k. By group independence the elasticity of the market size frontier is equal to 1. To characterise the strategic interactions of firms within each group and integrate their effects on aggregate market outcomes, a dual approach—using either price or quantity as the decision variable of firms—and an *ad hoc* form of conjectural variations are adopted. The FOC for profit maximisation of each firm is augmented by the perceived induced effect on P^k (resp. on X^k) of a strategic move in price (resp. in quantity) by firm i, that is, $\lambda^k \partial P^k / \partial p_i^k$ (resp.

[7] The *granularity hypothesis* concerning the macroeconomic effects of the behaviour of large firms, was introduced by Gabaix (2011) to explain aggregate fluctuations when all productivity shocks are firm-level idiosyncratic shocks. It was then applied to a multi-sector model of international trade by di Giovanni and Levchenko (2012).

[8] Gravity equations have been introduced by Tinbergen (1962) for analysing bilateral trade flows.

$\lambda'^k \partial X^k / \partial x_i^k$), with the conduct parameter λ^k (resp. λ'^k) taken as uniform across firms in group k. The corresponding markup formulas are given, using our notation, by

$$\mu_i^{k*} = \frac{1}{\left(1 - \lambda^k \alpha_i^{k*}\right) s^k + \lambda^k \alpha_i^{k*}} \quad \left(\text{resp. } \mu_i^{k*} = \left(1 - \lambda'^k \alpha_i^{k*}\right) \frac{1}{s^k} + \lambda'^k \alpha_i^{k*}\right), \tag{1.7}$$

that is, a weighted harmonic (resp. arithmetic) mean of $1/s^k$ and 1, the relative weight on 1 being $\lambda^k \alpha_i^{k*}$ (resp. $\lambda'^k \alpha_i^{k*}$), with the coefficient $\lambda^k \in [0, 1]$ (resp. $\lambda'^k \in [0, 1]$) parameterising the set of equilibria between monopolistic competition and price (resp. quantity) competition and so playing the role of competitive toughness $\theta^k \in [1/2, 1]$ (or its dual $\theta'^k \in [1/2, 1]$) in our markup formulas.

The preceding discussion was based on the assumption of Cobb-Douglas preferences across groups. But we could go a step further and, under more general preferences, assume that firms act *as if* their group were independent, hence not taking price and quantity feedback effects into account. This would be a way to *approximate* oligopolistic equilibria, meaning that the firms in group k conjecture the income Y^k left to be spent in their group and that this conjecture has to be verified at equilibrium. We get accordingly the following new definition:

Definition 5 An *oligopolistic equilibrium with conjectured incomes* is a K-tuple of triples $\left(\mathbf{p}^{k*}, \mathbf{x}^{k*}, Y^{k*}\right)_{k=1,\ldots,K} \in \mathbb{R}_+^{2n+K}$ such that, for any $i = 1, \ldots, n_k$ and any $k = 1, \ldots, K$,

$$\begin{aligned} \left(p_i^{k*}, x_i^{k*}\right) \in \arg & \max_{\left(p_i^k, x_i^k\right) \in \mathbb{R}_+^2} \left(p_i^k - c_i^k\right) x_i^k \\ \text{s.t.} \quad & x_i^k \leq H_i^k\left(p_i^k, \mathbf{p}_{-i}^{k*}, X^k\left(x_i^k, \mathbf{x}_{-i}^{k*}\right)\right) \\ \text{and } e^k\left(p_i^k, \mathbf{p}_{-i}^{k*}, X^k\left(x_i^k, \mathbf{x}_{-i}^{k*}\right)\right) & \leq Y^{k*}, \end{aligned} \tag{1.8}$$

with $X^k\left(\mathbf{x}^{k*}\right) = D^k\left(\mathbf{p}^*, Y^*\right)$, $Y^* = L + \sum_{k=1}^K \sum_{i=1}^{n^k} \left(p_i^{k*} - c_i^k\right) x_i^{k*}$ and no rationing of the consumer.

As before, we can derive the equilibrium markup formula :

$$\frac{p_i^{k*} - c_i^k}{p_i^{k*}} = \frac{\theta_i^{k*}\left(1 - \alpha_i^{k*}\beta_i^{k*}\right) + \left(1 - \theta_i^{k*}\right) p_i^{k*}x_i^{k*}/Y^{k*}}{\theta_i^{k*}\left(1 - \alpha_i^{k*}\beta_i^{k*}\right) s_i^{k*} + \left(1 - \theta_i^{k*}\right) p_i^{k*}x_i^{k*}/Y^{k*}} \equiv \mu_i^{k*}. \tag{1.9}$$

This is again a weighted harmonic mean of the reciprocals of the two demand elasticities of x_i^k with respect to p_i^k at $\left(p_i^{k*}, x_i^{k*}\right)$. That demand elasticity is s_i^{k*} for the market share frontier. But for the market size frontier that demand elasticity is simply 1, since the expenditure in group k is pre-determined. An apparent new term is the equilibrium budget share $p_i^{k*}x_i^{k*}/Y^{k*}$ replacing the elasticity $\alpha_i^{k*} \equiv \epsilon_i X^k\left(\mathbf{x}^{k*}\right)$. However, under homothetic separability,[9] the equilibrium budget share coincides with α_i^{k*}, so that formula (1.9) is a particular case of the general formula (1.3).

2 Multiproduct Cournotian Competition

Our objective in this section is to illustrate the main obstacles in developing a theory of general equilibrium with imperfect competition.[10] For that purpose we shall concentrate on the Cournot approach as applied to each group, assumed to produce a single homogeous good. These obstacles are of two types. First, from the modeler's point of view, combining the difficulties inherent to oligopoly theory, already present in a partial equilibrium context, with those of general equilibrium theory leads easily to intractability and to the introduction of extortionate assumptions to get existence. Second, from the players' point of view, it may be unrealistic to suppose that they are able or willing to take into account all the

[9] More generally, by the consumer's first-order condition,

$$\frac{p_i^{k*}x_i^{k*}}{Y^{k*}} = \alpha_i^{k*}\epsilon_X e^k\left(\mathbf{p}^{k*}, X_k\left(\mathbf{x}^{k*}\right)\right),$$

where $\epsilon_X e^k\left(\mathbf{p}^{k*}, X_k\left(\mathbf{x}^{k*}\right)\right)$ is not necessarily equal to one.

[10] For a survey, see for instance Hart (1985), Bonanno (1990) or d'Aspremont et al. (1999).

conceivable interactions, however weak, that might concern them. Gabszewicz and Vial (1972) in defining a concept of general equilibrium in this context, the *Cournot-Walras Equilibrium*, have adopted the Cournot approach and have brought to light these two kinds of obstacles. After reviewing this concept, we shall propose another concept, the *Cournotian monopolistic competition equilibrium*, in trying to reduce these obstacles.

Two Concepts of General Equilibrium

A first way to generalise the Cournot solution is to consider the demand function as the vector-valued function $\mathbf{D}$ of the vector of prices $\mathbf{P} = (P_k)_{k=1}^{K}$ of all the homogeneous goods

$$\mathbf{D}(\mathbf{P}, Y) = \left(D^1(\mathbf{P}, Y), \ldots, D^k(\mathbf{P}, Y), \ldots, D^K(\mathbf{P}, Y)\right), \tag{2.1}$$

with $P_k > 0$ and $D^k(\mathbf{P}, Y) \geq 0$ for $k = 1, \ldots, K$, and Y denoting the consumer's income. In each group k there are N_k firms each producing the same homogeneous good. Each producer i in the k-th group, denoted I_k, can offer a quantity x_i^k produced at a cost $C_i^k\left(x_i^k\right)$. The two features that characterise Cournot's approach are kept but adapted to the multiproduct case. In each group the law of one price applies. As already mentioned, "the price is necessarily the same for each proprietor" supplying competitively the same market (Cournot, 1838, p. 88). The second feature is here achieved by making the strong assumption that the demand system can be inverted. For each k, there is a well-defined function D_k^{-1} such that

$$P_k = D_k^{-1}\left(\sum_{i \in I_1} x_i^1, \ldots, \sum_{i \in I_k} x_i^k, \ldots, \sum_{i \in I_K} x_i^K, Y\right) \text{ if and only if}$$

$$\sum_{i \in I_k} x_i^k = D^k(P_1, \ldots, P_k, \ldots, P_K, Y). \tag{2.2}$$

This is not an innocuous requirement. From a general equilibrium point of view this is equivalent to requiring the existence of a unique Walrasian equilibrium for each quantity choices of the producers. Following this line one obtains the following definition:

Definition 6 Under perfect substitutability within each group, a *Cournot-Walras equilibrium* is a vector $\widetilde{\mathbf{x}} \in \mathbb{R}_+^N$ (with $N = \sum_k N_k$) such that, for each firm i in each group k,

$$\widetilde{x}_i^k \in \arg\max_{x_i^k \geq 0} x_i^k D_k^{-1} \left(\sum_{j \in I_1} \widetilde{x}_j^1, \ldots, x_i^k + \sum_{j \in I_k \setminus \{i\}} \widetilde{x}_j^k, \ldots, \sum_{j \in I_K} \widetilde{x}_j^K, \widetilde{Y} \right) - C_i^k \left(x_i^k \right),$$

with $\widetilde{Y} = L + \sum_{k=1}^K \sum_{i=1}^{N_k} \left(\widetilde{P}_k \widetilde{x}_i^k - C_i^k \left(\widetilde{x}_i^k \right) \right)$.

We shall not introduce assumptions ensuring existence of such an equilibrium.[11] The Cournot-Walras concept is very demanding not only in terms of assumptions, but also in terms of the conjectures that each firm is required to make about others' actions, i.e. the equilibrium quantities produced by all its competitors in each one of the K groups, aggregated within each group. If we refer to the alternative Ψ-equilibrium approach used in Chap. 2, generalised to the present context, with a pricing scheme allowing each firm to manipulate the market prices while maximising its residual demand, the Cournot-Walras equilibrium is equivalent to a Ψ-equilibrium such that *all* prices are fully manipulable by each firm (see d'Aspremont et al., 1997, for an application to an exchange economy).[12] A way to simplify these conjectures is to replace the overall pricing scheme by K schemes depending each upon the signals sent out by the firms of

[11]For references see Roberts and Sonnenschein (1977), Novshek and Sonnenschein (1978), Mas-Colell (1982), Gary-Bobo (1989), Codognato and Gabszewicz (1993). A recent paper by Azar and Vives (2020) also aims at building a tractable general equilibrium model of oligopoly with large firms, as we do, but using Cournot-Walras equilibrium and allowing for ownership diversification.

[12]The pricing scheme would now be a continuous increasing function Ψ from $\mathbb{R}_+^N$ to $\mathbb{R}_+^K$, associating with each vector of price signals $\boldsymbol{\psi}$ a single market price vector $\boldsymbol{\Psi}(\boldsymbol{\psi})$. As mentioned in Chap. 2, the essential property to get the Cournot solution is the manipulability of the market prices by each individual producer.

a sole group and to adopt the Cournot solution in each group k when $N_k \geq 2$. To be specific, in each group k, each firm i manipulates the price P_k so as to obtain the monopoly solution on the residual group demand curve $D^k\left(P_k, \mathbf{P}^*_{-k}, Y^*\right) - \sum_{j \neq i} \mathbf{x}^{k*}_{-j}$, taking as given its direct rivals' equilibrium quantities and the equilibrium market price of each other good. We thus obtain the outcome of a *Cournotian monopolistic competition equilibrium*, a concept introduced in d'Aspremont et al. (1991a,b, 1997) and applied in d'Aspremont et al. (1995): producers play Cournot in the markets for their own products, taking other goods prices as given.

Definition 7 Under perfect substitutability within each group, a *Cournotian monopolistic competition equilibrium* is a K-tuple of $(1+N_k)$-tuples $\left(P^*_k, \mathbf{x}^{k*}\right)_k$ in $\mathbb{R}^{K+N}_+$ such that, for any i in group k,

$$\left(P^*_k, x^{k*}_i\right) \in \arg \max_{\left(P_k, x^k_i\right) \in \mathbb{R}^2_+} P_k x^k_i - C^k_i\left(x^k_i\right)$$

$$\text{s.t. } x^k_i + \sum_{j \in I_k \setminus \{i\}} \mathbf{x}^{k*}_j \leq D^k\left(P_k, \mathbf{P}^*_{-k}, Y^*\right), \tag{2.3}$$

with $Y^* = L + \sum_{k=1}^{K} \sum_{i \in I_k}\left(P^*_k x^{k*}_i - C^k_i\left(x^{k*}_i\right)\right)$ and no rationing of the consumer.

Of course, a Cournotian monopolistic competition equilibrium coincides with an oligopolistic equilibrium in the situation characterised by perfect substitutability prevailing in all K groups. Formula (1.5) applies with $\boldsymbol{\theta}' \equiv 0$ ($\theta'^{k*}_i = 0$ for every firm i in every group k). Notice also that the Cournotian monopolistic competition equilibrium becomes a regular price equilibrium (or a monopolistic competition equilibrium in the Chamberlin small group case) when $N_k = 1$ for any k.

This concept has several variants, according to the degree of aggregation in the firms' conjectures. An individual firm is assumed "large" in its own group. But how "small" it is in the rest of the economy may vary. In the above definition each individual firm cannot consider the effects of its strategic choices either on the market prices of the other groups or on total income Y, an economy-wide variable treated as given (no Ford effects). One could go further and explicitly assume some economy structure rationalising the type of conjectures postulated in Definition 7. For example in Costa (2004), the number of groups is taken to be large enough so as "to rule out feedback effects from the macroeconomic variables." The most appropriate assumption in that respect is to assume a continuum of groups (and goods), as in Neary (2003, 2016) and Costa and Dixon (2011).[13] Costa and Dixon suppose a modified symmetric CES aggregator and a continuum $[0, K]$ of goods:

$$\widetilde{X}(\mathbf{X}) = K^{\frac{1-\lambda}{1-\sigma}} \left(\int_0^K X_k^{(\sigma-1)/\sigma} dk \right)^{\sigma/(\sigma-1)},$$

where $\sigma > 1$ is the constant elasticity of substitution and $\lambda \in [0, 1]$ is the love for variety parameter ($\lambda = 1$ in the Dixit-Stiglitz case). The corresponding Marshallian demand for group k can be derived as

$$D^k\left(P_k, P\left(\mathbf{p}\right), D\right) = \left(\frac{P_k}{P\left(\mathbf{p}\right)}\right)^{-\sigma} \frac{D}{K^{1-\lambda}},$$

where $P\left(\mathbf{p}\right)$ is the price aggregator

$$P\left(\mathbf{p}\right) = \left(\frac{1}{K^{1-\lambda}} \int_0^K P_k^{1-\sigma} dk\right)^{1/(1-\sigma)}, \qquad (2.4)$$

and D is the aggregate demand (which depends on the form of U and on total income Y).[14] Since individual firms are infinitesimally small in the overall economy, these two macroeconomic variables can rationally be taken as given by each one of them when choosing the quantity produced: at a Cournotian monopolistic competition equilibrium, each firm i in

[13]We also make this assumption in d'Aspremont et al. (2010).

[14]In Costa and Dixon (2011), aggregate demand is supposed to depend also on government public expenditure. Their purpose is to examine the effects of fiscal policy.

group k maximises its profit taking as given the equilibrium quantities of the other firms in its group (as in Cournot) as well as the equilibrium value of the price aggregator and of the aggregate demand. The inverse demand perceived by firm i in group k is

$$D_k^{-1}\left(x_i^k + \sum_{j \in I_k \setminus \{i\}} x_j^k, P, D\right) = \left(\frac{D}{K^{1-\lambda}}\right)^{\frac{1}{\sigma}} \left(x_i^k + \sum_{j \in I_k \setminus \{i\}} x_j^k\right)^{-\frac{1}{\sigma}} P.$$

With a continuum of groups, when $N_k = 1$ for any k, the Cournotian monopolistic competition equilibrium reduces to a Chamberlinian monopolistic competition equilibrium (the large group case).

Neary (2003, 2016) also assumes a continuum of groups (and goods), say the interval $[0, 1]$, and introduces a special case of additive and quasi-homothetic preferences, that he calls "continuum-quadratic preferences":

$$\widetilde{X}(\mathbf{X}) = \int_0^1 \left[aX_k - \frac{b}{2}X_k^2\right] dk,$$

with $a > 0$ and $b > 0$. The labour market, to which the representative consumer supplies inelastically L units of labour (entailing no disutility) is perfectly competitive. The FOC for the consumer utility maximisation programme with budget constraint $\int_0^1 P_k X_k dk \leq Y$ leads to the inverse demand function,

$$P_k = \frac{1}{\lambda}\left[a - bX_k\right],$$

where the budget constraint Lagrange multiplier λ denotes the marginal utility of income. This is computed to be a function of the price distribution and of total income,[15] both economy-wide elements which cannot be affected by an infinitesimal firm. Accordingly, firms take λ as given so that, using the marginal utility of income as numeraire, one can put $\lambda = 1$ (as it is the case when preferences are quasi-linear). Finally,

[15] $\lambda = \left(a\mu_1^P - bY\right)/\mu_2^P$, with μ_1^P and μ_2^P the first and second moments of the price distribution.

the economy-wide wage rate w, also taken as given, determines the unit cost in each group k (say $c_k(w) > 0$) and the Cournot solution in each group can be easily computed, since with fixed λ the perceived inverse demand is linear.[16] Equating total labour demand and supply the wage rate can be derived and we obtain the Cournotian monopolistic competition equilibrium, or in other words Neary's General Oligopolistic Equilibrium (GOLE).[17]

A Linear-Quadratic Application

To illustrate simply the difference between the Cournot-Walras equilibrium and the Cournotian monopolistic competition equilibrium we will now adopt the quasi-linear framework and consider the particular case of a linear demand for each group k:

$$D^k(\mathbf{P}) = \max\left\{0, a_k - \sum_{h=1}^{K} b_{kh} P_h\right\}, \tag{2.5}$$

with $a_k > 0$. Denoting $\mathbf{a}$ the column matrix $[a_k]_k$ and $\mathbf{B}$ the $K \times K$ matrix of coefficients, the system clearing all oligopolistic markets can be written as

$$\mathbf{X} = (\mathbf{a} - \mathbf{BP}), \tag{2.6}$$

where $\mathbf{P}$ and $\mathbf{X}$ are the column matrices of prices and quantities of different goods with, for each k, $X_k = \sum_{i \in I_k} x_i^k$. Assuming that the matrix $\mathbf{B}$ is positive definite, the inverse $\mathbf{B}^{-1} = \left[\beta_{kh}\right]$ is also positive definite and we obtain

$$\mathbf{P} = \mathbf{B}^{-1}(\mathbf{a} - \mathbf{X}) \tag{2.7}$$

[16] As well noted by Neary (2016), the distinction between treating λ parametrically and treating λ as endogenously determined "corresponds to the distinction between "perceived" and "actual" demand functions in the general-equilibrium formalisation of Chamberlin (1933) by Negishi (1961)."

[17] See Colacicco (2015) for more discussion and a survey of various applications of the GOLE approach to topics in international trade.

so that we get the inverse demand function

$$D_k^{-1}\left(\sum_{j\in I_1} x_j^1, \ldots, \sum_{j\in I_k} x_j^k, \ldots, \sum_{j\in I_K} x_j^K\right) = \mathbf{e}_k \mathbf{B}^{-1}(\mathbf{a}-\mathbf{X}), \qquad (2.8)$$

where $\mathbf{e}_k = (0, \ldots, 0, 1, 0, \ldots, 1)$ with 1 the k-th component. It is well known that this system of demand can be derived from the assumption of a representative consumer with preferences represented by a quadratic quasi-linear utility function

$$U(\mathbf{X}, z) = \boldsymbol{\alpha}'\mathbf{X} - \frac{1}{2}\mathbf{X}'\mathbf{B}^{-1}\mathbf{X} + z \qquad (2.9)$$

where z is the quantity of the numeraire good, $\mathbf{X} = [X_k]_{k=1}^K$ denotes the quantities of the different goods, $\boldsymbol{\alpha} = \mathbf{B}^{-1}\mathbf{a}$ and where $\mathbf{X}'$ is the transpose of matrix $\mathbf{X}$. Since $\mathbf{B}^{-1}$ is positive definite, U is strictly concave and the first-order condition for an interior solution for the consumer's programme is sufficient and delivers the inverse demand function $\mathbf{D}^{-1}$.

As for the producers, we assume, for $k = 1, \ldots, K$, that each $i \in I_k$ bears the same constant marginal cost $c_k > 0$. Computing first the Cournot-Walras equilibrium, we have for every k and for every firm $i \in I_k$, the following programme:

$$\max_{x_i \geq 0} \left(\mathbf{e}_k \mathbf{B}^{-1}(\mathbf{a}-\mathbf{X}) - c_k\right) x_i. \qquad (2.10)$$

The corresponding first-order condition is

$$\mathbf{e}_k \mathbf{B}^{-1}(\mathbf{a}-\mathbf{X}) - c_k - \beta_{kk} x_i = 0, \qquad (2.11)$$

for every k and every $i \in I_k$. By symmetry (since firms are identical in each group), the Cournot-Walras equilibrium $\widetilde{\mathbf{x}}$ should satisfy for every i in I_k

$$\widetilde{x}_i = \frac{\sum_{j\in I_k} \widetilde{x}_j^k}{n_k} = \frac{\widetilde{X}_k}{n_k}. \qquad (2.12)$$

Introducing the diagonal matrices $\operatorname{diag}[\beta_{kk}] = [\beta_{kk}\mathbf{e}_k]_k$ and $\operatorname{diag}[n_k^{-1}] = [n_k^{-1}\mathbf{e}_k]_k$, we can rewrite the first-order conditions at equilibrium, in matrix notation:

$$\mathbf{B}^{-1}\left(\mathbf{a} - \widetilde{\mathbf{X}}\right) - \mathbf{c} - \operatorname{diag}\left[\beta_{kk}\right]\operatorname{diag}\left[n_k^{-1}\right]\widetilde{\mathbf{X}} = \mathbf{0}. \tag{2.13}$$

Finally, we get the following expression for the Cournot-Walras equilibrium total quantities:

$$\widetilde{\mathbf{X}} = \left(\mathbf{B}^{-1} + \operatorname{diag}\left[\beta_{kk}\right]\operatorname{diag}\left[n_k^{-1}\right]\right)^{-1}\left(\mathbf{B}^{-1}\mathbf{a} - \mathbf{c}\right). \tag{2.14}$$

To compare, let us now compute the Cournotian monopolistic competition equilibrium $(\mathbf{P}^*, \mathbf{x}^*)$ with, for each group k and for each $i \in I_k$, $\left(P_k^*, x_i^{k*}\right)$ maximising $(P_k - c_k)\, x_i^k$ under the residual demand constraint

$$a_k - b_{kk}P_k - \sum_{h \neq k}^{K} b_{kh}P_h^* - x_i^k - \sum_{j \neq i} x_j^{k*} \geq 0, \tag{2.15}$$

taking as given the equilibrium quantities of other firms in its group and the equilibrium market price in each other group. Using again symmetry and matrix notation, with the diagonal matrix $\operatorname{diag}[\beta_{kk}]$, the first-order conditions at equilibrium can be written as

$$\mathbf{a} - \mathbf{X}^* - \left(\mathbf{B} + \operatorname{diag}\left[b_{kk}\right]\right)\mathbf{P}^* + \operatorname{diag}\left[b_{kk}\right]\mathbf{c} + \operatorname{diag}\left[n_k^{-1}\right]\mathbf{X}^* = 0. \tag{2.16}$$

Since $\mathbf{X}^* = \mathbf{a} - \mathbf{B}\mathbf{P}^*$, the (within-group-symmetric) monopolistic competition equilibrium can simply be expressed as

$$\mathbf{P}^* = \left(\mathbf{B} + \operatorname{diag}\left[n_k\right]\operatorname{diag}\left[b_{kk}\right]\right)^{-1}\left(\mathbf{a} + \operatorname{diag}\left[n_k\right]\operatorname{diag}\left[b_{kk}\right]\mathbf{c}\right), \tag{2.17}$$

an expression involving only elements of the matrix $\mathbf{B}$, whereas the Cournot-Walras equilibrium expression for $\widetilde{\mathbf{X}}$ only involves elements of $\mathbf{B}^{-1}$. To compare the two concepts, we can use $\mathbf{P}^* = \mathbf{B}^{-1}(\mathbf{a} - \mathbf{X}^*)$, to get from the first-order conditions:

$$\mathbf{X}^* = \left(\mathbf{B}^{-1} + \operatorname{diag}\left[b_{kk}^{-1}\right] \operatorname{diag}\left[n_k^{-1}\right]\right)^{-1} \left(\mathbf{B}^{-1}\mathbf{a} - \mathbf{c}\right). \tag{2.18}$$

By comparison with the corresponding expression for the Cournot-Walras equilibrium, the simplification is that the matrix $\operatorname{diag}\left[\beta_{kk}\right]$ of the diagonal elements of the matrix $\mathbf{B}^{-1}$, is replaced by the matrix $\operatorname{diag}\left[b_{kk}^{-1}\right]$ of the reciprocals of the diagonal elements of the matrix $\mathbf{B}$.

Existence of a Cournotian Monopolistic Competition Equilibrium

We now present a set of assumptions ensuring existence of a Cournotian monopolistic competition equilibrium. The first assumption is that, for each group of firms, and for any given uniformly bounded prices in the other groups, there exist a Cournot equilibrium satisfying the same bound. With some additional restrictions (such as imposing a reservation price for each good), several theorems on the existence of Cournot equilibrium could satisfy this assumption (see, e.g. Novshek 1985, or Amir 1996).

A1 There is a positive number $\overline{P}$ such that, for every good k, and for every given $\mathbf{P}_{-k} \in \left[0, \overline{P}\right]^{K-1}$, there is a Cournot equilibrium $\mathbf{x}^{k*} \in \mathbb{R}_+^{N_k}$ within the group I_k satisfying $D^k\left(P_k^*, \mathbf{P}_{-k}\right) = \sum_{i \in I_k} x_i^{k*}$ for $P_k^* \in \left[0, \overline{P}\right]$.

The next two assumptions ensure inter-group strategic complementarity in Cournot equilibrium prices.

A2 For every k, the demand function D^k is twice continuously differentiable and

$$\frac{\partial D^k(\mathbf{P})}{\partial P_k} < 0, \frac{\partial\left(-\epsilon D^k(\mathbf{P})\right)}{\partial P_k} > 0 \text{ and for all } h \neq k,$$
$$\frac{\partial\left(-\epsilon D^k(\mathbf{P})\right)}{\partial P_h} < 0 \text{ if } D^k(\mathbf{P}) > 0.$$

A3 For every good k, either (i) $C''_{ki} \equiv 0$ for all $i \in I_k$ or (ii) $C''_{ki} \geq 0$ for all $i \in I_k$, and $\partial D^k(\mathbf{P}) / \partial P_h \geq 0$, for every $h \neq k$.

We then have the following result:

Proposition 9 *Assumptions* ***A1–A3*** *imply the existence of a Cournotian monopolistic competition equilibrium.*

Proof By **A1**, for every k and every $\mathbf{P}_{-k} \in \left[0, \overline{P}\right]^{K-1}$, there exists a Cournot solution $\mathbf{x}^{k*}(\mathbf{P}_{-k})$ with corresponding price $P_k^*(\mathbf{P}_{-k}) \in \left[0, \overline{P}\right]$ satisfying the first-order condition

$$-\epsilon D^k\left(P_k^*(\mathbf{P}_{-k}), \mathbf{P}_{-k}\right) = \frac{x_i^{k*}(\mathbf{P}_{-k})}{X^{k*}(\mathbf{P}_{-k})} \frac{P_k^*(\mathbf{P}_{-k})}{P_k^*(\mathbf{P}_{-k}) - C'_{ki}\left(x_i^{k*}(\mathbf{P}_{-k})\right)}$$

for every active firm $i \in I_k$. Take $\mathbf{P}^0_{-k}$ and $\mathbf{P}^1_{-k}$ in $\left[0, \overline{P}\right]^{K-1}$ such that $P_h^0 \leq P_h^1$ for all $h \neq k$. We want to show that

$$P_k^*\left(\mathbf{P}^0_{-k}\right) \leq P_k^*\left(\mathbf{P}^1_{-k}\right).$$

Suppose the contrary. Then, by **A2**, $-\epsilon D^k\left(P_k^*\left(\mathbf{P}^0_{-k}\right), \mathbf{P}^0_{-k}\right) > -\epsilon D^k(P_k^*\left(\mathbf{P}^1_{-k}\right), \mathbf{P}^1_{-k})$ and, under (i) of **A3**,

$$\frac{P_k^*\left(\mathbf{P}^0_{-k}\right) - C'_{ki}}{P_k^*\left(\mathbf{P}^0_{-k}\right)} > \frac{P_k^*\left(\mathbf{P}^1_{-k}\right) - C'_{ki}}{P_k^*\left(\mathbf{P}^1_{-k}\right)} \text{ for all } i \in I_k,$$

implying

$$\frac{x_i^{k*}\left(\mathbf{P}_{-k}^0\right)}{X^{k*}\left(\mathbf{P}_{-k}^0\right)} > \frac{x_i^{k*}\left(\mathbf{P}_{-k}^1\right)}{X^{k*}\left(\mathbf{P}_{-k}^1\right)} \text{ for all } i \in I_k, \tag{2.19}$$

a contradiction (since $X^{k*} = \sum_{i \in I_k} x_i^{k*}$). Under (ii) of **A3**, $\partial D^k(\mathbf{P})/\partial P_h \geq 0$, for every $h \neq k$, and under **A2**, $\partial D^k(\mathbf{P})/\partial P_k < 0$, so that $X^{k*}\left(\mathbf{P}_{-k}^0\right) < X^{k*}\left(\mathbf{P}_{-k}^1\right)$. To avoid (2.19), we should have

$$\frac{x_i^{k*}\left(\mathbf{P}_{-k}^0\right)}{X^{k*}\left(\mathbf{P}_{-k}^0\right)} \leq \frac{x_i^{k*}\left(\mathbf{P}_{-k}^1\right)}{X^{k*}\left(\mathbf{P}_{-k}^1\right)} \text{ for some } i \in I_k,$$

hence $x_i^{k*}\left(\mathbf{P}_{-k}^0\right) < x_i^{k*}\left(\mathbf{P}_{-k}^1\right)$, leading again to a contradiction since $C_{ki}'' \geq 0$.

Since $\left[0, \overline{P}\right]^K$ is a complete lattice with respect to the natural order "$\geq$" and since we have shown that the function $\mathbf{P}^* : \left[0, \overline{P}\right]^K \to \left[0, \overline{P}\right]^K$ is monotone increasing (or isotone), by Tarski's theorem (1955) there is a fixed point. This proves the existence of Cournotian monopolistic competition equilibrium. □

In fact, by Tarski's fixed-point theorem, we know more. The set of equilibria is a complete lattice and it has a greatest and a least element.[18]

3 Group-Specific Competitive Conduct

We have exploited different degrees and types of separability with the main objective of reinforcing the general equilibrium flavour of the model. This was by considering groups of goods that are linked by close relations of substitutability or complementarity but also by introducing

[18]There is a large literature on the use of the Tarski fixed-point theorem to study imperfect competition: Topkis (1979), Frayssé (1986), Vives (1990, 1999), Milgrom and Roberts (1990), Amir (1996). For a survey, see Amir (2005).

simplified interactions between groups, the main applications in view being in macroeconomics and international trade. But our model is flexible enough for another objective. This is to focus on the partial equilibrium dimension, looking more closely, within an industry, at the interaction of groups (or subgroups) of firms characterised by similar degrees of competitive toughness, allowing to consider many issues arising in industrial organisation. This is a fruitful line of research both from the empirical and from the theoretical viewpoint, as this section wants to illustrate.

An Empirical Application

To show the empirical applicability of our approach in that regard we briefly describe the recent study of Sakamoto and Stiegert (2018). As they argue, the present approach "provides a way to measure empirically market power while overcoming some of the inherent difficulties highlighted in the previous literature." For that purpose they exploit two main features of the methodology we have adopted. The first is the weak separability of preferences as a preliminary assumption to be tested, allowing to decompose the representative consumer's budget allocation into multiple stages and to calculate various elasticities of substitution. The second feature is the simplicity of the finally obtained equilibrium markup formula based on these elasticity estimates and used for the estimation of the firms' competitive toughness. As compared to the New Empirical Industrial Organisation approach when extended to differentiated products (e.g. Nevo, 1998), the present approach is more parsimonious in the parameter space: the number of conduct parameters increases linearly with the number of goods and not with the square of the number of goods. But it keeps the flexibility of the NEIO approach, since the conduct parameters to be estimated are continuous, in contrast to the so-called "menu approach," where a menu of models to be tested (say Bertrand vs. collusion) is fixed in advance. As Schmalensee (2012) points out, "the best way forward may be to attempt to develop and employ parsimonious parameterisations in the spirit of the "conjectural variations" approach that can provide reliable reduced-form estimates of the location of conduct along the in-between range of incomplete collusion"' (p. 172).

The empirical analysis of Sakamoto and Stiegert is based on the retail market for differentiated varieties of caffeinated ground coffee in the US. They suppose the industry is divided into two groups, a dominant group and a competitive fringe. Their main objective is to estimate the competitive toughness of the firms belonging to the dominant group. To determine the composition of this group they test various preference structures for weak separability. Among those, only one is not rejected, having the two largest national brands (Folgers and Maxwell House, with more than 50% market share) in the dominant group. Hence this is a two-group oligopolistic sector with demand determined by the following representative consumer's utility function:

$$U(\mathbf{x}, z) = \widetilde{U}\left(\widetilde{X}(\mathbf{x}), z\right) = \widetilde{U}\left(X\left(X^1\left(\mathbf{x}^1\right), X^2\left(\mathbf{x}^2\right)\right), z\right).$$

Accordingly, in order to estimate demand, a three-stage demand system can be used[19] to determine three sets of estimation equations, corresponding to three levels of consumer decisions, between expenditure on coffee and the numeraire (top level), between the dominant group and the fringe (middle level), and finally between Folgers and Maxwell House (bottom level). As a result of this demand analysis, the different elasticities of Table 3.1 in this chapter can be computed, say for $k = 1$, the dominant group, $\alpha_i^1 \equiv \epsilon_i X^1\left(\mathbf{x}^1\right)$, $\beta_i^1 \equiv \epsilon_X H_i^1\left(\mathbf{p}^1, X^1\left(\mathbf{x}^1\right)\right)$ as well as the intra- and inter-sectoral elasticities of substitution s_i^1 and σ_i^1. Then using the generalised method of moments, an estimator of the competitive toughness and marginal costs parameters can be defined using the markup formula (1.3) presented in Proposition 8. We refer to Sakamoto and Stiegert (2018), sections 3 and 4, for the detailed discussion of the empirical model, the data, and their identification strategies to deal with the endogeneity of prices and markups. Their results show the advantage of adopting a flexible approach over adopting a menu approach assuming either the Bertrand-like or the pure collusive benchmark. The

[19] For multistage demand systems see Hausman et al. (1994).

competitive toughness parameters of both firms in the dominant group can be estimated using the advocated procedure. These parameters, θ_1^1 and θ_2^1, are not statistically different from zero (collusion),[20] but statistically different from 1/2 (Bertrand-like competition) and from 1 (monopolistic competition). As Sakamoto and Stiegert conclude, "these results suggest very strongly that Folgers and Maxwell House operated in a collusive pricing structure."

The Limit of Collusion: An Example

The case analysed by Sakamoto and Stiegert (2018) involves an oligopolistic sector divided into two groups, a dominant group and a competitive fringe (a typical situation according to Hottman et al. 2016). Although the dominant firms "operated in a collusive structure," it is not clear that pure collusion is enforceable. To examine the reasons for that, we will now develop this example in more general terms on the basis of the same weak separability assumption, with preferences represented by $\widetilde{U}\left(X\left(X^1\left(\mathbf{x}^1\right), X^2\left(\mathbf{x}^2\right)\right), z\right)$, where 1 and 2 are the dominant group and the competitive fringe, respectively.

For explicitness, we will use the following representative consumer's utility function:

$$U(X, z) = \frac{b^{1/\sigma}}{1 - 1/\sigma} X^{1-1/\sigma} + z, \tag{3.1}$$

with $b > 0$ and $\sigma > 1$, where

$$X = \left(X_1^{(s-1)/s} + X_2^{(s-1)/s}\right)^{s/(s-1)} \text{ and } X_k = \left(\sum_{i=1}^{n_k} \left(x_i^k\right)^{\left(s^k-1\right)/s^k}\right)^{s^k/\left(s^k-1\right)} \tag{3.2}$$

[20] Folgers' competitive toughness parameter is estimated in a tight range between 0.12 and 0.14. Maxwell House's toughness parameter are in an equally tight range of 0.02–0.04.

for $k = 1, 2$, with $s > 0$ and $s^k > 0$. Because of homothetic separability of X, indirect utility is also separable, with the corresponding CES price indices as arguments:

$$P = \left(P_1^{1-s} + P_2^{1-s}\right)^{1/(1-s)} \text{ and } P_k = \left(\sum_{i=1}^{n_k} \left(p_i^k\right)^{1-s^k}\right)^{1/\left(1-s^k\right)}. \tag{3.3}$$

For firm i in group k, the Hicksian demand is then

$$H_i^k\left(\mathbf{p}_i^k, X_k\right) = \left(p_i^k / P_k\right)^{-s^k} X_k, \tag{3.4}$$

and the Marshallian demand for the composite good produced in this group is

$$\widetilde{D}^k\left(P_k, P_{-k}, Y\right) = \left(P_k / P\right)^{-s} \min\left(Y/P, bP^{-\sigma}\right). \tag{3.5}$$

In the computations that follow, we will focus on the case of a high aggregate income $Y > bP^{1-\sigma}$. In this case, part of the income is spent in the numeraire sector[21] maintaining the general equilibrium flavour.

At equilibrium, the elasticities of the market share and market size frontiers at their intersection are equal in absolute value to $1/s^k$ and $1/\sigma^{k*}$ as defined above (see Table 3.1). By homotheticity, $\epsilon_i X^k\left(\mathbf{x}^{k*}\right) = \epsilon_i P^k\left(\mathbf{p}^{k*}\right) = \alpha_i^{k*}$ and with $\alpha_k = \epsilon_k P = P_k^{1-s} / \left(P_1^{1-s} + P_2^{1-s}\right)$, we get (assuming $Y^* > bP^{*1-\sigma}$)

$$\sigma^{k*} = \left(1 - \alpha_k^*\right) s + \alpha_k^* \sigma, \tag{3.6}$$

so that σ^k is a function of P_1 and P_2.

In the empirical application of the last subsection the competitive group was characterised by a large number n_2 of firms, each with a

[21]If $Y/P < bP^{-\sigma}$, all the income is spent in the oligopolistic sector, because the marginal utility of the composite good X deflated by its price P is larger than the corresponding marginal utility of the numeraire good, namely 1.

small market share. This should be associated with a weak market power and possibly a high degree of competitive toughness. However, it is not necessary to assume the limit case where $\theta_i^2 = 1$ for any firm i in group 2 (or $n_2 \to \infty$). In that limit case the equilibrium markup[22] reduces to $\mu_i^2 = 1/s^2$, leading to the equilibrium price $p_i^2 = c_i^2/\left(1 - 1/s^2\right)$. Although the constraints on market size are still present (they must be satisfied at equilibrium), the *equilibrium* prices in the competitive group reflect the sole market share constraints and are, *de facto*, independent from the collusive firms' decisions. The interesting case, from a general equilibrium point of view, is consequently the one where competitive firms' toughness is high but not maximal and where their number n_2 is not arbitrarily large.

As to the dominant group with $n_1 = 2$, we want to show that the limit case where $\theta_1^{1*} = \theta_2^{1*} = 0$ (full collusion) and $\mu_1^{1*} = \mu_2^{1*} = 1/\sigma^{1*}$ may not be feasible. To illustrate this issue we shall assume symmetry ($c_1^1 = c_2^1 = c^1$). The collusive price $\widehat{p}^1$ (the same, by symmetry, for both collusive firms) verifies the condition of tangency of the market size frontier and of the isoprofit curve through the potential equilibrium point in the space $x \times p$:

$$\frac{\widehat{p}^1 - c^1}{\widehat{p}^1} = \frac{\left(2^{1/\left(1-s^1\right)}\widehat{p}^1\right)^{1-s} + P_2^{1-s}}{\left(2^{1/\left(1-s^1\right)}\widehat{p}^1\right)^{1-s}\sigma + P_2^{1-s}s} = \frac{1}{\left(1-\alpha_1^*\right)s + \alpha_1^*\sigma} = \frac{1}{\sigma^{1*}}, \tag{3.7}$$

a condition which determines the collusive price as a function of the price P_2, of marginal cost c^1 and of the elasticities of substitution s^1, s and σ. For the collusive price to be an equilibrium price, it must be compatible with the simultaneous satisfaction of the market share and the market size equations in the case of a symmetric profile. Symmetry is enough as regards the market share equation. As to the market size equation, it

[22] This markup is not necessarily close to zero, because the goods produced by competitive firms may be sufficiently differentiated among themselves to keep each producer so to say in its own dedicated niche.

determines the collusive quantity $\widehat{x}^1$:

$$\left(2^{s^1/(s^1-1)}\widehat{x}^1\right) = \left(2^{1/(1-s^1)}\widehat{p}^1\right)^{-s} b\left(\left(2^{1/(1-s^1)}\widehat{p}^1\right)^{1-s} + P_2^{1-s}\right)^{(s-\sigma)/(1-s)}, \tag{3.8}$$

taking the case of a high aggregate income ($Y^* > bP^{*1-\sigma}$). Now, is the symmetric collusive profile with $\left(\widehat{p}^1, \widehat{x}^1\right)$ for each dominant firm enforceable as an oligopolistic equilibrium, conditional upon the price index value P_2 for the competitive group?

To answer this question, we resort to a graphical illustration. We take $\widehat{p}^1 = \widehat{x}^1 = 1$ by an appropriate choice of units of the goods produced in the two sectors.[23] We represent in Fig. 3.1, by solid curves, the market share frontier (for $p^1 > 1$) and the market size frontier (for $p^1 < 1$), as

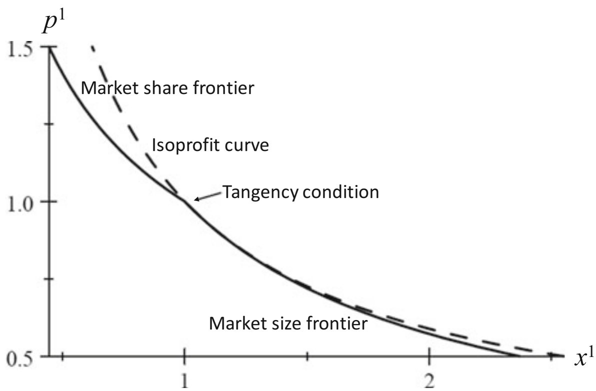

Fig. 3.1 Collusive equilibrium (for $s^1 = 2$)

[23]This amounts to choose the unit of the goods produced by the collusive firms so as to obtain the marginal cost

$$c^1 = \frac{2^{(1-s)/(1-s^1)}(\sigma-1) + P_2^{1-s}(s-1)}{2^{(1-s)/(1-s^1)}\sigma + P_2^{1-s}s},$$

and to choose the unit of the numeraire good so as to obtain the parameter value

$$b = 2^{(s^1-s)/(s^1-1)}\left(2^{(1-s)/(1-s^1)} + P_2^{1-s}\right)^{(\sigma-s)/(1-s)}.$$

well as the dashed isoprofit curve through their point of intersection, for the following parameter values: $s^1 = 2$, $s = 0.5$, $\sigma = 2$, $P_2 = 0.6$. The parameter values $c^1 \simeq 0.18$ and $b \simeq 9.2$ are chosen so as to ensure that $\widehat{p}^1 = \widehat{x}^1 = 1$ (see footnote 23). The collusive profile is clearly enforceable. Each collusive firm i maximises its profit under the two constraints at $(p_i^1, x_i^1) = (1, 1)$, when the other collusive firm chooses $\left(p_j^1, x_j^1\right) = (1, 1)$ and when the price index value of the competitive group is $P_2 = 0.6$.

Enforceability is however eventually lost if we start to increase the elasticity of substitution within the collusive group. A high elasticity of substitution, determining a strong demand response to a downward price deviation, makes such deviation attractive in terms of a larger profit. We represent such a situation in Fig. 3.2, drawn with the same parameter values, except $s^1 = 5$, where we see that somewhat higher isoprofit curves would not violate the market size constraint. This example thus shows that the collusive solution for group 1 is not always enforceable as an oligopolistic equilibrium.

This concludes this chapter aiming at keeping tractability while going beyond the two-sector Dixit-Stiglitz model. We have shown, in various ways, how introducing further separability in order to partition the oligopolistic sector into multiple groups allows to reinforce the general equilibrium flavour of the model without evacuating the strategic dimen-

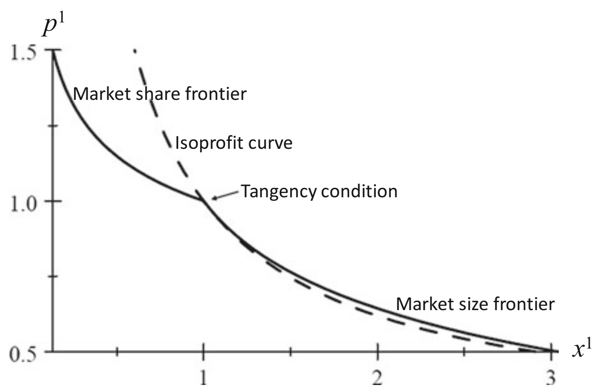

Fig. 3.2 Unenforceable collusive profile (for $s^1 = 5$)

sion of competition among the few. We turn now to different extensions and some more applications.

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4

Extensions

The purpose of this chapter is to show, by examples, how our canonical model can be extended to address some well-known issues and how the possibility of considering competitive toughness as a continuous parameter may open new perspectives and new insights. As a first extension, important for applications in macroeconomics or international trade, we explicitly introduce a labour market and examine in particular the possibility of "involuntary" unemployment (in Keynes' sense). This possibility is shown to result, with a fully adjustable wage, from the oligopolistic character of output markets. In the second extension we introduce a model with overlapping generations and study its distinct dynamic properties under strategic and non-strategic investment in capital. In the third extension, in a model of localised competition, we give another interpretation of our approach in terms of a delegation game.

The first extension proposed in the present chapter purports to add to the model an explicit labour market, up to now kept in the shadow within the competitive sector. Proceeding with an extensive analysis of the labour market which would address its specificities—wage bargaining in particular—would be beyond the scope of this book. And extending to the labour market the same treatment of competition as the one applied

C. d'Aspremont, R. Dos Santos Ferreira, *The Economics of Competition, Collusion and In-between*, https://doi.org/10.1007/978-3-030-63602-9_4

to the output markets, a procedure found in some early New Keynesian contributions (for instance, Cournot competition between syndicates in Hart, 1982, or monopolistic competition between households in Blanchard and Kiyotaki, 1987) would precisely miss those specificities, lending a symmetry to the working of output and labour markets that cannot be found in the real world. As an expository device, we have opted in favour of a fixed, but competitively adjustable, wage. Of course, the absence of a specific analysis of the labour market does not allow to study the types of unemployment that are found in partial equilibrium analyses of the labour market, like frictional unemployment or what Keynes calls "voluntary" unemployment, principally ascribable to *labour* market power. The adopted procedure throws however some light on one of the general equilibrium mechanisms at work behind "involuntary" unemployment: in spite of full adjustability of the money wage, strong *output* market power may prevent the labour market from attaining a full employment equilibrium.

The second extension of our model is addressed to investment. Modern macroeconomic models belonging to the Walrasian tradition, in particular those of the Dynamic Stochastic General Equilibrium (DSGE) type, concede a rather insignificant place to firms, degraded to the status of automata in perfectly competitive markets. Among these models even those of the New Keynesian brand, which acknowledge the market power derived from product differentiation, essentially ignore strategic interaction between firms: these are often assumed to form continua on the supply side of monopolistically competitive markets. And should this not be the case, the process of creation and destruction of firms is at least reduced to the fulfilment of the zero-profit condition and the intertemporal investment decisions replaced by the myopic hiring of capital services provided by the consumers, as a result of their own saving decisions.[1] This second extension is designed to give on the contrary a

[1]See, for instance, Christiano et al. (2005), where the representative household "makes a consumption decision and a capital accumulation decision, and it decides how many units of capital services to supply" (p. 11). Admittedly, their "assumption that households make the capital accumulation and utilization decisions is a matter of convenience. At the cost of more complicated notation, [they] could work with an alternative decentralization scheme in which firms make these decisions" (p. 13,

prominent place to investing firms. We come back to the limit case of the model described in Chap. 3, assuming a continuum of groups, but each one composed of a finite number of competitors producing the same homogeneous good, now in an economy with overlapping generations of households and firms, living for two periods. We compare the distinct dynamic properties of the model near the stationary solution in two cases, when investment is decided by the young saving representative household (in the spirit of most modern macroeconomic models) and when investment is strategically decided by young firms. In particular, robust self-sustained fluctuations only appear when investment is strategic, and under conditions which become stronger (eventually impossible to satisfy) as competition becomes tougher.

Contrary to the first two extensions, the last extension stays rather in the field of industrial organisation. The idea is to look at competitive toughness from another perspective, interpreting it as a managerial attitude. We first adapt our model to spatial competition, representing the ideal variety that each household wants to consume by a location in a product space *à la* Hotelling. An important advantageous difference with respect to the standard spatial competition model is that, thanks to the constraint on market size, our price-quantity approach reduces the possibility of undercutting which threatens the existence of a pure price equilibrium in the Hotelling game.[2] We then re-interpret competitive toughness as a managerial attitude which might be linked to the personality (more or less aggressive) of the manager or consciously chosen as a strategy of the firm. Contrary to the behavioural approach, we maintain that such a variety of managerial attitudes is compatible with profit maximisation. Finally, as in the delegation literature, we analyse a two-stage game where each firm owner chooses a managerial type at the first stage and managers compete in an oligopoly game at the second stage. Extending the oligopoly game to a delegation game and using a subgame perfection argument appears as a possible way to reduce the indeterminateness of oligopolistic equilibria.

n. 8). The results of such an alternative scheme may however be significantly different if strategic interaction is allowed for.

[2] See d'Aspremont et al. (1979).

1 Adding a Labour Market

The labour market has heretofore been left in the shadow, with rigid supply L, demand $\sum_{i=1}^{n} C_i\left(x_i\right)+z$ and competitive real wage equal to 1, the marginal productivity of labour in the numeraire sector. Dixit and Stiglitz (1977) suggest the interpretation of numeraire as leisure, which is consistent with our definition of oligopolistic equilibrium. Under this interpretation, labour supply must be viewed as addressing the sole oligopolistic sector, hence equal to $L-z=L-\left(Y-e\left(\mathbf{p}, D\left(\mathbf{p}, Y\right)\right)\right)$, becoming price- and income-elastic. Labour demand is accordingly restricted to $\sum_{i=1}^{n} C_i\left(x_i\right)$ and the wage 1 is then purely nominal, rather than real.

An Enlarged Canonical Model

We can however introduce labour as an additional good. In their seminal macroeconomic paper, Blanchard and Kiyotaki (1987) make consumers' utility depend upon the composite good produced in the monopolistically competitive sector, upon the real money balance and, negatively, upon labour. Labour being itself a composite good, the labour market is assumed to be monopolistically competitive too. Here, as in d'Aspremont et al. (1996), we shall however keep labour homogeneous and the labour market perfectly competitive. We shall further take the numeraire as a non-produced good, a way of lending to the economy some—although certainly not all—characteristics of a monetary economy.[3]

How do these assumptions impact our canonical model as defined in Chap. 2? As to the representative consumer's behaviour, we may keep intact the analysis of that chapter if we assume additive separability of the utility function with respect to labour supply l, namely $U\left(X\left(\mathbf{x}\right), z\right)-V\left(l\right)$, with V non-decreasing. In order to retrieve exactly, as concerns

[3]"The first characteristic […] is the fact that money has […] a zero, or at any rate a very small, elasticity of production. […] Money, that is to say, cannot be readily produced—labour cannot be turned on at will by entrepreneurs to produce money in increasing quantities as its price rises in terms of the wage-unit. […] Obviously […] the above condition is satisfied, not only by money, but by all pure rent-factors, the production of which is completely inelastic" (Keynes, 1936, pp. 230–231).

labour, the assumptions of Chap. 2, we will take $V(l) = 0$ for $l \in [0, L]$ and $V(l) = \infty$ for $l \in (L, \infty)$. However, there will be a difference as concerns income Y, since we shall now assume a consumer's endowment Z of the numeraire good, so that $Y = Z + wl + \Pi$, where w is the wage and Π the aggregate profit. At equilibrium, $l^* = \sum_{i=1}^{n} C_i\left(x_i^*\right)$ and $\Pi^* = \sum_{i=1}^{n}\left(p_i^* x_i^* - wC_i\left(x_i^*\right)\right)$, so that $Y^* = Z + \sum_{i=1}^{n} p_i^* x_i^* = Z + e\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)$.

In the canonical model of Chap. 2, the equilibrium income level and the associated scale of production are anchored to the labour endowment L. As the numeraire is produced under a linear technology and sold in a competitive market, full employment is necessarily attained at equilibrium, labour demand z^* in the competitive sector just adjusting to equalise the residual labour supply $L - \sum_{i=1}^{n} C_i\left(x_i^*\right)$.

Now, the anchor is provided by the endowment Z of the non-produced good, full employment resulting in principle from an appropriate wage adjustment. Notice indeed that the equilibrium price of good i is $p_i^* = wC_i'\left(x_i^*\right)/\left(1 - \mu_i^*\right)$, which can be made arbitrarily small by reducing w, under the condition however that the equilibrium markup μ_i^* is bounded away from 1. Otherwise, if μ_i^* tends to 1 for any i as w becomes small enough, "there may be *no* method available to labour as a whole whereby it can bring the general level of money-wages into conformity with the marginal disutility of the current volume of employment" (Keynes, 1936, p. 13), which is zero in our case if $l^* < L$. In other words, there may be *involuntary unemployment* in Keynes' sense. In the *General Theory*, the existence of involuntary unemployment is principally associated with an intertemporal coordination failure occurring in financial markets.[4] Here, the coordination failure stems from the oligopolistic character of output markets, but it does at least not result from wage rigidity of any sort. Dehez (1985), d'Aspremont et al. (1989a,b, 1990) and Silvestre (1990) have exploited this approach to involuntary unemployment under different regimes of competition.

[4] Dos Santos Ferreira (2014) suggests a formalisation of Keynes' approach to involuntary unemployment, as developed in the *General Theory*.

The Possibility of Involuntary Unemployment: An Example

The equilibrium markup μ_i^* is a weighted average of the equilibrium values of the intra- and intersectoral elasticities of substitution s_i^* and σ_i^*. But, when discussing the possibility of involuntary unemployment, it is principally the behaviour of the intersectoral elasticity of substitution, as the wage w and consequently the prices $\mathbf{p}^*$ of the produced goods are reduced, that must be considered. Rather than engaging in the analysis of the general case and just for the purpose of illustration, let us restrict our discussion to the simple case of a symmetric oligopoly under homothetic separability of consumer's utility.

More precisely, let the representative consumer's preferences be represented by $U\left(\sum_{i=1}^{n} x_i, z\right) - V(l)$, with U homogeneous of degree one and $V(l) = 0$ for $l \in [0, L]$, $V(l) = \infty$ for $l \in (L, \infty)$, and let the labour cost c of a unit of output be constant, uniform and positive. By homogeneity of U, $e\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right) = P\left(\mathbf{p}^*\right) \widehat{D}\left(P\left(\mathbf{p}^*\right), 1\right) Y^* \equiv \gamma\left(P^*\right) Y^*$, with $\gamma\left(P^*\right)$ denoting the budget share of the produced good. Since equilibrium income Y^* is the sum of the expenditure in the produced and non-produced goods, $\gamma\left(P^*\right) Y^*$ and Z respectively, we have

$$Y^* = \frac{Z}{1 - \gamma\left(P^*\right)}, \tag{1.1}$$

the equilibrium income being equal to the autonomous expenditure Z times the Keynesian multiplier $1/\left(1 - \gamma\left(P^*\right)\right)$. Output $\gamma\left(P^*\right) Y^*/P^*$ is feasible if and only if

$$\widehat{D}\left(P^*, \frac{1}{1 - \gamma\left(P^*\right)}\right) Z \equiv \frac{1}{P^*} \frac{\gamma\left(P^*\right)}{1 - \gamma\left(P^*\right)} Z \leq \frac{L}{c}, \tag{1.2}$$

with equality in the case of full employment. Notice that the LHS of this inequality is the demand for the produced good generated by the autonomous expenditure Z and augmented by the Ford effect as described by the Keynesian multiplier. We assume demand to remain a decreasing function in spite of incorporating Ford effects. Formally, at any price P,

$$\widehat{\sigma}(P) \equiv -\epsilon_P \widehat{D}\left(P, \frac{1}{1-\gamma(P)}\right) = 1 - \frac{\epsilon\gamma(P)}{1-\gamma(P)} = \frac{\sigma(P)-\gamma(P)}{1-\gamma(P)} > 0. \tag{1.3}$$

Therefore, assuming $\lim_{P\to 0} \widehat{D}(P, 1/(1-\gamma(P))) > L/Zc$ and $\lim_{P\to\infty} \widehat{D}(P, 1/(1-\gamma(P))) = 0 < L/Zc$, there is a unique price P^{FE} ensuring full employment.

Can the price P^{FE} be implemented as an equilibrium price? If the market for the produced good is perfectly competitive, that will always be the case, through the appropriate wage adjustment: $w = P^{\mathrm{FE}}/c$.

What about the oligopolistic market? Recall that the elasticity $-(dp_i/dx_i)(x_i/p_i)$ of the isoprofit curve through the symmetric equilibrium point $(x_i^*, p_i^*) = (x^*, P^*)$ is the markup $(P^* - wc)/P^*$ and that this markup can be expressed as a weighted average of the elasticities of the two frontiers defined by the market share and the market size constraints, here respectively 0 and $1/n\sigma(P^*)$ (without Ford effects) or $1/n\widehat{\sigma}(P^*)$ (with Ford effects). So, as previously formulated, we get (formula (3.9) in Chap. 2, in the symmetric case and omitting the prime symbol for simplicity):

$$\frac{P^* - wc}{P^*} = (1-\theta^*)\frac{1/n}{\sigma(P^*)} \text{ (or } (1-\theta^*)\frac{1/n}{\widehat{\sigma}(P^*)}\text{)}. \tag{1.4}$$

In a symmetric oligopolistic equilibrium, we then have:

$$P^* = \frac{wc}{1-\mu^*}, \tag{1.5}$$

$$\begin{aligned} &\text{where } \mu^* = \frac{1-\theta^*}{-n\epsilon_P \widehat{D}(P^*, 1)} \equiv \frac{1-\theta^*}{n\sigma(P^*)} \text{ without Ford effects} \\ &\text{or } \mu^* = \frac{1-\theta^*}{-n\epsilon_P \widehat{D}\left(P^*, \frac{1}{1-\gamma(P^*)}\right)} \equiv \frac{1-\theta^*}{n\widehat{\sigma}(P^*)} \text{ with Ford effects.} \end{aligned} \tag{1.6}$$

As an example, consider the Cournot case ($\theta^* = 0$). If $\mu^{\mathrm{FE}} \equiv 1/n\sigma\left(P^{\mathrm{FE}}\right) \leq 1$ (resp. $\mu^{\mathrm{FE}} \equiv 1/n\widehat{\sigma}\left(P^{\mathrm{FE}}\right) \leq 1$) as firms disregard (resp. consider) Ford effects, a wage w such that $w^{\mathrm{FE}} = \left(1 - \mu^{\mathrm{FE}}\right) P^{\mathrm{FE}}/c$ will support a unique equilibrium, at the Cournot price $P^{\mathrm{C}}\left(w^{\mathrm{FE}}\right)$, with full employment. Equilibria at higher prices will be unenforceable and equilibria at lower prices will be infeasible. The Cournot equilibrium has just taken the place of the competitive equilibrium, but wage adjustment continues to do the job. This will however not be the case as soon as $\mu^{\mathrm{FE}} > 1$, which is inadmissible.

To explore this issue, let us first redefine oligopolistic equilibria so as to allow for the rationing of consumer's labour supply. In the following, we will give a preference to equilibria with Ford effects, since the condition leading to involuntary unemployment, $\widehat{\sigma}\left(P^{\mathrm{FE}}\right) < 1/n$ or, equivalently, $\sigma\left(P^{\mathrm{FE}}\right) < 1/n + (1 - 1/n)\,\gamma\left(P^{\mathrm{FE}}\right)$ is then weaker.

Definition 8 A symmetric *oligopolistic equilibrium with Ford effects at positive wage* w is a pair $(P^*, x^*) \in [wc, \infty) \times [0, L/nc]$ such that, for any $i \in \{1, \ldots, n\}$,

$$\begin{aligned}
\left(P^*, x^*\right) \in \arg & \max_{(p_i, x_i) \in \mathbb{R}_+^2} (p_i - wc)\, x_i \\
\text{s.t.} \quad & p_i \leq P^* \\
\text{and } x_i + (n-1)\, x^* \leq & \widehat{D}\left(\min\left\{p_i, P^*\right\}, 1\right) \\
& \times \left(Z + \min\left\{p_i, P^*\right\}\left(x_i + (n-1)\, x^*\right)\right),
\end{aligned}$$

with the last constraint binding at (P^*, x^*).

Let us now formulate a proposition on the existence of *involuntary unemployment*, in the sense of unemployment emerging in equilibrium at any positive wage w, however small. Clearly, if $w > P^{\mathrm{FE}}/c$, all equilibria, even the competitive one, will display unemployment at that wage, but unemployment can then be seen as the consequence of a too high wage. Involuntariness of unemployment is by contrast the consequence of a *coordination failure*: even at an arbitrarily small wage, some equilibria—those characterised by a competitive toughness close to the lowest enforceable competitive toughness—will exhibit unemployment. Wage flexibility does not work in that case as a secure way to get rid of unemployment.

The argument underlying the proposition substantially refers to the Cournot regime, which is precisely the one with the lowest enforceable competitive toughness, zero, in the case of a homogeneous oligopoly. In addition, as the Cournot outcome coincides in this case with the quantity equilibrium outcome, taking a single strategy variable into account somewhat simplifies the argument.

Proposition 10 *Assume that, for any* $P \in (0, \infty)$, $\widehat{\sigma}(P)$ *is positive and increasing, with* $\lim_{P\to 0} \widehat{\sigma}(P) < 1/n$ *and* $\lim_{P\to\infty} \widehat{\sigma}(P) > 1$.[5] *Then, for* L/cZ *large enough, there exist symmetric oligopolistic equilibria with Ford effects and unemployment at any positive wage* w.

Proof The first step of the proof concerns existence of a symmetric oligopolistic equilibrium with Ford effects at some positive wage w. Take a firm facing a sole active constraint, the one on market size. This firm can be seen as maximising in P the profit obtained from its residual demand, namely $\Pi\left(P, \overline{X}\right) = (P - wc)\left(\widehat{D}(P, 1/(1-\gamma(P)))\,Z - \overline{X}\right)$, where $\overline{X}$ is the aggregate quantity chosen by its competitors. The corresponding first-order condition $\partial_P \Pi\left(P, \overline{X}\right) = 0$ can be expressed as

$$\frac{P - wc}{P} = \frac{1 - \overline{X}/X(P)}{\widehat{\sigma}(P)},$$

with $X(P) = \widehat{D}(P, 1/(1-\gamma(P)))\,Z$ and $\widehat{\sigma}(P) = (\sigma(P) - \gamma(P))\,/\,(1-\gamma(P))$, verifying our equilibrium markup formula for a symmetric profile and for $\theta = 0$. The second derivative of the profit function

[5] More generally, if $\widehat{D}(P, 1) > 0$ for $P < \overline{P} < \infty$ and $\widehat{D}\left(\overline{P}, 1\right) = 0$, we assume instead: $\lim_{P\to\overline{P}} \widehat{\sigma}(P) > 1$.

$\Pi\left(\cdot,\overline{X}\right)$ is

$$\partial^2_{PP}\Pi\left(P,\overline{X}\right)=-\frac{P-wc}{P}\frac{\widehat{\sigma}\left(P\right)}{P}X\left(P\right)\left[\epsilon\widehat{\sigma}\left(P\right)-\widehat{\sigma}\left(P\right)+\frac{P+wc}{P-wc}\right].$$

When the first-order condition is satisfied, the expression between brackets is equal to

$$\epsilon\widehat{\sigma}\left(P\right)+\left(\frac{\widehat{\sigma}\left(P\right)}{1-\overline{X}/X\left(P\right)}-1\right)+\frac{\overline{X}/X\left(P\right)}{1-\overline{X}/X\left(P\right)}\widehat{\sigma}\left(P\right)>0,$$

so that $\partial^2_{PP}\Pi\left(P,\overline{X}\right)<0$. Hence, the profit function $\Pi\left(\cdot,\overline{X}\right)$ is strictly quasi-concave, since decreasing at any critical point. It is also increasing for P close to wc and decreasing for large enough P (as $\lim_{P\to\infty}\widehat{\sigma}\left(P\right)>1$ implies $\lim_{P\to\infty}\gamma\left(P\right)=0$). So, given w, there exists a symmetric Cournot equilibrium at the Cournot price $P^{\mathrm{C}}\left(w\right)$ provided the realisability condition $cX\left(P^{\mathrm{C}}\left(w\right)\right)\leq L$ (or $P^{\mathrm{C}}\left(w\right)\geq P^{\mathrm{FE}}$, with $cX\left(P^{\mathrm{FE}}\right)=L$) is satisfied. The pair $\left(P^{\mathrm{C}}\left(w\right),X\left(P^{\mathrm{C}}\left(w\right)\right)/n\right)$ is then a symmetric oligopolistic equilibrium with Ford effects parameterised by $\theta^*=0$. Symmetric oligopolistic equilibria with Ford effects at the same wage and at lower prices, with each firm constrained in its market share (hence with $\theta^*>0$), will also exist for any P at least equal to $\min\left\{wc,P^{\mathrm{FE}}\right\}$. An adjustment of w leading to $P^{\mathrm{C}}\left(w\right)=P^{\mathrm{FE}}$ would leave a unique symmetric equilibrium at that wage, the Cournot one, with full employment. However, P^{FE} may become arbitrarily small as L/cZ indefinitely increases, so that, by our assumption on $\lim_{P\to0}\widehat{\sigma}\left(P\right)$, $\widehat{\sigma}\left(P^{\mathrm{FE}}\right)<1/n\leq\widehat{\sigma}\left(P^{\mathrm{C}}\left(w\right)\right)$ for any wage w. This implies $P^{\mathrm{C}}\left(w\right)>P^{\mathrm{FE}}$, with unemployment whatever the wage at symmetric equilibria close enough to Cournot, that is, parameterised by small enough competitive toughness θ^*. $\square$

We illustrate this result in the following diagram (Fig. 4.1), where x and P are measured along the horizontal and vertical axes, respectively. The highest decreasing curve represents the demand addressed (under symmetry) to the individual firm as a function of the price P, or rather

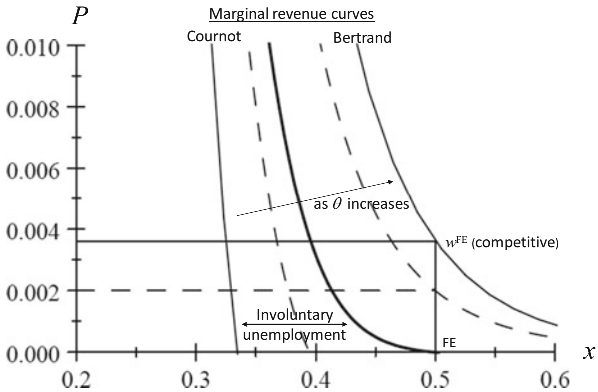

Fig. 4.1 Involuntary unemployment

its inverse $X(P)/n \longmapsto P$.[6] Available labour to produce one unit of the good is $L/c = 0.5$, so that the solid horizontal line represents the competitive full employment equilibrium wage. The lowest decreasing curve represents the marginal revenue of the Cournot competitor when all firms produce equal quantities: $X(P)/n \longmapsto P(1 - 1/n\widehat{\sigma}(P))$. The five decreasing curves belong to a family parameterised by the competitive toughness θ: $X(P)/n \longmapsto P(1 - (1-\theta)/n\widehat{\sigma}(P))$, with θ increasing from 0 (Cournot, the lowest curve) to 1 (Bertrand, the highest one).

The thick curve, intersecting the horizontal axis at $L/c = 0.5$, constitutes a watershed. For any tougher competition regime (one is represented by the higher dashed curve), there is a wage (corresponding to the dashed horizontal line) entailing full employment at equilibrium. By contrast, for any softer competition regime (one is represented by the lower dashed curve), there is equilibrium unemployment at any wage, however small.

To conclude, a last remark on the robustness of our result concerning the possibility of involuntary unemployment. This possibility has been challenged by Schultz (1992) as soon as we move from an atemporal

[6]We use the following specification of the inverse demand: $1/\left(X^8 + 10X\right)$, and take $n = 4$ and $L/c = 0.5$.

or temporary context to an intertemporal one, more specifically to an overlapping generations (OLG) economy under rational expectations. Notwithstanding, we have shown that the conditions for emergence of involuntary unemployment as characterised in the terms of Proposition 10 can be easily recreated in such an economy (d'Aspremont et al., 1991).[7]

2 The Dynamics of Investment Decisions: The Importance of Being Strategic

This second extension of our model, aiming at the study of intertemporal investment, is dynamic, using an overlapping generations version of a Dixit-Stiglitz economy. Investment is assumed to be decided by the young representative household in Sect. 1, where we explore the dynamics resulting from two conventional ways to close the model, assuming (i) a fixed number of firms and (ii) a variable number of firms endogenously determined by the zero net-profit condition. By contrast, in Sect. 2, investment is strategically decided by the young oligopolistic firms. As shown in Sect. 3, robust limit cycles can appear in the sole second case of strategic investment.

Investment as Decided by the Consumer

Following d'Aspremont et al. (2015), we want to contrast the consequences of this conventional approach to firm behaviour with those of a strategic investment approach. In order to introduce intertemporal decisions, we will convert a Dixit-Stiglitz economy (with labour as the numeraire good) into an overlapping generations economy where two representative consumers, one young the other old, coexist in each period.

[7] This OLG extension of our analysis was further exploited by d'Aspremont et al. (1995b) to discuss policy implications of involuntary unemployment in the above sense, and by d'Aspremont et al. (1995c) to show how producers' market power can favour the emergence of coordination failures and endogenous fluctuations.

The utility function of the young consumer at period t is

$$U\left(X_t, X_{t+1}\right) - vl_t = X_t^{\alpha} X_{t+1}^{1-\alpha} - vl_t, \text{ with } 1/2 < \alpha < 1 \text{ and } v > 0. \tag{2.1}$$

The variable X may represent the volume of a composite consumption good produced by a continuum of groups of producers of homogeneous varieties of that good (see the discussion on Cournotian monopolistic competition in Sect. 2 of Chap. 3). This justifies the adopted treatment of the number of those producers in each group (an average) as a continuous rather than a discrete variable. For simplicity, we shall however refrain from explicitising the disaggregated X and shall simply neglect the "integer problem" in the following.

Maximising the utility function under the two budget constraints $P_t X_t + K_t \leq l_t$ and $P_{t+1} X_{t+1} \leq r_{t+1} K_t$, where K_t is the capital accumulated in period t and r_{t+1} is the factor of return on capital expected to be obtained in period $t+1$, we get by first-order conditions:

$$X_t = \frac{\alpha l_t}{P_t}, \; X_{t+1} = (1-\alpha)\, l_t \frac{r_{t+1}}{P_{t+1}}, \tag{2.2}$$

and $\alpha X_t^{\alpha-1} X_{t+1}^{1-\alpha} - vP_t = 0$, implying $l_t \in (0, L)$ if and only if

$$v = \frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{P_t^{\alpha} \left(P_{t+1}/r_{t+1}\right)^{1-\alpha}}, \tag{2.3}$$

a formula that we will interpret as a condition on the factor of return on capital.

Notice that by the budget constraint $P_t X_t + K_t \leq l_t$ we are identifying the investment good with the numeraire good, rather than the consumption good. In other words, investment in period t takes the form of labour accumulated as human capital. Human capital accumulated in period $t-1$ allows to reduce the unit labour cost in period t: $c_t = K_{t-1}^{-\beta}$, with $\beta > 0$ measuring the economies of scale (the underlying production function is $X = K^{\beta} l$, so that the output elasticity is $1+\beta$).

We suppose production in period t to be performed by n_t identical firms and consider symmetric oligopolistic equilibria. Since demand is unit-elastic, formula (3.9) of Chap. 2 becomes simply (with symmetry and omitting the prime symbol)

$$\mu_t = \frac{P_t - c_t}{P_t} = \frac{1-\theta}{n_t}. \tag{2.4}$$

The symmetric oligopolistic equilibrium price at t is consequently

$$P_t = \frac{K_{t-1}^{-\beta}}{1-\mu_t} = \frac{K_{t-1}^{-\beta}}{1-(1-\theta)/n_t}. \tag{2.5}$$

Competitive toughness θ may of course differ across periods. However, we shall not assume any specific dynamics of this variable, which will be accordingly taken as exogenous and constant, characterising a particular regime of competition. Equality of marginal utility and disutility of labour thus imposes *on the household side*, as a perfect foresight equilibrium condition, the following value of the factor of return on capital (using formula (2.3)):

$$r_{t+1} = \rho \left(\frac{v}{\alpha} \frac{K_t^{-\beta}}{1-\mu_{t+1}} \right) \left(\frac{v}{\alpha} \frac{K_{t-1}^{-\beta}}{1-\mu_t} \right)^{\rho} \equiv R^h \left(\mu_t, \mu_{t+1}, K_{t-1}, K_t \right), \tag{2.6}$$

where $\rho \equiv \alpha / (1-\alpha)$ is an index of the young consumer's preference for the present. But the factor of return on capital is also constrained *on the firms side* by the realised aggregate profit $\Pi_{t+1} = \mu_{t+1} A_{t+1}$:

$$r_{t+1} = \mu_{t+1} \frac{A_{t+1}}{K_t} = \mu_{t+1} \frac{\alpha l_{t+1} + r_{t+1}(1-\alpha) l_t}{K_t} = \mu_{t+1} \frac{\rho K_{t+1} + r_{t+1} K_t}{K_t}. \tag{2.7}$$

Hence,

$$r_{t+1} = \frac{\rho}{1/\mu_{t+1} - 1} \frac{K_{t+1}}{K_t} \equiv R^f \left(\mu_{t+1}, K_t, K_{t+1}\right). \tag{2.8}$$

From now on, two conventional ways of approaching investment dynamics can be explored. One is to fix the number of firms, viewed as the exogenous constant n. The markup $\mu = (1-\theta)/n$ is then a constant, which converts the equation $\ln R^h(\mu, \mu, K_{t-1}, K_t) = \ln R^f(\mu, K_t, K_{t+1})$ into the second-order log-linear difference equation:

$$\begin{aligned} \ln K_t &= (1-\beta)\ln K_{t-1} - \rho\beta \ln K_{t-2} + b, \text{ with} \\ b &= (1+\rho)\ln(v/\alpha\mu) - \rho\ln(1/\mu - 1). \end{aligned} \tag{2.9}$$

The solutions to this equation converge to the steady state $K^* = e^{b/\beta(1+\rho)} = (v/\alpha\mu)^{1/\beta}(1/\mu - 1)^{-\rho/\beta(1+\rho)}$, monotonically if $1 < \rho < (1-\beta)^2/4\beta$, or oscillatorily if $(1-\beta)^2/4\beta < \rho < 1/\beta$. As the preference for the present ρ increases, the dynamic system undergoes a Hopf bifurcation at $1/\beta$, the equilibrium oscillations becoming explosive for $\rho > 1/\beta$. Two remarks to conclude this case of a constant number of firms: first, log-linearity excludes robustness of the cycles obtained at $\rho = 1/\beta$; second, the regime of competition, while affecting the steady state through the markup μ, has no impact at all on the investment dynamics.

Another conventional way to close the model is to assume a variable number of firms endogenously determined by the zero net-profit condition $\Pi_{t+1} = K_t$, hence $r_{t+1} = 1$ for any t. In this case, Eq. (2.6) constraining on the consumer's side the return on capital, defines an autonomous dynamic system in the price P_t:

$$P_{t+1} = \frac{1}{\rho}\left(\frac{\alpha}{v}\right)^{1+\rho} P_t^{-\rho}. \tag{2.10}$$

Since $\rho > 1$, all solutions to this equation are explosive, except the stationary one: $P^* = \rho^{-1/(1+\rho)}(\alpha/v)$. In order to avoid explosive solutions and keep the price stationary, the markup $\mu_{t+1} = (1-\theta)/n_{t+1}$ must verify in equilibrium:

$$\mu_{t+1} = \frac{P^* - K_t^{-\beta}}{P^*}. \tag{2.11}$$

Finally, Eq. (2.8) constraining the return on capital on the firms side becomes the first-order non-linear difference equation:

$$K_{t+1} = \frac{K_t}{\rho\left(P^* K_t^{\beta} - 1\right)}. \tag{2.12}$$

The logarithmic derivative of K_{t+1} with respect to K_t evaluated at the steady state $K^{**} = \left((v/\alpha)(1+1/\rho)\,\rho^{1/(1+\rho)}\right)^{1/\beta}$ is equal to $1-(1+\rho)\beta$, so that the solutions to this equation converge to the steady state, monotonically if $1 < \rho < 1/\beta - 1$, non-monotonically if $1/\beta - 1 < \rho < 2/\beta - 1$. As the preference for the present ρ increases, the dynamic system undergoes a flip bifurcation at $2/\beta - 1$, the equilibrium fluctuations becoming explosive for ρ larger than this value. Investment dynamics are again independent from the regime of competition, and so is now the steady state.

Investment as Decided by the Firms

In order to consider investment decisions made by firms rather than by the young consumer, we assume two periods in the life of the n_t firms born at t: the investment period t and the production period $t+1$. In addition, we assume that the investment effects are at least partially appropriated by the firm, in other words that there are internal as well as, possibly, external economies of scale. Formally, if k_{it} is the investment decided by firm i at t, its unit cost of production at $t+1$ is $c_{it} = k_{it}^{-\eta}\prod_{j\neq i} k_{jt}^{-(\beta-\eta)/(n_t-1)}$, with $0 < \eta \leq \beta$. Of course, at equilibrium capital demand must be equal to capital supply: $\sum_{j=1}^{n_t} k_{jt} = K_t$.

Let us now consider the two-stage game played by the n_t firms born at t. At the second stage (in period $t+1$), for an expenditure A_{t+1}, hence a price $P_{t+1} = A_{t+1}/X_{t+1}$, the markup formula for firm i gives

$$\frac{A_{t+1}}{X_{t+1}}\left(1-(1-\theta)\frac{x_{it+1}}{X_{t+1}}\right) = c_{it+1}. \tag{2.13}$$

By adding over all the n_t firms, supposed to be active at the second stage, we have:

$$\frac{A_{t+1}}{X_{t+1}}\left(n_t-(1-\theta)\right) = \sum_j c_{jt+1}, \text{ hence}$$

$$1-(1-\theta)\frac{x_{it+1}}{X_{t+1}} = \frac{\left(n_t-(1-\theta)\right)c_{it+1}}{\sum_j c_{jt+1}}$$

At the first stage, given the expected expenditure A_{t+1}, firm i maximises in k_{it} the profit (net of the investment cost)

$$\left(\frac{A_{t+1}}{X_{t+1}} - c_{it+1}\right)x_{it+1} - k_{it} = \frac{A_{t+1}}{1-\theta}\left(1-\frac{\left(n_t-(1-\theta)\right)c_{it+1}}{\sum_j c_{jt+1}}\right)^2 - k_{it}. \tag{2.14}$$

By computing the corresponding first-order condition, we can determine the investment decision:

$$\begin{aligned} k_{it} &= 2A_{t+1}\frac{n_t-(1-\theta)}{(1-\theta)(n_t-1)}\left(1-\left(n_t-(1-\theta)\right)\chi_{it+1}\right)\chi_{it+1} \\ &\quad\times\left(1-\chi_{it+1}\right)\left(n_t\eta-\beta\right), \end{aligned} \tag{2.15}$$

where $\chi_{it+1} \equiv c_{it+1}/\sum_j c_{jt+1}$. Notice that a positive investment by firm i requires the internal economies of scale η appropriated by the firm to be larger than the extent β/n_t of the external economies of scale that it can recover without investing.

At a symmetric equilibrium (with $\chi_{it+1} = 1/n_t$ and $K_t = n_t k_t$), aggregation of individual investments given by Eq. (2.15) leads to the *the first-stage equilibrium condition*

$$K_t = 2A_{t+1}\left(1 - \frac{1-\theta}{n_t}\right)\left(\eta - \frac{\beta}{n_t}\right). \tag{2.16}$$

Investment as decided by the firms is increasing in the internal economies of scale η and in competitive toughness θ. Now, for the expectation A_{t+1} to be correct, we apply *the second-stage equilibrium condition* (2.8) and obtain:

$$A_{t+1} = \rho K_{t+1} + r_{t+1}K_t = \rho K_{t+1} + R^f\left(\mu_{t+1}, K_t, K_{t+1}\right)K_t = \frac{\rho K_{t+1}}{1-\mu_{t+1}}, \tag{2.17}$$

so that a *subgame perfect equilibrium condition* of the two-stage game played by the n_t firms is

$$K_t = 2\frac{\rho K_{t+1}}{1-\mu_{t+1}}\left(1 - \frac{1-\theta}{n_t}\right)\left(\eta - \frac{\beta}{n_t}\right) \tag{2.18}$$

or, using again (2.8),

$$r_{t+1} = \frac{1}{2\left(n_t/\left(1-\theta\right)-1\right)\left(\eta - \beta/n_t\right)} \equiv R^s\left(n_t\right). \tag{2.19}$$

We see that an increase in the number of firms n_t has, through firm investment decisions, two negative effects on the factor of return on capital, one by decreasing the markup $\mu_{t+1} = \left(1-\theta\right)/n_t$, the other by decreasing the externality β/n_t.

We can now formulate a two-dimensional dynamic system, integrating the two stages of firms' investment and production decisions together with the consumer's saving decision, by taking

$$R^s(n_t) = R^h\left(\frac{1-\theta}{n_{t-1}}, \frac{1-\theta}{n_t}, \frac{K_{t-1}}{n_{t-1}}, \frac{K_t}{n_t}\right) \text{ and}$$
$$R^s(n_{t-1}) = R^f\left(\frac{1-\theta}{n_{t-1}}, K_{t-1}, K_t\right),$$

which leads to the two first-order non-linear difference equations in (K_t, n_t), respectively

$$\frac{(1-\theta)\, K_t^{\beta} n_t^{-\beta}}{n_t\eta-\beta} = 2\rho\left(\frac{v}{\alpha}\right)^{1+\rho}\left(\frac{K_{t-1}^{-\beta} n_{t-1}^{\beta}}{1-(1-\theta)/n_{t-1}}\right)^{\rho} \tag{2.20}$$
$$2\rho K_t = \frac{K_{t-1}}{\eta-\beta/n_{t-1}}$$

The second of these equations alone leads to the following steady state value of the number of firms: $n^* = 2\rho\beta/(2\rho\eta-1)$, increasing in the external and decreasing in the internal economies of scale. The steady state value K^* can then be easily computed from the first equation:

$$K^* = n^*\left(\frac{2\rho\,(v/\alpha)^{1+\rho}}{1-\theta}\frac{n^*\eta-\beta}{(1-(1-\theta)/n^*)^{\rho}}\right)^{1/(1+\rho)\beta}. \tag{2.21}$$

The Possibility of Limit Cycles

Strategic investment decisions lead in our model to a two-dimensional dynamic system (one dimension more than under the zero-profit condition), which is non-linear (by contrast with log-linearity under constancy of the number of firms), and involving both external and internal economies of scale. Fluctuations were not excluded under investment decisions made by the consumer, but they could not be self-sustained and robust (being linked to precise parameter values) as those we can obtain under strategic investment decisions. In the following, we concentrate on the existence of a limit cycle, for parameter values in a neighbourhood of a Hopf bifurcation, and abstain from proceeding to the complete analysis of the system (2.20).[8] By log-linearising this system at the steady state (K^*, n^*), we obtain by differentiation:

$$\begin{bmatrix} d\ln K_t \\ d\ln n_t \end{bmatrix} = \begin{bmatrix} 1 & -(2\rho\eta - 1) \\ \frac{(1+\rho)\beta}{2\rho\eta+\beta} & \frac{2\rho\eta-1}{2\rho\eta+\beta}\left(\frac{\rho(1-\theta)}{2\rho\beta-(1-\theta)(2\rho\eta-1)} - \beta\left(1+\frac{\rho}{2\rho\eta-1}\right)\right) \end{bmatrix} \begin{bmatrix} d\ln K_{t-1} \\ d\ln n_{t-1} \end{bmatrix}. \tag{2.22}$$

The determinant of this Jacobian matrix is

$$D(\eta) = \rho\frac{2\rho\eta - 1}{2\rho\eta + \beta}\left(\frac{1-\theta}{2\rho\beta - (1-\theta)(2\rho\eta - 1)} + \frac{2\beta(\rho\eta - 1)}{2\rho\eta - 1}\right), \tag{2.23}$$

an increasing function of η, measuring the internal economies of scale, which can be taken as bifurcation parameter: as η increases, the dynamic system undergoes a Hopf bifurcation when $D(\eta^{\mathrm{H}}) = 1$, if the trace $T(\eta^{\mathrm{H}}) < 2$.

Now, η is restricted to belong to an interval $[\underline{\eta}, \overline{\eta}]$. Indeed, by (2.14) and (2.15), profit non-negativity imposes at the steady state: $\eta \geq 1/2\rho + \beta/(1+\rho)(1-\theta) \equiv \underline{\eta}$. Also, as $n^* \geq 2$, $\eta \leq \min\{1/2\rho + \beta/2, \beta\} \equiv \overline{\eta}$. Notice that the inequality $\underline{\eta} \leq \overline{\eta}$ places an upper bound on the admissible competitive toughness: $\theta \leq (\rho - 1)/(\rho + 1)$. Necessary and sufficient conditions for the existence of a Hopf bifurcation are:

$$D(\underline{\eta}) < 1 < D(\overline{\eta}) \text{ and } T(\eta^{\mathrm{H}}) = 2 - \frac{\beta(2\rho\eta - 1)(1 + 1/\rho)}{\frac{(1-\theta)(2\rho\eta-1)}{2\rho\beta-(1-\theta)(2\rho\eta-1)} + 2\beta(\rho\eta - 1)} < 2. \tag{2.24}$$

[8] A complementary development can be found for the Cournotian case in d'Aspremont et al. (2015).

The inequality $D\left(\underline{\eta}\right) < 1$ translates into the condition

$$\beta < \frac{1}{\rho}\left(\frac{(1+\rho)^2(1-\theta)}{2\rho} + 1\right), \tag{2.25}$$

which is strengthened as the competitive toughness increases. The inequality $D(\overline{\eta}) > 1$ translates into the condition

$$\rho > \frac{2\beta - \frac{1-\theta}{1+\theta} + \sqrt{\left(2\beta - \frac{1-\theta}{1+\theta}\right)^2 + 4\beta^2(1+\beta)}}{2\beta^2} \text{ if } \rho\beta \geq 1,$$

$$\rho > \beta\left(1 + 2\rho(2-\rho\beta)\right)\frac{(2\rho\beta-1)\theta+1}{(2\rho\beta-1)(1-\theta)} \text{ if } \rho\beta \leq 1, \tag{2.26}$$

which is again strenghtened as the competitive toughness increases. Thus, the set of parameter values (β, ρ) which are compatible with the existence of a Hopf bifurcation shrinks as competition becomes tougher.

In Fig. 4.2, we give a representation of this set in the parameter space $\beta \times \rho$, as the region delimited by thick curves, for the minimum competitive toughness $\theta = 0$ (Cournot competition). The higher curve corresponds to $D(\overline{\eta}) = 1$ and the lower curve to $D\left(\underline{\eta}\right) = 1$. It is easy to check that the condition on the trace is satisfied for any admissible η, larger than $\underline{\eta}$.

The thin solid curve represents the corresponding (degenerate) region for the maximum admissible competitive toughness $(\theta = (\rho-1)/(\rho+1))$, for which the interval $\left[\underline{\eta}, \overline{\eta}\right]$ degenerates and η ceases to work as bifurcation parameter. The dashed curve $(\rho = 1/\beta)$ also represents a degenerate region, corresponding to the values of (β, ρ) which support a Hopf bifurcation in the case of investment decisions made by the consumer and under a constant number of firms.

The reader may be interested to notice that, by replacing aggregate investment K_t by individual investment $k_t = K_t/n_t$, we may easily transform the present dynamic system into an equivalent system of two first-order non-linear difference equations in (k_t, n_t). The log-linearised

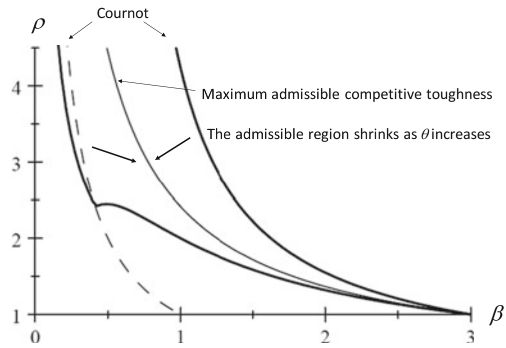

Fig. 4.2 Parameter values compatible with a Hopf bifurcation

transformed system at the steady state $(K^*/n^*, n^*)$ has as a particular case a discrete version of the Lotka-Volterra *predator-prey* equation system

$$\begin{bmatrix} d \ln k_t \\ d \ln n_t \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} d \ln k_{t-1} \\ d \ln n_{t-1} \end{bmatrix}, \tag{2.27}$$

obtained for the following parameter values: $\rho = 2\beta = 4\eta^{\mathrm{H}} = \sqrt{6} \simeq 2.45$ and $\theta = 0$ (Cournot).[9] Firm creation and investment play in this system the role of predator and prey, respectively. Firm creation dn_t/n_t is fed by former cost-reducing investment dk_{t-1}/k_{t-1} but, by squeezing the forthcoming markup, it discourages future investment dk_{t+1}/k_{t+1} (the Schumpeterian effect). An invariant 4-period cycle ensues. Other parameter value configurations entailing existence of a Hopf bifurcation outside the Lotka-Volterra case lead to higher period limit cycles, but the interaction of the two countervailing effects is still underlying the resulting investment dynamics.

[9] With these parameter values, the matrix equation (2.22) can indeed be rewritten as

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d \ln k_t \\ d \ln n_t \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d \ln k_{t-1} \\ d \ln n_{t-1} \end{bmatrix}.$$

By a direct computation, we then obtain the system (2.27).

3 Localised Competition and Delegation

We distinguished in Chap. 1 two functions of product differentiation from the consumers' point of view: to respond to the potential consumers' taste for variety and to adapt different products to different types of consumers. In the preceding chapters we have adopted the representative consumer approach, hence emphasising the taste for variety. We now adopt another approach, emphasising by contrast the specific ideal variety favoured by each element of a set of heterogeneous consumers. In the Launhardt (1885) and Hotelling (1929) specification, this ideal variety is represented by a location in the product space.

A classical distinction is made between two polar types of product differentiation: One type is *vertical differentiation*, whereby goods can be ranked according to some quality unanimously recognised by consumers so that, when supplied at the same price, the highest ranked product captures the whole market. The other polar type, on which we will focus in the following, is *horizontal differentiation*, whereby such unanimous ranking is absent and, thanks to heterogeneity in preferences or to the taste for variety, all products may have a positive demand when offered at the same price.

The Launhardt-Hotelling Model: A Price-Quantity Approach

Suppose that the product space is a closed interval on the real line. In this interval, each consumer is located at some point (the consumer's address) representing his ideal product. We assume consumers to be uniformly distributed over the interval $[0, 1 + 2a]$. There are two producers, each located at some point (the producer's address) with, say, producer 1 located at point a and producer 2 located at point $1+a$, both producing the same homogeneous good. We denote p_1 and p_2 the (mill) price of producer 1 and 2 respectively. For each producer i there is a transportation cost t_i per unit of distance to deliver the product to a consumer. In a general (not necessarily geographical) interpretation of the product space, this represents a mismatching cost. Each consumer buys one (and only one) unit of the good to the producer offering the minimum delivered price (mill price plus transportation cost) if it is less than some reservation price $v > 0$, assumed identical across consumers. Each firm is supposed to produce under the same cost conditions.

In the Hotelling model, the production cost is linear with marginal cost normalised to zero, the reservation price is infinite (contrary to Launhardt) and the (linear) transportation costs are identical (also contrary to Launhardt). Hotelling, though, unlike Launhardt, introduces in his model a first stage where producers choose their locations strategically. Here, we will remain in the fixed-locations case.

The more general Launhardt model, with different transportation costs, has the advantage of capturing both types of differentiation, vertical and horizontal (see Dos Santos Ferreira and Thisse, 1996). Vertical differentiation results from different transportation costs for the two products. A product which is cheaper to transport may be seen as a product of higher quality. When both producers are located at the same point and announce the same mill price, the producer with the higher transportation cost is selling nothing. Both producers can however be selling to customers for whom the delivered price is lower than v provided the producer with the higher transportation cost sets a lower price. If the transportation costs are identical, then there is horizontal differentiation only. When offered at the same mill price, both products have a positive demand.

Another early precursor of Hotelling spatial competition model is the model of Ellet (1839) concerning sellers of transportation services competing in prices (i.e. competing in tolls per ton transported). In his book, Ellet analyses in fact two models. The first is close to Hotelling second-stage model, but short of simultaneous profit maximisation by the sellers (hence without introducing an equilibrium concept). The other, which looks at the case where the two goods are complements, offers a solution to simultaneous profit maximisation (hence a Cournot-Nash equilibrium), giving the same result as Cournot's *concurrence* model of two complementary monopolies facing linear demand.

In this section we extend the Hotelling model in the direction of Launhardt by assuming differences in transportation costs and a finite reservation price. The introduction of horizontal product differentiation by Hotelling can be seen as a way to keep prices as the leading strategic variables but, at the same time, avoid the destructive conclusion of Bertrand's critique. We shall however go further and enlarge the model to price-quantity competition.

Take the Hotelling spatial duopoly with a continuum of consumers, each likely to buy one unit of the product, uniformly distributed over the interval $[0, 1 + 2a]$. We start by deriving the demand addressed to each of two firms for the brand it produces of the same product, brand 1 located at point a and brand 2 located at point $1 + a$. The market is thus divided into 3 segments: on the left, the interval $[0, a]$, which is the *captive* segment of firm 1; in the middle, the interval $[a, 1 + a]$ shared by the two firms, which is the *contested* segment; on the right, the interval $[1 + a, 1 + 2a]$, which is the *captive* segment of firm 2. The travel costs, borne by the consumers, differ according to the market segment considered. The consumer located in firm i's captive segment bears a travel unit cost t_i; the consumer located in the contested segment bears a travel unit cost t whatever the brand he is addressing, which excludes vertical differentiation.

Different scenarios are possible depending on the values of the parameters. We shall concentrate on the scenario where each firm sells both on its captive segment (which it does not completely cover) and on the contested segment (together covered by the two firms). The scenario we are focusing on is illustrated in Fig. 4.3.

A customer located on the captive segment for brand i at a distance $d \in [0, a]$ from firm i, will purchase that brand if the *delivered price* $p_i + t_i d$ does not exceed the reservation price v. The quantity demanded to firm i on its captive segment is consequently $(v - p_i)/t_i$. We assume

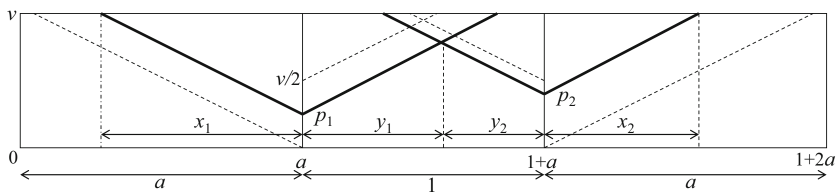

Fig. 4.3 Market segments and demands for the two brands

$\max\{v/t_1, v/t_2\} < a$, so that the captive segments are never completely covered, their sizes being determined by v/t_1 and v/t_2, independently in fact of any reference to a. As to the contested segment, a customer located at distance $d \in [0,1]$ from firm i will only purchase brand i if the delivered price $p_i + td$ does not exceed the delivered price $p_j + t(1-d)$ for the other brand, that is, if $d \leq \left(t + p_j - p_i\right)/2t$. The demand addressed to firm i on the competitive segment is consequently $\left(t + p_j - p_i\right)/2t$. We assume $v/t > 1$ so as to exclude the case of coexisting local monopolies (setting the price $v/2$) with market areas that are disconnected or juxtaposed (if $v/2 + t/2 \geq v$).

So, in our scenario, the demand $D_i\left(p_i, p_j\right)$ to firm i (the sum of the demands $D_{i0}\left(p_i\right)$ on its captive segment and $D_{ij}\left(p_i, p_j\right)$ on the contested segment) and the aggregate demand $\overline{D}\left(p_i, p_j\right) = D_i\left(p_i, p_j\right) + D_j\left(p_j, p_i\right)$ are respectively

$$D_i\left(p_i, p_j\right) = \frac{v - p_i}{t_i} + \frac{t + p_j - p_i}{2t} \text{ and} \tag{3.1}$$
$$\overline{D}\left(p_i, p_j\right) = 1 + \frac{v - p_i}{t_i} + \frac{v - p_j}{t_j}.$$

This description of the demand for each brand allows us to adapt the definition of an oligopolistic equilibrium formulated in a more general framework (now restricted to the duopoly case and to zero production costs), with each firm maximising its profit in price-quantity pairs (p_i, x_i) under market share and market size constraints. The market share constraint can be simply written as $x_i \leq D_i(p_i, p_j)$ and can be seen as a *price competitiveness* condition, imposing the consistency of the planned output x_i with the list price p_i, given the list price p_j of firm j. Binding alone for both firms, this constraint leads to a price (Bertrand-like) equilibrium. The market size constraint involves the residual demand taking into account the rival's sales target x_j: firm i considers that it will sell the quantity $\left(\overline{D}(p_i, p_j) - x_j\right)^+ \equiv \max\left\{\overline{D}(p_i, p_j) - x_j, 0\right\}$ left by firm j. We define accordingly the concept of oligopolistic equilibrium for the Launhardt-Hotelling model.

Definition 9 An *oligopolistic equilibrium* is a tuple $\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right) \in \mathbb{R}_+^4$ such that, for any i,

$$\begin{aligned} \left(p_i^*, x_i^*\right) \in \arg & \max_{(p_i, x_i) \in \mathbb{R}_+^2} p_i x_i \\ \text{s.t.} \quad & x_i \leq D_i\left(p_i, p_j^*\right) \\ & \text{and } x_i \leq \overline{D}\left(p_i, p_j^*\right) - x_j^*. \end{aligned} \tag{3.2}$$

We want to emphasise that we are not imposing in this definition, as we did in the former definitions of oligopolistic equilibrium, that the two constraints be satisfied as equalities. Indeed, all the tuples solving the programmes (3.2) for both firms have necessarily that property and are consequently eligible as equilibria. As before, we can use this two-constraint programme to obtain a convenient parameterisation of the set of oligopolistic equilibria. Letting $\lambda_i, \nu_i \geq 0$ be the Lagrange multipliers respectively associated with the market share and the market size constraints in firm i's programme, we get as first-order conditions at equilibrium:

$$\begin{cases} x_i^* + \lambda_i \partial_i D_i\left(p_i^*, p_j^*\right) + \nu_i \partial_i \overline{D}\left(p_i^*, p_j^*\right) = 0 \\ p_i^* - \lambda_i - \nu_i = 0 \end{cases}, i, j = 1, 2, i \neq j. \tag{3.3}$$

With competitive toughness parameter $\theta_i = \lambda_i/(\lambda_i + \nu_i) \in [0, 1]$, the two first-order conditions can be merged into the condition (for $i, j = 1, 2$ and $i \neq j$):

$$x_i^* + \theta_i p_i^* \partial_i D_i\left(p_i^*, p_j^*\right) + (1 - \theta_i)\, p_i^* \partial_i \overline{D}\left(p_i^*, p_j^*\right) = 0 \tag{3.4}$$

or, equivalently,

$$D_i\left(p_i^*, p_j^*\right) + p_i^*\left(\partial_i D_i\left(p_i^*, p_j^*\right) + (1 - \theta_i)\, \partial_i D_j\left(p_j^*, p_i^*\right)\right) = 0, \tag{3.5}$$

which in our scenario can be written as

$$\frac{v - p_i^*}{t_i} + \frac{t + p_j^* - p_i^*}{2t} - \left(\frac{1}{t_i} + \frac{\theta_i}{2t}\right) p_i^* = 0. \tag{3.6}$$

This condition leads to a complete parameterisation of oligopolistic equilibria, one for each $\boldsymbol{\theta} = (\theta_1, \theta_2)$, with

$$p_i^* = \frac{1 + 2v/t_i + p_j^*/t}{1 + 4t/t_i + \theta_i} t, \, i, \, j = 1, 2, \, i \neq j, \tag{3.7}$$

where we see that p_i^* is an increasing function of price p_j^* and of the captive segment size[10] v/t_i. It is a decreasing function of the ratio $\tau_i \equiv t/t_i$ of the travel unit costs on the contested segment with respect to the captive one.

Given $\left(\theta_i, \theta_j\right)$, we can solve these equations for the equilibrium prices (if an oligopolistic equilibrium exists for those parameter values), getting:

$$p_i^* \left(\theta_i, \theta_j\right) = \frac{\left(1 + 4\tau_j + \theta_j\right)(1 + 2v/t_i) + \left(1 + 2v/t_j\right)}{(1 + 4\tau_i + \theta_i)\left(1 + 4\tau_j + \theta_j\right) - 1} t, \, i, \, j = 1, 2, \, i \neq j. \tag{3.8}$$

The equilibrium price $p_i^* \left(\theta_i, \theta_j\right)$ of firm i is decreasing with respect to its own competitive toughness θ_i and to that θ_j of its competitor, so that

$$p_i^* (0, 0) = \frac{t}{2} \left(\frac{v}{t} + \frac{1 + 2\tau_j}{\tau_i + \tau_j + 4\tau_i\tau_j}\right) \tag{3.9}$$

is an upper bound for $p_i^* \left(\theta_i, \theta_j\right)$.

As this bound is higher than the monopoly price $v/2$, the condition for the contested segment to be always covered must be strengthened with respect to the condition $v/t > 1$, in order to ensure that, for each firm i,

[10] Hence, the firm with a larger captive segment (less exposed to competition) sets, *ceteris paribus*, a higher price and is, in this sense, less "aggressive" (see Narasimhan, 1988, p. 435).

$p_i^*(0,0) + t/2 < v$, i.e.,

$$\frac{v}{t} > 1 + \frac{1 + 2\max\{\tau_1, \tau_2\}}{\tau_1 + \tau_2 + 4\tau_1\tau_2}. \tag{3.10}$$

Another condition that has to be introduced so as to ensure that the two firms are always active competitors at equilibrium on the contested segment is that $p_i^*(1,0) + t > p_j^*(0,1)$ (for $i, j = 1, 2$ and $i \neq j$), i.e.,

$$\frac{v}{t} < 6 + 8\min\{\tau_1, \tau_2\}. \tag{3.11}$$

This condition imposes that, when the toughest competitor confronts the softest one, the former's delivered price at the rival's address be higher than the latter's mill price. It plays the role of a no-undercutting condition because of the market size constraint. As is well known, the possibility of undercutting threatens the existence of a Nash equilibrium of the Hotelling pricing game. The constraint on market size introduces a crucial difference between our oligopoly game and a pure price competition game. Figure 4.4 illustrates this difference.[11] The thick broken line represents the two constraints on the admissible strategies of firm i. The upper piece, the market share frontier, is part of the graph of demand $D_i(p_i, p_j)$, which is extended as a dashed broken line. In the Hotelling price game, firm i would choose the point $(4, 0.4)$, which gives access to the higher isoprofit curve, undercutting its rival. By contrast, in our oligopoly price-quantity game, the firm cannot go beyond the market size frontier (the lower piece of the thick broken line), so that undercutting is forbidden. With the role played by the market size frontier, we retrieve of course the essential difference in firm conduct between Cournot and Bertrand.[12]

We can now formulate the following existence proposition.

[11]The figure is computed with the parameter values $v = 1$ and $t_1 = t_2 = t = 0.4$ and the competitor j strategy $p_j = x_j = 0.8$.

[12]For an approach based on "pricing schemes" see d'Aspremont et al. (1995a).

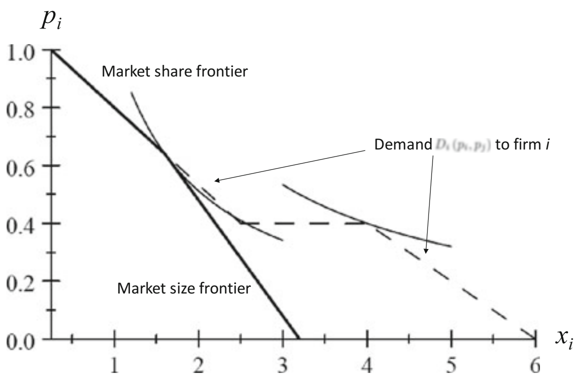

Fig. 4.4 Undercutting and the market size constraint

Proposition 11 *Under the following restriction on the parameter values:*

$$1 + \frac{1 + 2\max\{\tau_1, \tau_2\}}{\tau_1 + \tau_2 + 4\tau_1\tau_2} < \frac{v}{t} < 6 + 8\min\{\tau_1, \tau_2\},$$

there is an oligopolistic equilibrium

$$\left(\left(p_1^*(\boldsymbol{\theta}), D_1\left(p_1^*(\boldsymbol{\theta}), p_2^*(\boldsymbol{\theta})\right)\right), \left(p_2^*(\boldsymbol{\theta}), D_2\left(p_2^*(\boldsymbol{\theta}), p_1^*(\boldsymbol{\theta})\right)\right)\right)$$

for any $\boldsymbol{\theta} \in [0,1]^2$, *with* $p_i^*(\boldsymbol{\theta})$ *given by (3.8)), such that both firms are active competitors on the contested segment.*

Proof The two constraints in the programme (3.2) of each firm define a convex admissible strategy set. Equations (3.8) are an expression of first-order conditions (3.6) for the maximisation of each firm profit. These conditions are clearly sufficient for a local maximum, by concavity of the demand to the firm. The assumption ensures that at prices $p_1^*(\boldsymbol{\theta})$ and $p_2^*(\boldsymbol{\theta})$ both firms are active competitors on the contested segment for any $\boldsymbol{\theta} \in [0,1]^2$. □

Given the equilibrium prices $p_i^*\left(\theta_i,\theta_j\right)$ and $p_j^*\left(\theta_j,\theta_i\right)$, the corresponding profit of firm i is

$$\Pi_i\left(\theta_i,\theta_j\right) = p_i^*\left(\theta_i,\theta_j\right) D_i\left(p_i^*\left(\theta_i,\theta_j\right), p_j^*\left(\theta_j,\theta_i\right)\right) \quad (3.12)$$

$$= \left(\frac{\left(1+4\tau_j+\theta_j\right)\left(1+2v/t_i\right)+\left(1+2v/t_j\right)}{\left(1+4\tau_i+\theta_i\right)\left(1+4\tau_j+\theta_j\right)-1}\right)^2 \frac{2\tau_i+\theta_i}{2}t,$$

decreasing in θ_j, and unimodal with respect to θ_i. Maximisation of this profit with respect to θ_i gives:

$$\arg\max_{\theta_i\in[0,1]} \Pi_i\left(\theta_i,\theta_j\right) = \frac{4\tau_j+\theta_j}{1+4\tau_j+\theta_j}, i=1,2, i\neq j, \quad (3.13)$$

a result which is central for the developments of next subsection.

A New Interpretation of the Model: Delegation

We want now to look at competitive toughness from another perspective. Competitive toughness can be interpreted as a managerial attitude. Managers may adopt different attitudes towards their rivals for different motives and depart from the strict profit-maximising objective.[13] Under separation of ownership and control, the objective of the firm may be seen as a monetary compensation given to the CEO. For example, in the marketing management literature dealing with "aggressiveness," the managerial attitude is to be seen as a constructed feature of the organisation.[14] It might be a good strategy for a firm to hire a CEO with a particular personality.[15] In the delegation literature, it is shown that owners can gain in profit by having managers with objectives distorted

[13] References to such observation in empirical and experimental studies of behavioural economics can be found in the survey of Armstrong and Huck (2010).

[14] See, e.g., Covin and Covin (1990) and Venkatraman (1989).

[15] Of course the same objective can be attained through an appropriate monetary compensation, independently of the personality of the manager. The model is valid for the two interpretations.

according to their types, such as a linear combination of profit and sales Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), or of own and competitor's profits (Miller and Pazgal, 2001, 2002, and Cornand and Dos Santos Ferreira, 2020), or of profit and market share (Jansen et al., 2007). Typically, this can be used as a commitment device in a two-stage "delegation game": each firm owner chooses a managerial type at the first stage; managers compete in an oligopoly game at the second stage.

In this subsection we introduce delegation but in such way that the managerial attitude is still compatible with profit-maximising behaviour.[16] But, in contrast to what is done in this literature, we keep our price-quantity approach instead of looking at the different conclusions obtained when only one strategic variable is privileged. In our approach, introducing a first stage endogenises the choice of competitive toughness, and hence eventually the choice of the competition regime, possibly associated with a strategic variable (as in price and quantity equilibria).

For each firm i, we suppose the managerial objective to be a convex combination (with respective weights θ_i and $1-\theta_i$) of two components. The first component (corresponding to an aggressive, Bertrand-like, attitude) reflects pure price competition and ignores the quantity targeted by the competitor. The second component (corresponding to a conciliatory, Cournot-like, attitude) takes this quantity into account and anticipates the profit of firm i according to its residual demand. On this basis we can introduce a class of managerial objective functions allowing for a large variety of managerial styles (i.e. with varying degrees of competitive toughness). For every $\boldsymbol{\theta} \in [0,1]^2$ a general managerial objective can be written as follows:

$$\pi_i^{\theta} = \theta_i p_i x_i + (1-\theta_i)\, p_i \max\left\{\min\left\{x_i, \overline{D}(p_i, p_j) - x_j\right\}, 0\right\},$$
$$\text{s.t. } x_i \le D_i(p_i, p_j). \tag{3.14}$$

Notice that, when the managerial objective functions are maximised by both firms, $x_i = D_i(p_i, p_j)$ for $i, j = 1, 2$ $(i \neq j)$, so that $\pi_i^{\theta}\left(p_i, x_i, p_j, x_j\right) = p_i D_i(p_i, p_j)$: the managerial objective coincides

[16] We follow d'Aspremont et al. (2016).

at equilibrium with firm's profit. Also, given $\boldsymbol{\theta} \in [0,1]^2$, we can define a corresponding $\boldsymbol{\theta}$-game with, as strategies of each firm i, the price-quantity pairs $(p_i, x_i) \in \mathbb{R}_+^2$ such that $x_i \leq D_i(p_i, p_j)$ and as payoffs of each firm i the managerial objective functions π_i^{θ}. The first-order conditions for the maximisation of these payoffs coincide with the first-order conditions (3.4) associated with an oligopolistic equilibrium parameterised by $\boldsymbol{\theta}$. Thus, under the conditions of Proposition 11, an equilibrium of the $\boldsymbol{\theta}$-game is an oligopolistic equilibrium.

By choosing adequately the managerial attitudes of the producers, as characterised by the parameter $\boldsymbol{\theta}$, we obtain the outcomes of standard oligopolistic solutions. A simple inspection of (3.14) shows that, at one extreme, if $\theta_1 = \theta_2 = 1$ (competitive toughness maximal for both competitors), we get the Hotelling pure price competition game, with equilibrium price $p_i^*(1,1)$ easily computed from formula (3.8). Also, at the other extreme, if $\theta_1 = \theta_2 = 0$ (competitive toughness minimal for both competitors), we get a Cournot-like competition game, each firm maximising its profit on the basis of its residual demand.

As to pure quantity competition, since choosing the quantity x_i in order to maximise the profit of firm i given the quantity x_j chosen by firm j amounts to maximise $p_i D_i(p_i, p_j)$ under the constraint $D_j(p_j, p_i) = x_j$, we may stand on the first-order conditions:

$$D_i(p_i^{\mathrm{q}}, p_j^{\mathrm{q}}) + p_i^{\mathrm{q}} \left(\partial_i D_i(p_i^{\mathrm{q}}, p_j^{\mathrm{q}}) - \partial_j D_i(p_i^{\mathrm{q}}, p_j^{\mathrm{q}}) \frac{\partial_i D_j(p_j^{\mathrm{q}}, p_i^{\mathrm{q}})}{\partial_j D_j(p_j^{\mathrm{q}}, p_i^{\mathrm{q}})} \right) = 0,$$
$$\text{for } i, j = 1, 2,\ i \neq j. \tag{3.15}$$

Comparing with Eqs. (3.5) and using (3.1), we see that Eqs. (3.15) characterise an oligopolistic equilibrium for

$$\theta_i^{\mathrm{q}} = 1 + \frac{\partial_j D_i(p_i^{\mathrm{q}}, p_j^{\mathrm{q}})}{\partial_j D_j(p_j^{\mathrm{q}}, p_i^{\mathrm{q}})} = \frac{\tau_j}{\tau_j + 1/2}, \text{ for } i, j = 1, 2,\ i \neq j. \tag{3.16}$$

If travel cost t on the contested segment tends to zero $(\tau_1, \tau_2) \to (0, 0)$, so that $\left(\theta_1^{\mathrm{q}}, \theta_2^{\mathrm{q}}\right) \to (0, 0)$: the quantity equilibrium tends to Cournot equilibrium as horizontal product differentiation vanishes.

An interesting observation is that Stackelberg (1934) equilibria (say with firm 1 the leader, and firm 2 the follower), whether in prices or in quantities, under a regime parameterised by the follower's competitive toughness θ_2, can be determined as oligopolistic equilibria, by equating the leader's competitive toughness θ_1 to the one which maximises $\Pi_1(\cdot, \theta_2)$, namely $\theta_1^{\mathrm{S}}(\theta_2)$ as given by formula (3.13). In the case where Stackelberg firms compete in prices, the follower sets p_2 so as to maximise $p_2 D_2(p_2, p_1)$, which amounts to taking $\theta_2 = 1$, the maximal competitive toughness. Since Π_1 is a decreasing function of θ_2 we have a situation of first-mover *disadvantage*. But here sequentiality does not play a role. The first-mover disadvantage in the sequential game is replaced by the follower's tougher competition or a more aggressive attitude in the simultaneous game.[17]

In the case where Stackelberg firms compete in quantities, the follower solves an optimisation programme in x_2 (given x_1). Referring to Eq. (3.16), the solution amounts to firm 2 taking $\theta_2^{\mathrm{q}} = \tau_1/(\tau_1 + 1/2)$. Hence, the leader's choice of q_1 can be retrieved from the maximisation in θ_1 of $\Pi_1\left(\theta_1, \theta_2^{\mathrm{q}}\right)$, that is, according to (3.13),

$$\theta_1^{\mathrm{Sq}} = \theta_1^{\mathrm{S}}\left(\frac{\tau_1}{\tau_1 + 1/2}\right) = 1 - \frac{1/2 + \tau_1}{1/2 + 2\tau_1 + 2\tau_2 + 4\tau_1\tau_2}. \tag{3.17}$$

If $\theta_1^{\mathrm{Sq}} > \theta_2^{\mathrm{q}}$ (always true if $\tau_2/\tau_1 = t_1/t_2 \geq 1/2$, i.e., if the ratio of market powers in the two captive segments is not too unfavourable to the leader),

[17] Contrary to the conventional presentation of the Stackelberg duopoly as a two-stage game, where a pre-determined leader plays at the first stage and a pre-determined follower at the second (see, e.g., Vives, 1999), we do not find sequentiality in Stackelberg (1934) either. Instead, "A views the behaviour of B either as dependent on or independent of his own behaviour. [...] Since each supplier can have each of the two types of position, the price formation structure [...] is thus imperfect" (Stackelberg, 1934 [2011], p. 17). The supplier that views his rival's behaviour as dependent "will examine all profit maximisation options and implement the best one. We can say that [he] dominates the market" (*ibid.*, p. 16). The Stackelberg *leader* is not the *first mover* but the supplier exerting *market dominance* ("*Marktherrschaft*").

the leader's profit is higher than the follower's (first mover *advantage*). Now the first-mover advantage in the sequential game is replaced by the leader's more aggressive attitude in the simultaneous game. Moreover, as $\left(\theta_1^{\mathrm{Sq}}, \theta_2^{\mathrm{q}}\right) < \left(\theta_1^{\mathrm{Sp}}, \theta_2^{\mathrm{p}}\right)$, both profits are higher in a Stackelberg quantity equilibrium than in a Stackelberg price equilibrium.

However, in the context of separation between ownership and management, sequentiality is important since the type of the manager, represented by the competitive toughness parameter, may be chosen by the owner at a preliminary stage. This generates a two-stage *delegation game*. At the first stage, the owners of each firm i hire a manager with competitive toughness θ_i and, at the second stage, these managers play the corresponding θ-game. For simplicity, let us start by taking the Hotelling case of uniform travel costs, with $t_1 = t_2 = t$. In this case, the competitive toughness associated with the pure quantity game θ_i^{q} given by (3.16) is simply $2/3$ for both firms and the first-stage payoff given by (3.12) reduces to

$$\Pi_i\left(\theta_i, \theta_j\right) = \frac{(2v+t)^2}{2t} \frac{(2+\theta_i)\left(6+\theta_j\right)^2}{\left((5+\theta_i)\left(5+\theta_j\right)-1\right)^2}.$$

The equilibrium competitive toughness can be seen to be $\theta_1^* = \theta_2^* = 2\left(\sqrt{2}-1\right) \simeq 0.828$, between the competitive toughnesses characterising price competition ($\theta^{\mathrm{p}} = 1$) and quantity competition ($\theta^{\mathrm{q}} = 2/3$). At the subgame perfect equilibrium of the two-stage delegation game the profit $\Pi\left(\boldsymbol{\theta}^*\right)$ is lower than the quantity competition profit but higher than the price competition profit.

These equilibria can all be represented in the $\theta_1 \times \theta_2$ space, as depicted in Fig. 4.5. With any point $\boldsymbol{\theta} \in [0,1]\times[0,1]$ we associate the pair of prices $p^*(\boldsymbol{\theta})$ (given by (3.8)) of the equilibrium of the θ-game. The relations (3.13) determine the reaction functions $\theta_i = R_i(\theta_j)$, $i = 1, 2$, $i \neq j$, of the first stage of the delegation game: they are represented by the curves R_1 and R_2, the intersection of which gives the equilibrium (θ^*, θ^*) of this game.

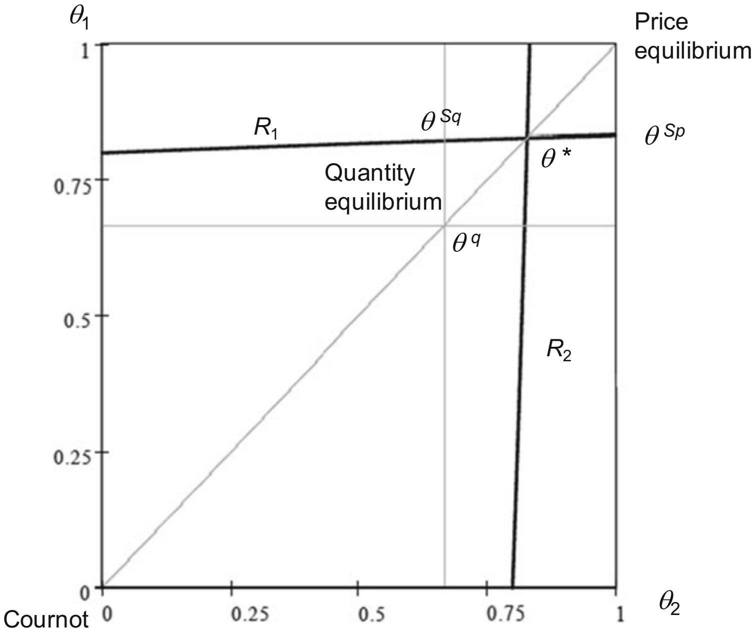

Fig. 4.5 Set of θ-equilibria under uniform travel costs

The competitive toughness pairs $\boldsymbol{\theta}$ leading in the family of θ-games to the Cournot solution[18] as well as to the price and quantity equilibria are represented by the points 0, 1 and θ^{q} of the first diagonal, respectively. Also, the competitive toughness pairs associated with Stackelberg equilibria when firms compete in prices or in quantities are represented by the points θ^{Sp} and θ^{Sq}, respectively, on the reaction curve R_1 of the leader, namely firm 1.

Coming back to the general case, with $t_i \neq t$, and using the equations given by (3.13), computation of the equilibrium competitive toughness

[18] It can be noted that the Cournot solution coincides with the collusive solution in the symmetric case. This can be checked by examining the first-order conditions (3.5), where the partial derivatives with respect to p_i of the two demand functions are both multiplied by p_i^*. In the first-order conditions for joint profit maximisation, they are each multiplied by the own price (hence, $\partial_i D_j$ by p_j^*). However, in the symmetric case where the two prices are equal, this difference is immaterial.

$\theta_i^*\left(\tau_i, \tau_j\right)$ of each firm i gives:

$$\theta_i^*\left(\tau_i, \tau_j\right) = \sqrt{\frac{2\left(\tau_i + \tau_j + 4\tau_i\tau_j\right)}{\left(1 + 2\tau_j\right)} + 4\tau_i^2} - 2\tau_i, i, j = 1, 2, i \neq j, \tag{3.18}$$

The equilibrium value θ_i^* is an increasing function of both its arguments, although more responsive to the competitor's ratio τ_j. A reduction of the travel unit cost t (the two brands becoming more substitutable) will thus decrease the competitive toughness of both firms, moderating the decline in profits. In other words, if competition becomes structurally more intensive, owners compensate this adverse occurrence by softening conduct, as they hire less aggressive CEOs. Inversely, a reduction of the travel unit cost t_i on the captive segment of firm i, increasing the size of firm i's captive segment, will induce an increase in the competitive toughness of both firms, but higher for firm j. At the limit, as τ_i (and a) become indefinitely large, we obtain the Stackelberg price equilibrium outcome, with firm j as the follower, choosing $\theta_j^* = 1$, and firm i as the (disadvantaged) leader.[19]

4 As a Conclusion

In this last chapter, we have presented several extensions of the approach to oligopolistic competition that this book is defending. There are of course many more extensions to be explored, both theoretically and empirically. But we hope to have convinced the reader that it is possible to overcome the obstacles that have slowed down the application of oligopoly theory to so many fields in economics where general equilibrium features are crucial. The proposed methodology does not wash out all strategic (and game-theoretic) interactions in integrating firms'

[19] Using the words of Deneckere et al. (1992), the firm with the largest captive segment becomes a price leader "with a large loyal consumer base" and provides a "price umbrella" to the follower.

behaviour, as is the case in models of perfect or monopolistic competition, and yet it remains tractable. The key element to make this approach successful is the basic simplification that Dixit and Stiglitz (1977) path-breakingly proposed, that is, to assume separable preferences. Exploiting this assumption more systematically allows for drastically simplified firms' conjectures. Finally, the famous (or infamous) "indeterminacy problem" in oligopoly, crystallised in the price-quantity dichotomy, can be turned into an advantage. Equilibrium conditions lead to an empirically testable formula depending on continuous conduct parameters that measure each firm's competitive toughness and that can be estimated.

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