Mathematical Model of Integration of Cyber-Physical Systems for Solving Problems of Increasing the Competitiveness of the Regions of the Russian Federation

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Abstract A mathematical approach to the creation of an integrated mathematical model of the socio-economic system for the study of various aspects of the development of the competitiveness of the region on the basis of the principles of building cyber-physical systems is proposed. For this, a modification of the Kolmogorov-Chapman equations has been developed, which allows one to describe the integration of enterprises into clusters and use fragmentary data on the state of indicators of the structures under study. A graph of cause-and-effect relationships of dangerous combinations of events is created. The results of model approbation are carried out. An example of numerical modeling is given, showing a high probability of timely response to the occurrence of elementary events included in the minimum sections of the graph. The results of the work are intended for use in the development of mathematical models of advising systems for monitoring and countering the violation of the sustainable functioning of an enterprise using modern mathematical models and information technologies.

Keywords Industry 4.0 · Cyber-physical systems · Competitiveness · Regions of the russian federation · Causal graph · Minimal sections · Kolmogorov-chapman equations

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1 Introduction

The modern period of socio-economic development corresponds to a series of various successive crises. At the same time, one crisis entails others, such as, for example, the problems caused by the coronavirus pandemic, which initiate many economic, social, psychological, and other interrelated problems interacting in explicit causal products and in hidden, latently affecting the environment. Often it is latent connections that have a decisive influence on the dynamics of the system as a whole. Therefore, the task of identifying latent causal relationships is urgent [1–3].

The problem is complicated by the formation of new industry 4.0 [4–6] and a change in the technological order. The processes occurring in the world are of a systemic nature and testify to the uneven, cyclical nature of economic development and the need to detect emerging basic technologies, the use of some innovations in order to relatively quickly overcome alternating crises corresponding to the minimum of the great Kondratyev wave. One of the most noticeable trends in the change in the technological structure is the digitalization of the economy and all spheres of society [7–9]. The modern information technology concept of cyber-physical systems [10–13] implies the integration of computing resources into real objects of any kind, including industrial complexes, socio-economic systems, and biological objects. In cyber-physical systems, information, and communication technologies (ICT) are distributed throughout the system and synergistically linked with its constituent elements [14].

According to the researchers, the prospects for building cyber-physical systems, as well as the formation of Industry 4.0 on their basis, affect the interests of society. Therefore, the creation of such systems should be investigated in technical, as well as in broader social, cultural, and economic aspects, including taking into account the increase in the competitiveness of various regions [15]. The use of cyber-physical systems extends to many types of human activity, including different complexes: defense, industrial, transport, commercial, and energy, as well as various types of life support systems from medicine to smart homes and cities, as well as many economic systems. At the same time, when creating these systems, it is necessary to take into account and identify cause-and-effect relationships, which is especially important when building competitive regions of the Russian Federation on the principles of cyber-physical systems. At the same time, it is believed that the creation of full-fledged cyber-physical systems in the future will lead to changes in interaction, including the economic sphere, similar to the construction and implementation of the World Wide Web in everyday life. In the current conditions of revolutionary changes, the study of the processes of increasing the competitiveness of regions is one of the key tasks of forecasting and management. One of the obvious ways of research is the development of mathematical models based on nonlinear differential equations.

2 Mathematical Modeling Based on the Kolmogorov-Chapman Equations

One of the approved methods for studying failures in technical systems, the tree of failures, and the system of Kolmogorov-Chapman equations has been successfully used to model processes in socio-economic systems [16, 17]. In the latter case, the special development of the cause-and-effect graph is carried out like a tree of events. Logical Boolean operations are introduced into the vertices of the graph, which are consequences of causes. Then the possible minimum cross-sections are revealed as sets of sequentially realized events. An event graph is built for each of the minimum sections. For each graph of events, a system of Kolmogorov-Chapman equations is constructed and solved. For each of these sets of events that form the minimum cross-sections, the dynamics of the probabilities of the combined events are obtained. The rest of the events are then excluded by the model. For them, other minimal sections, state graphs, systems of equations, and solutions are constructed. As a result, sets of separate possible solutions are considered for joint consideration of the system.

The difficulty in applying this model is, firstly, the need to declare clear dependencies of cause-and-effect relationships, which is very difficult and often incorrect for socio-economic systems. Secondly, a significant increase in the number of equations. For 4 reasons, $2^4 + 1 = 32$ equations are obtained, for 100 reasons—2^{101} equations, which is quite difficult for calculations based on a mathematical model. Thus, when building a model for the integration of cyber-physical systems, it is necessary to solve the following tasks:

(1) take into account the intersection of the studied processes;
(2) create a system of nested models to reduce the number of equations in the system;
(3) to allow the structure to develop naturally, taking into account the effects of causes that are absent in the field of study.

This problem of developing a mathematical model of a complex cyber-physical system can be solved only on the basis of nonlinear dynamics methods. In accordance with the tasks set, we will present the model of the economy of the region of the Russian Federation, as a certain socio-economic environment that allows providing the infrastructure of interaction between clusters of various organizations in the region. The initiative to increase the competitiveness of the region is based on the leadership policy that unites the leading organizations and allows the development of the infrastructure of the socio-economic environment of the region based on information and communication technologies (ICT), linking all the constituent elements (Fig. 1).

To create a mathematical model of such a structure, we will be based on a relatively simple model of the Kolmogorov-Chapman equations. Let the models of individual organizational structures be based on a system of balanced indicators [18] for which a cause-and-effect graph is built that combines the 4 branches of risk indicators of the threat of loss of the organization’s ability to work: finance, market, internal

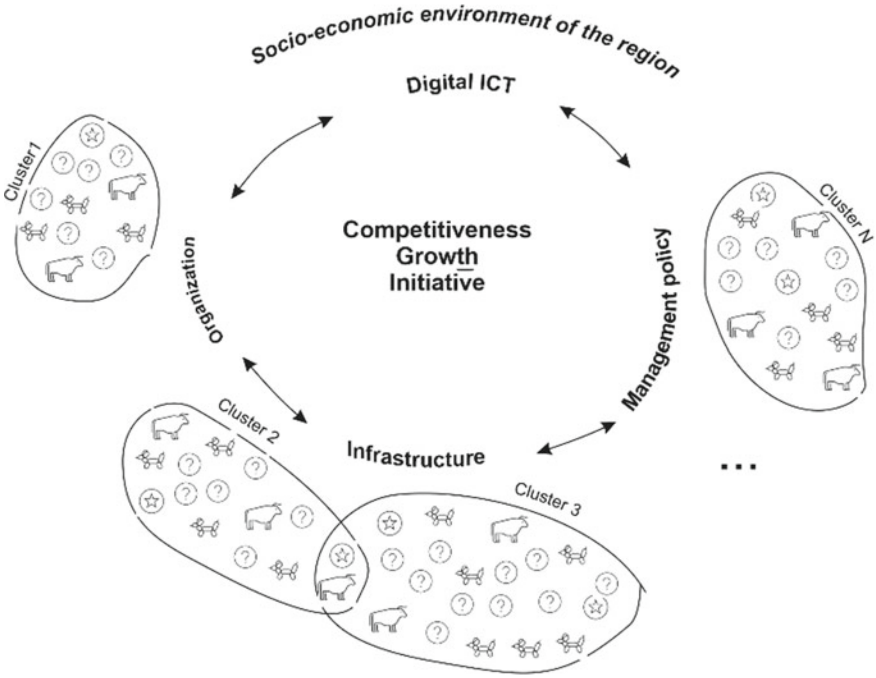

Fig. 1 An integrative model for describing the growth opportunities of the region's competitiveness

business processes, training, and growth. In the processes of the organization's functioning, risks arise from the impossibility of implementing the main processes, for example, industrial business processes, as a result, the risk of systemic disruption of the enterprise's activities and the implementation of the root top of the risk tree increases.

To implement the goal of monitoring the identification of dangerous combinations of risks for the successful functioning of an enterprise, it is advisable to use the system of linear differential equations of Kolmogorov-Chapman:

$$\frac{dP_0(t)}{dt} = -\sum_{j=1}^{k} \lambda_j P_0(t) + \sum_{j=1}^{k} \mu_j P_j(t);$$

$$\frac{dP_i(t)}{dt} = -P_i(t)\pi_i^- + \sum_{j=0}^{2^k-1} \pi_{ij}^+ P_j(t);$$

$$\frac{dP_{2^k-1}(t)}{dt} = \sum_{j=1}^{k} \lambda_j P_{2^k-k+j-2}(t). \tag{1}$$

This mathematical model is applicable to determine the probabilities of realizing the minimum cross-sections under certain conditions; it consists of 2 k equations for the functions $P_0(t), \ldots, P_2^{k-1}(t)$—the probabilities of events corresponding to the vertices of the cause-and-effect graph of the risk tree. The disadvantages of using such a model to characterize various socio-economic processes are the above-noted lack of taking into account the cross-section and mutual influence of risk events, a large number of 2^k equations taking into account the possibility of implementing k reasons, and the absence of a visible prospect of increasing the complexity of the modeled part of the system.

3 Modeling Statistics

Let us ask ourselves a question: what functions describe statistics? Consider the statistical data for the Saratov region of the Russian Federation. Let's take the data on the factor "Volume of innovative goods, works, services", which contain, in percentage terms, information on the total volume of goods that are shipped, as well as services and works [17] (Table 1).

Let's approximate these data and get the following function:

$$\begin{aligned}\text{Volume InnProd} = {} & 0.102866 + 1.23901e^{-x^2} - 0.0582672\text{Cos}(x) \\ & + 0.0331014\text{Cos}\left(x^2\right) - 0.0377882\text{Sin}(x) \\ & + 0.0362681\text{Cos}(10x)\text{Sin}(x).\end{aligned} \quad (2)$$

The results of approximation (solid line) and statistical data (points) are shown in Fig. 2. If you use this pattern, you can build the following extrapolation curve (Fig. 3). The obtained dependence allows us to assume: on the one hand, the value of the Volume InnProd factor, which has fallen to close to zero, will remain fluctuating near zero for a very long time. This may be so, if we recall, for example, the last few decades of the development of the South African Republic. However, on the other

Table 1 Characteristics of innovative products (Volume InnProd) by volume, services, and works as a percentage of the total volume of goods shipped, as well as services and works in the Saratov region

Year	Volume InnProd	Year	Volume InnProd	Year	Volume InnProd
2000	0.49	2006	0.080	2012	0.103
2001	0.06	2007	0.036	2013	0.111
2002	0.20	2008	0.175	2014	0.054
2003	0.17	2009	0.328	2015	0.140
2004	0.189	2010	0.211	2016	0.000
2005	0.15	2011	0.001	2017	0.000

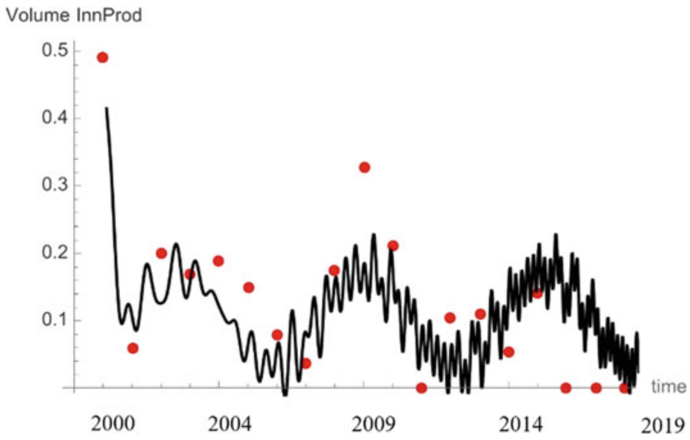

Fig. 2 Characteristics of the value of innovative goods, as well as services and works as a percentage of the total volume in the Saratov region for the period 2000–2018 according to [17], marked with red dots. The extrapolation results are shown by the solid line

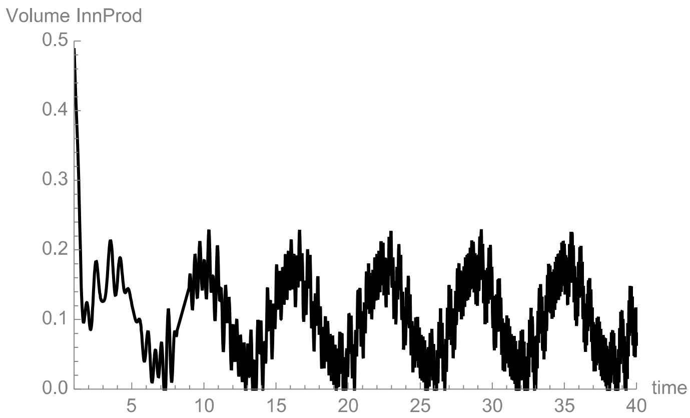

Fig. 3 Results of extrapolation of statistical data on the value of innovative services, goods, works as a percentage of the total volume in the Saratov region for 40 years, starting from 2000

hand, we can safely say that the period over which the extrapolation was carried out is too short. In addition, the Volume InnProd factor is dependent on many other components. It is not isolated and should be considered as a result of interaction with other factors of the region's development. Also, it should be taken into account that the region is an open system and interacts with the environment, consisting of other regions and the governing influences of the government.

It is important to note that the observed process is oscillatory. Let's carry out the transformation and look at the expression obtained as a result of statistical data interpolation. The resulting expression is complex-valued:

$$
\begin{aligned}
\text{Volume InnProd} = {} & 0.102866 + 1.23901e^{-x^2} - 0.0582672\text{Cos}(\text{x}) \\
& - (0.0291336 - 0.0188941i)e^{i\text{x}} \\
& - 0.00906701ie^{-i9\text{x}} + 0.00906701ie^{-i9\text{x}} \\
& + \ 0.00906701ie^{-i11\text{x}} - 0.00906701ie^{i11\text{x}} \\
& + 1.23901e^{-\text{x}^2} + 0.0165507e^{-i\text{x}^2} \\
& + 0.0165507e^{i\text{x}^2}
\end{aligned}
\tag{3}
$$

Let us assume that it is possible to build a certain model that describes the dynamics of statistically observed factors of the region's competitiveness. Let this model be based on the well-proven model of the Kolmogorov-Chapman equations.

4 Modification of the Kolmogorov-Chapman Equations

Let's modify the mathematical model for predicting combinations of risks for an organization. First, consider the replacement of probabilities in the nodes of the event tree with status functions (SF) [18]. Here SF characterizes the status, which is a set of characteristics that are inherent to the subject or object and determine its position in the system. In this case, an SF will be called a mapping that establishes a rule for determining the correspondence of some ordered pair of arguments [18]. An ordered pair of arguments is a characteristic of the probability of an event occurring at a graph node and its direction along an adjacent edge, which can be interpreted as an action or reaction. Let's introduce the binary values of the characteristics at the nodes. We will estimate the probability of an event occurring at a node as low and high, and the direction to the root top (up) or away from it (down). In this case, there will be 4 possible estimates of the state in the node of the event tree: {low, down}; {low, up}; {high, down}; {high, up}. The use of SF as characteristics of the top of the cause-and-effect graph allows us to combine λ_j—the values of the intensities of the occurrence of an event π_j and μ_j—the intensity of counteraction to this event, which is used in the system of Kolmogorov-Chapman equations. Intensities describing the "direction of action" λ_j, π_j, μ_j, and the probabilities P_i of these events in complex-valued status functions. In this case, the real characteristic of the function describes the classical probabilities of occurrence of events at the vertices of the causal graph, the imaginary characteristic corresponds to the angle of rotation of the vector and allows you to represent the "direction of action".

In addition, such a representation allows one to take into account interference effects. If two causes A and B are in states described by the attributed functions f_A and f_B and if these states interact, then the result of the interaction is described as.

$$P = |f_A - f_B|^2. \tag{4}$$

Such a mathematical description reflects the features of various interdependent processes, as well as the internal structure of various real objects that are subject to research. These include, for example, the fundamentals and elements of the firm's balanced scorecard. These include financial performance, customer relationships, internal business processes, and personnel training and development indicators [19]. Moreover, they are internally interconnected. A common description of these states allows one to obtain a probability distribution of the following form:

$$P = |f_A|^2 + |f_B|^2 \tag{5}$$

The methods used for estimation to obtain estimates introduce errors in these measurable states. This, in turn, influences the principle of fact-based governance. The use of the so-called complex functions for assessment will eliminate this discrepancy.

Note that SFs are attributed to the state of the object, similar to membership functions (FP) in the fuzzy set theory (TNM). They also depend on the base variable entered, such as r. However, unlike FP, the choice of which is determined by the conditions of the problem and the simplicity of presentation, the set of used SFs is formed as a set of orthonormal functions that form a set of orthogonal basis functions. The introduction of SF changes the interpretation of the calculation results. The probability is not limited by the supremum FP = 1, has a similar restriction on the integral SF = 1. To implement this condition, we introduce not a normalization equation as in the system of Kolmogorov-Chapman equations, but a restriction on the growth of the function into the equation. This limitation is introduced by analogy with the limitations of innovation diffusion models [20]. Then in the equations for P0 the 2nd term of the equation appears under the integral—P0 (2). The third term of the equation under the integral allows us to introduce the mixing of the SF, the implementation of which is described in the equation with the rest of the reasons for the set of reasons under study. Such a mathematical representation of the cross interaction of the processes under study is characteristic of synergistic models in the equations of chemical and biochemical kinetics. It is also useful in assessing the health of components in a production system that uses digital twin technology [21, 22].

Consider a model with nonlinear growth and mixing constraints introduced into the Kolmogorov-Chapman equations for the probabilities of realizing events introduced into the vertices of the event graph. The first term remains in the equations, the growth restriction is introduced in the second, and mixing is carried out in the third, which occurs nonlinearly. We obtain equations of the form (5):

$$\frac{dP_0(t)}{dt} = \int_{-1}^{1} \left(\sum_{j=1}^{k} P_0(t)\big(\psi_j(r) - P_0(t) - \psi_j^*(r) P_j(t)\big) \right) dr$$

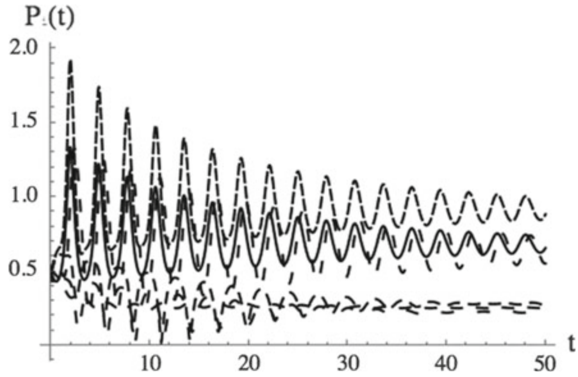

Fig. 4 Description of the results of digital modeling based on the modification of the Kolmogorov–Chapman equations. Nonlinearity is introduced for various initial conditions used: 1—P (0) = 0.5, 2—P (0) = 1. In this case, the values of the probabilities of other different events are taken equal to 0.5

$$\frac{dP_i(t)}{dt} = \int_{-1}^{1} \left(\sum_{j=0}^{2^k-1} P_i(t) \left(\psi_i^* - \psi_j P_j(t) - \psi_j P_i(t) \right) \right) dr$$

$$\frac{dP_{2^k-1}(t)}{dt} = \int_{-1}^{1} \left(\sum_{j=1}^{k} P_{2^k-k+j-2}(t) (\psi_j - \psi_{2^k-k+j-2} P_{2^k-k+j-2}(t)) \right) dr \quad (6)$$

where $P_i(t)$ are complex-valued SFs.

Numerical simulation of the system of Eq. (4) makes it possible to obtain a certain oscillatory process tending to an equilibrium stable state (see Fig. 4). The results obtained suggest that studies based on the proposed equations obtained on the basis of the modification of the Kolmogorov-Chapman equations for SF can be promising as a basis for describing cyber-physical systems that are a sub-system of the general socio-economic system of the region. This raises the complexity of the obtained results of digital modeling of interpretation for social applications. Therefore, when investigating a dangerous confluence of events, it is required to compare models for the same behavior of the system in a certain sense. It is desirable to form proactive management decisions based on forecasting [23].

5 Conclusions

Thus, a mathematical approach to the creation of an integrative model of the socio-economic system by studying the opportunities and risks of developing the competitiveness of the region is proposed. The model is based on a modification of the Kolmogorov-Chapman equations and implies the integration of individual enterprises into clusters with the possibility of using disparate fragmented data on the state of indicators of the structures under study and sections of the graph of cause-and-effect relationships of dangerous combinations of events. The preliminary approbation of the model has been carried out. An example of numerical calculations is presented, showing a high probability of timely response to the occurrence of elementary events included in the minimum sections of the graph. Their occurrence and development lead to the realization of the event of the graph vertex. The results of the work are intended to be used in the development of mathematical models of advising systems for monitoring and countering the violation of the sustainable functioning of an enterprise using modern mathematical models and information technologies.

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