Using Interval-Valued Fuzzy Sets for Recommending Groups in E-Learning Systems

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Abstract. To obtain required effects of Web-based learning process, teaching environment should be adjusted to student needs. Differentiation of the environment features can be received by grouping learners of similar preferences. Then each new student, who joins the community, should obtain the recommendation of the group of colleagues with similar characteristics. In the paper, we consider using fuzzy logic for modeling student groups. As the representation of each group, we assume fuzzy numbers connected with learner attributes ranked according to their cardinality. Recommendations for new students are determined taking into account similarity of their dominant features and the highest ranked attributes of groups. The presented approach is examined, for students described by learning style dimensions. The method is evaluated on the basis of experimental results obtained for data of different groups of real students.

Keywords: Recommender system · Fuzzy logic · Interval-valued fuzzy sets · Groups modeling

1 Introduction

Nowadays there is a big need of Web-based education process. However, its performance depends on the degree the learning environment is adjusted to learners needs. Dividing students into groups of similar preferences and tailoring learning resources appropriately may help to achieve assuming learning outcome. However, the group assignment of each new learner should guarantee his similarity to the members of the recommended group. Effectiveness of recommendations depends on accuracy of group modeling. However, if our knowledge concerning students' traits is imperfect one has to apply tools for describing uncertain or imprecise information. As the most consistent with human being decision making process Shakouri and Tavassoli [1] mentioned fuzzy logic methods, which are based on the fuzzy set theory [2].

A concept of a fuzzy set has been used in [3] for group recommending in e-learning systems. A fuzzy group representation has been defined with the use

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of linguistic terms corresponding to attribute values. The definition of fuzzy set contains a membership function which is a mapping $R \rightarrow [0,1]$. It indicates the membership grade by assigning to each element of a universe of discourse a number from the unit interval. However, in some circumstances traditional fuzzy sets (type-1 fuzzy sets) may appear an insufficient tool. Values of membership degrees are not always unique as it is required by the concept of the type-1 fuzzy set. To capture this kind of uncertainty one can apply its extension, known as an interval-valued fuzzy set, in which the membership grade is expressed by means of a closed subinterval of the interval $[0,1]$. The idea was proposed by Zadeh [4]. A place of interval-valued fuzzy sets among other extensions of the concept of fuzzy sets was shown in [5].

The current paper extends the idea presented in [3] by applying interval-valued fuzzy sets. The proposed method is examined for student traits based on their learning style dimensions. It is validated, on the basis of experiments, done for real students' clusters. The performance of the considered method is evaluated by experiments done on real student data. The results are compared with the ones obtained using traditional fuzzy sets. The remainder of the paper is organized as follows. The relevant research is described in the next section. Section 3 presents the basic notions related to interval-valued fuzzy sets. Then, the methodology for building recommendation based on probabilistic representation of groups is depicted. Section 5 discusses applying of interval-valued fuzzy sets in the considered problem. Section 6 focuses on application of the proposed methodology into attributes based on learning style dimensions. In the following section some experimental results are presented and discussed. Finally, concluding remarks and future research are outlined.

2 Related Work

Recommender systems are considered to be an important tool for improving e-learning courses. Numerous works discuss their adaptation to e-learning environments. A survey of the state-of-the-art in e-learning recommended systems was presented in [6]. Various challenges and techniques were discussed in [7]. An educational recommender system should take into account different preferences of learners and so it should be highly personalized [8]. Some authors proposed applying personal learning styles for building recommended systems [9,10]. Qomariyah and Fajar [9] implemented a system based on a logical approach. The proposed method helps students to choose the best material according to their preferences. Nafea et al. [10] elaborated a recommender algorithm for recommending personalized learning objects. Their approach is based on the Felder and Silverman learning style model [11]. Special attention was paid to using recommendations for group learning. Christodoulopoulos and Papanikolaou [12] discussed several factors that should be considered while assigning learners into groups. They investigated the choice of the best algorithms for creating student clusters. Masthoff [13] described using of group modeling and group recommendation techniques for recommending to individual users. A survey of the state-of-the-art in group recommendation was presented in [14].

In the design of e-learning systems researchers considered the use of fuzzy logic. Authors used fuzzy sets to describe reality by means of linguistic terms, which are close to human nature. Several researchers examined possibilities of using fuzzy logic for student modeling. Different aspects of the use of fuzzy techniques in e-learning were shown in [15]. Hogo [16] proposed applying fuzzy clustering methods for evaluation of e-learners behaviour. Similar analysis was presented in [17]. Limogneli and Sciarrone [18] applied fuzzy student modeling for personalization of e-learning courses. Goyal et al. [19] proposed student modeling based on learning style, personality and knowledge level which was evaluated with the use of the intuitionistic fuzzy approach. In [20] authors defined two metrics based on fuzzy logic for evaluation of different personal strategies. Modeling of educational data containing fuzzy classes for student performance assessment was applied in [21]. Using fuzzy logic for evaluation and classification of students performance was presented in [22]. Salmi et al. [23] used fuzzy sets in fuzzy evaluation of e-learning courses. Chen and Wang proposed methods for evaluating students' answerscripts based on interval-valued fuzzy grade sheets [24]. In order to improve recommendation of e-learning courses Lin and Lu [25] created an intuitionistic type-2 fuzzy inference system. A bidirectional approximate reasoning method based on interval-valued fuzzy sets was proposed in [26]. Fuzzy inference has also been used by Gogo et al. [27] in an inference engine which recommends relevant learning content to learners. In [28] Lu proposed a fuzzy matching method to find suitable learning materials. To handle uncertainties connected with e-learning environments and students Almohammadi et al. [29] proposed a type-2 fuzzy logic system which is able to estimate the engagement degree of students for both remote and on-site education.

3 Interval-Valued Fuzzy Sets

Since the first presentation of the fuzzy set theory [2] a number of its extensions have been proposed. One of them is the theory of interval-valued fuzzy sets.

Definition 1. *Let R be a universe of discourse. An interval-valued fuzzy set A in R is a set of ordered pairs:*

$$A = \{< x, \mu_A(x) >: x \in R, \quad \mu_A(x) : R \to \mathrm{Int}([0,1])\}, \tag{1}$$

where $\mu_A(x) = [\mu_{A_L}(x),\ \mu_{A_U}(x)]$ is an interval-valued membership function, Int$([0,1])$ stands for the set of all closed subintervals of $[0,1]$: Int$([0,1])$ $= \{[a,b] : a, b \in [0,1]\}$.

Thus, the mapping $R \to [0,1]$ occurring in the definition of an ordinary fuzzy set has been replaced with the mapping $R \to \mathrm{Int}\ [0,1]$. Each element x of R is associated with two membership functions $\mu_{A_L}(x)$ and $\mu_{A_U}(x)$, which are the bounds of the membership interval $\mu_A(x) = [\mu_{A_L}(x),\ \mu_{A_U}(x)]$. The basic characteristics of interval-valued fuzzy sets are defined with the use of the border type-1 fuzzy sets A_L and A_U which are determined by functions $\mu_{A_L}(x)$ and

$\mu_{A_U}(x)$. In the presented approach we use the cardinality concept. For a finite universe of discourse $R = \{r_1, r_2, \ldots r_n\}$ the cardinality $card(A)$ of an interval-valued fuzzy set A is defined by the interval $[card(A_L), card(A_U)]$, where:

$$card(A_L) = \sum_{x \in R} \mu_{A_L}(x),\ card(A_U) = \sum_{x \in R} \mu_{A_U}(x),\ R = \{r_1, r_2, ..., r_n\}\,. \quad (2)$$

Wu and Mendel define the cardinality concept as [30]

$$card(A) = \frac{1}{2} \sum_{x \in R} (\mu_{A_L}(x) + \mu_{A_U}(x)). \quad (3)$$

A support of an interval-valued fuzzy set A is determined by supports of A_L (lower support) and A_U (upper support):

$$\begin{aligned} supp(A)_L &= supp(A_L) = \{x \in R : \mu_{A_L}(x) > 0\}, \\ supp(A)_U &= supp(A_U) = \{x \in R : \mu_{A_U}(x) > 0\}. \end{aligned} \quad (4)$$

The lower support is included in the upper support: $supp(A)_L \subseteq supp(A)_U$. A closeness measure, between two interval-valued fuzzy sets A and B, denoted by $\approx (A, B)$, is expressed by a subinterval of [0, 1], with the bounds:

$$\begin{aligned} \approx (A, B)_L &= \sup_x \min(\mu_{A_L}(x), \mu_{B_L}(x)), \\ \approx (A, B)_U &= \sup_x \min(\mu_{A_U}(x), \mu_{B_U}(x)). \end{aligned} \quad (5)$$

4 Recommender System

Let us consider objects described by a set U of N categorical attributes $U = \{X_1, X_2, \ \ldots\ , X_N\}$. Domains of attributes, denoted by $D(X_i)$, $i = 1, ..., N$, are finite sets. Let us denote by p_i cardinality of $D(X_i)$: $p_i = card(D(X_i))$. Thus, $D(X_i) = \{x_{i,1}, x_{i,2}, \ \ldots x_{i,p_i}\}$. An object O is represented by a tuple t in the form

$$O = (t(X_1), t(X_2), ..., t(X_N)), \quad (6)$$

where $t(X_i)$ denotes a value of X_i and $t(X_i) \in D(X_i)$.

Let us assume that there exist different groups of objects of similar features GO_k, $k = 1, ..., M$ with the set U of attributes. For each attribute one can determine a distribution of values occurring in a given group. Thus, one can indicate dominant values for every attribute. Probability $P_{i,j}$ that objects of the group GO_k are characterized by a certain value $x_{i,j}$ of X_i can be expressed by the following formula:

$$P_{i,j} = card(\{O \in GO_k : t(X_i) = x_{i,j}\})/card(GO_k), \quad x_{i,j} \in D(X_i). \quad (7)$$

The probabilistic representation of the group can be used to classify new objects to the closest groups. In order to find an appropriate group for a given object one should determine matching degrees. Let a tuple

$NO = (ox_1, ox_2, ..., ox_N)$, $ox_i \in D(X_i)$ represent a new object. The matching degree, of NO to GO_k, for the attribute X_i, denoted as S_i, is computed as the proportion of objects O belonging to the group, $O \in GO_k$, such that $O(X_i) = ox_i$ to the size of the group. Thus it equals $P_{i,j}$. The total matching degree S for the group is a minimal value of S_i, $i = 1, ..., N$:

$$S = \min_i S_i. \tag{8}$$

A group with maximal S should be chosen for NO. According to this methodology a value of S strongly depends on the "worst" attribute. For example, if $S_i = 0$ for a certain attribute X_i then also $S = 0$, regardless of other matching degrees. The described way of recommendation assumes that all attributes are of the equal importance. However, if an attribute with a low matching degree is less important the rejection of a given group could be unjustified. The choice may be improved by introduction of weights. Let $w_i \in [0, 1]$ denote the grade of importance of X_i. It is assumed that $\max_{1,2,...,N} w_i = 1$. The total matching degree takes the form:

$$S = \min_i \max(1 - w_i, S_i). \tag{9}$$

For the most important attributes $w_i = 1$. If $w_i = 1$ for every i the total matching degree is expressed by formula (8). If $w_i = 0$, then the attribute X_i in not considered.

5 Interval-Valued Fuzzy Sets in Building Recommendations

The recommendation procedure does not take into account closeness relationships which may be associated with elements of attribute domains. If the neighbouring values are close to one another the change of matching degrees should be considered. Otherwise, the recommendation result may be unsatisfactory.

Assumption of sharp boundaries between elements of attribute domains impose a unique qualification to the corresponding category. In the paper we introduce imprecision to the definition of the group representation. The existing uncertainty is modeled by means of interval-valued fuzzy sets.

Let us consider attribute X_i with $D(X_i) = \{x_{i,1}, x_{i,2}, \ ... x_{i,p_i}\}$. Let elements of $D(X_i)$ be linguistic terms represented by the following interval-valued fuzzy sets $FX_{i,j}$:

$$FX_{i,j} = \{< x, \mu_{FX_{i,j}}(x) >: x \in D(X_i), \ \mu_{FX_{i,j}}(x) : D(X_i) \rightarrow \mathrm{Int}([0, 1])\}, \tag{10}$$

where $i = 1, ..., N$ and $j = 1, ..., p_i$. Let $FX_{i,j_L}(x)$ and $FX_{i,j_U}(x)$ be lower and upper membership functions of $FX_{i,j}$, respectively. According to (5) the degree of closeness between interval-valued fuzzy sets $FX_{i,j}$ and $FX_{i,j+1}$ is an interval with the following bounds:

$$\begin{aligned} &\approx (FX_{i,j}, FX_{i,j+1})_L = \sup_x \min(\mu_{FX_{i,j_L}}(x), \mu_{FX_{i,j+1_L}}(x)), \\ &\approx (FX_{i,j}, FX_{i,j+1})_U = \sup_x \min(\mu_{FX_{i,j_U}}(x), \mu_{FX_{i,j+1_U}}(x)). \end{aligned} \tag{11}$$

For every group GO_k, $k = 1, ..., M$ one can define fuzzy sets of objects $FO_{i,j}$ with corresponding values of attributes:

$$FO_{i,j} = \left\{ < O, \mu_{FO_{i,j}}(O) >: O \in GO_k,\ \mu_{FO_{i,j}}(O) : GO_k \to \mathrm{Int}([0,1]) \right\}. \quad (12)$$

The membership function of $FO_{i,j}$ is as follows:

$$\mu_{FO_{i,j}}(O) = \mu_{FX_{i,j}}(O(X_i)). \quad (13)$$

As the representation of the attribute X_i for the group GO_k, $k = 1, ..., M$, we will consider probability $P_{i,j}$, $i = 1, ..., N$; $j = 1, ..., p_i$ that objects from GO_k, are characterized by the linguistic term $x_{i,j}$ represented by interval-valued fuzzy set (10). Probability $P_{i,j}$ belongs to the following interval:

$$P_{i,j} \in [card(FO_{i,j_L})/card(GO_k), card(FO_{i,j_U})/card(GO_k)] \quad (14)$$

In further considerations the definition (3) will be used. Thus

$$P_{i,j} = \frac{card(FO_{i,j_L}) + card(FO_{i,j_U})}{2 * card(GO_k)} \quad (15)$$

Matching degrees of a new object $NO = (ox_1, ox_2, ..., ox_N)$, to GO_k for respective attributes equal to the corresponding values of $P_{i,j}$. The total matching degree is expressed by formula (9).

Let us assume, that there are M groups, then the whole process of recommendation building will take place in the following way:

[Input]: A set of M groups GO_k, containing objects described by N nominal attributes; a new object NO;
Step 1: For each group $GO_k, k = 1, 2, ..., M$ find its representation according to (15)
Step 2: For the object NO find the group GR_{rec} with the maximal value of the matching degree (9);
Step 3: Recommend GR_{rec} to the object.

6 Student Group Recommendation

For the purpose of the evaluation of the proposed method, we will consider a student model based on dominant learning styles [31]. We will apply Felder and Silverman [11] model, where learning styles are described by means of 4 attributes representing preferences for 4 dimensions from among excluding pairs: active or reflective (L_1), sensing or intuitive (L_2), visual or verbal (L_3), and sequential or global (L_4). A student without dominant preferences is called as balanced. The model takes the form of a vector SL of 4 integer attributes: $SL = (sl_1,\ sl_2,\ sl_3,\ sl_4)$, where $sl_i \in \{l_1, l_2, ..., l_{12}\}$, $l_j = 2j - 13$. Attribute values belong to the set of odd integers from the interval $[-11, 11]$, that represent student preferences. They are determined on the base of ILS questionnaire [32], filled by students.

Negative values of sl_1, sl_2, sl_3, sl_4 mean scoring for active, sensing, visual or sequential learning styles, respectively. Positive attribute values indicate scoring for reflective, intuitive, verbal or global learning styles. Values $-5, -7$ or $5, 7$ mean that a student learns more easily in a learning environment which favors the considered dimension; values $-9, -11$ or $9, 11$ mean that learner has a very strong preference for one dimension of the scale and may have real difficulty learning in an environment which does not support that preference.

For creating of the fuzzy group representation we will define in the domain of each attribute interval-valued fuzzy sets F_j with the border type-1 fuzzy sets F_{j_L} and F_{j_U}. Let us assume that the lower membership function $\mu_{F_{j_L}}(x)$ takes the value 1 for $x = l_j$ and 1/2 for neighbouring elements. The lower supports of F_j are the following sets:

$$\begin{aligned} supp(F_1)_L &= \{l_1, l_2\},\ supp(F_{12})_L = \{l_{11}, l_{12}\}, \\ supp(F_j)_L &= \{l_{j-1}, l_j, l_{j+1}\},\ \text{for } j = 2, 3...11. \end{aligned} \tag{16}$$

The lower membership functions are as follows:

$$\mu_{F_{1_L}}(x) = -(x+7)/4,\ \text{if } x \in \{l_1, l_2\}, \tag{17}$$

$$\mu_{F_{12_L}}(x) = (x-7)/4,\ \text{if } x \in \{l_{11}, l_{12}\} \tag{18}$$

and for j = $2, 3, ..., 11$

$$\mu_{F_{j_L}}(x) = \begin{cases} (x - 2j + 17)/4, & \text{if } x \in \{l_{j-1}, l_j\} \\ -(x - 2j + 9)/4, & \text{if } x \in \{l_j, l_{j+1}\} \end{cases}. \tag{19}$$

The upper support of F_j contains more elements. We will assume that the upper membership function $\mu_{F_{j_L}}(x)$ takes the value 1 for $x = l_j$, 3/4 for l_{j-1} and l_{j+1}, 1/2 for l_{j-2} and l_{j+2} and 1/4 for l_{j-3} and l_{j+3}. The upper supports of F_j are the following sets:

$$\begin{aligned} supp(F_1)_U &= \{l_1, ..., l_4\},\ supp(F_{12})_U = \{l_9, ..., l_{12}\}, \\ supp(F_2)_U &= \{l_1, ..., l_5\},\ supp(F_{11})_U = \{l_8, ..., l_{12}\}, \\ supp(F_3)_U &= \{l_1, ..., l_6\},\ supp(F_{10})_U = \{l_7, ..., l_{12}\}, \\ supp(F_j)_U &= \{l_{j-3}, ..., l_j, ..., l_{j+3}\},\ \text{for } j = 4, 5...9. \end{aligned} \tag{20}$$

The upper membership functions are as follows:

$$\mu_{F_{1_U}}(x) = -(x+3)/8,\ \text{if } x \in \{l_1, l_2, l_3, l_4\}, \tag{21}$$

$$\mu_{F_{1_U}}(x) = (x+3)/4,\ \text{if } x \in \{l_9, l_{10}, l_{11}, l_{12}\}, \tag{22}$$

$$\mu_{F_{2_U}}(x) = \begin{cases} (x+17)/8, & \text{if } x \in \{l_1, l_2\} \\ -(x+1)/8, & \text{if } x \in \{l_2, l_3, l_4, l_5\} \end{cases}, \tag{23}$$

$$\mu_{F_{11_U}}(x) = \begin{cases} (x-1)/8, \text{ if } x \in \{l_8, l_9, l_{10}, l_{11}\} \\ -(x-17)/8, \text{ if } x \in \{l_{11}, l_{12}\} \end{cases}, \tag{24}$$

$$\mu_{F_{3_U}}(x) = \begin{cases} (x+15)/8, \text{ if } x \in \{l_1, l_2, l_3\} \\ -(x-1)/8, \text{ if } x \in \{l_3, l_4, l_5, l_6\} \end{cases}, \tag{25}$$

$$\mu_{F_{10_U}}(x) = \begin{cases} (x+1)/8, \text{ if } x \in \{l_7, l_8, l_9, l_{10}\} \\ -(x-15)/8, \text{ if } x \in \{l_{10}, l_{11} l_{12}\} \end{cases} \tag{26}$$

and for j $= 4, 5, ..., 9$

$$\mu_{FL_{i,j_U}}(x) = \begin{cases} (x-2j+21)/8, \text{ if } x \in \{l_{j-3}, l_{j-2}, l_{j-1}, l_j\} \\ -(x-2j+5)/8, \text{ if } x \in \{l_j, l_{j+1}, l_{j+2}, l_{j+3}\} \end{cases}. \tag{27}$$

Let fuzzy sets F_j represent linguistic terms f_j corresponding to attribute values. Thus, lower C_L and upper C_U closeness degrees between elements of attribute domains are as follows:

$$\begin{aligned} C_L(f_j, f_{j-1}) = C_L(f_j, f_{j+1}) = 1/2, \\ C_L(f_j, f_{j-k}) = C_L(f_j, f_{j+k}) = 0 \text{ if } k > 1 \\ C_U(f_j, f_{j-1}) = C_U(f_j, f_{j+1}) = 3/4, \\ C_U(f_j, f_{j-2}) = C_U(f_j, f_{j+2}) = 1/2, \\ C_U(f_j, f_{j-3}) = C_U(f_j, f_{j+3}) = 1/4, \\ C_U(f_j, f_{j-k}) = C_U(f_j, f_{j+k}) = 0 \text{ if } k > 3 \end{aligned} \tag{28}$$

The membership functions of interval-valued fuzzy sets $FSL_{i,j}(SL)$ of students with corresponding values of attributes L_i are determined by formulas (17–19) and (21–27). Probability $P_{i,j}$, $i = 1, 2, 3, 4$, $j = 1, ..., 12$, that students of the group GS, are characterized by the linguistic term f_j with respect to the attribute L_i equals

$$P_{i,j} = \frac{card(FSL_{i,j_L}) + card(FSL_{i,j_U})}{2 * card(GS)} \tag{29}$$

Let $jmax_i$ denotes the index j of f_j for which $P_{i,j}$ is maximal. As the fuzzy group representative we will consider four sets Rep_i, $1 \leq i \leq 4$, consisting of 3 elements, $Rep_i = \{rep_{i,1}, rep_{i,2}, rep_{i,3}\}$, such that

$$rep_{i,1} = f_1,\ rep_{i,2} = f_2,\ rep_{i,3} = f_3,\ \text{if } jmax_i = 1, \tag{30}$$

$$rep_{i,1} = f_{10},\ rep_{i,2} = f_{11},\ rep_{i,3} = f_{12},\ \text{if } jmax_i = 12 \tag{31}$$

and for $jmax_i = 2, 3, ..., 11$

$$rep_{i,1} = f_{jmax_i - 1},\ rep_{i,2} = f_{jmax_i},\ rep_{i,3} = f_{jmax_i + 1}. \tag{32}$$

For the new student $NSL = (nsl_1, nsl_2, nsl_3, nsl_4)$, and each group GS_k, $k = 1, ..., M$ we can define a recommendation error Err_k as follows:

$$err_{k,i} = \begin{cases} 1 & \text{if } nsl_i \notin Rep_i \\ 0 & otherwise \end{cases}, \tag{33}$$

$$Err_k = \sum_{i=1}^{4} err_{k,i}. \tag{34}$$

7 Experiments

The performance of using interval-valued fuzzy sets for recommending student groups has been checked by experiments done on the real data sets. The effectiveness of the proposed method have been evaluated by recommendation error defined by (34) and compared to the results obtained by application of type-1 fuzzy sets presented in [3]. Tests have been conducted for different numbers of groups of different sizes and qualities.

We have considered two different datasets of real students' data represented by their dominant learning styles according to SL model (see (16)). The first set containing data of 194 Computer Science students from different courses has been used for building groups of similar students. The second set has comprised dominant learning styles of students, who were to learn together with their peers from the first dataset and whose data was used for evaluating the performance of the proposed recommendation method. This set consists of 31 data of students studying the master course of Information Systems in Management. The method of collecting learning styles data was described with details in [33].

The groups were created by different clustering techniques to obtain clusters of disparate structures and sizes. There were considered clusters built by three well known algorithms: partitioning - K-means, statistical - EM and hierarchical Farthest First Traversal (FFT) [34]. Such approach allows to examine the considered method for groups of different structures. To enable analysis of the performance of the proposed technique we investigated different data divisions, taking into account 3, 6 and 7 clusters, what enabled differentiating numbers and sizes of groups considered for recommendations. Recommendation accuracy has been measured by considering an error defined by (34). Additionally, every case has been examined to check if there exists better group for recommendation than the suggested one. The detail results of quantitative analysis for different group structures are presented in Table 1. The first two columns present clustering method and the number of clusters. Next columns show respectively the percentage of students of exact match (Err = 0), and of the ones for whom recommendation error was equal to $1, 2, 3, 4$. The results did not show dependency between clustering technique, group sizes and the percentage of properly assigned recommendations. The number of students of exact match was in most of the cases greater than the ones with recommendation error equal to 3. Mostly, recommendation errors take values 1 or 2. The average weighted error belongs to the interval $\langle 1.32; 1.58\rangle$. An error equal to 4 concerned only 2 students, whose characteristics significantly differ from their peers. These students should be considered separately as outliers. Finally, all the students obtained the best group suggestions.

Table 1. Quantitative analysis for different group structures

Schema	Cl. no	Err = 0	Err = 1	Err = 2	Err = 3	Err = 4
KM	3	12.90%	35.48%	32.26%	19.35%	0%
	6	16.13%	41.94%	25.80%	9.68%	6.45%
	7	19.35%	38.71%	35.48%	3.23%	3.23%
EM	3	19.35%	32.26%	29.03%	16.13%	3.23%
	6	19.35%	32.26%	22.58%	22.58%	3.23%
	7	12.90%	41.93%	38.71%	6.45%	0%
FFT	3	19.35%	25.81%	41.93%	12.90%	0%
	6	19.35%	25.81%	38.71%	12.90%	3.23%
	7	19.35%	29.03%	41.94%	9.68%	0%

In the next step recommendation effectiveness, of the considered method (IVFS), was compared to the techniques using traditional fuzzy sets (TFS) to build group representations. Table 2 presents values of the average weighted errors, regarding error values and the respective number of students, for the both of the techniques. In the case of 10 from 12 clustering schemes the average weighted error values are less for recommendations build by using the current method. What is more, in all the considered clustering schemas, 1 to 4 students haven't obtained the best recommendations while traditional fuzzy sets have been applied.

Table 2. Average weighted error of the two methods

Schema	Cl. no	IVFS	TFS
KM	3	1.58	1.61
	6	1.48	1.32
	7	1.32	1.32
EM	3	1.516	1.45
	6	1.58	1.70
	7	1.39	1.45
FFT	3	1.48	1.68
	6	1.55	1.58
	7	1.42	1.48

8 Concluding Remarks

In the paper, fuzzy logic for building group recommendations for students was considered. We use interval valued fuzzy sets to build group representations. The proposed method shows good performance for students described by dominant learning styles. Experiments done for data sets of real students and different group structures showed that for all of the students the system indicated the best possible choice of colleagues to learn together. The comparison to the technique based on traditional fuzzy sets showed the advantage of the proposed method.

Future research will consist in further investigations of the recommendation tool, examination of other attributes and including to recommendations student historical activities as well as making group creating process more dynamic, by adding new learners each time the recommendation is accepted.

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