Confidence Intervals for the Difference Between the Coefficients of Variation of Inverse Gaussian Distributions

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Abstract. The aim of this study is to propose confidence intervals for the difference between the coefficients of variation of inverse Gaussian distributions based on the generalized confidence interval (GCI), the adjusted generalized confidence interval (AGCI), the bootstrap percentile confidence interval (BPCI), and the method of variance estimates recovery (MOVER). The performances of the proposed confidence intervals were evaluated using coverage probabilities and average lengths via Monte Carlo simulation. The results showed that the GCI and AGCI methods were higher than or close to the nominal level in all cases. For small sample sizes, MOVER was better than the other methods because it provided the narrowest average length. The performances of all the approaches were illustrated using two real data examples.

Keywords: Bootstrap · Coefficients of variation · Generalized confidence interval · Inverse Gaussian distribution · Method of variance estimates recovery

1 Introduction

The inverse Gaussian distribution is used to describe and analyze positive and right-skewed data and has been applied in useful applications in a variety of fields, such as cardiology, pharmacokinetics, economics, medicine, and finance. The inverse Gaussian distribution was first derived by Schordinger [1] for the first passage of time of a Wiener process to an absorbing barrier and has been used to describe the cycle time distribution of particles in the blood [2]. Liu et al. [3] used data sets from an inverse Gaussian distribution applied to a lifetime model for reliability analysis. Banerjee and Bhattacharyya [4] applied this distribution in a study of market incidence models, while Lancaster [5] used it as a model for the duration of strikes, and Sheppard [6] proposed applying it for the time duration of injected labeled substances called tracers in a biological system. For more informations and applications, Chhikara and Folks [7], Krishnamoorthy and Lu [8], Tian and Wu [9], Lin et al. [10], and Ye et al. [11].

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The coefficient of variation, which is the ratio of the standard deviation to the mean, can be used to measure data dispersion with non-homogeneous units. It has been used in many fields, such as biology, economics, medicine, agriculture, and finance. Many researchers have proposed confidence intervals for the coefficient of variation. Wongkhao et al. [12] presented confidence intervals for the ratio of two independent coefficients of variation of normal distributions based on the concept of the general confidence interval (GCI) and the method of variance estimates recovery (MOVER). Banik and Kibria [13] estimated the population coefficient of variation and compared it with bootstrap interval estimators. Mahmoudvand and Hassani [14] proposed an unbiased estimator to construct confidence intervals for the population coefficient of variation of normal distribution. Hasan and Krishnamoorthy [15] proposed two new approximate confidence intervals for the ratio of the coefficients of variation of lognormal populations using MOVER and the fiducial confidence intervals method. Sangnawakij et al. [16] proposed two confidence intervals for the ratio of coefficients of variation of gamma distributions based on MOVER with the methods of Score and Wald. Thangjai and Niwitpong [17] constructed confidence intervals for the weighted coefficient of constructed of two-parameter exponential distributions based on the adjusted method of variance estimates recovery method (adjusted MOVER) and compared it with the general confidence interval (GCI) and the large sample method. Yosboonruang et al. [18] proposed confidence intervals for the coefficient of variation for a delta-lognormal distribution based on GCI and the modified Fletcher method.

As mentioned previously, there have been many comprehensive studies on confidence intervals for the difference between the coefficients of variation of inverse Gaussian distributions. However, few researchers have investigated confidence intervals for the parameters of inverse Gaussian distributions. For instance, Ye et al. [11] proposed confidence intervals for the common mean of several inverse Gaussian populations when the scalar parameters are unknown and unequal. Tian and Wilding [19] presented confidence intervals for the ratio of the means of two independent inverse Gaussian distribution. Krishnamoorthy and Tian [20] developed confidence intervals for the difference between and ratio of the means of two inverse Gaussian distributions based on GCI. The purpose of the current study is to establish new confidence intervals for the difference between the coefficients of variation of inverse Gaussian distributions based on GCI, the adjusted generalized confidence interval (AGCI), the bootstrap percentile confidence interval (BPCI), and MOVER.

The organization of this paper are as follows. Section 2 provides preliminaries for the difference between the coefficients of variation of inverse Gaussian distributions. Simulation studies are presented in Sect. 3. Section 4 presents empirical studies. Finally, concluding remarks are summarized in Sect. 5.

2 Confidence Intervals for the Difference Between the Coefficients of Variation of Inverse Gaussian Distributions

Let X = (X_1, X_2,..., X_n) be an independent random sample of size n from the two-parameter inverse Gaussian distribution, IG(μ, λ), is defined as

$$f(x,\mu,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{\frac{1}{2}} exp\left\{-\frac{\lambda\,(x-\mu)^2}{2\mu^2 x}\right\}, x>0, \mu>0, \lambda>0, \tag{1}$$

where μ and λ are the mean parameter and the scale parameter. The population mean and variance of X are define as

$$E(X) = \mu \tag{2}$$

and

$$Var(X) = \mu^3/\lambda. \tag{3}$$

Then, the coefficient of variation of X is expressed by

$$\omega = CV(X) = \sqrt{\frac{\mu}{\lambda}} \tag{4}$$

and the difference between of coefficients of variation is defined as

$$\eta = \omega_X - \omega_Y = \sqrt{\frac{\mu_X}{\lambda_X}} - \sqrt{\frac{\mu_Y}{\lambda_Y}}. \tag{5}$$

2.1 The Generalized Confidence Interval (GCI) Method

Weerahandi [21] introduced the generalized confidence interval (GCI) method. The concept of this method is to the definition of generalized pivotal quantity (GPQ). Suppose that X is a random sample from a distribution having the parameter (θ,δ), where θ is the parameter of interest and δ is the nuisance parameter.

Definition 1. *Let* $X = (X_1, X_2, ..., X_n)$ *be the observed value of* X *and the probability density function of* $(X; x, \theta, \delta)$ *is* $R(X; x, \theta, \delta)$*. The generalized pivotal quantity* $R(X; x, \theta, \delta)$ *satisfies the following two properties:*

(a) The probability distribution of the function $R(X; x, \theta, \delta)$ *is independent of unknown parameters.*
(b) The observed value of $R(X; x, \theta, \delta)$*,* $X = x$*, does not depend on nuisance parameters.*

Therefore, the $100\,(1-\alpha)\,\%$ two-sided GCI for the parameter of interest is defined as $[R(\alpha/2), R(1-\alpha/2)]$, where $R(\alpha/2)$ and $R(1-\alpha/2)$ are the $100(\alpha/2)-th$ and $100(1-\alpha/2)-th$ percentile of $R(X; x, \theta, \delta)$.

Ye et al. [11] developed the generalized confidence interval based on general pivotal quantities for the mean and scale parameter of the inverse Gaussian distribution. Suppose that k is independent populations of the inverse Gaussian distributions with mean parameters μ_i and scale parameters λ_i, $i = 1, 2, .., k$. Let $X_{i1}, X_{i2}, ..., X_{in_i}$ be the random sample from $IG(\mu_i, \lambda_i)$, $i = 1, ..., k$. Form the ith population, the maximum likelihood estimators (MLEs) of μ_i and λ_i can be found as

$$\hat{\mu} = \bar{X}_i, \qquad \hat{\lambda}_i^{-1} = \frac{1}{n_i}\sum_{j=1}^{n_i}(X_{ij}^{-1} - \bar{X}_i^{-1}), \tag{6}$$

where $\bar{X}_i = \sum_{j=1}^{n_i} X_{ij}/n_i$. Let $V_i = \hat{\lambda}_i^{-1}$, it is well known that $\bar{X}_i$ and V_i are mutually independent random variables. Note that

$$\bar{X}_i \sim IG(\mu, n_i\lambda_i), \qquad n_i\lambda_i V_i \sim \chi^2_{n_{i-1}}, i = 1, .., k, \tag{7}$$

where χ^2_m denotes as the Chi-square distribution with m degrees of freedom. It is easily proved that $(\bar{X}_i, V_i)$ forms a set complete sufficient statistics for (μ_i, λ_i). From Eq. (7), the generalized pivotal quantity R_{λ_i} for λ_i based on the ith sample is defined as

$$R_{\lambda_i} = \frac{n_i\lambda_i V_i}{n_i v_i} \sim \frac{\chi^2_{n_{i-1}}}{n_i v_i}, i = 1, ..., k, \tag{8}$$

where v_i denotes as the observed value of V_i. The distribution of R_{λ_i} is free of any unknown parameters and the observed values only relate to the parameter λ_i. Therefore, R_{λ_i} is the generalized pivotal quantity forλ_i.

According to Ye et al. [11], the generalized pivotal quantity R_{μ_i} for μ_i based on the ith sample is defined as

$$R_{\mu_i} = \frac{\bar{x}_i}{|1 + \frac{\sqrt{n_i\lambda_i}(\bar{x}_i - \mu)}{\mu\sqrt{\bar{x}_i}}\sqrt{\frac{\bar{x}_i}{n_i R_{\lambda_i}}}|} \sim \frac{\bar{x}_i}{\left|1 + Z_i\sqrt{\frac{\bar{x}_i}{n_i R_{\lambda_i}}}\right|}, \tag{9}$$

where $\sim$ denotes as "approximately distributed" and $Z_i \sim N(0, 1)$. $\bar{x}_i$ and v_i are the observed values of $\bar{X}_i$ and V_i. The approximations in Eq. (9) are derived by using Theorem 2.1 given by Chhikara and Folks [7]. Using the moment matching method, it can be shown that$\sqrt{n_i\lambda_i}(\bar{X}_i - \mu)/\mu\sqrt{\bar{x}_i}$ has a limiting distribution of $Z_i \sim N(0, 1)$. Note that the observed value of R_{μ_i} is μ_i. Therefore, R_{μ_i} satisfies conditions (a) and (b) in Definition 1. However, R_{μ_i} is an approximate generalized pivotal quantity for μ_i based on the ith sample.

Therefore, the generalized pivotal quantities for difference between the coefficients of variation are given by

$$R_\eta = R_{\omega_X} - R_{\omega_Y} = \sqrt{\frac{R_{\mu_X}}{R_{\lambda_X}}} - \sqrt{\frac{R_{\mu_Y}}{R_{\lambda_Y}}}. \tag{10}$$

Then, the $100(1-\alpha)\%$ two-sided confident interval for the difference between the coefficients of variation based on generalized confidence interval method is given by

$$CI_{(GCI)} = (L_{(GCI)}, U_{(GCI)}) = (R_\eta(\alpha/2), R_\eta(1-\alpha/2)), \tag{11}$$

where $R_\eta(\alpha/2)$ and $R_\eta(1-\alpha/2)$ are the $100(\alpha/2)\%$ and $100(1-\alpha/2)\%$ percentiles of the distribution of $R = R(X; x, \theta, \delta)$, respectively.

The following algorithm is used to construct the generalized confidence interval:

Algorithm 1.

Step 1. Generate x_j and y_j , $j = 1, 2, ..., n_i$ from the inverse Gaussian distribution.

Step 2. Compute $\bar{x}_j$, $\bar{y_j}$, $\hat{v}_{x_j}$ and $\hat{v}_{y_j}$.

Step 3. For $t = 1$ to T.

Step 4. Generate $\chi^2_{n_i-1}$ from chi-square distribution and $Z \sim N(0, 1)$.

Step 5. Compute R_{λ_i} from Eq. (8).

Step 6. Compute R_{μ_i} from Eq. (9).

Step 7. Compute R_η from Eq. (10).

Step 8. End t loop.

Step 9. Compute $R_\eta(\alpha/2)$ and $R_\eta(1-\alpha/2)$.

2.2 The Adjusted Generalized Confidence Interval (AGCI) Method

According to Ye et al. [11], We can use a similar method in GCI method for calculating the difference between the coefficients of variation η. $R_{\tilde{\lambda}_i}$ can be computed by the generalized pivotal quantity for $\tilde{\lambda}$ based on ith sample shown in Eq. (8). Then, Krishnomoorthy and Tian [20] presented an approximate generalized pivotal quantity for $R_{\tilde{\mu}_i}$ based on the ith sample, as follows:

$$R_{\tilde{\mu}_i} = \frac{\bar{x}_i}{\max\left\{0, t_{n_i-1}\sqrt{\frac{\bar{x}_i v_i}{n_i-1}}\right\}}, \tag{12}$$

where t_{n_i-1} denotes the t distribution with $n_i - 1$ degrees of freedom. Therefore, the denominator in Eq. (12) may be zero when t_{n_i-1} obtains the negative value. $R_{\tilde{\mu}_i}$ is an approximate generalized pivotal quantity.

Therefore, the generalized pivotal quantities for the difference between the coefficients of variation are given by

$$R_{\tilde{\eta}} = R_{\tilde{\omega}_X} - R_{\tilde{\omega}_Y} = \sqrt{\frac{R_{\tilde{\mu}_X}}{R_{\tilde{\lambda}_X}}} - \sqrt{\frac{R_{\tilde{\mu}_Y}}{R_{\tilde{\lambda}_Y}}}. \tag{13}$$

Then, the $100(1-\alpha)\%$ two-sided confident interval for the difference between the coefficients of variation based on generalized confidence interval method is given by

$$CI_{(AGCI)} = (L_{(AGCI)}, U_{(AGCI)}) = (R_{\tilde{\eta}}(\alpha/2), R_{\tilde{\eta}}(1-\alpha/2)), \tag{14}$$

where $R_{\tilde{\eta}}(\alpha/2)$ and $R_{\tilde{\eta}}(1-\alpha/2)$ which are the $100(\alpha/2)\%$ and $100(1-\alpha/2)\%$ percentiles of the distribution of $R = R(X; x, \theta, \delta)$ can be obtained from the concept of Algorithm 1.

2.3 The Bootstrap Percentile Confidence Interval (BPCI) Method

Efron and Tibshirani [22] introduced the bootstrap percentile method. The bootstrap is a re-sampling method for assigning measures of accuracy to statistical estimate a random selection of resamples from the original sample with replacement. Let x be a random sample of size n from the inverse Gaussian distribution. Suppose that $x = x_1, x_2, ..., x_n$ is a random sample of size n from the inverse Gaussian distribution. Sampling is replaced by $x^* = x_1^*, x_2^*, ..., x_n^*$, which can be obtained by the bootstrap sample with B times. When the re-sampling bootstrap sample is operated, the difference between coefficients of variation is then calculated.

The $100(1-\alpha)\%$ two-sided confidence interval for the difference between the coefficients of variation based on the bootstrap percentile confidence interval is defined by

$$CI_{(BPCI)} = (L_{(BPCI)}, U_{(BPCI)}) = (\eta^*(\alpha/2), \eta^*(1-\alpha/2)), \tag{15}$$

where $\eta^*(\alpha/2)$ and $\eta^*(1-\alpha/2)$ are the $100(\alpha/2)\%$ and $100(1-\alpha/2)\%$ percentiles of the distribution.

Algorithm 2.

Step 1. Generate$X_1, X_2, ...X_n$ from the inverse Gaussian distribution

Step 2. Obtain a bootstrap sample $X^* = X_1^*, X_2^*, ..., X_n^*$ from Step 1.

Step 3. Compute η^*

Step 4. Repeat Steps 2 and 3, B times.

Step 5. Compute $\eta^*(\alpha/2)$ and $\eta^*(1-\alpha/2)$

2.4 The Method of Variance Estimates Recovery (MOVER)

Gulhar et al. [23] proposed the confidence interval for a coefficient of variation of X which is

$$(l_x, u_x) = \left(\frac{\sqrt{n-1}(\hat{\omega_x})}{\sqrt{\chi^2_{1-\alpha/2,n-1}}}, \frac{\sqrt{n-1}(\hat{\omega_x})}{\sqrt{\chi^2_{\alpha/2,n-1}}} \right) \tag{16}$$

and the confidence interval for a coefficient of variation of Y is defined as

$$(l_y, u_y) = \left(\frac{\sqrt{n-1}(\hat{\omega_y})}{\sqrt{\chi^2_{1-\alpha/2,n-1}}}, \frac{\sqrt{n-1}(\hat{\omega_y})}{\sqrt{\chi^2_{\alpha/2,n-1}}} \right), \tag{17}$$

where $\chi^2_{1-\alpha/2,n-1}$ and $\chi^2_{\alpha/2,n-1}$ are respectively the $100(\alpha)\%$ -th and $100(1-\alpha)\%$ -th percentile of the chi-square distribution with $n-1$ degrees of freedom.

Donner and Zou [24] introduced the confidence interval estimation for the difference of parameters of interest using by MOVER. The lower limit and upper limit are given by

$$L_\eta = \hat{\omega}_x - \hat{\omega}_y - \sqrt{(\hat{\omega}_x - l_x)^2 + (u_y - \hat{\omega}_y)^2} \tag{18}$$

and

$$U_\eta = \hat{\omega}_x - \hat{\omega}_y - \sqrt{(u_x - \hat{\omega}_x)^2 + (\hat{\omega}_y - l_y)^2}, \tag{19}$$

where $\hat{\omega}_x$ and $\hat{\omega}_y$ are denoted in Eq. (5), l_x and u_x are denoted in Eq. (16), and l_y and u_y are denoted in Eq. (17).

Then, the $100(1-\alpha)\%$ two-sides confidence interval for the difference between the coefficients of variation of inverse Gaussian distribution based on the MOVER is given by

$$CI_{MOVER} = (L_{MOVER}, U_{MOVER}) = (L_\eta, U_\eta), \tag{20}$$

where L_η and U_η are defined in Eqs. (18) and (19), respectively.

3 Simulation Studies

A Monte Carlo simulation studies out to evaluate the coverage probabilities and average lengths of the confidence intervals for the difference between the coefficients of variation of inverse Gaussian distributions based on GCI, AGCI, BPCI, and MOVER. The simulations were run by using R statistics programming language. In the simulation, The sample sizes were (n_x, n_y) = (5, 5), (5, 10), (10, 10), (10, 30), (30, 30), (30, 50), (50, 50), (50, 100), and (100, 100); $\mu_x = 0.5$, $\mu_y = 0.5, 1$, $\lambda_x = 10$ and $\lambda_y = 1, 2, 5, 10$. The nominal confidence level is at 0.95. The number of simulation replications for each situation was 10,000 replications, 1,000 bootstrap samples and 5,000 pivotal quantities for GCI and AGCI. The confidence interval which has the coverage probability was greater than or close to the nominal confidence level and the shortest expected lengths are chosen.

The estimated coverage probability and estimated average length for this simulation study are respectively given as:

$$CP = \frac{c(L_{(g)} \leq \eta \leq U_{(g)})}{M}, \qquad AL = \frac{\sum_{i=1}^{m}(U_{(g)} - L_{(g)})}{M}, \tag{21}$$

where M is the number of simulation replications, and $c(L_{(g)} \leq \eta \leq U_{(g)})$ is the numbers of simulation replications for η which lies within the confidence interval.

The following algorithm is used to construct the coverage probability for the difference between the coefficients of variation:

Algorithm 3.

Step 1. For a given $M, m, n_1, n_2, \mu_x, \mu_y, \lambda_x$, and λ_y.

Step 2. For $g = 1$ to M.

Step 3. Generate $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_n$ from inverse Gaussian distribution.

Step 4. Use Algorithm 1, Eq. (14), Algorithm 2, and Eq. (20) to construct lower and upper limits for GCI, AGCI, BPCI, and MOVER, respectively.

Step 5. If $(L_{(g)} \leq \eta \leq U_{(g)})$, set $P_{(g)} = 1$; else set $P_{(g)} = 0$.

Step 6. Calculate $U_{(g)} - L_{(g)}$.

Step 7. End g loop.

Step 8: Compute the coverage probability and the average length.

The results in Table 1 showed that the difference between the coefficients of variation, the coverage probabilities of GCI method and AGCI method were greater than or equal to the nominal level in all cases. However, the average lengths of the AGCI method were shorter than the GCI method. For small sample sizes, the coverage probabilities of the MOVER were close to the nominal confidence level of 0.95 and the shortest average length.

4 An Empirical Study

Herein, we illustrate the methods used to computation of confidence intervals proposed.

Example 1: The data were given by Mudholkar and Hutson [25] for the consecutive annual flood discharge rates of the Floyd river at James, Iowa. The data are as follows:

In 1935−1944 : 1460, 4050, 3570, 2060, 1300, 1390, 1720, 6280, 1360, 7440.

In 1945−1954 : 5320, 1400, 3240, 2710, 4520, 4840, 8320, 13900, 71500, 6250.

The summary statistics of data are $n_1 = 10$, $n_2 = 10$, $\hat{\mu}_1 = 3063$, $\hat{\mu}_2 = 12200$, $\hat{\lambda}_1 = 6529.078$, $\hat{\lambda}_2 = 6434.176$, $\hat{\omega}_1 = 0.6849$, $\hat{\omega}_2 = 1.3770$, and the difference between the coefficients of variation $\hat{\eta} = -0.6921$. Based on the 95% two sided confidence interval for the difference between the coefficients of variation using GCI method was (−0.0051, 0.0148) with interval length of 0.0199; AGCI method was (−0.0045, 0.0144) with interval length of 0.0189; BPCI method was (−1.4848, 0.3342) with interval length of 1.8189, and MOVER was (−1.8489, 0.0182) with interval length of 1.8667. Therefore, the results confirm that the simulation results for difference between the coefficients of variation are not different from the results of the previous study.

Example 2: The real data were provided by Eilam et al. [26]. The plasma bradykininogen levels were measured in healthy subjects, in patients with active Hodgkin's disease and in patients with inactive Hodgkin's disease. The outcome variable is measured in micrograms of bradykininogen per milliliter of plasma. The data are as follows:

Active Hodgkin's disease: 3.96, 3.04, 5.28, 3.40, 4.10, 3.61, 6.16, 3.22, 7.48, 3.87, 4.27, 4.05, 2.40, 5.81, 4.29, 2.77, 4.40.

Inactive Hodgkin's disease: 5.37, 10.60, 5.02, 14.30, 9.90, 4.27, 5.75, 5.03, 5.74, 7.85, 6.82, 7.90, 8.36, 5.72, 6.00, 4.75, 5.83, 7.30, 7.52, 5.32, 6.05, 5.68, 7.57, 5.68, 8.91, 5.39, 4.40, 7.13.

The summary statistics of data are $n_1 = 17$, $n_2 = 28$, $\hat{\mu}_1 = 4.2418$, $\hat{\mu}_2 = 6.7914$, $\hat{\lambda}_1 = 50.8394$, $\hat{\lambda}_2 = 86.0289$, $\hat{\omega}_1 = 0.2889$, $\hat{\omega}_2 = 0.2810$, and the difference between the coefficients of variation $\hat{\eta} = 0.0079$. Based on the 95% two sided confidence interval for the difference between the coefficients of

Table 1. The coverage probability (CP) and average length (AL) of 95% two-sided confidence intervals for the difference between the coefficients of variation of inverse Gaussian distribution: $(\mu_X, \lambda_x) = (0.5, 10)$.

n_x	n_y	(μ_y, λ_y)	Coverage Probability (Average Length)			
			CI_{GCI}	CI_{AGCI}	CI_{BPCI}	CI_{MOVER}
5	5	(0.5, 1)	0.9710 (2.777)	**0.9518 (1.6723)**	0.6611 (0.6904)	0.9423 (1.5387)
		(0.5, 2)	0.9669 (1.7457)	0.9552 (1.2727)	0.7599 (0.5161)	**0.9555 (1.1874)**
		(0.5, 5)	0.9570 (1.0560)	0.9514 (0.9264)	0.9055 (0.3696)	**0.9590 (0.8749)**
		(0.5, 10)	0.9547 (0.8214)	0.9518 (0.7605)	0.9654 (0.3021)	**0.9593 (0.7204)**
		(1, 1)	0.9837 (4.4063)	**0.9551 (2.2717)**	0.5931 (0.9684)	0.9310 (2.0541)
		(1, 2)	0.9744 (2.7700)	**0.9568 (1.6717)**	0.6578 (0.6907)	0.9498 (1.5356)
		(1, 5)	0.9641 (1.5205)	0.9556 (1.1721)	0.7907 (0.4728)	**0.9577 (1.1003)**
		(1, 10)	0.9583 (1.0541)	0.9537 (0.9252)	0.9059 (0.3692)	**0.9594 (0.8749)**
5	10	(0.5, 1)	0.9769 (1.3175)	**0.9585 (0.9814)**	0.8482 (0.5851)	0.9460 (0.9528)
		(0.5, 2)	0.9667 (0.9175)	0.9553 (0.7973)	0.9129 (0.4353)	**0.9543 (0.7717)**
		(0.5, 5)	0.9550 (0.6986)	0.9504 (0.6475)	0.9363 (0.3190)	**0.9550 (0.6205)**
		(0.5, 10)	0.9515 (0.6173)	0.9484 (0.5787)	0.8641 (0.2690)	**0.9537 (0.5475)**
		(1, 1)	0.9874 (2.3690)	**0.9566 (1.2685)**	0.7904 (0.8328)	0.9285 (1.2216)
		(1, 2)	0.9777 (1.3214)	**0.9564 (0.9850)**	0.8527 (0.5881)	0.9444 (0.9568)
		(1, 5)	0.9620 (0.8517)	0.9511 (0.7577)	0.9288 (0.4014)	**0.9528 (0.7325)**
		(1, 10)	0.9513 (0.6978)	0.9462 (0.6465)	0.9270 (0.3190)	**0.9515 (0.6190)**
10	10	(0.5, 1)	0.9758 (1.1737)	**0.9532 (0.8416)**	0.7937 (0.5659)	0.9376 (0.8096)
		(0.5, 2)	0.9682 (0.7317)	0.9533 (0.6285)	0.8427 (0.4098)	**0.9506 (0.6115)**
		(0.5, 5)	0.9601 (0.4817)	0.9548 (0.4536)	0.9059 (0.2862)	**0.9554 (0.4450)**
		(0.5, 10)	0.9568 (0.3863)	0.9523 (0.3723)	0.9307 (0.2340)	**0.9559 (0.3662)**
		(1, 1)	0.9867 (2.2265)	**0.9502 (1.1421)**	0.7431) (0.8128)	0.9103 (1.0868)
		(1, 2)	0.9773 (1.1674)	**0.9505 (0.8379)**	0.7827 (0.5666)	0.9341 (0.5081)
		(1, 5)	0.9670 (0.6529)	0.9559 (0.5792)	0.8695 (0.3726)	**0.9534 (0.5649)**
		(1, 10)	0.9583 (0.4819)	0.9510 (0.4536)	0.9054 (0.2868)	**0.9538 (0.4451)**
10	30	(0.5, 1)	0.9783 (0.5631)	**0.9535 (0.4708)**	0.9251 (0.3991)	0.9429 (0.4678)
		(0.5, 2)	0.9654 (0.4173)	**0.9501 (0.3824)**	0.9300 (0.2985)	0.9466 (0.3739)
		(0.5, 5)	0.9548 (0.3269)	0.9490 (0.3133)	0.8996 (0.2240)	**0.9501 (0.3075)**
		(0.5, 10)	0.9551 (0.2932)	**0.9505 (0.2835)**	0.8548 (0.1944)	0.9497 (0.2758)
		(1, 1)	0.9908 (0.8725)	**0.9550 0.6089)**	0.8971 (0.5749)	0.9145 (0.6042)
		(1, 2)	0.9765 (0.5634)	**0.9518 (0.4709)**	0.9178 (0.3996)	0.9352 (0.4686)
		(1, 5)	**0.9599 (0.3864)**	0.9489 (0.3602)	0.9293 (0.2747)	0.9482 (0.3565)
		(1, 10)	0.9575 (0.3276)	0.9489 (0.3140)	0.9075 (0.2243)	**0.9527 (0.3082)**
30	30	(0.5, 1)	0.9813 (0.5018)	**0.9532 (0.4053)**	0.8842 (0.3759)	0.9295 (0.4002)
		(0.5, 2)	0.9691 (0.3357)	**0.9502 (0.3011)**	0.9063 (0.2673)	0.9415 (0.2986)
		(0.5, 5)	0.9606 (0.2237)	0.9531 (0.2140)	0.9247 (0.1848)	**0.9505(0.2132)**
		(0.5, 10)	0.9569 (0.1802)	0.9520 (0.1754)	0.9344 (0.1499)	**0.9500 (0.1749)**
		(1, 1)	0.9898 (0.8297)	**0.9514 (0.5579)**	0.8655 (0.1500)	0.9493 (0.1749)
		(1, 2)	0.9801 (0.5023)	**0.9501 (0.4056)**	0.8872 (0.3750)	0.9296 (0.4004)
		(1, 5)	0.9670 (0.2997)	**0.9525 (0.2749)**	0.9124 (0.2416)	0.9450 (0.2729)
		(1, 10)	0.9588 (0.2242)	0.9529 (0.2144)	0.9262 (0.1853)	**0.9508 (0.2136)**

(*continued*)

Table 1. (*continued*)

n_x	n_y	(μ_y, λ_y)	Coverage Probability (Average Length)			
			CI_{GCI}	CI_{AGCI}	CI_{BPCI}	CI_{MOVER}
30	50	(0.5, 1)	0.9785 (0.3824)	**0.9534 (0.3156)**	0.9213 (0.3105)	0.9265 (0.3141)
		(0.5, 2)	0.9699 (0.2652)	**0.9537 (0.2409)**	0.9355 (0.2239)	0.9469 (0.2402)
		(0.5, 5)	0.9618 (0.1880)	0.9519 (0.1806)	0.9386 (0.1610)	**0.9536 (0.1802)**
		(0.5, 10)	**0.9505 (0.1585)**	0.9458 (0.1544)	0.9238 (0.1354)	0.9438 (0.1537)
		(1, 1)	0.9911 (0.6119)	**0.9506 (0.4278)**	0.9050 (0.4548)	0.9003 (0.4242)
		(1, 2)	0.9798 (0.3832)	**0.9513 (0.3161)**	0.9188 (0.3102)	0.9278 (0.3147)
		(1, 5)	0.9651 (0.2399)	**0.9510 (0.2224)**	0.9329 (0.2044)	0.9446 (0.2221)
		(1, 10)	**0.9553 (0.1871)**	0.9476 (0.1798)	0.9328 (0.1607)	0.9451 (0.1794)
50	50	(0.5, 1)	0.9803 (0.3729)	**0.9525 (0.3729)**	0.9088 (0.3019)	0.9235 (0.3021)
		(0.5, 2)	0.9700 (0.2499)	**0.9511 (0.2253)**	0.9193 (0.2136)	0.9390 (0.2242)
		(0.5, 5)	0.9601 (0.1671)	**0.9510 (0.1601)**	0.9358 (0.1457)	0.9478 (0.1598)
		(0.5, 10)	0.9570 (0.1344)	**0.9517 (0.1310)**	0.9401 (0.1199)	0.9478 (0.1308)
		(1, 1)	0.9933 (0.6070)	**0.9522 (0.4201)**	0.9001 (0.4527)	0.8994 (0.4159)
		(1, 2)	0.9816 (0.3720)	**0.9508 (0.3040)**	0.9110 (0.3027)	0.9235 (0.3017)
		(1, 5)	0.9681 (0.2241)	**0.9538 (0.2063)**	0.9255 (0.1938)	0.9421 (0.2054)
		(1, 10)	0.9608 (0.1671)	**0.9527 (0.1602)**	0.9363 (0.1478)	0.9486 (0.1599)
50	100	(0.5, 1)	0.9801 (0.2650)	**0.9516 (0.2209)**	0.9344 (0.2281)	0.9278 (0.2204)
		(0.5, 2)	**0.9670 (0.1855)**	0.9478 (0.1695)	0.9372 (0.1659)	0.9377 (0.1694)
		(0.5, 5)	0.9616 (0.1342)	0.9548 (0.1293)	0.9433 (0.1219)	**0.9501 (0.1291)**
		(0.5, 10)	**0.9548 (0.1150)**	0.9488 (0.1121)	0.9297 (0.1041)	0.9474 (0.1118)
		(1, 1)	0.9926 (0.4175)	**0.9552 (0.2979)**	0.9283 (0.3369)	0.9009 (0.2971)
		(1, 2)	0.9811 (0.2648)	**0.9511 (0.2209)**	0.9377 (0.2281)	0.9296 (0.2205)
		(1, 5)	0.9655 (0.1689)	**0.9533 (0.1573)**	0.9424 (0.1522)	0.9448 (0.1572)
		(1, 10)	0.9581 (0.1341)	**0.9507 (0.1292)**	0.9391 (0.1218)	0.9481 (0.1290)
100	100	(0.5, 1)	0.9823 (0.2555)	**0.9514 (0.2102)**	0.9276 (0.2208)	0.9249 (0.2094)
		(0.5, 2)	0.9721 (0.1721)	**0.9551 (0.1556)**	0.9391 (0.1554)	0.9441 (0.1552)
		(0.5, 5)	0.9596 (0.1148)	**0.9518 (0.1102)**	0.9404 (0.1068)	0.9463 (0.1101)
		(0.5, 10)	0.9563 (0.0923)	**0.9510 (0.0900)**	0.9428 (0.0869)	0.9487 (0.0899)
		(1, 1)	0.9931 (0.4113)	**0.9504 (0.2899)**	0.9182 (0.3302)	0.8932 (0.2883)
		(1, 2)	0.9840 (0.2554)	**0.9536 (0.2101)**	0.9270 (0.2205)	0.9239 (0.2095)
		(1, 5)	0.9673 (0.1540)	**0.9523 (0.1421)**	0.9399 (0.1405)	0.9425 (0.1418)
		(1, 10)	0.9599 (0.1190)	**0.9515 (0.1103)**	0.9410 (0.1068)	0.9474 (0.1101)

variation, the results showed that GCI method $CI_{GCI} = (0.0188, 0.1497)$ with interval length of 0.1309, AGCI method $CI_{AGCI} = (0.0213, 0.1447)$ with interval length of 0.1234, BPCI method $CI_{BPCI} = (-0.1245, 0.1274)$ with interval length of 0.2519 and MOVER method $CI_{MOVER} = (0.1697, 0.2873)$ with interval length of 0.2873. Therefore, the results from above examples support our simulation results.

5 Conclusions

The new confidence intervals for the difference between the coefficients of variation of inverse Gaussian distributions based on GCI, AGCI, BPCI, and MOVER were presented. The performances of these confidence intervals were assessed in terms of their coverage probabilities and the average lengths. The results obtained from the GCI and the AGCI methods were satisfactory in all cases.

However, the AGCI method was better than the GCI method in terms of the average length. For small sample sizes, MOVER provided the shortest average length. Meanwhile, based on the findings of this study, BPCI is not recommended because the coverage probabilities were under the nominal confidence level in all cases.

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References

1. Schrödinger, E.: Zür theorie der fall-und steigversuche an teilchen mit brownscer bewegung. Physikaliche Zeitschrift **16**, 289–295 (1915)
2. Wise, M.E.: Skew distributions in biomedicine including some with negative powers of time. In: Patil, G.P., Kotz, S., Ord, J.K. (eds.) A Modern Course on Statistical Distributions in Scientific Work. NATO Advanced Study Institutes Series (Series C – Mathematical and Physical Sciences), vol. 17, pp. 241–262. Springer, Dordrecht (1975). https://doi.org/10.1007/978-94-010-1845-6_18
3. Liu, X., Li, N., Hu, Y.: Combining inferences on the common mean of several inverse Gaussian distributions based on confidence distribution. Stat. Probab. Lett. **105**, 136–142 (2015)
4. Banerjee, A.K., Bhattacharyya, G.K.: Bayesian result for the inverse Gaussian distribution with an application. Technometrics **21**(2), 247–251 (1979)
5. Lancaster, T.: A stochastic model for the duration of a strike. J. Roy. Stat. Soc. Ser. A (General) **135**(2), 257–271 (1972)
6. Sheppard, C.W.: Basic Principles of the Tracer Method. Wiley, New York (1962)
7. Chhikara, R.S., Folks, J.L.: The Inverse Gaussian Distribution. Marcel Dekker, New York (1989)
8. Krishnamoorthy, K., Lu, Y.: Inference on the common mean of several normal populations based on the generalized variable method. Biometrics **59**(2), 237–247 (2003)
9. Tian, L., Wu, J.: Inferences on the mean response in a log-regression model: the generalized variable approach. Stat. Med. **26**, 5180–5188 (2007)
10. Lin, S.H., Lee, J.C., Wang, R.S.: Generalized inferences on the common mean vector of several multivariate normal populations. J. Stat. Plan. Infer. **137**(7), 2240–2249 (2007)
11. Ye, R.D., Ma, T.F., Wang, S.G.: Inference on the common mean of several inverse Gaussian populations. Comput. Stat. Data Anal. **54**, 906–915 (2010)
12. Wongkhao, A., Niwitpong, S., Niwitpong, S.: Confidence intervals for the ratio of two independent coefficients of variation of normal distribution. Far East J. Math. Sci. **98**(6), 741–757 (2015)
13. Banik, S., Kibria, B.M.G.: Estimating the population coefficient of variation by confidence intervals. Commun. Stat. Simul. Comput. **40**, 1236–1261 (2011)
14. Mahmoudvand, R., Hassani, H.: Two new confidence intervals for the coefficient of variation in a normal distribution. J. Appl. Stat. **36**(4), 429–442 (2009)
15. Hasan, M.S., Krishnamoorthy, K.: Improved confidence intervals for the ratio of coefficients of variation of two lognormal distributions. J. Stat. Theory Appl. **16**(3), 345–353 (2017)

16. Sangnawakij, P., Niwitpong, S.-A., Niwitpong, S.: Confidence intervals for the ratio of coefficients of variation of the gamma distributions. In: Huynh, V.-N., Inuiguchi, M., Denoeux, T. (eds.) IUKM 2015. LNCS (LNAI), vol. 9376, pp. 193–203. Springer, Cham (2015). https://doi.org/10.1007/978-3-319-25135-6_19
17. Thangjai, W., Niwitpong, S.: Confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions. Cogent Math. **4**, 1315880 (2017). https://doi.org/10.1080/23311835.2017.1315880
18. Yosboonruang, N., Niwitpong, S.-A., Niwitpong, S.: Confidence intervals for the coefficient of variation of the delta-lognormal distribution. In: Anh, L.H., Dong, L.S., Kreinovich, V., Thach, N.N. (eds.) ECONVN 2018. SCI, vol. 760, pp. 327–337. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-73150-6_26
19. Tian, L., Wilding, G.E.: Confidence intervals of the ratio of means of two independent inverse gaussian distributions. J. Stat. Plan. Infer. **133**(2), 381–386 (2005)
20. Krishnamoorthy, K., Tian, L.: Inferences on the difference and ratio of the means of two inverse Gaussian distributions. J. Stat. Plan. Infer. **133**, 381–386 (2008)
21. Weerahandi, S.: Generalized confidence intervals. J. Am. Stat. Assoc. **88**(423), 899–905 (1993)
22. Efron, B., Tibshirani, R.J.: An Introduction to Bootstrap. Chapman & Hall/CRC, Boca Raton (1993)
23. Gulhar, M., Golam Kibria, B.M., Albatineh, A.N., Ahmed, N.U.: A comparison of some confidence intervals for estimating the population coefficient of variation: a simulation study. Stat. Oper. Res. Trans. **36**, 45–68 (2012)
24. Donner, A., Zou, G.Y.: Closed-form confidence intervals for functions of the normal mean and standard deviation. Stat. Methods Med. Res. **21**, 347–359 (2010)
25. Mudholkar, G.S., Hutson, A.: The exponentiated Weibull family: some properties and a flood data application. Commun. Stat. Theory Method **25**(12), 3059–3083 (1996)
26. Eilam, N., Johnson, P.K., Creger, W.P.: Bradykininogen levels in Hodgkin's disease. Cancer **22**, 631–634 (1968)