Chapter 7
Learning to Teach Mathematics: How Secondary Prospective Teachers Describe the Different Beliefs and Practices of Their Mathematics Teacher Educators

Margaret Marshman

In secondary initial teacher education, mathematics teacher educators (MTEs) generally include mathematicians who teach the mathematical content courses and mathematics educators who teach the mathematics curriculum and pedagogy courses. In part because of this, mathematics teaching and learning in schools is usually different from mathematics in university. We also know that the way in which teachers teach is influenced by their beliefs. Prospective teachers' beliefs are influenced by their previous experiences of learning mathematics as a school student, the MTEs who teach them, the curriculum documents that they study, and their practicum experiences. This chapter begins by defining beliefs and then reviews the literature of beliefs about mathematics and its teaching and learning. The beliefs about mathematics, and mathematics teaching and learning of Australian MTEs and secondary mathematics prospective teachers are documented. The chapter explores how prospective teachers negotiate the different beliefs and practices of their MTEs and the impacts of this on the ways in which they plan to teach. This chapter reports on a study in which MTEs and prospective teachers were initially surveyed about their beliefs about mathematics and mathematics teaching and learning. Follow-up interviews further explored MTEs' beliefs, their decision-making about the pedagogy they used, the links between their practices of mathematics and their teaching, and the links with the practices of mathematics in schools. Interviews with the prospective teachers asked about how they were taught mathematics, how they were taught to teach mathematics, and how they negotiated any differences between the way they were taught mathematics and the way they were taught to teach mathematics.

There is a growing body of research on the influence of teachers' beliefs on their teaching practice and how these beliefs influence their students' beliefs about

M. Marshman (✉)
School of Education, University of the Sunshine Coast, Sippy Downs, QLD, Australia
e-mail: mmarshma@usc.edu.au

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mathematics and their capability to learn mathematics (Grootenboer, 2008; McLeod, 1992; Mosvold & Fauskanger, 2014; Pajares, 1992). Despite this, there is not a clear definition of the concept of beliefs (e.g. Pajares, 1992). Generally, beliefs are seen as personally held assumptions which predispose the person to a particular type of action (Rokeach, 1968). Philipp (2007) defined beliefs as:

> psychologically held understandings, premises, or propositions about the world that are thought to be true. … Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions towards action. (p. 259)

Here beliefs will be used in the same way as Ajzen and Fishbein (1980) and Beswick (2005, p. 39), who maintain that a belief is "anything that an individual regards as true".

It is generally accepted that personally held beliefs are organised into some sort of structure (Green, 1971; Rokeach, 1968). Beliefs can exist in relatively independent clusters (Green, 1971), which can help to explain why individuals can hold seemingly contradictory beliefs about the discipline of mathematics, school mathematics, and how mathematics is best learned (Beswick, 2005, 2012; Jorgensen, Grootenboer, Niesche, & Lerman, 2010; Philipp, 2007). Beliefs cannot be directly observed and need to be inferred from people's words and actions (Grootenboer & Marshman, 2016; Pajares, 1992). Leatham (2006) described mathematics teachers' beliefs as a sensible system in which an individual's beliefs make sense to them, are internally consistent to them, and fit with their other beliefs. It does not, however, necessarily follow that an individual can express their beliefs or even be aware of them (Leatham, 2006).

We can infer someone's beliefs from their actions, but we cannot know with certainty which belief(s) they were acting on. When a teacher's actions appear to be inconsistent with the beliefs they have been inferred to have, it may be that we have "either misunderstood the implications of the belief, or that some other belief took precedence in that particular situation" (Leatham, 2006, p. 95). This can be complicated by tacit and powerful personal and social reasons (e.g. satisfying the schools' ethos or fitting in with a social group). Due to the contextual and clustered nature of beliefs, individuals may express different beliefs depending on the situation or context.

7.1 Beliefs About Mathematics and Mathematics Teaching

Teachers' beliefs about the teaching and learning of mathematics and the social context in which they teach, along with the degree to which teachers think about and reflect on their teaching, will determine what happens in their classroom (Ernest, 1989b). Ernest described three different views of mathematics: instrumentalist, Platonist, and problem-solving, each of which can be related to learning and teaching mathematics. From an instrumentalist perspective, mathematics is a collection of procedures, facts, and skills, and the teacher is an instructor whose role is to enable students to master the procedures and skills by carefully following the textbook or prescribed procedure. The Platonist view defines mathematics as a structured, unchanging body of knowledge that is discovered rather than created. Hersh (1997) described the Platonist view as follows: "mathematical entities exist outside space and time, outside thought and matter, in an abstract realm independent of any consciousness, individual or social" (p. 9). The teacher with a Platonist view is an explainer whose role is to support students to build conceptual understanding. The problem-solving view of mathematics is that it is human creativity and invention that drives a dynamic, growing field within a social and cultural context. In this view, the teacher is a facilitator helping students become confident problem-posers and problem-solvers (Ernest, 1989b).

Dreyfus and Eisenberg (1986) investigated the aesthetic value of mathematics and recommended that teaching include the "aha" of problem-solving and that "considerations of two or more solution paths could bring practical benefits by developing a familiarity with different solution methods, and deeper conceptual understanding" (p. 9). Schoenfeld and Herrmann (1982) explored differences in problem-solving by experts and novices, pre- and post-problem-solving course, showing that following the course, their students "perceived problem relatedness more like the experts" (p. 484).

Burton (1999) interviewed 70 mathematicians from the United Kingdom and Ireland who described mathematics as making sense of the world, seeing the connections between mathematics and the "real" world and between the different aspects of mathematics. Most mathematicians noted the collaborative or cooperative cultural climate of their research, describing mathematics as "personally- and culturally/socially-related" (Burton, 1999 p. 139). Many applied mathematicians and statisticians explained that "[y]ou know when you know, because it works, or, sometimes, because you can create a picture which convinces you" (Burton, 1999, p. 134). Mathematicians described mathematics as "a world of uncertainties and explorations, and the feelings of excitement, frustration and satisfaction, associated with these journeys, but, above all, a world of connections, relationships and linkages" (Burton, 1999, p. 138). This model of mathematics fits well with learners at any level but does not fit with the transmission model of teaching mathematics "where mathematics is presented to learners in disconnected fragments … [which] deprives them of the very pleasure of which these research mathematicians speak - the pleasure of making a connection" (Burton, 1999, p. 139). Although the interviews Burton conducted were not specifically about teaching, many mathematicians said they did not think much about their teaching, nor did they convey to their students "the struggle and the pleasure … of doing mathematics" (p. 140). According to the mathematicians, students needed to learn mathematics before they could begin mathematising, which Burton (1999) described as "objective mathematics they, as teachers, thrust towards reluctant learners" (p. 20).

Mura (1993, 1995) surveyed mathematicians and mathematics educators in Canada, asking open questions about their view of mathematics. Although many were reluctant to respond to philosophical and historical questions, mathematicians' definitions of mathematics were concerned with the design and analysis of models abstracted from reality; logic, rigour, accuracy, and reasoning; and the study of axiomatic systems (Mura, 1993). The most common themes to which mathematics educators alluded were patterns, logic, and models of reality (Mura, 1995). The views of mathematics educators and mathematicians differed in that the former were more concerned with patterns and mathematicians with logic. It may be that mathematics educators' views align with Schoenfeld's (1992) influential definition of mathematics as:

> an inherently social activity in which a community of trained practitioners (mathematical scientists) engage in the science of patterns – systematic attempts based on observation, study, and experimentation to determine the nature of principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics"). (p. 34)

Carlson and Bloom (2005) studied how mathematicians solve problems and their emotional responses to doing so. For these mathematicians, it was important to make sense of problems and to manage their frustration and anxiety. Danish mathematicians chose mathematical problems strategically (Misfeldt & Johansen, 2015), ensuring problems contributed to their "identity as a mathematician" (p. 368), were interesting and potentially fruitful, fitted within their skills and competencies, and commanded an audience. Similarly, school teachers chose interesting problems for which their students had the skills and competencies and were potentially fruitful (Misfeldt & Johansen, 2015).

More recently Brandt, Lunt, and Meilstrup (2016) surveyed US and Canadian mathematicians and mathematics educators, asking them to rank, according to importance, processes used in doing mathematics. For lower-level mathematics courses at university (college algebra, trigonometry, or calculus), mathematicians identified problem-solving, acquiring content knowledge, and acquiring informal logical reasoning, whereas mathematics educators identified problem-solving, conjecture/generalisation/exploration, and making connections. For higher-level mathematics courses (abstract algebra, number theory, or topology), mathematicians valued proving, acquiring content knowledge, and conjecture/generalisation/exploration, whereas mathematics educators identified conjecture/generalisation/exploration, proving, and problem-solving (Brandt et al.). These mathematicians and mathematics educators described "doing mathematics" as investigating problems, looking for patterns, and understanding the mathematical ideas of others. "[s]imply mimicking procedures or reciting phrases with no understanding was not doing mathematics. Instead, doing mathematics required some understanding of the underlying mathematical principles" (Brandt et al., 2016 p. 765).

Lockwood, Ellis, and Lynch (2016) maintained that although students do not need to be aware of mathematicians' practices, those teaching them do. They showed that understanding how mathematicians think with examples is useful in teaching "to help [undergraduate] students learn to think critically about how they can draw upon examples as they engage in exploring and proving conjectures" (p. 194). Leikin, Zazkis, and Meller (2018) interviewed four mathematicians who taught prospective teachers as part of larger cohorts. While these mathematicians acknowledged that only some of the mathematical content, problem-solving strategies, and techniques of proof would be used by teachers in classrooms, they believed that the mathematical language, distinctions between problem-solving strategies and algorithms, the beauty of mathematics, mathematical history, understanding the meaning of theorems and definitions, and abstraction would be useful for school teachers (Leikin et al., 2018). However, these mathematicians were more interested in preparing professional mathematicians than teachers, which suggests that either they considered their roles as MTEs as less important than their role as educators of prospective mathematicians or they had not considered the possibly different needs of prospective teachers (Leikin et al., 2018).

Australian curriculum documents (e.g. *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority (ACARA), n.d.); *Mathematics K-10 Syllabus* (Board of Studies New South Wales, 2012)), as with international curriculum documents (e.g. Council of Chief State School Officers, 2010; Ministry of Education, Singapore, 2012), are informed by a constructivist view of learning where the teacher's role is "to facilitate, on the part of students, significant cognitive restructuring that goes beyond merely adding to and adjusting existing constructions" (Beswick, 2005, p. 4). This encourages a problem-solving pedagogy in a supportive classroom environment (Cobb, Wood, & Yackel, 1991).

7.2 This Study

This project used a mixed-methods methodology. A quantitative survey of Australian MTEs' and prospective secondary teachers' beliefs about mathematics and mathematics teaching and learning was conducted in 2017. MTEs and prospective teachers were then interviewed in order to explore in more depth the responses given in the survey.

The survey included demographic questions and 26 5-point Likert scale items, the aim of which was to elicit responses (strongly disagree to strongly agree) about the participants' beliefs. These items were replicated from Beswick's (2005) survey of teacher beliefs – *Beliefs about mathematics, its teaching and its learning.*

The online survey was sent to mathematicians, statisticians, and mathematics educators who were involved in initial teacher education programmes in Australia, inviting them to participate. Eighty-two academics (out of 120 who started the survey) completed all items. The respondents represented 35 Australian universities and 5 international universities, while 3 were seeking employment and 3 were retired. The overseas MTEs all reported having previously taught Australian secondary prospective mathematics teachers. Forty-nine (60%) were male, 33 (40%) were female, and the median age was 46 years. Sixty respondents (73%) taught mathematics content courses only, 8 (10%) taught mathematics pedagogy only, and 14 (17%) taught both pedagogy and mathematics (though it was likely that in most cases, this was not necessarily mathematics content courses but mathematics content as part of their pedagogy courses or to prospective primary teachers). The qualifications of the respondents included PhD in mathematics (44, 54%), PhD in education (12, 15%), PhD in mathematics and a Graduate Diploma in Education (GDE) (11, 13%), Master's or Honours in mathematics (7, 9%), Master of Education (3, 4%), and initial teacher education qualifications (5, 6%).

The online survey was also sent to prospective secondary mathematics teachers at three universities in south east Queensland. Twenty-five (of 39) prospective secondary mathematics teachers responded to all the statements. Nineteen were studying an undergraduate programme that included both mathematics and education courses, and six were undertaking a postgraduate programme in which only education courses were studied. The six in this last category had completed mathematics courses as part of a previous qualification.

The survey data were analysed using SPSS and included descriptive statistics and one-way between groups ANOVA with Bonferroni post hoc tests, eliminating those items that violated the Levene test for homogeneity of variance. Queensland survey respondents were invited to participate in a semi-structured interview to further explore their beliefs about mathematics and its teaching and learning.

Of the seven MTEs interviewed, five taught mathematics, one taught mathematics education, and one taught both mathematics education and mathematics content courses. The following questions were used as part of semi-structured interviews which were audio-recorded, transcribed, and analysed to identify concepts and themes related to MTEs' practices of doing and teaching mathematics and the ways in which mathematics is taught in schools:

1. Will you please describe how you teach mathematics in a lecture and a tutorial?
2. How would you describe any perceived differences (if any) between the way mathematics is practised and the way mathematics is taught?
3. How would you describe any differences between how mathematics is taught in schools and university?

Seven prospective teachers also participated in semi-structured interviews. Of these, six were studying an undergraduate qualification, and one was studying a postgraduate qualification after spending 10 years in the workforce in a non-teaching role. The questions to which the seven prospective teachers responded were:

1. How would you describe the difference, if any, in the way you are taught mathematics and the way you are taught to teach mathematics?
2. Do you feel any tension between the ways you are taught mathematics and the way you think you learn it best?
3. How would you best describe the different ways your lecturers and tutors view mathematics? Do you ever find it confusing? Please explain.

Burton's (1995) theoretical framework for knowing mathematics, which she developed and tested in her study of research mathematicians (Burton, 1999), was used to further analyse the interviews and to test the applicability of the model to MTEs and prospective teachers. The model consists of five categories:

- Person- and cultural/social-relatedness
- Aesthetics
- Intuition and insight
- Different approaches (particularly to thinking)
- Connectivities (Burton (1995, 1999, p. 122))

Person- and cultural/social-relatedness recognises that knowing mathematics is "a function of who is claiming to know, related to which community, how that knowing is presented, what explanations are given for how that knowing was achieved, and the connections demonstrated between it and other knowings" (Burton, 1995, p. 287). Intuition and insight refer to being able to understand the idea instinctively and aesthetics to the beauty of the mathematics and different approaches to recognition of the different ways that ideas can be represented. Connectivities are the links between the mathematics at hand and other areas of mathematics, and/or with real-world data. In the following section, the survey results and discussion are organised around the major item categories in Beswick's survey, namely, participants' beliefs about mathematics, learning mathematics, and teaching mathematics.

7.3 Survey Results and Discussion

7.3.1 Beliefs About Mathematics

Most MTEs and prospective teachers (96%) agreed or strongly agreed that mathematics was a "beautiful, creative and useful human endeavour" and "both a way of knowing and a way of thinking", whereas only 10% of MTEs and 20% of prospective teachers agreed or strongly agreed that mathematics is "computation". These responses, shown in Table 7.1, indicate that participants were inclined to have problem-solving views (Ernest, 1989a, b) of mathematics as a discipline. The findings are consistent with those of Grigutsch and Törner (1998) that "mathematicians view mathematics as a discovery and understanding process" (p. 29).

Table 7.1 Survey responses on beliefs about mathematics collapsed into a three-point scale

		Educators			Prospective teachers		
No.	Item	D	U	A	D	U	A
9	Mathematics is a beautiful, creative, and useful human endeavour that is both a way of knowing and a way of thinking	0 0%	3 4%	79 96%	0 0%	1 4%	24 96%
20	Mathematics is computation	68 83%	6 7%	8 10%	14 56%	6 24%	5 20%

Note: *D* strongly disagree or disagree, *U* undecided, *A* strongly agree or agree

7.3.2 Beliefs About Teaching Mathematics

Table 7.2 summarises the combined MTE and prospective teacher responses to the survey items about teaching mathematics. Instances of 90% or more agreement have been highlighted for ease of viewing. Most MTEs (at least 83%) disagreed or strongly disagreed with traditional teaching methods (as reflected in Items 22, 23, 25, and 26 in Table 7.2) as did at least 80% of prospective teachers. Traditional teaching methods, including telling students how to solve mathematical problems,

Table 7.2 Survey responses about teaching mathematics (Beswick, 2005)

		MTEs			Prospective teachers		
	Item	D	U	A	D	U	A
13	Justifying the mathematical statements that a person makes is an extremely important part of mathematics	1 1%	2 2%	79 96%	1 4%	3 12%	21 84%
10	Allowing a student to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur	1 1%	3 4%	78 95%	2 8%	4 16%	19 76%
15	Teachers can create, for all students, a non-threatening environment for learning mathematics	5 6%	13 16%	64 78%	1 4%	0 0%	24 96%
11	Students always benefit by discussing their solutions to mathematical problems with each other	7 9%	18 22%	57 70%	3 12%	3 12%	19 76%
12	Persistent questioning has a significant effect on students' mathematical learning	5 6%	22 27%	55 67%	2 8%	6 24%	17 68%
14	As a result of my experience in mathematics classes, I have developed an attitude of inquiry	8 10%	20 24%	54 66%	2 8%	9 36%	14 56%
19	Mathematical material is best presented in an expository style: demonstrating, explaining, and describing concepts and skills	25 30%	23 28%	34 41%	7 28%	4 16%	14 56%
26	If a students' explanation of a mathematical solution doesn't make sense to the teacher, it is best to ignore it	76 93%	5 6%	1 1%	25 100%	0 0%	0 0%
22	I would feel uncomfortable if a student suggested a solution to a mathematical problem that I hadn't thought of previously	75 91%	1 1%	6 7%	20 80%	3 12%	2 8%
23	It is not necessary for teachers to understand the source of students' errors; follow-up instruction will correct their difficulties	75 91%	3 4%	4 5%	24 96%	0 0%	1 4%
25	It is important to cover all the topics in the mathematics curriculum in the textbook sequence	68 83%	7 9%	7 9%	17 68%	6 24%	2 8%
24	Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics	54 66%	21 26%	7 9%	17 68%	4 16%	4 16%

Note: *D* strongly disagree or disagree, *U* undecided, *A* strongly agree or agree are opposed to the social constructivist conceptions of learning mathematics for understanding by actively building on previous knowledge and experience espoused by the National Council of Teachers of Mathematics (NCTM) (2000). Items 10–15 describe teaching strategies that aim to support students to construct knowledge. MTEs and prospective teachers generally agreed or strongly agreed with these statements, particularly "allowing students to struggle" (Item 10), with which 95% of MTEs and 78% of prospective teachers agreed, and "the importance of justifying statements" (Item 13), with which 96% of educators and 84% of prospective teachers agreed.

Almost all prospective teachers (96%) and 78% of MTEs agreed or strongly agreed that the creation of a "nonthreatening environment" (Item 15) was desirable, and there was general agreement (67% of educators and 68% of prospective teachers) as to the importance of "persistent questioning in learning" (Item 12). Sixty-six per cent of MTEs and 56% of prospective teachers believed they had "developed an attitude of inquiry because of classroom experiences" (Item 14). However, 41% of MTEs and 56% of prospective teachers agreed that mathematics is learned best when taught using an "expository style" (Item 19), while 30% of MTEs and 28% of prospective teachers disagreed that was the case.

Overall, the survey results suggest that the respondents shared beliefs about the importance of supporting students to construct their own knowledge. However, both MTEs and prospective teachers were, on average, less comfortable with the use of questioning and less inclined to agree that they developed an attitude of inquiry in the classroom.

7.4 Beliefs About Learning Mathematics

Responses to items concerning beliefs about learning mathematics are summarised in Table 7.3. Items 1–3 and 5–8 describe approaches to learning mathematics that are consistent with Ernest's problem-solving view of mathematics. At least 90% of MTEs and prospective teachers agreed with statements that teachers needed to "motivate students to solve their own problems" (Item 1), to "give students opportunities to reflect on and evaluate their own mathematical understanding" (Item 3), and that "ignoring the mathematical ideas that students generate themselves can seriously limit their learning" (Item 2). Similarly, at least 90% of MTEs and at least 80% of prospective teachers agreed that "effective mathematics teachers enjoy learning and 'doing' mathematics themselves" (Item 5), and 88% agreed that "knowing how to solve a mathematics problem is as important as getting the correct solution" (Item 6). There was somewhat less agreement with the statement that teachers should be "fascinated with how students think" (Item 7) and "providing interesting problems to be investigated in small groups" (Item 8).

Only 2% of MTEs and 20% of prospective teachers agreed that "telling students the answer was an efficient way of facilitating mathematics learning" (Item

Table 7.3 Survey responses on beliefs about learning mathematics (Beswick, 2005)

			MTEs			Prospective teachers		
	Item	Number	D	U	A	D	U	A
6	Knowing how to solve a mathematics problem is as important as getting the correct solution		1 1%	1 1%	80 98%	2 8%	1 4%	22 88%
3	It is important for students to be given opportunities to reflect on and evaluate their own mathematical understanding		2 2%	1 1%	79 96%	0 0%	1 4%	24 96%
4	It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other		1 1%	2 2%	79 96%	0 0%	0 0%	25 100%
1	A vital task for the teacher is motivating students to solve their own mathematical problems		4 5%	1 1%	77 94%	1 4%	0 0%	24 96%
5	Effective mathematics teachers enjoy learning and “doing” mathematics themselves		0 0%	5 6%	77 94%	3 12%	2 8%	20 80%
2	Ignoring the mathematical ideas that students generate themselves can seriously limit their learning		4 5%	4 5%	74 90%	0 0%	2 8%	23 92%
7	Teachers of mathematics should be fascinated with how students think and intrigued by alternative ideas		5 6%	10 12%	67 82%	1 4%	3 12%	21 84%
8	Providing students with interesting problems to investigate in small groups is an effective way to teach mathematics		4 5%	23 28%	55 67%	2 8%	4 16%	19 76%
16	It is the teacher’s responsibility to provide students with clear and concise solution methods for mathematical problems		19 23%	24 29%	39 48%	1 4%	7 28%	17 68%
17	There is an established amount of mathematical content that should be covered at each grade level		20 24%	22 27%	40 49%	1 4%	6 24%	18 72%
18	It is important that mathematics content be presented to students in the correct sequence		21 26%	23 28%	38 46%	1 4%	2 8%	22 88%
21	Telling the students the answer is an efficient way of facilitating their mathematics learning		61 74%	19 23%	2 2%	18 72%	2 8%	5 20%

Note: *D* strongly disagree or disagree, *U* undecided, *A* strongly agree or agree

21), and 74% of MTEs and 72% of prospective teachers disagreed with this statement. A quarter of MTEs and 8% of prospective teachers were undecided. Together these survey responses suggest the prevalence, among MTEs and prospective teachers, of a belief in the value of problem-solving for mathematics learning.

7.4.1 Differences Between the Beliefs of Subgroups of MTEs and Between MTEs and Prospective Teachers

The data were analysed for differences between MTEs who taught only mathematics content courses, MTEs who indicated that they taught both mathematics content and mathematics courses, MTEs who taught pedagogy only, and prospective teachers. Mean responses were also compared for MTEs categorised according to their highest mathematics and/or mathematics education qualification. The groups along with the numbers and percentage of MTEs in each group were as follows: no PhD (15, 18%), PhD in mathematics education (12, 15%), PhD in mathematics (44, 54%), and PhD in mathematics as well as a Graduate Diploma in Education (GDE) (11, 13%).

Survey responses differed among MTEs, depending on their teaching responsibilities and qualifications, as well as between some of these subgroups of MTEs and prospective teachers. Sixty (73%) MTEs taught only mathematics or statistics content courses, while eight (10%) taught only pedagogy courses. Fourteen (17%) taught both discipline content and pedagogy. It is unclear whether those who said they taught both mathematics and pedagogy taught separate content courses and pedagogy courses or whether they taught mathematics content as part of education courses primarily aimed at teaching prospective teachers how to teach mathematics.

Two of the items in Table 7.4 that concern beliefs about learning mathematics showed a statistically significant difference ($p < 0.05$) between some MTEs and prospective teachers (PSTs in Table 7.4). MTEs who were mathematicians had a higher mean agreement than did prospective teachers to Item 5: "teachers enjoy learning and 'doing' mathematics themselves". This suggests, unsurprisingly, that the prospective teachers tended to see themselves as teachers of mathematics rather

Table 7.4 Differences in beliefs of educators categorised according to their teaching responsibility and prospective teachers (PSTs)

	Statistic	MTECs	MTEPs	Both	PSTs	F (*p* value)
Effective mathematics teachers enjoy learning and "doing" mathematics themselves (5)	Mean SE	4.58^{b} 0.08	$4.75^{a, b}$ 0.16	$4.43^{a, b}$ 0.20	4.04^{a} 0.20	3.923 (0.011)
Mathematical material is best presented in an expository style: demonstrating, explaining, and describing concepts and skills (19)	Mean SE	$3.37^{a, b}$ 0.13	2.38^{a} 0.32	$2.79^{a, b}$ 0.24	3.48^{b} 0.22	3.708 (0.014)
Mathematics is computation (20)	Mean SE	$1.80^{a, b}$ 0.13	$2.00^{a, b}$ 0.27	1.57^{a} 0.25	2.52^{b} 0.19	3.898 (0.011)

PST prospective teacher, *MTEC* Content only MTEs, *MTEP* Pedagogy only MTEs

[a, b]Mean values within a row with unlike superscript letters are significantly different ($p < 0.05$). For example, for Item 5, the (Bonferroni-adjusted) t-test results show a small p comparing "teaching content" with "prospective teachers", but neither was different from "teaching content and pedagogy" and "teaching pedagogy"

than doers of mathematics. MTEs had a lower mean agreement than did prospective teachers as to the value of using an "expository style" of teaching (Item 19).

Each of the four groups, on average, disagreed that "mathematics is computation," (Item 20) but those who taught both mathematics content and pedagogy had a significantly lower mean agreement than did the prospective teachers. The MTEs tended to have more problem-solving belief about mathematics than did the prospective teachers.

7.5 Differences Related to MTEs' Qualifications

There were six items about learning mathematics for which there were statistically significant differences ($p < 0.05$), as shown in Table 7.5. Educators with a mathematics PhD had a lower mean agreement with the statement "ignoring students' mathematical ideas can limit their learning" (Item 2), compared to those who also had a GDE. MTEs with a PhD in mathematics education had a higher mean agreement than those with either no PhD or a mathematics PhD with Item 8: "providing students with interesting problems to investigate in small groups". Those with no PhD had a higher mean agreement for there being a "set amount of mathematical content to cover at each level" (Item 17). MTEs with a mathematics education PhD had a lower mean agreement than those with no PhD with "mathematics must be presented in the correct sequence" (Item 18). MTEs with a mathematics PhD and those with no PhD had a stronger mean agreement than mathematics educators that "mathematics should be presented in an expository style" (Item19).

These results suggest that gaining postgraduate education qualifications, either a PhD or a PhD and a GDE, provided MTEs with an opportunity to reflect on how

Table 7.5 Differences in beliefs of MTEs according to qualifications

Abbreviated item and number	Statistic	PhD M, S	PhD Ed	PhD M and GDE	No PhD	F (p-value)
Ignoring students' mathematical ideas can limit their learning (2)	Mean	4.02[a]	4.58[a, b]	4.82[b]	4.20[a, b]	3.228
	SE	0.144	0.149	0.122	0.262	(0.027)
Students with interesting problems to investigate in small groups (8)	Mean	3.68[b]	4.50[a]	4.36[a, b]	3.53[b]	4.729
	SE	0.121	0.195	0.203	0.322	(0.004)
An established amount of content to be covered at each level (17)	Mean	3.41[a, b]	2.75[a]	3.00[a, b]	3.87[b]	3.090
	SE	0.157	0.329	0.270	0.256	(0.032)
Content should be presented in the correct sequence (18)	Mean	3.57[a]	2.50[a, b]	3.45[a, b]	3.40[b]	3.451
	SE	0.154	0.314	0.312	0.254	(0.020)
Mathematics is best presented in an expository style (19)	Mean	3.39[b]	2.50[a]	2.55[a, b]	3.53[b]	5.130
	SE	0.135	0.359	0.247	0.236	(0.003)

M mathematics, *S* statistics, *Ed* education

[a, b]Mean values within a row with unlike superscript letters are significantly different ($p < 0.05$). For example, for Item 19, the (Bonferroni-adjusted) t-test results show a small *p* comparing "a PhD in mathematics or statistics" with "a PhD in education", but not between the others

mathematics is learned, such that their mean agreement for statements about mathematics learning consistent with problem-solving views of mathematics was higher than those of MTEs with mathematics PhDs or no PhD. Institutional pressures may mean that MTEs with mathematics PhDs spend less time than other groups reflecting on and developing their teaching since they are typically employed as research mathematicians.

7.6 Interviews with MTEs and Prospective Teachers

Interviews with three MTEs were conducted to deepen understanding of the survey data. Burton's (1995, 1999) categories for knowing mathematics (person- and cultural/social-relatedness, aesthetics, intuition and insight, different approaches (particularly to thinking), and connectivities) were used to identify ways in which these MTEs knew mathematics. Understanding how MTEs "know mathematics" may help with inferring their beliefs about mathematics and its teaching and learning. Following that, prospective teachers' responses to interview questions about their experiences of being taught mathematics at school, and learning to teach mathematics at university, are reported. In these sections, italics are used to show direct quotes from the interviews and to highlight Burton's (1995, 1999) categories.

7.6.1 The Case of Ryan

Ryan was a pure mathematician who believed the *aesthetics* of mathematics were important. He described mathematics as beautiful and bringing joy: "*There's a famous mathematician by the name of Hardy who said that all mathematics should be beautiful. There was no room for ugly mathematics. I would add … that the experience of doing mathematics should be one that brings joy.*"

This enjoyment extended to his teaching which he described as quite expository, although he wanted the students to understand:

> I enjoy … presenting problem solutions to students

Ryan thought that *intuition* was important and that one needed time for thinking. To him the practices of mathematics involved looking at problems and thinking about them before putting pen to paper:

> … this idea of being able to look at a problem and then sometimes deciding that the best way to advance the problem is to walk away from it for a while and just let the mind percolate on the challenges of the problem. …

This was reflected in his description of his teaching. He said that he "*advise[d] students to do [walk away from the problem for a while]*". Ryan's preference for using pen and paper *(*"*I much prefer to use pen and paper*"*)* was also reflected in his teaching with the use of the document camera to project his writing onto the screen for students to read. He believed this allowed students time to think as it slowed the pace of the lecture.

> I use the document camera to write out a fresh set of notes... writing at the speed of thought makes a much better connection … I encourage students to have a folio of worked solutions, so they have not a model solution but solution models.

Ryan talked about the traditional *culture* of mathematics – that is, "*the long tradition of mathematics to think logically with precision and without ambiguity*" – as something he shared with his postgraduate students rather than undergraduate students. He believed he was trying to cater to a range of students with diverse disciplinary backgrounds in engineering, science, and education, students with a wide range of abilities.

7.6.2 The Case of Paul

Paul was an MTE who taught mathematics pedagogy to prospective teachers. During the interview, he did not talk about his beliefs about mathematics. He began his courses by talking with his prospective teachers about:

> … what it might involve, teaching in a secondary school … get them to see the world from the eyes of a student.

Paul thought that it was important for prospective teachers to have a variety of activities ("*different approaches*") in their toolkits and that they considered how their students learned and that they adapted their teaching accordingly. He explained:

> [w]hat I would hope, is for any teacher, that they would think about how kids learn and then try and develop their pedagogy as best they can, based on what they think is going to work for their kids. … I'd encourage them to have as much variety [as possible].

Paul told his prospective teachers that sometimes they would use explicit teaching methods but that doing activities was valuable. For example, in his class for prospective senior secondary teachers, he had them use calculus to model the motion of Hot Wheel cars:

> I tend to show things which are more based around mathematical modelling and activities and different ways of approaching the mathematical ideas. … when we do an introduction to calculus for example, we will start by giving them Hot Wheel sets … do an experiment of sorts and then try and answer a question which will involve them using some sort of calculus. … [we] use the mathematical model to solve the problem. … I try not to give them answers. I just prompt them and ask questions.

Paul talked about the importance of building *insight*, which for him involved "*helping them to generalise some of the ideas*".

7.6.3 The Case of Sam

To Sam, an applied mathematician who taught mathematics courses to student cohorts that included prospective teachers, doing mathematics involves *different approaches* and *connectivities*. He talked about exploring different mathematical ideas and ways of working as he made decisions within a structured environment:

> … [When doing mathematics, one needs to be] prepared to play around a little bit with different ideas and ways of working that might shed different information on the same problems … you can have structure which still allows you choice and forces you to make decisions and then work out the consequences of those decisions. It gives you an understanding of what it is you're looking at.

Originally, Sam said that he had used a very transmissive way of teaching, using traditional lectures in which his students were given the information. He believed this was necessary because limited information was available from other sources. He described his teaching as:

> … running through the notes, running through examples with not much interaction from the students.

Not comfortable with how he had been teaching, Sam worked with another MTE who made him realise he could change the *culture and social relations* in his lectures and tutorials and how he taught. The MTE whom he consulted supported him by providing a variety of different examples of ways in which he could change his teaching. He said that, as a result, his classes became more interactive; students were encouraged to solve problems, to interact with the person sitting next to them, and to participate in whole-group discussions. When describing his working with the MTE, he said that she made:

> … me realise I had choices that I could make about how I was teaching. … she [the mathematics educator] was actually able to provide me with different ways that I could achieve that.

> [Now] I try to give students opportunities to talk during lectures. I'll give them a problem or ask them a question and ask them to discuss it with the person next to them or do a bit of work and then talk to their neighbour about what they did.

From their collaboration, Sam and the other MTE developed a mathematics course for prospective teachers designed to help students develop an understanding of the sociocultural and historical development of mathematical concepts and a deeper understanding of school mathematics and its connections to quantitative disciplines. One task was to regularly critique passages of the textbook they used in order to analyse the robustness of the mathematics as it was presented. Sam described his pedagogical approach in this context as follows:

> [w]e definitely try to model doing mathematics the way we would want them to think of mathematics as teachers and for them to think about the way we do our maths … A very typical example is to get the students to work with a textbook and for them to critique a passage in the textbook, a section or a bunch of questions for their robustness, if you like, in terms of presenting the mathematics.

7.6.4 Discussion of the MTE Cases

Ryan linked his beliefs about mathematics – that it was beautiful (Hardy, 2012) and involved working with problems, using pen and paper, and taking time to think – with his teaching. The constraints inherent in teaching large cohorts of undergraduate students from a range of programmes, and with varied abilities and mathematical backgrounds, meant that he used a more expository form of presenting problem solutions, reserving his knowledge of the traditions of mathematics for his postgraduate students.

In contrast to Ryan's expository style of mathematics teaching, Paul believed that it was important that prospective teachers understood how students felt. He encouraged prospective teachers to use different approaches to teaching, to think about how students learn, and to develop their pedagogy from there. Paul's prospective teachers performed experiments and developed mathematical models to solve problems and build insights that helped them to generalise.

Having collaborated with another MTE, Sam now linked his belief that mathematics involves exploring different ideas and ways of working – with his teaching. Students in his classes solved problems and interacted with others, building a mathematical *culture*. Teaching collaborations between MTEs working primarily as mathematicians and MTEs working exclusively with prospective teachers, such as that in which Sam was involved, are internationally rare (Fried, 2014). The sociocultural differences between these groups of MTEs can act as a boundary between them (Akkerman & Bakker, 2011). Akkerman and Bakker identified four processes that could lead to learning at the boundary between disciplines: (1) *identification*, whereby the specific ways of working of the two communities are challenged, (2) *coordination* of practices or perspectives through discussion to allow movement between the two worlds, (3) *reflection* on the differences of ways of working, and (4) *transformation* leading to significant changes. Sam used *identification* as he recognised the different ways of teaching in the two disciplines (university mathematics and mathematics education) and *coordination*, where dialogue with a mathematics educator allowed him to use some of the practices of educators, and this led to *transformation* of his practice.

These dialogues led to the creation of the course "Mathematics content for lower secondary school teaching" which was jointly taught by a Sam, a mathematician, and his MTE collaborator, as well as an awareness on Sam's part of how the pedagogies used in schools could be adapted for use in a university context.

7.6.5 Prospective Teachers' Views on Mathematics Teaching

This section presents the beliefs about mathematics teaching of five prospective teachers that they expressed in their interviews. One, Tim, was studying a 1-year Graduate Diploma of Education after having done some mathematics tutoring at university. Another, Dan, was studying a Bachelor of Education programme majoring in mathematics. The three other prospective teachers were studying for combined Bachelor of Education/Bachelor of Science degrees with mathematics as a teaching area.

The prospective teachers described differences between the way they were taught mathematics at university and the way they were being taught to teach mathematics. These related to methods of presenting material, assumptions about prior knowledge, ways of working with the students to help them to understand the mathematics, catering for the diversity of students in a class, the context in which content was presented, and the relevance of the content. Generally, the prospective teachers described being taught university mathematics via traditional lectures and tutorials in which information and worked examples were presented with the textbook as an important part of the process. Doug, for example, recounted that at university:

> the teacher [would be] up the front presenting the information and it's up to you to interpret it and try and make sense of it.

Tim described a similar approach that he experienced as:

> very text book heavy. Here's the content. Here's what this means. Here, go practice it.

When considering how they were being taught to teach mathematics, the prospective teachers described being encouraged to use a more problem-solving approach, encouraging and guiding students to work towards their own solutions so as to build their understanding. Doug explained his experience of learning how to teach mathematics as follows:

> We [were] taught to approach things teaching as a problem-solving approach, so to get students to try and discover things on their own and not just give them the data but guide them towards finding their own solutions.

The prospective teachers also discussed being taught to cater for the diversity of their students by considering their background knowledge rather than assuming that students had the prior knowledge that the curriculum might suggest. They described being urged to take time to explain concepts and to work with their students:

> … when you're taught to teach maths you can't assume what a student knows. We're taught to explain a lot more and to break it down a lot more … to make sure everyone is following each step if that makes sense … when you're a teacher you're taught to appreciate different learning styles and present things in ways that are relevant to the whole range of students.

(Dan)

As the prospective teachers were negotiating the different approaches they saw modelled in their mathematics courses and advocated in their mathematics education courses, they were thinking about how they wanted to teach. They believed it was important to engage their students by connecting the mathematics to the real world and giving their students a reason to be doing the mathematics by showing them its relevance. The aim, as Max put it, was:

> to try and develop a deeper understanding initially, connecting through the real world and connecting it to a reason for learning it.

For some prospective teachers, this different way of teaching was exciting. For example, Jett said:

> When I first started doing the maths education courses … [the mathematics education lecturer] would do sample lessons like inquiry lessons. I definitely thought that that was a way better way to learn

At university, the prospective teachers had experienced mathematicians presenting the mathematics to them. It was up to them how they engaged with and practised the material. In contrast, they were being taught to teach mathematics by engaging and motivating students to construct their own understanding of the mathematics. The prospective teachers were aware of the difference between how mathematics was taught at university and how they were being taught to teach it and described the tension between being taught to teach one way but being taught in a very different way. Anne described the two contexts as follows:

> [Some MTEs] were happy to allow discussions about different content and the way we should consider explaining or the activities that we choose or introduce to students to help synthesise or corroborate that knowledge.

Whereas being taught mathematics:

> One of my other course co-ordinators was very much of the, "You will write this down and you will understand it from having it written down and practicing it, and that's the only way you're going to learn it."

The prospective teachers appeared to accept whichever way they were taught, and because they were capable mathematics learners, they had the motivation and resources to find extra information and get help as needed. For example, Anne stated that:

> I can gain some understanding from that [the mathematics written down during the lecture] but, I will often take that further myself, in my own study time … I do have to engage in other learning practices to try and synthesise that knowledge.

They also believed that the way they were taught mathematics reflected the teacher's (MTEs with mathematics PhDs) beliefs. As Max put it:

> I think there are different interpretations and different views on it [how to teach mathematics]. I don't find it confusing because I see it as their makeup. There're university lecturers here that are more from analytical and more engineering backgrounds. Then you've got ones that are more from theoretical, pure mathematics backgrounds. To me it makes sense that they would have slightly different outlooks on it.

Some prospective teachers discussed their concern that in some of their classes, other university students in their cohort could not cope with the lecturing style. Doug explained this as follows:

> I imagine with a lot of students they'd have a lot of trouble just trying to put together that raw data they're given and actually understanding everything behind it.

Frustration was expressed that at one university, mathematics for engineering was taught without any consideration of prospective teachers and the mathematics and statistics they would be teaching in schools. Dan, for example, noted that:

> … I found the second year of uni extremely frustrating. Because we were learning maths well beyond what we needed to which in some way is useful but there's so much attention based on it. We weren't really covering all the maths in the curriculum too. We were just focusing on a small part of it. Things like statistics. We didn't touch statistics at all or very rarely in the two years of maths. … it's taught more like engineering rather than from the perspective of a maths teacher.

7.7 Conclusions

Students' beliefs about mathematics are directly influenced by their teachers' classroom practices (McLeod, 1992; Mosvold & Fauskanger, 2014). Therefore, one would expect that the practices of MTEs – teaching content or teaching pedagogy – would affect the beliefs of prospective teachers about mathematics teaching and learning. The survey of MTEs and prospective teachers reported in this chapter showed that generally they held problem-solving views of mathematics (Ernest, 1989b), consistent with Grigutsch and Törner's assertion (1998) that "mathematicians view mathematics as a discovery and understanding process" (p. 29).

In the main, respondents in this study held problem-solving views of teaching mathematics, which involved "allowing students to struggle" and highlighted "the importance of justifying statements". However, there were aspects of the problem-solving view with which not all respondents were comfortable, for example, the use of questioning.

Generally, prospective teachers believed that the aims of teaching mathematics at university were different from the aims of teaching mathematics in schools. At university, mathematicians presented the mathematics to the students, and the way in which they engaged with and practised the material was up to them. This contrasted with the way in which they were taught to teach mathematics to school students. The prospective teachers accepted the tension between how they were taught mathematics and how they were taught to teach mathematics. They acknowledged that they had the motivation and resources to find extra support to help them build their mathematical knowledge, but future research in this project will explore the nature of the beliefs that these prospective teachers take into schools and the extent to, and ways in, which the various groups of MTEs with whom they have worked may have influenced them.

The findings presented in this chapter suggest that preparing prospective teachers needs to include discussions on beliefs about mathematics and its teaching and learning, in which teachers are encouraged to reflect on the differing beliefs that may underpin the teaching of the various MTEs they have encountered and to consider how this may have shaped their own beliefs and could influence their own teaching. This would help prospective teachers develop the resilience and confidence to negotiate future school environments in which there are likely to be tensions between their beliefs about mathematics, and mathematics teaching and learning, and those of colleagues.

In Sam's case, engaging in discussion with an MTE with a different disciplinary background led to changes in his beliefs and teaching practices, resulting in a new approach that was more supportive of student learning. Boundary dialogues (Goos & Bennison, 2018) such as that in which Sam engaged can enable MTEs to identify and understand the practices of other MTEs and to reflect on their beliefs about mathematics and its teaching and learning. Such conversations could allow differing groups of MTEs to work together in order to develop greater coherence for the prospective teachers across their studies. Ultimately, improving the education of prospective teachers is likely to result in better outcomes for school students.

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References

Ajzen, I., & Fishbein, M. (1980). *Understanding attitudes and predicting social behavior*. Englewood Cliffs, NJ: Prentice-Hall.

Akkerman, S., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (n.d.). *Australian curriculum: Mathematics.* Retrieved 11 May 2018, from http://v7-5.australiancurriculum.edu.au/mathematics/content-structure

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal, 17*(2), 39–68.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147.

Board of Studies New South Wales. (2012). *Mathematics K-10 syllabus*. Retrieved 7 Jan 2019, from https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/learning-areas/mathematics

Brandt, J., Lunt, J., & Meilstrup, G. R. (2016). Mathematicians' and math educators' views on "doing mathematics". *Primus, 26*(8), 753–769.

Burton, L. (1995). Moving towards a feminist epistemology of mathematics. *Educational Studies in Mathematics, 28*(3), 275–291.

Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics, 37*, 121–143.

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics, 58*(1), 45–75.

Cobb, P., Wood, T., & Yackel, E. (1991). A constructivist approach to second grade mathematics. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 157–176). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Council of Chief State School Officers (2010). *Common Core State Standards*. Retrieved from http://www.corestandards.org/Math/

Dreyfus, T., & Eisenberg, T. (1986). On the aesthetics of mathematical thought. *For the Learning of Mathematics, 6*(1), 2–10.

Ernest, P. (1989a). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*(1), 13–33.

Ernest, P. (1989b). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249–254). London: Falmer Press.

Fried, M. N. (2014). Mathematics and mathematics education: Searching for common ground. In M. Fried & T. Dreyfus (Eds.), *Mathematics and mathematics education: Searching for common ground* (pp. 3–22). Dordrecht, The Netherlands: Springer.

Goos, M., & Bennison, A. (2018). Boundary crossing and brokering between disciplines in pre-service mathematics teacher education. *Mathematics Education Research Journal, 30*(3), 255–275.

Green, T. (1971). *The activities of teaching*. New York: McGraw-Hill.

Grigutsch, G., & Törner, S. (1998). *World views of mathematics held by university teachers of mathematics science*. Retrieved from the University of Duisburg-Essen website: https://duepublico.uni-duisburg-essen.de/servlets/DerivateServlet/Derivate-5249/mathe121998.pdf

Grootenboer, P., & Marshman, M. (2016). *Mathematics, affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Singapore: Springer.

Grootenboer, P. J. (2008). Mathematical belief change in preservice primary teachers. *Journal of Mathematics Teacher Education, 11*(6), 479–497.

Hardy, G. H. (2012). *A mathematician's apology*. Cambridge: Cambridge University Press.

Hersh, R. (1997). *What is mathematics, really?* Oxford: Oxford University Press.

Jorgensen, R., Grootenboer, P., Niesche, R., & Lerman, S. (2010). Challenges for teacher education: The mismatch between beliefs and practice in remote indigenous contexts. *Asia-Pacific Journal of Teacher Education, 32*(2), 161–175.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*(1), 91–102.

Leikin, R., Zazkis, R., & Meller, M. (2018). Research mathematicians as teacher educators: Focusing on mathematics for secondary mathematics teachers. *Journal of Mathematics Teacher Education, 21*(5), 451–473.

Lockwood, E., Ellis, A. B., & Lynch, A. G. (2016). Mathematicians' example-related activity when exploring and proving conjectures. *International Journal of Research in Undergraduate Mathematics Education, 2*(2), 165–196.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan.

Ministry of Education, Singapore. (2012). *Mathematics syllabus secondary one to four*. Retrieved from the Singapore Ministry of Education website: https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/mathematics-syllabus-sec-1-to-4-express-n(a)-course.pdf

Misfeldt, M., & Johansen, M. W. (2015). Research mathematicians' practices in selecting mathematical problems. *Educational Studies in Mathematics, 89*(3), 357–373.

Mosvold, R., & Fauskanger, J. (2014). Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education, 8*(2–3), 43–61.

Mura, R. (1993). Images of mathematics held by university teachers of mathematical sciences. *Educational Studies in Mathematics, 25*(4), 375–385.

Mura, R. (1995). Images of mathematics held by university teachers of mathematics education. *Educational Studies in Mathematics, 28*(4), 385–399.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age.

Rokeach, M. (1968). *Beliefs, attitudes and values: A theory of organisational change*. San Francisco, CA: Jossey-Bass.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 8*(5), 484–494.