# Chapter 22
# Who Are We as MTEs: And How Do We Learn and Develop?

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## 22.1 Background and Introduction

Throughout Themes 2 and 3 of this book, for me, issues of *identity* are very strong. Who *are* "we" as MTEs? Particularly, are we learners? Are we teachers? Are we researchers? Are we all three of these? And how do our identities sit alongside those of the teachers with whom we work in a complexity of contexts and cultures? There will be many more questions as I seek to unravel the richness of relationships and issues that have been revealed in these chapters.

In 1998, the first issue of JMTE, the *Journal of Mathematics Teacher Education*, was published. This was a strong acknowledgement that teacher education in mathematics had become an important field for research. Building on a long(ish) history of research into the learning of mathematics in schools and higher education, and a rather shorter history of mathematics teaching at these levels, it had become clear that these two fields of research interest were incomplete without consideration of how teachers come to know how to work with students to support "effective" learning of mathematics (Simon, 2008). In many countries, by this time, teacher education programmes were in place to educate or in some cases "train" new teachers of mathematics or to contribute to the professional development of practising teachers. In JMTE, much of the research into mathematics teacher education was conducted by the people who were responsible for teaching the prospective or practising teachers. In many cases, these people, the MTEs, were also the researchers seeking to illuminate this field of education. Research addressed how the new teachers learn to teach mathematics in a variety of programmes and contexts led by MTEs. However, few of the papers submitted to JMTE or other mathematics education journals raised

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questions about the learning of the MTEs in their programmes for educating teachers to teach mathematics effectively to students at a range of levels (Chapman, 2008; Jaworski, 2001).

When the editors of JMTE at that time agreed to edit the (first) *International Handbook of Mathematics Education* (Wood, 2008), it seemed imperative to recognise this general omission in the research and to build on what a smallish number of scholars were doing to remedy it. Thus, the fourth of four volumes of the handbook was entitled *The Mathematics Teacher Educator as a Developing Professional* (Jaworski & Wood, 2008). Its chapters offered a richness of experience of MTEs' learning and development from working to educate prospective and practising teachers and researching the associated programmes and courses, their models and theories. I believe that what we find in this book is indicative that MTE learning and development has become an important area of research since 2008. I therefore thank Merrilyn Goos and Kim Beswick for the opportunity to read and respond to these accounts in Themes 2 and 3 of this book.

What I write here is related to my own knowledge and experience and my reflections on what I read. Readers can perhaps recognise my projecting from what I read into my own experience and my own interests, both practical and theoretical. I expect that this is what you are doing as you read the chapters.

## 22.2 Theme 2: Learning and Development as an MTE

In their prospectus for the special issue, the editors for this book set out their focuses for Theme 2 including the following paragraph:

> Mathematics teacher educators are also well positioned to learn from their research with teachers, even though this learning is often left unacknowledged and unarticulated (Jaworski, 2001). Chapman (2008) suggested that an explicit goal of mathematics teacher educators' research of their practice should be self-understanding and professional development. Reports of such studies, therefore, need to include how the teacher educator-researchers reflected, what practical knowledge they acquired, and how this knowledge impacted or is likely to impact their future behaviour in working with their students. This will allow such research to contribute to greater theoretical understanding about mathematics teacher educator learning and to the improvement of practice.

This paragraph was insightful in its projection into the chapters of Theme 2 of this book, which together provide a most interesting and illuminative response.

I start my commenting with reference to the first chapter in the 2008 book, written by Martin Simon, who refers to "Two categories of Mathematics Teacher Education":

> Teacher professional development efforts can be sorted into two categories, those with process goals only and those that have content and process goals (Simon, 2008, p. 18).

According to Simon, the second category "consists of courses and workshops for teachers in which teacher educators aim to promote particular mathematical and

pedagogical concepts, skills, and dispositions" (p. 18). The first category seems less well defined. Simon illustrates with reference to "the Japanese Lesson Study model … and programmes based on teacher inquiry or teacher research" (p. 18). He writes further:

> There are no a priori learning goals for teachers involved in these programmes (other than learning the processes of inquiry, reflection, etc.) (p. 18).

The premise behind the programmes in the first category is that teachers' engagement in reflective, inquiry-based, professional activity, including research, supports their professional development. I was curious to see whether the focuses of MTE involvement in teacher professional development discussed here would fit these two categories, and what might go beyond this, to new areas of study. A subtle difference here is that our focus is mainly on MTE learning as differentiated from teacher learning, although teacher learning remains a factor in several cases. First, the two categories.

Category 2: *teacher educators working with teachers in a course or workshop aimed to promote teachers' mathematical and pedagogical concepts, skills and dispositions.*

I start with Category 2 because I find it the one that is perhaps the most clear to recognise. In this book, teachers in several of the chapters learn to teach in some initial teacher education, or prospective teachers' programme or course, engaged in sessions or workshops led by MTEs. In Theme 2, we find such a setting in the chapters of Bissell, Brown, Helliwell and Rome; Brown, Brown, Coles and Helliwell and Ingram, Burn, Fiddaman, Penfold and Tope, in all of which the MTEs work in educating prospective teachers in a nationally guided initial teacher education programme in the United Kingdom. Learning and development for these MTEs is related to their work with teachers in such a programme and to their own reflections on their activity with teachers. Chapters of Nolan and Keazer; Olanoff, Masingila and Kimani; and Osborn, Prieto and Butler all relate the learning of MTEs to their teaching of teachers in a range of courses, in, respectively, Canada, the United States and Australia. The courses vary among "Culturally relevant pedagogies" (Nolan and Keazer), "Mathematical knowledge for teaching" (Olanoff et al.) and the "Disciplinary teaching of science, or mathematics, or statistics" (Osborn et al.). While all of these chapters vary in the ways in which MTEs' learning is related to their teaching of teachers in the prospective teacher course or programme, their commonality is that their learning is essentially related to their responsibility to the programme and their desire to develop their own activity with the teachers. I see a difference here from many articles in this category in the past that the MTEs address their own learning as much as, if not more than, they evaluate the teachers' learning.

Category 1: *there are no* a priori *learning goals for teachers involved in these programmes (other than learning the processes of inquiry, reflection, etc.).* (Simon, 2008, p. 18).

This category is much harder to define or determine. The four other chapters, in Theme 2, are harder to group – all have some elements that relate them to Simon's (2008) Category 1, although in different ways. Chapter 15, by Bakogianni, Potari,

Psycharis, Charalambos, Spiliotopoulou and Triantafillou, is located within a large EU project (Mascil) which promoted research and development into inquiry-based learning and the connection of school mathematical activity with authentic workplace situations. Situated in Greece, ten teacher educators (mathematics and science) worked with groups of teachers to explore inquiry-based learning and authentic workplace learning with their students, in the spirit of Lesson Study approaches, and to share experiences and issues among their own MTE inquiry community. A characteristic of this programme concerns the large number of teachers and MTEs involved and the corresponding complexity of issues arising. Chapters 9 and 12 both involve two MTEs, one experienced and one novice, learning together in a specially focused project. Chapter 12 (Sikko and Grimeland) offers a self-study of two mathematicians developing their own practice in working with teachers, prospective and practising, exploring what a mathematician needs to learn to become an MTE. Chapter 9 (Van Zoest and Levin) offers multiple layers of MTE learning, in the United States, through "Artefact-enhanced Collegial Inquiry (ACI)" between two MTEs teaching teachers in a "methods" course. Chapter 17, rather different from the other chapters, focuses on educational structure in China and explores the roles of two groups of educators, the MTEs and MTRs, mathematics teacher researchers, who work both separately and collegially to support prospective or practising teachers.

An issue here is that while these chapters address MTEs' learning and development in relation to teachers' learning in some course or programme, the course or programme is very much in the background, subsumed in the more prominent focuses. These focuses have the "process goals" of which Simon writes and can be seen to involve research and development into new approaches and ways of thinking such as inquiry-based and workplace learning or how to teach mathematics in ways such that pedagogy supports mathematical concept development.

*A question we might ask here is what are the similarities and differences between our learning as MTEs and the learning of teachers, both in content and in process, in the programmes in which we teach? How do our responsibilities in these programmes limit our learning opportunities?*

As I read these chapters, in both categories, I became strongly aware of MTEs *reflecting* on their activity and learning in relation to their work with teachers, often in narrative style. I was reminded of a special issue of *ZDM Mathematics Education* which I edited with Rongjin Huang, addressing collaborations between mathematics teachers and MTEs, in which we observed the following:

> Reflective practice emerges as a principal goal for effective development and is linked to teachers' and didacticians' engagement with inquiry and research. (Jaworski & Huang, 2014, p. 173).

We drew on the work of Orit Zaslavsky (2008) who had emphasised that among the enormous and multifaceted demands on MTEs, in terms of knowledge and quality, the overarching demand is for MTEs (like teachers) to be reflective practitioners. The seminal work of Donald Schön (1987) comes to mind, in which he writes of reflection *on*, *in* and *for* action, where the *action* is that of the professionals

engaging with their particular practice. Stephen Kemmis (1985), building on John Dewey (1933), has written about reflection as not only something quiet and personal but also being *action oriented* and *critical*. In our case, here, we are thinking of the practice of taking action as an MTE. I believe that, our over recent years, we have seen a transition through MTEs supporting *teacher* reflection to acknowledgement of our own reflection leading to us developing insight, awareness and knowledge. The literature has many examples (e.g. Chapman, 2008; Goos, 2014; Jaworski, 2001; Zaslavsky, 2008).

*As MTEs, how does our reflection on, in or for practice affect overtly our learning and development and ongoing action in practice?*

### 22.2.1  Reflection and Voice

In this book, many of the chapters offer narratives, dialogues or personal statements of MTEs in which their reflections provide readers with insight into the nature of their practice and the issues whose critical consideration *on* or *of* this practice leads to a learning that is compelling and motivating *for* or *in* future practice. Space limits what I can focus on particularly, but a few examples seem in order.

1. In Chap. 11 (Bissell et al.), Alistair writes the following:

   Despite having cared so much about my opportunities to listen, I found that in this case I wasn't interested in the responses that were coming back from the teachers – I was only waiting for the responses that were in the plan for the day, which felt immediately uncomfortable (p.14).

2. In Chap. 16 (Nolan and Keazer), Kathy writes the following:

   Prior to this moment, I had made assumptions that this PT probably had experiences similar to mine, but this experience taught me that there is always more to know about my students. This left me with the conviction that I must mine knowledge out of my own students and build on that knowledge as I teach them about mathematics and CRP.

In these two examples, we find the writer reflecting on his or her intentions, actions and, possibly, implications for the future that arose from the reflection. In both cases, I find myself saying, "YES, I recognise this, I've been there," which leads to my own reflections on my own practice.

In the third example, I was struck by the layers of learning that emerged from a classroom event.

3. In Chap. 9 (Van Zoest and Levin), we gain insight from a three-way interaction, over time, between MTE (LVZ), MTE (ML) and teacher Karry: During Karry's teaching, a mathematical explanation seems to cause confusion, and when discussion stalled, LVZ intervened:

   LVZ (to the class): It seemed like we were on to something and then all of a sudden it started to go awry—go wrong—like sometimes math problems do. [slight pause].
       Have you ever had that happen? You have a great idea and you're just cooking along and then all of a sudden it's just not working anymore? [LVZ laughs reassuringly.]

In a written reflection, Karry makes reference to her experience of the intervention:

> Karry (written reflection): Another place I could have been able to fully articulate better towards a clear mathematical goal is when one student's way went awry. What I have learned is that I need to know when to "funnel" and when to "focus" ….
>
> However, after multiple attempts to redirect to what I noticed was wrong with the student's train of thought and kind of beating around the bush, this is when I should have switched from "focus" to some form of "funnelling".

Subsequent analysis reveals a range of issues; LVZ's choice of action, to intervene in the class; the focus of the intervention; analysis of various data from the event; and the wisdom of switching from focus to funnelling. Discussion between LVZ (expert MTE) and ML (novice MTE) leads to MLs becoming aware of the complexity of issues for both teacher Karry and MTE LVZ. I urge you to read the full account.

In all three examples, I was made aware of my own entering into the experience described and feeling for myself how it might have been for me in that situation. As John Mason has so vividly expressed (e.g. 2002), it is by entering into moments in our own experience, recognising and *noticing* issues we have faced ourselves, that we gain the possibility to externalise and to re-enter such experiences, both in and outside the event. Having access to the voice of the teacher or MTE (spoken or written) both enables us to *hear* that person and also *to enter the experience* in our own practice.

These examples of reflection draw attention to the matter of "voice." We can talk *about* what we do, or what others do, but *giving voice to someone* is about having that person speak (or write) their own reflections on what they have experienced and learned and how this does or can affect their work in the future. In the ICME 13 survey on "Teachers working and learning through collaboration," in our reading of many relevant papers, we reported that the teachers' voice was largely absent. What we read was written by researchers talking *about* the teachers with whom they worked and their learning and development (Robutti et al., 2016). As Chapman (2008) pointed out, in research reports from researchers who were also MTEs, the MTE voice was rarely heard reflecting *on their own learning and development*. Rather, we read what they observed of the teachers they studied and their analysis of the resulting development of teaching practice. In recalling an experience, talking *about* the experience is very different from talking *in* or *from* the experience, giving voice to the thoughts and feelings the experience created.

*Do we, or how do we, engage with a methodology of promoting or giving voice to our teachers and to ourselves as a device for learning and development?*

## 22.2.2   Collaboration and Inquiry

Although not always labelled as such, many of the chapters discuss "communities of inquiry" within a research and development setting. For example, in Chap. 15 (Bakogianni et al.), a group of 11 MTEs formed a community of inquiry to share

experiences and learn from each other's practice with regard to supporting teachers to link mathematics and workplace situations in inquiry-based learning. In the examples from Chaps. 9 and 16, we see two pairs of MTE colleagues each acting as a small community of inquiry. The idea of community of inquiry (CoI) is that a group of practitioners (in these cases, the practitioners are MTEs) collaborate to explore activities and issues from their practice for mutual support, learning and development. They literally *inquire into* their activities and address the issues raised; this gives rise to identifying and exposing issues and tensions relating to inquiry in practice. Two quotations illustrate such tensions related to issues of teachers' and researchers' collaboration and the role of teacher as researcher:

> The involvement in supporting the teachers was a learning experience, teachers, educators and students, we are all learners. This is what we are doing. We are learning how to communicate (Chloe, 4th meeting).
>
> Teachers have to be reinforced to communicate through the platform between themselves… to inquire by themselves… to search for resources (Sofia, 5th meeting).

The words used to express relationships and issues are revealing of the speaker's focus – on their own learning or on the teachers' learning, or on both.

Chapter 12 (Sikko and Grimeland) presents a "self-study" in which two mathematicians form a community of inquiry to address what a pure mathematician needs to learn in order to become an MTE. Reflection and collaboration are evident in their inquiry. The nature of the self-study involved the two inquirers in lengthy discussions drawing on literature, a range of artefacts including lecture notes and presentations, conversations with other colleagues and personal reflections. They write the following:

> As an MTE you need to inquire into your own practice. This includes inquiring into the choice of models and representations, trying out new approaches, not being "locked" into one particular way of doing things but instead continuing to reflect upon your own practice.

One example illustrates the ways in which mathematics and pedagogy became linked:

> An example is the concept of division, where neither of the authors was aware of the distinction between partitive and quotitive models of division prior to moving into teacher education. For the mathematician, this distinction is not important … For the teacher, on the other hand, … the question is rather how to be able to help pupils extend their understanding of division from division of integers to division involving fractions, and which representations and models are helpful in this extension.

The idea of community of inquiry (CoI) can be linked closely to theory of community of practice (CoP; Jaworski, 2006; Wenger, 1998). In Chap. 8 (Olanoff et al.), we are told that three MTEs (two novices and one experienced) form a CoP to develop their mathematical knowledge for teaching teachers and improve their teaching of mathematics content courses for prospective elementary school teachers. With relation to CoP (Wenger, 1998), they write, "Through our mutual engagement and shared repertoire (e.g., reflections, memos, tasks, lesson plans), we came to realise the importance of looking deeply at the underlying mathematics behind the representations, algorithms, and definitions that we use." However, they also talk of inquiry, referring particularly to Cochran-Smith and Lytle's (2009) "inquiry as

stance." As I read the chapter, appreciating reflective writing from the two novices on how the CoP theory had influenced their personal learning and development, it was not until reflections from the experienced MTE that inquiry became explicit in the reflection. The following quotations illustrate:

> Dana: Participating in the CoP helped me to develop my pedagogical content knowledge, specifically knowledge of content and students. … writing down what happened with my students and thinking about how to help them construct knowledge and see problems with their work helped me make connections and figure out ways to help my students in the future. Being able to share the experience with the other members of the CoP also helped me develop my own knowledge in a way that reflecting on my own would not.
>
> Patrick: I believe the co-teaching/observation experience and writing a memo is really helping me reflect on how I can make this course a better course for the students. By reflecting on the students' struggle, the goal of the activity and my actions as an instructor combined with my observation in Jo's class, I am getting an opportunity to think about my teaching more than I would normally have done.
>
> Joanna: I benefited from the mutual engagement of having other people to think carefully about how to support PTs in understanding the mathematical concepts underpinning the procedures they would be teaching in the future. For example, the CoP with Dana and Patrick caused me to rethink how I engaged PTs in thinking about tasks involving probability. … I also learned by observing Patrick and Dana teach and saw some things that they did (e.g., how Patrick engaged his students in thinking about necessary and sufficient conditions for definitions of plane figures) that provided me with new insight into my own teaching practices. … I have changed my practice as a result of participating in the CoP as I am more intentional in approaching my teaching through a stance of inquiry.

In these statements, although "inquiry" is not articulated until the very end, I believe these MTEs are all inquiring into their own teaching using a number of tools to aid them and finding out more about themselves and the teaching approaches they engage with. The position of "inquiry as stance" lies behind their statements, even if not uttered specifically. We see again here how collaboration and inquiry are important for these MTEs in researching their own developments in practice. Although not part of the theoretical basis of this chapter, I would encourage these researchers and readers of this chapter to consider the theoretical underpinnings of CoP and *inquiry as stance* as constituting a *community of inquiry* linking reflection and development through collaborative inquiry.

*In what ways do we use theoretical terms like community, practice, collaboration or inquiry in our reflections and communications? What can they offer us for learning and development?*

### 22.2.3   Theoretical Underpinnings

In the above examples, we see MTEs using *community of practice* as a theoretical basis for their inquiry. As Lawrence Stenhouse (1984) reminded us, *research is systematic inquiry made public*. Transitioning from CoP to CoI is a recognition that the inquiry basis of activity is fundamental to this research. The significance of acknowledging inquiry as a theoretical element of the community activity is that it

brings with it the construct of "critical alignment." Wenger (1998) speaks of three elements of belonging to a CoP*: engagement, imagination and alignment*. We engage with the practice alongside our co-participants, we use imagination to guide our own trajectories in the practice and we align with the norms and expectations within the practice. While CoI draws on many of the postulates within CoP, including "engagement" and "imagination," the concept of "alignment" is tricky. Thinking of *teaching* as the practice under consideration, with the goals of supporting student learning (of mathematics), and then *aligning with* the practice as it exists might support elements of practice that many professionals would like to change (e.g. rote learning). However, some elements of practice are deeply ingrained in what schools and teachers do and have been doing for years; they cannot easily be changed overnight; the concept of *alignment* supports the lack of change. Thus, in a CoI, theoretically, the postulate of alignment is modified to become "critical alignment" (Jaworski, 2006). Critical alignment is a theoretical basis for the inquiry of change. In practice, it means that we do not align uncritically. By inquiring into our practice as we engage with it, we consider what ideally we should like to do and see: we *look critically* at the status quo, discuss with our colleagues and seek ways to bring in the changes that can lead to the outcomes we would like to see. Of course, this might be a lengthy process involving cycles of innovation and reflection through which we learn about what is possible as well as desirable. For example, in the quotations above, Karry might carry out her intention to "funnel rather than focus" and then discover other issues or tensions in the outcome, and further attention to the elements of funnelling could be necessary, prompting further innovation. In practice, the inquiry of critical alignment can be lengthy and challenging, requiring much reflection, sharing with supportive colleagues and willingness to sustain uncertainty, a significant process of learning and development and, ultimately, sustainable outcomes.

Other theoretical perspectives are used by researchers in these chapters. For example, in Chaps. 10 and 11, by Brown et al. and Bissell et al., researchers use *enactivism* as their theoretical foundation. I am reminded that Sandy Dawson (1999) referred to enactivism in practice as "a path laid while walking" (p. 148); literally, we achieve the path we want, as MTEs, teachers and students, as we engage with the practice of doing what we do and, I add this, looking critically at what we achieve. This suggests that the path laid while walking might be seen as a form of critical alignment in practice. Dawson quotes Bakhtin in writing, "we are completely responsible for our actions and it is in knowledge garnered through embodied action that ethical responsibility lies" (p. 149). He raises issues with those who judge teachers and teaching as "wrong" with associated claims of what *should* be done in classrooms – the "right" thing. He writes, "Part of the motivation behind the development of the enactivist view is a questioning of current views of the nature of knowledge development and acquisition in the mathematics classroom" (p. 149). As an alternative, he proposes the following:

> Consider for a moment a different approach … one based on becoming aware of what you are doing without judging it. … mathematics teacher educators and mathematics teachers could move from a culture based on judgment to one based on possibility (p. 148).

For me, this is entirely consistent with critical alignment.

Bissell et al., in Chap. 11, write of enactivism as "Using what we have done previously in a new environment will be followed by adapting when what happens is not effective or good-enough for the situation" and "Identifying feelings of being uncomfortable and staying with the detail of what happened can support our learning by opening up new possibilities for acting." As we read on and encounter Alistair and his transition from being a mathematics teacher to becoming an MTE, we see how these words relate to the actions, experiences and feelings he reports.

In Chap. 10, Brown et al. acknowledge that enactivism guides the processes that they use as MTEs and, indeed, underpins the design of their teacher education course (for prospective teachers). They write, "the processes we use as MTEs to develop our practices are the same as those our prospective teachers are offered to develop their practices." However, enactivism as a guiding force is less upfront in this chapter than in Chap. 11 (for some of the same researchers). Here, in Chap. 10, several further constructs are offered to describe/explain teaching/learning developmental processes. The first is *awareness*. With reference to a number of well-known scholars, awareness is used as a "synonym for consciousness," as "the world experienced by the person" and as "a core action or function that must be present in order to learn." They claim "Only awareness is educable," suggesting that "this is the chief role of the mathematics teacher, while keeping open the ways in which it might happen." They quote Dave Hewitt (2001) as follows:

> By educating awareness the mathematician inside a student is being educated, which would not be the case if everything were treated as if it were to be memorised. Awareness informs decisions and how to act using information which is known. (p.38).

Brown et al. exemplify awareness as follows: "an awareness of counting squares covered by a shape might allow attention to be drawn to a definition of area; and an awareness of tangents to a curve might allow attention to be drawn to stationary points of the curve." Course design takes account of layers of awareness as MTEs and prospective teachers follow the same processes in teaching and learning, but in educating teachers' awareness, MTEs need to become aware of the awarenesses of the teachers in relation to the teachers' awarenesses of mathematics, not forgetting of course their own awarenesses – a complex set of relationships. I see these theories or constructs, enactivism and awareness, as integral to the nature of the project, providing a philosophy and methodology underpinning the research. As well as the construct of awareness, these researchers refer to other theoretical constructs including *metacommunication, second-person perspectives* and *experiences to issues to action*. I leave it to readers to follow these up in the chapter and to link them to the overarching perspectives of enactivism.

A range of other theoretical perspectives are evident in other chapters. For example, in Chap. 14, Ingram et al., within a broad sociocultural perspective, refer to "two theoretical models of professional learning": Clarke and Hollingsworth's "interconnected model of professional growth" and Merrilyn Goos's adaptation of Valsiner's *zone theory*. Here, the sociocultural perspective seems to be a philosophy underpinning the activity and research, while the models are used as a lens to examine or analyse the ways in which professional growth changes the context in which

growth occurs. This seems a different use of theory from that in enactivism, awareness and other perspectives above. The models here are used in the analytical process to make sense of the data, rather than to provide constructs in the developmental process itself.

Two chapters use the concept of *boundary crossing* between communities of practice. The concept of boundary crossing seems to me to be both integral to the developmental process and a tool or provision of tools for analysis. Bakogianni et al., in Chap. 15, see *boundary crossing* between different practices as a way to address learning, using Akkerman and Bakker's (2011) four mechanisms: identification, coordination, reflection and transformation. These they apply to *boundary objects* such as curriculum materials, representations, school or workplace records that facilitate interactions and crossings at the boundaries. They recognise many "tensions" arising for MTEs from perspectives and activity across the various communities within the project; they use the lens of boundary crossing to analyse MTEs' tensions and to bring the work of MTEs and researchers closer to the teachers' and students' reality. Sikko and Grimeland, in Chap. 12, use the concept of boundary crossing to explore relationships between communities of mathematicians and mathematics teacher educators, using the learning mechanisms already mentioned above. They used Jaworski's (2003) framework for analysing teaching-learning development in co-learning partnerships and overlaid it with learning mechanisms in the boundary between mathematics and mathematics education. Thus, we see the framework used as an analytical tool, whereas the theory of boundary crossings seems to be both a developmental and an analytical tool.

Finally, three chapters report theoretical principles closely related to the philosophy of mathematics learning and teaching espoused by the researchers. For Nolan and Keazer, in Chap. 16, theory provides a basis for their course on *Culturally Relevant Pedagogies* in which they study their own teaching practice. They write, "the theoretical premise of our research and teaching as discussed in this chapter is grounded in efforts to disrupt and decolonise NUC [Near-Universal, Conventional] mathematics." Their desire is to challenge dominant discourses of "training" and "preparation" in mathematics education and the notion that mathematics is value-free and culturally neutral. In their teaching practice, they seek what can be seen theoretically as a pedagogy of opposition and a mathematics education that privileges issues of power and social justice. Their study draws on reflections and narratives from their own teaching and the dialogue that emerges between them as they look critically at tensions and dilemmas in a practice that embodies the theoretical principles on which it is based.

Situated within a broadly constructivist paradigm, Osborn et al. (Chap. 13) focus on collective identity relating to collective agency among MTEs working across disciplinary boundaries, here specifically mathematics and statistics. Researchers see collective identity both as a gestalt in their focus on identity and as having multiple layers of significance for the study, addressing the question "Do we, the project team, indeed have a collective identity?" In addressing this question, they take a narrative, storytelling approach, analysing narratives of individuals to discern commonalities of rapport and appreciation. They noticed differences between seeing themselves as members of the project team and separately and historically as MTEs,

the latter perhaps challenging the construct of collectivity more than the former. However, their attentions to project legacy indicated a collective desire to form a continuing community of practice.

Van Zoest and Levin, Chap. 9, started from (consistent) perspectives of "Inquiry as stance" (from Cochran-Smith & Lytle, 2009) and "Inquiry as a tool" (from Jaworski, 2006) to address their own development as MTEs. Their approach to collaborative practice – *Artefact-enhanced Collegial Inquiry* (ACI) – emerged from their early experience of putting inquiry into practice as well as from their guiding literature. Thus, ACI was embedded in their practice and also provided a framework for analysing data with three phases of inquiry-based activity. Their roles as experienced and novice MTEs (similar to those in the chapter of Sikko and Grimeland) and their associated learning were both differently significant and commonly rewarding, enhanced through the ACI framework.

This panorama of theoretical perspectives seems broadly to be distinguishable in three ways: theories or theoretical constructs that guide developmental processes internally and allow a critical questioning of developmental outcomes, theories or theoretical constructs that provide an external analytical process to make sense of the outcomes of developmental processes and lastly theories or theoretical constructs that do both.

*We have seen here a range of theoretical perspectives, their uses in research and for development by MTEs for themselves and their teachers. In what ways, if at all, do we see the theoretical perspectives and the outcomes of research activity to be related?*

### 22.2.4   Methodology

While it seemed important to address some of the nuances as well as the detail of theoretical perspectives in these chapters, I am somewhat daunted when I consider doing the same for methodology. Thus, I have decided to focus on a few things that I have noticed that seem to permeate several of the chapters and some things that I think extend our ways of presenting ourselves to our MTE community as a whole.

One of the first things I noticed was the use of first names in reporting from the data. This was very obvious when the names used were names of chapter authors. For example, in Chap. 10 (Brown et al.), we meet, in the order of years of experience, Laurinda, Alf, Tracy and Julian who are both the MTEs reflecting on their practice and the authors of the chapter. Here, we gain insight into the personal narratives of these practitioners, reflected in the phrase "revealing the lived experience," which is achieved by both personal storytelling and what the authors call "second-person" interviewing. In Chap. 9 (Van Zoest and Levin), initials are used, so we meet authors LVZ and ML reflecting on their own learning and that of their teacher students. I have mused on what difference of effect it makes, revealing ourselves in first-person reflections addressing our learning and development versus a more distanced, third-person passive voice. In the former, we treat our own experiencing both as individuals with personal identity and as representatives of our

(international) community of MTEs. In the latter, we try to offer a distanced, perhaps more rational, perspective, perhaps seeking for greater objectivity but missing the emotional and psychological impact that we experience.

*How much are we prepared to reveal of our own perceptions and perspectives and our learning from them, capturing vividly our issues, tensions or contradictions, and to what extent are we more comfortable with presenting a general or common rationality, objectively argued? Where are we most likely to find one or the other?*

Sikko and Grimeland, Chap. 12, refer to John Mason's work in stating the following:

> Whereas in mathematics, knowledge is built by adding new theorems to old, education is a journey of self-discovery where each new traveller has to re-experience, re-learn, re-express and re-integrate what previous generations have learned.

My own view, and one I have pursued myself in a number of publications, is that our willingness to reveal our personal learning and its associated challenges (the "lived experience") can be powerful in discerning insights and issues deeply germane to our community and especially instructive for its novices.

This brings me to the methods and modes by which we share our experiences and analyse their significance for the learning and development of our students and colleagues and as elements of wider theoretical understandings and practical guides. Again, I notice pervasiveness, this time of the use of narrative accounts, stories, either as data for further analysis or as a narrative analytical style. In the chapters here, we see both, and in some cases, it is hard to separate them. For example, in Chaps. 8 and 14 (Olanoff et al. and Ingram et al.), I think we see a form of narrative analysis, while in Chaps. 11 and 16 (Bissell et al. and Nolan and Keazer), we see raw narrative. When I say "raw," I don't mean it has not been worked on, but I see a (lengthy) story told in the "I" form, rather than selected extracts juxtaposed to illustrate some key analytical construct. Both are, of course, important analytical forms. I hurry to emphasise that these are my own views and that the authors might disagree. Let's say these are conjectures for consideration. I recognised the chapter by Bakogianni et al. as being different methodologically from some of the others. Here, the participants have pseudonyms which label extracts from their contributions in the project meetings. I can see that the large number of participants in the project and associated issues of confidentiality possibly influenced this choice.

*How do we choose the modes through which to share our personal experiences and learning? How do we want the chosen mode to influence the response of others to what we try to convey?*

### 22.2.5   Learning from the Literature

I expect that we all encourage our students to read, read and read again. It goes without saying that becoming familiar with the literature in our research areas and beyond is a principal plank in our research methodology. All chapters,

unsurprisingly, include substantial referencing of the literature. Indeed, when we review papers, the literature review is both an indication that the author has attended to theory and research relevant to their focus and personally informative for the reviewer. In some of the chapters here, we see direct reference to encouraging our students to read. For example, in Chap. 14, Ingram et al. outline the professional development which a master's programme provides for practising MTEs, offering different kinds of stimulus which include "directed readings." We see some of the value of "directed" readings in the following quotation:

> Clare, who had long assumed that many prospective teachers had fixed ideas about mathematics as a subject and about the process of teaching and learning mathematics, was anxious to find ways of stimulating more active discussion – and thereby potential re-evaluation – of their ideas. Further thought about this issue was stimulated by two readings: one that demonstrated how deep-rooted these beliefs are … and another which suggested that such beliefs might be held consciously or unconsciously …. The reading, as a stimulus in the external domain, prompted Clare to reflect on her existing beliefs … which were strengthened, giving her the confidence to suggest changes in practice to her team of tutors.

In Chap. 12, Sikko and Grimeland, as mathematicians and MTEs, one experienced and one novice, found a reading group and the literature it addressed extremely valuable as seen in the quotation below:

> Attending an organised "reading group" on topics of mathematics education research, and research methods in the field, made a big contribution to her understanding of the nature of research in mathematics education and about relevant questions in mathematics education research. The group was led by "more knowledgeable others" in the form of more experienced colleagues, including the first author. … the readings in the form of journal papers and book chapters played a role as boundary objects. In this way, the second author became a participant in a community in which she was able to build a basis of knowledge that would have taken much longer to develop in a less organised setting, as experienced by the first author. Both authors found the reading group an opportunity to discuss research literature at the appropriate level in a community open to questions of any kind, providing learning for both the newcomers and the mentors.

As well as the significance of learning through reading the literature, these two examples emphasise the importance of some more formal approach to this reading: in the first case as an integral part of an accredited course with required reading (in this case a master's course) and in the second case as part of a reading group which provides both support and structure as well as recommended reading. Support through others of varying degrees of experience and structure through the course or reading group provide building blocks for all participants.

### 22.2.6   Another Chapter

In the sections above, I believe I made reference to all chapters in Theme 2, except for one, Chap. 17 (Wu and Cai). This chapter provides a fascinating introduction to teacher education in China. It provides an account of educational stages and their content in China, with a particular focus on the education of teachers, leading to a

detailed discussion of MTE activity and development. In fact, the chapter emphasises important distinctions between three kinds of MTE with different roles and developmental routes: university-based MTEs, school-based mathematics mentor teachers and MTRs, mathematics teacher researchers. Together, these three groups, despite their different names, fulfil the roles of MTE that correspond to MTE roles in the chapters above.

Graduates with a bachelor degree progress to become teachers, later possibly mentor teachers or MTRs; some graduates progress through master's studies to become teachers and possibly mentor teachers or MTRs; master's graduates can also progress to become university-based MTEs. The system is complex providing a range of education and support for teachers. The system has a long history of research in schools, where MTRs lead research activity in which teachers engage to explore and learn from teaching-learning experiences. University MTEs design and implement mathematics teacher education programmes in the university for both prospective and practising teachers.

Master's programmes include courses in mathematics, so both MTRs and MTEs are well qualified mathematically; MTEs, with doctoral qualifications, are knowledgeable in educational theory. However, there is little support for either group in the roles they are expected to fulfil in educating teachers. A research study, described in the chapter, surveyed university-based MTEs, working mainly in mathematics departments, focusing on the challenges they faced in their teaching of pedagogical courses. Perhaps unsurprisingly, the results showed more challenges in teaching pedagogically related courses than teaching undergraduate mathematics, which links directly to what we learn from Sikko and Grimeland in Chap. 12. It would be great to read some personal reflections from the Chinese MTEs.

More telling for those of us in westernised educational contexts is the reported difference between the teaching and expertise of MTEs and MTRs. While the MTEs excel in theoretical knowledge but have little practical pedagogically focused teaching experience, MTRs have the experience of being teachers themselves and have developed research expertise through their experience as researchers in schools and classrooms. The two groups are complementary in their education, experiences and qualifications and, seemingly, could learn much from working together. It would have been interesting to read more chapters from such backgrounds, perhaps with MTEs and MTRs inquiring into and reflecting on their developing activity and its challenges.

*I am impressed by the roles and facets of MTEs' activity and the challenges they face as revealed in these chapters. I wonder if the experiences revealed by others illuminate or challenge the situations and contexts we experience ourselves?*

## 22.3 Theme 3: Methodological Challenges in Researching MTE Expertise, Learning and Development

As in my beginnings in addressing Theme 2, I extract what seems to be a guiding paragraph, from the editors' prospectus, focusing on Theme 3:

Zaslavsky and Leikin (2004) introduced the role of *mathematics teacher educator educator* to describe a person responsible for the development of mathematics teacher educators. This introduces a new "layer" that could be seen as analogous to mathematics teachers researching their students, and mathematics teacher educators researching mathematics teachers. MTE educators could thus be the appropriate people to research mathematics teacher educators. In reality, however, those who take the role of MTE educators often are also mathematics teacher educators and hence, as was the case in the study reported by Zaslavsky and Leikin (2004), likely to be involved in the milieus that they are researching as well as personally engaged with the same issues with which their research subjects (mathematics teacher educators) are grappling.

While recognising that the authors of chapters included under Theme 2, and discussed above, are in many ways "likely to be involved in the milieus *that they are researching* as well as personally engaged with the same issues with which *their research subjects* (mathematics teacher educators) are grappling," this section of the book includes just three chapters explicitly. My first challenge was to think about how these three were "different" from the ten chapters in Theme 2. I decided to start by addressing overtly the roles of the researchers and those of their research subjects.

In Chap. 18, by Oates, Muir, Murphy, Reaburn and Maher, in Tasmania, Australia, the researchers are the authors of the chapter, who are MTEs in a teacher education programme educating prospective primary school teachers. These researchers' research subjects are themselves, as a group, working within their teacher education programme and exploring the factors that underpin decisions they make as MTEs. In this respect, their activity fits within Martin Simon's second category as discussed above. In contrast, while the researchers in Chap. 19, Rojas, Montenegro, Goizueta and Martínez, in Chile, are teacher educators; their research subjects are both the teachers who participated in the MTEs' courses on modelling and themselves in action with these teachers. The teachers were addressed through a questionnaire and interviews focusing on their experiences of participation in the MTEs' courses; thus, we might, again, locate the activity in Simon's second category. In Chap. 20, by Arzarello and Taranto, in Italy, the complexity of participation and relationships between the authors, researchers, MTEs and researcher-teachers makes this indeed a complex milieu in which to distinguish researchers and research subjects. It is also hard to locate in relation to Simon's categories, but I tend to see it also in Category 2 since the learning of the MTEs is related to the MOOC courses for teachers as well as the other practitioners in their construction.

One of the guiding questions raised by Arzarello and Taranto is the following: "What are the dilemmas and opportunities associated with researching ourselves as MTEs?" They regard this question as a self-referential problem creating dilemmas for MTEs. In the Italian tradition, they claim that the figure of *researcher-teacher* can be a solution for this problem. They tell us that the researcher-teacher is a common role: "In our case, the researcher-teachers are in-service [practising] teachers who have been collaborating with our research group in mathematics education for several years." They see researcher-teachers as "brokers" in the divide between teachers and university researchers. I found it interesting to compare the role of researcher-teacher in the Italian tradition with that of MTRs, mathematics teacher

researchers, in the Chinese tradition, articulated in Chap. 17, although I have not the space to follow this up here.

This question, posed by Arzarello and Taranto, and the associated self-referential "dilemmas" led me to look more closely at the chapter of Oates et al. who, like researchers in several of the chapters in Theme 2, conduct research into their own practices as MTEs. Oates et al. make the following statement:

> It was clear to us all that our PCK as MTEs plays a significant role in underpinning the decisions we make about course and unit design, and, at this time, we lacked an appropriate theoretical approach to analyse these effects. We decided to deepen the extent of the review, to document and interrogate the process we undergo in the collective redevelopment of our units and explore theoretical bases for the decisions we make.

Thus, I see the "theoretical bases" being, potentially, a way in which these researchers overcome the self-referential problem raised by Arzarello and Taranto. For Oates et al., the two theoretical perspectives are *activity theory*, rooted in the work of Vygotsky and A. N. Leont'ev and developed by Yrio Engeström among many others, and *professional capital*, arising from Strober's notion of *human capital*. The research to which these perspectives are applied involved a study of these researchers' review of their *Bachelor of Education* course for educating primary school teachers. This review followed a survey of the previous cohort of students (prospective teachers) concerning the value and alignment of content and assessment in three core mathematics units, addressing how effective these units were in preparing students to teach mathematics. Their review began in a meeting to discuss the outcomes of the survey, which led to a recognition of a complexity of factors which deserved further attention. Thus, the data for their study consisted of recordings of further meetings and focus group interviews which were analysed through a discourse analysis to identify emerging themes. The chapter reveals these themes with reference to anonymised quotations from members of the MTE team.

The authors write, "Meeting four raised some questions about these themes, for example, whether they might be legitimately emerging from the data compared to what we were predisposed to look for." I see this question as addressing directly the dilemmas raised by Arzarello and Taranto. Readers will follow this up specifically in the chapter itself; however, I quote briefly on the effect of one of their theoretical perspectives, that of activity theory. The authors write the following:

> Reflection on the themes using activity theory enabled us to interpret aspects of the activity and to explore the complexities that underlie our actions. The review meetings and interviews uncovered our activity to the extent that we can begin to see how we are influenced by the different aspects in the system. Hence, we now have a deeper view of the factors involved in determining the outcome of the activity, that is, the possible redesign to better develop effective pedagogical knowledge for teaching mathematics with primary preservice teachers.

Thus, we see that the researchers' use of activity theory helped them to see beyond their own extraction of themes, to a revealing of factors and relationships that took them more deeply into their own thematic analyses.

This use of external theory to allow an alternative way to inspect self-referential outcomes of research is clearly one of the focuses of Theme 3. Arzarello and

Taranto, focusing on the development of massive open online courses (MOOCS) for educating prospective teachers, also used two theoretical perspectives to analyse the contribution of MTEs and of the researcher-teachers to the preparation of the MOOCs. MOOCs are designed to engage students at a distance, and it is up to the students to design their own course through elements provided in the MOOC. The theoretical perspectives used here are meta-didactical transposition (MDT), as developed originally by Ferdinando Arzarello and colleagues, and Valsiner's zone theory as developed by Merrilyn Goos. The author, Eugenia Taranto, had modified these theoretical perspectives, adapting first MDT to the MOOC environment to produce MOOC-MDT and then networking with zone theory to produce MOOC's zone theory. Details are in the chapter. A key element of MDT "uses the notion of *broker* as a professional who belongs to more than one community and makes possible exchanges between them," a role similar to that of brokers in Wenger's community of practice theory and Engeström's activity theory. *MOOC's zone theory* adapts the brokering role to the nature of the MOOC, with the roles of MTEs in relation to the MOOC's content and structure being analysed through zones of free movement and promoted action mediated by the researcher-teachers as brokers. My brief account cannot do justice to the complexities here, but the point I want to make is that theory is being used here both to explain and to examine the ways in which MTEs both contribute to the MOOC but are also distanced from it by the brokering. The authors see this theoretical mediation as a "fresh approach to addressing the self-reference dilemma experienced by MTEs who wish to analyse their own learning and development." Thus, in both Chaps. 18 and 20, we see theory playing a methodological role in the self-referential dilemma acknowledged in both chapters.

In contrast, in Chap. 19 (Rojas et al.), the methodological challenges relate to theories of *modelling* which guide how MTEs "model" processes and actions for teachers in the teachers' learning and subsequently teaching of mathematics. Aims of the modelling process are for teachers not only to gain access themselves to the mathematical concepts being taught but also to be aware of what the MTE is doing that contributes to their learning and can be used subsequently in their teaching of students in school classrooms. These processes involve a complexity of factors – whether the MTE as teacher is aware of modelling for her students, the extent to which the modelling is explicit for the MTE, the extent to which students are overtly aware of the modelling processes and how modelling in a university context can be transferrable into school classrooms. Research has focused on all these elements in addressing both prospective teachers' perspectives and MTEs' perspectives. The chapter proposes "a new methodological challenge" that a community of practice involving prospective teachers and their MTEs might together explore modelling practices and their contribution to the learning of both the MTEs and the teachers. The authors conclude with the following words:

> we believe that reconceptualising research on modelling from a more integrated, holistic perspective should take into account the complementary roles of mathematics teacher educators and prospective teachers and how they complement each other in the challenge of learning to teach mathematics.

It strikes me that we might see here a community of inquiry involving MTEs and their prospective teachers inquiring together into the ways in which MTEs' modelling is effective in enabling teachers to develop their own learning and their teaching practices.

Related to all three papers in this section, we might ask the following:

*What are the challenges that we MTEs face as teachers of teachers in the learning of mathematics and how we teach mathematics? How do these compare with our teachers' learning? What theoretical perspectives guide our inquiry, enable us to look critically at the self-referential nature of our own learning and development and allow our knowledge and practice to develop objectively?*

## 22.4   In Conclusion

I have been inspired by my reading of chapters in these themes, and although I have exceeded my word limit, I have left unsaid much of what I have learned from my reading. It has been my pleasure and privilege to engage with some of the key ideas in these chapters and offer them here. Perhaps drawing attention to what has stimulated me to consider and question might lead you to delve more deeply into what is written and follow your own threads in these texts. Please be aware that what I say here is my own version of what is written, my responsibility, not that of the authors. I heartily recommend that you read the chapters themselves for a full enjoyment of the richness that is offered.

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