Chapter 15
Mathematics Teacher Educators' Learning in Supporting Teachers to Link Mathematics and Workplace Situations in Classroom Teaching

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15.1 Introduction

The chapter aims to provide insight into the development of mathematics teacher educators' (MTEs') professional learning by reflecting on their attempts to facilitate primary and secondary school teachers' professional development in the context of a European-funded project, Mascil (Mathematics and Science for Life). The Mascil project brought together 18 partners from 13 countries in order to promote inquiry-based learning (IBL) and connect school mathematical activity and authentic workplace situations (Mascil project, 2013, https://mascil-project.ph-freiburg.de/). To achieve these goals, in Greece, professional development (PD) activities were designed where science and mathematics teachers collaborated in groups to design, implement and analyse lessons in the spirit of lesson study approaches (Hart, Alston, & Murata, 2011). MTEs were predominantly academic researchers, teachers with Master's studies or school mentors in mathematics or science education.

The major challenge of the project for the MTEs' group was to link workplace situations with mathematics teaching in the context of PD activities. Although workplace settings can be seen as rich and meaningful contexts for students' mathematical understanding (e.g. Hoyles & Noss, 2001; Wake, 2014), connecting these contexts to classroom teaching appears to constitute a complex task for mathematics teachers (Nicol, 2002; Potari et al., 2016; Triantafillou, Psycharis, Potari, Zachariades, & Spiliotopoulou, 2017). Additionally, the linkage of workplace situations and

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mathematics teaching has been studied mostly in vocational school settings (e.g. Bakker, 2014). Thus, very little is known as to how innovations of that kind can be introduced into existing educational contexts and teaching realities. As regards the Mascil project and the Greek educational context, the formalistic view of the official curriculum, the lack of accessibility to workplace settings and resources and teachers' unfamiliarity with inquiry approaches constituted some added concerns for the national MTE group. Moreover, the collaboration between participants from different teaching subjects, although this might be seen as creating meaningful PD opportunities for all of the participants (Frykholm & Glasson, 2005), was a rather demanding task for the MTEs' group to deal with in the Greek educational context, where cooperation among teachers is not encouraged. The MTEs' own professional differences in terms of research, teaching and experiences in educating teachers provided some extra challenges that the MTEs' group had to deal with.

All the above complexities that stem either from the project itself or the Greek educational context provide a challenging site for MTEs' learning, the study of which can contribute to the growing research field focusing on forms of knowledge, competencies and challenges related to MTEs' practice and development (Ball & Even, 2009; Jaworski & Huang, 2014; Jaworski & Wood, 2008). The present study aims to trace the path of a group of MTEs endeavouring to support teachers through PD activities to employ inquiry-based teaching approaches targeting the connection between mathematics and workplace situations. Particularly, we want to investigate the following research questions:

What are the MTEs' concerns expressed in the design and enactment of the PD activities?

What emerging tensions were faced by the MTEs, and how did these tensions contribute to their professional learning?

15.2 Literature Review and Theoretical Background

Facing the challenge of supporting the Mascil project ideas in the Greek context, our work is framed by the term reflective practitioners (Shön, 1987) in two directions, namely, examining the role of teachers as well as the role of MTEs in the development of teaching practice. We view teachers as co-producers and consequently co-responsible in the research process as well as in the development of scientific knowledge (e.g. Ponte & Chapman, 2006). Teachers in this respect are key stakeholders (Kieran, Krainer, & Shaughnessy, 2013), advancing their role to be informed by research findings, to design and evaluate teaching material, to investigate their own practices and to use their own teaching experiences to produce new research findings. Such a demanding role can be cultivated and supported in collaborative contexts, where mathematics teachers, or mathematics teachers and researchers, co-learn in developing teaching practice, such as communities of inquiry (Jaworski, 2006; Potari, Sakonidis, Chatzigoula, & Manaridis, 2010), lesson study (Huang & Bao, 2006) or action research settings (McNiff, 2010).

Jaworski and Huang (2014) emphasised reflective practice as a principal goal for effective development of both mathematics teachers and mathematics educators. They also discussed the competences that mathematics educators need in order to be reflective in what they do. Such competences include being self-aware, reflective and articulate in action and able to explain tacit knowledge of teaching but also comprehensive, rich and deep knowledge, based on theory and theory testing in practice (Smith, 2005). Moreover, it is important for MTEs to develop adaptability; to cope with problems, dilemmas and problem situations; to select and use appropriate tools and resources for teaching; as well as to learn from the study of practices (Zaslavsky, 2008).

An important part of the limited research focusing on forms of knowledge, competences and challenges that are related to professional learning of MTEs is based on contexts where teachers and researchers are collaborators and co-learners in developing opportunities for students' learning and additionally on large-scale programmes of teachers' PD. To bridge the gap between research findings and the actual needs of teaching practice, Goos (2014) reported on ways in which this can be achieved by teachers' and researchers' collaboration. She considers mutuality and complementarity as central in developing expertise within the communities of both teachers and researchers and hence theoretical and practical knowledge in mathematics education. Potari et al. (2010) reported on conflicts and tensions in a 4-year collaboration between secondary school teachers and academic researchers that gradually led to an apprenticeship of both groups in inquiring into mathematics teaching and to a self-understanding and reconceptualisation of mathematics teaching and PD. Research findings thus indicate that collaboration between researchers and teachers, despite constituting a fruitful ground for the professional learning of both groups of participants, is also a terrain of continuous challenges and emerging demands that need to be addressed by the community of academic researchers. The increased demands concerning MTEs' practice have turned the lens of research towards their own professional development (e.g. Krainer, 2008). However, studies that provide relevant empirical evidence related to the development of learning in the case of mathematics teacher educators are still scarce.

This chapter aims to contribute to this open and unexplored field of discussion concerning the development of MTEs by investigating their professional learning as teacher educators in the context of an innovative project.

When MTEs and mathematics teachers collaborate to develop teaching, each brings to the emerging community new forms of mathematics learning and teaching discourse and practice. MTEs might bring the critical and reflective stance and modes of discourse that are valued within the academic community, whereas teachers can bring craft knowledge about pedagogical practices and the sociocultural contexts of their classrooms. Together, these two groups of participants can learn new ways of thinking about their practices and simultaneously create new forms of discourse and practice about mathematics learning and teaching, that is, new communities. These communities, while potentially powerful tools for developing pedagogical practice, may also introduce tensions into the PD experience. These tensions are often due to the mismatching and even conflicting goals of the practice itself but also of the activity within which the “old” and the emerging communities are situated. Focusing on the different goals of the practice, sources of tensions can be identified in the participants’ efforts to align their practice, while taking the view of the different goals of activity, tensions can be traced in participants crossing boundaries between different practices (Wenger, 1998).

Boundaries are dynamic constructions denoting co-location of practices and co-existence of competing discourses. Efforts by individuals or groups at boundaries to restore continuity in action or interaction across practices trigger dialogical engagement and collective reflection, compelling people to reconsider their assumptions and look beyond what is known and familiar (Akkerman & Bakker, 2011). Through collaboration/negotiation at boundaries between different practices, new and hybridised ideas and practices emerge where mutual understanding of shared tasks and problems develops (Edwards & Fowler, 2007). Described as boundary crossing (Engeström, Engeström, & Kärkkäinen, 1995), this process involves moving into unfamiliar territories and requires cognitive retooling. People who cross boundaries are called brokers or boundary crossers, and they are simultaneously members of multiple communities, while objects that cross boundaries are called boundary objects (Akkerman & Bakker, 2011). These objects can be, for example, curriculum materials, representations, school or workplace records that facilitate interactions and crossings at the boundaries.

Boundary crossing between different practices is seen as a way to address learning through four mechanisms: identification, coordination, reflection and transformation (Akkerman & Bakker, 2011). These mechanisms concern the different ways in which learning can occur when people interact with, move across and participate in different practices.

1. Identification: Boundary crossing can lead to a renewed insight into what the different practices concern.
2. Coordination: Boundary crossing can also lead to establishing minimal routine exchanges between two practices so as to facilitate transitions.
3. Reflection: Reflection involves going deeper into the specificities of two practices (perspective-making) and learning to consider one practice by taking on the perspective of the other practice (perspective-taking).
4. Transformation: Transformation leads to changes in practices or even the creation of a new practice that stands between the established ones.

15.3 Methodology

15.3.1 The Context of the Study

The context of this study is the European project Mascil aiming at supporting teachers in using IBL and workplace situations in mathematics and science teaching. In Greece, 11 MTEs (academic researchers, teachers and mentors) with different research and teacher education experiences worked for 1 academic year with 13 groups each comprising about 10 mathematics and science practising teachers (1–2 groups for each educator), who were meant to collaborate in developing shared teaching experiences. The PD activities aimed at promoting both the development of teaching in the direction of the innovative ideas of Mascil, as well as teachers' continuous reflection. Instructional materials in the form of exemplary tasks were provided by the project as a basis for teachers' designs (http://www.fisme.science.uu.nl/publicaties/subsets/mascil/). These tasks were available to the teachers through the project website. However, the teachers could modify them according to their teaching goals or even design new ones aligning with the same philosophy. MTEs could also use a teacher education toolkit provided by the project involving ideas and strategies for organising the PD activities. MTEs used this tool as a resource to design the PD activities, especially during the initial meetings with the teachers. In addition, a communication platform for teachers was available, although this was not widely used in Mascil implementation in Greece.

Overall, the Mascil project aimed to offer professional development to a large number of mathematics and science teachers in the participating countries. Most of the developed resources were translated into the language of each country. Although the project had specific goals, as mentioned in the introduction to this chapter, teacher educators and teachers in every country were flexible in using these resources and adapting them to their national educational context.

15.3.2 The Group of MTEs

In this chapter, our focus is on the group of MTEs of which the authors were members. The profile of each participant is briefly presented in Table 15.1. Sophia was the coordinator of the programme.

Although the group of MTEs consisted of science and mathematics teacher educators, we refer to them as MTEs due to our special focus on mathematics teaching practice.

MTEs collaborated for a period of 1 academic year (October 2014 to June 2015) to develop a mutual plan for the PD activities. We collected data consisting of audio and video recordings based on MTEs' discussions in their meetings (five in total lasting about 3 hours each). A brief description of the focus of the discussion in each meeting is presented in Table 15.2.

15.3.3 Data Analysis Process

The analysis of the data was based on grounded theory approaches (Charmaz, 2006), and it was carried out in two steps. Firstly, following an inductive content analysis approach, we investigated the main concerns of MTEs and issues triggering the group's attention and described them through a systemic network (Bliss, Monk,

Table 15.1 MTEs' profiles

Participants	MTEs' professional status	Research interests
Sofia	University teachers	Development of mathematics teaching and learning and teacher development with experiences on the use of mathematics in workplace situations and its transfer into mathematics teaching
Tim		Teaching and learning calculus in secondary and undergraduate education
Jason		Design of learning environments for mathematics with the use of digital tools and IBL approaches
Ben		Development of mathematics teaching and learning and teacher development in primary education
Elsa		Development of science teaching and learning and teacher development
Anna	Postdoctoral researcher in mathematics education	Use of mathematics in workplace situations and its transfer into mathematics teaching
Diana	PhD student in Mathematics Education	Development of teaching and learning of statistics in secondary education
Ken	Mentor (public schools' advisor offering practice-based professional support to teachers at school)	Mathematics learning in primary and secondary education
Marko and Chloe	Secondary mathematics teachers with Master's degrees in Mathematics Education	Curriculum development and action research
Rose	Secondary science teacher with Master's degree in Science	

Table 15.2 Brief description of the MTEs' meetings

Meeting	Main focus of the discussion
First	Familiarisation with the Mascil ideas and development of resources for the introductory meeting with the teachers
Second	Sharing insights from PD experience and adjusting the PD design
Third	Developing structures to facilitate teacher collaboration and co-design
Fourth	Sharing insights from the teachers' classroom implementations and developing materials to promote teacher reflection
Fifth	Connecting the Mascil ideas with the actual practice. Issues related to the classroom reality, the Greek context, the PD aims, the project's sustainability

& Ogborn, 1983). The network presents the different dimensions in the emerging concerns that co-exist throughout the MTEs' discussions; some of them appeared early, others later.

In the second step of the analysis, we identified tensions inherent in various categories of concerns in the systemic network. Tensions indicated either explicit divergent views among the MTEs or dilemmas implicit in these views. Considering that the identified tensions indicated a boundary, each tension was described, coded and traced in the data in different instances in which it appeared, and it was characterised in terms of the participants, the practices involved and boundary objects. In the present paper, we present two dominant tensions throughout MTEs' discussions: (a) authenticity of workplace situations versus classroom teaching and (b) high versus low degree of teacher autonomy. Next, we coded the process of MTEs' dealing with the boundaries inherent in these tensions by using the four types of learning at the boundaries.

15.4 Results

15.4.1 MTEs' Concerns

Figure 15.1 shows the categorisation of MTEs' concerns. Two categories appear: making sense of how workplace situations and IBL can be linked to mathematics teaching and the enactment of workplace situations and IBL in PD meetings. The categories and subcategories are discussed through illustrative examples below.

15.4.1.1 Making Sense of How Workplace Situations and IBL Can Be Linked to Mathematics Teaching

How to link workplace situations with mathematics teaching was a central issue in all MTEs' meetings, while IBL was discussed in a less extensive way. Workplace situations were seen by some MTEs through the use of tasks based on realistic or scientific contexts: "The ideal would be to have workplace situations related to physics or chemistry and to be able to solve problems in this area and somewhere there will be mathematics." (Jason). Some MTEs with experience from research on mathematics in workplace situations emphasised the need to sustain workplace authenticity in the classroom. Nevertheless, a number of issues that need to be considered seriously when exploiting workplace situations for classroom teaching emerged. One concern related to the complexity of the workplace context indicated by the unfamiliarity of context, representations, symbols and language. For example, Ben argued that "The student needs to learn extra things from the workplace context". Anna addressed complexity and limited accessibility of the workplace context: "We said that these authentic examples take you out from what you are

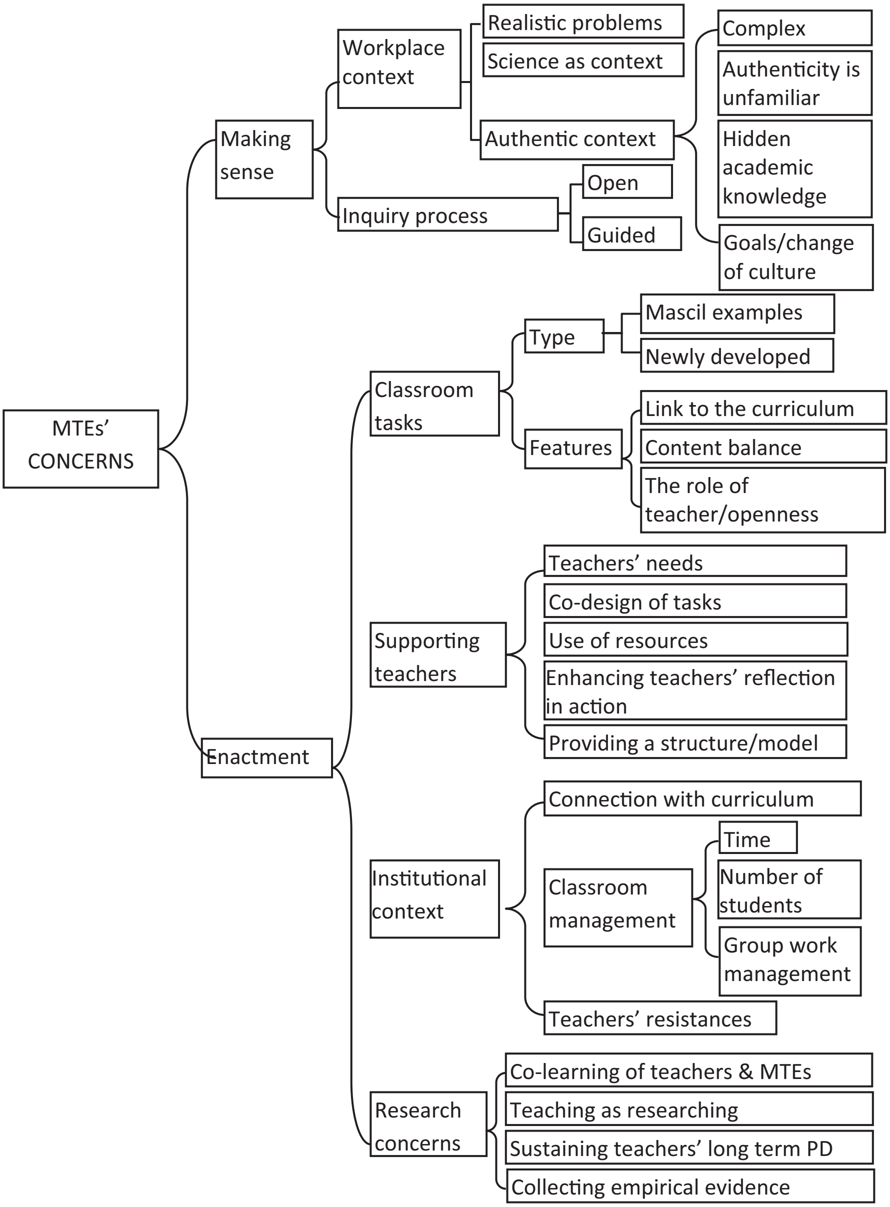

Fig. 15.1 Concerns that emerged during MTEs' meetings (the Bar ([) notation signifies that all the categories are mutually exclusive, while the Bra ({) notation signifies that the categories might co-exist)

familiar with. You see it and you say that I do not want it. Why do I need to understand what they say here?" She also pointed out that mathematics is hidden in the workplace context, but she considered it as a challenge to inquiring into mathematics: "The workplace setting, because it hides a lot of academic mathematics, it gives itself elements of inquiry. This hidden thing helps the inquiry. What is hidden? Why does this relation hold?" Yet another concern was the distance between the workplace culture and the established culture of school mathematics teaching, the former being seen as inferior to the latter: "What shall we do when the teacher says that the tasks that you give me are technical things? I have high goals for my students" (Marcos, 1st meeting).

IBL was considered a familiar construct both for MTEs and teachers: "IBL is more familiar to teachers and teacher educators than workplace settings. Thus, it gives us a basis for developing our PD activities" (Marcos, 1st meeting). This explains the limited focus on IBL in the initial meetings of MTEs. It was initially seen through the use of open tasks and then more related to workplace situations as the process of discovering the hidden mathematics.

15.4.1.2 Enacting Workplace Situations and IBL in PD Meetings

MTEs' concerns in the design and implementation of the PD programme were related to the use of classroom tasks, ways of supporting teachers, the role of the institutional and classroom context and the MTEs' research focus.

The nature of classroom tasks became the focus of the discussion from the beginning, referring to the type and the features of the task. MTEs wondered to what extent the tasks developed in the context of the project could be used in PD meetings and in the classroom. The example below illustrates the above concern: "Even in Mascil tasks, the workplace context is not integrated in a realistic way. It is role playing. In a few cases where the workplace context appears in a realistic way, it seems to exist as an idea" (Diana, 2nd meeting).

Another concern was whether the teachers themselves could develop their own tasks aligned to the project's perspective. The example that follows reveals MTEs' exchange of ideas to motivate teachers in developing their own tasks.

> Teachers in my group proposed a task referring to factors that are related to AIDS. One science teacher sent me some ideas, but he was not able yet to propose a specific task… How can we support teachers to complete their own designs based on contexts that are not familiar to us? (Anna, 4th meeting)

The link between the tasks and the curriculum; the content balance between science, mathematics and workplace situations; and the role of the teacher in designing authentic or open tasks were concerns addressed in the discussions. The following extract illustrates the above concerns:

> When I proposed the Photovoltaic task [an exemplary Mascil task] in the first PD meeting, the mathematics teachers were very negative in using this task… they could not see any mathematics there. In the second meeting though, a science teacher proposed some very nice ideas about this particular task (Tim, 4th meeting)

Supporting teachers in the PD meetings was another concern of the MTEs throughout the discussions. Identifying teachers' needs, and supporting them in the design of tasks and lessons, we considered what kind of literature readings and specific examples from the workplace setting could be helpful. Also, we cared about promoting teacher collaboration in PD meetings and especially the co-design of tasks between science and mathematics teachers. The following example is characteristic of how co-design could result in an interesting experience for the participating teachers:

> The Earthquake task designed together by mathematics and science teachers indicated how mathematics is used for the study of earthquakes. In this task the students had to play the role of a seismologist responsible for studying the main features of a specific earthquake, for example the epicentre. (Jason, 5th meeting)

After teachers' initial classroom implementation, MTEs' concerns were related to how to enhance teachers' reflection on teaching. A characteristic example is Sofia's concern for supporting teachers' reflection:

> I asked teachers to present the reports from their lessons in the meetings. What they wrote was somehow descriptive, I posed questions on what they noticed… But finally the discussion was between them and me…. so, I suggested them some research articles on teacher noticing (Sofia, 5th meeting).

Providing structures/models was a basis for mathematics teachers to make sense of the IBL dimension of teaching and of its connection to workplace settings. The example below is characteristic of the above concern:

> It is very difficult to identify the mathematics in an authentic context. But when teachers start from a workplace situation I insist to return to it at the end of the lesson. To follow the process of modelling, I return to the context, I reflect on it and I move forward … is the solution we found reasonable? (Anna, 5th meeting)

Institutional factors were addressed through MTEs' interaction with the teachers in PD meetings and in the school. Connections to mathematics curriculum and classroom management (short teaching time, large number of students in the classroom, complexity of the group work setting) were concerns that emerged and were debated. Many of these concerns were also expressed by teachers indicating their resistances to designing and using inquiry and authentic tasks in their classroom. In the following extract, two MTEs discuss the connection of a specific task with the curriculum on the basis of teachers' expressed doubts about the appropriateness of the task.

> Anna: They [teachers of group 10] are working now on a new task. I suggested to them, "the tournament of ping-pong". I found it in the Mascil toolkit. You see, teachers in vocational schools find Mascil activities as complex and they look for something simpler… I consider this is a good example…
>
> Chloe: This task has been also considered by the teachers in my group. However, during the discussion they claimed that it is not related to mathematics at all…, they considered it as a quiz and not connected to the school curriculum. They said that they could use it in the future when combinatorics will be taught. I liked it and I spent time on it, but when I discussed with teachers, all of them were very negative, asking what mathematics are involved in this? (4th meeting)

Issues central to research in mathematics teacher education beyond the specific project emerged in almost all the meetings and guided MTEs' actions. These issues were teachers' and researchers' collaboration, the role of teacher as researcher and the sustainability of teachers' professional development. The following extracts illustrate two of the above concerns:

> The involvement in supporting the teachers was a learning experience, teachers, educators and students, we are all learners. This is what we are doing. We are learning how to communicate (Chloe, 4th meeting).

> Teachers have to be reinforced to communicate through the platform between themselves… to inquire by themselves… to search for resources (Sofia, 5th meeting).

Finally, the data collection process (i.e. observation of PD meetings and classroom sessions, interviews with teachers and artefacts produced by students and teachers) was another of MTEs' concern throughout the meetings. This empirical evidence was important to our practice as researchers and teacher educators.

15.4.2 Tensions and Attempts to Deal with Them

In this section, we will describe and analyse two emerging tensions. The first one is related to the role of the authenticity of workplace situations in mathematics teaching, while the second one concerns the degree of guidance offered to teachers in different phases of the PD activities. Below, we exemplify these tensions through different instances of PD meetings indicating the boundaries that were encountered, the practices that were involved and the boundary crossing that occurred.

15.4.2.1 Tension: Authenticity of Workplace Situations Versus Classroom Teaching

In the first three meetings, the MTEs attempted to conceptualise the workplace-related innovation and think of ways of introducing it to the teachers. Divergent views were expressed as regards the potential of authenticity in workplace-based classroom tasks. Supporting views considered the importance of using authentic tasks in mathematics teaching as a means to promote inquiry, motivate students and develop students' mathematical meanings through rich representations. The rather sceptical views concerned the complexity of the workplace context, the different epistemological nature of school and workplace mathematics and the pedagogical difficulty of linking these two in the context of PD and mathematics classroom. For example, Chloe, a mathematics teacher, supports the use of workplace situations in teaching as a basis for inquiry: "Since the workplace context hides a lot of academic mathematics, it gives itself elements of inquiry". On the other side, Tim, a mathematician and mathematics education researcher, sees inquiry in mathematics and mathematics teaching as not necessarily related to workplace context: "Inquiry in mathematics does not necessarily involves a realistic context". Similar debates took place throughout the first two meetings.

In the following extract, we see a debate between Sofia and Ben. Sofia supports the view that workplace contexts can promote students' understanding, while Ben points out the complexity of the workplace context to an outsider (teacher or student). In particular, Sofia emphasises the importance of linking informal and formal learning and the flexibility of representations of workplace situations and practices that can be compared with the formal mathematical representations; so in this way, students' informal activity can gradually be mathematised and eventually lead to more formal mathematical activity. Ben challenges this development by arguing on the complexity of the workplace context:

> Sofia: First, the students can see a flexibility in the representations which can be found in the workplace context and through the connection between the formal and the informal mathematics that the workplace context offers, maybe develop more flexible problem solving strategies and decision making.
>
> Ben: What I do not understand is in what ways the school can exploit the informal knowledge for making connections and build coherent mathematical knowledge. Do we have some tasks?
>
> Sofia: I think that we have. When we say that the knowledge is hidden and the formulas and the symbols are different, it gives me the opportunity to discuss in what ways the typical formula differs, and this helps me to get the meaning of the formula. The formula is not something else than an expression of a relation. (1st meeting)

These views appear to be mainly originating from the research and teacher education practices of the participants. The supporting views about the connection between workplace situations and classroom teaching were mainly expressed by participants who were members of the research community involved in projects related to mathematics and science at work (Anna, Elsa, Sofia). Views doubting this connection were expressed by participants with research on primary mathematics teaching (Ben) and university mathematics teaching (Tim). MTEs supporting the use of authentic workplace situations in the classroom tended to indicate means (e.g. resources) and procedures (e.g. problem-solving strategies) allowing workplace practices and mathematics teaching practices to potentially cooperate efficiently in the classroom. This group of MTEs facilitated boundary crossing between research on workplace mathematics and mathematics teaching as coordination. MTEs questioning the connection of workplace situations to classroom teaching consider the distance between the two practices in terms of their epistemological and pedagogical differences. This engages them in an identification process where MTEs become uncertain of the possibility of crossing the two practices.

The aforementioned tension started to become less distinct in the last three meetings when MTEs interacted with the teachers in the PD meetings and visited mathematics classrooms to observe teachers' implementation of tasks in the spirit of the innovation. Discussing how to support teachers to further develop their teaching practice and to develop professionally led to reformulation of the meaning of workplace and IBL. For example, Ben, who initially doubted the connections of workplace contexts to primary mathematics classrooms, argues after working with the teachers:

> For me, the main issue in the primary school is how to make connections with the world of work. One way to overcome this problem is to look for important things of any human activity. For example, it is very important to search what social workers do to support unaccompanied refugee children … we do not know anything about these children's cultural or mathematical background. (Ben, 3rd meeting)

> In our team… the task was collaboratively devised, taught by one of the teachers and observed by the others. The teachers thought that the process of working together would imitate professionals' collaboration at work to reach an outcome …. Mathematics in human activities and actions that are of importance. Workplace in authentic terms! Back to something that was raised in the beginning (Ben, 4th meeting).

Although Ben belongs to the second group, here he highlights the potential of boundary crossing by broadening the meaning of "workplace" to involve a range of human activities. Building on his research perspective characterised by inclusive mathematics teaching and teacher collaboration, he appears to coordinate the mathematics teaching practice with the workplace context.

Teachers' difficulties in enacting the innovation in the classroom made MTEs aware of the complexity involved in relation to the existing educational context. Even Anna, who was in favour of using authentic workplace tasks in mathematics teaching, appears to reconsider her view in light of the inferiority attributed to practice as against theory in the Greek educational system and the wider society:

> What I understand is that the workplace context does not fit to the classroom's world! It is true that there exists this view in the Greek reality, that is, that the workplace context is a realm of practice far away from school…inferior to it. What a worker does is more practical/ practice oriented… That is, I think it has to do with the whole philosophy of the system, not alone the educational system. This explains why teachers have difficulty to integrate workplace situations into their teaching practice. (Anna, 4th meeting)

Along similar lines Jason, a researcher in mathematics education, was challenged by the teachers in his PD group as to whether authentic workplace contexts can promote challenging mathematical ideas (content) for students:

> …when the teachers raised questions related to whether this is trivial mathematics, I was not sure what to do or how to respond… There are organisational issues here … There is pressure on the teacher educator… I felt that I should provide answers compatible to the innovation but also operational! Hence, the issue of what workplace and IBL is acquiring less importance! (Jason, 4th meeting)

Anna and Jason, with rather little experience as teacher educators, were challenged by two dipoles, theory versus practice and context versus mathematics content, respectively. The PD practice mediated through these dipoles supported them to reconsider the relation between workplace situations and mathematics teaching in terms of systemic and epistemological features (perspective-making). The boundary crossing is also evidenced in the development of their awareness of the complexity surrounding mathematics teaching and teachers' work framed by these features (perspective-taking). It could be claimed that the learning mechanism is evident here.

Summarising, MTEs' attempts to understand the relation between workplace situations and classroom teaching brought to the fore tensions that progressively faded. The tension that we examined here stemmed from the multi-membership of MTEs in current and prior communities (research, teaching, educating teachers), and dealing with it facilitated boundary crossing. Different types of boundary crossing include identification, coordination and reflection, which supported MTEs to develop awareness about epistemological, pedagogical and systemic features shaping the meaning of the innovation.

15.4.2.2 Tension: High Versus Low Degree of Teachers' Autonomy

Although there were not strong divergent views among MTEs promoting opposing ways to work with teachers (high versus low degree of guidance), this tension seemed to underlie MTEs' decision-making. This was evident in the selection of appropriate resources for the teachers, the role attributed to the teachers in task design, the management of the diversity of the teachers' groups and the ways of supporting teachers' reflection. The MTEs were not sure about the level of teacher autonomy in designing tasks and lessons connecting workplace contexts and school mathematics. There was some debate in the group as to whether this responsibility can be given to the teachers from the beginning or if the MTEs should provide in the first PD meetings more direct ways of how this integration can be facilitated. For example, Ken, a school advisor, pointed out teachers' needs for some guidance before being involved in designing tasks for their lessons:

> What will we do if teachers want us to propose to them tasks related to the workplace context? We could discuss with the teachers some of the tasks coming from research and then to start to explore the emerging issues together. This might help them to start to develop some tasks. (Ken, 1st meeting)

Taking a similar view, Chloe suggested the provided Mascil tasks as a starting point in the PD meetings to smooth teachers' engagement in exploiting authentic workplace situations in their designs: "In the first meeting we can start with a Mascil task and in the second meeting we can support teachers to explore more authentic situations" (Chloe, 1st meeting).

In the second meeting, MTEs brought experiences from their first interaction with teachers and reported teachers' preferences to design their own tasks in the spirit of the Mascil project. MTEs started to develop more elaborated ideas about how to support teachers in their attempts to design their own tasks. Jason considers teacher collaboration as an important condition to engage teachers in developing and sharing ideas as a basis for their didactical designs:

> Collaboration is very important. Even if they have initial ideas I do not think that they will have a full idea of what they will finally implement. We [as teacher educators] concentrate on two of the proposed ideas so as the teachers to have time until the next meeting to communicate these ideas. It is not good to provide five strictly defined ideas. I think it is more

important to cultivate a culture of discussion and communication around the final formulation of the tasks. (Jason, 2nd meeting)

In the above extracts, Ken and Chloe refer to mathematics education research and in particular to workplace mathematics and look for tools that could facilitate boundary crossing between research and mathematics teacher education. In this direction, Mascil tasks or other authentic workplace situations seem to play the role of boundary objects between the research and the teacher education practice. Jason builds on both his research practice and teacher education practice in early PD meetings. He targets the same boundary crossing by suggesting teacher collaboration. In terms of boundary crossing mechanisms, these actions indicate learning through coordination.

Task design or choice and its classroom implementation and/or management were shown to be central components of teaching practice in the first classroom implementations. These implementations were rather informative for MTEs as regards teachers' needs for support to enact their designs to facilitate mathematical inquiry and connections to workplace situations. A rather "instrumental" teaching approach adopted by the teachers became evident, characterised by a vague conceptualisation of IBL and connection to the workplace. It seems that a boundary was raised between the targeted project innovation and the existing teaching reality. This boundary challenged MTEs to reconsider their goals and actions to promote teachers' autonomy. They started to modify their working agenda for PD activities and to appreciate the need to extend the provisional resources, recognising their limited functionality in PD meetings. This became evident in the last two meetings where MTEs were able to describe clearly PD strategies and resources so as to make innovation accessible to teachers through reflection and PD.

For instance, in the fourth meeting, two main literature-driven ideas were discussed and used in developing schemes of action for PD activity: co-learning contexts for teachers and teacher educators and teaching as researching. A distinct feature of these ideas is that MTEs seem to degrade their role as "experts" and take a more global consideration of all the participants in PD meetings as "learners". This allows them to reflect more deeply on their approaches and use their PD experiences as the basis for combining teacher education and research-informed actions to facilitate teachers' PD.

In different parts of the data, MTEs refer to reflection as a PD practice and co-learning activity: "We need to help teachers develop ways of reflecting on their own teaching practice. However, what is a good practice like?" (Rose, 4th meeting). MTEs questioned their role as evaluators targeting participatory ways to engage teachers in reflecting on their own and/or other teachers' practices:

> We clearly cannot tell the teacher whether it was good or not. At this stage, I would say "what do you think? What was it that you didn't like? What was it that you didn't like?" Because we don't have the role of an evaluator … I would like to ask them to bring in the meeting a critical incident of their lesson and discuss it … I do not know whether we can determine (some) minimum elements expected to be there for the practice to be innovative! Because it depends too much on the group, its enthusiasm … Let them bring us something that was important for them... We can also present something that impressed us. (Sofia, 4th meeting)

The idea of teaching as researching was further promoted and concretised in the last two MTEs' meetings where central directions of action have been proposed: studying literature, sharing experiences and ideas and reflecting on teaching practice. These actions were discussed in relation to the identification of structures for helping teachers to reflect on their practice, indicating a much deeper concern of MTEs as regards teachers' PD in the long run. The above points are shown in the following extract where Ben tackles directly the theory-practice problem in mathematics teacher education: "We need to give them a framework to think, how to discuss what they did, which is not necessarily easy. A structure that they can modify as they please, which will contribute to the way they understand their PD" (Ben, 4th meeting).

In the last meeting, discussion about providing PD structures for supporting teachers targeting a balance between autonomy and guidance indicated a gradual distancing from the innovation itself and its usage. MTEs referred more explicitly to PD approaches reported in the literature, but now they connected them to their own practice as teacher educators in more specific/operational terms. The following extracts explicate two such approaches:

> Because I was worried I guided the teachers to organise a scheme of three phases. In the first phase, a familiarisation with the workplace context is taking place… whatever is this, the storekeeper, etc. in order to see the agents and understand how it works. In the second phase, to give the tasks, to see what this profession is about and in the third phase to enter in this profession and practice as a professional… to become an apprentice… This is like an agenda to follow. (Ben, 5th meeting)

> When they start from a workplace context, I insist to return to it at the end of the lesson. To follow the process of modelling. (Anna, 5th meeting)

The above extracts indicate relations between research, teacher education and mathematics teaching. In devising a practice-informed scheme of action for PD activity, several features are employed, some driven by the relevant research literature and the project's objectives and resources and others by MTEs' recent experiences in working with teachers. The research-informed ideas they expressed act as boundary objects among these three practices attempting to normalise the integration of workplace and IBL into mathematics teaching and make it part of the everyday teaching. MTEs take into account teachers' perspectives concerning mathematics teaching and professional needs and link them to their research and teacher education practice. This reflection process is characterised by an openness to take up teachers' perspectives to look at MTEs' own practice (perspective making/perspective taking).

Summarising, the tension concerning the level of teachers' autonomy in designing their lessons and the way that MTEs dealt with it throughout the PD meetings revealed several instances of boundary crossing. As in the previous tension, the multi-membership of MTEs in different communities (research, teacher education and mathematics teaching) influenced boundary crossing in terms of the practices involved and their professional learning. The dominant crossing was between research and mathematics teacher education where Mascil tasks and research-informed structures and constructs operated as boundary objects. The intersection of mathematics teaching, research and teacher education practices facilitated the emergence of reflection processes (perspective-making/perspective-taking) allowing a smooth integration of workplace contexts and IBL in PD and actual classroom teaching. These processes were characterised by the development of MTEs' awareness of the existing contradiction between teachers' autonomy and the targeted innovation and of teacher education strategies closer to mathematics teachers' needs.

15.5 Conclusion

In this chapter, we studied a group of MTEs as they designed and enacted PD activities to support teachers adopt IBL approaches and make connections between workplace situations and mathematics teaching. A number of concerns emerged regarding the meaning of IBL and workplace in mathematics teaching and the design and enactment of PD activities. Making sense of the connections between workplace situations and classroom teaching required MTEs to develop understanding about the complexity of the workplace context and the specificity of mathematics in it, as well as the differences between school and workplace culture (Nicol, 2002). Designing and enacting PD activities generated many concerns as the targeted innovation was rather new for both MTEs and teachers. Selecting/designing classroom and PD tasks, finding ways to support teachers, overcoming institutional constraints and linking research to PD were challenges faced by MTEs throughout their work with the teachers.

We focused on two dominant tensions, namely, the teaching potentiality of authentic workplace-based classroom tasks and the level of autonomy in teachers' work. These tensions brought to the fore three main practices enacted in the MTEs' meetings: research, mathematics teaching and teacher education. MTEs' participation in these practices and the negotiation of perspectives among them fuelled the raising of boundaries and facilitated boundary crossing including the learning mechanisms of identification, coordination and reflection. This process supported MTEs to develop a deeper awareness of meaning and of the materialisation of the targeted innovation and resulted in the development of their own professional learning. The objects that seemed to facilitate the process of boundary crossing were the tasks and objectives of Mascil, the authentic workplace situations and the relevant research literature regarding mathematics teaching and mathematics teacher education.

The work of MTEs, especially in large scale programmes, constitutes a very complex and challenging research issue which requires various skills and competences for which there are no professional programmes to support them (Jaworski & Huang, 2014; Smith, 2005; Zaslavsky, 2008). The complexity stems from the MTEs' activity, the specificities of the innovation and issues related to the large-scale character of the programme. The results revealed the multifaceted and systemic character of teacher education programmes targeting educational innovations. The MTEs' close collaboration and the high degree of teacher autonomy seemed to be crucial both for supporting the integration of innovation into actual practice as well as for MTEs' learning and professional development. Our study contributes to the existing literature in two ways. Firstly, the systemic network provides a tool that can help MTEs and researchers to gain deeper insights into the difficulty of bringing teaching innovations into mathematics classrooms and developing ways to bridge the gap between research and practice (Boaler, 2008). Secondly, the lens of boundary crossing to analyse MTEs' tensions and how they were dealt with offers a way to highlight the role of different practices in MTEs' professional learning becoming visible through the continuous transitions of MTEs across them. Becoming aware of how these crossings can be facilitated seems to bring the work of MTEs and researchers in mathematics education closer to the teachers' and students' reality.

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