

Chapter 10

Working with Awareness as Mathematics Teacher Educators: Experiences to Issues to Actions



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10.1 Introduction

We, the authors of this chapter, teach, or have taught, on a 1-year, postgraduate, initial teacher education course in the UK. We have a range of experiences in this role from 27 years to 7 years to 2 years to having just started. Each of us taught mathematics for over 10 years, in secondary schools, before coming to the university mathematics teacher educator (MTE) role. In this chapter, we explore both how we work with our prospective teachers and how we work together in becoming more comfortable in the MTE position. We believe that some of our ways of working are both unusual and powerful, in terms of the learning of our prospective teachers. We offer them here, in the context of discussions related to our planning of MTE teaching sessions, in the spirit of “expanding the space of the possible” (Davis, 2004, p.184). These discussions are, of course, in part for ourselves, part of our praxis as MTEs working together to develop awarenesses that we use, enacting our planning. In putting together this writing, we illustrate that the processes we use as MTEs to develop our practices are the same as those our prospective teachers are offered to develop their practices. These processes have emerged from the way learning is seen within an enactivist perspective and underpin the design of the teacher education course. We believe experts and novices can learn in the same way through staying with the detail of their practices and attending to “what is the same and what is different” to expand their range of possibilities to act (Brown & Coles, 2011, p.866).

After brief discussions of the important ideas for us of (1) “awarenesses”, (2) “metacommunication” and (3) “second-person perspectives”, each of the authors of the chapter, in order of years of experience from most to fewest, offers a discussion

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of their planning for a session at the university with prospective mathematics teachers on the course. There is then a section of reflecting, where similarities and differences are discussed and analysed.

10.2 Background Ideas

10.2.1 *Working with Awarenesses*

An important word for us, which is in the title of this chapter, is “awareness”, which was made into a countable noun by Gattegno (1987): “awarenesses” (p.25). A conviction that is expressed strongly at various stages of our MTE course is that there is no one model of good mathematics teaching. Planning does not focus, therefore, so much on a model, or even models, of mathematics teaching but rather on creating opportunities to develop awareness. In one form, awareness can be taken as experiential and self-referential. In this sense, “a person’s awareness is the world as experienced by the person” (Marton & Booth, 1997, p.108), and there are similarities here to the use of awareness as a synonym for consciousness and as a framing of levels of articulation of mental states (Winkielman & Schooler, 2011). Here, though, we make use of the work of Gattegno (1970, 1987) and use “awareness” to indicate specifically the potential for and enabling of activity. In this sense, awareness is used to describe a core action or function that must be present in order to learn (Mason, 2008), so that, for instance, an awareness of counting squares covered by a shape might allow attention to be drawn to a definition of area and an awareness of tangents to a curve might allow attention to be drawn to stationary points of the curve (Wheeler, 1975). In particular, Gattegno (1987) describes a necessary condition for being a mathematician as the “awareness of relationships” (p.26) and, further, suggests it is when we become aware of such an awareness that we move forward, or, as we would say, we learn.

A teacher of mathematics can become engaged in a project of offering contexts in which learners’ experience provokes them to make connections, giving the possibility of new actions; the assertion of Gattegno (1970) that “only awareness is educable” suggests this is the chief role of the mathematics teacher while keeping open the way in which it might happen. A movement into awarenesses as “that which enables action” (Mason, 2011, p.43) can be a powerful enabler for classroom practice (Coles, 2013). As Hewitt (2001) says:

By educating awareness the mathematician inside a student is being educated, which would not be the case if everything were treated as if it were to be memorised. Awareness informs decisions and how to act using information which is known. (p.38)

The focus for teachers becomes the awarenesses that are present and might be brought to mind in their students. They are recognising when and how students experience the shifts in attention that indicate becoming “aware that what used to be attended to was only part of a larger whole” (Mason & Davis, 1988, p.488). This attention requires that teachers become aware of their own awarenesses, of what is

present in the classroom and what is not, allowing action on their part (e.g. offering or not offering further prompts, new questions or different heuristics). By extension, the focus for MTEs is to become aware of the awareness of awarenesses, a guiding principle in planning for this course, in allowing both MTEs and prospective teachers to expand their possibilities for action. To illustrate this extension, imagine a school student in a classroom, who acts in a way that indicates they have not considered negative numbers as possible solutions to a particular problem. As a teacher, becoming aware of (in this instance, the absence of) an awareness might lead to a comment such as “you seem to be considering positive numbers only” to explore whether there is any awareness of the possibility of using negatives. Now imagine a prospective teacher, in a teacher education session, displaying exactly the same behaviour. In this case, the MTE might want to follow up any comment about the mathematics (which comes from a position of awareness of awareness of the mathematics) with a comment such as “so, as a teacher, how will you work with your students so they are able to question the assumptions they make in problem-solving?”. Now the comment is coming from a position of an awareness that the teacher needs to be operating with an awareness of mathematical awarenesses (present or absent) of their students.

10.2.2 *Metacommunication*

We follow Bateson’s (1979) use of “metacommunication” (p.107) to denote communication that is *about* communication. Bateson (1972) was among the first to bring to our attention the distinction between message and metamessage. He suggested that message and metamessage interact in meaning making and metacommunication and claimed that an essential function of metacommunication is to direct interpretation, as frames within which the speaker’s comments are to be understood. In this chapter, we refer to our own use of verbal metacommunication when something is said *about* the communication that is taking place. As MTEs, we use verbal metacommunication explicitly in response to what our prospective teachers are saying in sessions we are running, to point to a range of ways of behaving as learners of mathematics, mathematics teachers and in schools. We believe their own explicit metacommunication will help them establish their mathematics classrooms, through pointing to the range of behaviours they value from their students and that they believe will support the learning of mathematics. In the imagined examples at the end of the previous section, the comments to both the school student and the prospective teacher would be examples of metacomments.

In a classroom, the teacher’s metacomments are about their students’ learning of mathematics (while the teacher is learning about the students’ learning about mathematics). For MTEs, our metacomments are about our prospective teachers’ learning of how to teach mathematics; our learning is about their learning (to teach mathematics, as well as about mathematics). A teacher’s or MTE’s metacommenting (rather than, say, directly answering, or offering leading hints) can act as a powerful mechanism to establish desired patterns of working as a group. A metacomment

may require an awareness of what is absent. For instance, a common pattern for novice teachers is that when they talk about lessons they have just taught, their attention is only on themselves and what they did or did not do. As an MTE, awareness of the absence of discussion of the school students might provoke a metacomment. A metacomment about observing a desired behaviour can equally be powerful in establishing that behaviour as something others might do, or that might be done again (e.g. see Coles, 2013, for more illustrations of this phenomenon).

10.2.3 Second-Person Perspectives

Drawing on roots in introspection and phenomenology, Varela (who, along with Maturana, is one of the influential figures in the birth of enactivism, where knowing and doing are equivalent) offers the notion of “gestures of awareness” (Varela & Scharmer, 2000, p.1), in the process of elucidating first-person experience. His gestures are “suspending”, “redirecting” and “letting go” (p.4), envisaged as a cycle that allows for learning from first-person experience. Suspending involves a break in our typical processes of sense-making in the world and may need an active determination not to be caught in habitual patterns of perception-action, for example, attending to the detail of a classroom event rather than evaluating. Redirecting is a process of directing attention towards something perhaps previously unnoticed, for example, provoked by the articulation of the awareness of another. Letting go refers to the gesture of non-attachment to previous modes of thinking-doing-being, to allow for a continuation of the cycle into suspension and redirection, for instance, in allowing a reinterpretation of an incident in a lesson that might have been experienced as “wrong” or “bad”, to accepting alternative views.

For us, what is particularly significant, in Varela’s characterisation of awareness, is the importance he places on the second-person perspective, the more experienced “other” who is able to recognise the awarenesses being elucidated during the cycle of suspending, redirecting and letting go (see Metz & Simmt, 2015, for a methodological use of the second-person perspective) and, as illustrated earlier, to recognise awarenesses that are, or are not, present. We cannot have access to each other’s first-person awareness. However, an empathic “second person”, who is an expert (in mathematics, or in teaching mathematics, or in the MTE role), is able to observe, not just externally. A second person who is an expert can recognise, empathise and become a “partner in the process” (Varela & Scharmer, 2000, p.7) of becoming aware. At the end of this chapter, we return to the theme of the second person, to elucidate the role of this “other”, through the stories that now follow.

In the next section, Laurinda describes the origins of a cycle we refer to as “experiences to issues to actions”, which informs all of our teaching on the teacher education course. Although not teaching on the course any more, she illustrates the ideas with an example of her own planning using this cycle. Following this section, the three MTEs currently teaching on the course, in each individual voice and in descending order of years of experience, offer accounts of their own teaching at the

university, focusing on the planning. These accounts are offered from a first-person perspective. We then come together in a concluding section to look across these accounts to draw out similarities, differences and implications.

10.3 A Way of Working: Experiences to Issues to Actions (Laurinda)

No idea is original. In planning for writing this chapter, *experiences to issues to actions* emerged as important for the three other authors of this paper as they discussed how they teach prospective teachers on this course. The tutors work with prospective teachers both in the university and on visits to observe them teaching mathematics in school. My immediate reaction to the emergence of the phrase was that I had worked with it having read a book published by Barbara Jaworski through The Mathematics Association (1991) (one of the associations supporting teachers of mathematics in the UK), documenting the work of a group that she chaired. I offer here a historical perspective to the idea leading to a related action, a story of how I planned using the cycle. Although not currently teaching on the course having retired, I had, at some point in the early 1990s, designed the course in its current form.

I had originally started to use the phrase in working on Master's mathematics education courses and wrote up the sessions in a chapter in a book *Liberating the Learner: Lessons for Professional Development in Education* (Claxton, Atkinson, Osborn, & Wallace, 1996). Although I had thought that I had taken the ideas from Jaworski, in the chapter appeared:

The way in which I planned to work in the session was by progressing from a consideration of *experiences*, via the formulation of *issues*, to the delineation of possible *actions*. The methodology is adapted from Jaworski (1991) [...]. (Brown & Dobson, 1996, p.214)

I had adapted the ideas but needed to see the original to know how. I asked Barbara Jaworski if she still had a copy of her book from 1991 since I could not now find my copy. She kindly posted a copy to me, and I looked for what had been the original stimulus. The whole book was called *Develop Your Teaching* (The Mathematical Association, 1991) and was written to support the professional development of teachers. The process was based on what were called “anecdotes”, which could be spoken or written and might provoke others to recall incidents. In the book (p.26) is a diagram for the ongoing work of a group of teachers. Anecdotes from a number of teachers lead to identifying issues, and classroom action is then implemented after which there is feedback into more anecdotes, and the process is then cycled. For anecdotes I had focused on spoken stories of experiences.

What follows is my planning using “experiences to issues to actions” for a session early in the teaching year of the course. Given that the session was repeated for many years, the discussion also illustrates how my own learning followed the same pattern of “experiences to issues to actions” as my awarenesses of using the activity and these particular tasks developed.

10.3.1 Story: Planning for the 4-Minute Workshop

The 4-minute workshop first appeared in my diaries on 26 September, 1991, and I then went on to lead the session during the first week of the new academic year (late September) for the next 25 years. As the person who had designed the course around “experiences to issues to actions”, I did not have to write this down in my planning. I looked for an experience, in this case an activity that the group would experience together, that would have many purposes given how early in the course the session was given. The previous year, in the summer term, the prospective teachers, in groups, had created resources that would fill a need for the partner schools of the project. One of these resources was a workshop to support teacher assessment in mathematics for low-achieving year 8 students (aged 12–13 years old). The resources had been placed on tables that were in a circle around the walls of the classroom with a resource island in the middle. Two chairs were placed at each table facing the wall. The two students sitting at each table were labelled A and B. Every 4 minutes, hence the name of the workshop, the teacher would say, “Move”, and the As went clockwise, and the Bs went counter-clockwise. In adapting this organisation as a session for prospective teachers, as the size of the group varied, I would add to or subtract from tasks in the original workshop so that there were enough tasks for pairs and perhaps, in some years with an odd number of participants, a singleton to be catered for.

This seemed a useful activity that would serve a whole range of purposes: introducing teacher assessment and supporting the prospective teachers in learning each other’s names when meeting and working with their peers. There would also be issues, such as some of the activities would not take an adult 4 minutes, and they would like and not like particular activities for different reasons. It is important to work with all the members of the group, as they will have to do with all colleagues in school; but this activity would work, given how they reacted to working with different people, to uncover aspects of themselves that being aware of would prove useful when they went to school, for example, being used to working on mathematics by themselves and not being comfortable working in a pair. My focus is on supporting their individual developing awarenesses through learning about how they interact with the tasks.

The activity illustrates “experiences to issues to actions”. The experience that we can all share is doing the workshop. Issues arise in reflecting together after the event, and we can then think through actions we could take as teachers to address those issues. From my perspective as leader of the activity, I get a lot of time to learn the individual behaviours of a new group of prospective teachers as they work.

Setting up the activity with the prospective teachers takes a lot of care and time because it includes giving them meta-tasks to work at, alongside the doing of the mathematics:

- Think about issues arising from working with another, and learn their name.
- Which tasks do you like and why, dislike and why?
- What if? Finding their own extension to the problem as written if they finish before the 4 minutes is up.

- What is each task assessing?

The activity is still done today, although the tasks are different. It is an example of an activity that takes quite a while to get ready on the day, prior to the arrival of the group, but once underway gives a lot of space for interaction and noticing of how individuals work, important for my learning of the individuals’ strengths and areas for development in the group. There are particular points I want to get across for each task, illustrated by the following three examples from the set of 15.

10.3.1.1 Task 1: Limitations We Put on Ourselves

Two triangles, cut out of card, are provided for the task below. The triangles are congruent, obtuse-angled and scalene.

Triangles

Using the two triangles can you make: a rectangle; a parallelogram; a kite; a pentagon; a hexagon?

What about a heptagon?

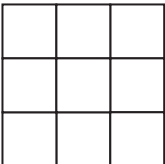
What is the biggest number of sides possible?

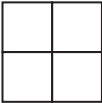
Observing prospective teachers doing this task, the first few can be made with sides of the same length touching corner to corner. Some pairs then get stuck. In the end, I would be looking for when this happens from the awarenesses built up through past experience and, when the issue is noticed, would then act, picking up one triangle and laying it across the other, overlapping. “Is that allowed?”, is often asked. In response to such a comment, I would metacomment, saying something about, “Beware of limitations you put on yourselves and notice them in your students”. Another limitation is to assume that “pentagon” means “regular pentagon”.

10.3.1.2 Task 2: What to Do When Students Have Finished?

How many squares?

How many squares can you find in this diagram? ➔



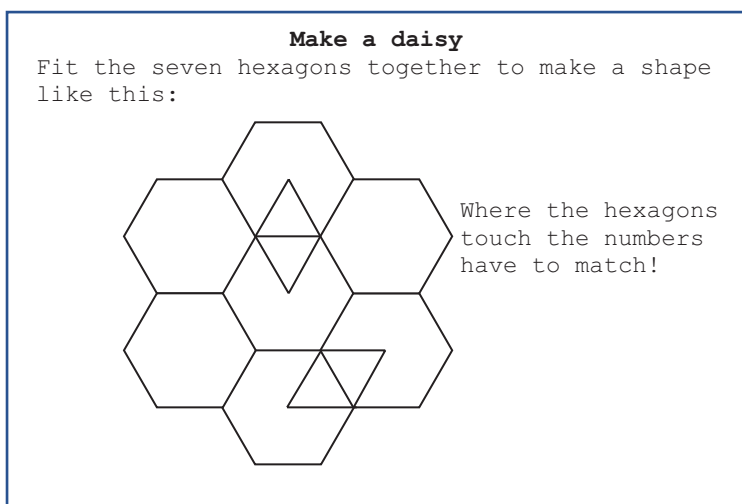


What about this one? ➔

Answers of 5 and 14 come quickly from the prospective teachers for the task above. It does not take them 1 minute to agree on those. Some pairs discuss what to do next, often generalising for a square of side n smaller squares. Some pairs think that they have finished and start to make notes on whether they like the problem or not. The previous experiences with this workshop led me to notice the issue, for me, of “having finished” and the related actions of the prospective teachers. I move to act.

“What will you do when students you are teaching say that they’ve finished?”, I ask. I ask them what the constraints are in the question. Small squares in a square? Only cases 2×2 and 3×3 . What if it’s not a square? What if it’s $n > 3$? Identifying what’s changing and then “what if-ing” and “what if not-ing” are strategies for generating new questions (Brown & Walters, 1969, p.38). This intervention gives the prospective teachers an action or something to offer to the students to whom they teach mathematics if they finish early. As an experienced MTE, my attention is fully on the prospective teachers’ learning, and I am learning about them while also acting to provoke their learning.

10.3.1.3 Task 3: What’s the Purpose of the Activity?



In designing Task 3, the original prospective teachers had the idea of using hexagons with fractions, decimals or percentages on each edge being fitted together so that the numbers matched. To make the original “daisy”, they drew hexagons in that pattern and then wrote numbers on the sides that they wanted the workshop to assess the equivalence of. Having cut up the hexagons and tried to fit them together again, they could not do it. Initially the group thought they would have to redo the making of the hexagons, but then one of them realised that for this task, the more practice that the students had on equivalent forms of fractions, decimals and percentages, the better, all with the spurious purpose of making the daisy. For some prospective teachers, this is their favourite activity and, for others, their least favourite because

they could not finish it in the time. The same discussion has happened after the workshop has finished down the years: Is the purpose of the activity to make the daisy, or to get more practice than their future students would probably be prepared to do if given a set of questions from the textbook?

In keeping the activities of the 4-minute workshop the same over years, I became more and more skilled at making points and noticing where to intervene with the minimal of fuss, for example, moving one of the triangles to show that fitting corners together did not matter or saying, “What are the variables? What could be changed?”, for the number of squares in the square problem. This is my learning. It seems to me that I am going through the same process as the prospective teachers in the sessions are. My *experiences* of teaching the activity repeatedly raise *issues* for me that become the focus of my observations leading to *actions* and *metacomments* that feed back into my teaching.

10.4 Current Stories and Discussions of Planning

10.4.1 Alf: Session on Using ICT

The use of ICT and technology in the classroom is an element of teaching practice that seems to change from year to year. We had timetabled a session of 90 minutes as an “Introduction to ICT” in the Autumn Term. I wanted to introduce prospective teachers to two software packages. This was a judgement made from wanting to give them some experience of comparison but also to allow enough time in the session (i.e. 45 minutes per package) where they could get deep enough into the package to hopefully mean they got a sense of its potential in the classroom and therefore had the motivation to explore one package in more depth in their own time (ultimately leading to them incorporating it into lessons).

Having one dynamic geometry package felt an easy choice. Just for ease of access to the software and also for the fact that it can act as a graphic tool, *GeoGebra* was the one I picked. For a second tool, I chose *Scratch* (which is a programming language developed out of the Logo microworld). This choice was made, perhaps partly because I know how I used to introduce children to work on *Logo* in a classroom and I could do the same introduction here. *Scratch* also links to programming, which is a relatively new focus in schools.

The two packages also felt important because having a contrast would allow me to make the focus of the session both learning the packages and saying this session was about them needing to choose one ICT package in which they were going to become expert over the year. The start would therefore be this “meta”-task, and hopefully they would like at least one package of the two. In the text below, for reasons of space, I focus just on the introduction to *Scratch*.

I planned to get the prospective teachers to clear a space at the front of the room and form a circle with chairs and sitting on desks. I put two desks in the space, making a square obstacle. I would ask for two volunteers, one to be a robot and one a controller. The controller has to direct the robot around the desks, but the robot has

a very limited vocabulary (that I will help moderate). This start forces prospective teachers to think themselves “into” the robot’s perspective. To turn requires a command of “left” or “right”, and moving needs a “forward” or “backward” command. The task gives prospective teachers an entry into the programming language of *Scratch/Logo*, and I will show how they can use exactly those commands to control their own “robot” on the *Scratch* screen. The challenge for prospective teachers would be to try and generate different regular polygons, followed by trying to cover the screen with one of them. I imagined I might introduce the prospective teachers to how to generate variables and how to repeat and perhaps how to set up recursive instructions, as they got into the task.

In this introduction, there is a task beginning I have used in the classroom. I use it here, not to model good practice, but because I believe this is an efficient and potentially energising way to get into working with the piece of software. Unlike the classroom, I do not have any particular areas of mathematical content I want the prospective teachers to work on. The aim at the university is to consider the potential for ICT in their teaching and for them to commit to one programme on which they will do more work. I planned to end the session with a discussion of these issues.

I would be on the lookout for any mathematical awarenesses exhibited or perhaps seeming to be lacking, in the prospective teachers, and would comment on the issues I noticed, as they arose (e.g. I recognise how “natural” it can seem to be to think the exterior angle of an equilateral triangle is 60 degrees and might comment on this as the error arises). I am also aware of being on the lookout for how the prospective teachers handle their own emotional reactions. I am aware that certain individuals will respond to using ICT in a heightened manner (e.g. highly positive or negative), and, again, I might act on my awareness to comment on an issue which is them needing to work with students in their own classrooms who may have the opposite reaction. There is also an important learning, for me, about the way in which the prospective teachers approach their learning. Finally, it is an aim of this beginning not to set up an expectation that “good” practice in their placement schools would involve the use of these, or other, ICT tools (which could lead to a sense of what they are offered at university not being relevant to the reality of classroom life).

10.4.2 Tracy: Session on “Algebra”

The timetable for the course has included many of the same session titles for a number of years. Some session titles suggest a focus on specific areas of the mathematics curriculum (e.g. probability, algebra, proof), and others imply a focus on issues from teaching (e.g. assessment, English as an additional language (EAL) issues, topic planning). The “Algebra” session was one such session that had featured on the timetable for many years. The session is scheduled for around 90 minutes and takes place in week eight of the course when the prospective teachers are 3 weeks into their first extended placement in school and they return to university for a week.

I planned to begin with the question, “What is algebra?” or “What is algebraic activity?”. By beginning with a list of how the prospective teachers are seeing algebra, the idea was to return to the list at the end and add to it in light of the activities done in the session. I saw this to be a way of demonstrating the expansion of an initial set of views through offering the prospective teachers a common experience on which to reflect.

In planning any university session, one awareness I have is not wanting to offer any one particular model of mathematics teaching, and in this case a particular model of teaching algebra, that might be seen as a model for prospective teachers to try out in school. In order to talk at a meta-level *about* the algebra activities (detailed below) worked on by prospective teachers in the session, I wanted to provide a framework. I imagined the framework could support a way of thinking and talking about the activities from a more neutral position. I decided to introduce a set of distinctions of algebraic activity from Kieran (2004, p.22), which consists of three types of algebraic activity: *generational activities* involving generating expressions, equations and expressions of generality from geometric patterns or numerical sequences; *transformational rule-based activities*, for example, factorising and simplifying expressions and solving equations which are predominantly concerned with equivalence; and *global/meta-level activities*, for example, an awareness of the structure of mathematics and constraints of problem situations, prediction, justification and proof (which are therefore not exclusive to algebra). I also envisaged that using a theoretical framework in this way might support the prospective teachers with their Master’s level thinking and writing, and I planned to make this link explicit to them. The prospective teachers would need to make sense of this framework through firstly reading it and then being asked how they are seeing these distinctions. I planned to give them time reflecting briefly on where they would place their own responses to the original question, “What is algebra?”, within this set of distinctions.

Given “Algebra” is one of our long existing session titles, the common feature of this session over the years is that the prospective teachers are offered a variety of algebra tasks (often by a variety of tutors – in this case, it was going to be Julian and me). There is therefore a pre-existing list comprising of different tasks that have been offered before over the years, some of which we used. However, *Painted Cube* was not on the pre-existing list. I was keen to use visualisation at some point over the year and an activity where algebraic symbolism can be drawn out directly from a structure “from geometric patterns” (Kieran, 2004, pp.22–3). *Painted Cube* is an old coursework task used when I was in the classroom over 10 years ago, so I was very familiar with it having used it many times since then. I planned to be explicit about the old coursework task context, so it felt real and again not about me and *my* classroom but a well-known, much used, task. The meta-task while working on each of the activities would be to consider which, if any, of Kieran’s headings is most fitting for that particular activity.

Usually, when I am going to teach a session involving a mathematical activity, part of my preparation is working on the mathematics. Given my familiarity with the problem and with the algebra, I spent some time practising the visualisation on

Alf and Julian. I was aware that, in working with a visualisation with a group of individuals, it is likely that some prospective teachers would see something quite differently from what I had intended. At the end of the visualisation of an $n \times n \times n$ cube made of cubelets ($1 \times 1 \times 1$) painted red on the outside, I planned to ask the following questions:

- How many cubelets are there with 3 red faces?
- How many cubelets are there with 2 red faces?
- How many cubelets are there with 1 red face?
- How many cubelets are there with 0 red face?

I imagined that these questions would be likely to expose any differences in what the prospective teachers were seeing and would provide an opportunity to offer the group an experience of what can happen if you choose to use visualisation, that is, working with the group immediately after the visualisation so that we all see the same. Having worked on these questions with the group to the point where we can agree on some answers, I planned to allow them to extend the problem for themselves. This idea of allowing the prospective teachers to follow their own lines of enquiry when working on a problem like this is something I would do in a number of different sessions. For me, this is about offering them an experience of being motivated through working on their own mathematical questions.

Having spent some time working on a series of algebra activities, I planned to end the session returning to the meta-task by asking the prospective teachers to consider the activities in light of Kieran's framework, where they would place each activity and why. Having experienced a number of different activities together, it felt important to return to their original thoughts about algebra as a way of expanding what they are thinking are possibilities for their classroom, not staying with their original ideas.

10.4.3 Julian: Session on "Assessment"

Before planning individually, we met as a team of three and looked at resources from the equivalent session in the previous year of the course. The assessment session was scheduled to last for 90 minutes, and I planned four main sections:

- (a) Beginning with school experiences of assessment
- (b) Collecting experiences as a small group and then as a whole group
- (c) Experiencing the use of a questioning and listening task as an opportunity for assessment
- (d) Implementing ideas to design an assessment activity for a defined purpose

I made use of activities that had been refined over many iterations of the course, choosing not to change the substance of these. Quite quickly, my planning became populated with phrases that I intended to speak, and, at some point, the planning activity became writing a script for the session. The primary intention behind this

scripting was to document what I would say and when but also to monitor what I would not say. A particular focus of these considerations was setting up the small group activity (step (b)), in which I would ask prospective teachers to work in groups of five (or six) to create a “poster” using a single sheet of flip chart paper, gathering their school-based observations of methods of assessment. It would have been possible to set up the group activity in any of a number of ways, and my thinking was concerned with how much to reflect with the group on the process of setting up the activity. I decided to draw attention to my instructions as a way of offering something to the group, but to leave the primary focus on thinking about assessment. Similar considerations applied to the mechanism used for sharing outcomes of each group, and a similar approach was used: drawing attention to the instructions while not inviting comments on the process.

In my script, I chose to adopt a feature of interaction I had noticed each of Alf and Tracy employ with the group, namely, use of a leading “So” at points of transition. My feeling about this detail was that it addressed an issue of stepping between the frames of the activity itself and of metacommenting. The verbal marker became a deliberate part of my delivery.

This was to be my first “solo” teaching session on the course. In addition to thinking about the group, I was also aware that in the room would be the two established tutors who would be able to offer reflections afterwards and that this would happen naturally as part of a debrief conversation between the three of us. These conversations take place routinely on days when we work with the mathematics group, over coffee and lunch, with an imminence that supports access to the experiences themselves.

A large part of the decision to use existing resources was my awareness of “experiences to issues to action” as an approach to the whole course that was well-understood and of great significance within the course. This awareness was informed by conversations with the other tutors in preparation for other sessions, in which attention was focused on the influence of experiences on the emergence of group and individual frames of reference.

During previous sessions with the group, I had adopted the practice modelled by the other university tutors of noting down what was said by the tutor leading the session. This activity had focused my attention on the language used and certain patterns of speaking. My feeling was that these patterns of speaking had a significance in forming spaces of attention in the room and guiding the attention of the students, as they do in school classrooms. In this way, the words and phrasing (the “So” that creates a space for commenting) took on a significance that matched, and perhaps exceeded, that of the “content”. This feels in keeping with an enactivist positioning, since it is in doing that we change our knowing. Moving to writing a script created a short-circuit to my own recalled experience as a prospective teacher; I have a clear sense of writing scripts for my lessons when first on placement as a prospective teacher on the course myself. Many of the same motivations run through both situations, although with a different balance. In both cases, I was processing my own reluctance to let details arrive in the moment, lest I say something that was not what I intended. (In my school-based classroom practice, after some 13 years,

scripting happens rarely now; generally, I would let ideas emerge from the members of the class or access descriptions I have used before.) There is a sense of freeing my attention to be on what is happening in the room, in the moment. This aspect was much more explicit and significant for me now than as a prospective teacher. I cannot ignore the personal significance of this being the first session I had led “solo” on the course, and, undoubtedly, some of my decisions were about taking control of my role in the session, of doing what I could in advance. Again, this is a counterpoint to my journey as a mathematics teacher, where I have worked on changing student perceptions of the locus of control within lessons.

I remember using these tasks as a prospective teacher myself, on this course. I have used some of the “listening” tasks with other teachers when in school, as a head of department. The mathematics in the activities has proved to be engaging, but the key aspect of using the activity is the quality of the listening (Ginsburg, 1981), so the mathematics needed to have sufficient complexity to provoke a need to reason (aloud) while providing opportunities to begin quickly. While I was struck by the similarities of the approaches I took as a beginning teacher and a beginning MTE, my awareness of my purposes in using the approaches was now in a different place, informed by considering “experiences to issues to actions” in discussion with the other university tutors. Through the processes and content of the session, opportunities were created for students to engage with practical issues related to assessment and to reflect on ways of being in the classroom. For me, the session gave clear opportunities to reflect on my own experiences as a beginning MTE.

10.5 Reflecting on Similarities and Differences in the Learning of Prospective Teachers and MTEs

This section will point to the way experiences, issues and actions work on the course, from the evidence of these stories, in the learning of both our prospective teachers and ourselves. We interweave discussion of metacommenting and second-person perspectives, before a final section returning to the theme of layers of awareness.

Experiences: There are a number of ways in which the word “experiences” is exemplified in the examples of planning above. Julian begins his session, related to assessment, with prospective teachers’ experiences of assessment in schools. The prospective teachers (in pairs or individually) have placements in different schools, and, although there is a national curriculum in place for mathematics in England, schools will have some similar assessment practices and some different ones. No one individual prospective teacher could have observed all the different assessment practices in their own school either, so there is an opportunity for a group of prospective teachers to share and, in this case, make a poster to illustrate the range they have discussed. Here the “experiences” are in different schools, but there are ways of working with these experiences in sessions at the university, one of which is

described in Julian's story. Another way "experiences" can be used is by the prospective teachers having a common experience of an activity that they can then use to discuss issues raised. Laurinda, Tracy and Alf's planning is for mathematical tasks that are used to raise issues. The 4-minute workshop is a range of mathematical tasks experienced for a short amount of time; Alf introduces an ICT package actively, and Tracy works with prospective teachers on the task *Painted Cubes*. There are many ways of using such common mathematical experiences, for instance, being able to extend awarenesses of a concept through application of a framework (e.g. for algebraic activity) and becoming aware, as with the 4-minute workshop, that, within the group of prospective teachers, as with a group of students in a classroom, the actual experience of doing the mathematics and how you feel about it is different from person to person (one likes the challenge of making the daisy; another gets frustrated at not completing the task in the time; another likes the way the activity gave lots of practice with number skills). As MTEs, our experiences are within the sessions we offer prospective teachers, noticing similarities and differences in their responses. These are alluded to in all the stories.

Issues: A number of ways of organising sessions to support prospective teachers sharing their experiences to raise issues exist on the course. When asked about planning, Laurinda commented that often, in travelling to the university to lead a session, she was focusing most on how many in a group today and how to organise the seating in the room. For Julian's posters, there might be five or six in a group. Another common grouping, the first session back after a block of school practice, is a reflecting team of three. In a reflecting team, each prospective teacher is given a fixed time to explore the detail of an experience, while the other two prospective teachers ask probing questions, helping to get at the issues arising from the experience. Pairs are used in the 4-minute workshop to highlight issues of working with others. In a further parallel between prospective teacher learning and our learning as MTEs, Julian, Tracy and Alf also act as a reflecting team for each other, making time to explore the detail of our own work with prospective teachers, raising issues and asking probing questions.

Expertise as an MTE allows the move from our experience of prospective teacher behaviours to the explicit raising of an issue. This can be observed in stories from Laurinda's planning (such as noticing participants *not* overlapping triangles in Task 1 of the 4-minute workshop) and Alf's ICT task (e.g. raising issues linked to emotional reactions to packages). Raising these issues is dependent on a second-person awareness. Both Laurinda and Alf notice particular awarenesses (present or absent) in part because they recognise times when such awarenesses are present or absent in themselves, when working on mathematics or when teaching. Tracy indicates her awareness of typical behaviours (e.g. that some prospective teachers will interpret her visualisation differently) and is perhaps on the cusp of wanting to use such occurrences as an opportunity for metacommenting about issues. Julian described the first "solo" session he had taught as an MTE. He therefore had no patterns of expected behaviours, from the prospective teachers, on which to draw, and it is to be expected that his reflections focus on his own learning (e.g. comparing his learning as an MTE to his learning as a teacher). Work as a reflective team can support the

placing of behaviours of prospective teachers in any particular session within a context or range of likely responses.

Actions: In the descriptions of planning there, are, of course, actions performed by prospective teachers and MTEs. The “actions” in the cycle “experiences to issues to actions” refer to actions that follow, and are linked to, the raising of an issue. So, for the prospective teachers in Laurinda’s story of the two triangles in Task 1, the significant “actions” will be what they do in their own classrooms, for example, in response to Laurinda’s prompt: “Beware of limitations you put on yourselves and notice them in your students”. As MTEs on the Bristol course, we are fortunate (compared to some other colleagues internationally) in having the opportunity to observe our prospective teachers, teaching in placement schools. So, while there will be no immediate way of knowing what “actions” (if any) a particular issue might provoke, over time we do get a sense of this movement.

As MTEs, our “actions” are related to the learning of the prospective teachers. At its most immediate, as described above, our “experiences” are of the learning of those prospective teachers. “Issues” are linked to our awareness of the behaviours of the teachers; and our actions are the making explicit, via metacommenting, of these issues. Our learning is therefore focused directly on the learning process of the prospective teachers and is linked to our second-person awareness of that process, of learning to teach and of learning mathematics. However, when we work as a reflective team of MTEs, debriefing each other’s experiences of teaching a session, there is a process we engage in which is much closer to what we offer our prospective teachers, for instance, when we invite them to work in groups of three, debriefing their experiences in schools. We might invite a story from our own (MTE) teaching (an “experience”, as Julian also provoked in his session about assessment) and then gather other similar (or different) stories from each other. Having gathered a collection of stories, we would then move to identifying the “issue(s)” raised (as Julian invited prospective teachers to do in creating a poster about assessment). From here we would then consider implications, that is, “actions”, for our own future practice – as we invite our prospective teachers to do, at the end of MTE sessions.

Our different experiences as MTEs also mean we can provide a second-person perspective for each other. We lay open, to each other, some of the “intelligent awareness” (Varela, 1999, p.32) behind our actions, and, in recognising and perhaps labelling some of the awarenesses of each other, we support further noticing. It is the second-person perspective that is often crucial to the “issues” phase of the cycle of learning, both for our prospective teachers and for ourselves. While it is possible to identify issues for ourselves, it can often take a more experienced and empathic “other” to recognise a similarity or pattern or connection.

10.6 Layers of Awareness

When we work on developing as MTEs, we do not have an image of ourselves as experts in teaching, transmitting our knowledge to our prospective teachers. The process of learning (for us, and our prospective teachers) is through awarenesses

that can be metacommented upon. Our planning is, therefore, focused not only on the content of the session, such as *Painted Cubes*, but also on the meta-tasks, which for *Painted Cubes* are related to using a framework for algebraic activities and creating a space in which the prospective teachers are expanding their own awarenesses of how algebra might look in their classrooms. Working with our awarenesses is directly linked to our metacommenting, pointing to gaps and patterns in our prospective teachers' learning. The students in classrooms work on their mathematical awarenesses; the prospective teachers use their awarenesses of mathematics and mathematics teaching and learning to support the learning of those students by offering experiences, observing, listening and commenting. As MTEs, we are working with our awareness of the awarenesses of teaching mathematics. Awarenesses can rarely be communicated or pointed to directly. An empathic, second-person perspective allows the non-judgmental arising of potential issues, linked to the behaviours of the other (be they in a classroom, a prospective teacher or an MTE) and therefore the possibility of metacommunication about those behaviours and the occasioning of new possibilities for action.

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