Chapter 5
Agent Based Model of Cross Media Reach of Advertising

Michał Kot and Bogumił Kamiński

Abstract In this paper we investigate how advertising communication strategy affects consumer buying decisions. We develop an agent based model that allows to compare how effectively different strategies of advertising budget location reach potential consumers. As effectiveness measures we use standard media metrics such as reach and frequency, but we define them for marketing campaigns that utilize many media vehicles (e.g. TV, Radio, Online). Model's parametrization and agents' features are based on results of a dedicated research study in order to obtain results valid for a population of Poland. We emphasize that our model has two unique features that are important in practical applications. Firstly, it allows us to measure not only aggregate reach and frequency but also the effectiveness of communication strategies against narrow sub-populations with high purchase potential. Secondly, one is able to assess and compare the whole distribution of projected reach of communication strategies, and in this way understand not only their expected outcome but also the associated uncertainty.

Keywords Agent based modelling · Media selection · Marketing science

Introduction

There is consent among researchers that marketing communication plays a significant role in influencing consumer decisions regarding product choice. It has been shown that advertising changes tastes in the short term [15] and builds brand equity, that reflects how well-known the brand is, in the long-term perspective [13]. Through brand equity building, advertising informs about product existence extending volume of considered baskets of goods [1] and is able to affect which product features

---

M. Kot (✉) · B. Kamiński
SGH Warsaw School of Economics, al., Niepodległości 162, 02-554 Warsaw, Poland
e-mail: michal.kot.sgh@gmail.com

B. Kamiński
e-mail: bkamins@sgh.waw.pl

P. Ahrweiler and M. Neumann (eds.), *Advances in Social Simulation*,
Springer Proceedings in Complexity,
https://doi.org/10.1007/978-3-030-61503-1_5

customers perceive as especially important, which has an impact on consumer choice function [20]. Strengthening brand equity leads to increased sales [16], brand differentiation [14] or price sensitivity [12].

Due to the importance of marketing communication in terms of affecting consumers emerges problem of assigning the budget to proper marketing techniques. It was a known problem in the beginning of the twentieth century when John Wanamaker formulated his quote ‘I know half the money I spend on advertising is wasted, but I do not know which half’ [22] and has been under researchers’ interest ever since [19], yet with the development of new media vehicles, especially digital ones, the problem got even more complex [11]. Moreover, as marketing expenditure is expected to grow by 5–7% yearly until 2022 totaling 792 billion dollars globally [9] the problem requires comprehensive investigation.

In this paper we develop a simulation based approach for media selection problem. We reproduce the population of agents-customers, assuming their heterogeneity, with a list of parameters describing socio-demographic and customers’ value features. We parametrize the agents and construct their individual media consumption stochastic functions based on data taken from the survey performed for the purpose of this research. As a result we are able to simulate the behaviour of a complex system of agents that is exposed to media communication. Our aim is to compare different media selection strategies in terms of their effectiveness. To evaluate the strategies’ expected outcomes, we use indicators based on reach and frequency among total agents’ population and also among agents with high buying potential reflected in their customer value.

Links between sales and marketing expenditure have been analyzed in the literature. Since Dorfman who has proven that only for non-perfect markets optimal advertising is not null [8] and Nerlove who has proven that in case of log-log model between demand and price optimal advertising should be at constant ratio versus sales [21] researchers focused on econometric inference of advertising to sales relationship. Two comprehensive meta analyses of econometric driven findings can be found in Assmus [2] and Vakratsas [23]. Functional dependency between advertising and sales has also been investigated by Little and Lodish with the usage of MEDIAC system [18], Zufryden with Mean Response function [25] or Liaukonyte with Causal Approach using econometric modelling [17].

A classic approach to media selection problem assumes using deterministic optimization techniques such as linear programming [5, 10], dynamic programming [26], nonlinear programming [3] or treating media selection as a multi choice knapsack problem [24]. In mentioned applications, the problem comes down to constrained maximization of combined media reach among given population. The combined reach formula used in the problem literature (either Sainsbury or Agostini formula) assumes that each media vehicle performs independently [6].

Unlike the literature standards, our approach is derived from the microfoundations. In our model agents have individual features that determine their personal attitude and habits towards advertising and as a result control the intensity of advertising that the agents are exposed to. Based on micro-level data gathered for each agent, our goal is to determine the global system values of variables measuring the absolute effect of the media campaign. This approach has four significant advantages versus classic solutions, e.g.. econometric based, that rely on aggregated metrics.

– *robustness*: It is robust to structural changes and can be used to simulate outcomes of such;
– *exact cross media reach*: It allows to calculate exact combined media reach as each agent has registered a complete history of advertising contacts, thus we are able to capture the interactions in which different media are consumed (we do not need a media independence assumption);
– *sub-population analysis*: It allows detailed analysis of segments of consumers in terms of their buying potential;
– *measurement of uncertainty*: It allows to analyze not only expected value of outcome's measures but also their distribution, thus we are able to capture the risk of different decisions.

In particular, according to our best knowledge and literature research, such an approach, combining the four above mentioned features, has not been used before in the media selection research. However, one should bear in mind that agent based approach is more time consuming as many simulation iterations are required in order to obtain robust results.

Aside from the main objective, we propose a novelty in terms of agent set creation (synthetic population generation) that incorporates the multivariate Bayesian sampling to cover the dependencies between agents' features.

The remainder of the paper is structured as follows: in section Model of Media Consumption we present the model in terms of variables, functional dependencies and mechanisms. In section Simulation Preparation we describe model parametrization and simulation setup and in section Simulation Results we present the results and discuss them in section Concluding Remarks. Due to paper's length limitations, we provide the description of key elements of the proposed model.

Model of Media Consumption

In this section we will cover the model of media consumption's elements, mechanics and configuration.

The model consists of heterogeneous agents that represent the consumers of media communication. All agents use all media vehicles with different frequency and therefore have different opportunity to see the advertisement in each of them. The simulation environment is assumed to consist of 6 media vehicles available, each denoted as M_j for $j \in \{1, \ldots, 6\}$, which are: TV, Radio, Print, Digital Display (static media form used in the Internet), Digital Video on Demand (audio-video ad format presented to consumer before the movie is loaded in the Internet) and Social Media (all ad formats used within Internet Social Media networks). Simulation is performed in a discrete time for a period of maximum T steps, with each step reflecting a single

Table 5.1 Agent's features description

Feature group	Feature name	Feature description
SD	Gender	Agent's gender
SD	Age	Agent's age group
SD	Education	Agent's level of completed education
SD	Income	Agent's household's net monthly income
SD	Location	Agent's household's location
MC	Media consumption **MCP**	Agent's probability of consumption of each media vehicle distribution
MC	Weekly frequency **WFC**	Agent's weekly frequency of consumption of each media vehicle distribution
MC	Daily frequency **DFC**	Agent's daily frequency of consumption of each media vehicle distribution
BP	Household size	Agent's household size
BP	Shopping frequency	Agent's frequency of shopping
BP	Ad potential	Agent's susceptibility to advertising
BP	Consumer value	Agent's buying potential

day. The agent set contains N agents each denoted by A_i, for $i \in \{1, \ldots, N\}$. All agents are described by a list of 12 socio-demographic (*SD*), media consumption related (*MC*) and buying potential (*BP*) features, as presented in Table 5.1. Population of agents and their features is initiated at the start of each simulation iteration and remains fixed throughout its duration. Each simulation run is divided into three phases: campaign's setup, campaign's execution and campaign's evaluation.

Campaign's setup phase is introduced by setting the advertising campaign budget B and picking the allocation strategy s_a. We assume B to be a non-negative number, while s_a is a vector of six non-negative numbers that sum up to 1, indicating share of budget assigned to each medium. Based on B and s_a a vector of budget spent on each medium under given strategy $\mathbf{B}(s_a)$ is calculated:

$$\mathbf{B}(s_a) = Bs_a \tag{5.1}$$

Budget assigned to each medium is expressed in terms of number of effective contacts. To obtain the total campaign's contacts vector $\mathbf{C}(s_a)$ we use Hadamard division of $\mathbf{B}(s_a)$ by vector of costs of generating a single contact in each medium $Cost$ (j denotes j-th element of the vector):

$$\mathbf{C}(s_a)_j = \mathbf{B}(s_a)_j / Cost_j \tag{5.2}$$

Having total volume of contacts for the campaign, we compute a plan of contacts for each simulation step $\mathbf{C}(s_a, T)$, which is a matrix. Elements of $\mathbf{C}(s_a, T)$ are denoted as C_{jt} (number of contacts in medium j in time t). In this paper we assume the plan to be a flat allocation of contacts therefore:

$$C_{jt} = \mathbf{C}(s_a)_j / T \tag{5.3}$$

Campaign's execution phase is a simulation of spreading of contacts available in each step and each media vehicle among agents. In order to achieve it, we assign each agent an opportunity-to-see metric, denoted as **M**, that is an array of elements M_{ijt}. Each **M** element is computed based on each agents **MCP**, **WFC** and **DFC** features describing individual media consumption presented in Table 5.1 in line with *MedCons* procedure presented in Algorithm 1. Based on individual computed opportunity-to-

Algorithm 1 Media consumption simulation high-level flow

```
procedure MEDCONS(t, i, j, MCP, WFC, DFC)
    for t = 1 to T do
        for i = 1 to N do
            for j = 1 to 6 do
                Sample MCP_ijt from MCP
                if rand(0, 1) < MCP_ijt then
                    Sample WFC_ijt from WCF
                    if rand(0, 1) < WFC_ijt then
                        Sample M_ijt from DFC
                    end if
                end if
            end for
        end for
    end for
end procedure
```

see in step t, for each media vehicle j there are C_{jt} agents being sampled with replacement with probability of choosing each agent proportional to M_{ijt}. Therefore, agents with higher media consumption frequency have higher probability to be sampled.

Campaign's evaluation phase summarizes the campaign performance with a list of effectiveness indicators:

– Multimedia contacts per agent (AC_i)

$$AC_i = \sum_{t=1}^{T} \sum_{j=1}^{M} C_{ijt} \tag{5.4}$$

– Multimedia reach for frequency $F = \{0, 1, 2, 3...\}$ ($MMR(F)$)

$$MMR(F) = 100 \frac{\sum_{i=1}^{N} [AC_i = F]}{N} \tag{5.5}$$

– Cumulative multimedia reach for frequency $F = \{1, 2, 3...\}$ ($CMMR(F)$)

$$CMMR(F) = 100 - \sum_{f=0}^{F-1} MMR(f) \tag{5.6}$$

– Cost per cumulative multimedia reach for frequency $F = \{1, 2, 3...\}$ ($CPCMMR(F)$)

$$CPCMMR(F) = \frac{\mathbf{B}}{CMMR(F)} \tag{5.7}$$

In summary section we evaluate the campaign's strategies using indicators $MMR(F)$ and $CPCMMR(F)$ while other presented metrics play supporting role, as they are used to calculate target metrics. For additional information please refer to the ODD: http://bit.ly/2MG34lz.

Simulation Preparation

In this section we discuss model parametrization and the simulation's setup.

Agents' Creation

For agents' creation and parametrization of their features we use data from the dedicated study provided by SW Research, one of the research agencies in Poland. The sample of 1016 respondents' answers has been collected in March 2019. Each respondent has been assigned a population's weight to reflect the structure of Poland's population. Marginal distributions of answers provided for socio-demographic questions have been presented in Table 5.2, all respondents' answers can be found in the ODD.

Each agent's creation process is divided into two phases: sampling of socio-demographic features and sampling of media consumption and buying potential features. We assume that socio-demographic features are sampled first, in line with the Algorithm 2, while features in the second phase are sampled conditionally, based on agents socio-demographic profile.

Due to the fact that socio-demographic features are not independent one should sample them taking into account their cross dependencies. We propose to sample the combination of socio-demographic features instead of sampling each feature independently from the marginal distribution. There exist 270 unique combinations of features (we will refer to those as *cells*), as for mentioned agents' characteristics we have 2, 5, 3, 3 and 3 potential answers available, respectively. However, one should consider the following problem. Firstly, sampling from marginal distribution does not take into account dependencies between features. Secondly, sampling from

Table 5.2 Socio-demographic features' marginal distributions

Variable	Class Id	Answer	Distribution
Gender	1	Male	0.4636
	2	Female	0.5364
Age	1	15–24	0.1772
	2	25–34	0.2776
	3	35–44	0.2323
	4	45–54	0.1378
	5	55+	0.1752
Education	1	Primary and lower	0.0984
	2	Secondary, incl. professional	0.4134
	3	Higher, PhD	0.4882
Income	1	Below 4.000 PLN	0.3189
	2	4.000–8.000 PLN	0.5472
	3	Over 8.000 PLN	0.1339
Location	1	Rural	0.3307
	2	City below 200.000 citizens	0.4183
	3	City over 200.000 citizens	0.2510

the survey data only would prevent us from sampling cells that do not appear in the research study, but may be present in population. To allow both, feature cross dependencies and positive probability of sampling each potential cell we propose using Bayesian sampling in line with the Algorithm 2.

Based on a conducted survey, out of 270 potential cells of features combinations, 215 had at least one respondent assigned, while 55 were null—not observed in the sample. Based on completed cells we have reconstructed the marginal distributions for each feature and as a result the prior probabilities of each cell. Afterwards, we mixed the prior with counts of respondents in each cell and obtained the posterior distribution.

Mixing data into prior has changed the distribution mostly for cells that were missing in the data set and had low prior probability of existence (e.g. combinations of young age and high income and education). Figure 5.1 shows the relation between λ and number of empty cells in the sampled population. We can use λ parameter to balance between importance of prior and survey's information. Note that the proposed algorithm produces the same marginal distributions as observed in the survey data independent on the value of λ, which influences only the level of cross-dependency between features.

Algorithm 2 Agent set sampling

1: Use Dirichlet distribution as a prior with K parameters indicating number of unique feature combinations and α_i, for $i = 1, 2, ..., K$ being concentration parameters calculated from marginal feature distributions, treated as independent variables

$$\alpha_i = \prod_{j=1}^{F} P(f_j) \in (0, 1)$$

$$\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_K)$$

Dirichlet distribution, $Dir(K, \boldsymbol{\alpha})$ with density function given as

$$f(x_1, ...x_K, \boldsymbol{\alpha}) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

$$B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{K} \alpha_i)}$$

2: Use survey data to update prior distribution with observed counts of each class β_i.

3: Obtain posterior distribution that is conjugate Dirichlet distribution with concentration hyperparameter $\boldsymbol{\theta}$. θ_i expected value is specified as

$$E(\theta_i) = \frac{\lambda\alpha_i + \beta_i}{\sum_{i=1}^{K} \lambda\alpha_i + \beta_i} \in (0, 1)$$

$$\boldsymbol{\theta} = (\theta_1, ..., \theta_K)$$

Where λ indicates the importance of prior in construction of posterior distribution (higher λ results in posterior identical to prior distribution)

4: Sample $\hat{\theta}_i$ from posterior distribution $Dir(K, \boldsymbol{\theta})$

5: Sample agents' population based on $\hat{\theta}_i$ realization

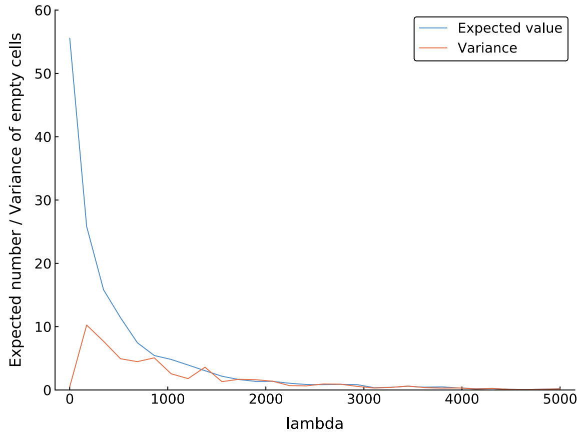

Fig. 5.1 Number of empty cells sampled from posterior distribution versus lambda

Table 5.3 Simulation parameters

Variable	Range
N	1000
T	30
B	[2000, 4000, 6000, 8000, 10000]
λ	1000
Cost	[5, 10]

Simulation Setup

Simulation algorithm has been tested using parameters presented in Table 5.3.

Parameters presented in Table 5.3 play supporting role in evaluation of allocation strategies s_a. We have created 126 s_a vectors that met the restrictions that each element is either 0.0, 0.25, 0.50, 0.75, 1.0 and sum of all vector's elements is equal to 1.0. Moreover, we created cost vectors as all possible combinations of 5 and 10 for each media vehicle, resulting in 64 vectors. Based on simulation setup there exist 40,320 potential combinations of variable values. Each combination has been iterated 5 times for 3 randomly created agents' populations which leads to a total number of simulations equal to 604,800. All computations have been evaluated using Julia language [4].

Simulation Results

In this section we present results of simulation described in Sects. Model of Media Consumption and Simulation Preparation.

Tables 5.4 and 5.5 present best five strategies in terms of building average effective reach $CMMR(1)$ and $CMMR(3)$, respectively. All strategies have been presented as a combination of a form of % of allocated budget (TV, Radio, Print, Display, VOD, Social Media) and refer to total budget equal to 8000. Results prove that TV shall remain a primary medium in the media mix, as all top strategies in terms of expected level of reach assume at least 50% of budget assigned to this media vehicle. However, results differ in case of $CMMR(1)$ and $CMMR(3)$, because in case of $CMMR(1)$ one should use wider media mix, with 2 or 3 media vehicles in media split, while to build $CMMR(3)$ one shall focus on 1 or 2 media vehicles.

The presented approach is robust of structural changes. It allows to predict the outcome of a strategy that has never been used before, e.g. adding digital media into media mix with the total budget increased by 50% as presented on Fig. 5.2. We emphasize that all strategies have been analyzed in terms of results' variability, measured by standard deviation. The analysis of uncertainty allows to draw the

Table 5.4 Best strategies in terms of generating CMMR(1)

Strategy	E(CMMR(1))	S(CMMR(1))
(50, 0, 0, 25, 0, 25)	37.85	1.37
(75, 0, 0, 0, 0, 25)	36.56	1.04
(75, 0, 0, 25, 0, 0)	35.65	1.36
(50, 25, 0, 0, 0, 25)	36.65	1.29
(50, 0, 0, 0, 25, 25)	34.84	0.98

Table 5.5 Best strategies in terms of generating CMMR(3)

Strategy	E(CMMR(3))	S(CMMR(3))
(100, 0, 0, 0, 0, 0)	28.12	0.73
(75, 0, 0, 0, 0, 25)	26.27	0.76
(75, 0, 0, 25, 0, 0)	25.71	0.68
(75, 25, 0, 0, 0, 0)	23.23	0.50
(75, 0, 25, 0, 0, 0)	22.93	0.59

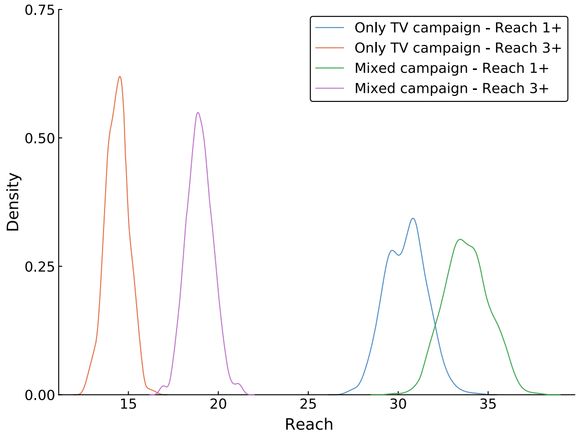

Fig. 5.2 What-if scenario evaluation example

conclusion that on average strategies generating higher levels of reach are more volatile than their less effective counterparts as shown on Fig. 5.3.

From the target group's perspective, agent based model allows to identify narrow subgroups with high buying potential to focus communication on them. A simplified presentation of such analysis is presented in Fig. 5.4, where on X axis we present agents' features combination: Education—Income—Location (e.g. 111 stands for Primary education and household net income below 2000 PLN and rural household location) and on Y axis we present Gender—Age combination (e.g. 11 stands for Male and age 15–24). Colours indicate buying potential index of a given cell and vary from dark violet (low) to yellow (high buying potential).

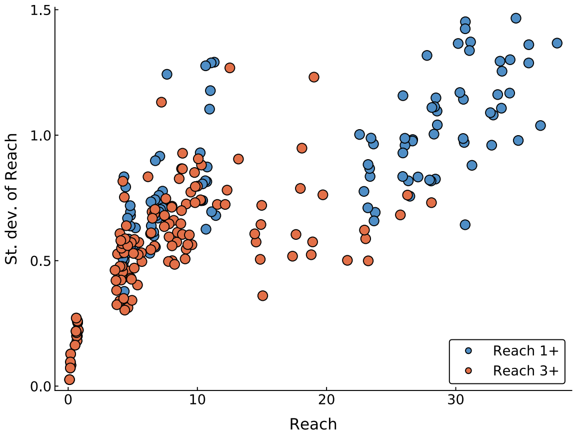

Fig. 5.3 Strategies mean reach versus st. dev. of reach

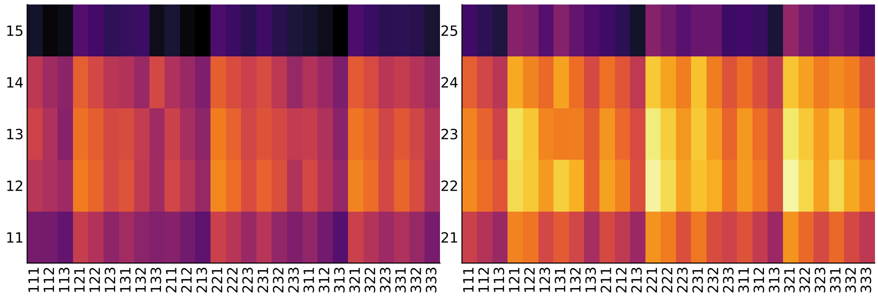

Fig. 5.4 Heatmap of target group based on buying potential(left—men, right—women)

Table 5.6 Cost of generating reach point per budget size

Budget	CPCMMR(1)	CPCMMR(3)
2000	258.22	1139.1
4000	366.10	1134.8
6000	460.28	1191.2
8000	551.24	1246.3
10000	644.55	1300.8

From the cost effectiveness's perspective each budget increase has diminishing impact on generated reach. On average, media budget of 2000 generates $CMMR(3)$ for 258 per reach point, while budget of 10,000 performs less effectively, for 665 per reach point. Detailed results have been presented in Table 5.6.

Concluding Remarks

In this paper we have presented an agent based model of media communication that allows to simulate potential advertising campaign's outcomes. Based on dedicated survey we have parametrized our model and obtained results which confirmed that campaign results in terms of media metrics are strongly dependent on the strategy of budget allocation. We have investigated population of agents seeking for especially important, from the buying potential perspective, subgroups. We plan to develop further our model with the strongest focus on:

- Extending media strategy scope with a possibility of non-flat budget assignment over time and setting longer campaign duration;
- Adding a possibility of targeting agents belonging to the desired target group;
- Extending analysis by adding value of media contacts, reflecting the fact that certain media contacts may be more persuasive than the others [7];
- Extending analysis with more complex uncertainty measures;
- Extending analysis how lambda parameter in agent sampling impacts results;
- Presenting the population of agents in a form of a network. Adding connections between agents and allowing them to communicate.

Acknowledgements The presented research was funded by Badania Młodych Naukowców BMN18/16/18 grant from SGH Warsaw School of Economics.

References

1. D.A. Ackerberg, Advertising, Learning, and Consumer Choice in Experience Good Markets: An Empirical Examination. International Economic Review **44**(3), 1007–1040 (2003). https://doi.org/10.1111/1468-2354.t01-2-00098
2. G. Assmus, J.U. Farley, D.R. Lehmann, How Advertising Affects Sales: Meta-Analysis of Econometric Results. Journal of Marketing Research **21**(1), 65–74 (1984). https://doi.org/10.2307/3151793
3. A.S. Belenky, An approach to planning an advertising campaign of goods and services. Computers & Mathematics with Applications **42**(6–7), 993–1008 (2001). https://doi.org/10.1016/S0898-1221(01)00215-2
4. J. Bezanson, A. Edelman, S. Karpinski, V.B. Shah, Julia A fresh approach to numerical computing. SIAM Review **59**(1), 65–98 (2017). https://doi.org/10.1137/141000671
5. D.B. Brown, M.R. Warshaw, Media Selection by Linear Programming. Journal of Marketing Research **2**(1), 83–88 (1965). https://doi.org/10.2307/3149343
6. J.M. Caffyn, M. Sagovsky, Net audiences of British newspapers - a comparison of Agostini and Sainsbury methods. Journal of Advertising Research **3**(1), 21–25 (1963)
7. H.M. Cannon, J.D. Leckenby, A. Abernethy, Beyond effective frequency: Evaluating media schedules using frequency value planning. Journal of Advertising Research **42**(6), 33–46 (2002). https://doi.org/10.2501/JAR.42.6.33
8. R. Dorfman, P.O. Steiner, Optimal advertising and optimal quality. The American Economic Review **44**(5), 826–836 (1954). https://doi.org/10.1007/978-3-642-51565-1_53
9. eMarketer Editors: eMarketer Releases New Global Media Ad Spending Estimates. https://www.emarketer.com/content/emarketer-total-media-ad-spending-worldwide-will-rise-7-4-in-2018 (2018), accessed: 2019/02/20

10. M.P. Haydock, Media allocation optimization. The International Journal of Applied Management and Technology **5**(2), 146–164 (2007)
11. H. Katz, The media handbook: A complete guide to advertising media selection, planning, research, and buying. Routledge (2016). https://doi.org/10.4324/9781315537870
12. A. Kaul, D.R. Wittink, Empirical Generalizations About the Impact of Advertising on Price Sensitivity and Price. Marketing Science **14**(3 (suppl.)), G151–160 (1995). 10.1287/mksc.14.3.g151
13. K.L. Keller, Brand synthesis: The multidimensionality of brand knowledge. Journal of consumer research **29**(4), 595–600 (2003). https://doi.org/10.1086/346254
14. A. Kirmani, V. Zeithaml, Advertising, perceived quality, and brand image. In: Aaker, D.A., Biel, A.L. (eds.) Brand Equity and Advertising. Advertising's Role in Building Strong Brands, chap. 10, pp. 143–162. Psychology Press (2013). 10.4324/9781315799537
15. K.J. Lancaster, A New Approach to Consumer Theory. Journal of Political Economy **74**(2), 132–157 (1966). https://doi.org/10.1086/259131
16. R.P. Leone, Generalizing What Is Known About Temporal Aggregation and Advertising Carryover. Marketing Science **14**(3 (suppl.)), G141–150 (1995). 10.1287/mksc.14.3.g141
17. J. Liaukonyte, T. Teixeira, K.C. Wilbur, Television advertising and online shopping. Marketing Science **34**(3), 311–330 (2015). https://doi.org/10.1287/mksc.2014.0899
18. J.D.C. Little, L.M. Lodish, A media planning calculus. Operations Research **17**(1), 1–35 (1969). https://doi.org/10.1287/opre.17.1.1
19. L.M. Lodish, M.M. Abraham, S. Kalmenson, J. Livelsberger, B. Lubetkin, B. Richardson, M.E. Stevens, How TV advertising works: A meta-analysis of 389 real world split cable TV advertising experiments. Journal of Marketing Research **32**(2), 125–139 (1995). https://doi.org/10.1177/002224379503200201
20. T. Meenaghan, The role of advertising in brand image development. Journal of Product & Brand Management **4**(4), 23–34 (1995). https://doi.org/10.1108/10610429510097672
21. M. Nerlove, K.J. Arrow, Optimal advertising policy under dynamic conditions. Economica **29**(114), 129–142 (1962). https://doi.org/10.2307/2551549
22. G. Rossolatos, "I know halfthe money I spend on advertising is wasted, but I do not know whichhalf (J. Wannamaker)": Semiotic answers to perennial brandingtroubles. Social Semiotics **23**(4), 545–560 (2013).10.1080/10350330.2013.799009
23. D. Vakratsas, T. Ambler, How Advertising Works: What Do We Really Know? Journal of Marketing **63**(1), 26–43 (1999). https://doi.org/10.1177/002224299906300103
24. H. Zhang, Y. Vorobeychik, A.D. Procaccia, Multi-channel marketing with budget complementarities. In: Proceedings of the 16th Conference on Autonomous Agents and MultiAgent Systems. pp. 1232–1240 (2017)
25. F.S. Zufryden, Media scheduling: a stochastic dynamic model approach. Management Science **19**(12), 1395–1406 (1973). https://doi.org/10.1287/mnsc.19.12.1395
26. F.S. Zufryden, On the dual optimization of media reach and frequency. The Journal of Business **48**(4), 558–570 (1975). https://doi.org/10.1086/295782