Kinematic Analysis of a 3T1R Fully Decoupled Parallel Manipulators Family

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Abstract. This paper analyses the kinematic behavior of a 4-DOF fully decoupled parallel manipulators family. These manipulators have a kind of motion known as Schönflies motion (SM). The kinematic analysis proposed in this paper takes into account the position analysis, differential kinematics, singularity positions, and workspace analysis of one representative parallel manipulator (PM) of such a family. Also, the advantages and applications of this type of parallel manipulators are discussed.

Keywords: Parallel manipulator · Kinematics · Schönflies motion · Singularity

1 Introduction

The kinematic analysis of PMs is a prolific field of study on robotics since a large number of them have been proposed and introduced in the past decades. Most of these manipulators have 6-DOF. However, there are several applications in which 4-DOF are sufficient [6].

Recently, many researchers proposed the employment of simpler kinematic structures for some application, that requires less than 6-DOF to accomplish the desired task. For example, for pick-and-place tasks, a broad exploited application on the modern industry needs only 4-DOF, where three translations are responsible for moving the object from a point to another and one rotation to orient the object. This kind of motion is known as Schönflies motion (SM) or 3T1R [6].

The isoconstrained or non-overconstrained PMs, are also increasingly popular for industrial applications. Lee and Lee [6] presented three different isoconstrained PMs for pick-and-place tasks. The authors developed the kinematic analysis, including the kinematic equations of motion, jacobian matrix, singularities analysis, workspace, and performance index. Kong and Gosselin [5] studied a quadratic 3T1R PM known as Quadrupteron. They presented the direct position analysis for the Quadrupteron. Moreover, a singularities analysis is presented. Taking the found results into account, the author illustrated the singularity free

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workspace regions, as well as the input conditions, in order to operate on the desirable regions.

The possibility of decoupling the end-effector's position and orientation is an attractive property on PMs. Di Gregorio [2] identified a group of 63 3T1R decoupled PMs architectures with three limbs, then he studied them adopting a unified approach. Carricato [1] presented a methodology using screw theory to synthesize the diagonal and constant forms to both direct and inverse jacobian matrix. He applied the methodology on a particular family of translational PMs, making their motions completely decoupled. Furthermore, their kinematic analysis is straightforward.

In the literature, some authors have discussed and proposed the type synthesis of architectures that perform the well known SM. One of the most thorough studies was proposed by Gogu [4], which exposed a total of 619 parallel architectures with SM. The architectures are classified into distinct families, e.g., fully-parallel topologies, overactuated topologies, topologies with decoupled motion, et cetera. PMs with 4-DOF SM applications are very diversified. Stepanenko et al. [10] presented a new 4DOF 3T1R PM for micromachining applications. This PM has a decoupled architecture and an elementary kinematic analysis.

Inspired by the diversified applications, this paper proposes a study of the fully decoupled family architectures with 4-DOF 3T1R PMs, generated in the type synthesis, proposed by Gogu [4]. Indeed, the proposed study is a kinematic analysis of such manipulators. This analysis focuses on position kinematics, velocity analysis, kinematic singularities, and workspace analysis. This paper aims to conduct a simple and straightforward kinematic analysis of one parallel architecture representing its whole PMs family. Such analysis helps to understand these PMs kinematics better, also some advantages are drawn.

This paper is organized as follows. In Sect. 2, the selected PMs family is discussed, also the selected architecture to represent the whole family is illustrated. In Sect. 3, the kinematic analysis as a whole is studied, namely the position kinematics, differential kinematics, kinematic singularities, and the workspace analysis. Finally in Sect. 4, some advantages and conclusions are presented.

2 The Fully Decoupled Schönflies Motion Family

The PMs family analyzed in this paper was synthesized by Gogu [4]. Gogu [3] also demonstrated a new formula for mobility, connectivity, redundancy, overconstraints, and evolutionary morphology. He adopted a unified approach for type synthesis, providing novel solutions for PM architectures.

This work focuses on the decoupled motion feature. The chosen family is referred to as "Fully-Parallel Topologies with Decoupled Schönflies Motions," henceforth called as "fully decoupled family." Here, only the simple limb topologies were considered. The family consist of ten architectures showed in Fig. 1.

These architectures have similar topologies, i.e., they are composed of translational, revolute, or a combination of both joints (cylindrical joint). Also, this group is composed of four spatial limbs, three of them are identical, and one is

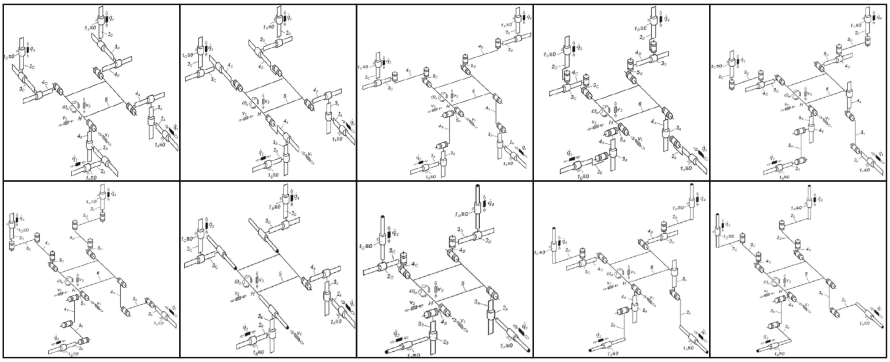

Fig. 1. Studied architectures group of the fully decoupled family. (Source:[4])

slightly distinct. The limbs have four or five joints. These limbs connect the fixed platform towards the moving one. Each of them is combined with one actuator connected on the fixed platform in a translational or cylindrical pair (just the translational motion is actuated on the cylindrical joints).

This group of architectures has 3T1R motion, that is, three translations along the Cartesian axes X, Y, and Z combined with one rotation along the X axis in this particular case. The translational velocity of the moving platform depends on one actuated joint velocity $\nu_i = \nu_i(\dot{q}_i)$, i =1, 2, 3 and the rotational velocity on two actuated joint velocities $\omega_\delta = \omega_\delta(\dot{q}_3, \dot{q}_4)$. The moving platform's action point is showed as a red dot in Fig. 2a. In other words, the adopted reference point used to analyze the kinematics depends only on each limb's actuation.

2.1 Representative Architecture for Kinematic Analysis

Considering the family particularities exposed in Sect. 2 and taking into account all ten architectures, from a kinematic point of view, they have the same actuation and action behavior. Thus, it is feasible to select one representative architecture for the kinematic analysis. Without losing generality, the chosen architecture, that represents the whole family, is shown in Fig. 2a, with its workspace volume in Fig. 2b, which will be better discussed in Sect. 3.

First, these architectures have 3T1R fully-decoupled motion, with no idle mobilities. Thus, their jacobian matrices are triangular. The moving platform rotation is along the same axis for all architectures. Furthermore, the actuators are always located on the translational joint. These properties make the position analysis, differential kinematics, singularity positions, and the workspace analysis the same for all ten architectures.

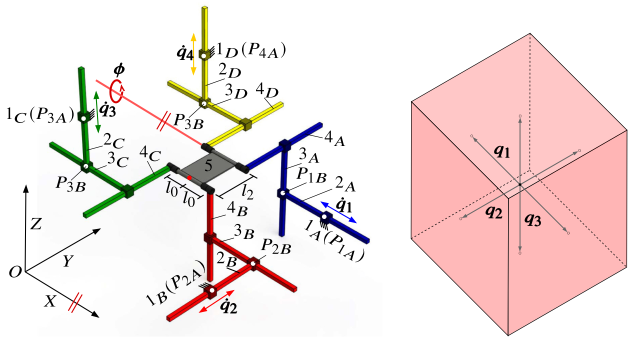

Fig. 2. (a) Selected fully decoupled PM, and (b) its workspace volume representation.

3 Kinematic Analysis

The kinematics of some PM is an important feature, producing pertinent data that can be assertively used to define its kinematic design. The proper use of the data provided by the kinematic analysis allows determining a set of geometric parameters. These parameters help achieve the manipulator's optimal performance, in terms of its workspace, accuracy, velocity, and so on [7].

3.1 Position Kinematics

A geometric approach is employed for kinematic analysis. The strategy consists of using generic dimensions, points, and vectors, strategically positioned along with the manipulator's limbs, respecting its morphology, the position kinematics can be calculated. A functional representation of all the four legs is shown in Fig. 3.

It is possible to see that the manipulator's legs functional representations have different perspective views. The first one represents the XZ plane, while the other ones represent the YZ plane. Some geometric parameters are thus adopted, i.e., the points P_{iA}, $i = 1, 2, 3, 4$ are located on the fixed links. Additionally, the points P_{iB} are representing 2_j, j = A, B, C, D links extremities (distal points), located near to the passive translational joints in each limb. Both P_{iA} and P_{iB} points were represented in Fig. 2a, for each limb as white dots.

Moreover, the $\dot{\boldsymbol{q}}_{\boldsymbol{i}}$ vectors are the translational velocities of the input links 2_j. Finally, l_0 is a generic geometric parameter of the moving platform defining its length along the X axis, while l_2 is the moving platform length along the Y axis. l_3 is a geometric variable that depends on the moving platform angle between the X axis, namely the ϕ angle[1]. All of the parameters, as mentioned above, can be seen in Fig. 2a.

[1] As cited in Sect. 2, the moving platform rotational velocity, including the rotation magnitude (moving platform inclination), depends on two actuated joints ($\dot{q}_3$,$\dot{q}_4$).

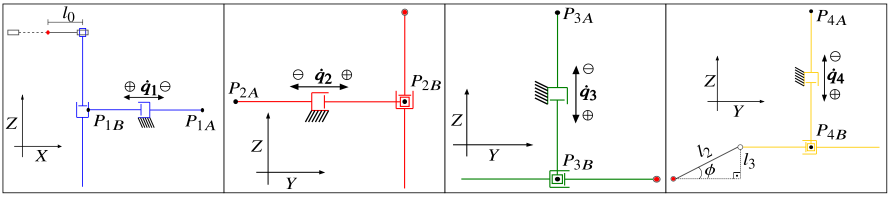

Fig. 3. Functional representation of all manipulator's limbs.

Once defined these geometric parameters, according to Fig. 3, it can be postulated the following equations:

$$P_{iA} = \begin{bmatrix} P_{iA_x} \\ P_{iA_y} \\ P_{iA_z} \end{bmatrix}, \begin{cases} P_{1B} = P_{1A} + q_1 \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ P_{2B} = P_{2A} + q_2 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{cases}, \begin{cases} P_{3B} = P_{3A} + q_3 \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ P_{4B} = P_{4A} + q_4 \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \end{cases}, \tag{1}$$

where P_{iA} is a point at the fixed link, so its coordinates have constant values, the values depend on the manipulator's dimensions. Also, the point P_{iB} is located on the same P_{iA} axis, so they have the same coordinates, varying only on the distance along one axis, which leans on the translational joint direction. The distance between both points relies on the translation magnitude of the input link 2_j.

Here, with Eq. 1, the position and orientation of the moving platform can be fully defined as presented in the below equations. In fact, the parameter l_3, was used to calculate the orientation as follows $l_3 = P_{4B_z} - \text{Plat}_z$. Plat_z is the last vector coordinate of the moving platform's action point. Then, one can substitute the values accordingly and calculate the ϕ using its inverse sine:

$$\text{Plat} = \begin{bmatrix} \text{Plat}_x \\ \text{Plat}_y \\ \text{Plat}_z \end{bmatrix} = \begin{bmatrix} P_{1A_x} - q_1 - l_0 \\ P_{2A_y} + q_2 \\ P_{3A_z} - q_3 \end{bmatrix}, \phi = sin^{-1} \left[\frac{(P_{4A_z} - q_4) - (P_{2A_z} - q_3)}{l_2} \right]. \tag{2}$$

3.2 Differential Kinematics

The jacobian matrix (J) represents the actuated joints velocities map; likewise, the same works for the end-effector [8]. The Jacobian can be achieved by differentiating the position kinematics equations. In PMs, the Jacobian can be derived as follows:

$$J_s \cdot \dot{q}_s = J_p \cdot \dot{q}_p \Rightarrow \dot{q}_s = {J_s}^{-1} \cdot J_p \cdot \dot{q}_p \text{ and } \dot{q}_p = {J_p}^{-1} \cdot J_s \cdot \dot{q}_s, \tag{3}$$

Further, l_3 is directly proportional to the moving platform inclination, since it represents the ϕ angle opposite side.

also considering $J_p = \frac{\partial F(s)}{\partial t}$ and $J_s = \frac{\partial F(p)}{\partial t}$, the subscript suffix "$p$" represents the primary variables, while the "s" one represents the secondary ones [9]. J_p and J_s are respectively inverse and direct Jacobian. Considering the Eq. 3, then applying it in Eq. (2), one can assertively derive these above equations:

$$\begin{cases} T_x = P_{1A_x} - q_1 - l_0 \\ T_y = P_{2A_y} + q_2 \\ T_z = P_{3A_z} - q_3 \\ l_2 \cdot sin\phi = P_{4A_z} - q_4 - P_{2A_z} + q_3 \end{cases} \Rightarrow \begin{cases} V_x = -\dot{q}_1 \\ V_y = \dot{q}_2 \\ V_z = -\dot{q}_3 \\ l_2 \cdot cos\phi \cdot \omega_\phi = -\dot{q}_4 + \dot{q}_3 \end{cases}. \tag{4}$$

The equation array on the left side means the displacements, while the right side refers to velocities and represents the time derivative. Therefore, adopting the Eq. (3), yields:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & l_2 \cdot cos\phi \end{bmatrix} \cdot \begin{bmatrix} V_x \\ V_y \\ V_z \\ \omega_\phi \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}. \tag{5}$$

3.3 Kinematic Singularities

The singular configuration appears when the rank of the Jacobian matrix drops, and this also can be detected from calculating the matrix determinant, i.e., $det(J) = 0$.

There are no inverse singularities, once the inverse jacobian matrix never has the determinant equal to zero ($det(J_p) \neq 0$).

The direct singularities come from the jacobian matrix J_s, that is $det(J_s) = 0 \rightarrow l_2 \cdot \cos\phi = 0 \rightarrow \cos\phi = 0$. It follows that these direct singularities occurs when the moving platform is oriented on 90° or −90° regarding the X Cartesian axis.

3.4 Workspace Analysis

To better understand the workspace (WS) of the selected architecture, a displacement study was elaborated similarly as done by Stepanenko et al. [10]. A CAD model was constructed employing the SOLIDWORKS® program, and this model is that one exposed in Fig. 2a.

To construct the model actuators and translational passive joints, a specific displacement was adopted, $q_1 = q_2 = q_3 = q_4 = q \Rightarrow 0$ mm $\leq q \leq 300$ mm. The rotation displacement magnitude on the revolute joints was also determined as $-45° \leq \phi \leq 45°$. Once the action point at the moving platform is defined, it is possible to import its CAD model coordinates along with the workspace. Thus, one can plot them and use an algorithm to calculate the whole workspace volume for an imposed moving platform inclination (ϕ).

The obtained results are exposed in Table 1, in which T_x, T_y and T_z are representing the maximum translational displacement along the Cartesian axes.

Table 1. Results obtained with the displacement study on CAD model.

Phi [°]	T_x [mm]	T_y [mm]	T_z [mm]	WS Volume [cm^3]
±45	300	235.57	144.44	10207.72
±33.75	300	262.93	177.78	14023.11
±22.5	300	283.25	215.81	18338.45
±11.25	300	295.77	257.08	22810.96
0	300	300	300	27000

In fact, the workspace volume morphology is a parallelepiped that varies its pattern between a perfect cube when $\phi = 0°$, into a generic parallelepiped when $\phi \neq 0°$. The first case for a perfect cube is seen in Fig. 2b.

To mathematically analyze the workspace in a comprehensive way, one can represent the moving platform on the YZ plane perspective, as shown in Fig. 4.

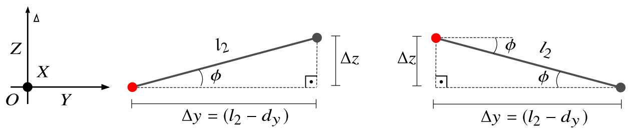

Fig. 4. Two possible situations for the moving platform orientation.

With these values, one can calculate the total translation displacement along the Y and Z axes by:

$$\begin{cases} T_y = (T_{y0} - d_y) \\ T_z = (T_{z0} - \|d_z\|) \end{cases} \Rightarrow \begin{cases} T_y = (T_{y0} - l_2(1 - cos\phi)) \\ T_z = (T_{z0} - \|l_2(sin\phi)\|) \end{cases}, \tag{6}$$

where d_y and d_z represent the dimensional discrepancy along the Y and Z axes, respectively. One can conclude that this disparity relies on the moving platform inclination.

The total workspace volume is given by:

$$\text{WS Volume} = T_x \cdot [T_{y0} - l_2(1 - cos\phi)] \cdot [T_{z0} - \|l_2(sin\phi)\|]. \tag{7}$$

4 Conclusions

This paper presented the kinematic analysis of the fully decoupled family proposed by Gogu [4], where the simple limb topologies were considered. The position and differential kinematics, singularities, and workspace volume have been studied.

The direct kinematic analysis shows a one-to-one relationship between the input actuated links and the end-effector position. This peculiarity makes the manipulator control easier, facilitating its path-planning calculation. As a consequence of the simple position kinematics, the differential one is also trivial, producing triangular smooth jacobian matrices, enabling an effortless manipulator velocities mapping.

Kinematic singularities showed themselves as very well-conditioned, the manipulator has only direct singularities at convenient orientations throughout the workspace. This fact aids in kinematic singularities avoidance on real applications.

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