

Chapter 1

A Dynamic Lot Sizing Model Under Vendor Managed Inventory (VMI)



Farshid Evazabadian, Regina Berretta, and Mojtaba Heydar

Abstract This paper explores the impact of Vendor managed inventory (VMI) on a decentralized supply chain by proposing two Mixed-integer linear programming (MILP) models for a dynamic lot sizing problem. The models describe the decision-making for lot sizing before and after the implementation of VMI. The proposed models highlight that VMI takes advantage of centralized decision-making, and can reduce the cost of the lot sizing by better synchronization of the decisions. Numerical results, provided to compare the efficiency of VMI with the traditional decentralized lot sizing, indicate significant cost reduction under VMI. A set of experiments is also designed to determine the impact on retailers' cost (ordering and holding) as well as vendors cost (setup and holding) on the gap.

Keywords Vendor managed inventory · Retailer managed inventory · Lot sizing · Mixed-integer linear program

1.1 Introduction

Vendor managed inventory (VMI) is a partnership based on the information shared between the retailer and the vendor (the manufacturer or the supplier) [3]. In VMI, the management of the retailer's inventory is transferred from the retailer to the vendor, after setting shelf space requirements or service level by the retailer [18]. Therefore, the retailer's role changes from managing the inventory to renting the inventory

F. Evazabadian (✉) · R. Berretta

School of Electrical Engineering and Computing, The University of Newcastle, Callaghan, NSW 2308, Australia

e-mail: Farshid.Evazabadian@uon.edu.au

R. Berretta

e-mail: Regina.Berretta@newcastle.edu.au

M. Heydar

Discipline of Mathematics, School of Natural Sciences, University of Tasmania, Hobart, TAS 7001, Australia

e-mail: Mojtaba.Heydar@utas.edu.au

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space [14, 18]. Both retailer(s) and the vendor benefit from VMI implementation. Vendors benefit from VMI by having direct access to the demand information and consequently having more accurate forecasts [12]. Retailers benefit from VMI by increasing their service level and product availability [5, 12], Yao et al. [22].

According to Marquès et al. [13], VMI has three main agreements: partnering agreement, logistical agreement and production and dispatch agreement. In *partnering agreement*, retailers and vendors decide how to collaborate with each other. In fact, they agree on issues such as information that should be shared, the periodicity of transferring information and the timescale of forecasting [13]. In *logistical agreement*, parties discuss issues related to transportation and delivery of products, such as determining the minimum delivery quantity and transport schedule [7, 13]. In *production and dispatch agreement*, the parties make decisions about production planning and shipment scheduling [13]. Production planning contains different stages, including aggregate planning, detailed planning and lot sizing [9]. Lot sizing is considered as an operational level decision that aims to identify the quantity and time of production that minimizes the sum of production, setup and holding costs [2, 9]. The early developments in lot sizing originate from the Economic order quantity (EOQ), proposed by Harris [8], and the Wagner-Whitin algorithm by Wagner and Whitin [6, 20].

In this study, we show how lot sizing can be modelled under VMI. A review related to the lot sizing under VMI is presented in the next section. In Section Mathematical Modelling, two mathematical models for the lot sizing problem are introduced, where only one works under VMI. In Section Numerical Experiments, two sets of numerical experiments are presented. In the first, the models are tested with a set of generated instances to show the range of cost reduction under VMI. Next, our study shows the impact of some of the parameters (retailer setup cost, retailer holding cost, vendor setup cost and vendor holding cost) on the VMI efficiency. Finally, the conclusion and directions for future research are presented in Section Conclusion.

1.2 Literature Review

Lot sizing models under VMI can be divided into static and dynamic models. By static models, we mean single-period and continuous timescale models, while dynamic models are multi-period models. In static models, it is assumed that the parameters of the problem, especially demand, are constant and do not change over the planning horizon [11]. However, these parameters can change over time in a dynamic model. Static models can also be divided into deterministic and stochastic models. According to these two factors: uncertainty of parameters (stochastic-deterministic) and variation of parameters over time (static-dynamic), four possible categories of studies can be defined as shown in Table 1.1.

The deterministic-static studies related to lot sizing under VMI first introduced by Yao et al. [22]. They developed two models based on EOQ to compare the total

Table 1.1 Classification of studies for lot sizing under VMI

	Static	Dynamic
Deterministic	Deterministic-Static	Deterministic-Dynamic
Stochastic	Stochastic-Static	Stochastic-Dynamic

cost of a supply chain under VMI and Retailer management inventory (RMI). Van der Vlist et al. [19] extended Yao et al. [21] work by considering shipment cost for the problem. Pasandideh et al. [16] presented an EOQ model with the backlog and discussed how the backlog can increase the profit of the supply chain. Sadeghi et al. [17] presented a model for a multi-vendor, multi-retailer and single warehouse, including a limit for the number of orders and inventory space.

Deterministic-static problems focus on the retailer-vendor competition. Yugang et al. [26] study a supply chain in which the product has two prices: wholesale price and end price, the former is determined by the vendor and the latter set by the retailer. Yu and Huang [25] considered a condition in the supply chain where there are two games between retailers and vendor. The first game is between retailers to gain market share and the second game is between vendor and retailers to achieve profit. They formulated the problem as two Nash games: vertical (between retailers) and horizontal (between vendor and retailers). Yu et al. [24] developed a model to identify which retailers (among several retailers) should be chosen for a VMI partnership to maximize the profit of supply chain.

Stochastic-static studies consider the condition that the values of some of the parameters are not known. In the earliest research in this area, Mishra and Raghunathan [14] considered a supply chain with two vendors and one retailer in which vendors produce two different but substitutable products (each vendor produces one product). Yao et al. [21] study the condition which the vendor, instead of holding a significant amount of the product as inventory, the vendor can pay the retailers to convert the lost sale to backorder when stock-out occurs. In another research, Yao et al. [23] extended their study by considering both backorder and lost sale. Nia et al. [15] developed a Fuzzy programming (FP) model to identify the order size in a supply chain in which multi-products are produced.

In contrast with single-period models, dynamic lot sizing models under VMI have gained less attention. All the dynamic studies fall into the deterministic dynamic category (Table 1.1). In one study, Jaruphongsa et al. [10] developed an MILP model for the incapacitated lot sizing problem, in which for each period, there is a time window, during which the demand must be satisfied. Al-Ameri et al. [1] developed an MILP model for integrated routing and lot sizing problem in a supply chain with multiple manufacturers and retailers. Archetti et al. [4] proposed an MILP model for integrated routing and lot sizing problem in a supply chain, in which there is one vendor, producing one product for several retailers over a time horizon.

As discussed, the majority of the studies related to lot sizing under VMI falls in the static category and there are a few studies in the dynamic area. Static models, due to considering the parameters constant, cannot be applicable for the real cases. Hence, in this study, we present a dynamic lot sizing model under VMI and show

how VMI can be different from a traditional supply chain. Moreover, we show the most affecting factors that differentiate VMI from a traditional supply chain.

1.3 Mathematical Modelling

We consider a supply chain including one manufacturer (vendor) and multiple retailers ($j = 1, 2, \dots, J$). There are multi-products ($i = 1, 2, \dots, I$) which each retailer has a specific demand over a time horizon ($t = 1, 2, \dots, T$). It is assumed that retailers are independent in terms of decision-making, and there is no competition between them for achieving a higher profit. In other words, the demand d_{ijt} for the product i of retailer j in period t is independent of the other retailers' demand. It should be noted that this work considers products with infinite shelf life which will not perish over the horizon. The retailer's costs include an ordering fixed cost sr_{ijt} that does not depend on the quantity of the order, a unit ordering cost o_{ijt} , a holding cost hr_{ijt} for each unit of product and a backorder cost br_{ijt} for each unit of product, if stock-out occurs. It is also considered an ordering lead time lr_{ij} , which is the time, measured in number of periods, between placing of an order and receiving the ordered products. The aim of each retailer is to minimize its total cost, including ordering, holding and backorder cost.

The vendor's costs are setup and production costs for each unit denoted by s_{it} and c_{it} , respectively, a holding cost for each unit (h_{it}), and if stock-out occurs, a backorder cost bs_{it} is incurred for each unit. In regards to capacity for production, it is considered that v_{it} is the necessary time to produce one unit of product i in period t , f_{it} is the setup time required to start production, and production lead time l_i indicates the time from initiations of production until the products become available. Similarly, to the retailers, the vendor aims to minimize its total cost, which includes setup, production and holding costs considering available time (cap_t) as a major limitation. It is assumed that vendor and retailers both have their own warehouse for holding the products.

In order to respond to the demand, two scenarios are considered. In the first scenario which is called RMI, retailers decide how to satisfy the demand. For this purpose, they set some orders and send them to the vendor and based on those orders the vendor plans the production. In the second scenario which is VMI, retailers share the information of demand with the vendor and transfer the decision-making to the vendor. It is assumed that at the beginning of the first period, depends on the scenario, retailers either share the demand information (VMI case) or send their orders over the time horizon to the vendor (RMI case).

In the RMI scenario, each party deals with its own optimization problem. Therefore, in the RMI scenario two mathematical models are required, one for each party (retailers and vendor). In RMI, first retailers make decision and afterwards vendor. However, in the VMI scenario, retailers share the demand information and all the decisions are made by the vendor. Hence, in the second scenario, there is only one mathematical model and all the decisions are made simultaneously in it.

First, we describe the mathematical models for the RMI scenario. In RMI there is one model for retailer j (Model R_j) and one for the vendor (Model V). The notation used is as follows:

Index	
i :	index for product $i = 1, \dots, I$.
j :	index for the retailer $j = 1, \dots, J$
t :	index for time period $t = 1, \dots, T$.
Parameters:	
sr_{ijt} :	fixed cost of ordering product i for the retailer j in period t
o_{ijt} :	unit ordering cost of product i for the retailer j in period t
hr_{ijt} :	unit holding cost of product i for retailer j at the end of period t
d_{ijt} :	demand of product i for retailer j in period t
lr_{ij} :	ordering lead time of product i for retailer j
br_{ijt} :	backorder cost incurred for each unit of product i when stock-out for retailer j occurs in period t
s_{it} :	setup cost incurred if product i is produced in period t
c_{it} :	unit production cost of product i in period t
h_{it} :	unit holding cost for the product i in vendor warehouse at the period t
v_{it} :	necessary time to produce one unit of product i in period t
f_{it} :	setup time required to produce product i in period t
l_i :	the production lead time of product i
bs_{it} :	the backlogging cost which is incurred for each unit of product when the stock-out for vendor occurs
M :	an upper bound for production
cap_t :	total production time available in period t

Decision variables:

$YR_{ijt} =$	If retailer j orders a batch of product i
$\begin{cases} 1 \\ 0 \end{cases}$	Otherwise
XR_{ijt} :	The order quantity of product i for retailer j in period t .
IR_{ijt}^+ :	Overstock inventory of product i for retailer j in period t
IR_{ijt}^-	Under-stock inventory of product i for retailer j that backlogged at the end of period t
$Y_{it} =$	If the vendor produces product i in period t
$\begin{cases} 1 \\ 0 \end{cases}$	Otherwise
X_{it} :	Production quantity of product i in period t
I_{it}^+ :	Overstock inventory of product i for the vendor at the end of period t

(continued)

(continued)

I_{it}^-	Under-stock inventory of product i backlogged for the vendor at the end of period t
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In the following, Models R_j (for the retailer j) and V (for the vendor) are presented.

Model R_j (for Retailer $j : 1, \dots, J$):

$$\min \sum_i \sum_t sr_{ijt} YR_{ijt} + \sum_i \sum_t hr_{ijt} IR_{ijt}^+ + \sum_i \sum_t br_{ijt} IR_{ijt}^- + \sum_i \sum_t o_{ijt} XR_{ijt} \quad (1.1)$$

s.t

$$IR_{ijt}^+ - IR_{ijt}^- = IR_{ijt-1}^+ - IR_{ijt-1}^- + XR_{ij,t-lr_{ij}} - d_{ijt}, i = 1, \dots, I; t = 1, \dots, T, \quad (1.2)$$

$$XR_{ijt} \leq \left(\sum_{t'} d_{ij,t'} \right) YR_{ijt}, i = 1, \dots, I; t = 1, \dots, T, \quad (1.3)$$

$$IR_{ijt}, XR_{ijt} \geq 0, i = 1, \dots, I; t = 1, \dots, T, \quad (1.4)$$

$$YR_{ijt} \in \{0, 1\}, i = 1, \dots, I; t = 1, \dots, T. \quad (1.5)$$

In model R_j , the objective function (1.1) aims to minimize the total cost of retailer j including ordering, holding and backorder costs. Equation (1.2) is the inventory balance for the retailer j . Constraint (1.3) are logical constraints that relate the decision variable X_{it} to the binary decision variable Y_{it} meaning that whenever an order quantity is greater than zero, variable YR_{it} must take on value 1. Constraints (1.4) and (1.5) show the domains of the variables.

Model V (for vendor):

$$\min \sum_t \sum_i s_{it} Y_{it} + \sum_t \sum_i h_{it} I_{it}^+ + \sum_t \sum_i bs_{it} I_{it}^- + \sum_t \sum_i c_{it} X_{it} \quad (1.6)$$

s.t

$$I_{it}^+ - I_{it}^- = I_{it-1}^+ - I_{it-1}^- + X_{i,t-l_i} - \sum_j XR_{ijt}, i = 1, \dots, I; t = 1, \dots, T, \quad (1.7)$$

$$X_{it} \leq MY_{it}, i = 1, \dots, I; t = 1, \dots, T, \quad (1.8)$$

$$\sum_i (f_{it} Y_{it} + v_{it} X_{it}) \leq cap_t, t = 1, \dots, T, \quad (1.9)$$

$$I_{it}, X_{it} \geq 0, \quad i = 1, \dots, I; t = 1, \dots, T, \quad (1.10)$$

$$Y_{it} \in \{0, 1\} \quad i = 1, \dots, I; t = 1, \dots, T. \quad (1.11)$$

In model V , the objective function (1.6) aims to minimize the sum of holding, backorder, production and setup costs for the vendor. The Eq. (1.7) is the inventory balance for the vendor. Constraint (1.8) guarantee that the solution have set up when it has production. Constraint (1.9) represent the capacity limitation for production. Constraints (1.10) and (1.11) show the domain of variables.

It should be noted that the decision variables XR_{ijt} in Model R are defined as an input parameter for Model V . Formulations (1.12)–(1.21) represents the mathematical model for VMI.

Objective function (1.12) minimizes the summation of retailers and vendor costs. The constraints (1.13)–(1.16) are similar to constraints (1.2)–(1.5) except the former is considered for just one retailer (retailer j), but the latter is considered for all the retailers simultaneously. Finally, constraints (1.17)–(1.21) are similar to the constraints (1.7)–(1.11).

It is observed that the RMI model is hierarchical, while the VMI model is integrated. In other words, VMI model aggregates all the objective functions as well as the constraints and centralizes the decision-making. Due to integration in the VMI model, the performance of the supply chain improves. This issue is discussed in the Lemma below.

Lemma: *VMI offers the optimal solution for the whole supply chain.*

Proof. Since under VMI, the objective function is the minimization of the total cost for the whole supply chain and all the constraints, including retailers and vendor constraints, are considered simultaneously, the obtained solution offers the minimum cost for the whole supply chain.

Since VMI offers the optimal solution for the supply chain, it is interesting to know how much the difference in total cost between RMI and VMI is.

Model VMI

$$\begin{aligned} \min \quad & \sum_i \sum_t \sum_j sr_{ijt} Y R_{ijt} + \sum_i \sum_t \sum_j hr_{ijt} I R_{ijt}^+ + \sum_i \sum_t \sum_j br_{ijt} I R_{ijt}^- + \sum_i \sum_t \sum_j o_{ijt} X R_{ijt} \\ & + \sum_t \sum_i s_{it} Y_{it} + \sum_t \sum_i h_{it} I_{it}^+ + \sum_t \sum_i bs_{it} I_{it}^- + \sum_t \sum_i c_{it} X_{it} \end{aligned} \quad (1.12)$$

s.t

$$\begin{aligned} I R_{ijt}^+ - I R_{ijt}^- &= I R_{ijt-1}^+ - I R_{ijt-1}^- + X R_{ij,t-lr_{ij}} - d_{ijt}, \\ i &= 1, \dots, I; j = 1, \dots, J; t = 1, \dots, T, \end{aligned} \quad (1.13)$$

$$XR_{ijt} \leq \left(\sum_{t'} d_{ij,t'} \right) YR_{ijt}, \quad i = 1, \dots, I; j = 1, \dots, J; t = 1, \dots, T, \quad (1.14)$$

$$IR_{ijt}, XR_{ijt} \geq 0, \quad i = 1, \dots, I; j = 1, \dots, J; t = 1, \dots, T, \quad (1.15)$$

$$YR_{ijt} \in \{0, 1\}, \quad i = 1, \dots, I; j = 1, \dots, J; t = 1, \dots, T, \quad (1.16)$$

$$I_{it}^+ - I_{it}^- = I_{it-1}^+ - I_{it-1}^- + X_{i,t-l_i} - \sum_j XR_{ijt}, \quad i = 1, \dots, I; t = 1, \dots, T, \quad (1.17)$$

$$X_{it} \leq MY_{it}, \quad i = 1, \dots, I; t = 1, \dots, T, \quad (1.18)$$

$$\sum_i (f_{it}Y_{it} + v_{it}X_{it}) \leq cap_t, \quad t = 1, \dots, T, \quad (1.19)$$

$$I_{it}, X_{it} \geq 0, \quad i = 1, \dots, I; t = 1, \dots, T, \quad (1.20)$$

$$Y_{it} \in \{0, 1\} \quad i = 1, \dots, I; t = 1, \dots, T. \quad (1.21)$$

1.4 Numerical Experiments

In the previous section, we showed that the total cost of lot sizing under VMI is less than or equals to the RMI optimal solution. One important question is how much the maximum possible cost reduction applied by the VMI is. To answer this question, a numerical approach is applied. A set of instances is generated based on the characteristics and parameters depicted in Tables 1.2 and 1.3, respectively. For generating capacity, the formula (1.22) is used. Each instance is solved under RMI and VMI, and the relative gap (the difference between the total cost under RMI and VMI) based on Eq. (1.23) is calculated.

Table 1.2 Instance characteristics

Group 1 of instance	
$I \times J \times T$	$S_1 = 1 \times 2 \times 10, S_2 = 1 \times 10 \times 10, S_3 = 1 \times 20 \times 10$ $S_4 = 10 \times 2 \times 10, S_5 = 10 \times 10 \times 10,$ $S_6 = 10 \times 20 \times 10$ $S_7 = 20 \times 2 \times 10, S_8 = 20 \times 10 \times 10,$ $S_9 = 20 \times 20 \times 10$
Total	90 (10 instances for each $S_i, i = 1, \dots, 9$)

Table 1.3 Parameters used to generate instances

Parameters	Interval
d_{ijt}	$U[0,100]$
sr_{ijt}	$U[100,200]$
o_{ijt}	$U[1,5]$
hr_{ijt}	$U[2,20]$
br_{ijt}	$U[4,40]$
s_{it}	$U[250,1000]$
c_{it}	$U[2,10]$
h_{it}	$U[1,10]$
bs_{it}	$U[2,20]$
v_{it}	$U[1.5,2]$
f_{it}	$U[150,200]$
cap_t	$1.2c_t$

$$c_1 = \left(\frac{\sum_i \sum_j \sum_{t=1}^{T-1} d_{i,j,t+1} \times v_{it} + \sum_{t=1}^{T-1} f_{it}}{(T - 1)} \right) \tag{1.22}$$

$$gap = \left(\frac{Z^{RMI} - Z^{VMI}}{Z^{RMI}} \times 100 \right). \tag{1.23}$$

Table 1.4 shows the gap between RMI and VMI for all the generated instances. It is observed that the range of cost reduction after VMI implementation is between 0 and 6.4%. Since the lot sizing cost can be very high, to reduce to even only 0.1% can lead to a remarkable saving for the parties.

Table 1.4 % Gap between RMI and VMI total cost for the generated instances

Instances												
	$I \times J \times T$	1	2	3	4	5	6	7	8	9	10	Average
Sizes	$1 \times 2 \times 10$	2.3	5.0	4.7	0.3	2.3	0.9	0	1.7	0.6	1.3	1.91
	$1 \times 10 \times 10$	1.0	0.5	1.1	1.4	6.4	1.4	1.7	0.6	5.3	1.5	2.09
	$1 \times 20 \times 10$	3.1	1.4	1.5	2.3	0.4	0.9	1.7	1.8	1.4	2.2	1.67
	$10 \times 2 \times 10$	1.3	3.7	2.0	1.8	1.9	2.1	2.6	2.4	1.9	2.8	2.25
	$10 \times 10 \times 10$	1.2	1.2	0.9	1.5	1.3	1.4	1.2	1.2	1.5	1.2	1.26
	$10 \times 20 \times 10$	2.0	1.1	0.9	1.2	1.5	1.4	1.4	2.0	1.3	1.0	1.38
	$20 \times 2 \times 10$	2.4	1.8	1.5	2.7	2.6	2.6	2.5	2.1	2.6	2.2	2.3
	$20 \times 10 \times 10$	1.0	1.2	1.7	1.2	1.1	1.2	1.5	1.0	1.3	1.3	1.25
	$20 \times 20 \times 10$	1.6	1.2	1.3	1.4	1.0	1.5	1.6	1.2	1.2	1.2	1.32

1.5 Analysis of Variation of Parameters

This section presents a study about the impact of different parameters on the gap between RMI and VMI. These parameters are: retailer holding cost (hr_{ijt}), vendor holding cost (h_{it}), vendor setup cost (s_{it}) and retailer fixed cost for ordering (sr_{ijt}). As mentioned in the previous section, each parameter is generated in a specific interval. In order to determine the impact of each parameter, we divide the interval into three equal sub-intervals and consider only the *low* and *high* intervals for these experiments. The low level refers to the first one third of parameters' interval, while the high refers to the last one third of the interval. For example for the parameter (hr_{ijt}) the low refers to the range [2, 8].

Since we are considering four parameters with two levels (high and low), we have 16 ($=2^4$) possible combinations which we consider each of them as a case. In order to analyse the impact of the parameters on the gap (between RMI and VMI), we generated two instances for each case in each size. In other words, for each case, we have 18 ($=2 \times 9$) instances. The average of gap for all of these 18 instances for each case is shown in Table 1.5. It should be noted in Table 1.5, L and H refer to low and high levels, respectively.

In order to assess the impact of one specific parameter (target parameter) on the gap, we need to compare the instances with the same parameter intervals, except the target parameter. For example, in order to analyse the impact of the retailer holding

Table 1.5 Average gap between RMI and VMI for each case

Case	Vendor setup cost	Vendor holding cost	Retailer setup cost	Retailer holding cost	Gap (%)
1	L	L	L	L	2.83
2	L	L	L	H	5.38
3	L	L	H	L	2.30
4	L	L	H	H	5.24
5	L	H	L	L	0.39
6	L	H	L	H	0.47
7	L	H	H	L	0.36
8	H	H	H	H	0.58
9	H	L	L	L	2.60
10	H	L	L	H	5.34
11	H	L	H	L	3.33
12	H	L	H	H	5.12
13	H	H	L	L	0.35
14	H	H	L	H	0.76
15	H	H	H	L	0.38
16	H	H	H	H	0.70

cost on the gap, we compare the instances which the other parameters are produced in the same interval, for example, cases 1 and 2, or cases 3 and 4. The instances which are required to be compared to assess the impact of each parameter are listed below.

- (a) Retailer holding cost: compared cases are $\{(1, 2), (3, 4), (5, 6), (7, 8), (9, 10), (11, 12), (13, 14), (15, 16)\}$,
- (b) Retailer setup cost (fixed cost of ordering): compared cases are $\{(1, 3), (2, 4), (5, 7), (6, 8), (9, 11), (10, 12), (13, 15), (14, 16)\}$.
- (c) Vendor holding cost: compared cases are $\{(1, 5), (2, 6), (3, 7), (4, 8), (9, 13), (10, 14), (11, 15), (12, 16)\}$.
- (d) Vendor setup cost: compared cases are $\{(1, 9), (2, 10), (3, 11), (4, 12), (5, 13), (6, 14), (7, 15), (8, 16)\}$.

Figure 1.1 shows the comparison of the results of paired cases when each parameter is set at its low or high level, respectively. In this figure, the Y axis shows the value of gap (%) for each case.

According to Fig. 1.1, the gap between RMI and VMI increases by increasing retailer holding cost and vendor's holding cost. Moreover, it shows that the vendor's and retailers' setup costs do not have a considerable impact on the gap of optimality. It is also possible to note that lower holding costs lead to lower optimality gap.

1.6 Conclusion

This paper presented two mixed-integer linear mathematical formulations for lot sizing under RMI and VMI paradigms. First, we developed a model (RMI) in which each member optimizes its own production plan individually. Then, in the second model (VMI), retailers share the demand information and transfer the decision-making to the vendor. Through a lemma, we showed that VMI always outperforms RMI and our numerical study showed VMI can reduce the total cost of supply chain by 6.42%, which can be a considerable value in a real-world scale. Our analysis of variation of parameters showed that the gap between VMI and RMI increases by an increase in holding costs of the retailer and the vendor and the setup costs do not have a considerable impact on the gap.

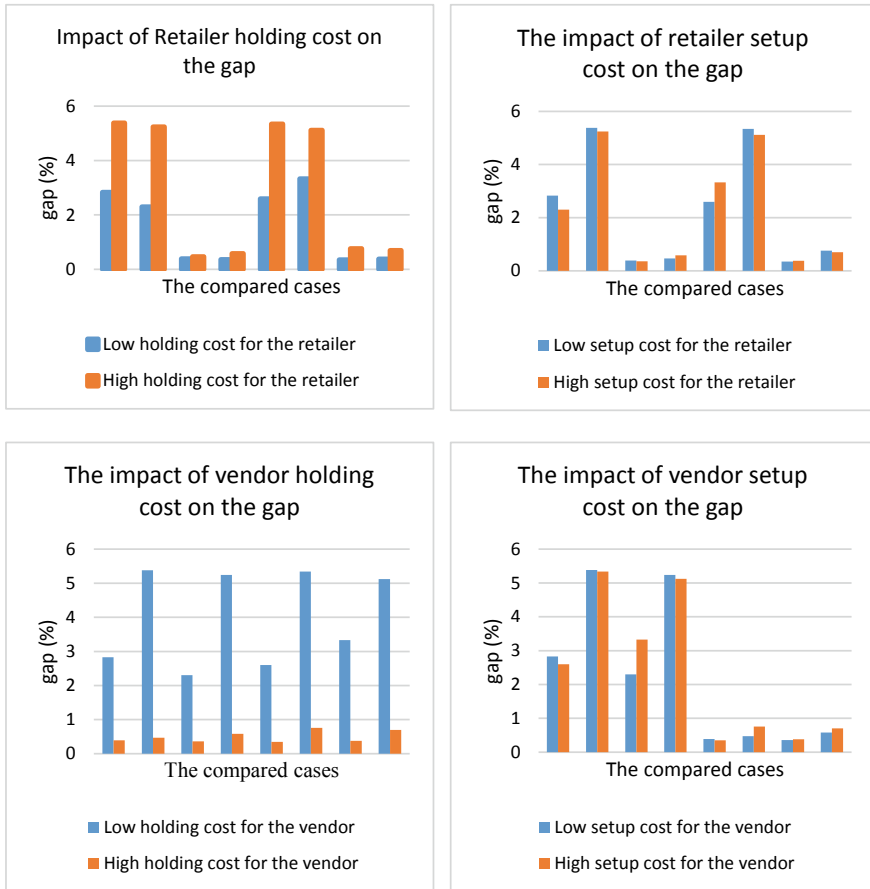


Fig. 1.1 The impact of parameters on the gap between RMI and VMI: **a** Retailer holding cost, **b** retailer setup cost, **c** vendor holding cost, **d** vendor setup cost

References

1. Al-Ameri TA, Shah N, Papageorgiou LG (2008) Optimization of vendor-managed inventory systems in a rolling horizon framework. *Comput Ind Eng* 54(4):1019–1047
2. Almada-Lobo B, Clark A, Guimarães L, Figueira G, Amorim P (2015) Industrial insights into lot sizing and scheduling modeling. *Pesquisa Oper* 35(3):439–464
3. Angulo A, Nachtmann H, Waller MA (2004) Supply chain information sharing in a vendor managed inventory partnership. *J Bus Logist* 25(1):101–120
4. Archetti C, Bertazzi L, Laporte G, Speranza MG (2007) A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transport Sci* 41(3):382–391
5. Çetinkaya S, Lee C-Y (2000) Stock replenishment and shipment scheduling for vendor-managed inventory systems. *Manag Sci* 46(2):217–232
6. Clark A, Almada-Lobo B, Almeder C (2011) Lot sizing and scheduling: industrial extensions and research opportunities. Taylor & Francis

7. Gröning A, Holma H (2007) Vendor managed inventory: preparation for an implementation of a pilot project and guidance for an upcoming evaluation at Volvo Trucks in Umea
8. Harris FW (1913) How many parts to make at once. *Factory Mag Manag* 10(2):135–136
9. Hennes J-C (1999) From the aggregate plan to lot-sizing in multi-level production planning. In: *Modeling manufacturing systems*, pp 5–23. Springer
10. Jaruphongsa W, Çetinkaya S, Lee C-Y (2004) A two-echelon inventory optimization model with demand time window considerations. *J Global Optim* 30(4):347–366
11. Khator SK, Leung LC (1997) Power distribution planning: a review of models and issues. *IEEE Trans Power Syst* 12(3):1151–1159
12. Kuk G (2004) Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes. *Inf Manag* 41(5):645–654
13. Marquès G, Thierry C, Lamothe J, Gourc D (2010) A review of vendor managed inventory (VMI): from concept to processes. *Prod Plan Contr* 21(6):547–561
14. Mishra BK, Raghunathan S (2004) Retailer-vs. vendor-managed inventory and brand competition. *Manag Sci* 50(4):445–457
15. Nia AR, Far MH, Niaki STA (2014) A fuzzy vendor managed inventory of multi-item economic order quantity model under shortage: An ant colony optimization algorithm. *Int J Prod Econ* 155:259–271
16. Pasandideh SHR, Niaki STA, Nia AR (2010) An investigation of vendor-managed inventory application in supply chain: the EOQ model with shortage. *Int J Adv Manuf Technol* 49(1–4):329–339
17. Sadeghi J, Mousavi SM, Niaki STA, Sadeghi S (2013) Optimizing a multi-vendor multi-retailer vendor managed inventory problem: two tuned meta-heuristic algorithms. *Knowl-Based Syst* 50:159–170
18. Sari K (2007) Exploring the benefits of vendor managed inventory. *Int J Phys Distr Logist Manag* 37(7):529–545
19. Van der Vlist P, Kuik R, Verheijen B (2007) Note on supply chain integration in vendor-managed inventory. *Decis Support Syst* 44(1):360–365
20. Wagner HM, Whitin TM (1958) Dynamic version of the economic lot size model. *Manage Sci* 5(1):89–96
21. Yao Y, Dong Y, Dresner ME (2007) Analyzing information-enabled stockout management under vendor-managed inventory. *Inf Technol Manag* 8(2):133–145
22. Yao Y, Evers PT, Dresner ME (2007) Supply chain integration in vendor-managed inventory. *Decis Support Syst* 43(2):663–674
23. Yao Y, Dong Y, Dresner M (2010) Managing supply chain backorders under vendor managed inventory: an incentive approach and empirical analysis. *Eur J Oper Res* 203(2):350–359
24. Yu Y, Hong Z, Zhang LL, Liang L, Chu C (2013) Optimal selection of retailers for a manufacturing vendor in a vendor managed inventory system. *Eur J Oper Res* 225(2):273–284
25. Yu Y, Huang GQ (2010) Nash game model for optimizing market strategies, configuration of platform products in a Vendor Managed Inventory (VMI) supply chain for a product family. *Eur J Oper Res* 206(2):361–373
26. Yugang Y, Liang L, Huang GQ (2006) Leader–follower game in vendor-managed inventory system with limited production capacity considering wholesale and retail prices. *Int J Logist: Res Appl* 9(4):335–350