

# There Is No Such Thing as a Free Lunch! Who Is Paying for Our Happiness?



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## 1 Introduction

In a report published by the Danish Ministry of Environment (HRI 2012) it is stated that “*it is no longer possible to imagine a future where the pursuit of happiness is not somehow connected to sustainability. As the human species continues its quest for happiness and well-being, more emphasis must be placed on sustainability and the interaction between sustainability and happiness*” and further “*there is a growing awareness of how sustainability and happiness can go hand-in-hand*”. However, the term happiness is not uniquely defined and a somewhat broad definition could be “the experience of joy, contentment, or positive well-being, combined with a sense that one’s life is good, meaningful, and worthwhile” (Lyubomirsky 2008). A more well-defined and structured index for happiness has been reported based on seven indicators (HI 2016, 2017, 2018):

1. GDP per capita is in terms of Purchasing Power Parity (GPD)
2. Social support (or having someone to count on in times of trouble) (SocSup)
3. The time series of healthy life expectancy at birth (LifeExp)
4. Freedom to make life choices (FreeCho)
5. Generosity (Gener)
6. Perceptions of corruption (PerCor)
7. The country’s own perception of doing better or worse than the hypothetical country Dystopia (Dys)

In a recent study, comprising the 157 countries included in the World Happiness Index study (HI 2016) these indicators were analyzed (Carlsen 2018) applying

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partial ordering techniques, disclosing, among other features that on an average basis the following 10 countries were found as the happiest countries: Iceland, Australia, Switzerland, Norway, New Zealand, Denmark, the Netherlands, Finland and Austria, whereas the bottom of the list displays Madagascar, Congo (Brazzaville), Egypt, Benin, Chad, Gabon, Burundi, Angola, Armenia and Yemen as the least happy countries, results that is somewhat different from the original study where the index is generated by a simple arithmetic aggregation of the 7 indicator values (HI 2016, 2017, 2018).

In today's world nothing is free, so the obvious question that arises is now: who is paying for our happiness? To some extent a study of the Happy Planet Index, which is focused on sustainable wellbeing for all and is based on 4 indicators, i.e., experienced wellbeing (EWB), life expectancy (LEX), inequality of outcomes (IoO) and the ecological footprint (EFP) (Jeffrey et al. 2016) may give some answers.

The present study focus on answering the above question by partial order analyses of the World Happiness Index and the Happy Planet index in parallel.

## 2 Methodology

The present paper describes how selected partial order tools may be applied in the evaluation of a series of countries taking several indicators simultaneously into account as an alternative to conventional methods to study MIS (Bruggemann and Carlsen 2012).

### 2.1 The Basic Equation of Partial Ordering

In its basis partial ordering appears pretty simple as the only mathematical relation among the objects is " $\leq$ " (Bruggemann and Carlsen 2006a, b Bruggemann and Patil 2011). The basis for a comparison of objects, here countries, characterized by the subset of indicators describing their performance in relation a) to happiness as well as b) to the planetary 'happiness' (vide infra). This series of indicators,  $r_j$ , characterizes the single countries. Thus, characterizing one country ( $x$ ) by a set of indicators  $r_j(x)$ ,  $j = 1, \dots, m$ , where  $m$  is the number of indicators, can be compared to another country ( $y$ ), characterized by the indicators  $r_j(y)$ , when

$$r_j(y) \leq r_j(x) \text{ for all } j = 1, \dots, m \quad (1)$$

Equation 1 is a very hard and strict requirement for establishing a comparison. It demands that all indicators of  $x$  should be better (or at least equal) than those of  $y$ . Further, let  $X$  be the subset of countries included in the analyses,  $x$  will be ordered higher (better) than  $y$ , i.e.,  $x > y$ , if at least one of the indicator values for

$x$  is higher than the corresponding indicator value for  $y$  and no indicator for  $x$  is lower than the corresponding indicator value for  $y$ . On the other hand, if  $r_j(x) > r_j(y)$  for some indicator  $j$  and  $r_i(x) < r_i(y)$  for some other indicator  $i$ ,  $x$  and  $y$  will be called incomparable (notation:  $x \parallel y$ ) expressing the mathematical contradiction due to conflicting indicator values. A set of mutual incomparable objects is called an antichain. When all indicator values for  $x$  are equal to the corresponding indicator values for  $y$ , i.e.,  $r_j(x) = r_j(y)$  for all  $j$ , the two objects/nations will have identical rank and will be considered as equivalent, i.e.,  $x \sim y$ . The analysis of Equation 1 results in a graph, the Hasse diagram. Hasse diagrams are unique visualizations of the order relations due to Equation 1.

## 2.2 The Hasse Diagram

The Eq. 1 is the basic for the Hasse diagram technique (HDT) (Bruggemann and Carlsen 2006a, b; Bruggemann and Patil 2011). Hasse diagrams are visual representation of the partial order. In the Hasse diagram comparable objects are connected by a sequence of lines (Bruggemann and Carlsen 2006a, b; Bruggemann and Patil 2011; Bruggemann and Münzer 1993; Bruggemann and Voigt 1995, 2008).

## 2.3 The More Elaborate Analyses

In addition to the basic partial ordering tools some more elaborate analyses have been used including average ranks (Bruggemann and Annoni 2014; Morton et al. 2009; De Loof et al. 2006; Lerche et al. 2003; Bruggemann et al. 2004; Bruggemann and Carlsen 2011) and sensitivity analysis (Bruggemann and Patil 2011; Bruggemann et al. 2014), the latter gives an insight in the relative importance of the included indicators (Bruggemann and Patil 2011; Bruggemann et al. 2014).

The average ranking is expressed as average height from bottom (min. Height = 1) to the top (max height =  $n$ , i.e., the maximum number of objects) (Bruggemann and Annoni 2014). The average rank is generated by calculating all linear order preserving sequences (set LE), the “linear extensions of the original partial order. From LE\_0 the statistical characterization for each object is obtained. For example the characterization is calculated as the average value an object has, taken all positions of this object within LE\_0, the averaged heights. It is clear that this procedure is computationally extremely difficult. Hence, approximations were developed.

For the sensitivity analysis (Bruggemann and Patil 2011; Bruggemann et al. 2014), let  $Q$  be the set of all indicators, then taken all indicators of  $Q$  leads to a partial order, which is called PO\_0. The corresponding set of linear extensions is denoted by LE\_0. Leaving out one indicator of  $Q$ , say  $r_j$ , then another partial order results, which is denoted as PO\_j.

Both partial orders can be described by an adjacent matrix, say  $A_0$  for  $PO_0$  and  $A_j$  for  $PO_j$ .

Taken the Euclidian Distance (squared) quantifies the role of indicator  $q_j$  in  $PO_0$ . This is a sensitivity measure for the indicators of set  $Q$ , describing the structural changes of the partial order leaving one indicator out. This is not immediately a measure of the sensitivity of the indicators for a ranking, because the ranking is per definition a linear order and here derived over many interim steps.

If a linear order is obtained by all orders in  $LE_0$ , the set of linear extensions taken from  $PO_0$ , then any  $PO_j$  will also lead to a corresponding set  $LE_j$ . And this set is the more differing from  $LE_0$  the larger the sensitivity is. Therefore the ranking due to averaged heights is as more affected by indicator  $r_j$  as larger its sensitivity is.

For detail information on the single tool the cited literature should be consulted as a detailed description is outside the scope of the present paper.

## 2.4 Software

All partial order analyses were carried out using the PyHasse software (Bruggemann et al. 2014). PyHasse is programmed using the interpreter language Python (version 2.6) (Ernesti and Kaiser 2008; Hetland 2005; Langtangen 2008; Weigend 2006; Python 2015) Today, the software package contains more than 100 modules and is available upon request from the developer, Dr. R.Bruggemann (brg\_home@web.de).

## 2.5 Indicators

The seven indicators applied in the World Happiness Index (HI 2016, 2017, 2018) has been stated above in the introduction.

As mentioned in the introduction, the Happy Planet Index (HPI), focussing on sustainable wellbeing is based experienced wellbeing (EWB), life expectancy (LEX), inequality of outcomes (IoO) and the ecological footprint (EFP), the latter being expressed in global hectares per capital. One global hectare is the world's annual amount of biological production for human use and human waste assimilation, per hectare of biologically productive land and fisheries. An approximate formula for calculating HPI is given by

$$HPI \approx \frac{LEX * EWB * IoO}{EFP} \quad (2)$$

The eventual calculation of HPI uses a somewhat more elaborate formula applying 'some technical adjustments are made to ensure that no single component

dominates the overall score' (Jeffrey et al. 2016), where inequality adjusted values of LEX and EWB are used and some scaling constants are incorporated.

$$HPI = \frac{0.452 * ((EWB_{IA} - 0.158) * LEX_{IA} + 3.951)}{(EFP + 2.067)} \quad (3)$$

The subscript IA denotes that the EWB and LEX indicators have been 'inequality adjusted' for economic inequalities in the countries. For details Jeffrey et al. (2016) and nef (2016) should be consulted.

It should be noted that in order to achieve a sensible ranking picture it is mandatory that all indicators included have the same orientation, e.g., the larger the better. Thus, in the case of the HPI the EFP indicator will be multiplied by  $-1$  in order to guarantee co-monotony with the EWB and LEX indicators.

## 2.6 Data

The data used for the analysis can be found in the appropriate cited reports (HI 2016; Carlsen 2018; Jeffrey et al. 2016). The full set of indicators and the complete set of countries (approx. 150) have been used for the calculations.

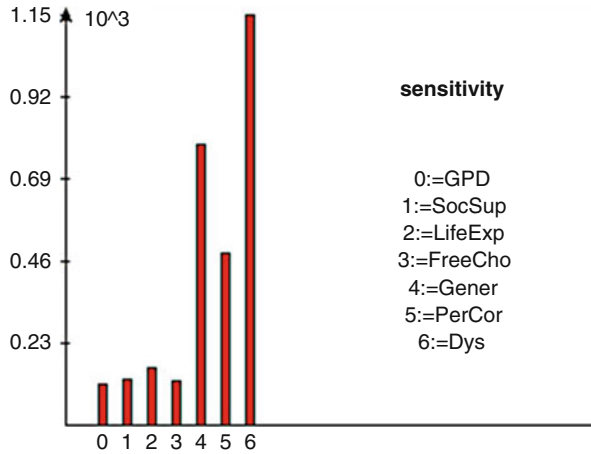
## 3 Results and Discussion

### 3.1 The World Happiness Index

Let us initially look at what makes us happy. Here we take the onset in the World Happiness Index (HI 2016, 2017, 2018). As mentioned in the introduction this index is calculated by a simple arithmetic aggregation of the 7 indicators mentioned above. Obviously, such an aggregation of data may lead to more or less strange results due to compensation effects (Munda 2008), roughly speaking adding apples and oranges getting bananas. Hence, in a recent paper (Carlsen 2018) the happiness index was revisited applying partial order methodology, among other things to disclose the relative importance of the seven indicators. In Fig. 1 the relative importance of the seven indicators are depicted as calculated applying the sensitivity module sensitivity23\_1 of the PyHase software package (Bruggemann and Patil 2011; Bruggemann et al. 2014) on the 2016 happiness index data (HI 2016).

The result summarized in Fig. 1 has in details been discussed by Carlsen (2018), a discussion that shall not be reproduced here. However, it is worthwhile to mention just 3 specific indicators, i.e., GPD, Gener and Dys, respectively.

First it can be noted that in an overall evaluation of happiness money, here expressed as the gross domestic product or more precisely as the purchasing power



**Fig. 1** Relative importance of the seven indicators used to generate the 2016 World Happiness Index (HI 2016)

parity (PPP), apparently plays only a minor role, actually displaying the lowest importance of the seven indicators. This is in agreement with the old myth that ‘money can’t buy you happiness’. Second it is, in the context interesting to look at the second most important indicator is generosity (Gener). Hence, if the GDP indicator is a measure of receiving/having it is immediately clear that to helping others and to give is a much more important factor for our happiness as pointed out in Acts 20:35 “It is more blessed to give than to receive”(KJBO 2016; see also McConnell 2010).

Third, it is immediately seen the Dys indicator appears as the most important factor in our perception of happiness. The Dys indicator reveals the single country’s own, obviously subjective perception of doing better or worse than the hypothetical country Dystopia, a country where it, roughly speaking, couldn’t be worse (HI 2016, 2017, 2018; Carlsen 2018). This dominance of the Dys indicator is not surprising. It has been nice expressed by Fyodor Dostoevsky: “The greatest happiness is to know the source of unhappiness“(Brainyquote 2001). In Table 1 the top-10 countries based on average ranking are shown. The numbers in parentheses after the single countries refer to the placement based on the HI for the years 2016–2018 (HI 2016, 2017, 2018; Carlsen 2018).

It can be noted (Table 1) that apart from a single case (Austria in 2016) the Top-10 countries based on an average ranking including all seven indicators fits reasonable well with the original HI. However, it also puts a question mark to the annual discussion in Danish news media that we are no longer the most happy people in the world (2017 and 2018) since Denmark based on the average ranking never was.

A short video presentation highlighting the main finding of the study can be found at <https://www.researchsquare.com/article/rs-113102/v1>.

**Table 1** Top-10 countries based on average ranking of the seven Hi indexes for 2016–2018. The number in parenthesis refer to the placement based on the HI for the years

	2016	2017	2018
1	Canada (6)	Switzerland (4)	Switzerland (5)
2	Iceland (3)	Iceland (3)	Norway (2)
3	Australia (9)	Norway (1)	Iceland (4)
4	Switzerland (2)	Canada (7)	Canada (7)
5	Norway (4)	Denmark (2)	Finland (1)
6	New Zealand (8)	New Zealand (8)	Australia (10)
7	Denmark (1)	Netherlands (6)	Denmark (3)
8	Netherlands (7)	Australia (10)	Netherlands (6)
9	Finland (5)	Sweden (9)	New Zealand (8)
10	Austria (12)	Finland (5)	Sweden (9)

**Table 2** Ecological footprint, inequality-adjusted life expectancy and wellbeing for the top-10 countries by the Happy Planet index

HPI Rank	Country	Footprint (gha/capita)	Inequality-adjusted life expectancy	Inequality-adjusted wellbeing
1	Costa Rica	2.84	72.62	6.79
2	Mexico	2.89	66.31	6.83
3	Colombia	1.87	63.10	5.72
4	Vanuatu	1.86	60.32	5.94
5	Vietnam	1.65	64.79	5.22
6	Panama	2.79	68.33	6.32
7	Nicaragua	1.39	63.44	4.76
8	Bangladesh	0.72	56.62	4.27
9	Thailand	2.66	66.35	5.98
10	Ecuador	2.17	64.09	5.52

### 3.2 The Happy Planet Index

Turning to the Happy Planet Index (HPI) a quite different picture develops. Let us first look at the top-10 and bottom-10 countries based on the HPI (Eq. 3).

In the top-10 countries Bangladesh is surprisingly found in the top-10, i.e., at rank 8 (Table 2). However, looking at the details (Table 2) the answer is found. Thus, although the Inequality-adjusted life expectancy (56.62) as well as the inequality-adjusted wellbeing indicators (4.27) are found relatively low also the ecological footprint for Bangladesh is extremely low, i.e. 0.72, which obviously led to the high ranking (cf. Eq. 3).

Turning to the bottom-10 countries based on HPI (Table 3) again some surprising results are seen. In general these countries have rather low Inequality-adjusted life expectancy and the inequality-adjusted wellbeing indicators which in combination with low ecological footprint (cf. Eq. 3) lead to the low rank. However, 3 countries appearing on this list (Table 5) are surprising, especially with regards to the

**Table 3** Ecological footprint, inequality-adjusted life expectancy and wellbeing for the bottom-10 countries by the Happy Planet index

HPI Rank	Country	Footprint (gha/capita)	Inequality-adjusted Life expectancy	Inequality-adjusted wellbeing
131	Burundi	0.80	33.01	3.03
132	Swaziland	2.01	31.81	4.44
133	Sierra Leone	1.24	28.18	3.98
134	Turkmenistan	5.47	48.33	5.12
135	Cote d'Ivoire	1.27	30.64	3.51
136	Mongolia	6.08	56.87	4.61
137	Benin	1.41	37.27	2.82
138	Togo	1.13	39.64	2.42
139	Luxembourg	15.82	78.97	6.70
140	Chad	1.46	27.32	3.67

ecological footprint. Thus, Turkmenistan (5.47), Mongolia (6.08) and, virtually out of scale Luxembourg (15.82). In the case of Luxembourg it is worthwhile to mention that one reason for the extreme ecological footprint may be sought for in the fact that the country is rather small ( $2.6 \text{ km}^2 \times 1000$ ) and dominated by the city Luxembourg. Hence, Luxembourg as a country may be regarded as urban area with a population density of 231 people per square kilometer (World Bank 2017) in contrast to the other much larger countries like, e.g., Mongolia with an area of  $1564.1 \text{ km}^2 \times 1000$  and a population density of 2 people per square kilometer (World Bank 2017) For these countries obviously a somewhat higher values for the Inequality-adjusted life expectancy and the inequality-adjusted wellbeing indicators cannot compensate for the high ecological footprint.

The data presented in Tables 2 and 3 and the associated discussion point at the importance of the ecological footprint (EFP). This is confirmed by looking at the relative importance of the 3 indicators, EFP, LEX and WB (Fig. 2).

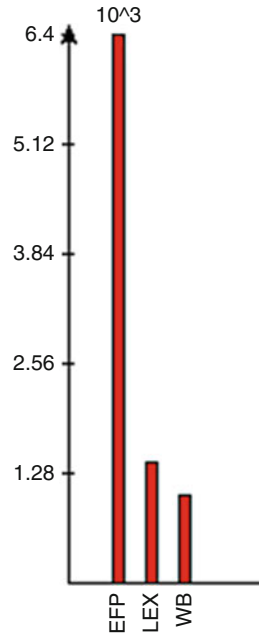
Not surprisingly an average ranking differ here significantly from the simple HPI ranking based on Eq. 3. In Tables 4 and 5 the top-10 and bottom-10 countries based on an average ranking applying the 3 HPI indicators (see Sect. 2.5) is shown. The original HPI calculated based on Eq. 3 is given in addition to the ecological footprint for the single countries. For comparison the result of the average ranking for the 10 countries based on the seven Hi indicators are shown. Denmark and Luxembourg are further included (Table 4) for comparison to the HI.

Immediately (Tables 4 and 5) is it noted that significant variations in the average HPI ranking compared to the average HI ranking prevail.

Looking at the ecological footprint as a key factor to the HPI it appears interesting to elucidate the variation in the average HPI ranking with a changed EFP. Using Luxembourg as a spectacular example it is found that a reduction of the Luxembourg EFP by 10 gha/capita moves the country from place 103 to place 39.



**Fig. 2** Relative importance of the three indicators used to generate the 2016 Happy Planet Index (Jeffrey et al. 2016)



**Table 4** Top-10 countries plus Denmark and Luxembourg based on average ranking of the HPI indicators

Rkav	Country	HPI	EFP	HI (Rkav)
1	Bangladesh	8	0.72	84
2	Costa Rica	1	2.84	20
3	Pakistan	63	0.79	47
4	Norway	12	4.98	5
5	Spain	15	3.67	32
6	Colombia	3	1.87	56
7	Tajikistan	25	0.91	54
8	Philippines	20	1.1	51
9	Vietnam	5	1.65	100
10	Nicaragua	7	1.39	23
50	Denmark	32	5.51	7
103	Luxembourg	139	15.82	21

For comparison the original HPI and the ecological footprint are given in addition to the average ranking of the same countries applying the HI indicators. All 2016 data

### 3.3 Including the Financial Aspect

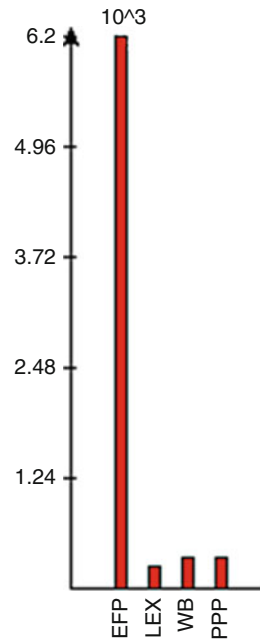
Now, with reference to the HI, it might be of interest to including the financial aspect. Thus, adding the Purchasing Power Parity (PPP) as a fourth indicator, PPP

**Table 5** Bottom-10 countries based on average ranking of the HPI indicators

Rkav	Country	HPI	EFP	HI (Rkav)
131	Gabon	120	2.02	153
132	Trinidad and Tobago	130	7.92	90
133	Benin	137	1.41	151
134	Estonia	118	6.86	87
135	South Africa	128	3.31	136
136	Djibouti	127	2.19	na
137	Latvia	121	6.29	82
138	Botswana	126	3.83	130
139	Turkmenistan	134	5.47	49
140	Mongolia	136	6.08	99

For comparison the original HPI and the ecological footprint are given in addition to the average ranking of the same countries applying the HI indicators. All 2016 data

**Fig. 3** Relative importance of the three original indicators used to generate the 2016 Happy Planet Index plus the Purchasing Power Parity (Jeffrey et al. 2016)



compares different countries’ currencies through a “basket of goods” approach. In Fig. 3 the relative indicator importance is visualized.

In excellent agreement with the HI it is seen that again the financial aspect plays a very minor role. However, not surprisingly inclusion of the PPP indicator does make some changes to the average HPI ranking both in the top-10 (Table 6) and the bottom-10 (Table 7). Of the more significant changes Norway, Denmark and Luxembourg can be mentioned (Table 6) where Norway climbs to the top rank, whereas Denmark climbs by 17 places and Luxembourg from 103 to 73,

**Table 6** Top-10 countries plus Denmark and Luxembourg based on average ranking of the original three

HPI (Rkav)	Country	HPI	EFP	HI (Rkav)
1	Norway	12	4.98	5
2	Spain	15	3.67	32
3	Colombia	3	1.87	56
4	Pakistan	36	0.79	47
5	Philippines	20	1.1	51
6	Uruguay	14	2.91	33
7	Bangladesh	8	0.72	84
8	Palestine	22	1.19	138
9	Netherlands	18	5.28	8
10	Costa Rica	1	2.84	20
37	Denmark	32	5.51	7
73	Luxembourg	139	15.82	21

HPI indicators plus the PPP indicator. For comparison the original HPI and the ecological footprint are given in addition to the average ranking of the same countries applying the HI indicators. All 2016 data

**Table 7** Bottom-10 countries plus Denmark and Luxembourg based on average ranking of the original three

Rkav	Country	HPI	EFP	HI (Rkav)
131	Benin	137	1.41	151
132	Trinidad and Tobago	130	7.92	90
133	Mauritania	117	2.54	125
134	Guinea	129	1.41	97
135	Niger	122	1.56	127
136	Djibouti	127	2.19	na
137	Estonia	118	6.86	87
138	Latvia	121	6.29	82
139	Turkmenistan	134	5.47	49
140	Mongolia	136	6.08	99

HPI indicators plus the PPP indicator. For comparison the original HPI and the ecological footprint are given in addition to the average ranking of the same countries applying the HI indicators. All 2016 data

**Table 8** Comparison between the four indicators for Norway and Luxembourg

HPI	Country	EFP	LEX <sub>IA</sub>	EWB <sub>IA</sub>	PPP
12	Norway	4.98	78.60	7.42	101,564
139	Luxembourg	15.82	78.97	6.70	105,447

in agreement with the relative high PPP for these countries. Hence, the PPP for Denmark, Norway and Luxembourg in 2016 were 57,636, 101,564 and 105,447 thousand USD, respectively (Jeffrey et al. 2016). For comparison the PPP for Bangladesh in 2016 was only 859 thousand USD (Jeffrey et al. 2016).

A direct comparison between Norway and Luxembourg is an exemplary case to illustrate the effects of the different indicators (Table 8).

The original HPI rank for the two countries are clearly having 3 positive contributions, i.e., LEX, EWB and PPP, respectively, and one significant negative contribution, i.e., the EFP. Assuming the latter for Luxembourg to be changed by 10 gha/capita the country changes its average HPI ranking from 73 (Table 6) to 24 again supporting the assumption the EFP is the main controlling factor.

## 4 Conclusions and Outlook

It has been revealed that the most important sub-indicator for our happiness as expressed by the analysis of the World Happiness Index appears to be the ‘Dystopia’ indicator, which is a rather subjective measurement that fits quite nicely with the Lyubomirsky definition of happiness (Lyubomirsky 2008) as “the experience of joy, contentment, or positive well-being, combined with a sense that one’s life is good, meaningful, and worthwhile” as well the Dostoevsky quote:” The greatest happiness is to know the source of unhappiness “(Brainyquote 2001). On the other hand it was found that the gross domestic product per capita in terms of purchasing power parity plays only an inferior role. This latter finding is found again looking at the Happy Planet index. Hence, introducing the GDP expressed as the Purchasing Power Parities (PPP) again discloses the minor role of financial wealth as a factor for sustainability in terms of happiness.

It has been demonstrated that the original ranking based on HPI is significantly different from that based on HI and a posetic based data analysis of the HPI dataset leaves no doubt that the culprit in this respect unequivocally is the ecological footprint, which point directly to the Sustainability Development Goal No. 12, i.e., Responsible consumption and production (SDG 2018). Of less importance for the average HPI ranking is inequality adjusted life expectancy and wellbeing that both increase the HPI. Here reference to Sustainability Development Goal No. 3, i.e., Good health and well-being and No. 10, i.e., Reduced inequalities, appears (SDG 2018) appropriate.

One serious question apparently remains: *Who is paying for our happiness?* The answer appears rather simple as it point to us. Hence, apparently through our (non-sustainable) exploitation of nature we let our planet pay for our happiness! This answer unequivocally leads to a further question: *Are we ready for a change?* The more optimistic answer is a maybe, as there might still be time. Let the words by Frederika Stahl (2015) from ‘The world to come’ close this:

I breathe you in  
 Soon you’ll be gone  
 Look at the mess you’re in  
 See what we’ve done

The more pessimistic, also expressed by Frederika Stahl is:

I breathe you in  
Kiss you one last goodbye  
We knew that we could save you  
But never really tried

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