

Chapter 2

Spatial Complexity in Nature, Science and Technology



*Science! True daughter of Old Time thou art!
Who alterest all things with thy peering eyes
(Edgar Allan Poe, 1809–1849, “Science”)*

Abstract Some characteristic domains of science and technology for which spatial complexity is significant are examined here. In cosmology, spatial complexity relates to the large-scale spatial inhomogeneities of the universe. In geography, earth sciences and ecology, it appears as one of the two major components of landscape complexity and thus affects ecosystem management and landscape protection. In physics, complex systems and fractals are two of the most characteristic fields of science for which spatial complexity is important, while in electronics, spatial complexity is of paramount importance, for the miniaturization of electronic devices, for broadening the capacity of electronic systems, processing times of spatial datasets, QR technologies etc. In the bio-medical sciences, spatial complexity is a key descriptor of melanomas and emerges as a decisive factor in some interpretations of MRI imagery, while the complexity of knotting of proteins is important for DNA analyses.

Keywords Spatial complexity · Geographical complexity · Landscape complexity · complexity and cancer · Fractals and Complexity · Complex systems · Geocomputation

2.1 Spatial Complexity in Cosmology

Nature in the universe has been articulated from infinites and finites.

“Ἄ φύσις δ’ ἐν τῷ κόσμῳ ἀρμόχθη ἐξ ἀπείρων τε καὶ περαινόντων”
(Philolaus, 470-385 b.C.,F1,Diog.Laert. 8.85)

Spatial complexity lies at the heart of the problem of the exploration of the large-scale structure of the universe, and of the “cosmic web” of spider web-like conglomerates of galaxies that are developed in it (Liebeskind et al. 2017). In cosmology,

the “horizon problem” consists in explaining the absence of a generally convincing explanation of why the universe is homogeneous at its very large scales (that is why its matter is allocated more or less homogeneously in space). According to the measurements carried out so far, the large-scale topology of our universe presents a “flatness”, in the sense that angles between any two intersecting lines are preserved (which would not be anticipated if it had a hyperbolic or a spherical shape). Yet, following string theory, our universe is a 2d surface (which may as well have infinite extent), that is called a “brane” (abbreviation for “membrane”). In some theories (i.e. the Horava-Witten), there are several branes, with dark matter possibly present in some of them. As one brane moves towards another, it is thought it can modify the other brane’s physical laws. Thus, the anticipated set of stable states of a “string landscape” is estimated to range in between 10^{100} and 10^{1000} and each such state is thought to be governed by different physical laws. The theory that different possible worlds might be emanating from simple geometric structures is examined in the context of string theory, particularly in the context of a multiplicity of universes, the “multiverse”, which, if exists, should have a truly unfathomable spatial complexity.

At a more detailed spatial scale however, the form, structure and distribution of galaxies in the universe are intricately related to spatial complexity, of which the importance for the development of large-scale structures in the universe is evident, given the presence of areas with large masses (i.e. $\sim 10^{15}$ solar masses of the “Coma” cluster) where “clusters of galaxies” prevail in sharp contrast to other large empty spaces. This *spatial inhomogeneity*, in tandem with the matter-antimatter asymmetry in our universe, has puzzled cosmologists. If a spatial object is homogeneous, then it is not as complex as an inhomogeneous one (of the same size). Perhaps, the large-scale spatial inhomogeneities that have been observed in the universe have never been better observed than from the spatial distribution of the cosmic microwave background radiation (CMB) which was emitted since the “big bang” and was dispersed all over the universe ever since.

Probably, the increasing complexification in the universe occurred in tandem with an increase in its entropy. In turn, the cause of the increase in entropy is believed to be the expansion, which, in turn, is related to the gravitational field. As the universe expands, its entropy increases and so does its complexity, but as it approaches its heat death, its complexity is expected to diminish dramatically. Alternatively the roots of changes in both energy and complexity may be sought in Helmholtz’s free energy, in the sense that the entropy difference increases, free energy decreases according to the change in temperature, so free energy grows entropy (Lineweaver et al. 2013). Understanding the gap between actual and maximum entropy in the universe is essential in order to explain the growth of the amazingly complex forms in it and the interplay between entropy and complexity manifests itself in the history of life on earth, the multiplicity of forms of life, and eventually, the rise of civilization itself.

Spatial complexity might also hold the key to unlock further secrets of the cosmic space. Specifically, the possibility of existence of life in extraterrestrial environments depends on particular geoindicators, one of which is the geological differentiation of a planet’s surface, which can be detected by means of remote sensing techniques (Jovian satellites for instance, display significant surface fragmentation and hence

higher spatial complexity than other satellites in our planetary system). These indicators resemble those used in spatial analysis and landscape ecology for the analysis of complex structures of terrestrial landscapes.

Eventually, understanding how entropy and spatial complexity change due to the expansion of the universe is a big question of its own, although the interplay between entropy and spatial complexity reverberates across spatial scales, from the large-scale structure of the universe down to shapes and patterns, both natural and human-made, on the face of the earth.

2.2 Spatial Complexity in Geography and Ecology

Vigorous branches falling one on another,
 Complexifications of petals, covers of leaves,
 complexes of fruits: thus was the language of plants
 “Ἐθαλλον οἱ κλάδοι, συνέπιπτον ἀλλήλοις ἄλλος ἐπάλλον,
 ἐγίνοντο τῶν πετάλων περιπλοκαί, τῶν φύλλων περιβολαί,
 τῶν καρπῶν συμπλοκαί: τοιαύτη τις ἦν ὁμιλία τῶν φυτῶν”
 (Achilles Tatius, 1st-2nd cent. b.C., “Leucippe and Clitophon”, 1.15)

Measuring the complexity of a surface constitutes a major challenge for many fields of the earth sciences, such as geospatial technologies and GIS. Geography, landscape analysis and landscape ecology constitute the prominent fields of “visible” applications of spatial complexity analysis (Papadimitriou 2010a, b; Papadimitriou 2012a, b, c), with repercussions for land use planning (Papadimitriou 2013). In geography, spatial landscape complexity can be perceived in various ways, by means of maps, satellite imagery or aerial photographs etc. and specific mathematical methods have been devised for this purpose (Papadimitriou 2002, 2009, 2012a) to measure this type of landscape complexity. In geomorphology, given the puzzling spatial complexity of landforms, it has also been suggested that *landscape complexity* should be one of the highest research priorities in geomorphology (Werner 1999; Fonstad 2006; Murray and Fonstad 2007; Goehring 2013; Tlidi et al. 2018).

Landscape complexity has also been identified as one of the key research priorities in landscape ecology (Wu and Hobbs 2002) and several studies have appeared over the last years in the literature, aiming at quantitative assessments of landscape complexity, while research has produced evidence of non-linear interactions in landscapes (Turchin and Taylor 1992; Pahl-Wostl 1995).

Recognizing the importance of the complexity for landscapes however, a classification of types of landscape complexity was made by Papadimitriou (2002; 2010a), who suggested that landscape complexity is of three basic types: (a) *spatial* (or “structural”), (b) *functional* and (c) *qualitative* (or semantic). The words “structure” and “function” have precise meanings in landscape ecology and the reader is

referred to classic texts of landscape ecology (i.e. Forman and Godron 1986) for their explanation.

Landscape ecology often resorts to combinations of already known landscape-ecological indices, such as “landscape patchiness”, “landscape diversity” and “landscape fragmentation”. Expectedly, such metrics should affect decisions related to land management. The “European Landscape Convention” requires assessments of “landscape fragmentation” (Llausas and Nogue 2012). This can be attributed to the widely held view that landscape fragmentation is held responsible for negative effects on ecosystems’ function, as several studies have documented in the literature of landscape ecology, from across different bioclimatic settings (Pütz et al. 2011; Gao and Li 2011; Bassa et al. 2012). Besides, landscape fragmentation often occurs in areas of rapid urbanization (Shrestha et al. 2012). Fragmentation is important in the complexity of predator–prey systems (Morozov et al. 2006), it is usually caused by human activity, and affects complexity not only structurally, but sometimes even functionally so (Briefer et al. 2010). Forest fragmentation poses considerable problems to successful landscape management plans, and constitutes an important factor in land use/land cover change analyses of its own (Andrieu et al. 2011; Mas et al. 2012). Besides, evidence from marine biological observations suggests (Hovel and Lipcius 2002; Hovel 2003) that some marine species (i.e. the juvenile blue) depend on environmental fragmentation.

The complexity of landscapes needs to be assessed for practical applications, such as land management, land use planning and forecasting changes (Papadimitriou 2012a, b, 2013). In geo-ecological spatial analysis, we need to explore spatially structured ecological interactions, but we also need methods to simplify spatial complexity, while taking into account combinations of nonspatial along with spatially explicit approaches. This is particularly interesting, considering the strong evidence that self-organization at the landscape level may emerge even from homogeneous landscapes, therefore forcing us to examine spatial complexity also in the context of its changes with time (Papadimitriou 2002, 2009, 2010a, b). One way to analyze spatial complexity is by means of spatial ecological networks in which the nodes of the network can be species or individuals and links can represent the direction of flows of mass or energy. In these cases, the complexity of a network can be measured i.e. by the number of connections per node or the randomness of connections per node (Fig. 2.1). The extent to which an ecosystem can maintain its functions even if some of its nodes are removed from it, reveals its stability (Papadimitriou 2013).

Spatial complexity also enters as an important parameter in geographical information systems (GIS) and geospatial technologies and in various practical applications (Batty 2005), because it directly impacts time and resources necessary to process geospatial data (although this impact may not always be measurable); relief modelling for instance, depends on terrain complexity. Spatial complexity is also important in qualitative spatial reasoning and in efforts to develop new generation GIS, as it modulates the efficiency of handling large repositories of big spatial data. For these reasons, we sometimes need to reduce the spatial complexity of large spatial databases in order to speed up computations.

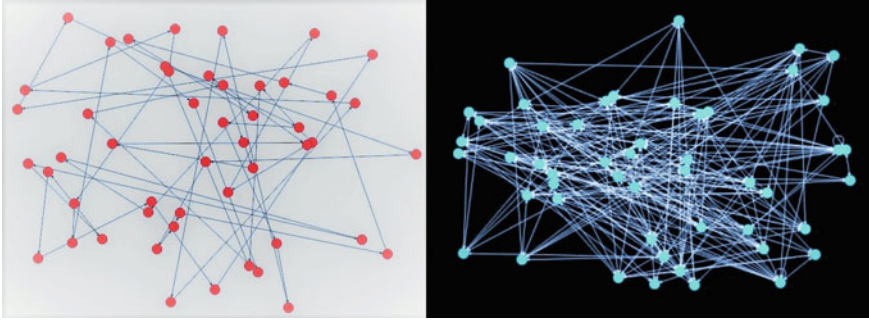


Fig. 2.1 Two networks, with the same number of nodes each (50), but with different number of links per node: there are two connections per node for all nodes in the network on the left. The network on the right is more complex, as it has both a higher number of links per node and a random number of links per node

Thus, comparing landscape ecology and geospatial technologies, it becomes evident that whether high spatial complexity is desirable and beneficial or not varies and depends on particular situations and conditions. In ecology and biogeography, high spatial complexity is desirable for ecosystems, as it is associated with “information-richness” in the geographical space (White and Engelen 1994) and “species richness” in ecology. In contrast to these, when we need to handle data *representing* the environment, ecosystems and geographical spaces, a high complexity of these data may not always be desirable (it can be so only up to an acceptable level that will allow us to process those data at the desired spatial resolution in order to fulfill our practical requirements).

2.3 Spatial Complexity in Physics and Electronics

You still have Chaos in you

“Ihr habt noch Chaos in euch”

(Friedrich Nietzsche, 1844–1900, “Also sprach Zarathustra” part 5)

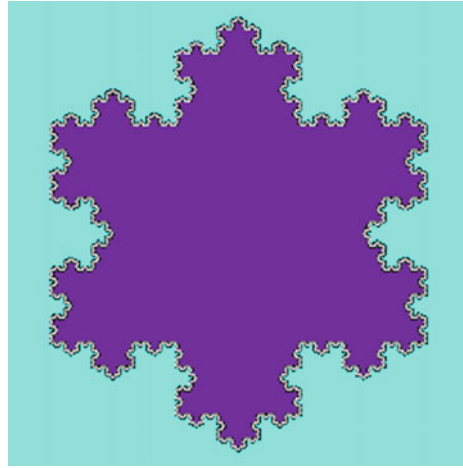
It is common place to say that the earth looked from above a spaceship appears like a “blue planet”, but zooming in to this all-blue image would reveal the immense variety of landscapes our blue planet is endowed with. Mapping out this variety of natural and artificial settings would certainly not result in “blue” color in them all. In fact, the color of each patch would depend on how far or how close we look at it from. The dependence of spatial analysis (and, inevitably, complexity also) on the spatial scale of observation has given rise to the growth of “fractal geometry”, “fractal landforms”, “fractal dimensions” etc. that have attracted great interest in physics and other sciences.

Spatial forms are often “*fractal*”, self-similar across different spatial scales and non-differentiable. Measuring “fractal dimension” is based on the fact that the length of the contour of a planar fractal object is proportional to the ruler’s length g used to measure the length of the object’s contour and the fractal dimension d , where d is the limit as the ruler’s length g grows infinitely small (Mandelbrot 1983; Normant and Tricot 1991):

$$d = 1 - \lim_{g \rightarrow 0} \frac{\log[\text{length}(g)]}{\log g} \quad (2.1)$$

A common fractal is the “*Koch curve*” which has a fractal dimension of $d = \log 4 / \log 3 = 1,262$ (Fig. 2.2).

Fig. 2.2 The “Koch curve” (or “Koch island” or “Koch snowflake”) has a fractal dimension of $d = 1,262$: a non-integer dimension, classifying this shape as an object with dimension in between 1 (lines) and 2 (planes)



Beyond physics, fractals have been examined in earth sciences, geography and ecology (among many other disciplines). Although they present a useful approach to describe how complex a boundary or a spatial form is with measurements becoming progressively more detailed, they serve as *indicators* of spatial complexity, but not as *measures*. However, the fact that increasingly more refined spatial scales of analysis reveal higher complexity is important and concerns many domains of scientific inquiry. Assessing the complexity of higher than one-dimensional forms is also a key issue in software measurement, computer graphics, information theory, neural networks, pattern recognition, materials science, physics of fractal objects and many more. For all these domains, cellular automata occupy a prominent position, because discrete models of complex processes have traditionally been based on cellular automata. These are automatic cellular evolutionary processes depending on a set of “states” S_1, S_2, \dots, S_n and a set of “transition rules” T_1, T_2, \dots, T_m , acting on these states. Each cell is found in one state only and that state is determined by the rules and the states its surrounding cells are in. Consequently, at time $t + 1$, the state of each cell, S_{t+1} , is determined from the transition rule T_i acting on the state of that

cell at time t : $S_t \xrightarrow{T_i} S_{t+1}$. The “rules” are simple algorithms acting on an array of cells in two dimensions. Thus, all cells interact with their neighbouring ones, either in the “rook” sense or in the “king” sense (as the rook’s or the king’s movements in chess). These neighborhood types are defined as neighborhoods around a point (x,y) , defined by the sets of cells surrounding the central cell. In the case of a von Neumann’s neighbourhood the interactions are described as: $N_4(x; y) = \{(x + 1; y); (x-1; y); (x; y-1); (x; y + 1)\}$ and in the case of Moore’s neighbourhood: $N_8(x; y) = \{(x + 1; y); (x-1; y); (x; y-1); (x; y + 1); (x + 1; y + 1); (x + 1; y-1); (x-1; y + 1); (x-1; y-1)\}$. Yet, there are other possibilities (Fig. 2.3) for constructing cellular automata,

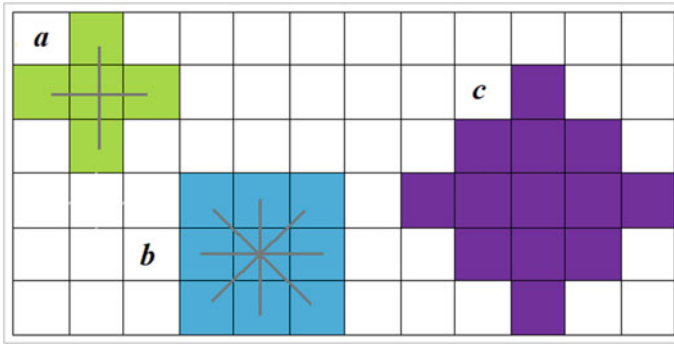


Fig. 2.3 Some common types of spatial arrangements in cellular automata: The central cell interacts with its four surrounding cells in “rook’s case” (a), its nine surrounding cells in “king’s case” (b) and with twelve cells around it in the case of a von Neumann neighbourhood (c)

with longer interactions than in the 9 cells surrounding the immediate neighbourhood of the central cells (as, i.e. in the case of a “5x5 Von Neumann neighbourhood”).

Cellular automata have been intensely explored in order to simulate the emergence of spatially complex forms in geography and ecology. In landscape research for instance, such applications range from the possibility to establish general algorithmic ecological laws, to the exploration of ecological processes such as niches, industrial ecologies, interspecies competition, latitudinal gradients and species diversity (Rohde 2005; Baynes 2009). In geography, they are used to model the progress of urbanization in the course of time, geomorphological processes such as run-off and soil erosion and landscape evolution. On the interface between ecology and geography, cellular automata can be used to simulate the ways by which landscapes change with the spatial propagation of natural hazards (i.e. forest fires) and to explore the complexity of spatial synchronization and self-organization. With cellular automata, among many applications, it is also possible to simulate the complexity of spatial synchronization processes (Satulovsky 1997) as well as phenomena of self-organization in space (Manrubia and Sole 1996; Malamud and Turcotte 1999).

Changes over time that are simulated by means of cellular automata may unveil pattern formation and self-organization, which are highly significant facets of complex system behaviors. Evidently, these changes emerge if the cellular automata are allowed to run for long times and thus become able to produce aggregates in

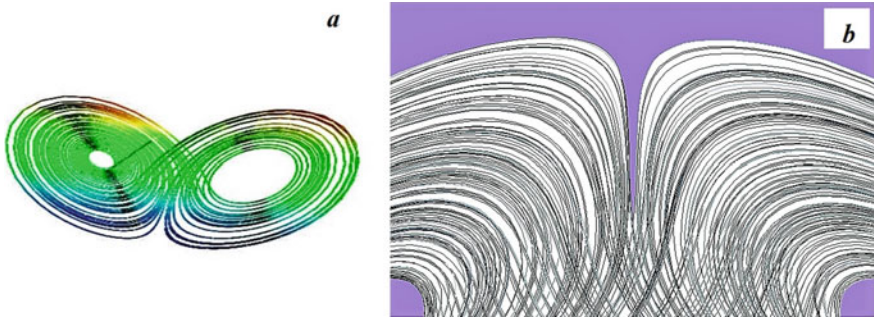


Fig. 2.4 Details (b) of the orbits (a) of the phase space of a widely known dynamical system, the *Lorenz attractor*. The finer the resolution of the observations, the more different orbits can be discerned

space. However, cellular automata (much like fractals) are useful to model and simulate spatially complex developments, scenarios, behaviors, processes and dynamics, but not to measure spatial complexity (describing a complex process as it evolves with time is different than actually measuring it). Further, they can be used to create spatially complex forms from simple rules, but this is a process that only evolves in time.

Spatial complexity can also be observed in the representation of the orbits of dynamical systems in the *phase space*. In this case however, we do not deal with spaces of the physical world but with mathematical constructs that are used to plot and represent the orbits of dynamical systems in time. Phase spaces are not spaces of the real world; they are mathematical constructs used to represent behaviors of systems of the real world. Yet, in terms of spatial complexity, it is interesting to notice that as some orbits are observed at increasingly finer scales, the denser they may appear. As an example, consider the detail in the orbits of the phase space of the system of differential equations that is famously known as the *Lorenz attractor* (Fig. 2.4), which represents (with some detail of abstraction) the atmospheric circulation by means of the “Lorenz system” of nonlinear differential equations (Lorenz 1963):

$$\begin{aligned}
 \frac{dx_1}{dt} &= a(x_2 - x_1) \\
 \frac{dx_2}{dt} &= bx_1 - x_2 - x_1x_3 \\
 \frac{dx_3}{dt} &= x_1x_2 - cx_3
 \end{aligned}
 \tag{2.2}$$

The variables x_1, x_2, x_3 are *not* coordinates of space; they are meteorological variables and therefore the space they define is a mathematical construct devised to help us visualize the system's behavior.

Spatial complexity also manifests itself in phenomena of “nonlocal coherence” (communication of behaviors, properties and processes by distance) that are sometimes observed in natural and social systems. Besides, coherence, coupling and synchronization emerge from models of nonlinear spatial dynamics (Casado 2001; Berglund 2007a, b) but the mechanisms by which these properties emerge in spatial ecosystems are largely unknown. It is probable however, that there exist processes leading prey-predator interactions to the “*edge of chaos*” through oscillators (Rai 2004), by making dynamical systems to appear as “*riddled systems*”. In such systems, as their naming suggests, it is difficult to distinguish basins of attraction and orbits in the phase space. If this is the case (as currently appears more likely), then we should not lose sight from the fact that such riddled systems may demand extremely more elaborate models than we already have at our disposition, particularly if they are spatially explicit.

Besides physics, spatial complexity is important in the analysis of “big geospatial data” (in image analysis, most commonly). In fact, the need to reduce image complexity has led to the development of various image compression protocols, such as jpeg. Other fields of technological applications are QR encodings, that have now become ubiquitous in algorithmics (Park et al. 2011), microchip construction, cryptography (Alvarez et al. 2012), materials science (Hyde and Schroder-Turk 2012) etc. The more complex a space is within the confines of a QR square, the more information it contains. Compare, for instance, the QR encoding of the author's name and surname with another QR representing additionally his degrees and mailing address (Fig. 2.5).



Fig. 2.5 QR-representations. Left: the author's name and title. Right: the author's name and title, along with his degrees and mailing address. The finer the (binary) detail in a QR, the more information is stored in it and, the finer the detail, the higher the spatial complexity

Differences in spatial complexity are also important for various functions of opto-electronic systems (radars, scanners etc. technologies), which aim to solve problems of detection, recognition, classification and identification of targets by using opto-electronic tracers (either multispectral or monochrome), radiometry in automatic target recognition, chromatic filters bearing Foreon technology etc., whatever the scanning method is used and embedded in them (i.e.serial “raster” scanning, parallel “linear array” scanning, “rosette scanning”).

2.4 Spatial Complexity in the Life Sciences

“All that man is, all mere complexities,
the fury and the mire of human veins”
(W.B. Yeats, 1865-1939, “Byzantium”, 1930)

One of the precursors of modern medicine, Paracelsus, in his “Labyrinthus Medicorum Errantium” (1538) used the term “complex” to mean the mixture or coexistence of properties and qualities, while correctly pointing out that the basic elements of nature are not complex, but simple (the “temperamenta”). Perhaps, nowhere in the bio-medical sciences becomes the significance of spatial complexity more explicit than in the identification of skin malignancies; melanomas in particular. The widely used “ABCDE method” is characteristic for spotting a dangerous melanoma (Fig. 2.6): Asymmetric form, rough Borders, many Colors, larger than 6 mm Diameter and Evolution (change in any of the above). Essentially, all these criteria (except for the last one that relates to time) are criteria of spatial complexity. Similar criteria are











	<i>Asymmetry</i>	<i>Boundary</i>	<i>Color</i>	<i>Diameter</i>	<i>Evolution</i>
<i>Benign</i>	 <i>Symmetric</i>	 <i>Smooth boundary</i>	 <i>One color</i>	 <i>Small diameter</i>	 <i>Non-evolving</i>
<i>Malignant</i>	 <i>Asymmetric</i>	 <i>Rough boundary</i>	 <i>Many colors</i>	 <i>Large diameter</i>	 <i>Evolving</i>

Fig. 2.6 The “ABCDE” criteria for identification of melanomas. A malignancy has a higher spatial complexity than a benign skin spot

used in other cancer malignancy categorization schemes (i.e. the Gleason 5-degrees scale for prostate cancer).

Yet, the significance of spatial complexity for medicine becomes explicit if one considers even more domains of the biomedical sciences, such as MRI imagery, histology, and DNA sequencing. Analyzing ultrasound imagery is an important field of everyday medical practice, more recently also combined with applications in telemedicine. Before proceeding to image analysis, histological observations (images) are converted to binary or multicoloured maps of raster (square) grids to make them suitable for further analysis. Then, successive images of the same tumour revealing its changes in spatial complexity indicate the degree of cellular differentiation and hence malignancy, with a predictability that can mount up to 90% (Tambasco et al. 2009). The potential of fractal geometry has also been explored as an indicator of cancer cell differentiation with high degree of success (Capri et al. 2006; Timbo et al. 2009). Besides, one of the most exciting fields of applications of fractal patterns seems to be the detection of tumour growth (Cross 1994, 1997; Cross et al 1994; Bash and Jain 2000; Esgiar et al. 2002). Estimating the increasing roughness of borders of cells and tissues as a discriminator of pathological situations has found applications in cardiology, osteoporosis detection and diagnosis of pulmonary diseases (Heymans et al. 2000).

But the usefulness of the concept of spatial complexity in biomedical sciences extends well beyond the domain of oncology. For instance, spatial complexity has been identified as a key issue in understanding brain function (Jia et al. 2018; Schulz et al. 2018), so another domain of application is brain MRI analysis. Neuroimaging using MRI (magnetic resonance imaging) is now ubiquitous and aims at detecting changes in texture in the spatial distributions of cell types and biochemical substances in the brain. The applications of such analyses are wide ranging, although the main body of research has thus far focused on neurodegenerative diseases, such as Alzheimer's disease and dementia, particularly with respect to cortical thinning. In such cases, spatial complexity has been correlated with a number of neurodegenerating diseases (Singh et al. 2006; Young and Schuff 2008).

From a topological perspective, knottedness and entanglement serve as measures of complexity of folded proteins (Taylor 2007). Some proteins form links that can be even more complex than knots and, interestingly, the link topology is characteristic of eukaryotic organisms only (Dabrowski-Tumanski and Sulkowska 2017). Furthermore, sequence complexity relates to the topology of proteins (Romero et al. 2001; Park and Levitt 1995; Edgar 2004) while various topologically interesting structures have been identified in them (toroids, solenoids etc.). Interestingly, the higher the knot complexity, the higher the probability of knotting of a physical substance. This has been experimentally proved in the case of ring polymers (Shimamura and Deguchi 2002). The importance of spatial complexity for DNA topology is exemplified by the role of solenoidal and plectonemic supercoils, involving topoisomerase enzymes, histones etc. (i.e. Vologodskii 1992; Champoux 2001; Bar et al. 2011).

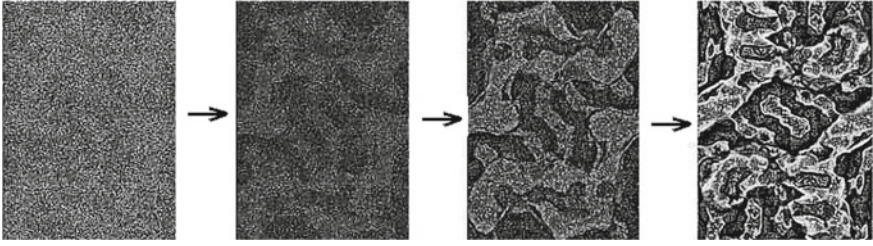


Fig. 2.7 A morphogenetic process resulting from Turing’s reaction–diffusion equations. The amorphous landscape at time 1 (top left) soon becomes slightly patterned after 5 time steps (top right), then more so after 10 steps (bottom left) and even more so after 100 time steps (bottom right)

Yet, the extent to which knot invariants can be used in order to derive estimations of complexity remains an open problem. In some cases, it does seem possible (Ricca 2012), as the number of crossings of a knot (a measure of a knot’s complexity) remains as the handiest simple measure of knot complexity that is also easily applicable to chemical and biochemical analyses (Vargas-Lara et al. 2017).

But spatial complexity does not leave its imprint on living beings at their DNA level only; patterns that appear on the skin or on the fur of animals are quite often interesting for their spatial complexity also (e.g. zebra strips, puma spots). Trying to explain how such patterns can be modelled and how spatially complex forms can be derived from simple differential equations has led to many researches ever since Turing proposed his set of equations of morphogenesis (Turing 1952):

$$\begin{aligned}
 \frac{dx_1}{dt} &= (a_1x_1 + a_2x_2^2 + a_3x_3^2) + D_1\nabla^2x_1 \\
 \frac{dx_2}{dt} &= (a_4x_2 + a_5x_2x_3 + a_6x_1x_2) + D_2\nabla^2x_2 \\
 \frac{dx_3}{dt} &= (a_7x_3 + a_8x_2^2 + a_9x_1x_2) + D_3\nabla^2x_3
 \end{aligned} \tag{2.3}$$

(where a_i are parameters and D_i are the diffusion coefficients) which has been the oldest but also one of the most successful models of self-organization in space (Fig. 2.7).

Other equations model other complex spatial pattern formation processes, i.e. spiral forms that are produced by the Complex Landau-Ginzburg equation (Fig. 2.8) of amplitude $f(u,t)$:

$$\frac{\partial f(u,t)}{\partial t} = (1 + ic_1) \frac{\partial^2 f(u,t)}{\partial x^2} + \varepsilon f(u,t) - (1 - ic_2) |f(u,t)|^2 f(u,t) \tag{2.4}$$

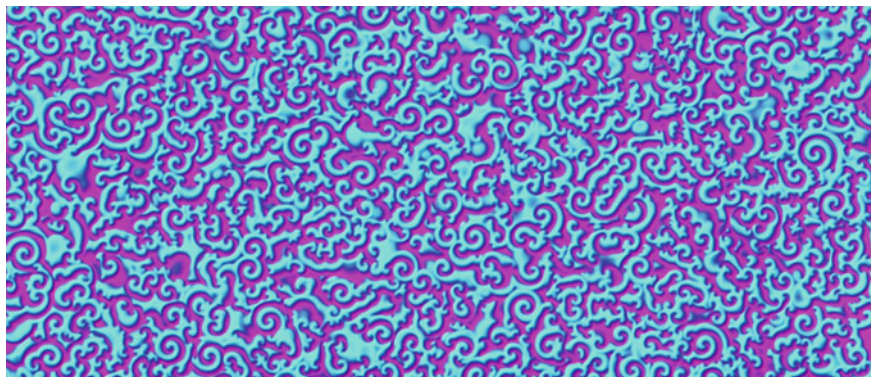


Fig. 2.8 Spirals resulting from modeling spatial diffusion, as calculated from a complex Landau Ginzburg equation

Fig. 2.9 Dendrites are a typical example of natural morphogenesis. Here, a digitally processed picture of a tree in the vicinity of the author's residence in Athens



where ε is the control parameter (instability incurs when $\varepsilon > 0$), c_1 stands for a coefficient measuring linear dispersion (the dependence of the wave frequency on the wave number) and c_2 represents the measure of nonlinear dispersion. This equation has been effectively used to describe ecological invasions affecting the parameters of oscillatory ecological systems (Reichenbach et al. 2008; Sherratt et al. 2009; Smith and Sherratt 2009).

Self-organization produces many spatially complex forms of plants and animals, both regular (geometric) and irregular (dendrites, filaments, aggregates etc.), whereas branching processes can also yield spatially complex dendritic forms of morphogenesis (Fig. 2.9).

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