



# On Temperature Variation of the Diabetic Foot

Ana Teixeira<sup>1</sup>(✉)  and Ana I. Pereira<sup>2</sup> 

<sup>1</sup> Mathematics Centre CMAT, Pole CMAT - UTAD, Sciences and Technology School, University of Trás-os-Montes e Alto Douro, Quinta dos Prados, Vila Real, Portugal  
ateixeir@utad.pt

<sup>2</sup> Research Centre in Digitalization and Intelligent Robotics (CeDRI), Instituto Politécnico de Bragança, Campus de Santa Apolónia, Bragança, Portugal  
apereira@ipb.pt

**Abstract.** This work aims to give an additional contribute to the development of an alternative diagnostic method to be applied to early detection of foot pathology in Diabetes Mellitus individuals. In this work, the main concepts related to the topic under study are introduced and a framework concerning the use of thermography to evaluate the temperature distribution in the feet is presented. Additionally, in this work, a mathematical model to characterise the plantar temperature distribution variation is presented and an optimization programming problem based on the nonlinear least squares approach is proposed. Some considerations about the two global non-linear optimization metaheuristic methods used to solve this model, namely a Hybrid Genetic Algorithm and a Hybrid Simulated Annealing, are also described. Thermal plantar images of non diabetic and diabetic individuals are used to test the approach. The numerical results obtained with both methods for the different regions of each foot are presented and analysed; the best results were obtained with the Hybrid Genetic Algorithm. Some preliminary conclusions were made.

**Keywords:** Least squares model · Diabetic foot · Thermography · Genetic Algorithm · Simulated Annealing Algorithm

## 1 Introduction

The human being is homoeothermic, thus being able to maintain the body temperature constant, regardless of the changes that happen in the environment. This feature is vital for the preservation of a constant environment within the human body with regard to functions and composition of fluids and tissues [1, 2]. There are several pathologies that alter the body temperature [2, 3] and, in particular, the temperature of the plantar region; Diabetes Mellitus, usually known as diabetes, is one of these diseases.

Diabetes Mellitus is a chronic illness that affects many people and it is estimated that its global cost will increase from 1.2 trillion euros in 2015 to between

1.9 and 2.3 trillion euros in 2030 [4]. Additionally, as the foot is the support for the locomotion of human beings, it is the most vulnerable part of a diabetic's body because it becomes the most common region for occurrence of complicated lesions, decreasing the quality of life of the individuals [5]. Thus, prevention and early diagnosis of diabetic foot in Diabetes Mellitus patients is a very pertinent and important subject of study.

Diabetic foot is a pathology developed by individuals with diabetes and it is characterized by a variety of foot injuries, like infection, ulceration and/or destruction of deep tissues that may be associated with neuropathy or vascular disease [6]. Although not all diabetics develop this pathology, those who develop it loose quality of life. In addition, ulceration of the diabetic foot causes serious medical consequences for the patient; for example, the development of ulcers can lead to infections, since ulcers can harbor bacteria and fungi. These infections, when severe, can lead to amputation of the lower limb [7]. Therefore, early identification and effective preventive methods for diabetic foot are essential. A significant number of studies have demonstrated that temperature variations in the plantar feet region may be related to diabetic foot problems [8–10]. Thus, the analysis and characterization of the temperature distribution on the plant of the foot can contribute to the early diagnosis of the onset of certain diseases, namely the diabetic foot, consequently preventing their appearance and increasing the probability of cure.

The use of thermography to examine the feet of diabetic individuals captured the interest of several researchers for some time now, e.g. [8–15]. Nevertheless, to the extent of our knowledge, Bento et al. [16,17], in 2011, were the first ones to apply mathematical optimization techniques and models to this topic, with the aim of finding a characterization of the temperature distribution of plant of the foot of healthy subjects. In a second phase, Carvalho et al. [18,19] continued the work of Bento et al., initiating the characterization of the temperature of the foot of healthy and diabetic individuals and comparing both of them; these two works used mathematical models composed of trigonometric functions. More recently, Contreras et al. [20] proposed a characterization of the plantar temperature distribution based on a probabilistic approach.

In the sequel of the work of Carvalho et al., the objective of this study continues to be the analysis and characterization of the temperature distribution variation of the plant of the foot of non diabetic and diabetic individuals, using thermographic images, as well as the presentation of a mathematical model that approximates these temperature distributions variation; thus, giving an additional contribute to the development of an alternative diagnostic method, using a painless and non-invasive tool, to be applied to diabetic individuals in order to early detect foot pathology.

The structure of this paper considers: in Sect. 2, some background information on the concepts: diabetes mellitus, diabetic foot and thermography is presented; in Sect. 3, the used mathematical model is explained; in Sect. 4, the optimization methods used to solve the model are described; in Sect. 5, the

numerical results are presented and analysed; and in the last section the main conclusions are drawn and future work presented.

## 2 Thermography and Diabetic Foot

### 2.1 Diabetes Mellitus

Insulin is a hormone produced in the pancreas and it is responsible for transporting the glucose present in the blood to the cells, where it is used as energy. Diabetes is a chronic disease, characterized by the occurrence of abnormalities in glucose metabolism, resulting in defects in insulin secretion, insulin action or both. There are three main types of diabetes: Type 1, Type 2 and Gestational diabetes. Type 1 is associated with autoimmune disease, that is, the human system attacks the beta cells of the pancreas that are responsible for the insulin production. Thus, the body becomes unable to produce the sufficient quantity of insulin and the individual needs daily doses of insulin to regulate blood glucose levels. Type 2 is associated with the most common form of diabetes. The body produces insulin but, due to the resistance developed by the body to this hormone, it does not act effectively. Over time, it leads to high blood glucose levels. The majority of the patients do not require daily insulin treatment, they only take oral medication that helps control blood glucose levels. Gestational diabetes happens to women when the hyperglycemia state is diagnosed during pregnancy and extra care is needed as there is the possibility of complications with the baby.

Diabetes leads to chronic hyperglycemia, that is diagnosed by elevated levels of glucose in the blood. Over time, hyperglycemia causes damage to various tissues of the body, which leads to the development of serious health problems that can compromise the life of the individual [21–23]. Thus, this disease requires early diagnosis and ongoing treatment to prevent serious complications, such as: heart, blood vessel, eyes, nerves and kidneys problems, as well as a high risk of developing infections that can lead to limb amputation.

### 2.2 Diabetic Foot

Neuropathy is one of the complications of diabetes; it affects the nerves, causing difficulty with sensations, movements and other aspects, depending on the affected nerve. It leads to the loss of foot sensitivity, sometimes resulting in deformation of the foot; a minor trauma (poorly adjusted footwear, walking barefoot or acute injury) suffered by an individual with this disease may precipitate chronic ulceration. Peripheral vascular disease is another consequence of diabetes, provoking obstructive atherosclerotic of the extremities of the body; as in the previous case, minor trauma in patients with this pathology may result in painful and chronic ulceration of the foot [24, 25].

Diabetic foot appears in diabetic patients as a result of damaging nerves and blood vessels problems that may increase the risk of ulceration, infection

and amputation [22]. In fact, the biggest problem associated with diabetic foot is ulceration [26], which represents the biggest medical, social and economic problem worldwide [5]. Inflammation is one of the first signs of foot ulceration, being characterized by redness, pain, swelling, loss of function and heat [9]. Diabetic foot ulceration occurs due to a variety of factors such as: neuropathy, peripheral vascular disease, foot deformity, arterial insufficiency, trauma, and decreased resistance to infection [23]. Usually, it results from a combination of two or more risk factors, being neuropathy and peripheral vascular disease the main factors that lead to the development of diabetic foot ulcers [5, 24].

The diabetic foot is classified according to the type of ulceration that it presents. Diabetic foot ulcers can be classified as neuropathic, ischemic or neuro-ischemic [27, 28]. Neuropathic ulcers are usually located in the metatarsal, but are also found on the fingertips, the dorsum of the foot and the plantar zone. They are the least painful for the individual. The surrounding skin shows loss of sensitivity and calluses. This type of ulcer comes from bone deformation caused by neuropathy. Ulcers that develop on the plantar surface are usually circular and perforate in appearance, often deep. The foot that presents this type of ulceration is considered warm. Ischemic ulcers are located in the plantar and dorsal zones of the feet, in the heel, in the big toe, in the medial surface of the head of the first metatarsus and in the lateral surface of the fifth metatarsus. They are black in color and the surrounding skin is pale, shiny and cold. This pathology is not associated with callus or bone deformation, instead it is associated with peripheral vascular disease, for example, atherosclerosis (occlusion of blood vessels). Ischemic foot is considered cold.

### 2.3 Thermography

All the bodies with temperature above absolute zero emit infrared radiation, also called thermal radiation. The total energy radiated by a surface is known as its emissive power. This emission rate is comprised between 0 and 1, and is obtained as the ratio of the energy radiated from the body's surface and the energy radiated by a blackbody for the same wavelength. The higher the value of the emissivity coefficient, the closer the emissivity of the body is to that of the blackbody, that is, the greater is its energy emission capacity. The emissivity of human skin is almost constant and equal to  $0,98 \pm 0,01$  for wavelength range of 2 to 14, thus, being almost an ideal blackbody [2, 29].

Medical thermography can be divided into four categories: contact electrical thermometry, cutaneous temperature discrimination, liquid crystal thermography and infrared thermography (IRT) [30]. This work will focus on IRT.

In IRT, the radiation emitted by the body is detected by a thermographic camera and the intensity of the radiation emitted is converted into a temperature. The image generated by the radiation emitted by the body is called a thermogram [31]. One of the great advantages of IRT is that it allows the acquisition of a large number of pixels in a short time and each pixel corresponds to a temperature of a specific point, that is, it has a high resolution [32].

IRT is increasingly being accepted by the medical community because it is a fast, painless, non-contact, non-invasive method, that enables the simultaneous monitoring of a large area. Additionally, it is a non-ionizing technique, without any repetitive use side effects and the color code of the thermograms is easy to interpret [3,31,33]. However, to obtain a quality thermographic image that allows a reliable analysis, certain requirements must be fulfilled, namely: the basic standards of the exam room must be guaranteed; the imaging system, the image acquisition and the image processing must follow a protocol; and, finally, the analysis of results has to follow some criteria in order to enable us to decide about the viability of the data. Moreover, some care should be taken with the patient as there are several factors that can affect body temperature [34].

Another reason that justifies the progressive increase of the use of IRT in the medical area (mainly for diagnostics) is the fact that in many pathological processes changes in body temperature occur [31]. Among others, IRT has applications in: diabetes mellitus; vascular disorder; breast cancer detection; muscular pain; and rheumatoid arthritis [35–40].

For patients with diabetes, IRT has been used to diagnose diabetic neuropathy, assess changes in body temperature and, in particular, to obtain a diabetic foot diagnose, through the analysis of the temperature distribution of the sole of the foot [31,33,41–44]. Although biochemical tests, as blood tests, are the most usual techniques to diagnose diabetes, Sivanandam et al. [45] concluded that IRT diagnosis have higher potential than the previous mentioned tests to achieve this goal.

Some studies using IRT have shown that patients with diabetes and neuropathy have higher plantar temperature than individuals without this pathology [46]; others suggest that thermographic foot patterns analysis allows diabetic patients to be screened for the risk of ulceration and that high temperatures are indicative of ulceration [10]. In most cases, the thermogram of a healthy individual shows a symmetrical butterfly pattern (one wing on each foot), where the highest temperatures are located.

The readers that are interested in deepening their knowledge in this topic should read the reviews presented by [42,47], where more than one hundred references can be found. Both papers classify the studies on the use of IRT in four different categories: i) independent limb temperature analysis, where the studies that perform a temperature analysis on each limb separately fit; ii) asymmetric analysis, that include the works that use the fact that in healthy subjects there is a contralateral symmetry in the skin temperature distribution and consider that an asymmetry in this distribution can be an indicator of an abnormality; iii) temperature distribution analysis, when the studies observe similar skin temperature distribution on the feet of healthy individuals comparing to varying temperature variation on diabetic individuals; and iv) external stress analysis, when the aim is the study of the reaction of the body thermoregulation system under the application of thermal and/or physical stress, such as putting the feet into cold water or running.

### 3 Mathematical Model

Bento et al. [16,17] were the first to apply optimization techniques and mathematical models to characterize the distribution of the temperature of the feet of healthy subjects, as well as to confirm the symmetric behavior of their temperature. With this objective, ThermaCamResearch program was used to convert each thermographic image into Matlab format file, obtaining the temperature matrix of each feet. In order to analyse the foot temperature in more detail, each foot was divided in three regions, like in Fig. 1; thus, the original temperature matrix was used to obtain six new temperature matrices, that represent the temperature in each one of the six regions presented in Fig. 1.

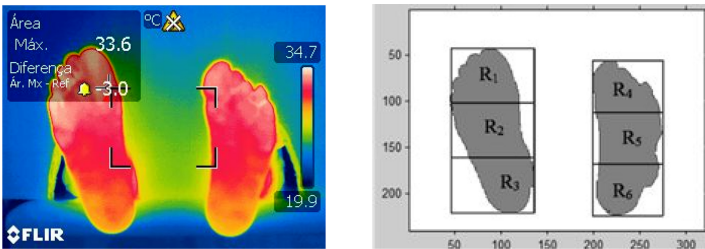


Fig. 1. Feet images

In [16], the author tried to identify the model that best fits the distribution of the temperature for each one of these six regions, minimizing the sum of the quadratic errors of the temperature distribution and the mathematical model. Initially, experiences with quadratic functions were performed, but after having noticed the existence of a wave pattern in the temperature matrix, the authors decided to use functions based on the basic trigonometric functions, concluding that the best model was based on the sum of squares of trigonometric functions.

Carvalho et al. [18,19] continued the work of Bento et al. and initiated the characterization of the temperature distribution of the feet of both healthy and diabetic individuals. Since the temperature values in the plantar foot differ from individual to individual, only the temperature variations presented by each individual are relevant. So, each one of the six different matrices corresponding to each one of the regions in Fig. 1 concern to the temperature variation and were normalized, considering zero as the minimum temperature value. Numerical experiences with models that use variants of trigonometric functions combinations were performed, by minimizing the sum of the quadratic errors between the temperature variation and the mathematical model; the best results were obtained with the models involving the sum of trigonometric functions and/or its squares.

The present study is a sequel of the one developed by Carvalho et al. and the analysis of the distribution of the temperature variation of (non)diabetic feet was carried out by using a mathematical model based in [18] and defined as

$$f(x, i, j) = x_1 \sin^2(ix_2 + jx_3 + x_4) + x_5 \cos^2(ix_6 + jx_7 + x_8) + x_9 \sin(ix_{10} + jx_{11} + x_{12}) + x_{13} \cos(ix_{14} + jx_{15} + x_{16}) + x_{17}, \quad (1)$$

where  $x = (x_1, \dots, x_{17})$  is the vector of the variables that represents the weights assigned to the different model components,  $(i, j)$  represents the pixels positions and  $f(x, i, j)$  represents the temperature variation at the  $(i, j)$  pixel position.

To characterize the temperature variation matrix,  $R_k^l$ , for each image  $l$  and each region  $k$ , the following nonlinear optimization problem is solved.

$$\min_{x \in \mathbb{R}^{17}} \sum_{i=1}^{M_k^l} \sum_{j=1}^{N_k^l} (r_{ij}^{k,l} - f(x, i, j))^2, \quad (2)$$

where  $l$  represents the image to be analyzed;  $k \in \{1, \dots, 6\}$ ;  $M_k^l \times N_k^l$  is the dimension of matrix  $R_k^l$ ;  $r_{ij}^{k,l}$  represents the temperature variation value in region  $k$  of image  $l$  in the pixels positions  $(i, j)$ ; and  $f$  is the function presented in (1).

## 4 Optimization Methods

In the works of Bento et al. and Soraia et al. different optimization strategies were applied and the best results were obtained with the Genetic Algorithm [16, 18, 19]. In the current work, the Hybrid Genetic Algorithm and the Hybrid Simulated Annealing Algorithm, described below, were used to find the minimum of the sum of the quadratic errors between the mathematical model and the foot temperature variation matrix.

### 4.1 Hybrid Genetic Algorithm

The Hybrid Genetic Algorithm (HGA) is based on the Genetic Algorithm (GA) method. The GA is a metaheuristic that simulates the behavior of nature based on an analogy with the mechanisms of natural genetics and is a random search technique with the aim of generating high-quality solutions to an optimization problem [48]. This method is implemented in MatLab software through the predefined *ga* function. This predefined function combines the GA with different strategies with the aim of finding an approximation of a global minimum of a (un)constrained problem. The *ga* predefined function has a large number of options, e.g., is possible to limit the number of iterations or the number of function evaluations, as well as to define different strategies to crossover and mutation procedures. These options are created by the command *gaoptimset*.

In this work a HGA was considered, by applying a local search after the GA procedure to improve the accuracy of the current solution; the Nelder-Mead method was considered as local search procedure.

### 4.2 Hybrid Simulated Annealing

The Hybrid Simulated Annealing (HSA) method is based on the Simulated Annealing (SA) method and uses a local procedure to improve the approximate solution provided by the SA. The SA method is a probabilistic technique for approximating the global optimum of a given optimization problem. The process develops in a set of iterations and, at each iteration, a new random point is generated. The distance from the current point to the new point is based on a probability distribution with a scale proportional to a given parameter [49]. This method is also implemented in MatLab software for unconstrained optimization and *simulannealbnd* is the function that defines it; the Nelder-Mead method was used as local search procedure.

## 5 Numerical Results

In the present work, problem (2), with  $f$  defined as in (1), was considered to analyse the distribution of the temperature variation of (non)diabetic feet. Two global optimization metaheuristic algorithms, a Hybrid Genetic Algorithm (HGA) and a Hybrid Simulated Annealing (HSA) were used to solve this problem, in order to analyse which method provides a better approximation of the foot temperature variation.

Additionally, the four non diabetic feet images (H 01, H02, H03, H04) and the four diabetic feet images (D 05, D 06, D 07, D 08), presented in Fig. 2, were used. These images were collected from Polytechnic Institute of Bragança and Centro de Saúde de Santa Maria - Bragança.

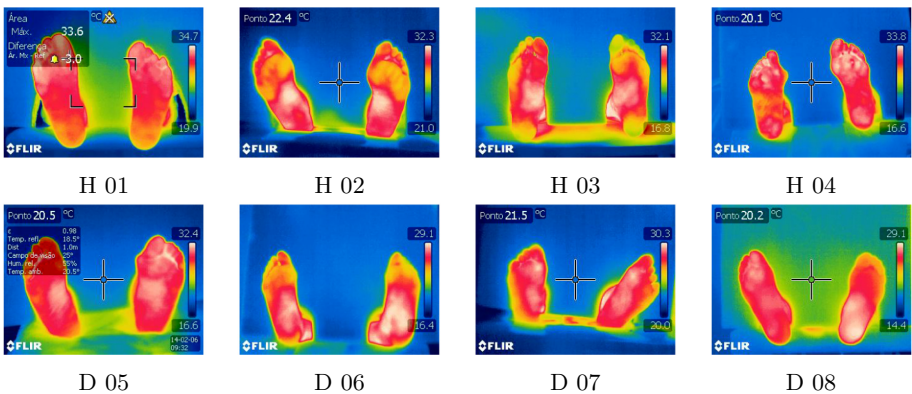


Fig. 2. Non diabetic and diabetic feet images

The numerical results were obtained using an Intel(R) Pentium(R) CPU, with 2.0 GHz, and 4 GB of RAM.



Since both the HGA and the HSA are stochastic and the starting point was generated randomly, the code was executed ten times for each region. Thus, ten possible solutions for the optimization problem (2) are obtained for each region. The computational results, obtained by both optimization methods for each one of the six regions of the eight analysed thermographic images (four of non diabetic individuals and four of diabetic individuals), are presented in Table 1. This table presents the information concerning the minimum value of the objective function of problem (2), Min, as well as the mean value, Mean, and the standard deviation, Std Dev, of the ten obtained solutions, for each one of the six regions.

As can be seen in Table 1, the HGA method obtained better results than the HSA method, for both diabetics and non-diabetic individuals. Not only the minimum values (Min) are lower with the HGA method, but also the same happens with the mean values. So, it is possible to conclude that HGA gives better results than HSA method. This conclusion is corroborated by the values of the standard deviations, which indicate that the ten objective values obtained by the HGA are more similar to those obtained by the HSA.

Furthermore, HGA is always faster than HSA, providing to the solution in a couple of minutes.

As the HGA performed better than HSA, some global considerations about the numerical results obtained using this method are presented in Table 2. This table presents the minimum (Min), maximum (Max) and the average (Mean) values in Table 1 for all regions; the range is also presented ( $\Delta$ ) as well as the relation between the minimum values obtained in all regions, where  $\#R_i$  represented the minimum value obtained by HGA method in region  $i$ .

Observing Table 2 is possible to conclude that the lowest values of minimum (Min), maximum (Max) and average (Mean) are obtained for diabetic images, which means that in these cases this model fits better in the pixel network of the thermographic image. With the obtained computational results, it is also possible to verify the symmetric behavior for the non diabetic images. This means that when it is obtained the best fit in Region 1 (comparing the results of R1, R2 and R3), then it is expected that the best fit will happen in Region 4 (when comparing R4, R5 and R6). This situation is described in reflection column, for example  $\#R3 < \#R2 < \#R1$  means that the best value in Region 3 is smaller than the value in the Region 2, and the last one is smaller than the value in the Region 1. Similar situation occurs in the right foot ( $\#R6 < \#R5 < \#R4$ ). This situation occurs in images H01, H02, H03 and D06. So, this preliminary results indicate that this symmetric behavior occurs more often in non diabetic foot images than in foot images of diabetic individuals. This situation is corroborated by health professionals that considered the temperature variation usually is similar in both feet.

**Table 1.** Numerical results for the eight thermographic images using HGA and HSA methods

Images	Methods		Regions					
			R1	R2	R3	R4	R5	R6
H 01	HGA	Min	4.0E3	1.3E3	6.8E2	3.0E3	1.1E3	6.7E02
		Mean	4.6E3	1.9E3	8.2E2	3.5E3	1.5E3	9.1E02
		Std Dev	1.2E3	6.5E2	1.1E2	6.2E2	9.8E2	4.8E02
	HSA	Min	1.1E4	4.6E3	2.0E3	6.7E3	3.4E3	2.5E03
		Mean	1.9E4	9.2E3	3.3E3	9.9E3	6.0E3	4.1E03
		Std Dev	7.0E3	3.1E3	1.6E3	5.1E3	1.5E3	1.1E03
H 02	HGA	Min	1.5E3	1.5E3	2.4E3	1.2E3	1.3E3	1.5E03
		Mean	1.8E3	1.6E3	2.9E3	1.3E3	1.5E3	1.9E03
		Std Dev	5.6E2	1.0E2	6.6E2	1.7E2	1.0E2	5.0E02
	HSA	Min	3.4E3	5.0E3	5.4E3	3.8E3	2.3E3	4.6E03
		Mean	4.8E3	7.4E3	8.3E3	4.5E3	6.5E3	8.4E03
		Std Dev	2.2E3	5.1E3	5.9E3	1.2E3	2.7E3	4.8E03
H 03	HGA	Min	2.4E3	5.9E3	4.6E3	1.1E3	7.1E3	3.4E3
		Mean	2.8E3	6.4E3	5.1E3	1.2E3	7.9E3	4.1E3
		Std Dev	5.2E2	8.5E2	4.9E2	4.2E2	8.8E2	6.7E2
	HSA	Min	5.0E3	5.1E3	9.4E3	3.2E3	6.5E3	8.9E3
		Mean	7.7E3	5.9E3	1.1E4	4.3E3	1.3E4	1.0E4
		Std Dev	4.4E3	3.1E2	3.4E3	1.8E3	1.4E4	1.7E3
H 04	HGA	Min	2.7E3	5.9E2	9.3E2	2.1E3	6.7E2	6.5E2
		Mean	3.0E3	7.0E2	1.1E3	2.7E3	8.9E2	6.9E2
		Std Dev	8.6E2	1.3E2	1.1E2	7.0E2	1.8E2	3.8E1
	HSA	Min	8.0E3	2.7E3	3.1E3	4.7E3	2.2E3	1.9E3
		Mean	9.5E3	3.7E3	4.1E3	6.4E3	3.6E3	3.1E3
		Std Dev	3.2E3	1.6E3	2.0E3	3.7E3	2.3E3	2.3E3
D 05	HGA	Min	1.4E3	1.1E3	7.9E2	1.5E3	4.3E2	5.9E2
		Mean	1.7E3	1.2E3	1.1E3	1.6E3	4.9E2	6.6E2
		Std Dev	5.4E2	1.2E2	2.0E2	1.1E2	4.7E1	9.9E1
	HSA	Min	4.4E3	1.3E3	3.6E3	5.5E3	2.5E3	3.0E3
		Mean	6.6E3	5.2E3	5.0E3	7.0E3	3.9E3	4.7E3
		Std Dev	3.1E3	5.1E3	2.1E3	2.4E3	3.8E3	3.6E3
D 06	HGA	Min	1.4E2	3.2E2	9.7E2	7.0E1	3.1E2	9.8E2
		Mean	1.8E2	4.7E2	1.2E3	1.7E2	4.5E2	1.2E3
		Std Dev	3.2E1	2.8E2	2.3E2	1.1E2	2.3E2	2.2E2
	HSA	Min	1.1E3	4.6E2	8.3E2	2.7E2	1.2E3	1.1E3
		Mean	2.4E3	1.2E3	1.4E3	4.2E2	1.7E3	2.2E3
		Std Dev	1.6E3	1.2E3	6.8E2	1.9E2	1.2E3	1.5E3
D 07	HGA	Min	7.3E2	3.9E2	1.1E3	1.0E3	1.5E3	1.3E3
		Mean	1.2E3	5.5E2	1.1E3	1.1E3	1.7E3	1.4E3
		Std Dev	1.2E3	2.6E2	4.3E1	1.6E2	1.4E2	2.0E2
	HSA	Min	4.6E3	3.5E3	3.5E3	7.0E3	1.1E3	2.3E3
		Mean	8.7E3	7.6E3	5.0E3	8.8E3	3.1E3	2.8E3
		Std Dev	3.6E3	5.2E3	3.3E3	2.9E3	2.6E3	1.7E3
D 08	HGA	Min	2.8E2	2.0E2	1.9E2	3.4E1	2.1E2	4.5E2
		Mean	3.3E2	2.8E2	2.7E2	1.2E2	2.2E2	5.9E2
		Std Dev	1.5E2	8.4E1	1.5E2	2.0E2	1.6E1	2.6E2
	HSA	Min	2.2E2	1.6E3	2.5E3	2.0E2	1.7E3	2.8E3
		Mean	3.7E2	1.7E3	4.2E3	3.5E2	4.3E3	6.3E3
		Std Dev	7.6E1	6.0E0	2.1E3	5.1E1	3.2E3	2.4E3

**Table 2.** Numerical analysis using HGA method

Images	Min	Max	Mean	$\Delta$	Reflection
H 01	6.74E02	3.95E03	1.78E03	3.28E03	#R3 < #R2 < #R1 #R6 < #R5 < #R4
H 02	1.23E03	2.37E03	1.57E03	1.14E03	#R1 = #R2 < #R3 #R4 < #R5 < #R6
H 03	1.05E03	7.06E03	4.07E03	6.01E03	#R1 < #R2 < #R3 #R4 < #R5 < #R6
H 04	5.94E02	2.68E03	1.27E03	2.09E03	#R2 < #R3 < #R1 #R6 < #R5 < #R4
D 05	4.32E02	1.52E03	9.70E02	1.09E03	#R3 < #R2 < #R1 #R5 < #R6 < #R4
D 06	7.04E01	9.77E02	4.65E02	9.06E02	#R1 < #R2 < #R3 #R4 < #R5 < #R6
D 07	3.93E02	1.52E03	1.00E03	1.13E03	#R2 < #R1 < #R3 #R4 < #R6 < #R5
D 08	3.39E01	4.47E02	2.28E02	4.13E02	#R3 < #R2 < #R1 #R4 < #R5 < #R6

## 6 Conclusions and Future Work

In this work, eight thermal plantar feet images (four images of non diabetic individuals and four images of diabetic individuals) were used to test the proposed mathematical model, which describes the distribution of the variation of the temperature of the feet. Two optimization techniques were used to minimize the sum of the square error between the results produced by the mathematical model and the observed temperature.

The numerical results indicate that, in general, the proposed mathematical model fits well the distribution of the temperature variation for images of diabetic individuals, when comparing with the results obtained with images from non diabetic individuals. Furthermore, is possible to confirm the symmetric behavior of the temperature variation on the images from non diabetic individuals. The results also indicate that the Hybrid Genetic Algorithm obtains a better solution when compared with the Hybrid Simulated Annealing method and is faster.

As future work, the authors intent to study more plantar feet images from non diabetic and diabetic individuals and test different mathematical models to approximate the feet temperature variation on both cases. This study can also be expanded to patients with other types of diseases that affect directly their feet.

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