Monopolistic Competition Model with Retailing

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Abstract. We study the two-level interaction "producer - retailer - consumer" in the monopolistic competition frame. The industry is organized by Dixit-Stiglitz type, the retailer is the only monopolist. The utility function is quadratic. The case of the retailer's leadership is considered. Two types of retailer behavior are studied, namely: with/without free entry conditions. It turned out that the government needs to stimulate the retailer by paying him subsidies to increase social welfare. A similar result was obtained with respect to consumer surpluses.

Keywords: Monopolistic competition · Retailing · Equilibrium · Taxation · Social welfare · Consumer surplus

1 Introduction

Now, there are many works on the vertical interaction between the producer and the retailer, which describe the impact of such interaction on the economy.

In Spengler's early work [1], the simplest case of Stackelberg game with two players is studied, one of the players is a leader while the other one is a follower. In the first step, the leader sets his price. Then the follower, having analyzed the actions of the leader, makes his move. As a result, the price increases twice by each monopolist, respectively, which results in a decrease in social welfare.

Further, two classes of models can be singled out: spatial Hotelling models [2] and models of Dixit-Stiglits [3] type.

A striking example of the first class of models is the Salop model [4] (a circular city model) with one manufacturer and several retailers, located along the circle (street) equidistant from each other. As a result, retailers and consumers interact, so that each consumer is served, and for retailers there is no need to unite with the manufacturer. This model was modified by Dixit [5], by introducing two-level production (up stream and down stream industries), i.e., now the retailer also has the means of production. In other words, the monopolist-producer sells the intermediate goods to retailers, who then process and sell them in the form of finished goods. In this case, the integration is justified, because it increases social welfare by optimizing the number of retailers.

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The second class of models is based on the idea of a representative consumer in the style of Dixit-Stiglits. In particular, Parry and Groff [6] use the CES function and show that the integration leads to a deterioration of social welfare. The work of Chen [7] can be considered as the next step in this direction. In his model, the monopolist-producer is considered, who first chooses the number of manufactured products (and, accordingly, the number of retailers). Further, a supply contract is concluded with each retailer separately, namely, wholesale prices per unit of goods and an initial one-time payment are established. Finally, the retailer sets the retail price. The main result is that the number of product names is less than "socially optimal".

The combination of models of the type of Hotelling and Dixit-Stiglits gives Hamilton and Richards model [8]. It considers two types of commodity varieties: competing supermarkets and competing products within each supermarket. It turned out that the increase in product differentiation does not necessarily lead to an increase in the equilibrium length of product lines.

In our paper, the industry of producers is organized according to Dixit-Stiglits type with quadratic utility [9], and the monopolist is the only retailer. The relevance of the chosen approach is determined by modern realities. The majority of sales today fall on well-developed supermarket chains. Selling their value to suppliers, retailers begin to dictate tough conditions to them. In order to sell products, manufacturers have to agree these conditions. We explored two types of behavior of the retailer, namely, with and without the conditions of free entry (zero-profit).

The paper is organized as follows.

In Sect. 2 we define the main assumptions of monopolistic competition, formulate the model, find the condition when the free entry happens and when not, see Proposition 1; describe the equilibrium, see Proposition 2.

In Sect. 3 we introduce the taxation of Pigouvian type and recalculate the equilibrium, see Proposition 3.

In Sect. 4 we get the equilibrium social welfare (see Sect. 4.1, Proposition 4) and consumer surplus (see Sect. 4.2, Proposition 5). Moreover, here we formulate the main result of the paper: the optimal taxation is negative, i.e., the best is to subsidize the retailer (see Sect. 4.3, Proposition 6).

We omit the proofs of Propositions 1–5 which are rather technical. Instead, the main Proposition 6 we prove carefully, see Appendix A.

Section 6 concludes.

Note that the paper continues the authors' research [10–12].

2 Model

Consider the monopolistic competition model with two-level interaction "manufacturer - retailer - consumer". As it is usual on monopolistic competition, we assume the assumptions (cf. [3])

– firms produce the goods of the same nature ("product variety"), but not absolute substitutes;

– each firm produces only one type of product variety and chooses its price;
– the number (mass) of firms is quite large;
– the free entry (zero-profit) condition is fulfilled.

In addition to product diversity, there are other products on the market, “numéraire”. Moreover, there are L identical consumers, each of them delivers one unit of labor to the market.

In this paper, we consider the situation when manufacturers sell products through a monopolist-retailer.

Consider the utility function in the case of linear demand, so-called OTT-function, proposed by Ottaviano, Tabucci, and Tisse [9]:

$$U(\mathbf{q}, N, A) = \alpha \int_0^N q(i)di - \frac{\beta - \gamma}{2} \int_0^N (q(i))^2 \, di - \frac{\gamma}{2} \left(\int_0^N q(i)di \right)^2 + A, \quad (1)$$

where

– $\alpha > 0, \beta > \gamma > 0$ are some parameters[1];
– N is a continuum of firms or the length of a product line, reflecting the range (interval) of diversity;
– $q(i) \geq 0$ is a demand function, i.e., the consumption of i-th variety, $i \in [0, N]$;
– $\mathbf{q} = (q(i))_{i \in [0,N]}$ is infinite-dimensional vector;
– $A \geq 0$ is the consumption of other, aggregated products (numéraire).

Now we formulate the budget constraint. Let

– $p(i)$ be the wholesale price of i-th product variety, i.e., the price in the case without retailer;
– $r(i)$ be the retailer's premium on the i-th product variety, so $p(i) + r(i)$ is the price of the i-th variety for the consumer;
– $w \equiv 1$ be the wage rate in the economy normalized to 1;
– P_A be the price numéraire.

Then the budget constraint is

$$\int_0^N (p(i) + r(i))q(i)di + P_A A \leq wL + \int_0^N \pi_{\mathcal{M}}(i)di + \pi_{\mathcal{R}}, \quad (2)$$

[1] Due to [9] (see p. 413), “... α expresses the intensity of preferences for the differentiated product, whereas $\beta > \gamma$ means that consumers are biased toward a dispersed consumption of varieties. ... the quadratic utility function exhibits love of variety as long as $\beta > \gamma$. ... for a given value of β, the parameter γ expresses the substitutability between varieties: The higher γ, the closer substitutes the varieties. When $\beta = \gamma$, substitutability is perfect.” Two quadratic terms ensure strict concavity in two dimensions: definite consumer's choice among commodities and between the two sectors. The main feature achieved by this constructions is that this utility generates the system of linear demands for each variety and linear demand for the whole differentiated sector.

where $\pi_{\mathcal{M}}(i)$ is the profit of firm $i \in [0, N]$ while $\pi_{\mathcal{R}}$ is retailer profit. The right side of (2) is Gross Domestic Product (GDP) by income, while the left side is costs.

This way, the problem of the representative consumer is

$$U(\mathbf{q}, N, A) \rightarrow \max_{\mathbf{q}, A}$$

subject to (2), where $U(\mathbf{q}, N, A)$ is defined in (1).

Solving this problem, we can find the demand function for each $i \in [0, N]$:

$$q(i) = a - (b + cN)(p(i) + r(i)) + cP, \tag{3}$$

where coefficients a, b, c are defined as

$$a = \frac{\alpha}{\beta + (N-1)\gamma}, \; b = \frac{1}{\beta + (N-1)\gamma}, \; c = \frac{\gamma}{(\beta - \gamma)(\beta + (N-1)\gamma)},$$

and P is a price index

$$P = \int_0^N (p(j) + r(j))dj.$$

Let

- d be marginal costs, i.e., the number of units of labor required to each firm to produce a unit of differentiated product;
- F be fixed costs, i.e., the number of units of labor required by each firm to produce differentiated product.

Then the problem of maximizing the profit of firm $i \in [0, N]$ is

$$\pi_{\mathcal{M}}(i) = (p(i) - d)q(i) - F \;\rightarrow\; \max_{p(i)}, \tag{4}$$

where $q(i)$ is (3).

Note that problem (4) is quadratic on $p(i)$.

Now let us formulate the problem of the retailer. Similar to the firm's problem (4), let

- $d_{\mathcal{R}}$ be marginal costs, i.e., the number of units of labor required to retailer to sale a unit of differentiated product of each firm;
- $F_{\mathcal{R}}$ be fixed costs, i.e., the number of units of labor required to retailer to sale the differentiated product of each firm.

This way the problem of maximizing the profit of the retailer is

$$\pi_{\mathcal{R}} = \int_0^N (r(j) - d_{\mathcal{R}})\, q(j)dj - \int_0^N F_{\mathcal{R}} dj \;\rightarrow\; \max_{\mathbf{r}, N}, \tag{5}$$

$$\pi_{\mathcal{M}}(i) \geq 0, \quad i \in [0, N], \tag{6}$$

where $\mathbf{r} = (r(i))_{i \in [0,N]}$.

Since the model is homogeneous (the firms are identical), it is possible to show that only two cases can happen, namely,

- in the solution of problem (5), (6), the profit of each firm $i \in [0, N]$ is positive: $\pi_{\mathcal{M}}(i) > 0$ (ignoring the free entry conditions[2]);
- in the solution of problem (5), (6), the profit of each firm $i \in [0, N]$ is zero: $\pi_{\mathcal{M}}(i) = 0$ (taking into account the free entry conditions).

In this paper, we study the Stackelberg equilibrium under the leadership of a retailer, i.e., the retailer maximizes its profit under the best response of firms.

We call the case of the retailer's leadership with ignoring the free entry conditions as RL, while we call the case of the retailer's leadership with taking into account the free entry conditions as $RL(I)$.

Let us describe these cases explicitly.

Case RL. In this case the retailer at the same time chooses trade markup $\mathbf{r} = (r(i))_{i\in[0,N]}$ and scale of product diversity N, correctly predicting a subsequent response from manufacturers. There are stages of solving the problem:

1. Solving the problem of a consumer, we get $q = q(i, p(i), r(i), N)$.
2. Solving the problem of the manufacturer

$$\pi_{\mathcal{M}}(i) = (p(i) - d)q(i, p(i), r(i), N) - F \ \to \max_{p(i)},$$

   we get $p = p(i, r(i), N)$ and $q = q(i, r(i), N)$.
3. Solve the problem of the retailer

$$\pi_{\mathcal{R}} = \int_0^N (r(i) - d_{\mathcal{R}})\, q(\mathbf{r}, N)di - \int_0^N F_{\mathcal{R}} di \ \to \max_{\mathbf{r}, N},$$

$$\pi_{\mathcal{M}}(\mathbf{r}, N) \geq 0.$$

Case RL(I). In this case the retailer first uses the free entry condition to calculate $N = N(\mathbf{r})$, given the subsequent manufacturers response and then maximizes their profits by $\mathbf{r}$:

1. Solving the problem of a consumer, we get $q = q(i, p(i), r(i), N)$.
2. Solving the problem

$$\pi_{\mathcal{M}}(i) = (p(i) - d)q(i, p(i), r(i), N) - F \ \to \max_{p(i)},$$

   we get $p = p(i, r(i), N)$, $q = q(i, r(i), N)$.
3. The free entry condition $\pi_{\mathcal{M}}(i, r(i), N) = 0$ gives $N = N(\mathbf{r})$.
4. Solve the problem of the retailer

$$\pi_{\mathcal{R}} = \int_0^N (r(i) - d_{\mathcal{R}})\, q(\mathbf{r})di - \int_0^N F_{\mathcal{R}} di \ \to \max_{\mathbf{r}}.$$

[2] Ignoring the free entry conditions means that we are somewhat expanding the traditional concept of monopolistic competition.

The question arises: when is case RL realized, and when is case $RL(I)$ realized? It turns out the answer is uniquely determined by the value

$$\mathcal{F} = \frac{F_{\mathcal{R}}}{2F}. \tag{7}$$

Proposition 1. *1. The case RL happens if and only if* $\mathcal{F} > 1$.
2. The case $RL(I)$ *happens if and only if* $\mathcal{F} \le 1$.

Now we can describe the Stackelberg equilibrium when retailer is leader. Let

$$\Delta = \sqrt{\frac{F}{\beta - \gamma}} > 0, \qquad \varepsilon = \frac{\beta - \gamma}{\gamma} > 0, \tag{8}$$

$$f = \sqrt{F \cdot (\beta - \gamma)} > 0, \qquad D = \frac{\alpha - d - d_{\mathcal{R}}}{\sqrt{F \cdot (\beta - \gamma)}}. \tag{9}$$

Proposition 2. *In cases* RL *and* $RL(I)$, *the equilibrium demand* q, *price* p, *markup* r, *mass of firms* N *and profit of retailer* $\pi_{\mathcal{R}}$ *are as in Table 1, where* $\mathcal{F}, \Delta, \varepsilon, f, D$ *are defined in (7)–(9).*

Table 1. The equilibrium

	q	p	r	N	$\pi_{\mathcal{R}}$
RL	$\Delta\sqrt{\mathcal{F}}$	$d + f\sqrt{\mathcal{F}}$	$d_{\mathcal{R}} + \frac{fD}{2}$	$\frac{\varepsilon}{2} \cdot \left(\frac{D}{\sqrt{\mathcal{F}}} - 4\right)$	$\frac{f^2}{4\gamma} \cdot \left(D - 4\sqrt{\mathcal{F}}\right)^2$
RL(I)	Δ	$d + f$	$d_{\mathcal{R}} + f \cdot \left(\frac{D}{2} + \mathcal{F} - 1\right)$	$\frac{(D - 2\mathcal{F} - 2)\varepsilon}{2}$	$\frac{f^2}{4\gamma} \cdot (D - 2\mathcal{F} - 2)^2$

3 Taxation

Let the government stimulate producers in the following way: let the retailer pay the tax τ from each unit of the sold product. Then profit of the retailer is modified as

$$\pi_{\mathcal{R}} = \int_0^N (r(i) - (d_{\mathcal{R}} + \tau))q(i)di - \int_0^N F_{\mathcal{R}} di.$$

The taxes collected are distributed among the producers by a one-time payment method (of Pigouvian type). The case of negative τ means that the retailer needs subsidies, paid from consumer taxes in the amount of $\tau \int_0^N q(i)di$.

Proposition 3. *With taxation* τ, *the equilibrium demand* q, *price* p, *markup* r, *and mass of firms* N *are as in Table 2, where*

$$S = -\frac{\tau}{2f} + \sqrt{\left(\frac{\tau}{2f}\right)^2 + 1} > 0 \tag{10}$$

while $\mathcal{F}, \Delta, \varepsilon, f, D$ *are defined in (7)–(9).*

Table 2. Equilibrium with taxation

	q	p	r	N
RL	$\Delta\sqrt{\mathcal{F}}$	$d + f\sqrt{\mathcal{F}}$	$d_{\mathcal{R}} + \frac{fD}{2} + \frac{\tau}{2}$	$\frac{\varepsilon}{2\sqrt{\mathcal{F}}} \cdot \left(D - \frac{\tau}{f} - 4\sqrt{\mathcal{F}}\right)$
RL(I)	ΔS	$d + fS$	$d_{\mathcal{R}} + \frac{f}{2} \cdot \left(D + \frac{2\mathcal{F}+1}{S} - 3S\right)$	$\frac{\varepsilon}{2S} \cdot \left(D - \frac{2\mathcal{F}+1}{S} - S\right)$

4 Social Welfare and Consumer Surplus. Optimal Taxation

In this section, we consider social welfare, consumer surplus and we calculate the equilibrium social welfare and equilibrium consumer surplus in two cases: with and without taxation. Besides, we compare two kinds of optimal taxation.

4.1 Social Welfare

Consider the function of social welfare W, a measure of the well-being of society.

$$W = \alpha \int_0^N q(i)di - \frac{\beta-\gamma}{2} \cdot \int_0^N (q(i))^2\, di - \frac{\gamma}{2} \cdot \left(\int_0^N q(i)di\right)^2 - \int_0^N (d + d_{\mathcal{R}})q(i)di - \int_0^N (F + F_{\mathcal{R}})di.$$

In the symmetric case, it has the form:

$$W = (\alpha - d - d_{\mathcal{R}})Nq - \frac{\beta-\gamma}{2} \cdot q^2 N - \frac{\gamma}{2} \cdot N^2 q^2 - (F + F_{\mathcal{R}})N.$$

Substituting the equilibrium from Proposition 2 and Proposition 3, we get

Proposition 4. *The equilibrium welfare is as in Table 3 and Table 4, where*

$$H = \frac{F \cdot (\beta - \gamma)}{2\gamma} > 0, \tag{11}$$

while $\mathcal{F}, f, D, S$ are defined in (7), (9)–(10).

4.2 Consumer Surplus

Consumer surplus (CS) is a measure of well-being that people derive from the consumption of goods and services. It is the difference between the maximum price a consumer is willing to pay and the market price:

$$CS = \alpha \int_0^N q(i)di - \frac{\beta-\gamma}{2} \cdot \int_0^N (q(i))^2\, di - \frac{\gamma}{2} \cdot \left(\int_0^N q(i)di\right)^2 - \int_0^N (p(i) + r(i))q(i)di.$$

Table 3. Social welfare without taxation

	W
RL	$\left(D - 4\sqrt{\mathcal{F}}\right) \cdot \left(\frac{3}{4} \cdot (D - 2\sqrt{\mathcal{F}}) - \frac{1}{\sqrt{\mathcal{F}}}\right) \cdot H$
RL(I)	$(D - 2\mathcal{F} - 2) \cdot \left(\frac{3}{4} \cdot (D - 2\mathcal{F}) - 1\right) \cdot H$

Table 4. Social welfare with taxation

	W
RL	$\left(D - \frac{\tau}{f} - 4\sqrt{\mathcal{F}}\right) \cdot \left(3D + \frac{\tau}{f} - 6\sqrt{\mathcal{F}} - \frac{4}{\sqrt{\mathcal{F}}}\right) \cdot \frac{H}{4}$
RL(I)	$\left(D - \frac{2\mathcal{F}+1}{S} - S\right) \cdot \left(3D - 3 \cdot \frac{2\mathcal{F}+1}{S} - S\right) \cdot \frac{H}{4}$

In symmetric case, it is

$$CS = qN \cdot \left(\alpha - \frac{\beta - \gamma}{2} \cdot q - \frac{\gamma}{2} \cdot Nq - (p + r)\right).$$

Proposition 5. *The equilibrium consumer surplus is as in Table 5, where* $\mathcal{F}, f, D, S, H$ *are defined in (7), (9)–(11).*

Table 5. Consumer surplus

	CS *(without taxation)*	CS *(with taxation)*
RL	$\frac{f^2}{8\gamma} \cdot \left(D - 4\sqrt{\mathcal{F}}\right) \cdot \left(D - 2\sqrt{\mathcal{F}}\right)$	$\left(D - \frac{\tau}{f} - 4\sqrt{\mathcal{F}}\right) \cdot \left(D - 2\sqrt{\mathcal{F}} - \frac{\tau}{f}\right) \cdot \frac{H}{4}$
RL(I)	$\frac{f^2}{8\gamma} \cdot (D - 2\mathcal{F} - 2) \cdot (D - 2\mathcal{F})$	$\left(D - \frac{2\mathcal{F}+1}{S} - S\right) \cdot \left(D - \frac{2\mathcal{F}+1}{S} + S\right) \cdot \frac{H}{4}$

4.3 Optimal Taxation

We consider two concepts of optimal taxation. Namely,

– maximization of welfare W with respect to τ which leads to optimal τ_W, it allows the government to determine optimal fiscal policy;
– maximization of consumer surplus CS with respect to τ, which leads to optimal τ_{CS}.

It seems nature to assume that "market exists at $\tau = 0$", i.e., the condition

$$N|_{\tau=0} \geq 0 \tag{12}$$

holds.

It turns out that in case RL, an explicit formula for τ_W can be found. Moreover, in both cases, RL and $RL(I)$, it is possible to determine the sign of τ_W. As to τ_{CS}, we were not able to find τ_{CS}, but in case $RL(I)$ we can to determine its sign. Finally, in case $RL(I)$ we can compare τ_W and τ_{CS}.

Let us summarize these findings in

Proposition 6. *Let condition (12) holds.*

1. *In case RL, the optimal tax from social welfare point of view is*

$$\tau_W = \left(\frac{2+\mathcal{F}}{\sqrt{\mathcal{F}}} - D\right) f < 0, \tag{13}$$

while the optimal tax from consumer surplus point of view is

$$\tau_{CS} = -\infty. \tag{14}$$

2. *In case $RL(I)$, the optimal tax, from social welfare point of view, is negative,*

$$\tau_W < 0, \tag{15}$$

while the optimal tax from consumer surplus point of view is

$$\tau_{CS} < 0. \tag{16}$$

3. *The optimal tax from social welfare point of view is the optimal tax from consumer surplus point of view is less than the optimal taxation from social welfare point of view, i.e.,*

$$\tau_{CS} < \tau_W < 0. \tag{17}$$

5 Numerical Example

The example below illustrates Proposition 6. Parameters:

$$\alpha = 12, \beta = 2, \gamma = 1, F = 1, d = 1, d_{\mathcal{R}} = 2, f = 1, D = 9, \varepsilon = 1, \Delta = 1, H = \frac{1}{2}.$$

For the case RL: $F_{\mathcal{R}} = 7$, $\mathcal{F} = \frac{7}{2}$.

For the case RL(I): $F_{\mathcal{R}} = 1$, $\mathcal{F} = \frac{1}{2}$.

Figure 1 shows that (13)–(17) holds.

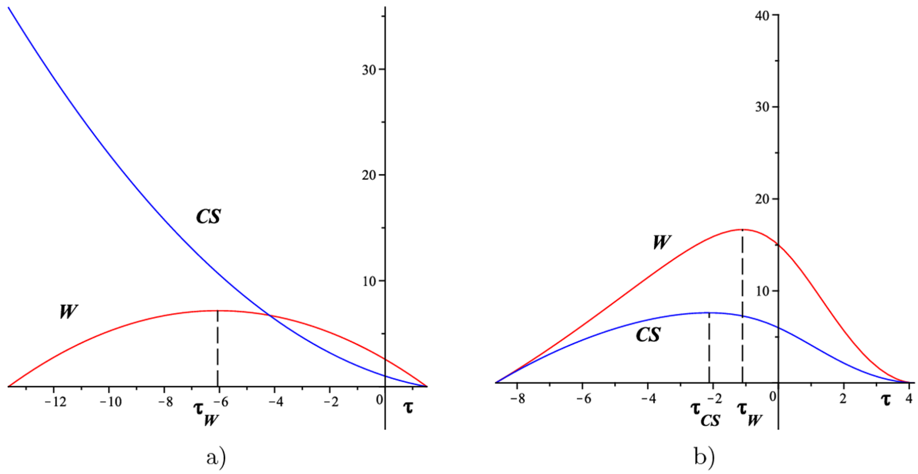

Fig. 1. Example: a) RL with taxation, b) RL(I) with taxation.

6 Conclusion

Our research is based on a theoretical model, which supports several conclusions.

1. The concentration of retailing per se (or, more precisely, the increase in bargaining power of the retailer against numerous manufacturers) generally *enhances* total welfare and, under realistic cost function, consumer surplus as well. The consumer gains operate either through lower retail prices, or through increasing variety, or both. From this viewpoint, the undertaken restriction of market shares looks unnecessary and even harmful.
2. The redistribution of profits to consumers in the form of Pigovian taxation levied on the retailer (with taxes proportional to the volume of sales) has been found *inefficient*. For the sake of total public welfare, the government should rather subsidize the volume of retailing, a policy which appears politically infeasible.

The economic forces leading to such (surprising for the Russian legislature) conclusions are more or less familiar to economists. Any consumer-goods industry produces many varieties of food, clothes, or other goods. Therefore it is organized as a "monopolistic competition" industry: each producer behaves as a monopolist for her brand, but entry is open. Each shop behaves monopolistically for similar reasons. Then, monopolistic behavior in production combined with a monopolistic retailer imply two-tier monopoly and a harmful "double marginalization" effect. When an increase in bargaining power on the retailer side occurs (the other side is too dispersed), it brings the bargaining relations closer to a vertically integrated industry, thus reducing the deadweight loss.

Of course, our theoretical conclusions need more thorough study from different viewpoints of reality, in particular from re-distributional considerations:

which groups of consumers may be affected positively or negatively by market concentration or its regulation (we did not take into account groups so far). Nevertheless, we provide several important hypotheses to be discussed in relation to current legislation. Our results also indicate the need for further research before imposing new market regulations[3].

Further increase in the retailer's bargaining power, in the sense of possible entrance fees levied on manufacturers, also *enhances* profit, consumer surplus and therefore total social welfare. Moreover, if the retailers were forced to pay from the government for the right to use entrance fees. By this or another similar redistribution tool, the additional retailer's profit could be transferred to consumers (through increasing public goods availability or decreasing taxes). Then such a welfare-improving pricing strategy as entrance fees could become more politically feasible. As to possible direct governmental regulation of retailing via capping the markup (not actually practiced so far), while this measure generally *enhances* total welfare and consumer surplus when the cap is tailored optimally (in the absence of entrance fees), it does *not* do so *as much as entrance fees combined with transfers to consumers.*

It seems interesting to extend this approach to the case of additive separable utility, as well as nonlinear cost functions, investments in R&D [13–16].

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A Appendix. Proof of Proposition 6

A.1 Optimal Taxation: The Case *RL*

We first consider the case RL, i.e.,

$$\mathcal{F} \geq 1. \tag{18}$$

[3] It seems that further increase in the retailer's bargaining power, in the sense of possible entrance fees levied on manufacturers, also *enhances* profit, consumer surplus and therefore total social welfare. Moreover, if the retailers were forced to pay from the government for the right to use entrance fees. By this or another similar redistribution tool, the additional retailer's profit could be transferred to consumers (through increasing public goods availability or decreasing taxes). Then such a welfare-improving pricing strategy as entrance fees could become more politically feasible. As to possible direct governmental regulation of retailing via capping the markup (not actually practiced so far), while this measure generally *enhances* total welfare and consumer surplus when the cap is tailored optimally (in the absence of entrance fees), it does *not* do so *as much as entrance fees combined with transfers to consumers.*

Let us note that in this case (12) means, see Table 1,

$$D \geq 4\sqrt{\mathcal{F}}. \tag{19}$$

Recall that, see Tables 2, 4 and 5,

$$N = -\frac{(\tau - \tau_1)\,\varepsilon}{2f\sqrt{\mathcal{F}}} \geq 0 \Longleftrightarrow \tau \leq \tau_1, \tag{20}$$

$$W = -\frac{(\tau - \tau_1)\,(\tau - \tau_2)\,H}{4f^2}, \tag{21}$$

$$CS = \frac{(\tau - \tau_1)\,(\tau - \tau_3)\,H}{4f^2}, \tag{22}$$

where

$$\tau_1 = \left(D - 4\sqrt{\mathcal{F}}\right) f, \qquad \tau_2 = \left(\frac{6\mathcal{F} + 4}{\sqrt{\mathcal{F}}} - 3D\right) f, \qquad \tau_3 = \left(D - 2\sqrt{\mathcal{F}}\right) f.$$

Note that, due to (18) and (19),

$$\tau_2 < 0 \leq \tau_1 < \tau_3. \tag{23}$$

Function (21) is strictly concave. Therefore, due to (20) and (23), we get

$$\tau_W = \frac{\tau_1 + \tau_2}{2} = \left(\frac{\mathcal{F} + 2}{\sqrt{\mathcal{F}}} - D\right) f.$$

Moreover, due to (19) and (18),

$$\tau_W \leq \left(\frac{\mathcal{F} + 2}{\sqrt{\mathcal{F}}} - 4\sqrt{\mathcal{F}}\right) f = \frac{(2 - 3\mathcal{F})\, f}{\sqrt{\mathcal{F}}} \leq -\frac{f}{\sqrt{\mathcal{F}}} < 0.$$

Thus, we get (13).

Besides, function (22) is strictly convex. Therefore, due to (20) and (23), we get (14).

A.2 Optimal Taxation: The Case $RL(I)$

Consider the case $RL(I)$, i.e.,

$$0 \leq \mathcal{F} < 1. \tag{24}$$

Let us note that in this case (12) means

$$D \geq 2 \cdot (\mathcal{F} + 1)\,. \tag{25}$$

Recall that, see Tables 2, 4 and 5,

$$N = -\frac{(S - S_1)\,(S - S_2)\,\varepsilon}{2S^2} \geq 0 \Longleftrightarrow S_1 \leq S \leq S_2, \tag{26}$$

$$W = \frac{(S - S_1)(S - S_2)(S - S_3)(S - S_4)H}{4S^2}, \tag{27}$$

$$CS = -\frac{(S - S_1)(S - S_2)(S - S_5)(S - S_6)H}{4S^2}, \tag{28}$$

where $S_1, S_2, S_3, S_4, S_5, S_6$ are (real due to (25))

$$S_1 = \frac{D - \sqrt{D^2 - 4 \cdot (2\mathcal{F} + 1)}}{2}, \qquad S_2 = \frac{D + \sqrt{D^2 - 4 \cdot (2\mathcal{F} + 1)}}{2},$$

$$S_3 = \frac{3D - \sqrt{9D^2 - 12 \cdot (2\mathcal{F} + 1)}}{2}, \qquad S_4 = \frac{3D + \sqrt{9D^2 - 12 \cdot (2\mathcal{F} + 1)}}{2},$$

$$S_5 = \frac{-D + \sqrt{D^2 + 4 \cdot (2\mathcal{F} + 1)}}{2}, \qquad S_6 = \frac{-D - \sqrt{D^2 + 4 \cdot (2\mathcal{F} + 1)}}{2}$$

and $S > 0$ is defined in (10). Since $\frac{\partial S}{\partial \tau} < 0$, S is monotone with respect to τ. Therefore, it is enough to examine the behavior of the functions (27) and (28) with respect to S. Note that, due to (24) and (25),

$$S_6 < 0, \quad 0 < S_5 < S_3 < S_1 \leq 1 < S_2 < S_4. \tag{29}$$

One has

$$\frac{\partial W}{\partial S} = \frac{H}{2S^3} \cdot \left(S^4 - 2D \cdot S^3 + 3D \cdot (2\mathcal{F} + 1) \cdot S - 3 \cdot (2\mathcal{F} + 1)^2\right), \tag{30}$$

$$\frac{\partial}{\partial S}(CS) = -\frac{H}{2S^3} \cdot \left(S^4 - (2\mathcal{F} + 1) D \cdot S + (2\mathcal{F} + 1)^2\right). \tag{31}$$

Due to (25), (29), and Descartes' theorem, the number of positive roots of the equation $\frac{\partial W}{\partial S} = 0$ is either three or one, while the number of positive roots of the equation $\frac{\partial}{\partial S}(CS) = 0$ is either two or zero. Due to Rolle's theorem, on each of the intervals $[S_3, S_1], [S_1, S_2], [S_2, S_4]$ there is a point at which $\frac{\partial W}{\partial S} = 0$, while on each of the intervals $[S_5, S_1], [S_1, S_2]$ there is a point at which $\frac{\partial}{\partial S}(CS) = 0$.

Therefore, $\frac{\partial W}{\partial S}$ has three positive roots, two of which do not lie in $[S_1, S_2]$, while $\frac{\partial}{\partial S}(CS)$ has two positive roots, one of which does not lie in $[S_1, S_2]$. Therefore, W has a single maximum S_W on $[S_1, S_2]$, while CS has a single maximum S_{CS} on $[S_1, S_2]$. Moreover, see (26),

$$\frac{\partial W}{\partial S} \begin{cases} > 0, & S \in (S_1, S_W), \\ < 0, & S \in (S_W, S_2), \end{cases} \qquad \frac{\partial}{\partial S}(CS) \begin{cases} > 0, & S \in (S_1, S_{CS}), \\ < 0, & S \in (S_{CS}, S_2). \end{cases}$$

Due to (25),

$$\begin{aligned}\frac{\partial W}{\partial S}|_{S=1} &= \frac{H}{2}\cdot\left(\left(6\mathcal{F}+1\right)D-2\cdot\left(6\mathcal{F}^2+6\mathcal{F}+1\right)\right)\\ &\geq \frac{H}{2}\cdot\left(2\cdot\left(\mathcal{F}+1\right)\left(6\mathcal{F}+1\right)-2\cdot\left(6\mathcal{F}^2+6\mathcal{F}+1\right)\right)=H\mathcal{F}>0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial S}\left(CS\right)|_{S=1} &= -\frac{H}{2}\cdot\left(1-\left(2\mathcal{F}+1\right)D+\left(2\mathcal{F}+1\right)^2\right)\\ &\geq -\frac{H}{2}\cdot\left(1-2\left(2\mathcal{F}+1\right)\left(\mathcal{F}+1\right)+\left(2\mathcal{F}+1\right)^2\right)=H\mathcal{F}>0.\end{aligned}$$

Therefore, due to (29), $S_W>1$ and $S_{CS}>1$. Hence, see (10),

$$\tau_W=f\cdot\frac{1-\left(S_W\right)^2}{S_W}<0,\qquad \tau_{CS}=f\cdot\frac{1-\left(S_{CS}\right)^2}{S_{CS}}<0.$$

Thus we get (15) and for the case (16).

A.3 Optimal Taxation: Comparison of τ_W and τ_{CS}

Now we get (17). For the case RL, (17) follows from (13) and (14).

Consider the case $RL(I)$. As we have got in Appendix A.2, on $[S_1, S_2]$, function W has a single maximum S_W, while function CS has a single maximum S_{CS}. To get (17), it is sufficient to show that $S_{CS}>S_W$, i.e., that

$$\frac{\partial W(S_{CS})}{\partial S}<0. \tag{32}$$

From (31), we get

$$D=\frac{\left(S_{CS}\right)^4+(2\mathcal{F}+1)^2}{(2\mathcal{F}+1)S_{CS}}. \tag{33}$$

Substituting (33) in (30), we get

$$\begin{aligned}\frac{\partial W\left(S_{CS}\right)}{\partial S} &= \frac{H}{2}\cdot\left(S_{CS}+\frac{3\cdot\left(2\mathcal{F}+1\right)-2\left(S_{CS}\right)^2}{\left(S_{CS}\right)^2}\cdot D-3\cdot\frac{\left(2\mathcal{F}+1\right)^2}{\left(S_{CS}\right)^3}\right)\\ &= -H\cdot\frac{\left(2\mathcal{F}+1-\left(S_{CS}\right)^2\right)^2}{\left(2\mathcal{F}+1\right)\left(S_{CS}\right)^2}<0,\end{aligned}$$

i.e., (32) holds. Thus, we get (17).

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