



Modeling the Compressive Behavior of Steel Fiber Reinforced Concrete Under High Strain Rate Loads

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Abstract. Concrete is a strain-rate sensitive material and shows relatively low ductility and energy dissipation capacity under high strain rate loads (HSRL) such as blast and impact, representative of terrorist attacks and accidents. Experimental research in the literature has evidenced that introducing steel fibers, into the concrete mixtures can significantly improve the concrete behavior under HSRL. Besides the experimental research, development of design models is an important aspect to provide more confidence for engineers to use SFRC in structural elements when subjected to HSRL. The existing design codes (e.g. CEB-FIP Model Code 1990 and fib Model Code 2010) propose models for the prediction of the strengths of concrete under different HSRL, but they are only function of strain rate. In this regard, the current paper deals with the improvement of design models in the fib Model Code 2010 for the prediction of the compressive behavior of SFRC by considering the effects of the important parameters such as volume fraction, aspect ratio and tensile strength of steel fibers, and concrete compressive strength, besides the strain rate effect. The developed artificial neural network mathematical model is calibrated and its predictive performance is assessed using a database collected from the existing compressive impact tests results on SFRC specimens.

Keywords: Steel fiber reinforced concrete · Design model · Drop-weight impact test · High strain rate load

1 Introduction

Concrete is a strain-rate sensitive material with low ductility and energy dissipation capacity under high strain rate loads such as blast and impact, representative of terrorist attacks and accidents. The experimental studies in the literature evidence that introducing various types of fibers, especially steel fibers, into the concrete mixtures can significantly improve the concrete behavior under high strain rate loads [1, 2]. In this context, SFRC has high impact resistance and energy dissipation capacity due to the reinforcement mechanisms provided by fibers bridging the crack surfaces. These mechanism, mainly those of fiber pull-out and snubbing effect at the fiber exit point, limit crack propagation and enhances the energy dissipation capacity.

The impact behavior of steel fiber reinforced concrete (SFRC) composite materials under compressive drop-weight tests is complex due to the nonlinear relationship between the impact force and other variables. In this regard, some efforts have been spent in order to establish an analytical model for the impact behavior of SFRC under impact tests [2]. In this area, proposing a design formulation to accurately predict the behavior of SFRC composite materials under impact loads considering the influence of effective parameters (i.e. volume fraction of steel fibers (v_f), aspect ratio of steel fibers (L/D), tensile strength of steel fibers (f_{ts}), and concrete compressive strength (f_{cs}), and shape of fibers is an issue that needs to be addressed. Moreover, there are still many other issues, such as sophisticated equipment to accurately measure the impact forces in these test types, deep analysis of failure mechanism, stress propagation in specimens, effect of the distribution of inertial forces in specimens and inertial force measurement. Meanwhile, such type of problem can be solved by some mathematical methodologies in the field of machine learning, for example, artificial neural network (ANN). ANNs have been used to solve some sophisticated problems in the field of civil engineering and provided excellent results [3].

In this regard, the present study focuses on proposing an analytical model with design framework for predicting the compressive behavior of SFRC composite materials under high strain loads using ANN method and considering the effective parameters.

2 Architecture of the ANN

A neural network is a concept born due to the scientific interest about artificial intelligence (AI) during in the middle of 1950s. Artificial neural networks are inspired by the architecture of the human central nervous system which consists in a large number of cells (neurons) working in parallel in order to facilitate decision-making in the most rapid way [4]. These simple units are connected each other by electrical stimulus named synapse. Similarly, the ANN offers the synaptic activity through a matrix of weight (numeric values), which is typically updated by the like-human learning process. The direct advantage of this approach is that neuron-computing devices do not have to be programmed, but on the contrary, the random choice of initial weights induces the learn-making from the process of adjusting the weights itself by reaching the minimum error of prediction. ANN composes of an input layer including the variables and an output layer. A defined number of layers are also added in between these input and output layers, called hidden layers. Commonly only one hidden layer is considered. However, the use of two or more hidden layers can sometimes significantly improve the performance of ANN [3–5].

In this study, the proposed mathematical model was developed in Python programming language to estimate dynamic increase factor (DIF) of SFRC composite materials under high strain rate compressive load. A large database was collected from the literature to use for the development of empirical model. This database includes the experimental results of compressive drop weight tests, split Hopkinson pressure bar (SHPB) tests and quasi-static tests on SFRC, which are the most current tests to assess the impact compressive behavior of this type of material. The analysed database

included 80 SFRC samples tested under a strain rate load of less than 30 s^{-1} and 296 SFRC samples under a strain rate load of higher than 30 s^{-1} , thus totally 376 SFRC samples was included in the database and utilized in this study. The effective parameters adopted in the proposed model including volume fraction of steel fibers (v_f), aspect ratio of steel fibers (L/D), tensile strength of steel fibers (f_{ts}), and concrete compressive strength (f_{cs}), were all reported for the specimens used in this database.

The database was randomly divided into two sub-databases for training (80%), and test (20%). Following the data division and preprocessing, the architecture of the hidden layer (including the number of hidden layers and the corresponding number of hidden nodes) was established by the usual method of trial and error (learning and training stages). Several trials were carried out, utilizing the whole database, in order to find the optimum number of nodes in the hidden layer offering the highest coefficient of correlation (R^2) with experimental data [3].

In brief, the network was designed by using four neurons as input layer, two neurons for hidden layer and one neuron as output layer. Transfer function in the hidden layer was sigmoid and in the output layer was linear. After designing the network and standardizing the value of the input parameters to improve the ANN-mathematical model and make the training faster, the network would be trained Fig. 1.

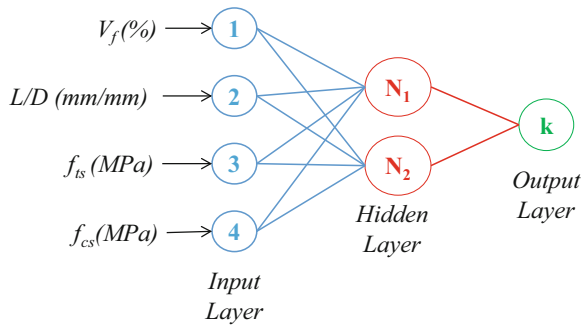


Fig. 1. Architecture of the comprehensive version of the proposed network.

3 Proposed ANN-Mathematical Model for Prediction of Dynamic Increase Factor (DIF)

The strength of concrete at high strain rate loads can increase significantly. To characterize the effects of strain rate on the compressive strengths of concrete, the dynamic increase factor (DIF), i.e. the ratio of dynamic to static strength, is generally proposed as a function of strain rates. In this regard, the CEB-FIP Model Code 1990 (MC1990) [6] and CEB-FIP Model Code 2010 (MC2010) [7] are proposed design formulations for estimating the DIF of concrete as a function of strain rate of loading. This section aims to first evaluate the performance of the proposed formulations in MC1990 and MC2010 to predict the compressive DIF of SFRC materials, and then, compare their predictive

performances with the corresponding performance of the developed ANN-mathematical model to predict the compressive DIF of SFRC. This model was developed based on modifying the proposed formulation in MC2010. In addition, a simplified closed form formulation derived from the developed ANN-mathematical model is proposed in the next section to predict the DIF of SFRC materials under compression.

Based on the MC1990, for a given strain rate, the compressive strength under high rates of loading are estimated from Eqs. (1) and (2). These power functions were established using appropriate underlying theory derived from thermodynamics and fracture mechanics analysis [6].

$$f_{cd}/f_{cs} = (\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{1.026\alpha} \quad \text{for } \dot{\epsilon}_c \leq 30s^{-1} \quad (1)$$

$$f_{cd}/f_{cs} = \gamma_s(\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{1/3} \quad \text{for } \dot{\epsilon}_c > 30s^{-1}$$

$$\log \gamma_s = 6.156 \alpha - 2$$

$$\alpha = 1/(5 + 9f_{cs}/f_{c0}) \quad (2)$$

where f_{cd} is the dynamic compressive strength under high rates of loading, f_{cs} is static compressive strength, $\dot{\epsilon}_c$ is compressive strain rate and $\dot{\epsilon}_{c0} = 30.10^{-6}s^{-1}$, and $f_{c0} = 10 \text{ MPa}$.

These formulations were proposed for two domains of strains, the first ranging from low to intermediate ($\dot{\epsilon}_c \leq 30s^{-1}$) and the other from intermediate to high rates ($\dot{\epsilon}_c > 30s^{-1}$). In a log (DIF) versus log($\dot{\epsilon}$) the relationship is bilinear with a change in slope around 30 s^{-1} . On the other hand, the proposed DIF in MC2010 for the compressive strength is given by:

$$f_{cd}/f_{cs} = (\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{0.014} \quad \text{for } \dot{\epsilon}_c \leq 30s^{-1} \quad (3)$$

$$f_{cd}/f_{cs} = 0.012(\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{1/3} \quad \text{for } \dot{\epsilon}_c > 30s^{-1} \quad (4)$$

where f_{cd} is compressive strength under high rates of loading, f_{cs} is the mean value of compressive strength of concrete, $\dot{\epsilon}_c$ is compressive strain rate in the range of $30 \times 10^{-6}s^{-1}$ to 300 s^{-1} and $\dot{\epsilon}_{c0} = 30.10^{-6}s^{-1}$.

These constitutive relations are valid for normal concrete, while for SFRC they need to be updated due to the steel fiber effects in concrete, by considering the effective parameters in the formulation. By inspiration of Eqs. (3) and (4), the alterations were conducted on the power of Eq. (3) proposed by MC2010 (k_1 in Eq. (5)) for the range of $\dot{\epsilon}_c \leq 30s^{-1}$ and on the constant coefficient of Eq. (4) proposed by MC2010 (k_2 in Eq. (6)) for the strain rates beyond the 30. In this regard, k_1 and k_2 parameters were derived from experimental database using Eqs. (5) and (6), and were adopted as output variable in ANN-mathematical model.

$$f_{cd}/f_{cm} = (\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{k_1} \quad \text{for } \dot{\epsilon}_c \leq 30s^{-1} \quad (5)$$

$$k_1 = \ln(f_{cd}/f_{cm})/\ln(\dot{\epsilon}_c/\dot{\epsilon}_{c0})$$

$$f_{cd}/f_{cm} = k_2(\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{1/3} \quad \text{for } \dot{\epsilon}_c > 30s^{-1} \quad (6)$$

$$k_2 = (f_{cd}/f_{cm})/(\dot{\epsilon}_c/\dot{\epsilon}_{c0})^{1/3}$$

k_1 and k_2 parameters were considered as a function of volume fraction of steel fibers (v_f), aspect ratio of steel fibers (L/D), tensile strength of steel fibers (f_{ts}), and concrete compressive strength (f_{cs}) for SFRC materials. Consequently, these four variables ($v_f, L/D, f_{cs}, f_{ts}$) were adopted in the input layer in the ANN-mathematical model. Two neurons were assumed in the hidden layer, and k_1 and k_2 parameters were considered as the output layer.

The different numbers of neurons in the hidden layer and the different transfer functions for hidden and output layers were accepted in the ANN-mathematical model to find an optimal network architecture offering highest coefficient of correlation (R^2) with experimental data. On the other side, since the main objective of this study is to propose a closed form design formulation derived from the ANN-mathematical model for the compressive DIF of SFRC materials, the neurons number adopted in the hidden layer was minimized.

The performance of the proposed ANN-mathematical model was verified against the experimental results for two range of strain rates $\dot{\epsilon}_c \leq 30s^{-1}$ and $\dot{\epsilon}_c > 30s^{-1}$. The plot of the experimental compressive DIF versus the corresponding ANN-mathematical model predictions for the database is shown in Figs. 2 and 3, and also, compared with the DIFs obtained from MC1990 and MC2010.

The best-fit line approximately aligns with the 45° benchmark proving a proper correlation between the experimental results and the predictions of the proposed ANN-mathematical model for the train and test data. The coefficient of correlation (R^2) of DIFs obtained from the ANN-mathematical model with the experimental results for training and test data are, respectively, $R^2 = 0.67$ and 0.70 for strain rates $\dot{\epsilon}_c \leq 30s^{-1}$, and $R^2 = 0.72$ and 0.78 for strain rates $\dot{\epsilon}_c > 30s^{-1}$ (Figs. 2 and 3).

However, the coefficient of correlation (R^2) of DIFs obtained according to MC1990 and MC2010 using Eqs. (1) and (3) for strain rates $\dot{\epsilon}_c \leq 30s^{-1}$ are, respectively, 0.13 and 0.07 for training data and 0.028 and 0.01 for test data. In addition, for strain rates $\dot{\epsilon}_c > 30s^{-1}$, R^2 of DIFs of MC1990 and MC2010 (Eqs. (2) and (4)) are, respectively, 0.41 and 0.23 for training data and 0.47 and 0.33 for test data. This comparison demonstrates that the proposed ANN-mathematical model performs significantly better than the current commonly used model MC1990 and MC2010 in the prediction of compressive DIF of SFRC materials.

Moreover, the experimental compressive DIF is plotted versus strain rate in Fig. 4 and compared with the compressive DIF obtained from MC1990, MC2010 and ANN-mathematical model. This figure also evidences the good predictive performance of the developed ANN-mathematical model for the compressive DIF of SFRC materials in comparison with the proposed formulations in MC1990 and MC2010.

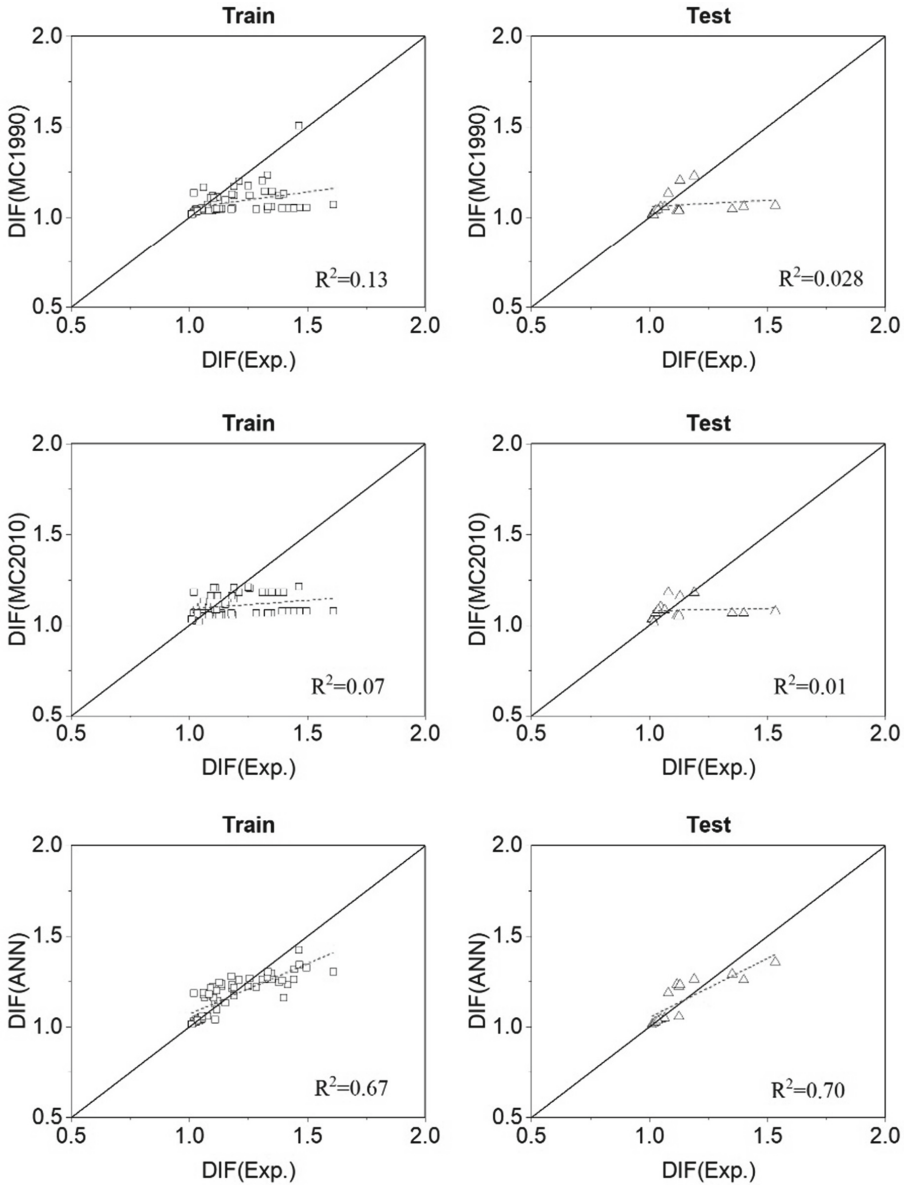


Fig. 2. Comparison of DIFs obtained from MC1990 and MC2010 and ANN-mathematical model with corresponding experimental values for $\dot{\epsilon}_c \leq 30s^{-1}$.

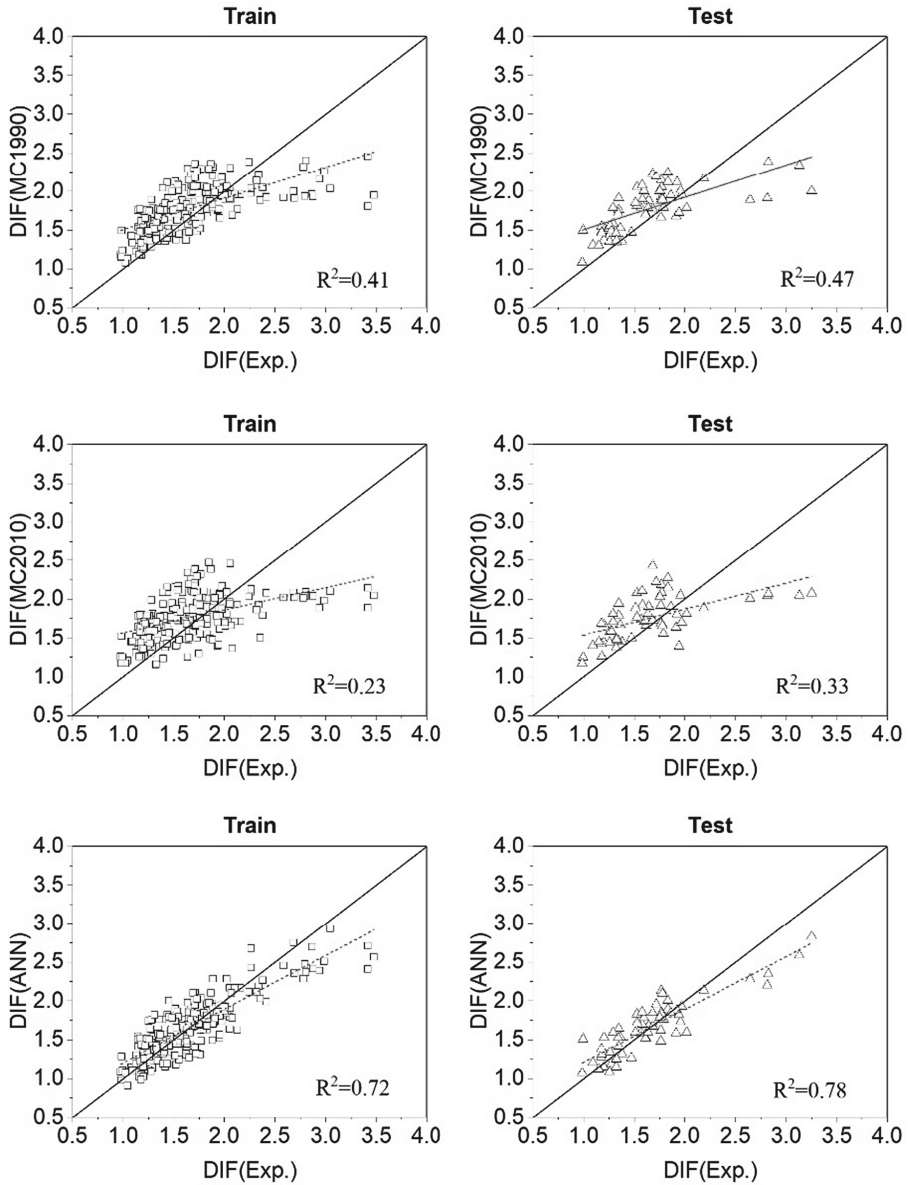


Fig. 3. Comparison of DIFs obtained from MC 1990 and MC 2010 and ANN-mathematical model with corresponding experimental values for $\dot{\epsilon}_c \leq 30s^{-1}$.

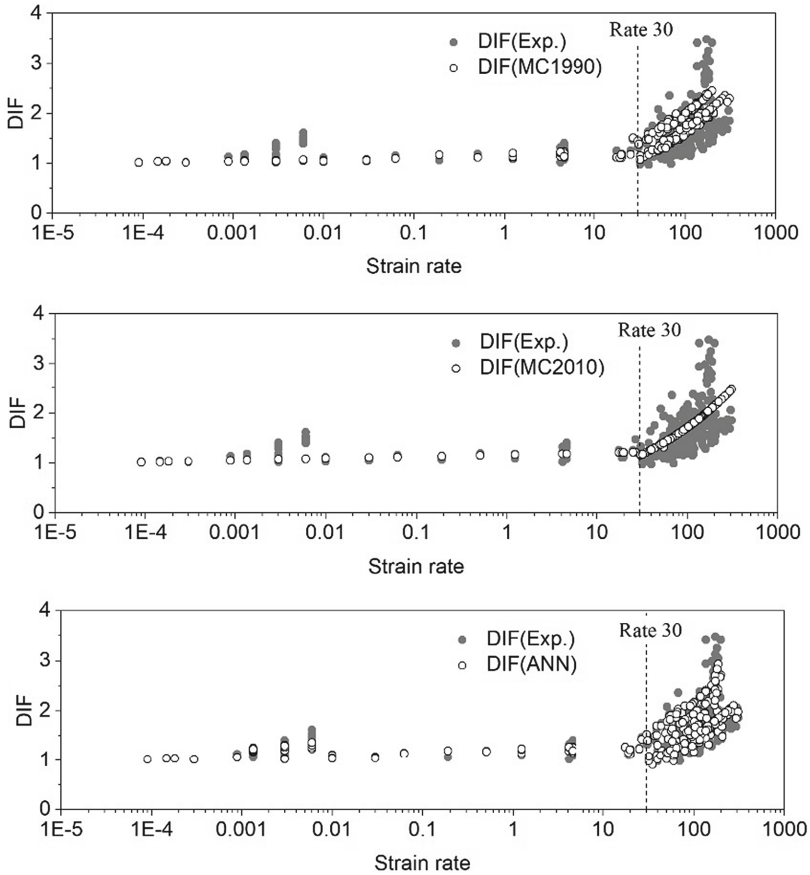


Fig. 4. Comparison of experimental DIF with DIF obtained from MC 1990, MC 2010 and proposed ANN-mathematical model.

4 Design Formulation Based on the ANN-Mathematical Model

It is worth noting that since the proposed ANN-mathematical model has been trained, it is ready to be adopted for predicting the DIF of SFRC under high strain rate compressive loads. Although the simulated results from the proposed ANN-mathematical model have a good agreement with the experimental data, it is inconvenient for engineers to use the networks for engineering design purposes. Since, the engineers need to have the fundamental knowledge of ANN and Python to be able to use the proposed model. Therefore, to make the proposed model more practical for direct application without performing ANN analysis, a functional-form equation could be explicitly derived from the trained networks by using input parameters and transfer functions and combining the weight matrix and the bias matrix (Fig. 5) [3, 5, 8]. The sigmoid transfer function was used in the hidden layer (see Eq. (7)) and linear transfer

function (see Eq. (8)) was used in the output layer. The procedure to develop the user-friendly equations based on the ANN-mathematical model to determine k_1 and k_2 parameters is represented in Fig. 5.

$$f(x) = 1/(1 + e^{-x}) \quad (7)$$

$$f'(x) = x \quad (8)$$

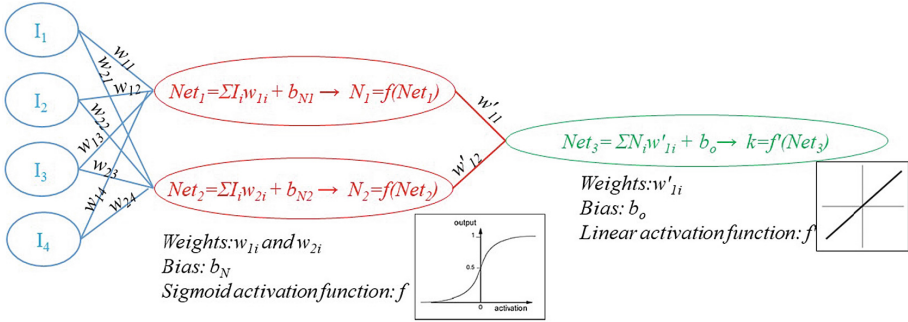


Fig. 5. Architecture of the proposed ANN-mathematical model

The equations derived from the ANN-mathematical model to determine k_1 and k_2 parameters to be used in Eqs. (5) and (6) for predicting the compressive DIF of SFRC composite materials are as follows:

$$k_1 = \frac{0.3}{1 + e^{-\alpha}} - \frac{0.3}{1 + e^{-\beta}} + 0.006 \quad (9)$$

$$\begin{aligned} \alpha &= -11.70v_f + 0.12(L/D) - 0.0007f_{is} - 0.09f_{cs} + 4.80 \\ \beta &= 11.05v_f + 0.22(L/D) - 0.0013f_{is} - 0.09f_{cs} - 0.94 \end{aligned} \quad (10)$$

$$k_2 = \frac{0.26}{1 + e^{-\alpha}} - \frac{0.26}{1 + e^{-\beta}} + 0.009$$

$$\begin{aligned} \alpha &= -49.43v_f + 0.04(L/D) - 0.0012f_{is} - 0.02f_{cs} + 1.68 \\ \beta &= -55.20v_f + 0.04(L/D) - 0.0013f_{is} - 0.02f_{cs} - 1.55 \end{aligned}$$

It worth to note that the proposed formulations are valid for the range of the parameters reported in the Tables 1 and 2.

Table 1. Range of the variables based on the collected database for $\dot{\epsilon}_c > 30s^{-1}$.

Input	Description	Unit	Range
v_f	Volume fraction of steel fibers	%	[0.3–6]
L/D	Aspect ratio of steel fibers	mm/mm	[20–125]
f_{is}	Tensile strength of steel fibers	MPa	[750–4300]
f_{cs}	Concrete compressive strength	MPa	[32–200]

Table 2. Range of the variables based on the collected database for $\dot{\epsilon}_c < 30s^{-1}$.

<i>Input</i>	<i>Description</i>	<i>Unit</i>	<i>Range</i>
v_f	Volume fraction of steel fibers	%	[0.5–3]
L/D	Aspect ratio of steel fibers	mm/mm	[39–81]
f_{ts}	Tensile strength of steel fibers	MPa	[900–2850]
f_{cs}	Concrete compressive strength	MPa	[32–160]

5 Conclusion

This study develops a new artificial neural network (ANN) mathematical model based on modifying the proposed formulation in fib model code 2010 (MC2010) to predict the dynamic increase factor (DIF) of SFRC composite materials under high strain rate compressive loads by considering the effective parameters (i.e.: volume fraction of steel fibers (v_f), aspect ratio of steel fibers (L/D), tensile strength of steel fibers (f_{ts}), and concrete compressive strength (f_{cs})). The compressive DIF of SFRC materials obtained from the developed ANN-mathematical model is compared to the corresponding DIF obtained from MC2010 and MC1990. A simplified closed form formulation derived from the developed ANN-mathematical model is proposed to predict the DIF of SFRC materials under compression. An overview of the developed model points out the following conclusions:

- A good predictive performance for the proposed ANN-mathematical model in terms of the compressive DIF of SFRC materials for both strain rates of $\dot{\epsilon}_c \leq 30s^{-1}$ and $\dot{\epsilon}_c > 30s^{-1}$ is demonstrated by comparing with the relevant experimental results.
- The proposed formulations in MC1990 and MC2010 for the compressive DIF of concrete cannot predict well the compressive DIF of SFRC material, due to the lack of parameters to consider the impact of steel fibers in concrete in these formulations.
- The proposed simplified closed form formulation with a design framework derived from the developed ANN-mathematical model can provide useful estimations of SFRC strength at high strain rate compressive loads (with the aim of designing the impact resistance of SFRC elements subjected to high strain rate loads such as blast and impact, representative of terrorist attacks and accidents which is an ongoing research project in the University of Minho).

The main objective of the next step of this research study is to extend the ANN-mathematical model to achieve a higher degree of accuracy in predicting the compressive DIF of SFRC materials by adding other important parameters, i.e. the shape of steel fibers, fiber orientation factor, and fiber efficiency factor, in the developed ANN-mathematical model.

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