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Francis Ysidro Edgeworth (1845–1926)

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1 Introduction

Francis Ysidro Edgeworth was born in Edgeworthstown in County Longford, Ireland. His large family background is fascinating, and has been richly described by Barbé (2010). His grandfather was the energetic and colourful Richard Lovell Edgeworth, whose life was documented in a two-volume memoir by his eldest daughter, the famous novelist Maria Edgeworth (1820); see also Butler and Butler (1927). Richard Lovell carried out many scientific and mechanical experiments, and was a member of the Lunar Society of Birmingham, whose members included James Watt, Matthew Boulton, Josiah Wedgwood, Joseph Priestley, Erasmus Darwin and Samuel Galton. In addition, Maria's scientific acquaintances included Humphry Davy, Alexander von Humboldt, William Herschel, Charles Babbage, Joseph Hooker and Michael Faraday. The marriage of Francis Ysidro Edgeworth's cousin Harriet Jessie Edgeworth (daughter of Richard Lovell's seventh and youngest son Michael Pakenham) to Arthur Gray Butler provided links with another eminent family. Furthermore, Butler's sister, Louisa, married Francis Galton, a cousin of Charles Darwin.

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Richard Lovell's sixth son, and seventeenth surviving child, was Francis Beaufort Edgeworth, who in 1831 met his wife, Rosa Florentina Eroles, from Catalonia and then aged sixteen, while on the way to Germany to study philosophy: they married three weeks later. Francis Ysidro was their fifth son. With his family background and considerable linguistic skills, Edgeworth had wide international sympathies.

Edgeworth was educated by tutors until 1862, when he entered Trinity College Dublin to study languages. His first association with Oxford came in 1867, when he entered Exeter College. After one term he transferred to Magdalen Hall, and then to Balliol in 1868, where in Michaelmas 1869 he obtained a First in *Literae Humaniores*. During the viva Edgeworth apparently replied, 'Shall I answer briefly or at length?', whereupon he spoke for half an hour to convert what was to be a Second into a First.

His career after graduation was varied. He was called to the Bar in 1877, the year in which his first book, *New and Old Methods of Ethics*, was published. Edgeworth applied unsuccessfully for a Professorship of Greek at Bedford College, London, in 1875, but later lectured there on English language and literature for a brief period from late 1877 to mid-1878. He had earlier lectured on logic, mental and moral sciences and metaphysics to prospective Indian civil servants, at a private institution run by a Mr Walter Wren. In 1880, he applied for a chair of philosophy, also unsuccessfully, but began lecturing on logic to evening classes at King's College London. Soon after the publication of his second book, *Mathematical Psychics*, in 1881, he applied for a professorship of logic, mental and moral philosophy and political economy at Liverpool. Edgeworth had to wait until 1890 to obtain a professorial appointment. This was at King's College London, where he succeeded Thorold Rogers in the Tooke Chair of Economic Science and Statistics. In the next year, he again succeeded Rogers, this time to become Drummond Professor, a position he held until his retirement in 1922, and Fellow of All Souls College, Oxford.

In addition to his work in economics, Edgeworth began a series of statistical papers in 1883, and was secretary to the British Association *Report on Index Numbers* (1887–1889). He was President of Section F of the British Association in 1889, a position he held again in 1922. Edgeworth's work on mathematical statistics took an increasingly important role. Indeed, of about 170 papers which he published, approximately three-quarters were concerned with statistical theory: many are collected in McCann (1996). He became a Guy Medalist (Gold) of the Royal Statistical Society in 1907 and was President of the Society from 1912 to 1914. His third and final book was *Metretike: or, The Method of Measuring Probability and Utility* (Edgeworth 1887); on his

statistics contributions, see Bowley (1928) and Stigler (1978). Near the end of his life, some of the vast stream of his economics papers were collected in three volumes of *Papers Relating to Political Economy* (Edgeworth 1925).

2 Edgeworth at Oxford

Edgeworth finally settled in Oxford at the age of 46 in one of the most illustrious British chairs in economics. In the same year, he also became the first editor of the *Economic Journal* and was editor or co-editor from its first issue until his death. He was buried in Holywell Cemetery, St Cross Church (next to Holywell Manor), which contains the graves of many notable Oxford people. Edgeworth has a professorship named after him at Nuffield College, Oxford. This distinction in economics is shared only with Nobel Prize winners, Sir John Hicks and James Meade (the other named professorship in Oxford is the Drummond at All Souls, but Drummond was not himself an economist).

At Oxford, Edgeworth was firmly established as the leading economist, after Marshall, in Britain. However, unlike Marshall at Cambridge, Edgeworth devoted little energy to improving the undergraduate teaching of economics. His influence at Oxford was described briefly by Bowley (1934: 123), and at greater length by Price (1946: 37) who complained that 'economics at Oxford looked like slumbering quietly or in effect at least must languish comparatively as it rested, so to say, inert in Edgeworth's keeping. There was no active stir of a resonant hive of busy students gathering honey under his helping regime'. Harrod said of his tutorials with Edgeworth, 'we used to sit side by side at a little table, and he'd go through my various diagrams' (Harrod quoted in Phelps Brown 1981: 662). It is indeed impossible to imagine, on the basis of his literary style, how Edgeworth could lecture clearly to undergraduates. He wrote always for fellow researchers, and even here his style was influenced by his attitude to the subject. As Price (1946: 35) argued, 'Edgeworth…convinced that Economics as he conceived it was so intrinsically hard a study that it could not possibly be made popularly plain…increased repellent difficulty'.

While Edgeworth was in no sense part of an Oxford group, Price (1946), Keynes (1933 [1972]) and Bowley (1934: 122) all stressed his generous hospitality, resulting in him having 'the widest personal acquaintance in the world with economists of all nations' (Keynes ibid.: 264). His complex character was described in the following terms by Keynes (ibid.: 265): 'He was kind, affectionate, modest, self-deprecatory, humorous, with a sharp and candid eye for human nature; he was also reserved, angular, complicated, proud,

and touchy, elaborately polite, courteous to the point of artificiality, absolutely unbending and unyielding in himself to the pressure of the outside world'.

He was said to have inherited 'the Edgeworthstown convention of rather formal good manners and conversation' (Butler 1972: 136). The poet Robert Graves (1960: 247) reported that Edgeworth avoided conversational English, persistently using words and phrases that one expects to meet only in books. One evening, T.E. Lawrence returned to All Souls College from a visit to London, and Edgeworth met him at the gate, asking, 'Was it very caliginous in the Metropolis?'; Lawrence replied gravely, 'Somewhat caliginous, but not altogether inspissated'.

3 Edgeworth's Approach to Economics

The obvious dominant characterised of Edgeworth's approach to economics is that it is mathematical, characterised by an original use of techniques, although he does not appear to have received a formal training in mathematics. However, he came to economics from moral philosophy. The central question of distributive justice, rather than simply the application of mathematics, dominated his attitude towards economics. His main argument was that mathematics provided powerful assistance to "unaided" reason, and could check the conclusions reached by other methods. For example, he suggested that 'he that will not verify his conclusions as far as possible by mathematics, as it were bringing the ingots of common sense to be assayed and coined at the mint of the sovereign science, will hardly realise the full value of what he holds' (Edgeworth 1881: 3).

The contrast between Edgeworth and Marshall was sharp. Although both men turned to economics from mathematics and moral philosophy, Marshall generally used biological analogies, and was concerned with developing maxims. In contrast, Edgeworth generally used mechanical analogies, and was more concerned with arriving at theorems. Pigou commented that, 'during some thirty years until their recent deaths in honoured age, the two outstanding names in English economics were Marshall…and Edgeworth … Edgeworth, the tool-maker, gloried in his tools … Marshall, on the other hand, had what almost amounted to an obsession for hiding his tools away' (Pigou quoted in Pigou and Robertson 1931: 3). Edgeworth's interest in the natural sciences often led him to make comparisons with scientific laws, and especially to show that the physical sciences also relied on abstraction and approximation.

Edgeworth argued carefully that the assumptions used in economics are often untestable, and he therefore took precautions against the accusation of "plucking assumptions from the air". He was conscious of the fact that the difficulty is in making the crucial abstractions which make the particular problem under consideration tractable, but which are not question-begging. His attitude to many a priori assumptions was influenced by his approach to statistical inference. He referred to, 'the first principle of probabilities, according to which cases about which we are equally undecided…count as equal' (Edgeworth 1881: 99). Thus, the appropriate assumption was that all feasible values, say of elasticities, were equally likely, until evidence is obtained or reference may be made to 'the consensus of high authorities' (Edgeworth 1925, ii: 391). This also illustrates Edgeworth's attitude to authority and his many allusions to the views of other leading economists. Price (1946: 38) referred to his frequent 'reference to authority for…support of tentative opinion waveringly advanced'.

Edgeworth was also prone to stressing negative results. For example, in discussing taxation, where the criterion of minimum sacrifice does not alone provide a simple tax formula, he stated:

> Yet the premises, however inadequate to the deduction of a definite formula, may suffice for a certain negative conclusion. The ground which will not serve as the foundation of the elaborate edifice designed may yet be solid enough to support a battering-ram capable of being directed against simpler edifices in the neighbourhood (Edgeworth 1925, ii: 261).

4 Early Work in Moral Philosophy

Edgeworth's first book, *New and Old Methods of Ethics*, published in 1877, was strongly influenced by the great Cambridge philosopher Henry Sidgwick. It examined in detail the implications of utilitarianism for optimal distribution. Edgeworth's original contribution was to apply advanced mathematics to this problem. His approach was dominated by utilitarianism, but the influence of contemporary psychological research and the impact of evolutionary ideas can also be seen here. Both aspects led to an explicit consideration of differences between individuals and changes over time.

On considering the major fierce debates in the second half of the nineteenth century between egoism, evolutionism, idealism and intuitionism, Edgeworth's brand of utilitarianism became extremely eclectic. It embraced the majority of other principles, except for those of the Hegelian idealists,

while regarding utilitarianism as the "sovereign principle". Writing of this book, Keynes (1933 [1972]: 257) commented that:

> Edgeworth's peculiarities of style, his brilliance of phrasing, his obscurity of connection, his inconclusiveness of aim, his restlessness of direction, his courtesy, his caution, his shrewdness, his wit, his subtlety, his learning, his reserve—all are there full-grown. Quotations from the Greek tread on the heels of the differential calculus.

Edgeworth generally distinguished between "impure" and "pure" utilitarianism. In the latter case, individuals are assumed to be concerned with the welfare of society as a whole. The former case in fact corresponds more closely with a short-term version of egoism. Economic exchange can usefully be analysed in terms of "jostling egoists", but he believed that ultimately individuals would evolve to become pure utilitarians. A reason for believing that individuals would make such a transition was later to be developed by Edgeworth in the form of his contractarian justification of utilitarianism as the appropriate principle of distributive justice.

Edgeworth's early utilitarianism was influenced by his wide knowledge of work in experimental psychology. In his books of 1877 and 1881, there are many references to the work of Joseph Delboeuf, Gustav Fechner, Hermann von Helmholtz, Ernst Weber and Wilhelm Wundt. These references occur in the context of the nature of utility functions and, although Edgeworth at this time was not aware of the earlier work of Stanley Jevons, the same range of work was also cited by Jevons. In 1877, Edgeworth explicitly suggested, in connection with Fechner, that an additive form would not be appropriate.

A further aspect of Edgeworth's utilitarianism is his attitude towards authority. An important issue for early utilitarians involved the nature of inductive evidence about the consequences of acts. Most people cannot know the full consequences of their acts, so that rules of moral conduct must be followed (in contrast with intuitionism where individuals are assumed to have immediate consciousness of moral rules). In arriving at such rules, the opinions of highly regarded individuals are taken to be credible even though it may not be possible to show conclusively that they are "correct". Edgeworth argued, for example, that 'we ought to defer even to the undemonstrated dicta and opinions of the wise, who have a power of mental vision acquired by experience' (Edgeworth 1925, ii: 149).

Edgeworth defined the problem of determining the optimal utilitarian distribution as follows: '[G]iven a certain quantity of stimulus to be distributed among a given set of sentients…to find the law of distribution productive of

the greatest quantity of pleasure' (Edgeworth 1877: 43). In treating this problem mathematically, he used Lagrange multipliers, without any explanation, and concluded that 'unto him that hath greater capacity for pleasure shall be added more of the means of pleasure' (ibid.). In using Lagrange multipliers, Edgeworth was also careful to discuss possible complications, referring to the possibility of multiple solutions and explicitly discussing corner solutions and inequality constraints.

Further complexities were then examined, where Edgeworth emphasised that utilitarianism implies equality of the 'means of pleasure' only under a special set of assumptions, and in the general case the prescribed solution will be some form of inequality. In a more general treatment of the problem, Edgeworth used the calculus of variations, but again provided the reader with virtually no help in following his mathematical argument. His analysis of the utilitarian optimal distribution was continued in his paper on "The Hedonical Calculus" (Edgeworth 1879), which was later reprinted as the third part of *Mathematical Psychics* (Edgeworth 1881).

5 Early Work in Economics

The turning point in Edgeworth's work was his introduction to Jevons in 1879 by a mutual friend James Sully, who in 1878 moved to Hampstead, London, where Edgeworth had lodgings in Mount Vernon and where Jevons also lived; see Sully (1918: 180, 223). Directly stimulated by Jevons's treatment of exchange, Edgeworth became interested in the problem of the indeterminacy of the rate of exchange, arising from the existence of only a small number of traders. This led rapidly to Edgeworth's second and most important book *Mathematical Psychics*: *An Essay on the Application of Mathematics to the Moral Sciences* (Edgeworth 1881), which was obviously written in a state of considerable enthusiasm for his new subject. Marshall's review began, 'This book shows clear signs of genius, and is a promise of great things to come' (Marshall quoted in Whitaker 1975: 265). Jevons began by stating that 'Whatever else readers of this book may think about it, they would probably all agree that it is a very remarkable one' (Jevons 1881: 581). However, this slim volume of 150 pages was long known only to a small group of experts, and it was not until the middle of the twentieth century that many of its central ideas began to be more fully appreciated.

Part 1 of *Mathematical Psychics* (Edgeworth 1881: 1–15) was devoted mainly to a justification of the use of mathematics in economics where precise data are not available. There is probably no other "apology" in the whole of economic

literature which compares with Edgeworth's plea for the application of mathematics. For example, when considering individual utility maximisation:

> Atoms of pleasure are not easy to distinguish and discern; more continuous than sand, more discrete than liquid; as it were nuclei of the just-perceivable, embedded in circumambient semi-consciousness. We cannot count the golden sands of life; we cannot number the "innumerable smile" of seas of love; but we seem to be capable of observing that there is here a greater, there a less, multitude of pleasure-units; mass of happiness; and that is enough (ibid.: 8–9).

Great stress was placed on comparison with Lagrange's "Principle of Least Action" in examining the overall effects produced by the interactions among many particles. The connection with Edgeworth's analysis of competition, involving interaction among a large number of competitors to produce a determinate rate of exchange, is central. The fact that in the natural sciences so much could be derived from a single principle was important for Jevons, but Edgeworth took this to its ultimate limit in arguing that the comparable single principle in social sciences, that of maximum utility, would produce results of comparable value. Referring to Laplace, he suggested (ibid.: 12) that '"*Mécanique Sociale*" may one day take her place along with "*Mécanique Celeste*", throned each upon the double-sided height of one maximum principle, the supreme pinnacle of moral as of physical science'.

Jevons's work in the *Theory of Political Economy* involved the application of mathematics to the analysis of exchange in competitive markets. The crucial development following Edgeworth's contact with Jevons was not simply the realisation that mathematics can be used to examine equilibrium in exchange. Rather, in his analysis, Jevons explicitly assumed, through his "law of indifference", that all individuals take equilibrium prices as given and outside their control. In using this law as 'one of the central pivots of the theory', Jevons (1957: 87) stated that 'there can only be one ratio of exchange of one uniform commodity at any moment'. His theory was explicitly limited to static equilibrium conditions and Jevons excluded the role of the number of competitors from his analysis via the awkward notion of the "trading body". This followed correspondence with Fleeming Jenkin, who could not see why two isolated individuals should accept the price-taking equilibrium; see Black (1977: 166–178). However, Jevons wished to consider the behaviour of two typical individuals in a large market.

In a section on "Failure of the Laws of Exchange", Jevons discussed cases in which some indeterminacy would result; for details of complex cases considered by Jevons, see Creedy (1992). His most notable example was house sales,

where it was suggested that indeterminacy would result from the discrete nature of the good being exchanged. A reviewer suggested instead that indeterminacy 'is really owing in our opinion to the assumed absence of competition' (Anonymous reviewer quoted in Black 1981: 157). It was this gap in Jevons's analysis which Edgeworth set out to fill. He examined how competition between buyers and sellers, through a barter process, leads to a "final settlement" which is equivalent to one in which all individuals act independently as price takers. As he later stated (Edgeworth 1925, ii: 453), 'the existence of a uniform rate of exchange between any two commodities is perhaps not so much axiomatic as deducible from the process of competition in a perfect market'. Edgeworth's highly original analysis is discussed in the following section.

6 Exchange, Contract and Indeterminacy

In modern economic analysis, the analytical tools invented by Edgeworth in 1881, such as the indifference map and the contract curve, are now used in a vast range of contexts. They were introduced by Edgeworth to examine the nature of barter among individuals. He wanted to see if a determinate rate of exchange would result in barter situations where it is assumed only that individuals wish to maximise their own utility, considered solely as a function of their own consumption. Given individuals' utility functions and their initial endowments of goods, would it be possible to work out a "determinate" rate of exchange at which trade would take place? Edgeworth's statement is as follows:

> The PROBLEM to which attention is specially directed in this introductory summary is: How far contract is indeterminate—an inquiry of more than theoretical importance, if it show not only that indeterminateness tends to [be present] widely, but also in what direction an escape from its evils is to be sought (Edgeworth 1881: 20; upper case in original).

Edgeworth began his analysis by taking the case of two individuals, A and B, exchanging quantities, x and y, of two goods. The framework is that described by Jevons, where the first individual holds all of the initial stocks of the first good, and the second individual holds all the stocks of the second good. Edgeworth wrote the utility functions of each individual in terms of the amounts exchanged, rather than consumed. He then immediately defined the general (rather than additive) utility function, the contract curve and indifference curves.

Following Edgeworth's introduction of the general utility function, he raised the question of the equilibrium which may be reached with, 'one or both refusing to move further'. In barter the conditions of exchange must be reached by voluntary agreement, or contract, between the two parties, and of course it is fundamental that egoists would not agree to a contract which would make them worse off than before the exchange. The question thus concerns the nature of the settlement reached by two contracting parties. He immediately answered that contract supplies only part of the answer so that 'supplementary conditions…supplied by competition or ethical motives' are required, and then wrote the equation of his famous contract curve (ibid.: 20–21).

The problem of obtaining the equilibrium values of x and y which, 'cannot be varied without the consent of the parties to it' was stated as follows: 'It is required to find a point (x, y) such that, in whatever direction we take an infinitely small step, [utilities] do not increase together, but that, while one increases, the other decreases' (ibid.: 21). The locus of such points, 'it is here proposed to call the contract-curve'. Edgeworth's alternative derivations of the contract curve involved the movement, from an arbitrary position, along one person's indifference curve. He stated, 'motion is possible so long as, one party not losing, the other gains' (ibid.: 23). Here, Edgeworth used the Lagrange multiplier method of maximising one person's utility subject to the condition that the other person's utility remains constant. After presenting the results for the two-person two-good case, Edgeworth (ibid.: 26) examined the contract curve in the case where three individuals exchange three goods. This involved an early use of determinants in economics.

The concept of the contract curve helps to specify a range of "efficient exchanges". The essential feature of the analysis from Edgeworth's point of view is that there is a range, rather than a unique point, so that 'the settlements are represented by an *indefinite number* of points' along the contract curve (ibid.: 29; italics in original). At any particular settlement, the rate of exchange is expressed in terms of the amount of one good which is given up in order to obtain a specified amount of the other good. Hence, the existence of a range of efficient contracts means that the rate of exchange (or effective price ratio) is "indeterminate". The rate achieved in practice depends on bargaining strength. This result led Edgeworth (ibid.: 30) to make his often-quoted remark that 'an accessory evil of indeterminate contract is the tendency, greater than in a full market, towards dissimulation and objectionable arts of higgling'.

Edgeworth argued that his analysis of indeterminacy in contract between two traders can be applied to a wide variety of contexts, including trade unions

and employers' associations. Having shown the possibilities of indeterminacy, Edgeworth went on to show how 'the escape from its evils' requires either competition or arbitration. He quickly moved on to the introduction of further traders.

In Edgeworth's problem of two traders exchanging two goods, the definition of a range of efficient exchanges along the contract curve is analytically separate from the question of whether or not two isolated traders would actually reach a settlement on the contract curve, through barter. However, these two aspects were not clearly separated by Edgeworth because at the beginning of his analysis he introduced his stylised description of the process of barter: this is the "recontracting" process. Edgeworth did not wish to assume that individuals initially have perfect knowledge. Instead, he supposed that 'there is free communication throughout a normal competitive field' (ibid.: 18). Knowledge of the other traders' dispositions and resources is obtained by the formation of tentative contracts, which are not assumed to involve actual transfers and can be broken when further information is obtained. Edgeworth introduced this in typical style, alluding to Alfred Tennyson's poem "Maud; A Monodrama": '"Is it peace or war?", asks the lover of Maud, of economic competition, and answers hastily: it is both, pax or pact between contractors during contract, war, when some of the contractors without the consent of others recontract' (ibid.: 17).

The recontracting process thus enables the dissemination of information among traders. It allows individuals who initially agree to a contract, which is not on the contract curve, to discover that an opportunity exists for making an improved contract according to which at least one person gains without another suffering. The importance of the recontracting process lies in the fact that it allows for Edgeworth's analysis of the role of the number of individuals in a market. With numerous individuals, the process makes it possible to analyse the use of collusion among some of the traders. Individuals can form coalitions in order to improve bargaining strength. Recontracting enables the coalitions to be broken up by outsiders who may attract members of a group away with more favourable terms of exchange.

Edgeworth's analysis was extremely terse. He introduced a second person *A* and a second person *B*, assumed to be exact replicas of the initial pair, with identical tastes and endowments. This simplification allows the same diagram to be used as in the case when only two traders are considered in isolation. Two basic points can be stated immediately. First, in the final settlement all individuals will be at a common point in the Edgeworth box. Second, the settlement must be on the contract curve. The first property arises because if

two individuals have identical tastes, their total utility is maximised by sharing resources equally.

The question at issue is whether the range of indeterminacy along the contract curve is reduced by the addition of these traders. Suppose with just one pair, the type-*B* trader has all the bargaining power and pushes the *A* trader to the limit of the contract curve where *B* obtains all the gains from trade. With the two pairs of traders no longer in isolation, the ability of a type-*A* trader to turn to someone else (or form a coalition), rather than deal with a single trader, means that the *B*s now compete against each other. The stylised process of recontracting with the two *B*s competing against each other will produce a final settlement with all traders at a common point on the contract curve, where the limit has moved inwards along the old contract curve. The analysis can be repeated by starting with an alternative situation whereby the *A*s are initially assumed to be able to appropriate all the gains from trade. This extreme point would no longer qualify as a point on the new contract curve. Hence, the introduction of the additional pair of traders means that the contract curve shrinks.

With many pairs of such traders, Edgeworth showed that a final settlement is on the contract curve, and looks just like a price-taking equilibrium. If there are multiple equilibria, the recontracting process causes the number of final settlements to shrink to the number of price-taking equilibria. For a discussion of utility functions involving multiple equilibria, and a comparison of bargaining, competitive and utilitarian solutions, see Creedy (1994a).

This argument relating to the shrinking contract curve, first established by Edgeworth, is often referred to as the Edgeworth limit theorem; for a more detailed exposition, see Creedy (1986). The fact that the price-taking solution is necessarily on the contract curve gives rise to what is now referred to as the "First Fundamental Theorem" of welfare economics, that a price-taking equilibrium is Pareto efficient. Furthermore, the use of price-taking, compared with recontracting, provides a considerable reduction in the amount of information required by traders. Given an equilibrium set, individuals only need to know the prices of goods, whereas in the recontracting process they have to learn a considerable amount of information about other individuals' preferences and endowments. However, Edgeworth placed most stress on the equivalence of the competitive price-taking solution with a barter process involving large numbers.

Given that coalitions among traders are allowed in the recontracting process, a price-taking equilibrium cannot be blocked by a coalition of traders, and the competitive equilibrium is robust. The argument that a process of bargaining among a large number of individuals produces a result which

replicates a price-taking equilibrium, allowing for the free flow of information using recontracting and enabling coalitions of traders to form and break up, is an important result that is far from intuitively obvious. The recontracting process can be said to represent a competitive process, and the contract curve shrinks essentially because of the competition between suppliers of the same good, although it is carried out in a barter framework in which explicit prices are not used (although rates of exchange are equivalent to price ratios).

The price-taking equilibrium, in contrast, does not actually involve a competitive process. Individuals simply believe that they must take market prices as given and outside their control. They respond to those prices without any reference to other individuals. However, the result is that the price-taking equilibrium looks just like a situation in which all activity is perfectly co-ordinated.

Edgeworth (1881: 28) also derived, from his indifference curves, the reciprocal demand curve, or offer curve, of each individual, although such curves (introduced by Marshall as diagrammatic representations of Mill's model of international trade) were then called 'demand-and-supply curves'. Edgeworth's contribution was to define offer curves in terms of indifference curves, 'the locus of the point where lines from the origin touch curves of indifference' (ibid.: 113). He mentioned them only briefly in the text (ibid.: 39), but the lack of emphasis is understandable, since in imperfect competition they are not relevant. When there is a lack of competition, giving rise to indeterminacy, there is nothing to ensure that individuals will trade on their offer curves and, as Edgeworth argued, 'the conceptions of demand and supply at a price are no longer appropriate' (ibid.: 31). It is this general preference, in favour of the analysis of barter in non-competitive situations, to which Marshall later objected.

7 The Utilitarian Calculus

Having shown how indeterminacy can be removed by increasing the number of traders, Edgeworth turned to consider the role of arbitration in resolving the conflict between traders, in a 'world weary of strife' (ibid.: 51). The need for arbitration was stated by Jevons as follows:

> The dispositions and force of character of the parties…will influence the decision. These are motives more or less extraneous to a theory of economics, and yet they appear necessary considerations in this problem. It may be that indeterminate bargains of this kind are best arranged by an arbitrator or third party (Jevons 1957: 124–125).

Edgeworth's statement of the same point was as usual rather less prosaic: 'The whole creation groans and yearns, desiderating a principle of arbitration, and end of strifes' (Edgeworth 1881: 51).

The principle of arbitration examined by Edgeworth was, not surprisingly, the utilitarian principle, which he had earlier used to examine optimal distribution. However, the new context of indeterminacy led him to a deeper justification of utilitarianism as a principle of distributive justice. Having arrived at this new link between "impure" and "pure" utilitarianism, Edgeworth had only to reorientate his earlier analysis of optimal distribution discussed above. His argument involved two steps. First, he showed that the principle of utility maximisation places individuals on the contract curve, because the first-order conditions are equivalent to the tangency of indifference curves. He exclaimed, 'It is a circumstance of momentous interest that one of the in general indefinitely numerous settlements between contractors is the utilitarian arrangement…the contract tending to the greatest possible total utility of the contractors' (ibid.: 53).

Edgeworth recognised that this result was not sufficient to justify the use of utilitarianism as a principle of arbitration. It is only a necessary condition of a principle of arbitration that it should place the parties somewhere on the contract curve. His justification of utilitarianism was as follows:

> Now these positions lie in a reverse order of desirability for each party; and it may seem to each that as he cannot have his own way, in the absence of any definite principle of selection, he has about as good a chance of one of the arrangements as another…both parties may agree to commute their chance of any of the arrangements for…the utilitarian arrangement (ibid.: 55).

The important point about this statement is that Edgeworth viewed distributive justice in terms of choice under uncertainty. He argued that the contractors, faced with uncertainty about their prospects, would choose to accept an arrangement along utilitarian lines. A crucial component of this argument, also clearly stated by Edgeworth in this quotation, is the use of equal a priori probabilities. The importance to him of this new justification of utilitarianism cannot be exaggerated. Indeed, the whole of *Mathematical Psychics* is imbued with a feeling of excitement generated by his discovery of a justification based on a social contract. This provided the crucial link between "impure" and "pure" utilitarianism in a more satisfactory way than his earlier appeal to evolutionary forces.

Edgeworth believed that he had provided an answer to an age-old question, stating, 'by what mechanism the force of self-love can be applied so as to

support the structure of utilitarian politics, neither Helvetius, nor Bentham, nor any deductive egoist has made clear' (ibid.: 128). Nevertheless, this argument was neglected until restatements along similar lines were made by Harsanyi (1953, 1955) and Vickrey (1960). The maximisation of expected utility, with each individual taking the a priori view that any outcome is equally likely, was shown to lead to the use of a social welfare function which maximises the sum of individual utilities. This approach is now described as "contractarian neo-utilitarianism".

In discussing the utilitarian solution as a principle of arbitration in indeterminate contract, Edgeworth did not indicate in 1881 that the utilitarian solution of maximum total utility could specify a position making one of the parties worse off than in the no-trade situation. This was later made explicit when, after proposing arbitration along utilitarian lines, he added, 'subject to the condition that neither should lose by the contract' (Edgeworth 1925, ii: 102). This possibility depends largely on the initial endowments of the individuals.

8 Later Work in Economics

After the publication of *Mathematical Psychics*, Edgeworth concentrated increasingly on mathematical statistics, in particular, on the problem of statistical inference but, following his appointment to the Drummond Chair at Oxford, he again made important contributions to economics, although this work mainly involved reactions to, and discussions arising from, the work of other authors. This section discusses a number of these issues.

8.1 Demand and Exchange

In the *Principles of Economics* (1890: Appendix F) Marshall included a brief discussion of Edgeworth's analysis of barter, and produced a figure showing the contract curve. During the following year, in the course of a review written in Italian, Edgeworth criticised Marshall for not having dealt sufficiently with the problem of indeterminacy. The basic problem was that Marshall, using a model in which a series of trades are allowed to take place at disequilibrium prices, believed he had shown that prices eventually settle at the price-taking equilibrium. However, the argument was not transparent. The adjustment process involves moving from the initial endowment point in a series of trades, where trading at "false" prices is allowed at each step. The

process must conclude with both individuals at a point on the contract curve. A feature of the process is the assumption that each stage or iteration of the sequence involves Pareto improvements: individuals trade only if it makes them better off. Furthermore, it involves trading at the "short end" of the market, that is, the minimum of supply and demand. This arises from the impossibility of forcing any individual either to buy or sell more than desired at any price. Starting from a disequilibrium price, trade takes place at the short end of the market, and endowments change. At the next trading stage, the price of the good with an excess supply must be lowered. At each trade, there is a Pareto improvement. The combination of Pareto-efficient moves at each stage, combined with an adjustment process such that an excess supply leads to a price reduction, and vice versa, produces a stable process that converges to an equilibrium somewhere on the contract curve. Interestingly, this type of sequence of disequilibrium trades was later used by Launhardt in examining total utility and price-taking (see Creedy 1994b).

Marshall believed that his assumption of an additive utility function, combined with the assumption that the marginal utility of one good is constant for both individuals, guaranteed a determinate price, if the good having constant marginal utility is money. This case was mentioned by Edgeworth (1925, ii: 317, fn. 1). The contract curve is a straight line parallel to the axis for the good with constant marginal utility, along which the rate of exchange is constant. So the equilibrium price does not depend on the sequence of trades. However, Edgeworth's point was that the total amount spent on the good remains indeterminate.

There was a later disagreement between Marshall and Edgeworth over the so-called Giffen good. In a book review, Edgeworth argued that 'Even the milder statement that the elasticity of demand for wheat *may* be positive, though I know it is countenanced by high authority, appears to me so contrary to a priori probability as to require very strong evidence' (Edgeworth 1909: 105; italics in original). The authority was of course Marshall (1890: 132), who replied directly to Edgeworth that I don't want to 'argue ... But... the matter has not been taken quite at random' (Marshall quoted in Pigou 1925: 438). Marshall gave a numerical example involving a journey travelled by two methods, where the distance travelled by the cheaper and slower method must increase when its price increases; for details, see Creedy (1990).

It was mentioned above that Edgeworth introduced the generalised utility function. An implication is that it allows for complementarity, although he did not explicitly consider this in 1881. It was used by Edgeworth in his paper on the pure theory of monopoly. The concept amounts to what is now called gross complementarity, defined in terms of cross-price elasticities. The first

major criticism came from Johnson (1913), who pointed out that the criterion is not invariant with respect to monotonic transformations of the utility function. His treatment was extended by Hicks and Allen (1934), so that the modern definition involves net complements in terms of compensated price changes. There is no symmetry between gross substitutes and complements as only the matrix of (compensated) substitution elasticities is assumed to be symmetric.

8.2 Monopoly and Oligopoly

In a paper first published in Italian in 1897, and not translated until the collected *Papers* (Edgeworth 1925), Edgeworth examined several problems relating to monopoly. He began with Cournot's (1838) example of the "source minerale" in which there are "two monopolists" (i.e. duopolists), each owning a spring of mineral water. It would be natural for Edgeworth to expect an indeterminate price in this "small numbers" context. Cournot arrived at a determinate solution for price and output, but Edgeworth showed that 'when two or more monopolists are dealing with competitive groups, economic equilibrium is indeterminate' (Edgeworth 1925, i: 116). He argued that '[A]t every stage…it is competent to each monopolist to deliberate whether it will pay him better to lower his price against his rival as already described, or rather to raise it to a higher…level for that remainder of customers of which he cannot be deprived by his rival' (ibid.: 120).

Edgeworth went on to define (what are now called) reaction curves and isoprofit lines, for variations in prices. However, it was not until Bowley's (1924) discussion that these matters began to be presented in a more transparent manner.

Edgeworth then considered the case of complementary demand within the context of bilateral monopoly, where the two goods are demanded in fixed proportions for use in the production of a further article. A feature is that he wrote the equations of the reaction curves and explicitly dealt with what came to be called conjectural variations, reflecting the extent to which one duopolist is expected to change price in response to changes made by the second duopolist. In discussing this problem, Edgeworth also introduced the concept of a "saddle point", which he called the "Hog's Back", indicating its importance for stability.

Walras (1874: 225) had introduced the concept of the entrepreneur who neither gains nor loses. This result applied only to the competitive equilibrium, where there are no incentives for entrepreneurs to enter any industry.

This does not of course mean that there are no profits, in the accounting sense, since the returns to homogeneous units of inputs of organisation and management services are subsumed in the costs of the firm. Edgeworth's criticisms of this concept of the no-profit entrepreneur, reproduced in his *Papers* (Edgeworth 1925, i), recognised that with Walras's assumptions there is nothing illogical about the argument. The theory simply means that nothing remains, 'after the entrepreneur has paid a normal salary to himself' (ibid.: 26). Furthermore, 'If [the general expenses] are taken into account, the argument becomes a fortiori. For why should not a substantial remuneration for the entrepreneur be included in the general expenses of the business' (Edgeworth 1925, ii: 469–470). Edgeworth's difference with Walras was to some extent "only verbal", but he was also unhappy with the idea that entrepreneurship is homogeneous and divisible.

8.3 Surveys of Taxation and International Values

In the 1890s, Edgeworth produced two surveys of considerable importance. These surveys, of the pure theory of taxation and of the pure theory of international values, were both published in the *Economic Journal* and subsequently reproduced (with alterations) in his *Papers* (Edgeworth 1925, ii). Each survey consisted of three separate parts. They represent his most serious attempts to produce any kind of synthesis of a branch of economic literature. Edgeworth began his taxation survey with the statement that 'The science of taxation comprises two subjects to which the character of pure theory may be ascribed; the laws of incidence, and the principle of equal sacrifice' (ibid.: 64). He then considered a variety of special cases and contexts of tax incidence. The framework for incidence analysis is the simple partial equilibrium approach, still used in many basic textbooks, in which the incidence depends on the relative values of supply and demand elasticities.

The approach to incidence analysis actually stemmed from Jenkin (1871/1872), who suggested that in general the price of the taxed good will either remain constant (in the extreme case of inelastic supply) or will increase. However, this result ignores interrelationships among commodities. Edgeworth showed that when such interrelationships are considered, there are circumstances in which the price of the taxed good will fall. When discussing this "paradox", Edgeworth reproduced his argument, which had in fact been explored in more detail in his paper on monopoly, published in Italian in the same year, 1897. Edgeworth first stated his "tax paradox" in the following terms:

> [W]hen the supply of two or more correlated commodities—such as the carriage of passengers by rail first class or third class—is in the hands of a single monopolist, a tax on one of the articles—e.g. a percentage of first class fares—may prove advantageous to the consumers as a whole … The fares for all the classes might be reduced (Edgeworth 1925, i: 139).

Edgeworth regarded this result as an example of a situation where 'the abstract reasoning serves as a corrective to what has been called the "metaphysical incubus" of dogmatic laisser faire' (ibid.; see also Edgeworth 1925, ii: 93–94). Essentially the two commodities must be substitutes in consumption and production, and the result arises partly because the monopolist has an incentive to increase the supply of the untaxed commodity. Edgeworth (ibid.: 63) also recognised that the result could occur in competitive markets. As with many of Edgeworth's original results, this tax paradox was not a subject of continuous development. Its main practical importance perhaps arises from the fact that it attracted the attention of Hotelling (1932); for further details, see Creedy (1988).

Edgeworth discussed the various sacrifice theories of the distribution of the tax burden, giving qualified support for progressive taxation. His attitude to taxation was similar to that of the major classical economists in that he rejected a benefit approach, on the argument that taxation is not an economic bargain governed by competition. Thus in his view the problem was to determine 'the distribution of those taxes which are applied to common purposes, the benefits whereof cannot be allocated to particular classes of citizens' (Edgeworth 1925, ii: 103). A principle of justice is thus required. His approach marks a crucial stage in the transition towards a welfare economics view of public finance, rather than using a special set of tax maxims such as those laid down by Adam Smith.

Not surprisingly, Edgeworth (1925, ii: 102–103) argued along neo-contractarian lines that the utilitarian arrangement would be accepted by individuals who are uncertain of their own prospects and take an equal a priori view of the probabilities. He suggested that

> each party may reflect that, in the long run of various cases…of all the principles of distribution which would afford him now a greater, now a smaller proportion of the sum-total utility obtainable…the principle that the collective utility should be on each occasion a maximum is most likely to afford the greatest utility in the long run to him individually.

Having established the use of utilitarianism as a principle of distribution justice, Edgeworth (ibid.: 103) succinctly argued that maximisation of total net utility reduces to the condition that the total disutility should be a minimum, and hence the marginal disutility of each taxpayer should be the same.

The implication is that if all individuals have the same cardinal utility function, after-tax incomes would be equalised. Edgeworth also recognised that if there is considerable dispersion of pre-tax incomes relative to the total amount of tax to be raised, where there is, 'not enough tax to go round' (ibid.), the equi-marginal condition cannot be fully satisfied unless there is a "negative income tax" which raises the incomes of the poorest individuals to a common level. Thus, 'the acme of socialism is for a moment sighted' (ibid.: 104). However, Edgeworth immediately considered the practical limitations to such high progressive taxation. The following quotation illustrates one of his favourite metaphors, his respect for Henry Sidgwick, his attitude to authority, his views on utilitarianism and the applicability of pure theory, and of course his unmistakable style:

> In this misty and precipitous region let us take Professor Sidgwick as our chief guide. He best has contemplated the crowning height of the utilitarian first principle, from which the steps of a sublime deduction lead to the high tableland of equality; but he also discerns the enormous interposing chasms which deter practical wisdom from moving directly towards that ideal (ibid.).

Among the various limitations, Edgeworth noted differences in individual utility functions, population effects, the disincentives to work, growth of culture and knowledge, savings, and of course the problem of evasion.

Edgeworth's survey of the pure theory of international values contributed to a change of emphasis in the approach to trade theory, despite the fact that it contained few original analytical contributions. Indeed, he said that 'Mill's exposition of the general theory is still unsurpassed' (Edgeworth 1925, ii: 20), and acknowledged further that '[W]hat is written…after a perusal of [Marshall's] privately circulated chapters…can make no claim to originality' (ibid.: 47). Edgeworth saw trade theory as an application of the general theory of exchange:

> The fundamental principle of international trade is that general theory…the Theory of Exchange…which…constitutes "the kernel" of most of the chief problems in economics. It is a corollary of the general theory that all the parties to a bargain look to gain by it … This is the generalised statement of the theory of comparative cost (ibid.: 6).

Thus the gains from trade are analogous to the gains from exchange in simple barter. Hence, trade theory is one more application of the general method of *Mathematical Psychics*. In directly applying the theory of exchange to that of trade, Edgeworth was content to use community indifference curves without clearly specifying how aggregation might be carried out. He said only that, 'By combining properly the utility curves for all the individuals, we obtain what may be called a collective utility curve' (ibid.: 293–294).

One of Edgeworth's criticisms of Mill (1848) was that the latter took as his measure of the gain from trade the change in the ratio of exchange of exports against imports. Thus Mill in this case 'confounds "final" with integral utility' (ibid.: 22). The same point had in fact been made by Jevons (1957: 154–156). However, Edgeworth, while preferring total utility, admitted that Mill was not otherwise led to serious error in using his own measure.

Edgeworth's survey was wide-ranging, though for later developments the most interesting parts are concerned with his elucidation of Mill's 'recognition of the case in which an impediment may be beneficial—or an improvement prejudicial—to one of the countries' (Edgeworth 1925, ii: 19). These cases would now be discussed under the headings of "optimal tariff" and "immiserising growth". In the case of an optimal tariff, a country acts as monopolist and imposes a price which enables that country to attain its highest indifference curve, subject to the other country's offer curve. However, this position is not on the contract curve. The detailed specification of the optimum tariff in terms of elasticities had to wait until Bickerdike (1906), Pigou (1908) and the later revivals of interest in the 1940s. Edgeworth's judgement of Bickerdike was that he had 'accomplished a wonderful feat. He has said something new about protection' (Edgeworth 1925, ii: 344).

Edgeworth did not support the use of such tariffs in practice. He acknowledged the possibility of retaliation. Also, for one nation to benefit itself at the expense of others 'is contrary to the highest morality … But in an abstract study upon the motion of projectiles in vacuo, I do not think it necessary to enlarge upon the horrors of war' (ibid.: 17, fn. 5). The 'highest morality' was, of course, the principle of utilitarianism.

9 Conclusion

After a varied beginning to his career, Edgeworth begin working and writing in economics when in his mid-thirties. In common with the majority of neoclassical economists, he pursued an academic career as a professor of

economics. Indeed, in a period which saw the rapid and widespread professionalisation of the subject, Edgeworth's academic position in Oxford was regarded as second only to that of Alfred Marshall. In spite of his wide range of reading and sympathies, Edgeworth's work was virtually all addressed to his fellow professional economists. He was uncompromising in his view that economics is a difficult subject offering only remote and nearly always negative policy advice. It may be said that his work was addressed to a small number of "fellow travellers" in the rarefied atmosphere of the "higher regions" of pure theory. However, Edgeworth imposed no geographical limitations and, with his considerable linguistic skills and international sympathies, was in contact with the majority of leading economists around the world.

The distinguishing feature of the neoclassical "revolution" was its emphasis on exchange as the central economic problem. The success of this shift of focus from production and distribution to exchange was closely associated with the fact that it had as its foundation a model based on utility maximisation. This allowed for a deeper treatment of the gains from exchange and the wider considerations of economic welfare. Schumpeter summarised the point by stating that utility analysis must be understood in terms of exchange as the central 'pivot' and 'the whole of the organism of pure economics thus finds itself unified in the light of a single principle' (Schumpeter 1954: 913). This is indeed the context in which Edgeworth's work in economics must be seen. Schumpeter's remark is merely a more prosaic expression of Edgeworth's view quoted above that '"*Méchanique Sociale*" may one day take her place along with "*Méchanique Celeste*", throned each upon the double-sided height of one maximum principle'. The central theme of Edgeworth's work is also clear in his revealing statement, taken from his Presidential Address to Section F of the Royal Society, that, 'It may be said that in pure economics there is only one fundamental theorem, but that is a very difficult one: the theory of bargain in a wide sense' (Edgeworth 1925, ii: 288).

With this perspective, the thread running through all Edgeworth's work in economics can be seen. His earlier mathematical analysis, of the implications of utilitarianism for optimal distribution, laid the foundation for his future research. The transition from *New and Old Methods of Ethics* to *Mathematical Psychics* was not a shift in major preoccupations but rather a change of emphasis. For Edgeworth, distribution was seen as an important concomitant of exchange, so that the analysis of contract became central. Edgeworth's emphasis on the indeterminacy—the inability of utility maximisation alone to determine the rate of exchange, only a range of efficient exchanges—which results

from the existence of a small number of traders, led to his path-breaking analysis of the role of numbers in competition, along with the efficiency properties of competitive equilibria.

The analysis of the utilitarian objective as an arbitration rule led Edgeworth directly to his social contract argument in explaining the acceptance of utilitarianism as a principle of social justice. It was the realisation of this justification of utilitarianism, using his newly developed analytical tools, which generated the excitement that is evident in his first work in economics. While *Mathematical Psychics* developed the techniques of indifference curves and the contract curve within the eponymous box diagram—tools which are now ubiquitous in economic analysis—Edgeworth himself was driven mainly by his ability to link the analysis of private contracts in markets to that of a social contract in which utilitarianism is the "sovereign principle". The integration of his analysis of barter, and the effects of the introduction of additional traders into the market, with the demonstration that the utilitarian arrangement prescribes a point on the contract curve of efficient exchanges and is acceptable to risk-averse traders, was to Edgeworth nothing short of momentous.

The results are of course highly abstract. In discussing their ultimate value, he suggested that:

> Considerations so abstract it would of course be ridiculous to fling upon the flood-tide of practical politics … It is at a height of abstraction in the rarefied atmosphere of speculation that the secret springs of action take their rise, and a direction is imparted to the pure foundation of youthful enthusiasm whose influence will ultimately affect the broad current of events (Edgeworth 1881: 128–129).

The intellectual pleasure derived from being able to draw together so many different subjects of analysis, and strands of Edgeworth's enormous range of learning, is clearly evident. However, it is precisely this wide field of vision, combined with the technical level and idiosyncratic style of writing, which made *Mathematical Psychics* so difficult for his contemporaries, and which continue to make the book seem so strange to the modern reader. When discussing, in *Mathematical Psychics*, the results of barter among a large number of competitors in a market, Edgeworth borrowed (without attribution) a line from Alexander Pope's *Essay on Man*, and described the market as, 'A mighty maze! but not without a plan'. This could just as appropriately be applied to Edgeworth's many contributions to economics.

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