

Chapter 31

Multiple-Model Fault Diagnosis Method for Gas Turbine Based on Soft Switch



Yunpeng Cao, Kehui Zeng, Shuying Li, Fengshou Gu, Yuandong Xu, and Bo He

Abstract Fault diagnosis based on a multiple-model (MM) approach is an analytical redundancy method. In this paper, we mainly focus on the model switch optimizing problem of the MM method. First, a MM method for gas turbine gas path fault diagnosis was proposed, and the gas turbine state space models are established based on analytical linearization which can simulate the nonlinear dynamic characteristics at the operating points. Then model soft switch method was proposed based on recursive Bayesian to solve the problem that the established state space model is accurate merely in the vicinity of the operating points, and generates the combined generic model for the full operating condition and make it smoother. Finally, the fault simulation was carried out on a marine gas turbine which shows that the proposed method can diagnose both single gas path fault and multiple gas path faults.

31.1 Introduction

Multiple-model (MM) provides the architecture for a bank of estimators or filters for isolation and identification of faults. It was first proposed by Magil [1] in the study of the optimal adaptive estimation of random processing of samples. In gas turbine related applications, Maybeck [2] were the first to apply MM methods based on the extended Kalman filter (EKF) to detect gas turbine actuator failure and sensor failure, then applied it on gas path fault diagnosis. In order to detect, isolate, and estimate gas turbine gas path fault, an IMM-GLR approach based on interacting

Y. Cao (✉) · K. Zeng · S. Li

College of Power and Energy Engineering, Harbin Engineering University, Harbin, China
e-mail: caoyunpeng@hrbeu.edu.cn

F. Gu · Y. Xu

Centre for Efficiency and Performance Engineering, University of Huddersfield,
Huddersfield, UK

B. He

College of Automation, Harbin Engineering University, Harbin, China

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MM and generalized likelihood ratio estimation were developed [3]. To overcome the coupling effect between the gas path fault and the sensor fault, an MM-based detection and estimation scheme for gas turbine sensor and gas path fault diagnosis is proposed [4]. In order to suitable for systems with significant gain variation due to nonlinearity, Strutzel FAM [5] introduces a new multi-model state-space formulation called simultaneous multi-linear prediction (SMLP), this method presents more accurate results than the use of single linear models while keeping much of their numerical advantages and their relative ease of development.

The MM based fault diagnosis method mainly needs to solve three problems, including the determination of the model set, the design of the filter and the estimation fusion. The determination of the model set is the main difficulty and key point, and it is also the basis for ensuring the accuracy of diagnosis. It is divided into two parts, firstly the establishment of the model set, followed by the selection and switching of multiple models in the running process. It is well known that one of the main difficulties in the modelling of gas turbine systems is the determination of the mathematical laws that describe these systems. However, obtaining a linear model simplifies the analysis of their dynamic behaviours, as well as the design of their control and surveillance strategies. Hadji [6] deals with a linearization strategy of the non-linear model, developed a novel method for modelling nonlinear dynamic variables of a gas turbine, the effective and equivalent linear approximation of its nonlinear model is realized, and the nonlinear dynamic model identification around the operating point obtained from the actual data is proposed. Qingcai Yang [7] proposed a nonlinear mode set automatic generation method that enables automatic generation of the modes of each level in the MM model. Pourbabae [8] extended the MM method based on multiple hybrid Kalman filters which represent an integration of a nonlinear mathematical model of the system with a number of piecewise linear models.

In this paper, the MM gas path fault diagnosis method for gas turbine based on the soft switch method is studied. In Sect. 31.2, the MM method is briefly introduced, including the principle of Kalman filter and the flow chart of gas path diagnosis based on the MM method. In Sect. 31.3, a method for establishing a general linear model of gas turbine based on piecewise linearization is studied. In Sect. 31.4, the soft switch method is proposed to generates the combined generic model at the full operating process and makes it smoother. Section 31.5 is the single fault and multiple faults simulation testing, and the conclusion is presented in Sect. 31.6.

31.2 Multiple-Model Gas Path Diagnosis Method

Figure 31.1 shows the gas turbine gas path diagnosis based on the MM method [9]. The basic idea of the MM method is to solve a current stochastic system [10, 11]. In order to estimate its current operating state, a set of models is constructed by establishing a hypothetical model of the current possible operating state of the

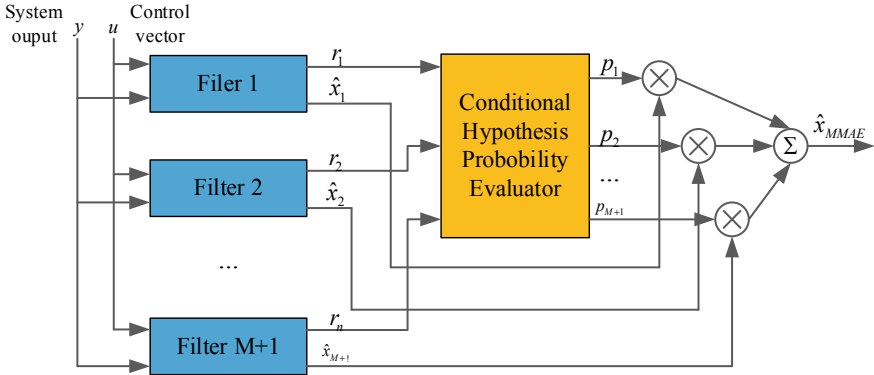


Fig. 31.1 Gas path diagnosis based on MM method

system, and the corresponding filtering is designed for each hypothetical model. By comparing the size of the filtered residuals, the degree of approximation of each hypothesis model in the model set with the actual operating state is characterized, and the filtering residuals are transformed into probability density by probability density function and the conditional probability of each hypothesis model is calculated by the recursive solution. The state estimate of each hypothesis model is fused as the current actual state estimate by the obtained conditional probability.

The MM method is characterized by the $(M + 1)$ sets of filters running in parallel, and M is the number of faults to be detected. The fault parameters a_i ($i = 1, 2, \dots, M + 1$) represent the mode of the system, and a_1 is the healthy mode. Assuming that the conditional probability $P_i(k)$ is the probability that the corresponding gas turbine operating state is a_i when the measurement parameter is Y_k at time k , that is:

$$P_j(k) = \Pr[a = a_j | Y(k) = Y_k] \tag{1}$$

The corresponding conditional hypothesis probability can be recursive by the Bayesian method using the value of the previous time and the conditional probability density function of the current measurement as shown in Eq. (2).

$$P_j(k) = \frac{f_{y(k)|a, Y(k-1)}(y_i | a_j, Y_{k-1}) P_j(k-1)}{\sum_{h=1}^M f_{y(k)|a, Y(k-1)}(y_h | a_h, Y_{k-1}) P_h(k-1)} \tag{2}$$

where $f_{y(k)|a,Y(k-1)}(y_i|a_j, Y_{k-1})P_j(k-1)$ is the currently measured Gaussian density function, and it is given by the following equation:

$$f_{y(k)|a,Y(k-1)}(y_i|a_j, Y_{k-1})P_j(k-1) = \frac{1}{(2\pi)^{q/2} \sqrt{|S_c^j(k)|}} \exp \left[\frac{-1}{2} (\gamma_c^j(k))^T (S_c^j(k))^{(-1)} (\gamma_c^j(k)) \right] \tag{3}$$

where $j = 1, 2, \dots, (M + 1)$, $\gamma_c^j(k)$ and $S_c^j(k)$ is the residual and covariance matrix of the $(M + 1)$ combination models associated with the fault parameter. The state of the system can be detected and isolated based on the maximum value of $P_j(k)$, whether it is healthy or faulty, and then we can locate the specific failure.

The selection of the filter is also very important. For the linear model used in this paper, the Kalman filter is selected for the calculation of multiple models. Equation (4) shows the general form of the state equation of linear discrete system:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + w_k \\ y_k = C_k x_k + D_k u_k + v_k \end{cases} \tag{4}$$

where x_k is the state vector, y_k is the output vector, and u_k is the control vector. There are specific definitions for the x, y, u , respectively, as described in Sect. 31.3. w and v are independent of the Gaussian white noise vector. While w denote the process noise and v denote the measurement noise, assuming they are Gaussian white noise and the covariances are Q and R , respectively.

The Kalman filter is applied to the analytical linearization model of the two-shaft gas turbine, as shown in the equation:

$$\begin{cases} \hat{x}_k = A_k \cdot \bar{x}_{k-1} + B_k \cdot u_k \\ \hat{y}_k = C_k \cdot \hat{x}_k + D_k \cdot u_k \end{cases} \tag{5}$$

where \hat{x} is the ‘‘predicted value’’, \bar{x} is the ‘‘update value’’.

$$\begin{cases} \bar{x}_k = \hat{x}_k + K_k(y - \hat{y}_k) \\ K_k = P_k^- C^T (C P_k^- C^T + R)^{-1} \\ P_k = P_k^- - K_k C P_k^- \\ P_k^- = A P_{k-1} A^T + Q \end{cases} \tag{6}$$

where K_k is the Kalman gain. $A, B, C,$ and D are the linear model matrix coefficients. P is the error covariance matrix.

31.3 Gas Turbine State Space Model

Since the two-shaft gas turbine is a complex nonlinear system, and its parameters have a strict dynamic relationship, so we can use the analytical method to obtain a linear model at operating points, then build the overall state space model [12].

The main components of the two-shaft gas turbine studied in this paper are shown in Fig. 31.2. The module is divided into six modules [13], namely, compressor, combustion chamber, turbine, compressor turbine and power turbine between the volume module (volume module 1), power turbine, the power turbine volume module (volume module 2), these modules have clear physical meanings and the corresponding entities, making the system module more intuitive.

The gas turbine continuous-time nonlinear model is:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \tag{7}$$

where x is the state variable, y is the output variable, u is the control variable. control variable u :

$$u = [wf \quad H_{mc} \quad H_{\eta c} \quad H_{mh} \quad H_{\eta h} \quad H_{mp} \quad H_{\eta p}]^T \tag{8}$$

output variable y :

$$y = [n_l \quad n_p \quad t_2 \quad p_2 \quad t_4 \quad p_4 \quad t_5 \quad p_5]^T \tag{9}$$

state variable x :

$$x = [n_l \quad n_p \quad t_3 \quad p_3 \quad p_4 \quad p_5]^T \tag{10}$$

Where $f(\bullet)$ and $h(\bullet)$ are system nonlinear functions. n_l is turbine speed, n_p is power turbine speed, t_2 is compressor outlet temperature, t_3 is combustion chamber outlet temperature, p_2 is compressor outlet pressure, p_3 is combustion chamber outlet

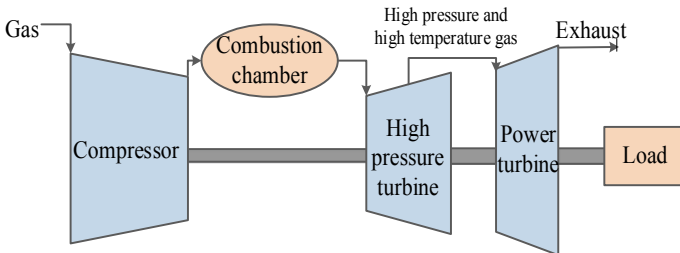


Fig. 31.2 Basic compositions of the two-shaft gas turbine

pressure, t_4 is turbine outlet temperature, p_4 is turbine outlet pressure, t_5 is the power turbine outlet temperature, and p_5 is the power turbine outlet pressure. H_{mc} , $H_{\eta c}$, H_{mlb} , $H_{\eta lb}$, H_{mp} and $H_{\eta p}$ are the fault parameters of the control variables. The corresponding types of faults are compressor mass flow reduction, compressor efficiency reduction, turbine mass flow reduction, turbine efficiency reduction, power turbine mass flow reduction, Turbine efficiency reduction.

The piecewise linearization is considered in this paper [14], so the steady-state deviation can be defined near the steady-state operating point. Linearizing the above-mentioned nonlinear model at a steady-state operating point $S_p : \{x_p, y_p, u_p\}$:

$$\begin{aligned} \Delta \dot{x} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + D\Delta u \end{aligned} \tag{11}$$

Where A, B, C, D are the Jacobian matrices, and The matrix coefficients of each module are solved by using the partial derivative method, replacing the intermediate variable and extracting common factor method.

$$A = \frac{\partial f(x, u)}{\partial x}, B = \frac{\partial f(x, u)}{\partial u}, C = \frac{\partial h(x, u)}{\partial x}, D = \frac{\partial h(x, u)}{\partial u}$$

31.4 Model Soft Switch Method

In order to smoothen the general model of the combined run-wide generation, we use soft switch between piecewise linear models. Each linear model is only valid around its associated operating point. Although the range of work has been increased by using a non-linear model rather than a steady-state variable, none of the piecewise linearization models is valid throughout the input variable range. Thus, each piecewise linearization model can obtain the validity function based on its normalized weight obtained by the Bayesian formula.

In this paper, the Bayesian method is used to calculate the weight. For this reason, the likelihood sequence and the covariance matrix generated by the Kalman filter bank are used to calculate the likelihood function of the j^{th} sensor pattern of the i^{th} operating region as follows:

$$\begin{aligned} \gamma^{(ij)}(k) &= Y(k) - \hat{Y}^{(ij)}(k) \\ S^{(ij)}(k) &= \text{cov}(\gamma^{(ij)}(k)) \end{aligned} \tag{12}$$

$$f^{(ij)}(\gamma^{(ij)}(k)) = \frac{1}{(2\pi)^{q/2} \sqrt{|S^{(ij)}(k)|}} \times \exp \left[\frac{-1}{2} (\gamma^{(ij)}(k))^T (S^{(ij)}(k))^{(-1)} (\gamma^{(ij)}(k)) \right] \tag{13}$$

Where $i = 1, \dots, L$, $j = 1, \dots, (M + 1)$; and L denotes the number of operating points, M denotes the number of faults to be detected, and q denotes the dimension of the measurement parameter. The message sequence $\gamma^{(i,j)}(k)$ is generated by the Kalman filter bank, whose mean is zero and is a Gaussian white noise process, and the covariance matrix $S^{(i,j)}(k)$ is calculated numerically. The normalized weight for the j^{th} sensor is updated recursively using the Bayes formula as follows:

$$w^{(i,j)}(k) = \frac{f^{(i,j)}[(\gamma^{(i,j)}(k))]w^{(i,j)}(k-1)}{\sum_{i=1}^L f^{(i,j)}[(\gamma^{(i,j)}(k))]w^{(i,j)}(k-1)} \quad (14)$$

The weights calculated above should be kept bounded to avoid them close to zero using the following formula:

$$\begin{aligned} \text{if } w^{(i,j)}(k) > \alpha, \text{ then } w^{(i,j)}(k) &= w^{(i,j)}(k) \\ \text{if } w^{(i,j)}(k) \leq \alpha, \text{ then } w^{(i,j)}(k) &= \alpha \end{aligned} \quad (15)$$

Where α is the design parameter, which is obtained by experiment, and its size has an effect on the speed of the model switch. The larger values of α result in the faster switch, while smaller values will result in slower conversion between piecewise linearization models. Here, take $\alpha = 0.001$ to switch the model. After calculating the normalized weight, the weighted new message sequence and the weighted covariance matrix of the general combinatorial model are designed by its piecewise linearization model and the associated normalized weights as follows:

$$\begin{aligned} \gamma_c^j(k) &= \sum_{i=1}^L w^{(i,j)}(k) \gamma^{(i,j)}(k) \\ S_c^j(k) &= \sum_{i=1}^L [w^{(i,j)}]^2 S^{(i,j)}(k) \end{aligned} \quad (16)$$

The process above is called the soft switch between piecewise linearization models. Where $\gamma_c^{(j)}(k)$ and $S_c^{(j)}(k)$ represent the weighted new message sequence and the covariance matrix for the healthy and faulty sensor modes. Thus, $(M + 1)$ Kalman filters run in parallel to form a multiple Kalman filter structure for diagnose.

At this point, the problem of determining the model set has been solved. The MM diagnose method based on the soft switch is applied to the two-shaft gas turbine. The Bayesian algorithm is used to design the soft switch, and the fault or health model of each operation mode is obtained. The probability density of each Kalman filter is obtained by using the hypothesis test algorithm, and the diagnose is realized by finding the maximum value.

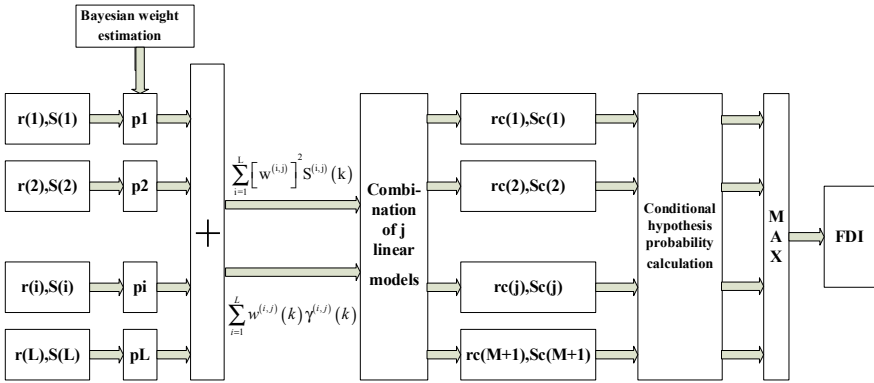


Fig. 31.3 Flow chart based on MM method and soft switch

Based on the design of Kalman filter banks and the application of soft switch in the MM approach, the flow chart based on the MM method and soft switch is showed in Fig. 31.3.

In practical applications, there are often more than one failure. The MM based fault diagnosis method established in the previous article is not suitable for multiple fault situations. To overcome this problem, we apply a layered structure to multiple fault diagnosis method. In this approach, a set of Kalman filters operate in parallel to detect and isolate the first fault and then activate the corresponding second-order Kalman filter, and the data-model it records is also updated to detect and isolate the second fault. Compared with the method of running all the models in parallel (single failure model and multiple failure models), this method can reduce costs and save time.

31.5 Case Study

The fault types that are simulated in this paper are all summarized as follows. The simulation tests of 3 different cases with a single fault and multiple faults were carried out respectively. And the single fault cases are divided into noiseless and noisy effects. $H_{mc} = 0.01$ in the table indicates the compressor mass flow fault with an amplitude of 0.01 and so on, and H_v indicates the noise fault parameter. Table 31.1

In this paper, p_1 to p_7 represent the probability of health status and the probability of fault types corresponding to H_{mc} , H_{nc} , H_{mh} , H_{nh} , H_{mp} and H_{np} respectively. The definition of those fault parameters are shown in Sect. 31.3.

Table 31.1 Simulation cases

	H_{mc}	$H_{\eta c}$	H_{mh}	$H_{\eta h}$	H_{mp}	$H_{\eta p}$	H_v
Case 1	0.02	0.02	0.02	0.02	0.02	0.02	0
Case 2	0.05	0.05	0.05	0.05	0.05	0.05	0.01
Case 3	0.01	0.01	0	0	0	0	0

31.5.1 Single Fault

(1) Single fault case without noise

In this section, we discuss the diagnosis result in the case of fault amplitude was 0.02, and the relative fuel flow = 0.95 (select the transition conditions here, so that the soft switch can detect and isolate the faults more stable). Figure 31.4 shows the fault amplitude of 0.02, and at $t = 20$ s, a failure occurred. Figure 31.4 (a) for the compressor mass flow (p2), Fig. 31.4 (b) for the compressor efficiency (p3) (other failures are similar, not enumerated).

The diagnose results under 0.02 fault amplitudes are shown in Table 31.2. t_d represents the detection time, and t_i represents the isolation time. It can be seen from the table that the detection and isolation can be completed in a short time, and there is no misdiagnosis of the situation. Therefore, the MM diagnose method based on the soft switch is of great accuracy.

(2) Single fault case with noise

In this section, we discuss the accuracy of the MM diagnose method based on the soft switch under the noise effect of the actual nonlinear model of the two-shaft gas turbine. The transition condition ($w = 0.95$) is also selected. In this section, the change in the size of the noise is achieved by adding an exceptional value, adding an exception to the output of the noise, with exception amplitude of 0.01. The partial measurement parameters after the injection of abnormal noise shown in Fig. 31.5, here the turbine speed (n_t) and power turbine speed (n_p) are presented.

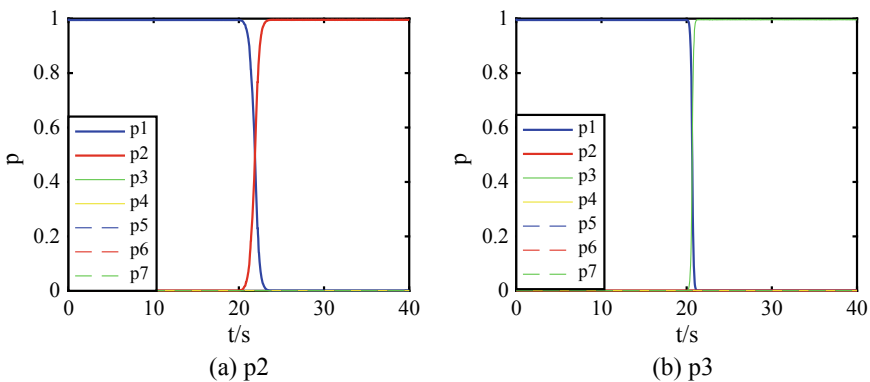


Fig. 31.4 Probability of each fault model under case 3

Table 31.2 Case 1 simulation results

Time	p_2	p_3	p_4	p_5	p_6	p_7
t_d (s)	0.86	0.34	0.14	1.28	0.20	0.32
t_i (s)	3.12	1.04	0.52	3.66	0.66	1.32

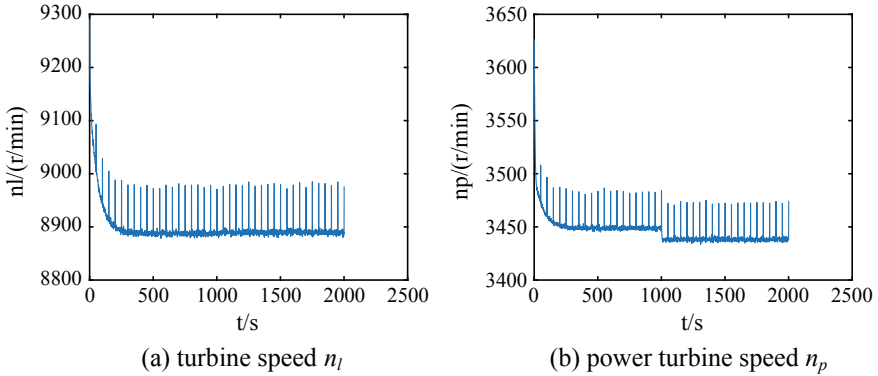


Fig. 31.5 Partial measurement parameters when $Hv = 0.01$

Table 31.3 Case 2 simulation results

Time	p_2	p_3	p_4	p_5	p_6	p_7
t_d (s)	3.08	1.14	0.48	3.54	0.64	1.22
t_i (s)	11.44	3.60	1.94	11.98	2.34	5.18

The diagnose result figures are similar to those in case1, and Table 31.3 is a list of the diagnose results for the presence of abnormal noise. After analysis, we can see that the MM diagnose method based on the soft switch can adapt to the condition of noise, and detect and isolate faults in a short time under the influence of noise.

31.5.2 Multiple Faults

This section verifies the accuracy of the MM diagnose method in the case of multiple faults, as well as the transition condition ($w = 0.95$). The first failure is the compressor mass flow failure. And the second failure occurs in the compressor efficiency. And both of the amplitude of the fault is 0.01 occurred at $t = 15$ s and $t = 20$ s, respectively. The results are shown in Fig. 31.6.

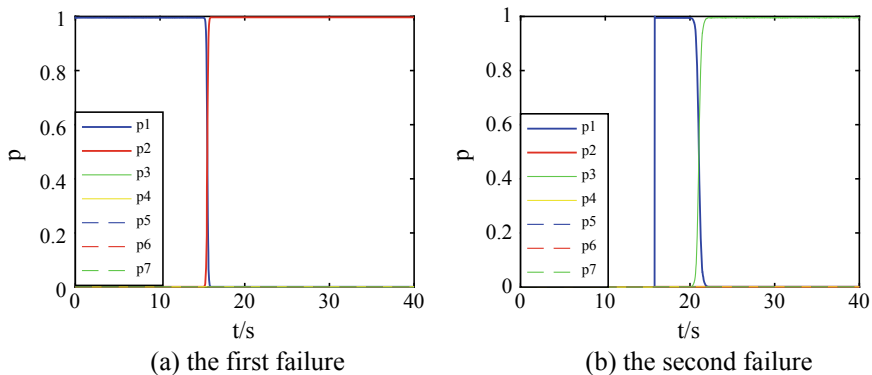


Fig. 31.6 Probability of each fault model under case 3

Table 31.4 Case 3 simulation result

	Fault detect result	t_d (s)	t_i (s)
First	p_2	0.34	0.80
Second	p_3	0.48	1.72

It can be seen from Fig. 31.6 that Fig. 31.6. (a) is the first diagnose result, (b) is the second diagnose result. The diagnose method detected the first fault at $t = 15$ s, that is p_2 , and it was isolated successfully. After isolated the first fault, the first Kalman filter group was expired, and the diagnose method transfer to the second one, which detected the second fault near $t = 20$ s, and rose to the main failure(p_3) to complete the isolation of the second fault. Table 31.4 shows the diagnose results for multiple faults.

From the analysis of Fig. 31.6 and Table 31.4, the MM diagnose method based on the soft switch can detect multiple faults accurately and isolate faults in a short period.

31.6 Conclusions

In this paper, a MM gas path fault diagnosis method for a two-shaft gas turbine is established and the model switching problems are optimized. Considering the nonlinear dynamic characteristics of gas turbines, a general state space model based on analytical linearization was established, which can directly calculate the linear model of the full operating process, and the nonlinear dynamic characteristics of the gas turbine are well simulated. The MM diagnosis method with a layered structure is proposed to detect and isolate gas turbines with a single fault and multiple faults.

According to the problem of switching the linear model between different operating points, the probability-based soft switch technique is used to determine the model according to the different proportion occupied by different piecewise linearized models to make the running area smoother. Finally, the fault simulation of the marine gas turbine is carried out. Through the fault simulation of the marine gas turbine, the verification results show that the MM diagnosis method based on the soft switch can diagnose single gas path faults and multiple gas path faults at different operating conditions.

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