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Mathematical Optimization for Efficient and Robust Energy Networks

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Preface

This book presents a collection of energy production and distribution problems identified by the members of the COST Action TD1207 “Mathematical Optimization in the Decision Support Systems for Efficient and Robust Energy Networks”. The aim of the COST Action was to coordinate the efforts of the experts in different fields, from academia and industry, in developing innovative tools for quantitative decision making and apply them to the efficient and robust design and management of energy networks.

At high level, energy systems—and related predictive and prescriptive business analytic (BA) problems—can be divided into two broad classes: Electrical Energy Systems (EES) and Energy Commodities Systems (ECS; mainly oil and natural gas). In the following two tables, a cross categorization about time horizon (Strategic, Tactical and Operational) versus types of optimization problems (Planning, Production, etc.) is presented for both EES and ECS. Sometimes in medium (tactical) and long term (strategic), the goals are similar (but not identical). On the contrary, as in the case of planning, these goals are inherently strategic and no medium (or short) term activities are considered. Also, some problems can be seen from different angles and depending on the actual structure of the electricity or gas system or markets. For instance, in production optimization problems for electricity markets, the network can be disregarded (or pretty much simplified), but if its management is considered, power plants cannot be done (unless a static Load Flow is of interest). Finally, we observe that while many of these problems could be considered as a single bigger problem, very often in the scientific literature—as well as in the industry practice—they are decoupled in a top-down or bottom-up approach depending on the focus, goals, data availability and, ultimately, the ability of the modellers. As just one example, take a large utility, its long-term gas portfolio optimization is coupled with fossil fuel power plants usage maybe in an electricity market environment.

The present work has been envisioned with the following three main goals:

- being a nimble while comprehensive resource of several real-life business problems with a categorized set of pointers to many relevant prescriptive problems for energy systems;
- being a balanced mix of scientific and industrial views;
- being so that it will evolve over time in a flexible and dynamic way giving, from time to time, a more scientific or industrial—or even political in a broad sense—weighed perspective.

The following tables provide an overview of the issues discussed in the book, organised separately for Electrical Energy Systems (Table 1) and Energy Commodities Systems (Table 2). The columns of the tables correspond to the chapters of the book in order to reveal the meaningful cross categorization.

A general knowledge of the underlying energy markets may be necessary to understand several of the terms involved in the description of the problems.

Table 1 Electrical Energy Systems

	Production and Demand Management	Network and Storage	Maintenance	Finance, Regulations, Politics and Market Design
Strategic Long Term	Energy Generation Capacity Expansion Planning (GEP) Network Expansion Planing (NEP) and Co-optimized GEP and NEP Long-Term Unit Commitment	An Overview of Network-Constrained Optimization Problems Problems of Network Expansion Planning Transmission Network Expansion Planning (TNEP) Distribution Network Expansion Planning (DNEP) Energy Storage System (EES) Siting and Sizing	Strategic Maintenance Transmission and Distribution Network Long-Term Maintenance	Long Term Electricity Bilateral Contracts Multilevel Modeling of Market Design Energy Policy Analysis Demand Response and Price Optimization
Tactical Medium Term	Unit Commitment (UC)	Energy Storage Operations Management	Medium Term Maintenance Scheduled Maintenance	Pricing Problem Derivative Pricing in Electricity Markets
Operational Short Term	Unit Commitment Under Uncertainty Balancing Markets and Non-programmable (Renewable) Power Coordination	Optimal Power Flow (OPF) Security-Constrained Optimal Power Flow (OPF) Optimal Transmission Switching (OTS) Optimal Network Islanding and Restoration Operations of Smart Grids	Nuclear Reloading Pattern Optimization	Combined Gas and Power Optimization European Electricity and Day-Ahead Markets

Table 2 Energy Commodities Systems

	Production and Demand Management	Network and Storage	Finance, Regulations Politics and Market Design
Strategic Long Term	Optimal Oil Wells Placement	Gas Pipeline Design District Heating Network Design Optimal Design of Energy Hubs and CCHP Systems	Evaluation of European Gas Market Designs Take or Pay (ToP) Contracts
Operational Short Term	Optimization of the Gas-Lift Process Total Gas Recovery Maximization Optimal Scheduling of Energy Hubs and CCHP Systems The Pooling problem	Operational Network and Storage Management Gas Network Flow Optimization Optimal Operation of District Heating Systems	Gas Balancing Market

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1 Demand Side Management

Demand Side Management (DSM) is usually considered as a process of energy consumption shifting from peak hours to off-peak times. DSM does not always reduce total energy consumption, but it helps to meet energy demand and supply. For example, it balances variable generation from renewables (such as solar and wind) when energy demand differs from renewable generation [278]. One of the limitation of electricity power is that generally, electrical energy cannot be stored because of a large number of economic or physical feasibility limits. Thus, it must be produced in the quantity needed. It is exactly the main objective of DSM—to equilibrate production and consumption of energy.

DSM originated after oil crisis in the 1970s. Then, energy demand relied on forecasts, which were often made with a ruler and double-log paper. In other words, demand side was largely disconnected from the market. Consumers were mostly simple users of energy sources. They received electricity from energy grid and paid for it. Gradually, this situation is changing. After petrol shock in 1973, Demand Side Optimization has become more important. Most countries tried to develop programs to reduce dependence on oil and to promote energy efficiency and alternative energy sources. Nowadays, energy consumers are more proactive. They want to optimize electricity consumption so as to reduce their expenses.

The process of DSM activities usually follows an integrated approach. DSM sends signals to end-use systems to shed load depending on system conditions. This allows for very precise tuning of demand to ensure that it matches supply at every period, reduces capital expenditures for the utility. Critical system conditions could be peak times, or in areas with levels of variable renewable energy, during times when demand must be adjusted upward to avoid over generation or downward to help with high needs. Consequently, the analysis and optimization on the demand side focuses on the involvement of the customer and fits to the vision of a customer centric energy grid. Unfortunately, many common control strategies [141, 142] fail.

According to literature [164], DSM can be divided into three categories:

- Energy efficiency means usage of less power due to more efficient load-intensive appliances such as water heaters, refrigerators, or washing machines.
- Strategic Load Growth refers to a general increase in energy consumption. Load growth may involve increased market share of loads which can be served by fuel switching from fuels to electricity such as heat pumps, induction cooker and microwave oven.
- Demand Response (DR) identifies the short-term relationship between price and quantity when the actions and interactions of substitutes and complements are considered. Currently, the term DR is used in a broad sense, in relation to electricity end-use, and is attributed to a wide range of control signals such as prices, resources availability and network security [256]. Figure 1 sums up DSM categories.

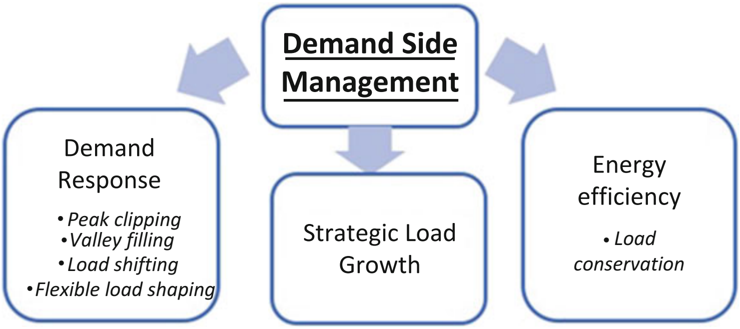

Fig. 1 Categorization of demand side management (based upon [256])

1.1 *Demand Side Management and Demand Response*

We define DR as part of DSM similar to [341] and [395], as the "voluntary changes by end-consumers or producers or at storages of their usual electricity/gas flow patterns in response to market signals such as time-variable prices, incentive payments" or beforehand given agreements between customers and third parties. Such pattern changes are possible due to flexibility on the demand side. Such flexibility might be provided for example through electrical or thermal storages where demand is decoupled from generation, but also from other flexible loads, such as EVs.

In electricity markets, traditional DSM programs are slowly getting replaced with DR programs. A good example of demand response implications (reducing electricity peak demand) is the introduction of "Time of Use Tariffs" in France. Its aim was to apply a fixed rate with different time units depending on hours and seasons. "Time of Use Tariffs" in France included three parts:

- "Green tariffs" (1956) for large firms or buildings (La Defense): Many prices options according to season/hour and localisation/use.
- "Off-peak hours" (1965) tariffs for residential market and business- special tariffs from 10 PM to 6 AM week days and on Sundays.
- "Peak day step back" (EJP) (1982) for residential market was introduced to decrease consumption at critical times (22 days of 18 h between 1st November and 31th March). It established high price during this period and low price for the rest of the year. Currently, EJP replaced by TEMPO (6 price's levels according to hour and season).

This program showed good results: "off-peak hours" tariffs reduced peak consumption by about 20% and customers with "Peak day stepping back" tariffs reduced their consumption by 50% during peak periods (4% of total residential consumption) [111].

More recently, it is possible to exploit temporal flexibility of the gas transport infrastructure for smart electrical power grid operation. Some of the compressor units of the gas networks are powered by electricity. The ratio of compression horsepower used by such compressors to total compression horsepower varies by countries, i.e., about 5% in the USA having about 1400 compressor stations [420], over 40% in France having 32 compressor stations [179, 412], and for UK predominant gas-driven turbines of 24 compressor stations are in process of being replaced by electricity units [319]. In peak power generation situations, the dispatchers of gas transmission system operators may start these compressor units already, in terms of the gas network operation itself, before compression is necessary to increase pressure. Thereby they load pipes with excessive gas for upcoming gas demands, without jeopardizing security of operations. Likewise, to a certain amount they may delay compression in situations with low energy generation. A third option is to choose between electrical and gas powered compressor units based on the electrical power situation. Thereby, in addition to Power-to-Gas technologies inducing synthetic fuels (see Sect. 7), coupling electrical power systems and natural gas infrastructure introduces new smart grid operation options. However, in order to ensure security of supply and safe gas network control, such options will need foresighted decision support systems (see Sect. 5 for more details on optimizing gas network flows).

1.2 Direct Load Control vs. Indirect Load Control

In general DSM and DR concepts can be distinguished between direct and indirect load control. Indirect load control implies an incentive, such as a price signal. Such signal might motivate the consumer to shift its consumption into times of lower prices. Direct load control rather means an agreement between the customer and a third party that allows the party to directly control the loads of the customer based upon the beforehand made agreement [230]. For field installations, the most promising solution which finds good acceptance in both research and industry is the automated demand response (OpenADR) protocol which is now a de-facto standard for DR concepts [325]. Several recent research activities that use mathematical optimization techniques for DR refer both to direct and indirect load control. These research topics are related to the optimization and coordination of the operation supply and demand units throughout a time horizon, e.g., an offline day-ahead scheduling under considerations of flexibility. The flexibility is achieved through temporal shifts over a Horizon T. Such problems are very generally known as the Portfolio Balancing problem.

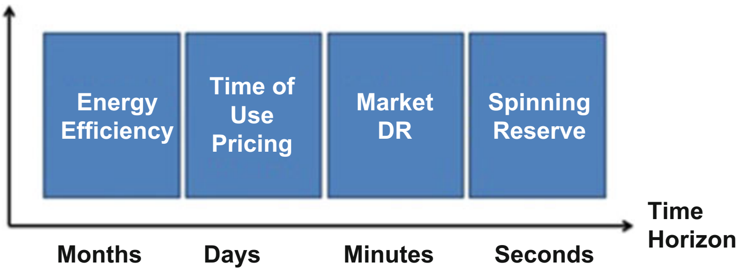

Fig. 2 Different time horizon in DSM, based upon [341]

1.3 Demand Side Management in Different Time Horizons (Short to Long Term)

The classic unit commitment problem is mainly short term, but can be solved also for medium and long term problems. As shown in Fig. 2, similarly as presented in [341], we can distinguish Demand Side Management according to its time line. Spinning Reserve in this context refers to primary and secondary and even tertiary control, which is usually done by power plants. However, in DSM, loads can be virtually aggregated and act as a negative spinning reserve for frequency control. The time horizon is seconds and minutes.

1.4 Integrated Demand Side Management

Nowadays, DSM technologies become increasingly feasible due to the integration of information and communications technology and the power system, new terms such as Integrated Demand Side Management (IDSM), or smart grid. Smart grid gives new opportunities of remote control services that allow the network operator to switch off high electricity consumption devices (for example, air conditioners, hot water tanks, heat pumps) for a limited period during peak demand without causing major issues for the consumer.

For example, French company Voltalis offers to residential customers, the 'Bluepod' box, a device which switches off electrical heating and space conditioning appliances. If demand exceeds electricity production, transmission network operator (Réseau de Transport d'Electricité, RTE) contacts Voltalis, which can withdraw demand in real time by modulating electricity consumption in many households via 'Bluepod' [324].

1.5 Challenges and Requirements for Demand Side Management and Demand Response in Optimization

The above mentioned general description of the portfolio balancing problem for city districts and neighborhoods incorporate several challenges both for the mathematical method and the overall approach. First, there is usually a high heterogeneity of participants and devices that must be taken into account. Residential buildings, but also industrial consumers might take part of the portfolio balancing. The load and flexibility of such units diversify within their granularity of time, their amplitude and their criticalness. Second, a city district contains in general a high number of participants and devices which lead to a computational intensive problem with an increasing portfolio size. Consequently, a mathematical optimization must be able to handle a large amount of heterogeneous participants. Third, referring to the concept of demand response and in particular to direct load control, it is an important requirement for the method to ensure data privacy. Fourth, the coordination within city districts usually needs to integrate both local (customer) and global (system) level objectives. In respect to this challenge the method requires an approach for both satisfying global and local objectives. Fifth, depending on the kind of installed devices on the demand side the mathematical optimization method might have to be able to take care of on/off devices leading to an Integer related problem formulation.

Research Papers and Solver Indirect load control on the demand side is for example studied in [370] and [309]. In particular [309] is a very recent example for showing the operation scheduling of Plug-in electric vehicles coordinated by an aggregator agent. The MILP is solved within GAMS Build 21.1.2. using the CPLEX 12.5.1 solver [1]. This research satisfies all of the mentioned requirements. As mentioned in the challenges above a central optimization becomes hard to solve with an increasing portfolio size. Indirect and direct load control for scheduling loads on the demand side by using a distributed algorithm is hence an active field of research. Consequently many research papers, such as [96, 228, 230, 252, 253, 357, 425] propose distributed optimization demand response techniques for (residential) energy demand side management. Decomposition methods such as in [228] and [310] use dual decomposition (DD) or the alternating direction method of multipliers (ADMM) such as in [230, 247]. For both DD and ADMM in particular challenges and requirements (1)–(4) are taken into account. Kuznetsova et al. [253] the residential demand side energy management in [310] for example used the matlab environment in combination with ILOG CPLEX 12.2 to solve the optimization problems. The ADMM problems were solved using CVX, a package for specifying and solving convex programs [51, 174]. Looking into integer related problem formulations authors in [187] propose a column generation approach for direct load control which is solved using the object-oriented Python Interface of Gurobi [184]. Research in [253] performs a decentralized robust ILP optimization for balancing a portfolio within a microgrid. The optimization uses the CPLEX package within Java. Both [187] and [253] are other examples for satisfying all mentioned challenges [230, 341, 370] and the resulting requirements. [249] uses a MILP formulation for the optimal control of a residential microgrid using the Gurobi solver as well through the object-oriented interface for Java. Further, authors in [48] perform a distributed optimization via a multi-agent system using the Java agent development framework (JADE) [220]. However, each local agent solves its own local MILP optimization using MOSEK [314].

2 Energy Generation Capacity Expansion Planning (GEP)

One of the major and difficult problems in the energy area that the European Union (EU) is facing today consists of estimating the timing for clean power generation technologies and electricity transmission expansion network at a pan-European level in a long term (e.g., 30 years time horizon). EU has established aggressive pollutant emission reduction targets: a 20% (resp. 27%) reduction in greenhouse gases with respect to 1990 levels by 2020 (resp. 2030) and an objective of 80% reduction by 2050 (Eurostat).

Mathematical optimization models and algorithms to address the above challenges in the electricity open market [68] are essential computerized tools for helping in the decision making to estimate the following key issues: feasible type and mix of power generation sources, ranging from less coal, nuclear and combined cycle gas turbine to more renewable energy sources (RES), namely hydroelectric, wind, solar, photovoltaic and biomass, among others; and timing for power generation plant farm site locations and dimensions. The solution should maximize different types of utility criteria and quantifying the benefits of using cleaner, safer and efficient (cheaper) energy.

In the past (up until 25 years ago), practically all over the world the energy sector was a very centralized one, where the electricity generating companies had a limited decision-making on generation expansion capacity planning . The energy prices were centrally decided as well as the main geographical areas where to service the energy demand. So, the maximization of profit functions was out of question and, then, the main goal was to minimize the net present value (NPV) of the global cost for the planning of the site, location and capacity of new energy generation sources in, first, hydropower, second, variety of thermal plants, and also nuclear generation. In the latter energy source, not too much room was left for the modeling tools to help in the decision making. On the other hand, the claims of other stakeholders (mainly, environmentalist ones) were not a strong issue given the strong regulation of the sector. The modeling could consider uncertainty on the main parameters that, by definition of the non-open market and, then, its strong regulation, was reduced to macro-economic and demographic factors that influence the energy demand to serve and the generation disruption. Additionally, the state-of-the-art on theory, modeling and algorithms for dealing with mathematical optimization under uncertainty (i.e., stochastic optimization) was not as advanced as it is today and, surely, will be in the future. Additionally, the computer performance was so low (until, say, 10–15 years ago) that the gigantic models that were needed to provide solutions to help the decision-making could not be considered.

Today, the situation has drastically changed. The computer performance is very high, and it seems that its exponential growth will continue at least in the near future. On the other hand, some stochastic optimization tools could be considered today as a sort of commodities ready to be used. Additionally, the energy sector is very different from what it was in the past. It continues being a crucial sector for the European economy as for the rest of the world. However, its market, without being fully an open one (something that, by definition, probably it cannot be), it allows higher freedom to the Generating Company (GenCo) for performing strategic energy generation capacity expansion planning in a long horizon. First, there is enough freedom on deciding the amount of power generation from the different plants/farms and, on the other hand, the energy price is not (fully, at least) decided by regulation. Second, the GenCo has very much freedom to decide the location, capacity and timing on new generation sources. Third, the macro-economic and demographic factors are not the only main factor to influence the demand, but the competitors' strategies are a major source of uncertainty. Fourth, the power to be generated by some new RES, mainly, wind and, in a lesser extent, solar and photovoltaic sources, is subject to a high uncertainty (that, on the other hand, it is difficult to formulate). Fifth, given the type of new energy sources, there is more variety on the location sites and generation capacity, which allows considering more opportunities for strategic planning. And, sixth, there are other stakeholders (environmentalists, among others) having different goals (whose directions are not the same sense as the GenCo) that, in some way, have to be considered in the decision making, plus some Government and EC directives, etc. All of that induce to consider multiobjective optimization.

So, the GenCo's aim has been moved from cost minimization to expected profit maximization along the time horizon. One of the interesting disciplines for problem solving is stochastic optimization, where the uncertainty of the main elements is represented (i.e., quantified) by a finite set of scenarios, ether in a two stage or a multistage scenario tree. Anyway, the problem modeling and algorithm development are big challenges, given the gigantic character of the instances of the problem. As an additional difficulty, big GenCos can influence some of the main uncertain parameters in the problem; say the energy price, among others. Then, the probability and value along the time horizon can be influenced by the decision-maker. In this case, the so-called decision-dependent probabilities [130, 171, 182] (also called endogenous uncertainty) should be considered. One of the potential tools consists of using stochastic mixed $0-1$ quadratic mechanisms; see [34], to be added to the modeling schemes presented next.

Multistage stochastic mixed $0-1$ optimization modeling for risk management should be considered. There are different approaches (some of them very recent ones) for energy generation planning, see [124, 421, 422], among others. The main parameters in the problem are uncertain and, so, a set of scenarios should be generated by considering the realization of the following parameters, at least: availability (and price, in case) of raw material for power generation: gas, fuel, water, wind, solar, etc.; electricity demand and prices at focal nodes in the energy network; operating hours per period of power generation technologies; CO2 emission permits; green Certificates prices for buying and selling in the market, and allowed bounds; power generation costs of different technologies; electricity loss of candidate power generation technologies; investment allocation bounding of the cost for the total power generation.

There is not a unique function-criterion to consider. Rather, it is a multicriteria problem, since the model considers the maximization of the NPV of the expected profit of the investment and consumer stakeholders' goals over the scenarios along the time horizon, subject to risk reduction of the negative impact of the solution to be provided by the optimization system in non-wanted scenarios, plus utility objectives of other stakeholders. Those other objectives include the power share of cleaner, safer and efficient energy accessible to all consumption nodes, cost investment from private and public institutions, generation network reliability, EC directives and EU Governments on environmental issues and others.

One of the difficult problems to deal with is the generation of a set of scenarios to represent the realization of the uncertainty as structured in a multistage scenario tree. Hence, a node in the tree for a given stage is related (in a one-to-one correspondence) to a group of scenarios that up to the stage have the same values in the uncertain parameters. Then, the solution for those scenarios should be unique for all stages up to the stage where the node in the tree belongs to, i.e., the so-named nonanticipativity principle is satisfied.

In the so-called Risk Neutral approach (RN), the function to maximize consists of the NPV of the expected profit along the time horizon over the scenarios with the following elements related to a given GenCo in the energy network: revenue from sale of electricity, revenue from sale (or, alternatively, cost from purchase) of Green Certificates, penalization of CO2 emissions, variable generation cost of thermal power plants, variable generation cost of renewable energy source power plants/farms (wind, solar, photovoltaic, biomass, etc.), periodic debt repayment of the investment on the new power plants/farms and new hydro power turbines, fixed power generation cost of available new plants/farms and new hydro power turbines, etc.

The gigantic character of the problem can be assessed by considering its dynamic setting (say, 30 years time horizon), the number and dimensions of replicated networks (i.e., hyper hydro valleys) in the time horizon for some big generator companies (e.g., EdF has 20+ valleys, some with 50+ elements, see [123], multiple choices in time and space of location and capacity decisions for the energy generation system of the given GenCo (current and candidate power generation plants/farms), and the representative scenario tree to consider for the uncertain parameters.

The aim of the Risk Neutral type model performs the maximization of the NPV of the expected profit. The main drawback of this popular strategy is that it ignores the variability of the profit over the scenarios, in particular the "left" tail of the profits (or big losses) of the non-wanted scenarios. For the problems with so high variability, there are some risk averse approaches that, additionally, deal with risk management.

Among them, the so-called time-inconsistent stochastic dominance (TSD) measure reduces the risk of the negative impact of the solution in non-wanted scenarios in a better way than others under some circumstances. See [9] for a computational comparison of some risk averse strategies, see also [10].

The TSD measure presented in [120, 126] for stochastic problems as a mixture of first- and second-order stochastic dominance strategies. It is a multistage extension of the two-stage strategies introduced in [115, 119], plus the consideration of hedging the solution against some types of negative impacts in non-wanted scenarios at selected stages along the given time horizon.

Then, the maximization of the NPV of the expected profit is subject to a scheme for risk management that consists of appending to the model a set of TSD constraints for given profiles at a stage subset for each function (including the objective one), such that a profile is given by the 4-tupla: threshold on the function value; maximum target shortfall on reaching the threshold that is allowed for each node in the scenario tree related to any of those stages; bound target on the probability of failure on reaching the threshold; and bound target on the expected shortfall.

As an alternative, the time consistent strategy proposed in [126] is so-called the expected stochastic dominance (ESD) measure. In ESD, however, the profiles are associated with the nodes of a modeler-driven stage subset for each function, where a profile consists of the 4-tupla: threshold of the function to be satisfied by any scenario in the group with one-to-one correspondence with the node; maximum shortfall of the value of the function that is allowed for any of those scenarios; upper bound target on the expected deficit (shortfall) on reaching the threshold that is allowed for that group of scenarios; and upper bound on the probability of failing to satisfy the threshold.

The rationale behind a time-consistent risk averse measure is that the solution value to be obtained for the successor set of a give node in the scenario tree for the related time consistent submodel, solved at the stage to whom the node belongs to, should have the same value as in the original model 'solved' at the beginning of the time horizon, [95, 129]. For the time consistent version of CVaR (Conditional Value−at−Risk), a very popular risk averse measure, see [95] and references therein.

It is worth to point out that, by construction, the time-consistent version of risk averse measure does not avoid the risk on non-desired shortfalls on reaching the thresholds for the given functions at intermediate stages of the (long) time horizon for the energy generation expansion planning problem. It is a challenge for problem solving, but both time-consistent and time-inconsistent versions of risk averse measures should be jointly considered in the same model.

The gigantic but well-structured multicriteria multistage stochastic nonlinear mixed integer (SMINO) problem with risk management cannot be solved up to optimality, see [120]. A realistic approach could consist of a combination of the following elements: sample scenario schemes, iterative algorithms for solving SMINO by sequential mixed 0-1 linear one; node-based decomposition algorithms; stochastic mixed 0-1 bilinear optimization solvers; and high performance computing. Another issue that needs further research is the scenario reduction due to the partition of the scenario set into strategic and tactical ones, see [120].

2.1 Optimization Methods

The GEP can be mathematically formulated as a high dimensional, nonlinear, nonconvex, mix-integer and highly constrained optimization problem with the least cost of the investment as the optimization criterion. The complexity of the problem rapidly increases if many practical constraints are taken into account.

Methods to solve the GEP can be generally categorized into two types: traditional mathematical programming methods and methods based on heuristic techniques. The traditional mathematical methods include Stochastic Nested Decomposition (SND), Dynamic Programming (DP), mix-integer programming (MIP) branch and bound, Benders' Decomposition, network flow methods, and others. See in [125] a review of decomposition algorithms for multistage stochastic problem solving. See also [85, 116, 117, 119, 126, 130, 464, 465].

3 Network Expansion Planing (NEP) and Co-optimized GEP and NEP

The objectives of electric-power Generation Expansion Planning (GEP) and Network Expansion Planning (NEP) problems are to determine the "optimal" selection of generation and network technologies (in a broad sense) and the right time and right place to construct (and/or dismiss) them, while ensuring (1) economic, (2) reliable, and (3) environmentally acceptable supply according to the predicted demand. Needless to say, these problems involve amount of money of the order of magnitude of the tens of billions EURO for large countries.

A typical GEP optimization model has (1) a planning horizon, (2) an economic (multi)-objective including the present value of the total cost and other components, (3) a long set of constraints including: capacity limitations, environment regulations, fuel costs, customer demands, fuel availability (for instance gas pipelines for CCGT) and mix diversification requirements, and (4) a set of decision variables representing the operating and expansion options (that depends on the perspective of the actor). To make the matter even more complicate, in the predominant market based models, the GEP and NEP problems must also take into account present, and future, market rules and incentives.

The GEP and NEP co-optimization problems can be defined as follows: co-optimization is the simultaneous identification of two or more classes of investment decisions within one optimization strategy. If co-optimization is used by a monopolist integrated utility, then its main result is the identification of joint GEP-NEP that are lower in cost than would be if GEP and NEP were developed separately. However, co-optimization can also be used within countries that are market based and where NEP is performed by one entity (TSO) while GEP is performed by others (GenCos). Co-optimization can be naturally the "one optimization strategy" that may consist of a formulation to solve a single optimization problem (e.g., for a GenCo in the GEP perspective, maximize expected profit subject to budget constraints) or it may consist of a formulation to solve an iterative series of optimization problems (i.e., sequential yet possibly coordinated generation and transmission planning). As an illustration , some goals of a GEP-NEP co-optimization are as follows:

- savings of transmission and generation investment and operating costs
- more efficient decisions concerning generation dismissals and repowering
- more appropriate treatment of intermittent resources
- efficient integration of non-traditional resources such as demand response, customer-owned generation, other distributed resources, and energy storage systems
- fuel mix diversification benefits
- improved assessment of the ramifications of environmental regulation and compliance planning
- reduced risk and attendant effects on resource adequacy and costs.

Historically, the practice was to attack the GEP problems first, and then the NEP ones. This approach was motivated because of (1) the complexity of the coupled problem(s), (2) the controllability of the traditional power plants with their different technologies (Nuclear, Coal, Steam Turbine, CCGT, Gas Turbine, Hydro Basin), and (3) the limited interregional power exchanges. However, by assessing both simultaneously to provide an integrated plan, it is possible to identifying attractive solutions that may not otherwise be considered. Doing so it is becoming more important, due to (1) the increasing penetration of non-programmable renewable resources, energy storage systems, distributed generation and demand response, and (2) the need for interregional energy transfers to take advantage of diverse and remote sources of power. For instance, it can be argued that the newly launched Price Coupling of Regions (PCR) in EU does not only enable to clear at European level the Day Ahead Markets in the short term, but it also gives the opportunity to consider at regional level the GEP-NEP problems in the longer terms. Thus, NEP are not necessarily the least-cost means of meeting those needs (considering both economic and environmental costs). Second, siting of new generation, including renewable sources, is influenced by the availability of transmission, so that different transmission expansion plans will ultimately result in different patterns and even mixes of generation investments.

In what follows GEP and NEP are considered as a single unified problem, and recent approaches are discussed.

First of all, notice that co-optimized GEP and NEP problems posed significant computational challenges. Computer resources available to planners before, say, 10 years ago were incapable of supporting the solutions of co-optimization models.

Fortunately, recent advances in computation methods have provided satisfactory solutions to co-optimized GEP and NEP problems with reasonable computation times, so now realizing savings by using co-optimization is a real possibility. Also, observe that in GEP-NEP optimization problems uncertainty is, of course, ubiquitous.

There are several challenging issues in the co-optimization of GEP and NEP. First, conflicting objectives: GEP can be driven by prices but the same principle may not apply to NEP (e.g. [364]). Second, power system constraints such as network flow limits, load demands, and reliability requirements [294], link the two planning problems, which introduce an additional dimension of difficulty in finding feasible and practical planning solutions. Third, one of the main obligations of expansion planners is to facilitate a fair and competitive market. The planner also has to take into account uncertainties associated with renewable energy, non-traditional generation resources such as microgrids, fuel costs, component outages (such as transmission lines, plants, and transformers), and customer behavior including demand response. The co-optimization of GEP and TEP becomes much more challenging when contemplating the full range of uncertainties relevant to expansion planning. Earlier attempts uses a Benders Decomposition-based approach developed to separate and coordinate the investment problem and operating subproblems (e.g. [344]). Reliability issues were assessed in terms of customer interruption functions in co-optimization models [266], allowing trade-offs between outage, investment, and operating costs. However, these earlier models were oversimplified and thus deemed impractical for market-based generation and transmission expansion planning.

In general, co-optimization is viewed as a bi or tri-level optimization problem for generation and transmission and iterative schemes have been used to coordinate the two planning problems. As an example, [29] presented a stochastic bilevel co-optimization model and transformed it into a single-level mathematical programming with equilibrium constraints. It is shown—as expected—that transmission expansion decisions significantly affect wind power capacity expansion even though investment cost in transmission expansion is much lower than that in wind power capacity. A recent study in [387] presented a co-optimization model that incorporated transmission congestion costs. It was also shown that distributed generation could mitigate congestion and defer transmission investments. A follow-up study in [388] proposed a co-optimization model which accounted for incentives offered to independent power producers (IPP).

As for data uncertainty, stochastic optimization was applied in [123, 273] to simulate random outages of system components. It was shown that even simple co-optimization models could result in significant savings when optimizing transmission and generation assets. Also, it was the main ingredients in order to consider alternative scenarios of future economic, regulatory, and technology developments.

GEP and NEP co-optimization models include both transmission expansion planning and generation planning for multiple years/decades and multiple locations/regions. This leads to computational challenges due to the fact that the details of power systems can greatly increase the size of the problem. In addition, nonlinearity, integer variables and uncertainties can add further complications. As discussed in the Network and Storage chapter, modeling of transmission flows by itself can be a very difficult non-linear program (the OPF with full AC representation). After adding investment expansion decisions, the problem becomes an even harder mixed-integer nonlinear program.

Several simplifications are therefore applied, such as aggregation of input data and model variables and simplification of dynamics and uncertainties (e.g., [383]).

Approaches to modeling aggregation include Location aggregation (e.g., aggregated region(s) instead of exact locations), and time period aggregation (e.g., multiple year instead of daily data) (e.g., [389]).

However, even if model aggregations and simplifications are effective for reducing computational complexity, models then lose fidelity and accuracy to some extent. Thus, it is desirable to solve large-scale and complicated problems. At the present time the two following approaches are probably the most appropriate ones: (1) trying to linearize everything by means of the many possible approaches eventually resorting to piecewise linear modeling, and (2) using decomposition approaches as those well known in the optimization community, say: SND, Benders Decomposition, Column Generation and Branch-and-Price. Today, there is a very extensive research on the topic, mainly for stochastic models.

4 Tactical Problems

Tactical problems can be seen as variants of the Unit Commitment (UC), which requires to optimally operate a set of hydro and thermal generating units, over a given time horizon in order to satisfy a forecast energy demand at minimum total cost. The generating units are subject to various technical restrictions, depending on their type and characteristics. The UC is typically a large-scale, non-convex complex optimization problem.

The particular optimization problems can be seen from the perspective of the multiple participants:

- **Monopolist systems**: here the actors are centralized in a single entity that manages the production, the transmission and distribution systems. Its main goal is a least cost schedule in order to supply load while respecting the several physical constraints. Depending on the power plants mix and level of details, several fuel unit constraints, hydro units (see below) and network constraints (e.g. voltage profile across the nodes, maximum active power flow across branches) are take into account. In increasing order of complexity we have the following problems:

  1. **Single centralized entity**:

     – Load Flow
     – Single Bus Economic Dispatch: only active power from units are optimized, status is assumed to be fixed, network is not considered

  - AC/DC Optimal Power Flow: only active power from units are optimized, status is assumed to be fixed, network is considered only with DC approximation or full AC equations
  - Short Term UC: both status and power of units can change, but network is not considered
  - Security Constrained UC: both status and power of units can change, network is considered (with DC/AC equations)

  As illustrated by Gambella et al. [163] on Canary Islands, within developed markets, such models are still in use in outlying islands, for instance.
- **Market based models**:

  1. **GenCo**: Its main goal is to maximize profit from selling energy and balancing capabilities. Depending on the size and risk profile, GenCos can approach the maximum profit maximization by considering different models, such as:

     - Pure Price Taker
     - Supply function equilibrium
     - Residual Supply
     - Cournot competition
     - Bertrand competition
     - Other more complicated models could include multi market maximum profit optimization models. In these models one tries to optimally allocate energy of power units among the different markets on cascade possibly with different clearing logic while respecting the operating—often multi-perioda—restrictions of the power units. Also, if zonal prices are considered by the electricity market, some form of arbitrage could be tried by GenCos with production plants geographically spread across the system.

  2. **Market Operators (MO)**: Its goal is to clear the (hourly) energy market solving a maximum welfare optimization problem. Depending on the market rules MO problems can have different additional peculiarities, such as portfolio bid (i.e. the GenCos are allowed to bid energy from a portfolio of generation units), zonal prices (i.e. MO problem includes zonal transmission constraints that potentially creates different prices for GenCos) and others. These peculiarities do change the form of the maximum welfare optimization problem ranging from middle scale Linear Programming to much more complex Mixed Integer Nonlinear Problems. From the beginning of 2015 the European Union started the so -called Price Coupling of Regions (PCR), a unified electricity market at European level that clears energy prices at EU level including the differences among previous national market rules.
  3. **Transmission System Operator (TSO)**: Its goal is to maintain overall system stability including network. This broad goal is achieved at different time scale, in the short term this basically amount of solving:

     - Residual Demand Offer Based SCUC (i.e. a residual demand SCUC, after energy market are cleared. This SCUC is based on the offer made by

GenCos to the TSO). This essentially is the goal of the Balancing Markets (BM).
  - Detailed OPF including full Alternate Current (AC) representation of the network laws.
  - Renewable coordination, since these types of production plants are subject to uncertainty, TSO in modern systems must take care of these issues in solving BM. This calls for specialized methodologies for the reserve requirements satisfaction.
  - Optimal Transmission Switching (OTS). Very recently TSO are investigating the possibility of opening (tripping out) some line of the High Voltage network in order to alleviate some constraints in the network itself. This problem must be solved in conjunction with SCUC or OPF and give raise to very complex optimization problems. (see also the Network and Storage chapter).

In both monopolist and market-based models of course production power plants dynamics have to be modeled in a correct way. In the short term, GenCos must consider these constraints in the most detailed way, some of the most important ones are sketched next.

- **Thermal units**: Thermal (including nuclear) power plant are modeled in a somehow detailed manner. Main constraints and objective function include:
  - quadratic cost curves possibly including some important (interdicted) valve point.
  - min and maximum stable production.
  - ramp rates and start up rates, possibly depending on the working points for bigger coal plants.
  - complex operating dynamics for Combined Cycle Gas Turbine (CCGT) that have several Gas Turbine (GT) coupled with Steam Turbine (ST).
- **Hydro Units**: Also, hydro units are modeled in a somehow detailed manner. Main constraints include:
  - Water-to-Power nonlinear relationships, for thin basin the bi-linear dependency of the basin level, together with the discharge, can be included. This severely complicates the models.
  - Complex cascade dynamics, including delays in the water flows from one basin to another. These delays can be also of different hours for big cascade and as a result their consideration strongly couples the decision variables along the time dimension.
  - Additionally a forecast of possible natural inflows must be considered, due to rain or snow melt in some situations.
- **Renewable non-programmable (i.e. wind and solar)**: These power plants do not actually have operational constraints but, due to their intermittency, the TSO (or the monopolist) must carefully forecast their production profile perhaps by geographical aggregation. In turns the inherent uncertainty in their (forecasted) schedule calls for stochastic-like approaches.

5 Unit Commitment (UC)

Unit Commitment (UC) has a research history of more than 50 years. UC models, being large-scale mixed-integer nonlinear optimization problems solution approaches, always have been inspired by ideas from different subdisciplines of optimization, with permanently adjusting “large-scale” to bigger and bigger numbers. In recent years, the integration of UC into energy optimization models which, themselves, already are large-scale, e.g., power flow or uncertainty management in production and trading, became a focal research topic.

A review of the first 25 years up to 1994 in the UC literature is presented in [392]. Some of the identified approaches later became major pathways in algorithmic unit commitment. On the one hand, some heuristic methods are also considered, such as Exhaustive Enumeration, Priority Lists, or Simulated Annealing, as well as mathematically rigorous methods from subdisciplines of optimization as Dynamic Programming, (Mixed-Integer) Linear Programming, Network Flows, and Lagrangian Methods. The computationally more demanding rigorous methods, on the other hand, yield provably optimal solutions or at least lower bounds allowing for gap estimates between objective function values of the best feasible solution found so far and lower bounds generated in the course of the algorithm.

Lagrangian Relaxation—As it Was In [392] it was granted a “clear consensus presently tending toward the Lagrangian Relaxation (LR) over other methodologies”. Indeed, still today LR offers flexible possibilities for relaxing constraints complicating the model, however, at the cost of having to solve repeatedly “close cousins” to the relaxed problem. The key features of LR applied to UC have been and still are:

- the relaxation of constraints inter-linking units, e.g., load coverage or reserve requirements, and arrival at single-unit subproblems,
- the dualizations of the relaxed constraints in the objective function by considering Lagrangian multipliers, so that the resulting problem so-called Lagrangian Relaxation(LR) is easier to solve than the original one,
- the solution of the convex, non-smooth Lagrangian dual whose objective-function value calculation benefits from reduction to solving single-unit subproblems and whose optimal value forms a lower bound to the optimal value of the UC problem, and
- the using of Lagrangian heuristics to obtain “promising” feasible primal solutions from the results of the dual optimization.

Lagrangian Relaxation—As it Is Fueled by improved bundle-trust subgradient methods for the Lagrangian dual and by permanent progress in “off-the-shelve” mixed-integer linear programming (MILP) software, up to the advent of market deregulation, two basic approaches developed which still today are widely used:

- LR, often in conjunction with SND, DP and heuristic methods for finding “promising” feasible solutions,

- direct solution (by branch-and-bound) of MILP formulations of UC by "off-the-shelve" solvers with linearization techniques for handling nonlinearities.

Lagrangian Relaxation—As it Will Be Rather than the transition to different time horizons, from short via medium to long-term, the new economic environment in the course of energy market deregulation poses research necessities and provides incentive to integrate UC and ED with load flow and uncertainty treatment, [160]. The latter is intended in the widest sense, from handling stochasticity to topics of mathematical equilibria in the context of power trading and bidding into power markets. In particular, this means to integrate UC into models which already are complex themselves.

Power Flow—Integrating UC and AC Load Flow This was considered utopic throughout the "Early Days", but now became possible by studying the quadratic nonconvex AC load flow equations from the viewpoint of SemiDefinite Programming (SDP). In [262] after relaxation of the rank condition, the solution to the dual of the remaining convex model allows to retrieve a primal solution often meeting the relaxed rank condition, and thus enabling to solve non-convex power flow optimization problems to global optimality. In turn, one can construct convergent solvers for unit commitment with SDP relaxations. See [163] for an example.

Power Flow—DC Model and Ohmic Losses The DC Load Flow Model provides a linear approximation of its AC counterpart by resorting to linear relations and avoiding variables in the space of complex numbers, see [153, 154]. The Ohmic Losses approximation, [376], provides the possibility to include power losses within the DC-approximation of an AC power system. Precise modeling of power losses turns out instrumental in congestion management when load dispatches or even commitments of units have to be revised to increase throughput of the grid under increased inflows of renewables.

Power Based UC Traditionally UC is modeled with time periods of 1 h and thus using only the same variable to represent the power and the energy, that is equal to power by time. However, this approximation could introduce difficulties to map the solution to an implementable power trajectory when also ramp-rate constraints are considered [435]. Models and solution algorithms considering separated power and energy variables are reported e.g. in [312, 433–435].

Polyhedral Methods Despite its success in combinatorial optimization, cutting plane methods based on polyhedral studies, either applied directly or enhancing branch-and-bound came to the fore in UC a bit more than 10 years ago, only. At this time, market deregulation enforced the need of solving UC in a competitive environment under incomplete information. In this way, solving UC problems became a subroutine in the treatment of more complex decision problems in electricity supply.

Today tight formulations are available for minimum-up/down contraints [263], for ramp constraints [91, 338, 342], for start-up costs [393], for start-up and shut−down limits [312, 312], for linearizing the convex quadratic power production cost [148, 151, 152], am ng others. Complete linear descriptions for all the feasible solutions of the single-thermal unit commitment problem have been recently provided in [150, 242]; both formulations are based on the DP algorithm presented in [149].

Tight formulations for crucial model ingredients and for complete polytopes arising in UC are also available. In particular, [263] give a best-possible formulation of minimum up and down times for power-generating units. Rajan and Takriti [358] give an alternative formulation with additional binary variables. Queyranne and Wolsey [353] extend this to handle maximum up and down times. Morales−España et al. [312] provide techniques for handling slow- and quick-start units (startup and shutdown power trajectories for slow-start units, and startup and shutdown capabilities for quick-start units).

Demand Side Management The interaction of growing distributed generation, with increased consumer flexibility and volatility of the input of renewable energy require a demand side management with unit commitment in a focal role and without neglecting the remaining determinates of the generation system. The market is invited to provide incentives for market participants to engage in Demand Side Management. Vice versa, engagement of market participants must be carried out in rational manner which, in turn, brings to the fore research at the interface of energy science and mathematics, with many open problems up to the present day. Last but not least, uncertainty of crucial model data remains a particular challenge.

6 Unit Commitment Under Uncertainty

In the presence of uncertainty, Unit Commitment (UC) either lives in a non-competitive or competitive environment. The former concerns the time before, the latter does it since deregulation of energy markets. Before deregulation, load has been the dominating entity that is prone to uncertainty [406]. After deregulation, UC-relevant sources of uncertainty have spread considerably: power input from renewables, power prices determined by bidding into power exchanges, competitors' actions at electricity markets. Yet UC is understood in a broader context than before; it rather is the scheduling of decentralized power supply with its small generating facilities than commitment of thermal let alone nuclear generation units. The integration of the production of renewable sources, due to its uncertainty, must be adequately addressed to avoid affecting the operational reliability of a power system. Generally, UC is a critical decision process that consists of an optimization problem to generate the outputs of all the generators to minimize the system cost. UC decisions are made once a day, 24 or more hours before the actual operation. The main principle in operating an electrical system is to cover demand for electricity at all times and under different conditions depending on the season, weather, and time. The common goal of UC formulations is to minimize the operating cost, while ensuring sufficient reserve to accommodate real-time realization of uncertainty. The main difference between models is the representation of this uncertainty. The deterministic UC formulation is a traditional solution in which the net load is modeled using a single forecast for each renewable output, and the associated uncertainty is managed using ad hoc rules (i.e., the generating units are committed to meeting the deterministic prediction, and the uncertainty is managed by imposing reserve requirements [289]). This approach is easy to implement in practice, but ad hoc rules do not necessarily adequately reflect uncertainty. While the mathematical apparatus is fairly well developed for exogenous uncertainty, the situation is completely different for endogenous uncertainty, i.e., with decision-dependent probability distributions. In case uncertainty is captured by probability measures, stochastic integer programming offers methodology for handling UC, both algorithmically and regarding structural understanding, see [67, 162, 384, 406, 464], among others, different approaches are used to manage the UC under uncertainty. The UC available approaches in the literature are as follows, see [231] and others:

6.1 Stochastic UC

Stochastic UC is based on probabilistic scenarios. A finite set of scenarios is generated with assigned weight for each scenario. The basic idea is to generate a large number of scenarios where each scenario represents a possible realization of the underlying uncertain factors. Stochastic UC is generally formulated as a two-stage problem that determines the generation schedule to minimize the expected cost over all of the scenarios respecting their probabilities. There is a difference between commitment and dispatch decisions: the first are the same for all the scenarios, the second are different for each scenario. The large number of scenarios in the model requires high computational demand for simulations. Similar scenarios are aggregated based on, for example, their probability or cost [231]. The structure of scenarios can be a number of parallel scenarios in a two-stage problem or a scenario tree in a multistage problem. Monte Carlo simulation [360] is often used to populate the scenarios based on probability distribution functions learned from historical data and to generate scenario trees based on stochastic processes. However, increasing the number of scenarios may lead to small improvements in the solution quality. Thus, Sample Average Approximation (SAA) [390] can be used to test the convergence of the solution. Scenario reduction techniques are used in the literature [109, 401] and [178]. The goal is to reduce the number of scenarios without sacrificing their accuracy to a large extent. An interesting approach is introduced in [465] and its UC application is presented in [464].

6.2 Robust UC

In Robust UC formulations, a deterministic set of uncertainty is used, instead of a probability distribution on the uncertain data. For example, the two-stage model in [41] has the first stage which finds the optimal commitment decision, and the second stage which generates the worst-case dispatch cost under a fixed solution from the first stage. The range of uncertainty is defined by the upper and lower bounds on the net load at each time period. In place of minimizing the total expected cost as in Stochastic UC, Robust UC reduces the worst costs to the minimum for all possible results of uncertain parameters [455]. These models produce conservative solutions, but they are better from a computational point of view because they can avoid incorporating a large number of scenarios. In the power system literature, Robust UC models have been used to address uncertainties from net electricity injection [453], wind power availability [225], demand-side management [454].

6.3 Interval UC

Interval UC formulations minimize the cost of covering the most probable load forecast by ensuring feasibility in the uncertainty range that is delimited with upper and lower bounds as in robust unit commitment formulations. There are strong differences between the stochastic optimization approach and the interval one, the model in the latter can be composed by three scenarios. In particular, the scenarios are: the central forecast, the upper bounds, and the lower bounds. However, the interval unit commitment can also be formulated as a two-stage problem where the optimal solution is found in the first stage and then tested in the second stage for feasibility. A method is proposed in [403]. To improve the advantages and reduce disadvantages of the models presented in the previous parts, **Hybrid UC** models have been proposed in the last years. Some of these models are unified stochastic and robust unit commitment formulation [451] as well as stochastic and interval unit commitment formulations [110]. Zhao and Guan[451] proposes a model able to achieve low expected total cost while ensuring the system robustness. Dvorkin et al. [110] proposes a model that applies the stochastic formulation to the initial hours of the optimization horizon and then switches to the interval formulation for the remaining hours.

7 Long-Term Unit Commitment

In the long term UC optimization models are applied to define tentative scheduling of the power plants over typically 1 year horizon in order to assess the producibility of a fossil fuel power plant and the tentative reservoirs management for hydro coupled with the (non programmable) uncertain production of renewable power plants. While for short time horizons, typically of 1 day or of 1 week, the pure short term UC problem (but not max profit UC in market related) can also be considered deterministic, for longer management horizons, a special emphasis must be put on the uncertain nature of data. In particular, on a yearly or more scale, reservoir inflows, demand, as well as availability of the plants cannot be considered deterministic. For instance in winter time customer demand can vary up to one GW per degree Celsius for big countries such as Italy, UK, France or Germany. On the other hand a rainy season can fill reservoirs and let the hydro production plants produce much more w.r.t. a dry season. Another crucial factor is related to renewables (wind and solar) power plant whose productivity fluctuations can be high. In the following we give insight to the different goals and constraints of the long term UC.

The main goal of the long term UC is to decide the production levels of the plants comprising the mix in such a way that the demand is satisfied at each time step and the production cost is minimized. The physical model typically considered is a stochastic or robust dynamical system for which the uncertain parameters are (a) the electricity consumption, (b) the availability rates of the thermal plants (either due to optimized scheduled maintenance or faults) and (c) the quantity of inflows received by the different reservoirs of the hydroelectric power stations. An additional goal of a long term UC could be a definition for a GenCo (or for the monopolist) of the gas long term (ToP) contract to be signed. As a reversed engineered problem also an optimized schedule for maintenance can be deduced.

- **Thermal units**. Thermal (including nuclear) power plant are modeled in a simplified manner w.r.t short term UC, main constraints include only min and maximum stable production and often simplified (e.g. linearized or constant) cost curves
- **Hydro Units**. Hydro units are modeled in a simplified fashion w.r.t. short term UC, for small basin production minimum is relaxed to 0 and very often cascade are aggregated to single production units
- **Renewable non programmable (i.e. wind and solar)**. These power plants do not actually have operational constraints but due to their intermittency the UC modeler should try to have a tentative forecast of their production profile perhaps by geographical aggregation. More importantly than in the short term cases the inherent uncertainty in their forecasted schedule in turns calls for stochastic-like approaches.
- **Electricity Demand**. Uncertainty in demand global values and profile shape are the most important data to deal with. Both the global demand and, separately, the demand profile are important to the solution of a long term UC. On the other hand this electricity demand uncertainty couples with the uncertainty of the Renewable non programmable units.

8 Balancing Markets and Non-programmable (Renewable) Power Coordination

The market share of renewable energy sources, and among them, intermittent energy sources, is increasing. These energy sources offer various features in terms of predictability. Tidal energy is highly predictable. Instantaneous or daily production of a photovoltaic installation may be somewhat random, but annual and seasonal productions are fairly predictable. Production of on-shore wind turbine is difficult to predict, regardless of the time scale.

This raises specific issues in order to ensure proper matching between supply and demand. In order to meet these issues, a number of solutions can be proposed:

- Diversity: Even though the production of an individual power unit may be hardly predictable, the overall production of a large number of units is usually much more predictable. Clustering non-correlated, or preferably, anti-correlated power units can improve significantly predictability.
- Storage: Energy storage is still expensive, costs are dropping steadily.
- Exchange: European grid markets include power exchange and bilateral contracts. This feature offers a large flexibility for balancing electricity demand and supply.
- Previsions: Accurate prevision models for production and consumption are a valuable support for grid management.
- Adapting demand to supply: A large number of electrical devices (thermal equipment, batteries, etc.) can support some power interruptions or delays in power supply without compromising the user's comfort. Adapting demand to supply, coupled with incentive pricing, may be a more relevant solution than the other way round.

The problem of balancing a market can be seen as an optimization problem in a competitive environment with uncertainty in resource availability and demand, or as a time-varying problem. Following much work on convex variants of time-varying optimization [39, 40, 88, 192, 394, 409], Liu et al. [271] presented an approach to the non-convex time-varying optimal power flows in the alternating-current model. Alternatively, it can be seen as regulation of a non-linear (and possibly time-varying) system in control theory. Either way, many standard methods fail, as shown by Fioravanti et al. [141] and Fioravanti et al. [142], and even those listed present only an initial take on an otherwise very open problem.

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1 Networks and Storage: An Introduction

The traditional view of electric power systems suggested that power cannot be efficiently stored and that the partially controllable supply and exogenous demand have to be matched at all times. The networks in electric-power systems are traditionally divided into the high-voltage "transmission" system and the lower-voltage "distribution" system. The transformers connecting the low-voltage distribution system to the high-voltage transmission system are often in so-called distribution substantions. From the point of view of the distribution system, the substation provides a source of electric power, and traditionally the only source. From the point of view of a transmission system, the aggregate demand of customers (in the traditional un-observable low-voltage distribution system) is "revealed" as voltage at the substations. The voltage in the transmission system then drives the power flow within the transmission system, as well as the generation of power at the (so-called synchronous) generators. This view is now being challenged by the increasing availability of demand response management, distributed generation, storage, and generators that cannot easily respond to voltage changes by changing power output. Still, electric power transmission is of paramount importance, and its physical shape is changing only very slowly.

1.1 The Physical Reality

Let us elaborate upon the physical reality of power transmission. The electric power transmission can be implemented using a variety of means, with the most common one being the overhead power lines. In overhead power lines, pylons are connected by (most often) multiple high-tension lines, each of which is typically made of aluminium wires wrapped around a steel core. Possibly, there may be additional sensors along the line, such as fiber optics for capturing the temperature gradient, or sensors measuring the magnetic field induced by the current flowing along the line. One typically utilises the alternating current (AC) in over-head power lines, as explained below.

As an alternative to overhead power lines, one can use underground and submarine power transmission, albeit at much higher investment costs and sometimes higher operational costs. First, the investment costs are higher, due to the needs for excavations, insulation, and power electronics. Underground or under sea, high-voltage (HV) cables require considerable amounts of insulation (often based on pressurised oil or polyethylene). Because alternating current allows only for very short lines (under 50 km), due to the high capacitance of the cable, one often uses high-voltage direct current (DC), which requires considerable investment in (and losses at) the power electronics involved in the AC-DC and DC-AC conversion (known as rectifiers and inverters, respectively). While the speed of the spread of HVDC cables beyond the situations, where they are deployed already, cf. the deployment of underground power transmission and distribution in Denmark and cables between Germany and Sweden (Baltic Cable), and Norway and the Netherlands (NorNed), remains unclear, it is clear that these require modelling both AC and DC transmission as well as the related power electronics, in many countries.

One connects multiple power lines at so-called substations. At the most basic, these can house a large slab of metal (e.g., copper), which is called a “bus”, onto which the power lines are physically attached and which equalises the voltage of the connected ends of the connected power lines. More often, one connects multiple transmission lines to multiple coils (so-called windings) of a transformer. The transformer can step up or steps down the voltage, in discrete steps, depending on how the windings are connected. There can be step-up transformers connecting a power station to the high-voltage transmission network (e.g., 400 kV), step-down transformers within the transmission network (e.g., to 220 kV and 110 kV), and then there are step-down transformers connecting the high-voltage transmission system to the distribution system (e.g., 55 kV). Traditionally, the settings of the transformer been limited to the step, and fixed for substantial periods of time, although this is changing (cf. FACTS devices below). Such a transformer would still often be referred to as a bus in an abstract view of buses connected by branches.

1.2 Models of Electric Power

Let us now elaborate upon the models of electric power. Although in general, the current and voltage are an arbitrary signal in both alternating and directed current systems, and one could hence use signal processing throughout, one often assumes the harmonic currents to simplify the modelling of real-world power systems. There, voltage, current, and power are sine waves with magnitudes, angular frequency ω, and π is the phase. (Notice that one can use Fourier transform to approximate any signal by sinusoids). Then, we have a closed-form solution for integral for the average power transmitted, which is equal to the product of the current and the voltage and the cosine of the phase. Together with the usual relationships of:

- ohmic heating (i.e., losses equal to the product of the resistance and the square of current),
- Kirchhoff’s current laws (e.g., sum of current injected is the sum of currents ejected, modulo losses),

one can formulate a variety of mathematical models for the harmonic currents, all of which are non-convex.

A key choice in formulating a mathematical model of harmonic currents is the choice of sine-waves to represent and the choice between polar and rectangular representation thereof. Generally speaking, using the rectangular representation, one can often derive a polynomial optimisation problem (POP), while using the polar representation, one obtains a problem with trigonometric constraints. In some cases, it may also be beneficial to combine both representations, especially when one considers piece-wise linearisations. The key choices studied so far include:

- polar power and polar voltage [69, 70, e.g.], where power generated at generators and all voltages (except for a reference bus) are employed
- rectangular power and polar voltage [294, e.g.]
- rectangular power and rectangular voltage [168, 262, e.g.], where power generated at generators and all voltages (except for a reference bus) are employed
- rectangular current and rectangular voltage [294, 335, e.g.], where currents and voltages are represented
- rectangular current injection.

We refer to [64] for a survey of the history of these formulations. We note that one may consider problem with trigonometric constraints and derive the polynomial optimisation problem [74] using substitutions and one of several well-known trigonometric identities. This way, one obtains many further polynomial optimization formulations.

1.3 Approximations

Considering the non-convexity, one often utilises approximations of a widely varying quality, and widely varying shape, dependent on the choices above. The simplest approaches assume the problem is convex, while it is clearly not, and apply gradient methods or Newton method directly to the non-convex problem. In this case, convergence guarantees can be obtained only for starting points within the vicinity of a local optima; recently, it has been shown that whether one is close enough is actually testable [270].

Without a starting point in the vicinity of a local optimum, one often considers either convex relaxations. or mixed-integer convex approximations (e.g., piece-wise linearisations), as in much of optimisation and control. In the most simple relaxation, known as the Direct Current (DC) model (but confusingly applied to AC systems or systems combining AC and DC transmission), the network structure is taken into account, including the capacity of the transmission links, but a simplified version of Kirchhoff laws is used so that the corresponding constraints become linear. In more sophisticated convex relaxations, one uses semidefinite programming (SDP) and second-order cone programming (SOCP). One should like to note that such sophisticated convexifications [168, 279, e.g.] can be made arbitrarily strong, i.e., with solution arbitrarily close to the solution of the non-convex problem, albeit at a major expense of computational power. Within mixed-integer convex approximations, one often considers piece-wise linearisations.

The most common convexifications include, depending on the choice of the variables:

- rectangular power and polar voltage can be piece-wise linearised in either an inexact and well-performing or asymptotically exact and rather less well performing fashion
- rectangular power and voltage yields very strong semidefinite-programming relaxations, and convergent hierarchies of semidefinite-programming relaxations
- rectangular current and voltage, which produces weaker convex relaxations, but may be suitable for the use in optimal transmission switching and network expansion planning, where the current may be set to zero without consider a high-degree polynomial
- rectangular current injection, which may again be suitable for the use in optimal transmission switching and network expansion planning, whenever the degree of a polynomial is less of a concern than the dimension of the system.

It should be noted that the convex and piece-wise convex approximations are an active area of research and many rules of thumb above may be invalidated yet. Finally, one sometimes uses the so called "transportation models", where network flows of units of energy are considered.

Let us now consider the time scale for the application of the approximation. Clearly, the changes to demand and (consequently) voltage are continuous. Some changes of limits on the power output may be continuous (e.g., wind power at low winds), while others may be discontinuous (e.g., where there is no momentum, e.g., when a wind turbine gets disconnected due to high winds). As in much of optimisation and control, one often considers a discretisation of time and classical batch-optimization algorithms that compute optimal operations based on a fully-specified input, valid at one point in time. One should note, however, that with the increasing volatility, this may seem inadequate. Novel algorithms that capture the inherent time-varying nature of the problem and leverage on-line optimization techniques as well as insights from control theory [39, 40, 87, 88, 192, 394, 409, 457] show a certain promise. One should like to point out that they present only an initial approaches to an otherwise very open problem, considering their use of crude convex approximations of the non-convex problem. The use of on-line non-convex optimisation [271] is nascent.

1.4 Looking Beyond

Going beyond the traditional view of power systems, energy storage [108] is a very active are of research within electrical engineering and materials engineering. Current large-scale implementations are based on pumped hydroelectric energy storage (PHES), which provides close to 40 GW of capacity in Europe and a similar capacity in the United States. Pilot projects involve lithium-ion batteries, cf. deployments in New England and Australia, lead batteries, sodium batteries, (super)capacitors, pumped storage underwater reservoirs, spinning rotary machinery (fly-wheels), compressed air, heavy-goods trains pushed uphill cranes lifting weights in the air or in a mine shaft, and many other suggestions. It should be noted that the reach of pumped hydro is limited to areas with the appropriate physical geography, while the pilot projects have not shown a system that would be clearly commercially feasible to operate at scale. See Sect. 6 It is hence not clear what shape and form energy storage would take, eventually.

Finally, in demand response management (DRM), one hopes to "emulate" energy storage by incentivising customers to amend their consumption in real time. We refer to Sect. 1 above. Although the first related policies have been proposed decades ago, large-scale deployments are still rather limited to, e.g., deferrable loads in industrial refrigeration. Still, DRM excites many, due to its zero losses, and hence costs bounded from below only by zero. One can construe a "virtual power plant" (VPP) being formed this way.

2 An Overview of Network-Constrained Optimization Problems

At a high-level, network-constrained problems of electric power systems can be characterised by the market environment they consider, and the time horizon. In vertically integrated systems the strategic electrical network management is performed in an integrated fashion by the monopolist, whereas in market-based ones, the responsibilities are split between the operators of the generating capacity (GenCos), operators of the high-voltage transmission system (TSO), lower-voltage distribution system (DSO), and possibly market operators and regulators. Long-term planning problems include:

- Network Expansion Problems: Expand the networks by constructing new branches and possibly removing old ones. Additionally, the decision of installing network technologies, together with their siting, can be considered in the expansion and reinforcement process.
- Energy Storage System (ESS) Siting and Sizing: Deciding the location and the size of an ESS, e.g. [320].
- Smart Grid Design: The actual design of a smart grid includes the siting and sizing of technologies that could enhance the observability and controllability of the system and include: Phasor Measurement Units (PMUs), Wide-Area Measurement Systems (WAMS), and notably Flexible Alternating Current Transmission Systems (FACTS).

As a sub-problem of a long-term problem, or independently over a shorter time horizon, one considers a variety of operations problem:

- Load Flow (LF): LF is actually not an optimization problem, but rather a calculation of the power flowing along an electrical network, once we have fixed

the generation schedule and the load in the substations. While not an optimization problem, it gives evidence on the networks operating points under different conditions. LF can also be used integrated in "what if" analyses.
- Optimal Power Flow (OPF): The OPF problem deals with the continuous-valued decisions within the optimization of the generating cost, and operations of renewable energy sources (esp. hydropower), considering the electricity grid. In considering the grid, OPF takes into account the non-linear Kirchhoff laws and the restrictions on power flow on each branch (transmission line) and voltage angles. Typically, the generation cost optimization is performed considering all the units status (on or off) fixed to a feasible status otherwise found. Similarly to the LF, OPF can also be used in a what-if analysis tool.
- Security Constrained (SC) Problems: Integrated problems, wherein one wants to consider a detailed set of constraints modelling reliability of the power plants and the grid, as well as the physics of the grid itself. Typically, the goal is to find a least cost schedule of production and flows that is also resistant to unpredictable fault of one of the components (power plant, network branch etc.). The n-1 security problem refers to a single fault. From a methodological standpoint one could consider n-k models with k faults, and some models in this direction have been presented. In practice, TSO tend to decouple OPF or unit commitment from n-k models, solving this latter problem by adding security requirements to an already quasi-fixed solution from SCUC [44].

In the following sections, we introduce these problems in turn.

One should like to note that the two horizons are not disconnected. It is very important to consider the operation and scheduling of generation and storage units already at design phase to determine the most convenient combination of technology selection and size. This is especially true when dealing with sizing of energy storage. Long-term storage systems have recently caught much attention due to their ability to compensate the seasonal intermittency of renewable energy sources. However, compensating renewable fluctuations at the seasonal scale is particularly challenging: on the one hand, a few systems, such as hydro storage, hydrogen storage and large thermal storage can be used to this purpose; on the other hand, the optimization problem is complicated due to the different periodicities of the involved operation cycles, i.e., from daily to yearly. This implies long time horizons with fine resolution which, in its turn, translates into very large optimization problems. Furthermore, such systems often require the integration of different energy carriers, including electricity, heat, and water. Exploiting the interaction between different energy infrastructure, in the so-called multi-energy systems (MES), allows to improve the technical, economic and environmental performance of the overall system [290].

To consider another such integrated problem, consider the discrete decisions (the so-called unit commitment problem) as a sub problem at design phase, which implies taking into account the expected profiles of electricity and fuel prices, weather conditions, and electricity and thermal demands along entire years. Moreover, the technical features of conversion and storage units should be accurately described. The resulting optimization problem can be described through a mixed integer nonlinear program (MINLP), which is often simplified in a mixed integer linear problem (MILP) due to the global optimality guarantees and the effectiveness of the available commercial solvers (e.g. CPLEX, Gurobi, Mosek, etc.). In this context, integer variables are generally implemented to describe the number of installed units for a given unit, whereas binary variables are typically used to describe the on/off status of a certain technology. Furthermore, decomposition approaches relying on heuristic algorithms for unit selection and sizing have been proposed. A comprehensive review of MINLP, MILP and decomposition approaches for the design of MES including storage technologies has been carried out by Elsido et al. [112]. However, independently of the implemented approach, significant model simplifications are required to maintain the tractability of the problem. Such simplifications include limiting the number of considered technologies, restricting technology installation to a subset of locations, analysing entire years based on seasonal design days or weeks, or aggregating the hours of each day into a few periods. Such integrated problems are a major direction for future research.

3 Problems of Network Expansion Planning

Network expansion planning (NEP) is one of the main strategic decisions in power systems and has a deep, long-lasting impact on the operations of the system. Relatively recent developments in power systems, such as renewable integration or regional planning, have increased considerably the complexity and relevance of this problem.

NEP has multiple criteria, albeit frequently combined into a single objective function, perhaps by considering the costs of the multiple criteria in a single monetary objective. The main criteria are usually: costs, environmental impact, market integration, and certain "exogeneous" factors. Costs are measured by the attributes such as investment and operating costs of the transmission decisions, but also operating costs of the system. In the cost criterion, one can also consider reliability. Environmental impact is determined by attributes such as the amount of renewable integration or curtailment avoided at system level and impact of the line construction. Market integration is accounted as the number of hours of market splitting. Social acceptance is an exogenous criterion and, nowadays, is a major concern of the current planning process and is the cause of many delays.

Among the current challenges to be addressed for the network expansion planning we can mention the following ones:

- coordination with GEP, as discussed previously. On the one hand, GEP is a deregulated business activity, while NEP is mostly regulated. On the other hand, generation investments can take around 3 years, while network expansion needs to be anticipated longer periods.

- renewable integration is one of the major drivers for investing in new transmission lines. Onshore and offshore wind power, and solar generation are renewable technologies currently being developed at large scale to meet the low-carbon electricity generation targets. A large part of this generation is located in remote areas far from the load centers requiring transmission reinforcements or new connections. Besides, the intermittent nature of these renewables introduces operational challenges and, from the network planning point of view, many varied operation situations should be considered.
- market integration is the current paradigm to achieve a competitive, sustainable and reliable electric system and the network is a facilitator in this process. The creation of an European internal market with strong enough interconnection capacity among the member states increases the scope of the planning process, from a national activity to a European scale.

A variety of mathematical optimization techniques are used for solving the network expansion planning.

Classical methods include linear, nonlinear and mixed integer programming methods. Linear optimization ignores the discrete nature of the investment decisions but still it can be useful is system is too large to be solved with discrete variables or a relaxed solution is good enough. A transportation or a direct-current (DC) load flow fit in this linear formulation. Nonlinear, in particular quadratic, models appear as a way to represent transmission losses. Finally, mixed integer optimization allows considering the integer nature of the decisions. If stochasticity in some parameters is included then models become stochastic and, therefore, decomposition techniques should be used for large-scale systems. Among them, Benders decomposition, Lagrangean relaxation and column generation are frequently used.

There is a vast array of academic literature on the subject. References [201, 261, 276] provide a good starting point. In the following two sections, we point to original research on Sects. 4 and 5.

4 Transmission Network Expansion Planning (TNEP)

Transmission network expansion concerns the expansion of the high-voltage part of the network. Market integration is the current paradigm to achieve a competitive, sustainable and reliable electric system and the network is a facilitator in this process. This is particularly true in the case of the EU. These challenges have been addressed in a vast array of both projects and papers. Thorough reviews of the academic literature on this topic can be found in [364, 387, 388].

A wide variety of models and the corresponding mathematical optimization techniques are used in solving the network expansion planning. Initially, models can be classified as either linear, non-linear, mixed integer linear (MILP), or non-linear (MINLP). Linear models, often based on transportation or direct-current (DC) load-flow, ignore the discrete nature of the investment decisions, but can be useful as an approximation. Nonlinear models, often quadratic, represent transmission losses, but still usually ignore the discrete nature of the investment decisions. MILP approaches allow for the integer nature of the investment decisions to be considered, but are restricted to an approximation of the non-linearity, either using piece-wise linearisations, or linearisations including the DC and transportation load-flow. If stochasticity in some of the parameters is considered, then models may become challenging to solve, and decomposition techniques are often used for large-scale instances. Finally, one may consider the full MINLP model: there, both the discrete and non-linear features of the problem are modelled faithfully, but the problem is challenging.

Further, one may consider a wide variety of objectives, although a single objective function is often obtained by combining the multiple criteria into one, e.g., by considering the monetary costs associated with each criterion, and minimising the total monetary costs across all criteria. The main criteria are usually: investment costs, costs of operations (OPEX), reliability issues, environmental impact, market integration factors, and rarely, other factors. While investment costs are often relatively straightforward to estimate, the operational expenses associated may be harder to estimate, especially considering the long planning horizon often considered. Similarly, the impact on reliability is often modelled only very approximately. Environmental impact is often evaluated in terms of the amounts of renewable integration made possible, or curtailment avoided at system level, in response to the line construction. Market integration is accounted as number of hours of market splitting. When the monetary costs of such approaches cannot be approximated, metaheuristic approaches may provide a sample of the feasible solutions, without any guarantees of their distance to optimality.

Considering GEP is a deregulated business activity, while NEP is mostly regulated at both national and super-national levels, one may also introduce market considerations explicitly. For example, one may consider an equilibrium in a pool-based market at one level, possibly including spot prices, and the transmission and generation expansion at another level. Such bi-level and multi-level models have been attempted, but often increase the complexity to a point, where real-life applicability is limited, considering the extent of many markets. In particular: Many super-national markets area already in operations. The eventual creation of a single European internal market with strong-enough interconnection capacity among the member states, for instance, increases the scope and complexity of the planning process.

Further, one may attempt to solve a problem combining the expansion of transmission (NEP) with the expansion of generation (GEP). Clearly, generation expansion has bearing upon network expansion, and vice versa. In particular, renewable integration is one of the major drivers for investing in new transmission lines. Onshore and offshore wind power, and solar generation are renewable technologies currently being developed at large scale to meet the low-carbon electricity generation targets. A large part of this generation is located in remote areas far from the load centres, and hence requires transmission reinforcements or new connections. Besides, the intermittent nature of these renewables introduces operational challenges and, from the network planning point of view, many varied operation situations should be considered. In such integrated problems, the size of the instances grows.

Within linear models, such as transportation or direct-current (DC) load-flow, general-purpose linear programming optimisation software is often used, based either on simplex or interior-point (barrier) methods. Often, it turns out to be challenging to devise a problem-specific method, whose performance improves upon the general-purpose methods. Still, in case of particularly large-scale instances, problem-specific decompositions such as column generation are used.

Within nonlinear models, often quadratic, a wide variety of methods is used, considering the limitations of the general-purpose non-linear programming optimisation software. Since 1990, interior-point methods have been most popular. First-order methods, including gradient and coordinate descent, and their stochastic variants, had been used prior to this and also very recently, inspired by their resurgence within machine learning.

Within MILP models, there has been much recent progress in general-purpose optimisation software based on branch-and-bound-and-cut. Often, modest instances considering either piece-wise linearisations or uncertainty, can be solved exactly using the general-purpose software.

Decompositions, such as Benders decomposition, Lagrangian relaxation or column generation are frequently used.

Within MINLP models, the methods are an active area of research, considering the limitations of the general-purpose non-linear programming optimisation software. Marecek et al. [294] surveys three convergent approaches, based on piecewise linearisation of certain higher-dimensional surfaces, based on the method of moments, and based on combining lifting and branching. The preliminary conclusion is that the combining lifting and branching may be the most promising.

We refer to [201, 239, 261, 276, 364, 387, 388] for detailed surveys. See [294] for the impact of the impact of the choice of model (AC vs. PWL vs. DC), [389] for an illustration of the impact of security of transmission constraints, [423] for an example of the impact of the uncertainty.

Software

Within two-stage approaches, there is a long tradition of work on decomposition methods [30, 344], although even a monolithic scenario expansion may be tractable [273, 383, 410], when AC and security of transmission constraints are ignored and the model of the network [383] is sufficiently coarse. The incorporation of market considerations [348, 364] complicates matters considerably. Within multistage approaches, there are very well-developed decompositions [5].

5 Distribution Network Expansion Planning (DNEP)

Network capacities were designed with a wide safety margin, so for a long time expansion planning in electrical energy systems was concentrated on generation expansion planning (GEP) with the goal of covering cumulative demand uncertainty based on averaged historic demand data in monopolistic environment for energy transmission. These were modelled as stochastic optimization problems with a one dimensional demand distribution represented by two-stage or multi-stage scenario trees that were generated by Monte Carlo methods. The models went to the limit of computational possibilities at any point in time, included binary decision variable, with a risk neutral approach and, then, only expected values in the objective function where considered in the time horizon over the scenarios. Very limited use was made of risk averse measures.

In order to solve the large-scale problems, decomposition methods played a central role, in particular the following methodologies:

- Two-stage Benders Decomposition (BD) for linear problems [37]. See [24, 275, 424], among many others.
- Multistage Benders Decomposition (BD) methodology for linear problems. See [45] among others.
- Two-stage Lagrangian Decomposition (LD) heuristic methodology. See [68, 115, 118, 172, 173, 267, 330, 332], among many others. See also [405, 421] for two surveys on the state-of-the-art of two-stage stochastic unit commitment, and using LD with bundle methods. See also [113, 371, 422] two-stage LD approaches with bundle methods applied to energy problems.
- Multistage Clustering Lagrangian Decomposition (MCLD) heuristic methodology. See [121, 126, 128, 287], among some others.
- Regularization methods. See [26, 267, 317, 368, 369, 386], among others.
- Progressive Hedging algorithm (PHA) for multistage primal decomposition. See [363, 438], among others.
- Nested Stochastic Decomposition (NSD). See [8, 86, 122, 127, 181, 246, 323, 345, 346, 367, 391, 463], among others.
- Multistage cluster primal decomposition. See [9, 10, 17, 34, 126, 287, 339, 377, 448], among others.
- Parallelized decomposition algorithms. See [8–10, 16, 26, 38, 269, 317, 339, 367, 377, 448], among others.

Today, new power production possibilities, technological developments and deregulation bring along several new sources of uncertainty with highly differing levels of variability. In addition to traditional demand, these are foremost dependencies on wind, market prices, mobile electricity consumers like cars, power exchanges on international level, local energy producers on distribution network level and, to a lesser extent, solar radiation. This introduces complex and volatile load and demand structures that pose a severe challenge for strategic planning in production and transmission and, on a shorter time scale, in distribution.

Networks may now be equipped with new infrastructure like Phase Measurement Units (PMUs) and other information technology in order to improve their cost efficiency. At the same time these upgraded networks should ensure high standards in reliability in their daily use and resilience against natural or human caused disasters. Companies now have teams devoted to the task of generating suitable planning data.

In optimization models, the emphasis has shifted to high dimensional stochastic data and to considering risk reduction measures instead of expected values. Computationally integrated models considering all relevant aspects are out of scope. Even for simplified models it is often difficult or not known how to provide stochastic data of sufficient quality [399]. Alternatives are then:

- robust optimization, where distributions are replaced by "easier" uncertainty sets [36],
- methods, where uncertainties are replaced by a kind of interval arithmetic equipped with scenario dependent probabilities [447],
- stochastic approaches: where some input data follow probability density function and some can be represented by fuzzy membership functions [397].
- information gap decision theory that aims at hedging against information errors [35, 355].

Methods for solving these stochastic optimization problems with binary decision variables employ the same decomposition approaches listed above, but much more care needs to be devoted to the properties of the decomposition. For risk averse measures in multistage models, methods are distinguished regarding their "time consistency" or "time inconsistency". So far, stochastic dynamic programming approaches are the most suitable ones for dealing with the time consistency property of risk measures, so that the original stochastic problem may be decomposed more easily via scenario clustering and cluster dependent risk levels.

In power generation optimization models for big companies the following are the issues of relevance, mainly addressed in the context of market competition:

- when and where to install how much new production capacity, mainly considering wind generators and thermal plants.
- how to extend or renew hydro plants and where to install what pumping capacities. Today, solar power is typically handled at the level of distribution networks.

In contrast, competition is not an issue for transmission and distribution network operators. Regulations on efficiency, reliability and resilience levels are the driving force in the following problems:

- when and where to install how much network capacity and information equipment,
- reducing transmission losses,
- reducing distribution losses (technical and detecting non-technical ones).

Challenges today and for the future comprise:

- The robust approach allows for safe optimization with uncertain data. What information can be extracted from these robust solutions e.g., on which additional data would be needed to improve the quality of the model?
- Several risk averse measures have been proposed, each with its advantages and disadvantages. How to make use of them in the best way?
- How to deal with endogenous uncertainty, i.e., with optimizing big player decisions that influence the probability distributions that are optimized over?
- How to construct hierarchical decomposition approaches in a consistent way?
- How to make use of high-performance computing (HPC, multi-core or Distributed) in decomposition approaches?
- How to integrate chance constraints (ICC), e.g., with respect to reliability or resilience?

General goals for future models include: increasing the level of integration; bringing models closer to reality by avoiding the excessive linearization of nonlinear aspects; reducing the gap between methods used in academia and those applied in practice; making use of new monitoring devices and communication systems; exploring the chances of cooperation between electric and other energy commodity systems.

On the software side, general-purpose stochastic optimization software still seems far away. Planning models are highly problem-dependent and off-the-shelf packages are not available. Companies use modelling languages like GAMS [398], AMPL, AIMMS, Python together with standard solvers to develop problem-specific approaches.

6 Energy Storage System (EES) Siting and Sizing

It is very important to consider the operation and scheduling of generation and storage units already at design phase to determine the most convenient combination (i.e., minimum objective function) of technology selection and size. This is especially true when dealing with selection, sizing and unit commitment of long-term, or seasonal, energy storage. Long-term storage systems have recently caught much attention due to their ability to compensate the seasonal intermittency of renewable energy sources. However, compensating renewable fluctuations at the seasonal scale is particularly challenging: on the one hand, a few systems, such as hydro storage, hydrogen storage and large thermal storage can be used to this purpose; on the other hand, the optimization problem is complicated due to the different periodicities of the involved operation cycles, i.e. from daily to yearly. This implies long time horizons with fine resolution which, in its turn, translates into very large optimization problems. Furthermore, such systems often require the integration of different energy carriers, e.g., electricity, heat and hydrogen. Exploiting the interaction between different energy infrastructure, in the so-called multi-energy systems (MES), allows to improve the technical, economic and environmental performance of the overall system [290].

In this framework, including the unit commitment problem already at design phase implies taking into account the expected profiles of electricity and gas prices, weather conditions, and electricity and thermal demands along entire years. Moreover, the technical features of conversion and storage units should be accurately described. The resulting optimization problem can be described through a mixed integer nonlinear program (MINLP), which is often simplified in a mixed integer linear problem (MILP) due to the global optimality guarantees and the effectiveness of the available commercial solvers (e.g., CPLEX, Gurobi, Mosek, etc.). In this context, integer variables are generally implemented to describe the number of installed units for a given unit, whereas binary variables are typically used to describe the on/off status of a certain technology. Furthermore, decomposition approaches relying on meta-heuristic algorithms for unit selection and sizing have been proposed. A comprehensive review of MINLP, MILP and decomposition approaches for the design of MES including storage technologies has been carried out by Elsido et al. [112]. However, independently of the implemented approach, significant model simplifications are required to maintain the tractability of the problem. Such simplifications include limiting the number of considered technologies, restricting technology installation to a subset of locations, analyzing entire years based on seasonal design days or weeks, or aggregating the hours of each day into a few periods.

7 Optimal Power Flow (OPF)

In the optimal flow problem, the costs of generation and transmission of electric energy is optimised, taking into account the active and reactive power generation limits, demand requirements, bus voltage limits, and network flow limits. In the alternating-current (AC) model, OPF is formulated as a non-convex optimisation problem (ACOPF) that is generally difficult to solve, due to the non-linear nature of the power-flow constraints. The problem was first formulated in 1962 and a large number of optimization algorithms and relaxations have been proposed [308, and references in] since then.

The directed-current optimal power flow (DCOPF) is a popular approximation based on the linear programming problem, which is obtained through the linearisation of the power flow equations. While DCOPF is useful in a wide variety of applications, a solution of DCOPF may not satisfy the non-linear power flow equations and hence the resulting solution may be infeasible and may be of limited utility.

Numerous heuristic algorithms were proposed for the OPF, including Newton-Raphson, Lagrangian relaxation, and primal-dual interior point methods. Although some of these algorithms can handle large-scale networks most them can only compute stationary point usually without assurance on the quality of the solution.

That is because most of the algorithms rely on first-order (Karush-Kuhn-Tucker) necessary conditions of optimality, which cannot even guarantee a locally optimal solution, in the non-convex problem, without considering the presence in the basin of attraction of a global optimum [270].

Alternatively, the OPF can be formulated as a non-convex quadratically constrained quadratic program, or more generally polynomial optimisation problem (POP). There, convex relaxations within second-order cone (SOCP) programming and semidefinite programming (SDP) can be applied. In contrast to the other proposed approaches, convex relaxations make it possible to check if a solution is globally optimal. If the solution is not optimal, the relaxations provide a lower bound and hence a bound on how far any feasible solution is from optimality. In particular, [27] proposed the first semidefinite programming relaxation for the ACOPF for general networks. Its strengthened versions [295] make it possible to find globally optimal solutions for several well-known instances. More recently, the moments and sum-of-square decomposition have been used [168, 250] to build hierarchies of improving SDP relaxations for a polynomial programming formulation of ACOPF. To overcome the computational complexity of using SDP and polynomial programming, sparsity has been exploited [168, 281] to simplify the SDP relaxation of the OPF. A number of challenges remain:

- To further improve the scalability of SDP relaxations, Alternating Direction Method of Multipliers (ADMM)-based computation can be used to solve sparse, large-scale SDPs [281].
- Alternatively, cheaper hierarchies are being investigated based on LP and SOCP relaxations. In the future, a combination of hierarchies mixing constraints from different cones may be envisioned.
- Another issue to address is development of techniques to certify infeasibility of optimal power flow instances.
- From an industrial point of view, dealing with incomplete data is one of the issues models and tools have to address. Aggregations of industrial data may lead to physically non-meaningful models, since some section of the power network are not represented in the data.

8 Security-Constrained Optimal Power Flow (OPF)

The security constrained optimal power flow (SCOPF) is an extension of the standard OPF which takes into account line outages that have an effect on the line flows. The SCOPF problem is modelled as a nonconvex mixed-integer non-linear, large-scale optimization problem, with both continuous and discrete variables. The optimization problem determines a generation dispatch with lowest costs while respecting the constraints, both under normal operating conditions and for specified disturbances, such as outages or equipment failures. A number of issues make the SCOPF much more challenging than the OPF problem: the significantly larger problem size, the need to handle discrete variables describing control actions (e.g. the start up of generating units and network switching) and the variety of corrective control strategies in the post-contingency states.

Similar to OPF problems, different solution approaches have been proposed to solve the SCOPF problem such as linear programming approximations and heuristics in addition to non-linear-programming based methods. For example, to obtain feasible solutions, [180] propose to adjust the generation levels with the commitment states obtained in the dual solution of the Langrangian relaxation [211].

9 Optimal Transmission Switching (OTS)

The Optimal Transmission Switching deals with changing the transmission network topology in order to improve voltage profiles, increase transfer capacity, and reduce the market power of some market participants. The topology is changed, primarily by the deliberate outage of some specific transmission lines. Further, one may also consider, the use of phase shifters (which change the angle difference between two adjacent buses) and other Flexible Alternating Current Transmission System (FACTS) devices (which can, among others, increase/decrease the impedance of two adjacent buses). The change in topology can be done by one or combination of the following actions:

- Deliberate outage of some specific transmission lines
- Adding phase shifters (these devices can change the angle difference between two connected buses)
- Adding Flexible Alternating Current Transmission System (FACTS) devices (these devices can increase/decrease the impedance of two connected buses in the system)
- Adding reactive series impedance (these devices can increase the impedance of two connected buses in the system) [400]

The idea of topology dispatch has been studied for several decades [170, 196, 299, 334], although it has gained much attention recently thanks to [143, 196], who have demonstrated how it can provide the electricity market with greater efficiency and competition. This idea was further developed in [197, 198, 373, 429] by not only considering the normal operation but also the N-1 contingencies and financial transmission rights (FTR) and Flexible Alternating Current Transmission System (FACTS) devices. The unit commitment problem constrained by transmission system is solved in [428].

Much of this early modelling work has been performed using linear programming (LP) approximations of the alternating-current power flow and can be applied to large-scale transmission systems. The present best LP formulations have been presented by Kocuk et al. [244] and Fattahi et al. [137].

Much recent work considers non-linear relaxations, in order to model the alternating-current transmission constraints without piece-wise linearisation. Jabr

[219] proposes an SOCP relaxation and [245] extends it. Marecek et al. [294] have experimented with the sparse variant of the method of moments for two formulations, lift-and-branch-and-bound using SDP relaxations, and certain piece-wise linearisations. Capitanescu and Wehenkel [65] and Sahraei−Ardakani et al. [372] study of heuristics based on non-linear optimisation. Generally, convergent methods considering the line-use decision within the alternating current model [219, 245, 294] have turned out to be challenging.

For mixed-integer linear-programming (MILP) models, there has been much recent progress in general-purpose optimisation software based on branch-and-bound-and-cut. Often, modest instances considering either piece-wise linearisations or uncertainty, can be solved exactly using the general-purpose software. Decompositions, such as Benders decomposition, Lagrangian relaxation, or column generation [428, 429] are frequently used beyond that.

For mixed-integer non-linear programming (MINLP) models, the methods are an active area of research [65, 83, 219, 245, 294, 372], considering the limitations of the general-purpose non-linear programming optimisation software. Marecek et al. [294] surveys three convergent approaches, based on piece-wise linearisation of certain higher-dimensional surfaces, based on the method of moments, and based on combining lifting and branching. The preliminary conclusion is that the combining lifting and branching may be the most promising.

See also Transmission expansion planning, which is structurally very closely related, although the uncertainty is often modelled differently. Note also one would often [429] like to expand the network knowing that one can perform switching later.

10 Optimal Network Islanding and Restoration

The power systems are usually subject to disturbances which may lead to loss of synchronization between groups of generators and possibly blackouts.

The system islanding refers to the condition, in which some areas of the transmission or distribution system are disconnected from the main grid, however the power supply continues in that region by local generating facilities. It may happen automatically, after some transmission lines are tripped by local relays to isolate the faulted region. The role of system operator is to optimally maintain the balance between the generation and demand in each island. The main idea is to reduce the total amount of load shedding to maintain such a balance and avoiding the blackout.

There are two types of islanding:

- **Intentional Islanding**: It is done to determine optimal splitting points (or called splitting strategies) to split the entire interconnected transmission network into islands ensuring generation/load balance and satisfaction of transmission capacity constraints when islanding operation of system is unavoidable [402]. It is considered as an emergency response for isolating failures that might propagate and lead to major disturbances [340].

- **Unplanned Islanding**: This is an unplanned condition which should be avoided [136]. The islanding detection techniques are applied to reduce the risk of this event. This phenomenon is due to line tripping, equipment failure, human errors and so on [268].

Studies [442] have shown that by intentionally splitting the system into islands wide-area blackouts could have been prevented for several large disturbance events, e.g., [419]. The objective would be to isolate the faulty part of the network in order to limit the spread of a cascading failure. *Intentional* islanding is therefore attracting an increasing amount of attention. Islands should be designed such that they are balanced in load and generation and have stable steady-state operating points that satisfy voltage and line limits. Further the action of splitting should not cause transient instability. Since this problem potentially involves a 0-1 decision for every line in the network the search space grows exponentially with the size of the network leading to a considerable computational challenge.

Most approaches in the literature deal with finding a pre-determined islanding strategy that could be implemented in case of a network fault irrespective of where the fault occurs. The simplest example of this is forming islands by only requiring that load and generation are balanced. In [232], a three-phase ordered binary decision diagram (OBDD) method is proposed that determines a set of islanding strategies. The approach uses a reduced graph-theoretical model of the network to minimize the search space for islanding; power flow analyses are subsequently executed on islands to exclude strategies that violate operating constraints, e.g., line limits.

An alternative strategy that aims to avoid transient problems is to split the network into electromechanically stable islands, commonly by splitting so that generators with coherent oscillatory modes are grouped. If the system can be split along boundaries of coherent generator groups while not causing excessive imbalance between load and generation, then the system is less likely to lose stability. Typically, these strategies additionally consider load-generation balance and other constraints; algorithms include exhaustive search [445], minimal-flow minimal-cutset determination using breadth-/depth-first search [436], and graph simplification and partitioning [441]. The authors of [226] note that splitting based simply on slow coherency is not always effective under complex oscillatory conditions, and propose a framework that, iteratively, identifies the *controlling* group of machines and the contingencies that most severely impact system stability, and uses a heuristic method to search for a splitting strategy that maintains a desired margin. Wang et al. [437] employed a power-flow tracing algorithm to first determine the domain of each generator, i.e., the set of load buses that ‘belong’ to each generator. Subsequently, the network is coarsely split along domain intersections before refinement of boundaries to minimize imbalances.

While several useful strategies exist for determining pre-planned islanding decisions, little attention has been paid to islanding in response to particular contingencies. If, for example, a line failure occurs and subsequent cascading failures are likely, it may be desirable to isolate a small part of the network—the impacted area—from the rest. A method that does not take the impacted area into account when designing islands may leave this area within an arbitrary large section of the network, all of which may become insecure as a result.

In [414] (for DC network constraints) and [415] (extended to AC constraints) the authors propose an optimization-based approach to system islanding and load shedding. Given some uncertain set of buses and/or lines, solving an optimization determines (1) the optimal set of lines to cut, (2) how to adjust the outputs of generators, and (3) which loads to shed. The authors assume that this is done intentionally under central control and not left to automatic safety devices. A key feature of the method is that any islands created satisfy power flow equations and operating constraints. Therefore, if a transiently stable path is followed from a pre-islanding state to the post-islanding operating point, the islanded network will be balanced and with minimal disruption to load.

The optimal network restoration is called to a class of actions taken by network operator to bring back the power system into its normal condition following a complete or partial collapse. Intentional system islanding can be one of these actions [73, 365], but generally the methods are only partially developed.

From a mathematical perspective the islanding MILP problem has similarities with the *transmission switching* problem [199] (cf. Sect. 9), in that the decision variable includes which lines to disconnect, while power flow constraints must be satisfied following any disconnection. Similar decision variables are also involved in *transmission expansion planning* [294] (cf. Sect. 4). All three approaches—expansion planning, transmission switching, and islanding—may be seen as network topology optimization problems with added power flow constraints.

11 Operations of Smart Grids

The smart grid paradigm improves upon the controllability and control of existing power systems. With the increased penetration of distributed production (solar, wind), energy storage (pumped storage, batteries, compressed air storage, and plug-in hybrid electric vehicles), transmission switching and controllable elements called FACTS (see below), power flows can be and need to be dynamically adjusted in order to improve reliability and efficiency. Also, a partial load shifting from peak hours to off peak hours is possible. Such opportunities also increase the complexity of the design and operations of the power system. A broad class of novel optimization problems hence emerges, with the focus varying power system to power system.

In power systems, where peak demand occurs in one season, while the peak generation from renewables occurs in another season [169], the focus has largely been on the improvements to the efficiency of power generation and reliability of power transmission under stress due to peak demand or peak generation from renewables. The improvements are made possible by the so called flexible alternating current transmission system (FACTS) devices [260], which are now routinely installed at generators, at the interconnection of one national transmission system (TS) with others, and elsewhere, such that the national transmission system operators (TSO) gain more control over the power flows in their TS [84, 359]. FACTS devices intended for steady-state operations include:

- load tap changer (LTC), thyristor-controlled load tap changers, which make it possibly to vary the tap ratio rapidly
- phase-shifters (PS), e.g. thyristor-controlled phase shifters, which make it possibly to vary the phase angle rapidly
- series capacitor (SC), e.g., thyristor-controlled series capacitor coupled in parallel with a thyristor-controlled reactor (TCR), makes it possible to smooth the output of the reactor with varying reactance
- interphase power controller (IPC), which makes it possible to control reactive and active power independently
- static VAR compensator (SVC), which is a source or sink of reactive power
- static compensator (STATCOM), which allows to control either the nodal voltage magnitude or the reactive power injected at the bus.

The availability of such devices underlies the corrective actions available in response to stress. To summarize the book-length treatment of [3]:

- when voltages are too low, one supplies reactive power (using STATCOM, SVC)
- when the voltages are too high, reactive power is absorbed (using STATCOM, SVC)
- when thermal limits are exceeded, load is reduced (using SC, IPC),
- when loop flows appear, series reactance is adjusted (using IPC, SC, PS),
- when power flow direction is reversed, phase angles are adjusted (using IPC, SC, PS).

It is hence believed that wider availability of FACTS devices will lead to an increased stability of power systems. The non-convex optimization problems combining efficiency and reliability objectives, decisions as to FACTS settings, and constraints of the alternating-current power flows remain a major challenge.

Especially in power systems, where peak demand and peak renewable generation occur within the same season, there is an additional focus on energy storage and demand response management. One report [351] estimates that the potential demand response capability was about 20,500 megawatts (MW) in the US, or 3% of total peak demand. This is obtained by combining a variety of readily deferrable loads, comprising:

- pumped energy storage, which has been introduced into a number of power systems since 1950s, and remains an important feature to the present day
- large industrial customers, e.g., in refrigeration, and gas networks operations, who are being converted to flexible contracts, allowing for load shedding

- charging of electric cars which could become a major load, eventually while many other loads may become deferrable, should the regulatory environment change such that retail prices vary over time and load control switches (e.g., remotely controlled relays or relays relying on price data, such as learning thermostats connected to domestic air conditioning) become widespread.

While many other loads may become deferrable, should the regulatory environment change such that retail prices vary over time and load control switches (e.g., remotely controlled relays or relays relying on price data, such as learning thermostats connected to domestic air conditioning) become widespread. In some regions, such as California, where the photo-voltaic generation facilities are widespread and the peak demand is due to the use of air conditioning, the resulting savings can be considerable. Notice, however, that a number of challenges remain. First, there is the issue of information provision: in many markets with dynamic pricing, customers do not have access to data on current prices. The immediate announcement of prices may lead to swings in the demand, whereas no announcement may make it impossible to reach the best possible efficiency. Second, the regulatory framework has to be compatible with the free markets. Third, if the decision making is to remain centralised, one needs to model the behaviour of the users. Because of the numerous difficulties of doing so, a number of mechanism design studies and distributed decision-making schemes have been proposed.

Overall, smart grids require both changes to the power systems' infrastructure, as well as changes to their control mechanisms, which require the generation, distribution, transmission, and consumption to be modelled jointly. Although much innovative thinking is required, any progress on solving the underlying problems (mainly LF, OPF, ONI and OTS) is still relevant.

12 Energy Storage Operations Management

12.1 Storage Systems

The increased awareness of the environmental impact and of the carbon footprint of all energy sources have motivated the recent widespread adoption of Renewable Energy Sources (RES). However, the intrinsic intermittent and not-schedulable nature of such naturally generated energy introduces a new source of uncertainty in the operation and planning of electric power systems. This poses a critical threat to the power grid since its stability relies on the balance between energy production and demand [251]. Therefore, as the installed capacity of RES keeps increasing, the need to compensate the fluctuations caused by non-dispatchable energy sources has become one of the most compelling drivers of research in the power-grid scientific community.

There are many ways to mitigate the variability of power generation from RES. On the one side, there have been many efforts in improving the accuracy of power generation forecasts from renewable sources. Most notably, recent efforts in this direction can be found in [21] and [443], and in the book [311]. Another possibility to handle the intermittent nature of RES is to use conventional (i.e., dispatchable) power plants as back-up to improve the resiliency and the flexibility of the overall mix of power plants in the power grid. Obviously, this solution brings back the pollution issues, associated with the usage of conventional power plants [283] and [439]. A further opportunity can be provided by hydro power plants, as they can respond quickly and absorb some of the energy fluctuations; however, hydro resources are limited by their availability and their unsuitability to handle frequent charge-discharge cycles.

According to the previous discussion, there is a general consensus that Energy Storage Systems (ESSs) may provide a viable way to systematically support power generation from RES, as they represent a cost-effective, flexible and quick tool to smooth and regularize intermittent power generation [32]. The next sections describe the main technologies employed to build storage devices, their main applications in power grids, and the main mathematical methods that are used to solve grid-related optimization problems when storage devices are also explicitly taken into account.

12.2 Technology

The physical characteristics of a storage system must be adapted to the particular service of interest. For instance, an ESS that has to provide primary frequency regulation will present different characteristics from one that is desired to provide the local supply to a private house. Accordingly, storage techniques can be divided into four categories [215]:

- Low power applications (e.g., transducers, private houses)
- Medium power applications (e.g., individual electrical systems, town supply)
- Peak levelling and network connection applications
- Power-quality applications

For the first two categories, we consider small-scale systems in which energy can be stored in the form of a flywheel (kinetic energy), fuel cells (hydrogen), or supercapacitors. The last two categories are instead large-scale applications and the most used technologies rely on storing the energy in the form of gravitational energy (e.g., hydraulic systems), thermal energy or compressed air. Finally, note that Electric Vehicles (EVs, either in terms of Fully Electric Vehicles or Plug-in Hybrid Vehicles) have been recently assimilated to ESS, due to their ability to behave as a battery when the vehicle is idly connected to the grid. Given the special characteristics of EVs (whose main purpose is clearly to serve as mobile vehicles, and not to serve as batteries), a specific and more detailed discussion about their usage is given here (add a link to the wiki entry to electric vehicles).

For a more detailed discussion on storage technology and their technical characteristics, we refer to [32, 215] and to the more recent [79, 459].

12.3 Benefits

The use of ESSs, due to their versatility and flexibility, can lead to a number of advantages for the power grid, both from a technical and an economic perspective. In what follows we list the main services that storage technology can bring. For a more detailed discussion the interested reader can refer to [32, 175, 215, 459], and especially to [459, Section 3].

- **Ancillary services**: ESSs can help regulate the active power supplied by non-dispatchable generation and provide primary frequency and voltage control, therefore improving the transient response of the power grid. This would remove the need to keep expensive dispatchable back-up power generation and would greatly facilitate the penetration of wind and solar power. Examples of technologies in the ESS for ancillary services segment are pumped storage for longer duration applications such as load following, reserve capacity and spinning reserves, or flywheels for high-power, short-duration applications such as frequency regulation.
- **Energy arbitrage**: ESSs would allow to purchase inexpensive electric energy, available during periods when prices or system marginal costs are low, to charge the storage system so that the stored energy can be used as a substitute for the expensive primary power used in peak-load power stations. Alternatively, ESSs could store excess energy production, which otherwise would be lost, from RES. A typical example would be Pumped storage. The principle is that during periods when demand is low, these stations use electricity to pump the water from the lower reservoir to the upper reservoir. When demand is very high, the water flows out of the upper reservoir and activates the turbines to generate high-value electricity for peak hours.
- **Network savings**: Power consumption during the day is characterized by high fluctuations, meaning that the minimum level of consumption is usually much lower than the maximum daily peak (especially during summer and winter). This leads to over sizing the production units and transmission lines, and the necessary equipment, that are tailored to absorb the peak demand. On the other hand, the usage of local supply in the form of ESS, would help compensating load variations and would make possible to operate transmission and distribution networks with lighter designs, closer to the average daily consumption rather than to the peak demand.

Due to the aforementioned diverse applications, the mathematical problems associated with ESSs that are of utmost interest for the power grid, correspond to their optimal siting (i.e., finding the most convenient location where to install them) and sizing within the power grid [459]. The next section reviews the most used techniques to address such problems.

12.4 Models

Different models, with different levels of accuracy, have been developed in the literature to model the functioning of storage devices. The level of detail usually depends on the particular application of interest, and in general on the level of detail with which other power grid devices have been modelled. When accurate models of the batteries are not required, in some cases simple first order linear equation may be used, [77, 139, 140, e.g.]. Such simple models can be used when one is not interested in the point-wise behaviour of the system (as the low-level electrical behaviour of the ESS is neglected) but, for example, when the aim of the study focuses on the effects of the transient behaviour of a power grid [139, 140]. More sophisticated and realistic models can be found in [336, 337], where many other low-level details of a storage unit are also taken into account (e.g., life cycle, ageing, dc link, specific technology).

While simultaneous determination of the optimal location and size of ESS is known to be a non-deterministic polynomial-time hard problem [459], yet different strategies have been adopted to tackle it. This includes the use of Monte Carlo simulations, more analytic approaches (like dynamic programming, mixed integer-linear programming and second-order cone programming), and certain heuristic methods.

Maintenance

F. Lacalandra, K. McKinnon, C. Helmberg, T. Triboulet, and E. De Klerk

1 Strategic Maintenance

Like any device or system, all electricity devices require periodical maintenance. Maintenance in electricity systems is a source of large costs; in the EU the maintenance costs amount to between 4 and 8% of the total sales turnover. In vertically integrated systems the strategic maintenance of power plants' and network's components is performed in an integrated fashion by the monopolist, whereas in those market based, these problems are responsibility of the GenCos and of the Transmission System Operator (TSO) respectively.

The maintenance activities are indeed complex even to classify. For instance if we define Preventive Maintenance in an abstract way as a general process carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning of an item, we can distinguish:

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- Scheduled maintenance: Preventive maintenance carried out in accordance with an established time schedule or established number of units of use. The maintenance is planned in advance.
- Condition based maintenance: Preventive maintenance based on performance and/or parameter monitoring and the subsequent actions. An example of condition based maintenance is when condition monitoring systems (CMS) are used to control the condition of the component or system, and thereby preventive maintenance is possible to perform.
- Opportunistic maintenance: Opportunistic maintenance refers to the situation in which preventive maintenance is carried out at opportunities. A typical example is when one component is out for maintenance and it is decided to take out another component for maintenance before failure. Such a decision would be based on a rational decision, e.g. by saving cost by performing several maintenance activities at the same time.

Basically the long term perspective coincides with a year frequency, and with this horizon in mind the maintenance refers to scheduled maintenance. In details the main goal of the maintenance processes in electrical systems are:

- Power plants long term maintenance, e.g. determining a schedule of plant outages aiming at minimizing various costs. The outage schedule must satisfy several constraints in order to comply with limitations on resources which are necessary to perform refueling and maintenance operations. When speaking about power plants we—of course—refer to any kind of power plant including wind, solar and hydro units.
- Transmission and Distribution network long term maintenance, e.g. determining a schedule of branches, transformers and other devices outages. Also in this case these outages must satisfy several security constraints and opportunity costs.

2 Transmission and Distribution Network Long-Term Maintenance

Considering the Transmission and Distribution Network long term Maintenance (TDNM), it is necessary to ensure that tripping out a branch for maintenance does not impact the network reliability and security. The TMS constraints are therefore globally the same as those for power plants, after all the system is unique (e.g., time windows for maintenance tasks, resource requirements, demand satisfaction, etc). Of course the equipment in this case are the network′s ones. It seems that TDNM has received less attention than power plant maintenance at least in the scientific literature. The network can be modeled as either a transportation model (i.e. without imposing Kirchhoff's law) or a more complex but more realistic DC power model or even with a full AC representation for some critical areas. The TSO has to coordinate the submitted schedules; the cheapest transmission lines and generators might be overloaded. If TDNM is not solved jointly with power plant maintenance, network constraints can be introduced once power plant maintenance is solved. Alternatively one can obviously see another example of a big single problem that the TSO might solve in a single process with iterative approaches like those described before.

For both classes of problems, we refer to [156] for an extensive review of the subject in the scientific literature.

3 Medium Term Maintenance

The medium term perspective covers periods from one to 3 months ahead. The main goals of the medium term maintenance processes in electrical systems are the same as the long term goals (see the final part of Sect. 1 on Strategic Maintenance). The shorter term maintenance problems are sub-problems of the long term problem and some decomposition approach is needed to coordinate the decisions on the different time scales. A simple approach is to respect the broad outline of the long term schedule and refine and adjust it as more information becomes available in the shorter term. Optimization oriented coordination approaches between different time horizon have also been proposed, see e.g. [298].

When dealing with shorter term problems it becomes of interest to consider *condition based* and *opportunistic* maintenance, i.e. when planning the maintenance schedule to use estimates of the condition of the equipment and the likelihood of it breaking down in the following months, and to allow modifications of the planned maintenance action and its duration once the maintenance of the equipment has begun and its true condition is revealed. Usually the condition of equipment is not directly observable, and in some cases there may be lot of uncertainty about this (e.g it may not be known how often a transformer has been overloaded since its last maintenance). Consequently the condition of the equipment and its prognosis, i.e. its expected time to failure, are highly stochastic and have to be quantified statistically from historic data for similar equipment [135]. The problem of making the best replace, repair or maintain decisions taking into account the uncertain condition of the equipment is a stochastic optimization problem. This problem is of interest both for the maintenance of single pieces of equipment (i.e. the *self scheduling maintenance problem*) and also for the whole system, where also unpredictability in demand has to be considered.

Dynamic programming is an appropriate technique for finding optimal solutions to both the stochastic and deterministic self scheduling maintenance problems. If the full system maintenance problem is decomposed into self scheduling problems for all single items of equipment, then what is needed is estimates of the cost to the whole electricity system of removing the equipment for maintenance or replacement and the cost of unexpected breakdown. If the whole maintenance process is centrally managed then Lagrangian methods analogous to those used historically for unit commitment are appropriate. In a decentralized system estimates of the costs for planned and unplanned unavailability need to be taken into account when negotiating the contracts between the individual equipment owners and the system operator.

4 Scheduled Maintenance

Froger et al. [156] provide an extensive survey on scheduled maintenance topics and research in the electricity industry. In a nutshell, maintenance scheduling has to decide which parts of the generating units or the transmission infrastructure to shut down during time windows with reduced energy demand at an acceptable failure risk so that profit losses/costs are minimal. In principle this requires to combine integer maintenance decisions with complex physical models on technical restrictions (ramping, power flow, etc.) as well as with stochastic models for the development of supply (e.g. due to wind and solar energy), demand and prices. Further restrictions include the necessary equipment and personnel. Because each single component is mathematically already a challenge, the main body of literature can be found in engineering journals while so far there is very limited coverage by mathematical journals.

In the past, solution techniques only considered a coarse discretization of the time horizon (weekly time steps) and the problem was decomposed into single production units (fossil fuel power plants, hydro-electric units,...). Stochastic aspects were considered at the unit level with scenario modeling for hydro inputs and marginal costs. For the optimization over a single unit, Dynamic Programming is a simple and efficient approach.

Nowadays, models are using a finer discretization (daily time steps). Technical coupling constraints between the different production units are incorporated (for e.g. limited resources to perform certain operations). The main solution methods are local search heuristics, decomposition approaches (Benders', Dantzig-Wolfe and Lagrangean Relaxation) and occasional Mixed Integer Programming (MIP) or Model Predictive Control (MPC) models.

In practice MIP approaches require small time windows for the schedule of maintenance. Local search approaches are less restrictive but don't provide proofs of optimality. A flurry of approaches for this problem have been developed in the 2010 ROADEF Challenge. There is much room for future work in mathematical methodology. Stochastic models should cover aspects like demands, renewable productions, delays in maintenance operations and availability of power plants (failures, efficiency,...). A highly desirable aim is to achieve stability of the computed schedule with respect to small modifications in the input. In deregulated markets, game theoretic aspects enter because an independent system operator must approve time windows in view of the proposals of several competitors.

5 Nuclear Reloading Pattern Optimization

In a nuclear reactor there are fuel rods of different ages. At the end of each fuel cycle, the oldest rods are moved to a spent fuel pool and are replaced by fresh rods. At this point, it is possible to make reallocate rods in the core leading to a combinatorial problem. This problem may be modeled as a Mixed Integer Nonlinear Problem (MINLP); see, e.g. [354]. The model includes dependent variables that describe physical properties such as neutron flux, burn-up, and yield. The neutron transport equations are converted to a set of algebraic equations using Green's functional theory, giving rise to a stationary description of the neutron flux in the core. The fuel burn-up is approximated by discretizing the differential equation; see e.g. [416] for details.

More physically accurate models are solved with Meta-heuristics search methods. There for each given reloading pattern the neutron flux etc. are calculated by the numerical solution of the relevant differential equation.

There is a symmetric problem to optimize the unloading process of fuel rods. The operations to determine are the placements of rods in the spent fuel pool and to optimize the manipulations of rods in the pool by automated handling systems.

Finance, Regulations, Politics and Market Design

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1 Overview

In order to find decisions in finance, regulations, politics including long term strategies for the electricity system and strategic planning on the industrial side, a holistic view on the overall energy system and markets is required at different

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levels of detail. These include mainly aggregated regional views of:

- power production units (differentiated according to technologies)
- electricity networks (ACDC)
- weather predictions and renewable feed-in
- demand forecasts (industry, trade sector, households)

and in different representations of markets (long term, spot, intraday, balancing) and participants producers (including productions of renewables)

- network operators
- consumers including demand side management
- policy and regulations
- traders and aggregators

The aim of a system modelling approach is to investigate the development of the system with respect to different time levels and corresponding questions (see Tables 1, 2 and 3).

Table 1 Strategic problems (10–50 years)

For policy makers	For producers
Development of long term pathways to energy transition	The perspective of long term market and general technology development
Defining long term targets of the electricity system depending on e.g. climate, emission and renewable targets	The effects of support schemes on the profitability of (renewable) production
Sensitivity analysis and system security aspects	The influence of political regulations and targets on the energy system
Investiagtions of different policy designs on their impact on the system	long term strategic portfolio effects

Table 2 Tactical problems

For policy makers and regulators	For producers	For traders and industries
System security and reserve capacities	Influence of regulations and market rules on dispatch	Medium term portfolio effects including long term bilateral contracts
	Influence on the political debate and market design issues	

Table 3 Operational problems

Network operators and regulators	Producers
Influence of balancing uncertainties in renewable feed-in and demand	Predictions of market development and revenues depending on weather and demand forecasts
	Support of optimal dispatch decisions
	Assesment of uncertainies and risks

2 Long Term Electricity Bilateral Contracts

Electricity markets offer a way to trade on a hourlydaily basis quantities of electricity at a given price. The short term producer's maximum profit problem has been discussed in another section. However for risk management reasons the producers may want to stipulate longer term bilateral contracts with third parts, i.e. (large) consumers. The problem of defining the amount and the price of such over the counter transactions can be seen as an simulation/optimization one. It is also a simulation problem since the long term horizon calls for the estimation of the future conditions of the spot market that remains an alternative. Although rare, a producer, especially a small one, may in fact want to go spot for all its capacity. The final goal is indeed maximizing the profit while maintaining a certain—quantity and price—risk.

2.1 Modeling and Algorithmic Considerations

From a modelling standpoint, the bilateral contract definition involves the price risk profile and the future conditions of the spot market. Moreover, the bilateral contracts are typically equipped with hourly profile (or blocks of hours) of demand from the counterpart. Therefore, the inclusion of some simplified technical constrains of the power plants must be considered (at least maximum capacity and ramp constraints). All in all, given a certain demand profile requested, the problem can define as variables price and quantity and try to optimize a custom objective function that takes into account the revenues, the costs, and the risk reduction, with respect to pure spot trading along the considered horizon.

3 Multilevel Modeling of Market Design

3.1 Redispatch-Based Electricity Trading

Many European countries have implemented a system of spot market trading of electricity that is redispatch-based [131]. Electricity is traded at power exchanges like the EEX in Leipzig, Germany. During these auctions, no or only a certain part of the technical and physical constraints of electricity transport through the transmission network are respected. For instance, in Germany, only a market clearing is imposed that yields the balance of traded production and consumption. As a result of this drastic simplification, spot market results do not have to be feasible with respect to actual transport through the transmission network. If this turns out to be the case, traded quantities have to be redispatched such that the resulting quantities can actually be transported. Different systems of redispatch rules are implemented in Europe, e.g., cost-based redispatch in Austria, Switzerland, and Germany or market-based redispatch in Belgium, Finland, France, or Sweden [176]. However, and independent of the actual redispatch system, this market design of spot market trading and redispatch yields a two-stage model that involves different agents and stakeholders like

- producers owning conventional power plants or facilities for producing power from renewables like sun or wind;
- consumers like municipal utilities or large industrial enterprises; and
- transmission system operators (TSO) that control and maintain the transmission network and organize the redispatch.

It is shown in the literature that this system of electricity market design may yield significant decreases in total social welfare; see [176, 205] and the references therein. Thus, the natural question arises if and how different markets can be designed that yield improved welfare outcomes. This question is currently an active field of research and involves the investigation of alternative systems like the introduction of zonal pricing [177, 205] or nodal pricing [205, 229].

From a mathematical point of view, the study of different market designs may introduce the regulator or state as an additional agent that decides on certain questions like, e.g., the specification of the actual price zones in zonal pricing or the specification or regionally differentiated network fees. Since the regulator or state anticipates the influence of his decisions on the actions of all other agents, such a rigorous mathematical modeling has important implications on the overall model, since the decisions of the regulating agent couples all other levels of the system, yielding a (typically mixed-integer) multilevel optimization [97].

These models are extremely hard to solve [98, 165, 427]. Hence, there is a political and social need to develop new mathematical theory and algorithms for solving realistic instances of these models.

4 Energy Policy Analysis

Energy planning requires the study of the interactions between the economy (at national or regional levels), the energy sector and the related impacts on the environment. Many countries do not hold indigenous energy resources becoming highly dependent on primary energy imports. In such cases, an increase of energy consumption which greatly relies on fossil fuels is frequently interweaved with economic growth, also leading to the exhaustion of finite resources and to Greenhouse Gas (GHG) emissions. To sum up, negative effects on economic growth and social welfare might be prompted as an outcome of energy and environmental policies. Henceforth, other evaluation facets besides economic concerns such as environmental and social welfare impacts should be explicitly considered in the appraisal of the merits of energy plans and policies to address energy problems in a societal perspective [22].

Thus, the assessment of the trade-offs between economic growth, energy demand/supply, as well as their corresponding environmental and social effects is particularly relevant for energy planners and decision-makers (DM) through the use of reliable tools for supporting the process of energy policy decision-making. In this context, the use of multiobjective programming models and methods combined with Input-Output (IO) analysis can be particularly appropriate for assisting in the process of Economy-Energy-Environmental (E3) policy design [333].

IO analysis is a top-down approach which can be intertwined with environmental satellite accounts provided by national statistical offices, allowing broad impact coverage of all sectors directly and indirectly involved with the energy sector. Furthermore, IO has influenced the outset of linear programming (LP) [430] and it may be considered as a simple particular case of LP [107]. The combined use of the IO methodology with LP models allows attaining value-added information, which would not be possible to achieve with the isolated use of both techniques. Inter/intra-sector relations entrenched in IO analysis allow obtaining the production possibility frontier. LP models enable selecting the level of activities which optimize a given objective function, satisfying the production sector relations imposed by IO analysis. Additionally, IO MOLP models allow assessing different efficient possibilities of production (i.e. output levels for each activity sector for which there is no other feasible solution that allows improving the value of a given objective function without worsening the value of, at least, other objective function) that can be reconciled with the competing axes of evaluation intrinsically at stake [333].

4.1 Strategic Problems

LP formulations of IO systems have been a normal part of standard texts since the 1960s [411]. The first IO LP models developed only addressed the economic system, but after the first oil crisis energy-environmental planning models started to play a prominent role.

IO analysis allows establishing an overarching framework to model the interactions between the whole economy and the energy sector, thus identifying the energy required for the provision of goods and services in an economy and also quantifying the corresponding pollutant emissions. Several indicators (either modelled as constraints or as objective functions) are obtainable with the application of IO LP/MOLP models specifically devoted to energy planning:

Economic

- Gross Domestic Product (GDP);
- Gross Regional Product (GRP);
- Gross Value of Production (GVP);
- Output levels;
- Private consumption;
- Balance of payments;

- Foreign-trade-balance;
- Gross value added;
- Public deficit;
- Production capacity;
- Exports and imports;
- Cost of the energy system;
- Employment;

Energy

- Energy imports;
- Energy use;
- Storage capacity;
- Security stocks for hydrocarbons;
- Wastes with energetic use;
- Efficient energy use;

Environmental

- GHG emissions (based on CO2, N2O and CH4 emissions);
- Acidifying substance emissions (based on SO2, NOx and NH3 emissions);
- Environmental discharges not related to fuel combustion;
- Wastes produced.

4.2 Integrated Energy Planning Models

Integrated energy planning (IEP) strives to account for the relevant strategic elements of the energy value-chain at a national level/regional level. IEP is intrinsically a multiobjective problem and when sustained by IO MOLP modelling tools, distinct alternative energy pathways can be assessed which can be consistent with different policy options. The solutions obtained help DMs to assess how energy requirements can be reduced without harming economic growth and socioeconomic development, allowing to understand the relationship (trade-off) between energy supply/demand and economic development/growth and corresponding environmental impacts.

IO MOLP models support IEP and provide help in the design of energy policies, namely guiding:

- the proposal of balanced energy policy configurations;
- the selection of appropriate technologies to meet energy demand;
- the suggestion of strategies to appraise the impacts of energy supply shortages/disruptions in an integrated manner;
- the development of procedures to assess the effects of nuclear power plant accidents, trade embargoes, and international conflicts, among others;
- reallocation of production problems;
- biomass production optimization;

- energy import resilience;
- energy-economic recovery resilience of an economy;
- energy efficiency planning.

4.3 Regional Energy Planning

Since the national energy supply/demand structure cannot reflect regional characteristics, regional energy planning is particularly relevant because it allows capturing each region's specifics, before being articulated with national energy planning.

IEP Models

Balanced regional policy configurations can be obtained by means of IO MOLP models. With the foregoing in mind, [315] presented a macro-level energy model aimed at minimizing the total cost of the energy system and its application to energy planning for three states in India (Gujarat, Kerala and Rajasthan). The methodology considers a reference energy system, an expanded IO table with disaggregated energy sectors and an LP model combined with a scenario analysis approach.

Cho [78] developed a model which includes the minimization of energy consumption and pollution, and the maximization of employment, being subject to the restriction of the range of outputs for twelve individual sectors considered, regarding the total output level of the Chungbuk economy. The impact multipliers (employment, pollution and energy consumption) are calculated and then combined with decision variables to form the objective functions of the MOLP model. The results of the model are able to illustrate how the regional production structure should be reorganized in order to become a more balanced one.

Assessment of Energy Shortage Impacts

The assessment of energy shortage impacts has been formulated as an LP problem in [265], where an energy flow matrix for Hawaii is built and the 1977 Hawaii IO table is used to evaluate each sector's direct energy intensity and total energy intensities.

The authors calculate shadow prices for different levels of gasoline availability with the use of an LP model and show that the solution thus obtained provides an efficient distribution of energy resources to various industry sectors during energy shortages.

IEP Models Under Dynamic Assumptions

Leontief [264] suggested the dynamic IO model where a new matrix describing the capital resources is considered, aimed at distinguishing different technological structures in different time frames. With this modelling formulation it is possible to account for the growth potential of an economy, since the final demand vector of the static IO model is replaced by a stock's coefficient matrix that is then multiplied by the anticipated increase of the output level between the present year and the following year. This new set of differential equations represents the dynamic relations of the IO model, allowing for the description and analysis of the economic growth process [264]. Based on this type of approach [462] applied an LP dynamic IO model considering the case of renewable energy industries, as well as the environmental policy instrument of emission taxes. In addition to exploring the relationships among Beijing's renewable energy, economy and environment, the model analyses the future trends of the economy and GHG intensity from 2010 to 2025. The objective function is the maximization of the total GRP from 2010 to 2025, being subject to constraints regarding material flow balance, value flow balance, electricity supply-demand balance, investment-savings balance and GHG emissions.

James et al. [221] suggested the combination of the IO model with a dynamic energy technology optimization model to compute the change in total energy demand and technological mix. The authors were able to identify through the use of the model part of the economic repercussions of technological change and inter-fuel substitution.

4.4 National Energy Planning

IEP models at the national level, explicitly incorporating the interactions of the energy system with the economy have been developed based on IO MOLP.

IEP Models

Hsu et al. [209] use the bicriterion NISE method for assessing the trade-offs between GDP and energy use in Taiwan. The solutions obtained represent simulated scenarios of aggressive, moderate and conservative policy alternatives. The evaluation of the outcomes is mainly centred on the economic performances resulting from the different policy alternatives and the energy requirements for supporting that performances.

The impacts of the electricity power industry can also be assessed by coupling IO with goal programming models. A goal programming model has been suggested in [18] to analyse the trade-offs among economic (generation cost minimization) and environmental (CO2 emissions minimization) objectives for the year 2000 in Japan's electricity power industry, which allows discussing the nature of the trade-off curve and the extent of power generation by source.

Antunes et al. [23] consider the TRIMAP interactive environment to analyse the interactions of the energy system with the economy in Portugal. Another version of this model with six objective functions (maximization of GDP, private consumption, self-power generation and employment, and minimization of energy imports and CO2 emissions) was proposed in [327] and solutions were obtained using the interactive STEM method. In [328] an interactive procedure to obtain solutions is employed based on a min-max scalarizing function associated with reference points, which are displaced according to the DM's preferences expressed through average annual growth rates. The objective functions considered in the model are: minimisation of acidification potential, maximisation of self-power generation, maximisation of employment, maximisation of GDP, and minimisation of energy imports.

Kravtsov and Pashkevich [248] suggested a three-objective LP model aimed at maximizing the GDP, minimizing the use of fuel and energy resources, and maximizing the foreign-trade balance. Solutions were computed using a weighted sum approach, with information on Belarus over the 1996–2000 period.

Hristu-Varsakelis et al. [208] optimized production in the Greek economy, under constraints relating to energy use, final demand, GHG emissions and solid waste. The effects on the maximum attainable GVP when imposing various pollution abatement targets were considered using empirical data. The results obtained quantify those effects as well as the magnitude of economic sacrifices required to achieve environmental goals, in a series of policy scenarios of practical importance. Because air pollution and solid waste are not produced independently of one another, the settings in which it is meaningful to institute a separate policy for mitigating each pollutant versus those in which only one pollutant needs to be actively addressed are identified. The scenarios considered represent a range of options that could be available to policy makers, depending on the country's international commitments and the effects on economic and environmental variables.

San Cristóbal [375] proposed an IO MOLP model combined with goal programming to assess economic goals (output levels), social goals (labour requirements), energy goals (reduction of coal requirements by 5%), environmental goals (reduction of total emissions of GHG and waste emissions by 10%). Solutions are obtained by considering the minimization of the total deviations from the goals.

de Carvalho et al. [92] proposed a hybrid IO MOLP model applied to the Brazilian economic system aimed at assessing the trade-offs associated with the maximization of GDP and the minimization of the total energy consumption and GHG emissions, considering the timeframe of 2017. The TRIMAP interactive environment was employed to grasp the trade-offs between these objective functions.

Assessment of Economic or Political Crises

The quantitative effects of economic or political crises can be assessed with IO MOLP models. Examples of such crises are nuclear power plant accidents, trade embargoes, and international conflicts. Kananen et al. [234] showed how a visual, interactive, dynamic MOLP decision support system can be effectively used with this aim in the Finnish economy. The IO MOLP model considers as objectives the maximization of private consumption, trade deficit and employment, and the minimization of the overall energy consumption.

Models Devoted to Reallocation of Production Problems

The reallocation of production problem can be formulated as a constrained optimization problem. Taking Greece as a case study, Hristu-Varsakelis et al. [207] considered the reallocation problem on a sector-by-sector basis, in order to meet overall demand constraints and GHG Kyoto emissions targets. The authors take into account the Greek environmental IO matrix for 2005, the amount of energy utilized and pollution reduction options. The model is aimed at maximizing total

GVP subject to upper bounds on energy use and pollution, lower and upper bounds on production, and lower bounds on the GVP of every activity sector.

Models Devoted to Biomass Production Optimization
IO MOLP models can be adjusted to include several alternative technologies. In this case, the LP formulation is able to handle the representation of alternative technologies [430]. This hybrid approach of linking detailed models with aggregated, economy-wide models is the current focus of research in Life Cycle Assessment (LCA). Following this approach, de Carvalho et al. [92] developed a hybrid IO framework coupled with LCA based estimates for two sugarcane cultivation systems, two first-generation and eight second-generation technology systems for bioethanol production scenarios. The integrated- or country-based assessment of the whole economic system has accompanied the process design and process-based analysis, supporting the identification of direct and indirect effects that can counterweight the benefits. The consideration of direct and indirect effects on the whole economic system is critical in policies and technological choices for prospective bioethanol production, since positive direct effects of first-generation and second-generation plants can be offset by indirect impacts on other sectors.

Energy Import Resilience
Energy import shortages may occur in various importing sectors and most of the times cannot be foreseen in advance. Models aimed at addressing energy import resilience can be used to simulate the impact of specified energy import losses on the sectoral production levels, and consequently, the final supply-demand balance. In this context, He et al. [194] developed an IO LP model that focuses on the connection between energy imports, industrial production technologies and capacities. The main value added rests on the possibility offered by the proposed model of appraising the worst-case scenario impact over a family of import loss scenarios. The impact of an energy import loss on the economy is the amount of final demand of goods that cannot be balanced by the given supply and production in the short run. An energy import resilience indicator is then defined, which essentially assesses the highest level of energy import loss possible to the economy. The methodological framework is also extended in order to encompass production capacity designs that allow reaching the maximum possible energy import resilience of a given IO structure.

Energy-Economic Recovery Resilience of an Economy
He et al. [195] proposed an IO LP to appraise the energy-economic recovery resilience of an economy by studying the interactions between energy production disruption, impacts on sectoral production and demands, and post-disruption recovery exertions. The developed model evaluates the minimum level of recovery investments necessary to reinstate production levels so that total economic impacts are tolerable over a specified post-disruption extent. It is presumed that disruptions are uncertain and can take place at different sectors and possibly simultaneously. The optimization model is then solved using a cutting plane method which involves computing a small sequence of mixed integer programming problems of reasonable

dimensions. Taking China's 2012 IO data as a case study, the study illustrates the model's ability to unravel vital inter-sectoral dependencies at different disruption levels. With this type of approach DMs become acquainted with relevant information regarding the appraisal and enhancement of the energy-economic resilience in a comprehensive manner.

IEP Under Uncertainty

The accurate specification of the coefficients of optimization models is a challenging endeavour in most real world problems since sometimes there is not enough information available. Moreover, the technical coefficients of the IO matrix may be subject to a considerable level of uncertainty. Uncertainty handling in the outline of IO analysis may be essentially based on three different approaches: the probabilistic approach, in which the probabilistic distribution functions associated with all the coefficients are presumably well known (e.g. [411, 440]); the interval approach (unknown but bounded coefficients), where the upper and lower bounds of the coefficients are considered without being associated with a structure of possibilities or probabilities (e.g. [223, 224]); and the fuzzy (or possibilistic) approach, in which membership functions are assigned to all uncertain coefficients (e.g. [62]). Therefore, IO LP/IO MOLP models explicitly handling uncertainty of the model coefficients have arisen in scientific literature.

Borges and Antunes [56] proposed an IO MOLP model with fuzzy coefficients in the objective functions and fuzzy right hand sides of the constraints for E3 planning in Portugal. Interactive techniques were used to perform the decomposition of the parametric (weight) diagram into indifference solutions corresponding to basic non-dominated solutions.

Oliveira and Antunes [329] and Oliveira Henriques and Antunes [331] have considered all IO MOLP model coefficients as intervals, then conveying information regarding the robustness of non-dominated solutions (that is, solutions that achieve desired levels for the objective functions across a set of plausible scenarios) under a more optimistic or pessimistic DM's stance. With the introduction of (direct and indirect) employment multipliers, this IO structure has been used to extend the interval MOLP to assess the trade-offs between economic growth (GDP), social welfare (employment), and electricity generation based on renewable energy sources [332].

Models Devoted to Biomass Production Optimization

Case studies based on electricity generation from biomass and ethanol production can be assessed to illustrate how the model determines optimal production levels of feedstock within each region, as well as optimal levels of trade between regions and imports from external sources. With this purpose, [407] presented a multi-regional fuzzy IO model to optimize biomass production and trade under resource availability and environmental footprint constraints. Uncertainty was only considered on the upper or lower bounds of the constraints.

Models Devoted to Energy Efficiency Planning
The introduction of a bottom-up approach into an IO MOLP model enables extending its application to the assessment of energy efficiency measures. This methodological framework combined with mathematical interval programming tools was followed in [333] to account for investment options aimed at improving the thermal properties of the building envelope (e.g., the insulation of external walls and roof, and the replacement of window frames and window glazing) in Portugal. The objective functions are the maximization of GDP, the building renovation investment, and the overall level of employment, being subject to several economic and environmental constraints.

Challenges of IEP Planning with MOLP IO Models
The main difficulty found in the studies carried out with these models rests on the availability of statistical information. In fact, the application of the IO approach in the framework of electricity generation can be a complex and challenging task since published IO tables do not allow assessing the environmental impacts that are likely generated from an increase in the demand for electricity generation from renewable energy and/or from conventional energy, but only the impact of an increase in demand for electricity in general. Published IO tables consider a single aggregated electricity sector, where generation, transmission, distribution and supply activities related to the production and use of electricity are included. Therefore, it is important to disentangle the different possible ways to tackle the disaggregation of the electricity sector.

Despite the typical limitations found when considering this type of approach, the power of IO analysis rests upon its capacity of depicting the technology of a country or region with enough accuracy to allow performing a real empirical study. In addition, IO analysis is a flexible tool that can be applied to a wide variety of problems, which can be used to modelling complex systems of economic and physical interrelations. In reality, IO analysis enables assessing any type of environmental burden caused by changes in the output of economic sectors once reliable data is used.

A broad range of (economic, social, energy and environmental) indicators according to coefficient scenarios and output levels attained for the activity sectors (industries) might thus be obtained with IO LP/MOLP models, which provide a useful planning and prospective analysis tool.

A major drawback usually mentioned in scientific literature relates to the static nature of the IO traditional matrix. However, the IO MOLP framework has evolved, explicitly encompassing the uncertainty handling of the model's coefficients, helping to overcome this particular limitation. This modelling approach could, nevertheless, benefit from the development of a dynamic, multi-period variant, relying on the integration of time-dependent technical parameters to account for technological learning curves and yield improvements, as well as incorporate game theoretical principles to accurately reflect the typical multi-agent's nature of the problem.

Another possibility for further enriching this modelling framework would be the development of tools for obtaining solutions considering the interaction with multiple planners/DMs with potentially conflicting views. The involvement of distinct stake-holders would bring new insights into the decision-making process at all stages, from model's definition to the evaluation of solutions.

5 Demand Response and Price Optimization

One of the main research objectives in Demand Response (DR) is the design and implementation of technologies and mechanisms to lower the electricity consumption via energy efficiency measures, and to improve the electricity consumption via demand shifting. Increasing energy efficiency requires a reduction of energy demand peaks by shifting part of the energy consumption in off-peak hours. This can be done via DR mechanisms and load control.

Demand shifting can provide a number of advantages to the energy system [94]:

- Load management can improve system security by allowing a demand reduction in emergency situations.
- In periods of peak loads even a limited reduction in demand can lead to significant reductions in electricity prices on the market.
- If users receive information about prices, energy consumption becomes more closely related to the energy cost, thus increasing market efficiency: the demand is moved from periods of high load (typically associated with high prices) to periods of low load.
- Load management can limit the need for expensive and polluting power generators, leading to better environmental conditions.

Potential benefits and implementation schemes for DR mechanisms are well documented in the literature. DR programs can be defined as methods to induce deviations from the usual consumption pattern in response to stimuli, such as dynamic prices, incentives for load reductions, tax exemptions, or subsidies. They can be divided in two main groups: price-based and incentive-based mechanisms [6, 7] and [341].

Price-based demand response is related to the changes in energy consumption by customers in response to the variations in their purchase prices. This group includes DR mechanisms like Time-of-Use (ToU) pricing, Real Time Pricing (RTP) and Critical-Peak Pricing (CPP) rates. If the price varies significantly, customers can respond to the price structure with changes in their pattern of energy use. They can reduce their energy costs by adjusting the time of the energy usage by increasing consumption in periods of lower prices and reducing consumption when prices are higher. ToU mechanisms define different prices for electricity usage during different periods: the tariffs reflect the average cost of generating and delivering power during those periods. For RTP the price of electricity is defined for shorter periods of time, usually 1 h, again reflecting the changes in the wholesale price of electricity. In RTP customers usually have the information about prices. CPP is a hybrid ToU RTP program. This mechanism is based on the real time cost of energy in peak price periods, and has various methods of implementation.

Incentive-based demand response consists in programs with fixed or time varying incentives for customers in addition to their electricity tariffs. Incentive-Based programs (IB) include Direct Load Control (DLC), Interruptible/ Curtailable service (I/C), Emergency Demand Response program (EDR), Capacity market Program (CAP), Demand Bidding (DB) and Ancillary Service (AS) programs. Classical IB programs include DLC and I/C programs. Market-Based IB programs include EDR, DB, CAP, and the AS programs. In classical IBP, customers receive participation payments (e.g. discount rate) for their participation in the programs. In Market-Based programs, participants receive money for the amount of their load reduction during critical conditions. In IC programs, participants are asked to reduce their load to fixed values and participants who do not respond can pay penalties based on the program conditions. DB are programs in which consumers are encouraged to change their energy consumption pattern and decline their peak load in return for financial rewards and to avoid penalties. In EDR programs, customers are paid incentives for load reductions during emergency conditions.

DR mechanisms and load control in the electricity market represent an important area of research at international level, and the market liberalization is opening new perspectives. This calls for the development of methodologies and tools that energy providers can use to define specific business models and pricing schemes.

Every actor in the electricity market has different objectives. For example, retailers and generators aim to maximize their own profit by reducing their costs. In contrast, customers would like their electricity bills as low as possible [425]. Game theoretical methods can also be used to capture the conflicting economic interests of the retailer and their consumers. Authors in [466] propose optimization models for the maximization of the expected market profits for the retailer and the minimization of the electricity cost for the consumer.

One implementation approach of DR mechanisms in the electricity market consists in defining economically and environmentally sustainable energy pricing schemes. In this field, optimization approaches to define dynamic prices have been proposed, and they focus on the definition of dayahead prices for a period of 24 h and for a single customer (or a single group of homogeneous customers). In [446], the response of a non-linear mathematical model is analyzed for the calculation of the optimal prices for electricity assuming default customers under different scenarios over a 24 h period. Yusta et al. [446] defines a model of an electric energy service provider in the environment of the deregulated electricity market. This problem studies the impact on the profits of several factors, such as the price strategy, the discount on tariffs and the elasticity of customer demand functions always over a 24 h period.

Consumers may decide to modify their load profile to reduce their electricity costs. For this reason, it is important to analyze the effect that the market structure has on the elasticity demand for electricity. Kirschen et al. [240] proposes an elastic model to characterize the demand-response behavior and load management with ToU programs and it describes how the consumers behavior can be modeled using a matrix of self and cross-elasticities. Aalami et al. [1] and [2] take into account also other schemes, and rely on the elastic model proposed in [240] to model the demand-response behavior. Torriti [413] assesses the impacts of ToU tariffs on a dataset of residential users in terms of changes in electricity demand, price savings, peak load shifting and peak electricity demand at sub-station level.

Response of the customers to the DR programs affects the daily load curve. Therefore, the Load Duration Curve (LDC) changes due to the responsiveness of the customers over a year and even the participation of the customers in DR programs can have considerable effects on the LDC [374]: the effects of DR need to be investigated over the daily time horizon. De Filippo et al. [93] has adapted elasticity model mentioned above to ToU based prices and considered scenarios over a 24 h period to better identify trends and assess how the characteristics of the market and the customers affect the consumption annual profiles.

Consumption and cost awareness has an important role for the effectiveness of demand response schemes for pricing optimization. Tanaka et al. [408] describes a system architecture for monitoring the electricity consumption and displaying consumption profiles to increase awareness. Ito [217] and Borenstein [55] study how customers respond to price changes, and which price indicators are more relevant in this respect.

6 Pricing Problem

Together with long term bilateral contracts, other—possibly additional—ways of managing various risks can be considered by a producer. Indeed he can also buy or sell financial instruments, such as derivatives. The simplest form of derivatives are the call and the put which may be specialized for the electricity commodity. They typically give the right (but not the obligation) to sell or buy a certain amount of energy at a given price. The price of this option is the strike price. Other, more sophisticated, options do exist, for instance a combination of both usually named as collar or other such as swing options. In choosing these options, two fundamental problems arise:

- From the selling side, the pricing, i.e. how much is the value of the instrument.
- From the buying side, the portfolio optimization, i.e. given a set of proposed derivatives, decide which one to buy and if/when to exercise them.

The pricing problem can be solved in a closed form with the well-known Black and Sholes (B&S) approach that has been criticized by various authors. However in the context of the electricity market more advanced pricing models may be useful. A recent and interesting approach is based on robust optimization models. Indeed, as the classical B&S approach, the option pricing problem aims to replicate an option with a portfolio of underlying (available) securities in each possible scenario, and therefore the robust valuation scheme proposed by some authors is natural and conceptually sound. Therefore one can use manageable robust optimisation linear programming problems, based on a dynamic hedging strategy with a portfolio of electricity futures contracts and cash (risk-free asset). The model can be used to find a risk-free bid (buyer's) price of the swing option.

7 Derivative Pricing in Electricity Markets

In portfolio theory, the most commonly used model for estimating the value of an option is the Black and Scholes model [49]. This model is based upon the assumptions of modern portfolio theory, where prices reflect all the available information. The Black and Scholes model gives the value of an option as a function of the spot price and the volatility of the underlying asset, the strike price, the time and the risk free rate. It is suitable for European options, while the estimation of an American option will require in addition to estimate the likelihood of early exercise (generally resulting from discontinuity events such as dividend distribution or bankruptcy).

Option pricing in energy markets raises specific issues, due to difficult storage and the existence of spot price models [204, 257]. For this reason, the time of exercise is a much more crucial parameter than in financial markets, and can be negotiated between the parties. Therefore, European and American options shall be treated separately. European options can be priced by predicting a spot price and using an approach similar to the Black and Scholes model. For American options, the Black and Scholes equation becomes an inequality whose solution can be approximated by robust optimization models [99, 134].

The ongoing transition from centralized architecture to interoperable grids managed by competing operators is expected to boost inter-grid transactions. The expected cost reduction in storage solution will offer to operators a more and more viable alternative to the sale of production surplus. These parameters must be taken into account in a model for derivative pricing.

8 Combined Gas and Power Optimization

Short-term scheduling of a combined natural gas and electric power system may be formulated as a two-stage optimization model and solved using mixed integer stochastic programming [210]. More stages could be considered and approached using the multi-stage stochastic programming. Benders decomposition [37] may be used to solve a nonlinear optimization problem.

A related problem is integration of the natural gas and electricity networks in terms of power and gas optimal dispatch [418]. A mathematical model of the problem may be formulated as a minimization of the integrated gas-electricity system operation cost with constraints involving the power system and natural gas pipeline equations and capacities. The problem may be solved using a hybrid approach combining evolutionary strategies and the Interior point method.

Another related problem is tri-multi-generation [76]. Various models exist for optimizing energy costs, annual costs and CO2 emissions. Optimization methods include linear programming, branch and bound, evolutionary algorithms for single- and multi-objective optimization.

9 European Electricity and Day-Ahead Markets

Besides long-term bilateral contracts, a large part of the production of electricity is traded in day-ahead markets where prices and exchanges of energy are determined for each time slot of the following day, typically an hour. Intraday and balancing markets are then meant to ensure security of supply and to balance positions taken in the day-ahead market which could not be maintained.

Pan-European Market
In Europe, the past decade has seen the emergence of a Pan-European day-ahead electricity market in the frame of the Price Coupling of Regions project (PCR), which is cleared using a common algorithm called Euphemia, handling peculiarities of the different kinds of bidding products proposed by national power exchanges. In a classical microeconomic setting, using supply and demand bid curves submitted by participants, a (convex) optimization problem for which strong duality holds, and aimed at maximizing welfare, yields a market equilibrium. The optimal dual variables then correspond to equilibrium market prices: one for each time slot and each bidding zone.

Non-convexity
These day-ahead markets are non-convex in the sense that participants are allowed to describe operational constraints such as minimum power output levels, and economic constraints such as start-up costs which must be recovered if a unit is started, rendering the primal welfare maximizing problem non-convex, mainly due to the introduction of binary variables.

Near-equilibrium
It can then easily be shown that most of the time, no market equilibrium with uniform prices could exist, where the use of uniform prices means that every bid of a given bidding zone and time slot is cleared at the same common market clearing price. The general approach throughout Europe is to use uniform prices, but to allow some non-convex bids to be paradoxically rejected in the sense that they would be profitable for the computed prices but are none the less rejected, ensuring the existence of feasible solutions, while enforcing all other market equilibrium conditions. This is classically modelled as a Mathematical Program with Equilibrium Constraints (MPEC), and handled by advanced branch-and-cut algorithms (such as Euphemia), see [280, 282, 297].

Part II
Energy Commodities Systems

Production and Demand Management

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1 Optimal Oil Wells Placement

The optimal oil wells placement problem, a crucial problem in reservoir engineering, consists in determining the optimum number, type, design, and location of oil wells to optimize the hydrocarbon production and the drilling costs.

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In industry, the decision to drill a well or not and its location is taken by reservoir engineers trusting their professional expertise. These decisions strongly relate to the understanding of the impact of different influencing engineering and geological parameters. However, such influence is very complex (nonlinear) and changing over time, thus a deep understanding of such phenomena requires more than human experience. Satisfying solutions could be provided by practitioners, but optimization methods can lead to improved configurations.

From a mathematical modeling viewpoint, the number of water injector and producer wells and the number of branches could be represented by integer variables. In addition, continuous variables as wells and branches design in the reservoir, the length of the branches, etc can be considered. The functions to optimize are generally computed from the outputs of a reservoir fluid flow simulator, costly in computational time: the outputs to optimize are the quantities of produced oil and water, and the quantities of injected water, needed for the production).

The two most widely considered objective functions are:

- maximize the quantity of produced oil;
- maximize the revenue of a wells configuration with Net Present Value (NPV) function. This function combines oil revenue, water management (water injection and separation), and drilling costs.

In both cases, given a wells location, the objective function value is provided by a numerical simulator. As we do not have access an analytic formula of the objective function, the problem is modeled as a Black-Box optimization problem. Hence, we have no information about the continuity, differentiability, or convexity of the objective function.

Constraints are generally physical ones, ensuring the practical realizability of the solution and the correct behavior of the simulator. A useful constraint is also the water cut constraint that consists in applying some reactive control on each producer to avoid producing much water which impacts negatively on the NPV. Such reactive control shuts off producers when the water cut, i.e., the ratio between the water rate produced and the sum of water and oil rates produced, is higher than a given threshold. It is also possible to add constraints during the production, e.g., produce a minimal quantity of oil for instance.

Thus, the oil well placement problem can be modeled as a Black-Box MINLP problem, a very challenging problem both from a theoretical and a computational viewpoint. Note also that, as no convexity assumption holds, one should perform some kind of global search to avoid being trapped in local minima.

In practice, nowadays well placement optimization is an iterative procedure that can be divided into the following procedures:

- Using engineering judgment, guess initial well(s) location.
- Use an optimization algorithm based on user-defined decision variables to suggest possible improved well location(s).
- Apply a reservoir response model to report to the optimization algorithm the performance of the proposed well locations.

- Include the effect of uncertainty in reservoir properties, economic factors, etc, which can be an optional step.
- Calculate the objective function (e.g. quantity of produced oil or NPV).
- Repeat steps 2–5 until stopping criteria (set by user) are met.

The approaches to problems 1–5, may differ in the optimization algorithm, reservoir response modeling technique, and available decision variables and constraints.

2 Optimization of the Gas-Lift Process

In the gas industry the key problem is the optimal gaslift with minimum energy consumption. The mathematical complexity of this optimization problem is connected with the matter that the corresponding control problem is of non-regular structure, boundary conditions of this problem include the control parameters. The gaslift method is of special importance at the initial period after the flowing of the oil fields [13, 14, 303]. The motion in the gaslift process is known to obey the hyperbolic nonlinear partial differential equations. Therefore, at gaslift operation of the borehole cavity the problem of optimization with boundary control is of special interest. However, with the original formulation of the problem of optimal control one encounters certain difficulties. The averagings of the hyperbolic equation describing the time profile of motion by the gaslift method are given here [13, 303]. It rearranges a partial derivative equation in the nonlinear ordinary differential equations. The strategy of constructing the objective quadratic functional with the use of the weight coefficients lies in minimizing the volume of the gas injected in the annular space and maximizing the desired volume of the Gas-Liquid Mixture (GLM) at the end of the lifter. In this case, the aim lies in solving the corresponding optimization problem where the volume of the injected gas which is used as the initial data and plays the role of the control action. The impossibility of using the standard methods to construct the corresponding controllers is a disadvantage of this approach. Yet, since at certain time intervals the boundary control is constant, the numerical data obtained can be readily compared with the production data. Using the method of time averaging, the partial derivative equations of motion of gas and GLM motion proposed in [13] are rearranged in the ordinary differential equations. The problem of optimal boundary control with the quadratic functional is formulated on the basis of the above considerations. The results obtained can be used to control the gaslift borehole cavity at oil extraction. For solution of the considered problem of boundary controls, the gradient method [303] is modified by describing the corresponding Euler-Lagrange equations [61].

3 Total Gas Recovery Maximization

In the short term operation, the most important problem is related to the total gas recovery maximization. In order to withdraw as much natural gas from a reservoir as possible, one option is to use waterflooding. This leads to the problem of finding an optimal water injection rate with respect to different objectives, such as the maximal ultimate recovery, or the total revenues. Indeed there are several objective functions due to different aspects of the problem.

Modeling and algorithmic considerations:

Consider two wells drilled on the surface of the gas reservoir, one for gas recovery and one for water injection. Therefore, let r(t) denote the withdrawal rate of gas which is bounded by the maximum rate of gas extraction rm(t). Through the water injection, well water is injected into the reservoir at the nonnegative rate s(t). This model assumes a constant g which is the ratio of gas entrapped behind the injected water to the volume of water at any time. The model aims at maximizing the ultimate gas recovery and can be posed in a nonlinear form. Some researchers discuss several other objective functions. For example, the objective function to maximize the present worth value of the net revenues for internal rate of return.

The application of concepts from systems and control theory to oil and gas production is the unifying idea behind the current research theme Production Systems and Subsurface Characterisation and Flow.

Past In the previous years, research and development was focused on three main areas:

1. The innovation of concepts for the hydrocarbons production process. This includes the application of smart wells, advanced, geophysical monitoring techniques, downhole treatment, the separation and conversion of substances and the injection of residuals (waste) [318, 432]. Closed-loop 'measurement and control' concepts from system theory will play an important role;
2. The development of an integrated 'real-time' dynamic simulation, inversion and validation environment for reservoir, well and processing facilities [233]. This environment will be used to test and evaluate newly developed technology from our groups and other sources. This environment is used as a learning environment and for work process analysis and optimisation;
3. Laboratory of innovation. The analysis and testing of methods, techniques and work processes to accelerate the process of innovation in the energy and production sector.

Present Currently, the application of concepts from systems and control theory to oil and gas production is the unifying idea behind the research themes production systems and subsurface characterisation and flow. By means of modelling, monitoring and control, the production systems theme aims at stabilising and optimising production in order to achieve production targets, which are being expected from an operator through long term contracts [145, 227].

Future: Smart Wells and Smart Fields Smart well technology involves down-hole measurement and control of well bore and reservoir flow. Drilling and completion techniques have advanced significantly over the last years and allow for the drilling of complex multi-lateral and extended reach wells, and the installation of down-hole inflow control valves, measurement devices for flow, pressure and temperature, and processing facilities such as hydro-cyclones in the well bore. Smart fields technology, also referred to as 'e-field' or 'digital oilfield' technology involves the use of reservoir and production system models in a closed-loop fashion [146]. The measurements may originate from sensors in smart wells, but could also involve simple surface measurements from conventional wells, or originate from other sources such as time-lapse seismics. Research in smart fields is now focused on the development of concepts and algorithms to improve hydrocarbon production though the use of systems and control theory. Future research will address the reservoir management aspects on time scales from months to many years, and in particular the development of techniques for closed-loop reservoir management. We are also developing methods to speed up the modelling and simulation part an order of magnitude [206]. For this reason we combine fast and robust iterative methods for large linear systems with Model Order Reduction insights originating from Optimal Control research. This combination has already led to very good results [102]. Various groups from the Delft University of Technology, Padua University and EPFL Lausanne collaborate in order to develop a new generation of simulators.

4 Optimal Scheduling of Energy Hubs and CCHP Systems

The future development of electric and thermal energy generation, transport and distribution relies on the exploitation of both conventional and renewable energy sources via a wide variety of energy conversion technologies; on the top of that electric and thermal energy storage could be utilized in order to match the demand with response exploiting more effectively the possible synergies between the installed units.

In this context Combined Heat and Power (hereafter CHP) power plants and engines are particularly attractive due to the higher efficiency when compared to conventional units generating only one energy commodity. CHP units can be classified into two main categories:

- one-degree-of-freedom units feature a single independent operating variable, the load (defined as the current fuel input rate divided by the maximum one), which controls the two energy outputs (e.g., electric and thermal power). As a result, for a certain power plant or engine load, it is not possible to vary the share of the two energy outputs according to customer needs. Examples of one-degree-of-freedom CHP units are internal combustion engines and gas turbines with waste heat boiler, backpressure steam cycles, and combined cycles with back-pressure steam turbine.

- Two-degree-of-freedom units feature two independent operating variables, the load and another one (such as a steam extraction valve) adjusting the share of the two energy outputs. Although these systems are more complex and typically more costly, the second control variable increase the operational flexibility of the unit. Examples are steam cycles with extraction condensing steam turbine (a steam extraction valve controls the steam bled from the turbine and used to provide heat to the customer).

It is worth noting that also more sophisticated units featuring three independent variables exist (e.g. CHP natural gas combined cycle with post firing and extraction-condensing steam turbine). Moreover, looking at the energy outputs, some units can be configured so as to cogenerate cooling power in addition to electricity and heat. Such units are called Combined, Cooling, Heat and Power (CCHP). Examples are units made by an internal combustion engine, a waste heat boiler and an absorption chiller (converting heat into chilling power).

Systems featuring several CCHP or CHP units may be integrated with other units such as boilers, heat pumps, and energy storage systems within so-called Energy Hubs. The sizes may range from few hundreds of kW for buildings to hundreds of MW for industrial users and or district heating networks.

Three main types of challenging optimization problems arise when dealing with such integrated systems:

- short-term scheduling, also called unit commitment,
- long-term operation planning,
- design or retrofit of the energy hub.

The short-term unit commitment problem can be stated as follows:

Given:

- the considered time horizon (e.g., 1 day, 2 days, 1 week) and an appropriate discretization into time periods (e.g., 1 h, 15 min),
- forecast of electricity demand profile,
- forecast of heating and cooling demand profile,
- forecast of ambient temperature,
- forecast of time-dependent price of electricity (sold and purchased),
- performance maps of the installed units,
- operational limitations (start-up rate, ramp-up, etc.) of units,
- efficiency and Maximum capacity of storage systems;

optimize the following independent variables:

- on/off of units,
- load of units,
- share among heat and power (only for two-degree-of-freedom units),
- energy storage level (hence charge/discharge rate) in each time period (for each energy storage system);

so as to minimize the operating costs (fuel + operation and maintenance + electricity purchase) minus the revenues from electricity sale for the given time horizon while fulfilling the following constraints:

- energy balance constraints for each time interval, e.g. electric energy, thermal energy, etc.,
- start-up constraints for each time unit, for each unit,
- ramp-up constraints for each time unit, for each unit,
- performance maps relating the independent control variables of the units with their energy outputs (e.g. output thermal power as a function of the load),
- a number of case-specific side constraints, e.g. maximum number of daily turns-on/off, for each unit; precedence constraints between units; minimum time unit permanence in on/off states, for each unit etc.

All constraints, except the performance maps of the units, can be easily formulated as linear equalities or inequalities. Performance maps of units are generally nonlinear and often not convex functions yielding to a nonconvex Mixed Integer NonLinear Program.

Due to the large number of variables, both integer and continuous, commercially available global MINLP solvers are not capable of finding the global optimum within reasonable time limits [404]. Besides genetic algorithms [236] or Tabu search [291] from late nineties or other solutions going from Lagrangian relaxation [57] to heuristic algorithms based on engineering practice for simple problems [46], the most effective approaches are based on the linearization of performance maps so as to obtain a Mixed Integer Linear Program (MILP) [307]. This allows to use efficient MILP solvers, such as Cplex [214] and Gurobi [184], and have better guarantees on the quality of the returned solution [404]. The performance maps of the machines can be linearized using either the convex hull representation [254] or classic piecewise linear approximations [89] of 1D [456] and 2D functions [46]; the latter kept into account also daily storage facing an large increase of computational effort, ranging from two to three orders of magnitude.

The so described problem assumes that forecasts of energy demands and prices are accurate and their uncertainty is limited. If data uncertainty needs to be considered, the short-term scheduling problem can be extended and reformulated either as a two-stage stochastic program [15, 66] or a robust optimization problem with recourse [313, 467].

As an additional challenge, when determining the optimal scheduling of CHP units, it is necessary to take into account of the European Union regulation for high efficiency CHP units [104]. If a CHP unit achieves throughout the whole year a primary energy saving index above a threshold value, incentives are granted. Being a yearly-basis constraint, it poses the need of considering the whole operating year as time horizon when determining the optimal scheduling of CHP units. The same requirement concerns energy hubs featuring seasonal storage systems [161] capable of efficiently storing energy for several months. Since tackling the scheduling problem for the whole year as a single MILP is impracticable, metaheuristics based on time decomposition to reach near optimal solutions in a reasonable amount of time have been proposed. Bischi et al. [47] proposed a rolling horizon algorithm in which the time horizon is partitioned into weeks. The extension of the MILP model from 1 day to 7 days may imply an increase of computational time from few sec for a single day to tens of minutes for the week (with MILP gap below 0.1%) but it allows to better manage the thermal storage system accounting for the weekly periodicity of the users' demand. Within the rolling-horizon algorithm, the weekly MILP subproblems are solved in sequence from the current week till the end of the year. The yearly-basis constraints related to the CHP incentives are included in each weekly MILP subproblem by estimating the energy consumption and production of the future weeks of the year with the corresponding typical operating weeks (previously determined and optimized). If the yearly basis CHP incentive constraints are not met, the rolling horizon algorithm is repeated considering a higher (less optimistic) energy consumption for the future weeks. Thanks to the decomposition of the operating year into weekly subproblems, the computational time required to optimize the whole year of operation with a tight relative MILP gap (0.1%) ranges from 1 day to 3 days, making the algorithm an effective scheduling and control tool for energy hubs featuring CHP units.

Finally it is worth pointing out that, due to growing industrial interest in the optimal operation of complex energy systems for providing cooling, heating and power (e.g., energy service companies, multi-utilities managing district heating networks as well as power plant operators), several tools are already available on the market [42].

5 The Pooling problem

The pooling problem arises in the chemical process and petroleum industries. It is a generalization of a minimum cost network flow problem where products possess different specifications (e.g. sulphur concentration). In a pooling problem, flow streams from different sources are mixed in intermediate tanks (pools) and blended again in the terminal points. At the pools and terminals, the quality of a mixture is given as the volume (weight) average of the qualities of the flow streams that go into them.

There are three types of tanks: inputs or sources, which are the tanks to store the raw materials, pools, to blend incoming flow streams and make new compositions, and outputs or terminals, to store the final products. According to the links among different tanks, pooling problems can be classified into three classes:

- Standard pooling problem: in this class there is no flow stream among the pools. It means that the flow streams are in the form of input-output, input-pool and pool-output.
- Generalized pooling problem: here, flow streams between the pools are allowed.
- Extended pooling problem: here, the problem is to maximize the profit (minimize the cost) on a standard pooling problem network while complying with con-

straints on nonlinearly blending fuel qualities such as those in the Environmental Protection Agency (EPA) Title 40 Code of Federal Regulations Part 80.45.

There are many equivalent mathematical formulations for the pooling problem, such as P-, Q-, PQ- and HYB- formulations, and all of them may be formulated as nonconvex (bilinear) problems, and consequently the problem can possibly have many local optima. More information about different formulations may be found in [183].

Despite the strong NP-hardness of a pooling problem in general, proved in [11], and even for problems with a unique pool, proved in [12], or with single-flow restriction, proved in [190], there are classes of pooling problems for which algorithms with polynomial running time exist; see e.g. [28, 53, 189, 191]. Furthermore, much progress in solving small to moderate size instances to global optimality has been made since 1978, when Haverly in [193] described the P-formulation and solved small standard pooling problems using recursive linear programming. A common approach is to construct good lower and upper bounds for use in a branch-and-bound framework; see e.g. [147]. To have tighter lower bounds, different methods have been proposed in the literature including Lagrangian approaches [4], (piecewise) linear relaxations [100, 101, 305], modification of polynomial optimization hierarchies [292], and convex nonlinear relaxations [274]. The first software that is developed specifically to solve pooling problems is called APOGEE [305], where the authors make use of an iterative piecewise linear relaxation, of which it is proved in [100] that the first iteration may result in a lower bound far from the optimal value.

Due to the high-complexity, different pooling problem instances have been collected in libraries such as [158], which are used as the test bed to assess the performance of newly developed solvers and algorithms for nonlinear optimization problems; see, e.g., [293, 304, 306].

Two interesting generalizations of the pooling problem are:

- more general networks where other types of units than pools are also present, e.g. units that extract pollutants. Mathematically, this generalisation falls within the framework of so-called wastewater management networks; see e.g. [235].
- treating the network topology as a decision variable, as done in [305].

Network and Storage

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1 Gas Pipeline Design

Natural gas is considered by many to be the most important energy source for the future. The objectives of energy commodities strategic problems can be mainly related to natural gas and deal with the definition of the "optimal" gas pipelines design which includes a number of related sub problems such as: Gas stations (compression) location and Gas storage locations, as well as compression station

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design and optimal operation such in [255]. Needless to say these problems involve amount of money of the order of magnitude of the tens of billions EUR and often these problems can be a multi-countries problem. From the economic side, the natural gas consumption is expected to continue to grow linearly to approximately 153 trillion cubic feet in 2030, which is an average growth rate of about 1.6% per year. Because of the properties of natural gas, pipelines were the only way to transport it from the production sites to the demanding places, before the concept of Liquefied Natural Gas (LNG). The transportation of natural gas via pipelines remains still very economical.

From an optimization standpoint, the gas pipeline design problems can be divided in the following main sub problems:

1. how to setup the pipeline network, i.e. its topology;
2. how to determine the optimal diameter of the pipelines;
3. how to allocate compressor stations in the pipeline network;

Typically, the mathematical programming formulations of these optimization problems contain many nonlinear/nonconvex and even nonsmooth constraints and objective functions because of the underlying physics of the gas flows that need to be considered. The classic constraints are the so-called Weymouth panhandle equations, which are a potential-type set of constraints and relate the pressure and flow rate through a pipeline.

As in many other situations, problems 1–3 are a single problem but a divide et impera principium is applied. Therefore the problems 1 and 2 are somehow determined via simulations and normally there are—in the first but also in the second problem—many economic drivers, and also political drivers when many countries are involved. From a technical point of view, problem 3, the allocation of the compressor stations, is probably the most challenging. Because of the high setup cost and high maintenance cost, it is desirable to have the best network design with the lowest cost. This problem concerns many variables: the number of compressor stations which is an integer variable, the pipeline length between two compressor stations, and the suction and discharge gas pressures at compressor stations. This problem is computationally very challenging since it includes not only nonlinear functions in both objective and constraints but, in addition, also integer variables.

In the case of transmission networks, existing infrastructure is already available, but needs to be expanded to increase the capacity. To this end, new pipelines are often built in parallel to existing ones, effectively increasing the diameter. On the other hand, for the exploitation of new gas fields or off-shore transportation, pipeline systems are designed from scratch with no predetermined topology. Capacity planning and rollout has a time horizon of several years. Accordingly, some optimization models consider multiple stages of network expansion. Many of the planning problems are formulated as mixed-integer non-linear programs (MINLP) with integer variables and nonconvex constraints. To solve these models directly, solvers apply outer approximation and spatial branching. Alternatively, the problem functions can be approximated piecewise linearly, yielding a mixed-integer linear program (MILP) formulation. A survey paper concerned with water networks is also relevant here [90]. Specialized algorithms make use of the fact that certain subproblems with fixed integer variables have a convex reformulation, which can be solved efficiently and used for pruning [212, 356].

The design of pipeline topologies from scratch is solved with a decomposition, where first a topology is fixed heuristically, and improved by local search. The pipeline diameters are then solved separately [366].

In the case that the network has a tree topology, Dynamic Programming has been applied, both for the choice of suitable pipe diameters [366] as well as compression ratios [58].

Another important aspect is how to treat varying demand scenarios. A finite number of different scenarios can be tackled using decomposition techniques [385]. When the network has a tree topology, also robust variants can be reduced to finitely many scenarios [362].

2 District Heating Network Design

In the current energy market context, District Heating (DH) has an important role, especially in countries with cold climate. DH often leverages on Combined Heat and Power (CHP) units, capable to reduce the consumption of primary energy to fulfill a given electric and thermal request, as well as on existing significant sources of heat generated by industrial processes or waste-to-energy heat generation. Additionally, heating networks will need to increase their flexibility in operation due to an increasing mix of renewable sources, both heat sources or green electricity utilized by heat pumps, distributed generation and smart consumers as well as DH operational temperature reduction and heat storage integration [277, 426].

From a management standpoint, the design of the district heating network is a strategic business issue, since it requires large investments due to the cost of materials and civil works for the realization of the network. Proper strategic design of the network (i.e. definition of the most convenient backbone pipelines to lay down) and tactical targeting of most promising potential customers both aims at maximizing the Net Present Value (NPV) of the investment.

Finding the extension plan for an existing (or eventually empty) DH network that maximizes the NPV at a given time horizon is a challenging optimization problem that can be stated as follows. Given:

- A time horizon (e.g., 15 years)
- A set of power plants, with specific operational limitations (maximum pressure, maximum flow rate,. . .);
- An existing distribution network, with information on the physical properties of the pipes (length, diameter,. . .);
- A set of customers already connected to the network with known heat demand;
- A set of potential new pipes that can be laid down;
- A set of potential new customers that can be reached;

find:

1. The subset of potential new customers that should be reached;
2. Which new pipelines should be installed;
3. The diameter of the new pipes

that maximize the NPV.

Research on modelling approaches for representing the behavior of the thermo-hydraulic network through sets of non-linear equations can be found in the literature (see for example [52] and [343]). Solving systems of non-linear equations is difficult and computationally expensive. For this reason, aggregation techniques of the network elements are often used to model large district heating networks, at the expense of some accuracy [258, 259, 272, 450, 452] and [272].

In [25], an integer-programming model is proposed for the optimal selection of the type of heat exchangers to be installed at the users' premises in order to optimize the return temperature at the plant. The authors achieve good system efficiency at a reasonable cost.

Bordin et al. [54] present a mathematical model to support DH system planning by identifying the most advantageous subset of new users that should be connected to an existing network, while satisfying steady state conditions of the thermo-hydraulic system. Bettinelli et al. [42] extend the model proposed by Bordin et al. [54] with the selection of the diameter for the new pipes and a richer economic model that takes into account

- Production cost and selling revenues;
- Cost for installing and activating new network links;
- Cost for connecting new customers to the network;
- Amortization;
- Taxes;
- Budget constraints.

Moreover, while the investment on the backbone pipelines is done on the first year, new customers are not connected immediately, but following an estimated acquisition curve (e.g., 25% the first year, 15%, the second year, ...). Hence, the corresponding costs and revenues have to be scaled accordingly.

The thermos-hydraulic model must ensure the proper operation of the extended network. The following constraints are to be imposed:

- Flow conservation at the nodes of the network;
- Minimum and maximum pressures at the nodes;
- Plants operation limit: maximum pressure on the feed line, minimum pressure on the return line, minimum and maximum flow rate;
- Pressure drop along the links;
- Maximum water speed and pressure drop per meter.

Continuous variables model pressures at nodes and flow rate on the links and binary variables model decisions on the connection of new customers, on the installation of new links, on the diameter choice and on flow direction on the links. The latter are necessary since DH networks contain cycles: the potential network usually corresponds to the street network. Thus, it is not possible to know the flow direction on the links a priori (at least not for all of them) and such decision must be included into the model. The pressure drop along a pipe is a non-linear function that depends on the flow rate and on the diameter of the pipe. This can be approximated using a piecewise linear function that translates into a set of linear constraints. The higher the number of segments in the piecewise linear function, the smaller will be the approximation error. At the same time, the number of constraints grows (there is one piecewise-linear function for each combination of pipe and diameter) and the solving time increases. To keep the number of segments small, while obtaining a good accuracy, breakpoints of the piecewise linear function can be concentrated in the most probable range of flow rate.

DH networks can be quite large (hundreds of existing and potential users, thousands of links) making it difficult to solve the full MILP directly. Solution methods developed in [42] approach the problem in three steps.

1. Solve the linear relaxation of the MILP model and use it to select water direction in all the pipes. Then, solve to integrality the MILP model, with the directions fixed, obtaining a first heuristic solution.
2. In the solution found at step 1, the conflict points, which are the nodes of the network where different water direction meet, are detected. The flow direction is released for the nodes close to conflict points, and the MILP model is solved again, obtaining a second heuristic solution.
3. The full MILP, initialized with the best solution found in the previous steps, is solved until either optimality or the time limit is reached.

The company Optit S.r.l. has developed a decision support system, in collaboration with the University of Bologna, based on the modelling mentioned above that has been successfully used in two of largest multi-utility companies operating in the Italian DH market. The application leverages on open source Geographical Information System (GIS) to allow a simple user interface and a number of plug-in tools to manage the specific optimization issue.

3 Optimal Design of Energy Hubs and CCHP Systems

The optimal design of energy hubs and combined cooling, heating and power (CCHP) systems consists in determining the energy technologies (i.e., power generation units and energy storage systems) to be installed and their sizes which minimize a certain cost function (e.g., the total annual cost given by annualized capital and operating expenditures) while providing electricity, heating and cooling power to a set of users. In the presence of multiple users and possible installation sites, it is necessary to determine the units to be installed in each site and the required energy network connections between sites.

The problem turns out to be a very challenging nonconvex MINLP [112] with a large number of binary variables, because it has to include not only the design variables (units selection and sizes) but also the operation variables and constraints for the whole system lifetime. Due to the variable energy demand profiles and electricity prices, the loads of the installed units must be continuously adjusted so as to meet the demands and maximize the revenues. Thus, when designing the system, the part-load performance and the operational flexibility (e.g. ramp constraints) must be evaluated for the set of expected operating conditions. As a result, in most formulations (see review in [112]), the design optimization problem includes also the operational/scheduling problem with a considerable increase of problem size and complexity.

The design problem is more complex than the scheduling problem not only because of the larger number of variables and constraints (design + scheduling variables) but also for the nonlinearity of the functions relating to units' sizes with energy efficiency (larger units feature higher energy efficiency [112]), and investment costs. The approaches proposed to tackle the resulting nonconvex MINLP problem can be classified in two main families:

1. linearization of all nonlinear functions so as to obtain a single large scale linear problem (MILP) [444] and [161].
2. decomposition of the problem into a design level (upper level or master problem) and a scheduling level (lower level) [138, 218] and [112].

At the upper level the selection and sizing of the units is optimized by either solving a simplified (and linear) design and operational problem [218] or using evolutionary algorithms [112, 138]. At the lower level, for each fixed design solution, the operational scheduling problem is solved.

In order to limit the size of the problem, it is possible to reduce the number of expected operating periods (i.e., days or weeks) by considering only the most representative ones (i.e., "typical days" [138] or "typical weeks" [112]). Starting from historical data of the users' energy demand, data clustering algorithms, such as the k-means algorithm [155, 188], can be effectively used to group similar operating periods (i.e., daysweeks with similar profiles of energy demands) into clusters and select a few representative demand profiles to be included in the design problem.

4 Operational Network and Storage Management

Originally natural gas was treated as a byproduct of crude oil or coal mining and was spared. The flares in the mining field were usually natural gas. Not until the introduction of pipelines did the natural gas become one of the major sources of energy. The earliest gas pipelines were constructed in the 1890s and they were not as efficient as those that we are using nowadays. The modern gas pipelines did not come into being until the second quarter of twentieth century. Because of the properties of natural gas, pipelines were the only way to transport it from the production sites to the demanding places, before the concept of Liquefied Natural Gas (LNG). The transportation of natural gas via pipelines remains still very economical, but it is highly impractical across oceans. Although the LNG market is burgeoning in high speed now, pipeline network remains the main transportation system for natural gas.

From the operational stand point, the main objective for the optimization model is to ensure optimal routing and mixing of natural gas. The objective for the model is to deliver the nominated volumes in the different import terminals within a time period. This objective can be reached in several ways, and in order to influence the operation of the network some penalties are introduced in the objective function. This is done to influence the impact of the following goals:

- Maintain planned production from the producers, where this is physically possible.
- Deliver natural gas which meets quality requirements in terms of energy content.
- Deliver within the pressure requirements of the contract.
- Minimize the use of energy needed in order to deliver the natural gas to the customers by minimizing the pressure variables.

The goal of the network and storage operation is to route the gas flow through the network in order to meet demand in accordance with contractual obligations (volume, quality and pressure). A set of constraints are therefore to be satisfied, the following list describes them:

- Production capacity: total flow out of a production node cannot exceed the planned production of the field in that node;
- Demand: the total flow into a node with customers for natural gas must not exceed the demand of that node;
- Mass balance for each node: this constraint ensures the mass balance in the transportation network;
- Pressure constraints for pipelines: this is probably the most important and complex constraint, since it calls for the satisfaction of the equation to describe the nonlinear relationship between flow in a pipeline as a function of input and output pressure. Normally this is done by using the Weymouth equation. This equation can be linearized through Taylor series expansion around a point representing fixed pressure into the pipeline and fixed pressure out of the pipeline respectively. Some physical pipelines between nodes where the distances are very limited can be modeled without pressure drops by the Weymouth equation simplifying part of the modeling of bidirectional pipelines.
- Modeling bidirectional pipelines: Sometimes a bidirectional flow must be ensured, so specialized constraints with binary variables must be inserted to model this to make sure that there only flows gas in one direction in the pipeline.

- Gas quality and blending: Gas quality is a complicating element because we have to keep track of the quality in every node and pipeline, and this depends on the flow. Where two flows meet, the gas quality out of the node to the downstream pipelines depends on flow and quality from all the pipelines going into the node

Apart from the pure network operation and optimization, also the storage must be taken into account in the whole operational problem. Indeed as a consequence of the liberalization process in the natural gas industry, the natural gas markets have become more dynamic. The spot markets and the possibility to trade gas in forward markets have increased the importance of gas storage. The main problem of the storage management is related to the simple fact that one wants to take advantage of the strong seasonal pattern in prices. Since the primary use of natural gas is for heating and production of electricity, the fundamental price determinant in the markets is the weather.

However, modelling the storage in a realistic way is not as simple as it may seem, in fact the maximum in—and outflow rates of the storage varies with the current storage level. The maximal injection rate is a strictly decreasing convex function of the storage level. Likewise the outflow rate can be given as a strictly increasing convex function of the storage level. Other concepts such as Cushion gas and Working gas must be considered in order to model the storage in a correct way.

All the variants of the network and storage operational problem can be complex MILP or MINLP, with typically non convex continuous relationships.

5 Gas Network Flow Optimization

A gas network has a number of entry and exit points. Shippers independently contract the right to use the network on these points. Only at the time of actual use, the combination of entries and exits is known. One of the questions is, if all possible future transport use by the shippers can be met.

Past In the past the situation of gas transport was merely static. So it was possible to take a long period (years) and use expert knowledge to generate severe realizations (these are called shipping variants) that should be considered to check whether a new contract can be honoured.

Present Currently a method is used, based on simplified models, to generate a limited set of shipping variants which should be considered when a new situation occurs. Since the changes in law and new energy sources lead to many more different situations such a method should be fast, robust, structured, objective, based on simple principles and generates a small set of shipping variants. The proposed method satisfies most of the requirements and reproduces known shipping variants obtained by expert knowledge.

Future Although the method works well for the current situation it is important to base the method on a firm mathematical basis. Furthermore it would be nice to reduce the number of shipping variants even more.

Open questions are:

- Which physical quantities, metrics and techniques should be used to find those transport conditions that determine the size of the infrastructure?
- Which techniques are available to sufficiently reduce the obtained set in order to find an exhaustive subset whose elements are mutually exclusive, given a required accuracy?
- What mathematical optimisation tools can be used to maximise the load and minimise the number of scenarios, given that all transport paths from entry to exit need to be covered?

For the problem a variety of optimization methods have been used: From linear programs to mixed-integer nonlinear programs. The choice of method depends first and foremost on the chosen model for the pressure drop in pipes and whether one uses a time-dependent model or not. Among the easy cases are the following: If the network is topologically simple, say a so-called gun barrel or tree-like network, then dynamic programming approaches are the state-of-the-art (see [71]). If one chooses to use a stationary model, then it can be reasonable to use an algebraic solution of a simplified system, a special case of these is known as the so-called Weymouth equation. The problem then is a mixed-integer nonlinear program which can be tackled directly with off-the-shelf MINLP solvers for small networks or using specialized methods for larger networks (see e.g. [213, 243]). Popular choices for methods include using piecewise linear approximations/linearizations to obtain MILP models [166, 167, 296], MPEC-based models (see [31, 380]). Neglecting the discrete decisions leads to NLP models which can be solved to local optimality (see [382]). But also these equations can be simplified even further. A possibility is to locally linearize them around a working point, an approach that is very successful in practice (see [203]). If one opts to use the full Euler equations in the instationary setting one obtains a mixed-integer PDAE-constrained optimal control problem that is intractable for current methods except for very simple networks. Another approach with high physical accuracy is to use a (sub)gradient-based approach on top of an accurate simulation tool (see [222] and [431]). In order to reduce the high degree of nonlinearity that results from the gas dynamics, different approaches simplify the full Euler equations to the isothermal case, i.e. assuming constant temperature of gas. Additionally, the usage of different discretization schemes for the underlying isothermal Euler equations results in different transient gas flow models. For example, [106] use a piecewise-linear representation of the nonlinearities leading to MILP models. Alternatively, [460] and [288] neglect the discrete nature of the active elements and solve the resulting NLP models. [63] present a new discretization scheme that admits to keep the algebraic structure of the stationary Weymouth equation which is used to obtain globally optimal solutions of the MINLP model.

6 Optimal Operation of District Heating Systems

Future power systems with a large penetration of fluctuating renewable energy production from wind and solar power generation call for demand flexibility. In Denmark, for instance, on average 44% of the power load in 2017 was covered by wind power production, and during several hours the wind production was well above 100% of the electricity load, which was possible partly because of the flexibility of the widely used DH systems.

Heating and cooling represent a huge part of the total energy consumption. According to 2014 Eurostat figures, in the EU around 30% of the primary energy is used to produce heat, and 40% is used for electricity, including electricity for heat production [241]. The dynamics and inertia of thermal systems and the low-cost storage capabilities for hot water, imply that DH systems are capable of playing an important role in the future intelligent and integrated energy system. As mentioned above, in Denmark DH systems already play a very important role in the integration of the fluctuating renewable energy production and for providing energy balancing services to the power grid.

Historically DH systems are often considered as single systems, but this is rather due to the historic emphasis on energy supply as subsystems of different supply sources (e.g., gas, coal, and electricity). However, today they act as a key element for integrating the different energy systems, and they provide some of the needed flexibility to the power system [300]. DH systems also provide an eminent possibility of using excess heat from e.g. industrial production and cooling in super markets.

This section briefly describes the operational optimization problems involved in various parts of DH systems, and some methodologies and tools for solving these problems will be indicated. For operational convenience we will split the discussion into a number of subsystems, which can provide flexibility to the overall DH system. Each subsystem leads to an optimization problem and calls for adequate related methodologies for providing the optimal operation. The optimal operation of the following subsystems are considered:

- DH plant, including production and storage facilities.
- DH network with the pipes and pumps.
- DH users, which might also consist of secondary distribution networks.
- DH connected heat pumps and boilers.

In general, DH systems often consist of a spectrum of different possibilities for heat production. For example, thermal solar plants are becoming increasingly popular nowadays. However, the solar energy production is often hard to predict, and hence this calls for methods like probabilistic forecasting and optimization under uncertainty [311]. Here methods like stochastic programming and stochastic control theory are obvious for solving the operation problem in near real time. Therefore, the operation of the total DH system can be considered as a set of nested stochastic programming and control problems, which are presented in more detail in the remainder of this section. In the following description only district heating will be considered, but almost the same principles can be used for district cooling.

Operation of DH Plants The portfolio of production units in a DH system, comprising of combined heat and power (CHP) plants and heat-only units, can be used to react to current state of the energy system and thus increase efficiency and reduce imbalances. In periods with high generation of intermittent renewable power resources, the generation can be shifted to heat-only units, which maybe even consume power (e.g. heat pumps) to lower the imbalance in the grid, while fulfilling the heat demand. In periods with less power production from wind and photovoltaic, CHP plants can provide power to the market while producing heat. Thus, the coupling of the operation of the district heating system to the electricity markets is important [277]. The key to reducing costs in the operational production is by considering all production units as a portfolio to make use of the flexibility. By optimizing the entire portfolio, the interplay between the units can be used to further reduce costs and increase income from the market. During the optimization several restrictions have to be considered, such as the capacities of the producing units and connected thermal storages as well as technical characteristics of the units (e.g. start up/shut down times and costs).

In recent years, the production of heat from the installed small CHP plants has slightly decreased in favor of heat-only units such as boilers and heat pumps, due to the reduction of the electricity prices. The design of today's electricity market forces CHP producers to present power production offers 1 day before the actual energy delivery. Consequently, forecast uncertainties in prices and heat demand must be considered for an optimal planning of DH systems. Furthermore, the above mentioned increase in solar thermal production introduces an additional source of uncertainty from the heat production side.

To efficiently operate this mixture of heat production units while reducing the operational costs, several optimization techniques such as MILP, Lagrangian relaxation, heuristics, or fuzzy linear programming have been proposed. However, the use of MILP prevails over the other methods due to the easy implementation of these programs in available commercial solvers. In addition, the formulation of two-stage stochastic and robust MILP problems allows the integration of uncertainty in the optimization problem yielding in better operation plans for CHP plants [103, 467]. The use of two-stage stochastic programs to optimize the heat production of different heat-only, storage and CHP units translates into more flexibility in the real-time operation [322]. Finally, stochastic programming has been proven to be an effective tool to make use of DH networks to integrate the uncertain production from renewable energy sources [200].

Operation of DH Networks The problem of determining the optimal operation of DH network relates to finding the optimal combination of flow and temperature profiles that provide the minimal operational cost. Pumping costs are, however, often an order of magnitude smaller than the costs related to the heat loss induced by having a too high supply temperature profile in the network [378]. Consequently, a reasonable control strategy for DH networks is to keep the supply temperature from the district heating plant as low as possible. This is in particular the case, if the heat production takes place at a CHP plant [284, 285].

The control of the temperature is subject to some constraints. For instance, the total heat requirement for all consumers must be supplied at any time and location, such that each individual consumer is guaranteed some minimum supply temperature. A lower supply temperature leads typically to large savings, since this implies lower heat losses from the transmission and distribution networks as well as lower production costs.

As described in [285], the optimal operational problem can be formulated as a stochastic problem which can be solved using dynamic programming. Furthermore, given probabilistic forecasts for the heat load, cf. [321], and stochastic models for the dynamics in the network, the problem can be described as a problem which can be solved using stochastic control theory.

A DH system is an example of a non-stationary system, implying that model parameters have to be time varying, e.g., the time-delay from the plants to the end-users is unknown and time-varying. Therefore, the methods used in conventional predictive control theory have to be modified [347]. The modified controllers have been incorporated in a software package, PRESS (HeatTO), developed at the Technical University of Denmark. PRESS (HeatTO) has been applied and tested, e.g. at Vestkraft in Esbjerg, Denmark, and significant savings have been documented [284].

Operation of DH End-Users The end-users in DH system can provide flexibility by storing energy in the thermal mass of the buildings or in a local water tank. In [186] it is shown how nested stochastic control problems can be defined such that the thermal mass of buildings can provide services to the future smart grid. This is further explained in [286].

Furthermore, in order to avoid, e.g., costly upgrades of the existing network in large cities, it will be more and more important to control the maximum energy used within a certain interval. This can be obtained by a control that directs the maximum flow towards specific end-users or districts.

Dynamic tariffs provide another option for enabling flexibility in DH networks. For instance, to reduce the peak consumption in the morning, an extra price or penalty can be utilized during peak hours.

Operation of Heat Pumps and Boilers It is suggested in e.g. [286] that dynamic electricity prices can be used to control the electricity consumption and hence to enable the needed flexibility for integrating large shares of fluctuating and intermittent renewable power generation.

Time-varying price signals are an example of a penalty signal that can be linked to the optimization and control problem in order to arrive at a cost optimal solution by demand response. Another example are real-time marginal CO_2 signals that can be used as a penalty signal linked to the optimization problem. Then the optimal solution will minimize the CO_2 emission associated with the optimal control or operation.

Different penalty signals will lead to different optimal solutions for the problem and the choice depends on the context or societal ambition. Three of the most obvious penalty signals are the following:

- **Real time CO_2**. If the real time (marginal) CO_2 emissions related to the actual electricity production is used as penalty, then the optimal control will minimize the total carbon emission related to the power consumption. Hence, the heat production provided by the heat pump or boiler will be *emission efficient*.
- **Real time price**. If a real time price is used as penalty, the objective is obviously to minimize the total cost. Hence, the optimal operation is *cost efficient*.
- **Constant**. If a constant penalty is used, then the controllers would simply minimize the total energy consumption. The optimal control will the provide a systems which is *energy efficient*.

It is clear that a DH system with controllers defined by an objective of minimizing the total emission would in general lead to an increased use of energy. However, this may happen during periods with, e.g., a large amount of wind power production and where the alternative would be to stop some wind turbines.

7 Gas Networks in Energy Systems Sector Integration

Transition of energy systems from fossil to renewable energy sources took a boost after 21st Conference of the Parties (COP21) in 2015, where participants of United Nations Framework Convention on Climate Change signed the Paris Agreement. They committed to reduce greenhouse gas emissions gradually until 2050 [417]. The aimed reductions in greenhouse gas emission in 2030 and 2050 are known as COP21 goals. To reach these goals there are several potential pathways and increasing the share of renewable energy sources (RES) in electricity production is one of them [361]. Regarding the share of electricity in final energy consumption and share of electricity production in greenhouse gas emissions [133, 216] as well as varying nature of RES, a holistic view to the energy production and transmission systems together with energy consuming sectors is required to reduce the greenhouse gas emission as aimed while maintaining the security of supply of energy. This brings the sector coupling notion into the scene.

The concept of sector coupling is defined by the International Renewable Energy Agency [326] as co-production, combined use, conversion and substitution of different energy supply and demand forms—electricity, heat and fuels. The readers are referred to [361] and [60] for extended literature review on sector coupling. As seen from the studies in the literature, the main challenge in modelling sector coupling is the computational complexity induced by integrating several models of sectors included in the study. Hence, the studies in the literature either include a smaller number of sectors, i.e., electricity and gas, or reduce the spatio-temporal span of the study, i.e., including only a single country or a restricted region [60, 81].

In a very recent study commissioned by European Parliament [132] sector coupling is separated in two groups.

- "end-user sector coupling" involves the electrification of energy demand while reinforcing the interaction between electricity supply and end-use [316].
- "cross-vector coupling" involves the integrated use of different energy infrastructures and vectors, in particular integration of electricity, heat and gas.

In this section, we focus on how other energy vectors are integrated into gas infrastructure in different scenarios for cross-vector sector coupling on the supply side. Gas networks' interaction with energy systems are twofolds.

- Gas networks as gas provider for gas-powered plants (GPPs): Gas networks main function is to transport gas from suppliers to the final consumers that include industry, household users as well as gas-fired power plants (GPPs). Hence, the main interaction between the gas networks and energy system is through the supply of gas to the GPPs that produce electricity. However, with the increase in share of RES that are stochastic in nature, in electricity production GPPs—as more agile electricity production facilities—have been used to balance the demand for electricity with a potential of resulting in a rapid and larger scale fluctuation in gas network demand then it used to be. This brought the question whether the gas demand required to produce electricity especially at peak demand times can be met from the gas suppliers, i.e., indigenous production sites, LNG facilities or other countries that the gas is imported, given the existing gas network infrastructure. State-of-the-art academic studies focusing on this question focus on very limited scales such as 5–10-node small networks designed for research or simplified small networks, i.e., 79-node UK network [20, 72, 80, 144, 302, 449]. On the other hand, Beulertz et al. [43], propose a flexible modeling framework including integration of other energy vectors to gas infrastructure, which is going to be tested by a case study aiming at investigating a multi-modal European energy concept. They use a stationary gas model to evaluate the feasibility of the optimal multi-model energy mix found in the case study using a multi-modal investment model, European unit commitment model and electricity grid model. In this context, gas networks serve as a flexibility option to the electricity network by their ability to store gas in pipelines and underground gas storage facilities connected to the gas network. The electric-powered compressor stations in gas network that use electricity to compress gas to increase the pressure of gas in the pipes of the network are other means of interaction in the context of security of supply of gas to GPPs (See Sect. 11).
- Power-to-gas: Power-to-gas (P2G) is an emerging technology that provides flexibility to energy system by converting surplus electricity produced by RES to hydrogen or synthetic methane, and feed it into the gas network. On the other hand, gas power stations convert gas into electricity in peak demand situations with not enough RES available. Thus, P2G lies in the interface of gas and

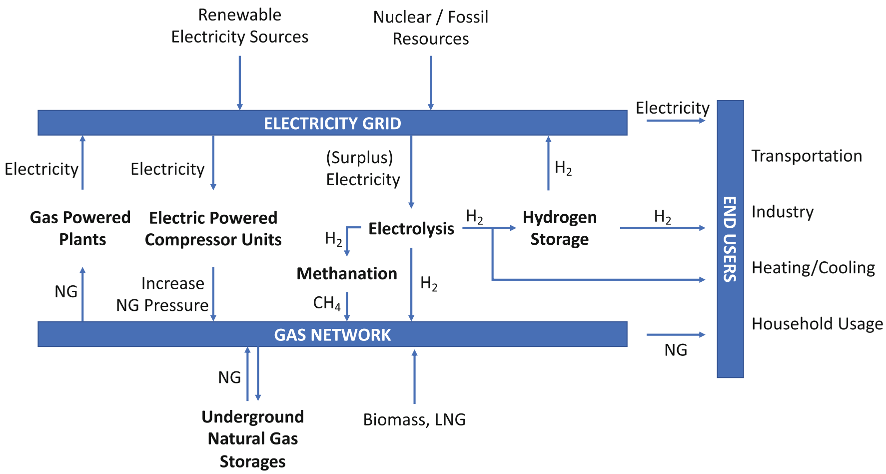

Fig. 1 Interaction of gas network and energy system through P2G

electricity network, where the amount of gas fed into the gas network is limited by the network's technical capacity and properties of the gas fed into the network. Interaction of gas network and energy system through P2G is presented in Fig. 1.

The main challenge for integrating other energy vectors into gas network models arise from the physical nature of gas and electricity. Gas flows in pipelines according to thermodynamic laws making it slow to move in the network with a velocity about 20 km/h, that is the speed of a bicycle. Hence, a demand in a gas network cannot be met instantaneously and the duration depends on the amount of the demand as well as the amount of gas in the network at a particular time. In addition, physical models for both gas networks (see Sect. 5) and electricity networks (see Sect. 7) possess non-linear natures that make them difficult to solve. For example, Clegg and Mancarella report about 140 min run time for optimal power flow model and about 100 min for transient gas network optimization model for monthly modeling of an equivalent 29-busbar system and a 79-node gas network [80]. Hence, the studies that evaluate the integration of gas networks and electricity networks use different approaches regarding the system scenarios and modeling approaches, as well as various spatio-temporal resolutions [20, 43, 72, 75, 80–82, 144, 202, 302, 449, 461].

The gas and electricity systems currently operate independent from each other and how the gas and electricity systems work depends on technological and operational developments in both sectors as well as the individual regulatory developments [316]. The studies that evaluate integrating electricity network models to gas networks use scenarios of different integration levels, which vary from scenarios involving separate operation representing current status-quo to fully coordinated operation, to evaluate the integration of gas networks and electricity networks.

- Separated operation of systems: They solve electricity network model and gas network model separately and use demand results from electricity network with gas network to find out whether a feasible solution exists in the gas network [43, 461].
- Interconnected gas network and electricity network systems: The electricity and gas network models are solved using iterative or sequential algorithms, in such a way that they use each other's results to fix/improve their solutions [20, 43, 80–82, 461].
- Integrated gas network and electricity network systems: They use a single model for finding a cost optimal operational setting for both gas and electricity models [20, 72, 144, 302, 316, 449].

From the modeling point of view, the integration is studied using detailed physical models of gas networks and electricity networks, or simple economic or energy flow model for at least one of the networks, i.e., simple economic model for electricity network and a detailed physical model for gas network [75], or vice versa [202].

In the studies where physical models are used for both systems, the gas network is modeled using either a stationary gas network flow model [43, 81, 82], or as transient network flow model on very limited scales such as 5–10-node small networks designed for research or simplified small networks, i.e., 79-node UK network [20, 72, 80, 144, 302, 449]. Although the former is practically less data demanding and computationally less expensive, it does not account for intra-day flexibility of the gas network [243], which is important to evaluate the feasibility of gas network operation subject to the fluctuations in gas demand caused by GPPs. In order to account for intra-day flexibility of gas networks, stationary models are augmented with linepack analysis [81].

ENTSO-G and ENTSO-E address P2G as a promising technology for integrating wind and photovoltaic production into the overall energy system, that is complementing other technologies like integration using the power grid, electric power storages and power to thermal storage [114]. For this matter, the EC and the European Council support the approach of implementing P2G facilities from the system perspective after a first assessment [114]. So that P2G is studied in the interface between gas and electricity networks in the context of integrating other energy vectors to gas networks.

Studies evaluating the potential of P2G technologies and pathways generally focus on the following three practicalities [352].

- Production: Efficient production of technologies and processes from electricity to gas are studied [33, 379].
- Distribution and transmission: There are some concerns about hydrogen injection in natural gas grid since hydrogen embrittlement can lead propagation of cracks in the iron and steel pipelines, hydrogen leakage is riskier than natural gas leakage, etc. [301]. However, it is generally agreed that low concentration of hydrogen in the natural gas grid has no serious safety issues. On the other hand, converting gas network to hydrogen networks and operation of hydrogen

networks that are separate from natural gas networks are also studied [105, 185, 396].

- End use: Household appliances or industrial machinery should be made suitable to using hydrogen blended gas. Heat pumps and transport vehicles using Hydrogen only are another potential use of Hydrogen generated by P2G [352, 379].

For extensive review of P2G studies in the literature, the readers are referred to [50, 185, 352]. [19, 43, 50, 80, 82, 349, 350] are examples for studies that model P2G when evaluating the feasibility of gas network and electricity network interconnection.

General practice in studies in literature that model P2G in the interface of gas and electricity networks is to assume that the gas network has an appropriate level of allowed hydrogen volumetric share in the gas network. This level imposes limits to amount of gas fed into the gas network by P2G affecting the dispatching of electricity production schedules in the electricity network side. Thus, restrictions to the application of P2G are implied to electricity grid models, although the level of allowed hydrogen into the gas grid changes among countries such that UK allows 1% whereas the limit in The Netherlands is 12% [352]. These restrictions imposed by limited transport capacities of the gas network and allowable amount of hydrogen are calculated by gas network models as linepack of the pipelines that depends on the volumetric gas flow and in gas network model the effect of hydrogen is not considered [349, 350, 352]. The reader, who is interested in effects of blended hydrogen in gas pipelines, is referred to [185] for energetic aspects of hydrogen through pipelines and percentwise mixing of hydrogen into a natural-gas bulk.

Finance and Regulations

M. Schmidt and F. Lacalandra

1 Evaluation of European Gas Market Designs

The Entry–Exit System. The liberalization of the European gas markets started in the 1990s and lead to the current situation in which European transmission system operators (TSOs) typically operate under the so-called Entry–Exit system (Directive 1998/30/EC, Directive 2003/55/EC, Directive 2009/73/EC). The timing of this system is as follows: The TSOs have to publish so-called technical capacities at every entry or exit point of the their network. Afterward, gas traders can book capacities that are bounded above by the corresponding technical capacity. The booking is a capacity right that ensures that the trader can inject (as an entry customer) or withdraw (as an exit customer) balanced amounts of gas up to the booked capacity. The latter process is called nomination and the TSOs have to be able to transport all possible nomination situation as they are (via the bookings) conformal to the published technical capacities.

The current entry–exit system can be addressed by mathematical modeling in various ways. From the perspective of the evaluation of this market design one is faced with multilevel models that are made up of the following levels:

1. Computation of technical capacities by the TSO,
2. Booking by gas traders,
3. Nomination by gas traders,
4. Transport by the TSO.

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For the ease of presentation we refrained from discussing secondary intra-day markets [237, 238].

The first mathematical challenge is the robustness that the TSO has to address when computing the technical capacities in level 1: All balanced nominations that are restricted by the bookings that themselves have to be in line with the technical capacities have to transportable by the TSO. Feasibility of transport depends on the physical model of gas flow and of the chosen models of technical entities of gas transport networks like compressor, control valve stations, filters, measurement devices, etc. The former is typically modeled by systems of nonlinear and hyperbolic partial differential equations (the Euler equations, cf., e.g., [59]) on a graph. The latter are mainly modeled by algebraic but highly nonlinear discrete-continuous equality and inequality systems. Details and further references can be found, e.g., in [157, 381]. Assuming the TSO's goal of cost-minimal transport of nominations, the levels 1 and 4 alone already lead to adjustable robust mixed-integer nonlinear optimization problems that are subject to hyperbolic partial differential equations on a graph.

Since the acting agents (TSO and gas traders) in this market game typically have different objectives one is readily confronted with multilevel optimization or complementarity problems in levels 2 and 3 that are intermediate levels in the overall equilibrium problem.

Possible further directions of research are the following. Although the mathematical model described so far is extremely challenging and far beyond the border of what can be solved with the current state of mathematical theory and algorithmic technology, there are still a lot of possible extensions of this setting. One possible extension is the consideration of uncertainty in the given setting; cf., e.g. [159, 458]. Typically, the exact gas demand is unknown before booking and nomination. Thus, both stochastic and robust optimization techniques may be employed to address this issue.

2 Take or Pay (ToP) Contracts

Take or Pay (ToP) contracts are a very common contract types in the oil as well as in the gas industry. In their simplest form they state that, once signed, a certain amount of commodity can be used or otherwise lost while already paid. Typically, these ToP contracts were historically used when new oil and, lately, gas pipelines were to be constructed in order to give some economic certainty to the pipelines constructor. However these type of contracts have evolved over time including several flexible clauses whose –optimal– usage from the buyer perspective, have rendered the operational portfolio problem, quite complex.

For instance, in a ToP a monthly amount and a total annual amount can be specified, therefore at least X% of the monthly amount has to be bought every month and at least Y% of the contracted annual amount for the year has to be bought. Hence, there might be some gas excess based on contracts of this type. Moreover some other clauses can be present, such as those named Make-Up. Such additional flexibility enables the buyer to “recover” some quantity after the ToP horizon have passed and the quantity has not been used. Of course the exercise of Make-Up clause can be at some cost, and typically have maximum amount of quantity. In real life Gas or oil contracts the set of clauses can be in the order of tens and interact with each other. The minimum or maximum amount of quantity applied to each clause can be a complicating factor. Indeed all the ToP contracts management deals with uncertainty from the demand that the buyer has to face.

From a modelling point of view, the portfolio management of ToP contracts with swing-like options such as Make-Up and others, is a complex, large scale, optimization problem with integer variables and a lot of source of uncertainty that cannot be neglected in modern models. The uncertainty, in turn, are of different types and produce either volume as well as price risks. For instance, price risk can be taken into account if the buyer consider alternatives of buying spot volumes on the market in future times.

3 Gas Balancing Market

In the short term, for natural gas there are other problems that can be considered. Here we discuss the newly designed balancing market in some Countries, i.e. natural gas markets where one typically wants to adjust its daily positions long or short. From an operational point of view we remark that, natural gas flows in the transmission system from one point to another on the network by virtue of the differential in pressure existing between those two points. Therefore also a short term balancing market must take into account the physical rules and cannot be a set of simple financial transactions. Moreover, by definition, it must include the gas TSO. In a daily balancing setting, for instance, at the end of each day (so called Gas Day), for any residual deviation between gas injections and withdrawals, shippers incur imbalance charges for the imbalanced volumes accumulated throughout the day in a given balancing zone, and not timely compensated. Because of this they may want to get closer to the balance, and this can be achieved by selling or buying some amount of gas with daily frequency. The gas balancing markets are typically built around the concept of uniform price as in the electric setting. However the clearing, and the consequent price, is typically unique in the day. Therefore the shippers are required to bid or offer a certain quantity at a certain price. The market is cleared accordingly with classic uniform price settings (i.e. intersecting the aggregated demand and the offer curve) and consequently the price is set and the transactions are cleared.

From the shipper point of view there are many possible alternatives of optimizing the bid/offer strategy, depending on the usage of the natural gas (e.g. residential, industrial or power production), the cost of imbalance, and the rest of the medium/long term alternatives of the portfolio they are managing. Therefore the balancing market can be views as another option to extract value in the gas value chain Modeling and algorithmic considerations:

From the modeling point of view, the optimal balancing market can be posed as a mixed integer optimization problems with uncertainty in the strategy of the competitor, and –as previously noted– other alternatives can be incorporated from existing contracts. Depending on the integration level with other contracts, and depending on the usage of the natural gas, the problem can be a small or medium size stochastic or robust MILP that can be solved with off the shelf tools.

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