Chapter 6
The Point of My Own Teaching

The previous chapter has discussed ways in which teacher education practice has changed in my own country such that student teachers spend most of their training period in school. The majority of teachers do a 1-year postgraduate course following a degree in their specialist subject, where there may be no pedagogically oriented focus. For mathematics secondary specialists, the time spent in many universities during that year on specifically mathematical themes is around 25 hours, and for those teaching primary mathematics as one of nine curriculum subjects, it is around 6 hours (Brown 2018). More prestigious universities with strong market positions can insist that schools follow their preferred approach, but there is still a need to comply with government requirements that specify the minimum number of hours to be spent in school during the training year. The majority of students entering teaching in England do so through universities more susceptible to market pressure centred on student recruitment where the influence of schools is much greater. In such settings schools can be stronger in setting the terms of the partnership arrangements, where universities and their students are seen as providing support to schools. The point for the purposes of this book is that mathematics education research does not necessarily have an opportunity to present its recommendations in such arrangements. Classroom practice in mathematics, certainly at primary level, more usually follows commercial schemes. The agency of individual teachers and of many teacher educators to make autonomous professional decisions is held in check. Inevitably these restrictions place limits on what mathematics can become in many schools.

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6.1 Spatial Apprehension[1]

Here I will describe some work that I have done with some students where I have explored my role as a mathematics teacher educator in situations where more opportunity to experiment is enabled. This follows up and develops discussion of some activities that I described in my book *Mathematics Education and Subjectivity* (Brown 2011). On this occasion, I provide some critical analysis carried out by students of their own work on various mathematical activities. I have a regular weekly class with a group of adults, typically in their 20s and 30s, from a range of professions, retraining to be mathematics teachers in British secondary schools. As non-mathematics specialists, they are offered the opportunity to develop their understanding of mathematics over a period of 6–9 months prior to entering the year-long, school-based postgraduate "training" course now typical in England and Wales. Such university Mathematics Enhancement Courses (MECs) offer a way to develop greater confidence in mathematics, seen from the perspective of how it might be taught in schools. For many people embarking on these courses, their own schooling was characterised by a mode of teaching centred on getting through exams. On some teacher education programmes, the mathematics input is so brief such a view of teaching might be difficult to shift. MECs can provide a luxury resort where more creativity can be possible revealing the delights of mathematics in an exam-free zone prior to the frenetic schedule of a teacher education course. MECs might then not so much be seen as basic training for future course survival but rather a vision for a more exciting career beyond, something to hold on to during the sometimes dark days of intensive training towards formal accreditation, as described in the previous chapter. Most of the units comprise straight mathematical content. The unit in the MEC featured in this chapter and the next is more designed to see the learning of mathematics as *research*, centred on the question: "What is it for someone to learn mathematics?" From day one of the course, the challenge is to pay attention to how we learn and how those around us in the session learn, with a view to developing a way of describing how the children to be encountered in school learn. Each of the 20 3-hour sessions comprises a mathematical activity to be explored with view to articulating how learning happens for those involved. The range of activities is broad to ensure that mathematics is experienced from many different perspectives and to resist the common expectation that school mathematics is just about doing exercises. My ambition as their teacher is to introduce them to a broad range of mathematical experiences prior to the intensity of the subsequent training year where a relatively reductive version of school mathematics will be encountered.

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[1] This section draws on material first published as:

Brown, T. (2016). Rationality and belief in learning mathematics. *Educational Studies in Mathematics. 92*(1), 75–90 and

Ballantyne, M., & Brown, T. (2016). Close encounters of the curved kind. *Mathematics Teaching 253*, 28–32.

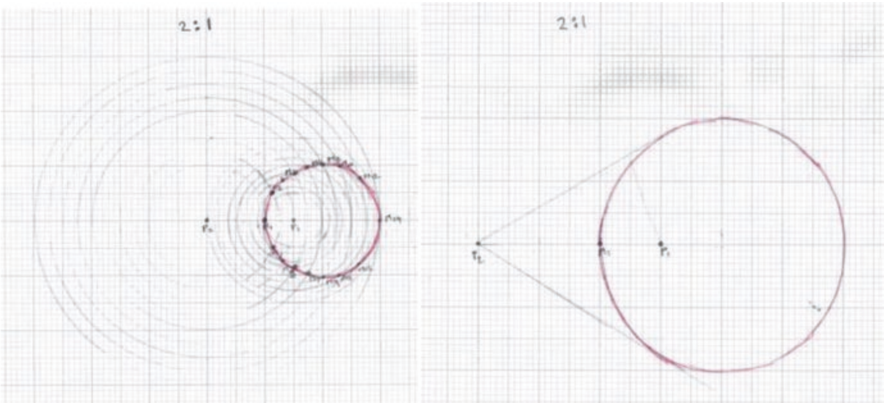

Fig. 6.1 Constructions of circles

In some activities directed towards enabling the students to develop their spatial awareness as a prelude to a more formalised approach to geometry, they were invited to explore various body movement activities. My intention here was to explore geometric entities from multiple perspectives, especially from inside very large versions of these configurations. In an example, a student was asked to position herself between two dots on the ground, which were about four metres apart, but where she was twice as far from one dot as she was from the other. She was asked to walk so that she was always twice as far from one dot as she was from the other. Alternative interpretations were provided as various students attempted this challenge. My hope was that there would be a lot of variety in the responses so that alternative ways of making sense of the situation could be shared and compared later. A few students produced drawings linking the points that satisfied the conditions, showing that they made a circular path (Fig. 6.1):

Another student reported a completely different experience in connection to the same problem. His response was to produce the following set of equations, seeing the same circular loci but in algebraic terms:

$$(x-12)^2+(y-0)^2=6^2$$

$$x^2-24x+144+y^2=36$$

$$x^2-24x+108+y^2=0$$

I checked that the formula is correct by solving when $y = 0$.

$$x^2-24x+108=0, x=6 \text{ or } 18$$

The issues became yet more complicated as the problem shifted to remaining twice as far from one dot as from the other in three-dimensional space. The challenge

Fig. 6.2 "Conceptualising" a three-dimensional object using string

provoked much ostensive gesturing alluding to points beyond immediate grasp and constructions out of string to confirm speculations (see Fig. 6.2).

This experience was written up at home as part of a diary package that would eventually be submitted for the course assessment. The course is set up at the outset as a place where we all research how people learn mathematics. Starting with ourselves and our own learning, we tell stories of our experiences towards building some sensitivity to understanding how similar situations can be experienced in very different ways and that our own learning can be enhanced by trying to see my problem through someone else's eyes. We explore our respective beliefs and expound the rationalities that link them. For example, two students in the same subgroup experiencing the same discussion documented the different ways in which they saw their colleagues had made sense of the problem:

> people do not visualise the same problem in the same way … each individual gave very different, but equally valid, explanations … for seeing a circle in 3D …: a penny being spun around at the end of a piece of string; modelling the shape with your hands; imagining being the origin of the circle (therefore being inside the shape) and what it would look like looking in each direction; imagining the shape being built up from the established points which were on the ground.

This student produced a drawing to show her own image (See Fig. 6.3).

> What was interesting was the different ways in which we described our thoughts and showed them to the group. N was thinking and demonstrating as if she was inside the shape.

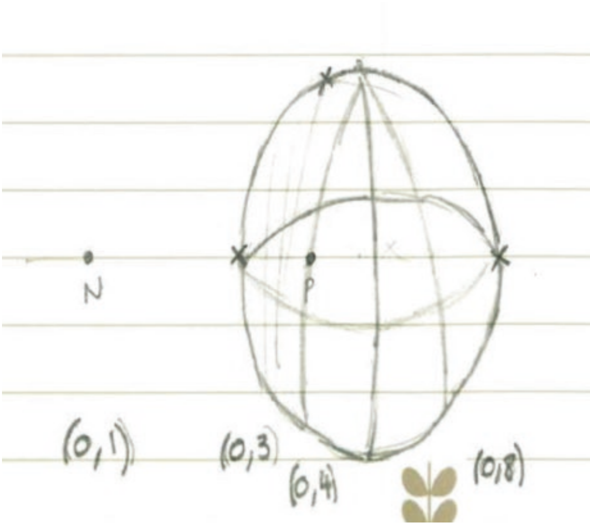

Fig. 6.3 A sphere

> S looked like she was thinking outside the shape. I thought of the shape spinning on a fixed axis, this fixed axis being along the line of the two fixed points; I was also visualising the shape on a 3D-type computer programme. J thought of it spinning on a fixed axis and used the idea of spinning of a coin to explain. We all agreed that the shape made would be a sphere.

This second student continues:

> Even though we all had different conceptualisations of what the curve looked like in 3D we could agree that we were talking about the same curve. This happened because each individual explained their visualisation and it fitted in with each of the other individuals' conceptualisation. For example, I imagined looking at the shape from the side seeing the established points and building it up from there. However, I could see that the image of a penny being spun fitted with my visualisation so could assume that person was having an equivalent (and yet different) visualisation to me.

So, a sphere is a sphere, but different people can experience it in different ways at different times (as a set of equations, drawing, as an imagined ostensible object, like this ball). The attempt by a student to reach out to someone else's description often required the intellectual challenge of comparing it with their own vision. "Every interpretation is partial, 'embedded' in an interpreter's ultimately contingent subjective position" (Žižek 2012, p. 359). Qualitative or perspectival dimensions supplement the mere fact of a sphere. Observing and making sense of the experience of others can further develop the qualitative or subjective experience of the sphere. For some there was a revelation in realising that the sphere could be understood or approached in many different ways and that the equivalence of these alternative approaches might be demonstrated. As their teacher, I opened many opportunities for ideas to be negotiated or exchanged. The students typically worked in small groups but regularly came together for whole class discussions. Every half an hour, I typically stopped the group so that they could engage in 5 minutes of private reflective writing to capture as “live data” how their understandings were developing. On some occasions students were invited to read their comments aloud with view to revealing the diversity of response. For their “homework” after the session students were urged to build a commentary of the lesson out of these pieces of reflective writing combined with their written mathematical work. The story of “how the learning took place” for the individual and for the different subgroups was emphasised as a major lesson objective. That is, the pedagogical story was valued as well as the mathematical story in the steep learning curve entailed in the student’s journey from seeing mathematics primarily as subject knowledge to seeing it as pedagogical content knowledge or as mathematics conceived from the point of view of a learner. The final package of material collected over 20 weeks allowed students to track their own learning history as evidenced in their evolving attempts to capture their mathematical experiences in reflective and increasingly critically analytical writing. In a separate study, I worked with a colleague in enabling graduate student teachers of English to document their own personal changes in relation to their subject area over a year as a key dimension of their course assessment (Hanley and Brown 2016).

On the occasion described, another student reported a more affective experience of being within the activity:

> On reflection, at that moment, I felt a real mix of emotions which combined many of the emotions that pupils face when asked to speak in class, in situations where they are not completely sure of what they are talking about. These involved almost a fear of saying the wrong thing, a desire to achieve the right answer, a wish not to appear foolish in front of the “teacher” and peers and also a concern over whether my explanation will be understood or even make sense.

This extract suggests that her mathematical experience was imbued with social, emotional and historical baggage. The imagination of the object in question, however, has real effect on the person’s image of his or her self in attempting to make tangible the object that is being sought. The person is reflexively constituted through their attempt to grasp it. The grasp reveals aspects of who they are. Thus, the activity was centred on each student exploring alternative subjective positions, on documenting connections to alternative formations of self, such as a physical self moving in space, a pedagogical self reflecting on the learning of others, a geometric self creating drawings, an algebraic self who is solving formulae and an emotional self commenting on how it felt relating to other students in the context of the supposed mathematical entities. But in building these images of oneself, one is alerted to territory that one can grasp in a tangible way and seemingly to spaces beyond reach that can only be pointed to or speculated. There are questions as to from where things are being seen. What am I seeing? Who, when or where am I precisely to be seeing it in that way? What had been movements of the body became elements of one’s comprehension of reality itself. The experience of the configurations became linked to how one felt at the time, a narrative of participation formalised for posterity, seemingly held in place by both rationality and belief at “the moment of pure subjective decision or choice which stabilizes a world” (Žižek

2012, p. 367) or implies a centre and specific sense of direction (Laclau 2007, pp. 66–83; Derrida 1978, pp. 278–294) or a new master discourse (Lacan 2007).

Each time these sessions are new for me in my role as the teacher as each individual account has unique features and a chosen storytelling style. Recent sessions have been enlivened by a move to the large brand new building that allows new spatial experiences – a huge internal atrium for suspending/stretching string in 3D, a drama studio that allows spotlighting, expansive flat space outside (where cold weather tempered the pleasure of making the curves very large), etc. My persistent ambition, the point of my teaching spelt out for my students, is for me to see things in new ways, to keep my teaching alive by enabling my students to provide new stimulus to each other and to me, to resist final versions that sum things up, towards recognising and perhaps specifying the limits of one's certainties and uncertainties. As Laclau (2007) argues:

> the abandonment of the aspiration to 'absolute' knowledge has exhilarating effects: on the one hand, human beings can recognize themselves as the true creators and no longer as the passive recipients of a predetermined structure; on the other hand, all social agents have to recognize their concrete finitude, nobody can aspire to be the true consciousness of the world. (p. 16).

In week 20 of 1 year's cohort, a challenge was set to carry out a photogenic activity for the university marketing department to show off our new education building. Matt Ballantyne, a student on the course describes his experience drawing on these photographic images (Figs. 6.4, 6.5, 6.6, and 6.7):

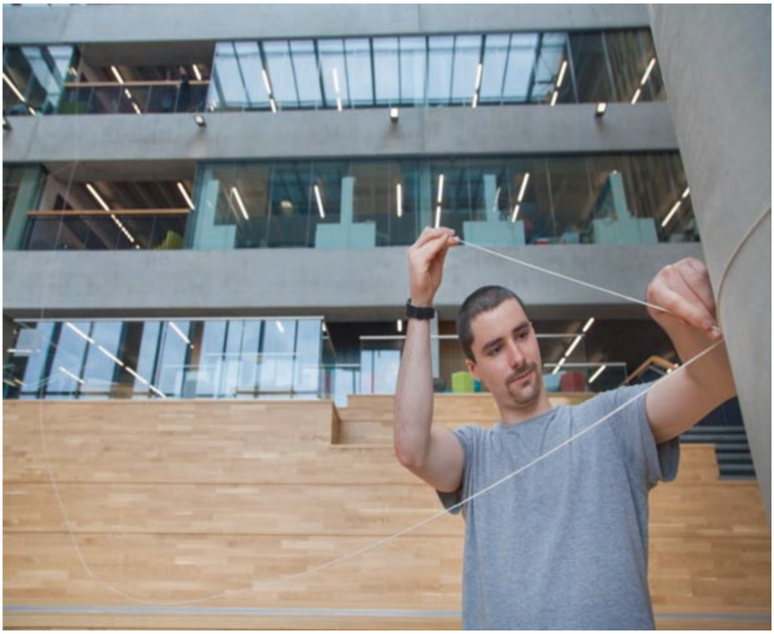

Fig. 6.4 Securing the end of the curve

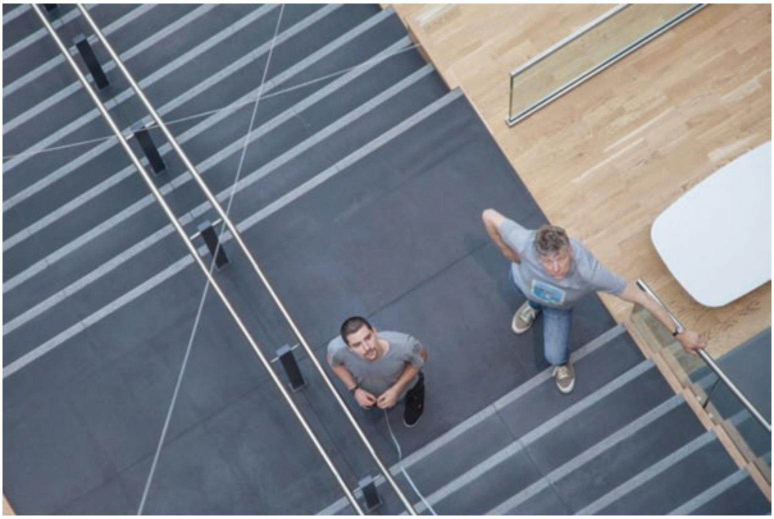

Fig. 6.5 The author with a student admiring the suspended double curve

> Today we would be venturing out of the classroom and down to the Spanish Steps. Our task to create two curved lines that descended and intersected at one specific point using a large ball of string. We had to develop a collective vision of how to work on the task. After some thinking, loosening, tightening, adjusting and altering we were all happy with the placement of the string. The two strings formed smooth curves that intersected at a single point. Now the real challenge began. The mapping and representation of what we had created. There were many obstacles to overcome when creating such a 3D image. There were many different measurements that we needed to make. We considered the measurements from the front looking at the balconies and floors above. Another group went to the side and looked at it with the balconies to their left and the entrance to their right looking along the Spanish steps. The last group pretended to be birds looking down on the site and seeing the points on the floor. The group reassembled and shared their results. Having collaborated, we were able to see if any measurements that were needed to map the curve were missing. When we were all happy. We had what we needed to create an accurate representation of the curve, so we headed upstairs to create our model. We believed that not one model but many would best represent the situation. One group decided to draw three different 2D images to show the 3D planes which we had measured. The other two decided to try a 3D model. One group went for a sketch of the curves. Our last team were probably the most audacious and decided to make an actual 3D model including little people. They carefully crafted a scale model of the Spanish steps with string and cardboard (See Fig. 6.7). A single task, undertaken by a group who have worked closely together for six months, had taken completely different turns and ended up in many different places. The thing I like most about the end results is that they are ultimately a complement to each other. (Ballantyne and Brown 2016).

Matt's description reveals the way in which his apprehension of the curves evolved through a succession of physical movements, measurement activity, visual perspectives, inspection of data and shared interactions and productions with other students.

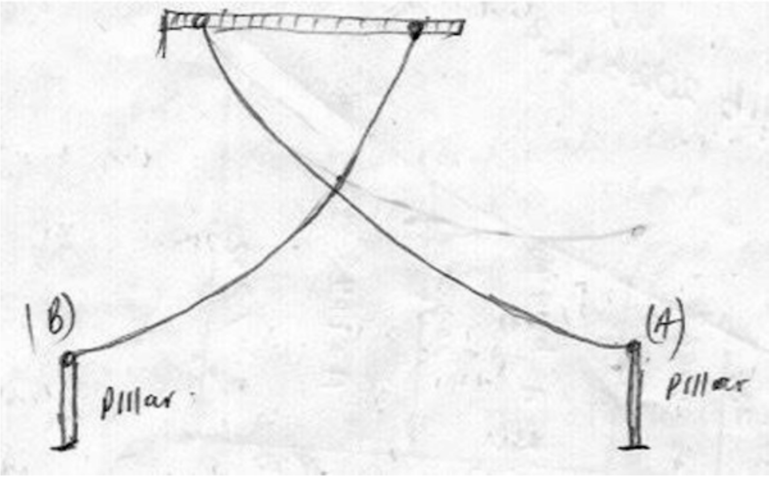

Fig. 6.6 Drawing of the suspended double curve

Fig. 6.7 Model of the suspended double curve

The description itself, later published in a teachers' journal, further processes Matt's comprehension of the mathematical entities concerned and stretches his capacity to articulate his understanding.

6.2 Apprehension of Planetary Movement[2]

Some of the bodily movement exercises conducted in our sessions involved acting out the relative configurations of earth, moon and sun and how these configurations might be seen from alternative perspectives, from deep space, from the surface of the earth, etc. (Heywood and Parker 2010). These configurations were enacted firstly with a globe for the earth, a small white sphere for the moon and a torch for the sun (Fig. 6.8). Later individuals took the place of the moon then spinning in relation to the earth (Fig. 6.9). The purpose of the sessions was to enable students to share their attempts to apprehend variously located mathematical objects, experienced as if navigating and orienting themselves inside big versions of the shapes concerned. That is, they told progressive stories of themselves, as apprehenders of the variously perceived spatial environments, developing technologies through which specific orientations could be achieved.

Some pieces of data derive from the research orientation of the sessions more generally. Everyone, students, tutors and visitors alike kept extensive records of their activity during each activity to better understand how the learning of mathematics happened. An extensive catalogue of video clips and photographs were collated, which were later used to build the written records that were produced. In the subsequent discussion, alternative approaches to framing school mathematical objects arose were suggested. From the teacher education perspective being taken, we sought to explore how alternative subjective positions can be productive of important qualitative aspects of the mathematical phenomena being portrayed. That is, these qualitative features, specific to the *world* in which the ideas were encountered, provided markers for observing and orienting the mathematical ideas being approached. School mathematics is typically encountered through qualitative features of the pedagogical worlds being entered (e.g. needing to make an argument to peers or tutors, representing ideas in different forms, reference to standard ways of depicting ideas for examination settings, etc.). Learning about the mode of embedding and working within it is part of the necessary learning required in many instances, especially those directed at supporting a specific purpose, e.g. conceptualising the moon that we can see as being on a circular (or is it elliptical) orbit.[3]

(i) The first piece of data comprises an extract from our written records collected during the activities as part of our own research:

> Kelly had brought some data with her, such as the exact duration of the day and the year, and it was apparent that her preparation for the activity was systematic, mentioning terms

[2] This section draws on material first published as: Brown, T., Heywood, D., Solomon, Y., & Zagorianakos, A. (2012). Experiencing the Space We share: rethinking objectivity and subjectivity. *Zentralblatt fur Didaktik der Mathematik: The International Journal on Mathematics Education*, *45*, 561–572.

[3] My students were far more graceful than the group of drunks doing a more complicated version of the exercise in Bela Tarr's movie *Werkmeister harmonies*: https://www.youtube.com/watch?v=_d5X2t_s9g8.

Fig. 6.8 The sun shines on a revolving world with the moon above

Fig. 6.9 Experiencing the movement of the earth in relation to the moon

> such as "aphelion" ("which is the point of orbit furthest from the sun; … which is going to be our winter"). As soon as the activity started Imogen said, 'the sun shines and the earth spins and when you don't have the sun on you it's night time'. Kelly pointed out that the length of a day is exactly 23.97 hours, reading it from the data that she had brought with her. … Imogen replied immediately that 'there is noon when the sun is at its highest, when you are closest to the sun'. She gave an example by choosing Saudi Arabia on the globe and turning it: 'if we look to Saudi Arabia, it is noon in Saudi Arabia, as it moves away the sun is sinking again and then it goes to night time and then this is the midnight, and then it gets early, the sun is rising, the sun is rising, it gets to the noon, the sun is at its highest point'.

(ii) Another piece of data comprises extracts of reflections from an experienced mathematics teacher within the team researching how mathematical objects result from pedagogical exchanges as part of his doctoral studies. During the session, he was observing the students but occasionally found himself drawn into discussions as the students had known that he was quite good at mathematics, and in turn he could not stop himself from revealing his knowledge. In the reflections, the teacher was exploring the consequences of these unexpected interventions from a pedagogical point of view. The extracts refer to the sequence above. They were chosen with view to showing how the teacher's reflections are revelatory of his own identification with conceptions of pedagogy and of scientific discourse.

The following comments indicate his pedagogic orientation:

- I was kind of prepared for it.
- I don't want to "spoil the fun of discovery".
- I responded with an expression of approval.
- I pretended to agree.
- I instinctively tried to break the rhythm, so I said something that wouldn't be much of a clue.
- I repeated what Kelly said, trying to sustain Kelly's conclusion as a base for the subsequent investigation.
- Without realizing it I was entangled in the group discourse in the way that I was initially trying to avoid.
- I fully understand that [was] my old reflex as a teacher.
- I had fallen in the trap of influencing the group, as I could not disengage myself from its activity and as I interrupted the group's interaction to some extent; I became a victim of my own devices.
- These comments however point to a "correct way" of seeing things:
- She was not using "aphelion" *the correct way*.[4]
- Using "aphelion" and "perihelion" *the right way*.
- Trying *to keep the level of the group discourse as advanced as possible*.

(iii) In the final extract, Kelly, Imogen and Rebecca (Fig. 6.2) share their apprehensions of how the moon moves in relation to the earth. They experienced some difficulty in communicating these apprehensions in words. Finally, they

[4] The teacher reports that at one stage: "Kelly mentioned the summer time, introducing the term 'aphelion'".

> enacted the orbit of the moon through bodily movements that seemed remarkably coordinated, with all three students moving in the same trajectory around a suspended sphere (the earth), where they each maintained a constant orientation to the earth throughout. Successive attempts interrupt each other:

K: Because we are on an angle of let's say this way I am looking at it… as we come round if we keep on that angle we only ever see my face, you never see the back of my head.
I: It doesn't matter whereabouts.
K: Yeah you split…
R: Kelly's focus stays on that ball so her body might be turning but she is still looking…
K: So you only ever see…
R: So, if someone is stood on there, they would only ever see my face, not the back of my head, otherwise I'd be going…
I: We must be right because we are all on the same wavelength. We all agree.
K: If I could spin myself like this …
I: The moon's just on an angle. That's what it is. Spin round double … see it's worked… best logic I've thought of.

The three examples comprise individuals displaying a range of pedagogical interests and attitudes towards notional mathematical objects. We have trainee teachers who oscillate between an unsteady grasp of the terminology and a more symbiotic immersion in the evolving world to which this terminology attempts to cling. This terminology is included in their own learning narratives within which meanings evolve. We have a teacher referencing his own interventions to established parameters. We have teacher educators in the background managing an activity towards influencing certain pedagogical results. We have researchers adopting more theoretical perspectives on how mathematical ideas are being framed. These alternative perspectives link to alternative conceptions of learning (discovery approaches, gravitation to correct understandings, creation of fresh perspectives, etc.) that variously construct and position mathematical objects and shape the apprehension of more or less familiar forms of knowledge. The enquiry in this chapter is specifically focused on how the participants variously identify with specific conceptions and how those identifications support teacher education ambitions, specifically those relating to building narrative around learning experiences. We cannot assume any sort of correct overview of the activity that took place nor be representative of the multitude of insider perspectives.

In the first extract, Kelly's experimental introduction of specific terminology is depicted as the consequence of advance preparation at home, preceding a more settled understanding of the parameters that framed the terms that she used. "Aphelion", as an embodiment of, or subjective perspective on, an ellipse, for example, was occasionally asserted as being linked to a position on an orbit closest to the sun, rather than furthest. Yet the bigger point is that the *world* that would host this term within a more secure scientific discourse was apparently not yet in place for her. Neither the host space nor the objects it allowed had been established. The technical term “aphelion” for Kelly was dislocated, floating in space as it were – its home had not yet been fully conceptualised as a point within an overarching spatial structure. Yet clearly, she was introducing the term to provisionally mark out the territory that she was seeking to better understand.

In the second piece, we view the events through brief extracts from the teacher’s extensive reflective writing where he indicates his own unexpected participation. The extracts point to a specific mediation and more or less obliquely depict his involvement in the activity. The teacher’s supposition of the task in hand is at least partially centred in a particular conception of the knowledge to be apprehended. Yet this interest is obscured by his own concern that he be an observer rather than a teacher. This is against the backdrop of Imogen, Kelly and Rebecca playing out alternative approaches to the task where they have prepared for the task differently and get to be convinced differently when they come together on the task. The routes from their understandings to their rationalities are different in each case. The teacher has an ambivalent role focusing primarily on understanding how the others are apprehending the task, where his own involvement in the proceedings is nevertheless having some impact. The ideas in question are manifested differently through the thoughts, action and speech of the people in question, in relation to a set of activities designed with certain pedagogical ambitions in mind. But the issue for this chapter is not with the relative merits of the perspectives achieved but with how mathematical objects derive from alternative subjective positions or modes of identification.

In the final example, the mathematical object in question is a circle (or ellipse) but where many qualitative dimensions of the pedagogical world supplement the students’ experience. The sensual perspectives assumed of this circle obscure its appearance as a clear cut geometric entity. The task was centred on being able to apprehend an orbit from various given perspectives, such that the students were challenged to situate themselves *within* and experience mathematically conceived space. The question of moving around this ellipse whilst maintaining the correct orientation further complicates the sharing of perceptions in words. The keenly felt perception of being on the “same wavelength” within shared movement, however, somehow reduced the need for a clear set of words – the agreement could be danced. Indeed, the desire for a correct set of words seemed destined to fail as the power of shared movement became far more evocative of the entity in question. (See also Roth and Thom 2008). The students are identifying with an experience that defies final capture in a symbolic form, but it also defies final capture of the students themselves in finished form. Their subjectivity is referenced to a lived experience, with no fixed relation of object (an elliptical orbit sought through a succession of fragmented sentences) to subjects (held by names and relations to other subjects). In a “real-life” context the affectivity of the space teaches the students to recognise their position in time and space through sensual clues, (e.g. shadows, direction of moon, darkness, temperature, reciprocities of sharing space with others). Their emergent spatial and temporal awareness, marked by these qualitative features, occurs as part of a layering of complex systems of relationships and spaces within constantly changing circumstances.

6.3 Lockdown Mathematics

This section of the book is the last piece to be written (in June 2020). Eight weeks into the 20-week course, England went into lockdown. We did not know that the eighth session would be the last and the course reverted to sharing hastily composed emails. After several false starts, we managed to get Zoom sessions working so long as my wife did not have her Zoom Pilates class at the same time. The new format became that I set a challenge first thing on the morning of the session and asked members of the group to do a one page write of their findings and to send this to everyone in the group. Figure 6.10 provides a brief extract from one of the pieces of work circulated by one of the students at this stage of the session. The variety was considerable among the ten students as other examples will show. When we congregated on Zoom at 11 am, each person talked about what they had done for a few minutes. This generated many ideas and questions. A key dimension of the discussion was to consider the investigation as a mathematical field of enquiry. The metaphor I use is for students to imagine that they are the finance minister facing a very complex situation but where it is known that various strategies work but in a conditional way. For example, we know that at the micro level a factory owner can reduce her production costs by paying the workers less so long as she can hold on to her workers. Trouble is that if every factory owner does that at a macro level, then workers cannot afford to buy the products, and the factory owner makes no profit (Harvey 2015, pp. 112–130). Consequently, the finance minister or factory owner needs to play several ideas off against each other to improve their respective successes. This contrasts with the approach to a school mathematical investigation described by Tanuj Shah at a link provided below where he solved the rectangle diagonal investigation outright, something that most school children could not do. Nevertheless, I have found that the activity is quite feasible with a mixed ability class of 11-year-olds, where there are many dimensions to the activity that allow all children to participate with various more qualitative aspects of the challenge and where sharing alternative perspectives is the chief pedagogical benefit. It was this attitude that I was seeking to explore with my adult students. The challenge was for each student to find out the way the task was understood by others. In presenting their thoughts, students need to articulate their understanding and in so doing strengthening and developing their understanding whilst enabling their partner to have access to these thoughts. But

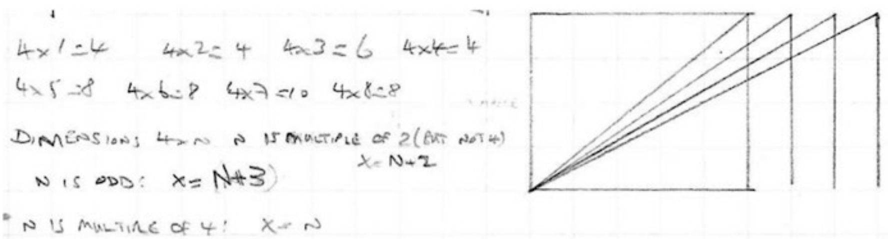

Fig. 6.10 Extract from notes shared by Khurram before Zoom meeting

Fig. 6.11 Dean doing a homework on curves. (Short film available on Youtube: https://youtu.be/6cN3oRcekPA)

mathematically, the task never really reaches an end if this approach is taken. It is not about reaching a final solution but rather exploring the what ifs of the situation as a way of building more sophisticated mathematical dialogue and independent thinking. The territory may never be fully conquered, but one's navigational skills in the field have been greatly enhanced. In the session, we talked and worked things out mentally until we felt our brains had reached capacity. The attempt at 3D cuboids was the final straw, and it was suggested that we retreat to our private space again so that we could write things down. I include below the text of an email that I sent to my students shortly after where I was explaining what I was attempting to do and summarising some of the points I thought we had discussed in the session. Last term I encouraged a write from the students every week. This term I am asking for four extended pieces of work of their choice over the 10 weeks of the second half of the course. I am anticipating that some of the students will choose to do an extended piece on this field of enquiry, where they will combine mathematical thinking, presentational quality, discussion of their own learning, how this contrasted with the work of others, etc. (Figure 6.11 provides a still from a film submitted by one student for his assessment at the end of the course).

> Last night I did a Google on "rectangle diagonal investigation" and got this:
>
> https://apfstatic.s3.ap-south-1.amazonaws.com/s3fs-public/07-tanuj_diagonal_investigation.pdf
>
> I did a screen shot of the first page and sent it to you. I have not done it for a few years and could not remember how productive it was as an activity, but I was hoping that I would be surprised.
>
> I was. I received a good range of work from you where the activity had been understood in a lot of different ways but with some core similarities.

Then the discussion made us all dig a bit deeper, and we came up with a range of questions that potentially defined multiple directions in which the activity could be further developed. This is PhD researcher in mathematics type mode, not the typical school mode where so often the teacher asks a well-defined question and gets a right or wrong answer back. In this activity there was no scope for being wrong, just seeing things in different ways, but with a firm footing in things we knew for sure. The algebra emerged as pathways become more familiar – as generalisations could be seen.

I was really taken by James' introduction of Pythagoras – something I had never thought of before. At first I thought James was a bit off track, but after a while I realised that James was seeing it in a more sophisticated way than I was. In short, for very large rectangles, the length of the hypotenuse gives you an approximation to the number of squares passed through. That provoked my question – how do you fix the dimensions so that you maximise the number of squares passed through?

Ellie had this great line: "different rules for each prime w and H value therefore infinite primes = infinite rules". Mind blowing, but is that true??

Ben's progression through successive heights for rectangles seemed to suggest that there was a pattern to how many squares were passed through in successive rows. Could that lead to a formula where you could find the maximum number of squares that could be passed through? We know the minimum number is equal to the longest side. But what about the maximum?

So we understand simple examples very well – but we are more foggy as cases become more complex – but we have resources to tackle those mysteries.

If you choose to write this up you could:

- Explore the limits of your own understanding now.
- Consider the differences between the approaches taken by different people.
- Pursue a line of mathematical investigation from things we have found out so far.
- Ask yourself the question – how is this activity stretching and developing our mathematical capabilities?
- If you were faced with a mixed ability class of 30 11-year-olds, how might you organise the lesson so that everyone is included for the whole lesson?