Fuzzy Formalization of Individual Quality Criteria for Quality Level Evaluation by Using Two-Level Optimization Model

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Abstract. The use of a multilevel model for evaluating the quality of instrument-making products assumes that the individual quality criteria are defined and quantified, since this overestimates the accuracy of obtaining the output result in the form of a numerical expression of the quality of the products produced; if we don't determine individual quality criteria, our decision support system will give us incomplete information. The system in fact needs to contain information about a manufacturer, a supplier, and other quality information. The objective is to develop a method for the quantitative identification of individual quality criteria for instrument-making products for a two-level model of the product quality assessment. Results: the problem of assessing the quality level from the point of view of the decentralization is considered, the target quality functions and areas of definition for each level of optimization are proposed, a method for quantitative identification of individual quality criteria is developed, and the ways to improve the developed method are proposed.

Keywords: Fuzzy sets · Decision-making · Quality assessment · Quality functions

1 Introduction

To meet the current needs of the consumer and manufacturer, a multilevel hierarchy of product quality indicators at the production stage requires identification of the nature of quality indicators. At the same time, the multilevel hierarchy of quality indicators in accordance with the concept of the product quality monitoring model [1] must meet the following requirements:

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- The behavior of the lower level should be limited to the requirements of the upper level.
- The quality function (main level) should not include indicators aimed at ensuring quality (sublevels), but these indicators should characterize the level of output quality (should act as restrictions).
- The hierarchy of quality indicators should include both qualitative and quantitative indicators.
- Quality indicators must meet the requirements of standardization, comparability, representativeness, sensitivity at threshold values, and the absence of duplication of quality indicators at each level.

Indicators of the developed product quality along with the supporting information will allow regulating the quality of product flexibly. In particular, it will allow managing (in frames of Quality management) the enterprise resource planning (ERP) system and the production process management system. Meeting the Industry 4.0 requirements will allow optimizing production chain processes, such as logistics, design, production, operation, and after-sales service, in terms of time, resources, and quality loss [2–4].

2 A Mathematical Model for Assessing of the Product Quality

Prior to identifying product quality indicators, it is necessary to describe a mathematical model for evaluating the product quality level. The mathematical model for evaluating the quality level will be based on a two-level linear optimization model, in which the first level is the leader, and the second level is the follower.

The assessment of the quality level is determined from the following set-theoretic definition of the model:

$$\langle Q, X, F_i, Y_i \rangle$$

where Q is the quality function (main level); X is the area for determining the numerical values of the quality function; F_i is the target functions (sublevels); and Y_i is the area for determining the values of the target function.

The finding of an optimal solution of the function (1) is carried out bottom-up: first, the optimal value of the sublevels $F_i(Y^i)$ are found, then these values are substituted in (1), and values for the master level are found.

The optimization problem for the top level, taking into account the restrictions imposed under the levels, looks like this [5, 6]:

$$\min_x \left\{ Q[y(x), x] : G[y(x), x] \leq 0,\ H[y(x), x] = 0,\ y(x) \in \psi(x) \right\} \tag{1}$$

where $y(x) = F_i$,$F_i : y(x) \in \psi(x)$,$\psi(x)$—polyhedron, domain of constraints such that ($Q : R^n \times R^m \to R,\ G : R^n \times R^m \to R^k,\ H : R^n \times R^m \to R^l$ for k indexes, there are restrictions with the sign "$\leq$", and for l th indexes, there are restrictions with the sign "$=$"). The target functions for the decentralization task, based on the requirements for the quality indicators monitoring model, are as follows. $Q(x, y_{1,2})$ is the effectiveness of the monitoring model, $F_1(x, y_{1,2})$ is the cost function for the quality, and $F_2(x, y_{1,2})$ is a supplier management function. Variables x are such that $x \in X \subset R^n$. The quality function is such that $Q : X \times Y_1 \times Y_2 \to R$, where y is quality criteria, so $y_i \in Y_i \subset R^{mi}$, and $F_i : X \times Y_i \to R$.

$$
\begin{aligned}
&\min_{x \in X} Q(x, y_1 = y, y_2 = z) = cx - d_1 y + d_2 z \\
&Ax + B_1 y_1 + B_2 y_2 \le b_1 \\
&\min_{y_i \in Y} F_1(x, y) = cx + d_1 y \\
&Ax + B_1 y \le b^/ \\
&\min_{z_i \in Z} F_2(x, z) = cx + d_2 z \\
&Ax + B_2 z \le b^{//}.
\end{aligned} \tag{2}
$$

The state of operation of the upper level is determined by the following output parameters (see Table 1).

Table 1. Output parameters of the status of the functioning of the top level.

Level	Name of the main indicator	Name of the private criterion
1	The effectiveness of the monitoring model	Level of the output of suitable products
		The degree of effectiveness of the developed warning measures
		The degree of effectiveness of newly implemented technologies and techniques
1.1	The cost function for the quality	The cost of quality assessment
		The cost of prevention of discrepancies
		The cost of removal of internal inconsistencies
1.2	Quality criteria such that	Timely delivery
		The timeliness of addressing complaints about the quality of products

3 Formalization of Unique Criteria for a Model for Assessing of Quality of Instrument-Making Products

To obtain an updated value of the proposed indicator, it is necessary to take into account the specifics of the production of instrumentation products, namely.

- structural complexity of the product after performing a certain technological operation is X_1;
- the shareof components with deviations by deviation is X_2;
- the shareof components with marriage certificates is X_3;
- the shareof components with acts of noncompliance is X_4;
- the shareof purchased items included in components (component parts of products) is X_5.

Structural complexity of the product x_1 is defined as the ratio of the complexity of the product at a certain operation (assigned index p) to the structural complexity of the product at the output (on the final operation, assigned index f):

$$x_1 = \frac{C_p}{C_f} = \frac{m_p}{n_p(n-1)} \times \frac{n_f(n-1)}{m_f}. \tag{3}$$

where C_p is the complexity of products for a particular operation; C_f is the complexity of the product output; m is the number of elements in the product, and n is the number of connections in the product.

Information on X_2, X_3, and X_4 will be recorded by counting cases of registration of permits for rejection, acts of marriage, and acts of noncompliance. The share of purchased items X_5 will be defined as the ratio of purchased items to the total number of used items in the product (design).

In the indicator “the degree of effectiveness of the developed preventive measures (for the previous period)”, the following criteria should be taken into account:

- the share of corrected (simple) technological operations is Y_1;
- the share of corrected (specially responsible) technological operations is Y_2;
- the share of the revised documents of the quality management system is Y_3;
- the percentage of corrected critical blocks in the product is Y_4;
- the percentage of corrected (noncritical) blocks in the product is Y_5;
- the percentage of corrected (not visible to the consumer) functions of the product is Y_5.

The indicator “degree of efficiency from newly introduced technologies” determines how successful the investment was, and should be determined by the following criteria:

- the share of the cost of developing a technological operation is Z_1;
- the share of expenses for the development of quality management system documentation is Z_2;
- the share of the cost of change of supplier is Z_3;

- the share of expenditure on adjustment of the design is Z_4;
- the share of costs for changing the functioning of the product is Z_5.

These elements of a cost must be taken into account when evaluating the single criterion "degree of efficiency from newly technologies introduced".

Let's set a linguistic variable to determine the level of output of suitable products. A linguistic variable is defined as follows:

$$\langle X_i, T(X_i), U, G, M \rangle,$$

where X_i is a name of the variable, $T(X_i)$ is a term-set of the variable X_i, U is the universal set, G is a syntactic rule, and M is a semantic rule.

The term set $T(X_i)$ for the linguistic variables "level of output of suitable products", "degree of effectiveness of developed warning measures (for the previous period)", and "degree of efficiency from newly introduced technologies" will be low value, average value, and high value. The universal set $U = [0, 1]$, and $G : M = [1 - u_t(u), (u_t(u))^2]$.

The following scale will be used in the rule database for the place of output value names:

- low value $x \leq 0, 25$;
- not the average value $0, 25 < x < 0, 45$;
- average $0, 45 \leq x \prec 0, 6$;
- low value $0, 6 \leq x < 0, 75$;
- high value $0, 75 \leq x < 0, 85$;
- very high value $0, 85 \leq x$.

The following rating scale will be used for the place of names of input values:

- low value $x < 0, 4$;
- average $0, 4 \leq x < 0, 8$;
- high value $0, 7 \leq x$.

The following functions will be used for input information:

- Z-shaped accessory function for $x < 0, 4$;
- Triangular accessory function for $0, 4 \leq x < 0, 8$;
- S-shaped accessory function for $0, 7 \leq x$.

To get a numerical expression of the quality indicator, the relative importance coefficients for each criterion must be determined. A mathematical model for determining relative importance coefficients is described as

$$\left\langle f_\Omega,\ K(X, Y, Z)_1^{n_1},\ \ldots,\ K(X, Y, Z)_m^1,\ \ldots,\ K(X, Y, Z)_m^{n_m},\ R_N \right\rangle$$

where f_Ω is the area of determining of the output values of relative importance coefficients, $K(X, Y, Z)_{1,\ldots,m}^{1,\ldots,n_m} = \left\langle K(X)_{1,\ldots,m}^{1,\ldots,n_m}, K(Y)_{1,\ldots,m}^{1,\ldots,n_m}, K(Z)_{1,\ldots,m}^{1,\ldots,n_m} \right\rangle$, criteria K_i is such that each corresponds to a set n_i (number of responses) of equally important criteria $K_i^1, \ldots, K_i^{n_i}$ and the non-strict preference ratio R_N.

The quantitative importance information ϑ included in $y(x)$ contains information about the degree of superiority of i, the criterion over other j criteria $i \succ^h j$, where h answers the question "how many times one criterion is more important than the other" [7, 8].

The implication operation is performed as $\mu_{A \wedge B \wedge C}$ $(x, y, z) = \min\{\mu_A(x), \mu_B(y), \mu_C(z)\}$. The Z-shaped function will be found for $a = 0$ and $b = 1$. The S-shaped function will be found for $a = 0$ and $b = 1$. The triangular function will be found for $a = 0$, $b = 0, 5$, and $c = 1$. After finding the fuzzy value, the truth of the statement will be found by the Zane function, where a is an upper bound of i th scale interval for output values, presented in the previous sections. For numerical formalization of the above criteria, it is necessary to determine the cost management model and its mathematical form. There are many models of cost management; the main models given in [9, 10] are the following: the system "standard-cost", target-costing method, the method of direct costing.

The most appropriate model for cost management is the model given in [11].

$$PR = \sum_{i=1}^{n} P_i Q_i - \left(\sum_{i=1}^{n} vc_i Q_i + FC \right) \tag{4}$$

where n is the number of types of products, P is the price of products of ith type, Q_i is a number of products of ith type, and vc_i are variable costs per unit of production of products of ith type, variable costs are vc $= \mathrm{U_{VC}}\, \mathrm{P}_{uVC}$, where U_{VC} is the specific consumption of the resource, and $\mathrm{P_{uVC}}$ is the price per unit.

To apply the model (4), the individual criteria for each particular criterion of the quality cost function must be defined. The costs of quality assessment are characterized by the following single criteria [12, 13]:

- The share of expenditure on critical functions that are visible to the consumer and the total costs is X_1';
- The share of costs for noncritical functions visible to the consumer to total costs is X_2';
- The share of costs for noncritical functions that are not visible to the consumer is X_3'.

The costs of preventing nonconformities are characterized by the following single criteria:

- The share of expenditure on critical functions that are visible to the consumer and the total costs is Y_1';
- The share of costs for noncritical functions visible to the consumer to total costs is Y_2';
- The share of costs for noncritical functions that are not visible to the consumer is Y_3'.

The costs of eliminating internal inconsistencies are characterized by the following single criteria:

- The share of costs eliminated by working with the supplier to the total cost of resolving internal inconsistencies is Z'_1;
- The share of costs included in the manufacturing technology to the total cost of eliminating internal inconsistencies is Z'_2.

In the case of the listed criteria in the model (4), it is necessary to substitute the argument from X',Y',Z' in the specific resource consumption and in the variable costs vc_i, as follows:$vc_i(X')$,$vc_i(Y')$,$vc_i(Z')$, $U_{VC}(X')$, $U_{VC}(Y')$, and $U_{VC}(Z')$.

A cost-ratio model approach will be used to calculate single criteria for the supplier management function. An analysis of this approach is presented in [14, 15]. The private criteria for the supplier management function are based on the following unique criteria [16–18]:

- the share of suppliers with a critical component of the total number of suppliers is X''_1, Y''_1, Z''_1;
- the proportion of outsourcing on the total number of suppliers is X''_2, Y''_2, Z''_2;
- the share of suppliers with imported products from the total number of suppliers is X''_3, Y''_3, Z''_3;
- the share of suppliers that make up 15% of the total cost of purchasing components to the total number of suppliers (products with high cost) is X''_4, Y''_4, Z''_4.

The timeliness of delivery is based on the formula [19, 20]:

$$b''_{i1}z_1 = \left[\frac{\sum_{i=1}^{n} z_1(t)_i}{m}\right] \times \frac{1}{z(t)_b} \times 1^{1-d_1}, \tag{5}$$

where $z_1(t)_i$ is the time needed to eliminate defects for customer satisfaction, m is a number of reference points, $z_1(t)_b$ is the time needed to eliminate defects for customer satisfaction under the contract, and d_1 is the ratio of the amount of the cost of components and the cost of applying them to their destination to the cost of the flaw detector.

Criterion d_2z_2 will be found by the formula:

$$b''_{i2}z_2 = \left[\frac{\sum\limits_{i=1}^{n} z_2(t)_i}{n}\right] \times \frac{1}{z_2(t)_b} \times 1^{1-d_2}, \tag{6}$$

where $z_2(t)_i$ is time points for delivering components, n is the number of reference points, $z_2(t)_b$ is the time to deliver components under the contract, and d_2 is the ratio of the amount of the cost of components and the cost of applying them to their destination to the cost of the flaw detector.

Criterion d_3z_3 will found by the formula:

$$b''_{i3}z_3 = z_3 \tag{7}$$

where z_3 is the relation of closed questions to the total number of questions.

The term set $T(X_i)$ for the linguistic variables "level of output of suitable products", "degree of effectiveness of developed warning measures (for the previous period)" and "degree of efficiency from newly introduced technologies" will be low value, average value, and high value. The universal set is $U = [0, 1]$, and $G : M = [1-u_t(u), (u_t(u))^2]$. The following scale will be used in the rule database for the place of output value names:

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- very high value $0, 85 \leq x$.

The following rating scale will be used for the place of names of input values:

- low value $x < 0, 4$;
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- high value $0, 7 \leq x$.

The following functions will be used for input information:

- Z-shaped accessory function for $x < 0, 4$;
- Triangular accessory function for $0, 4 \leq x < 0, 8$;
- S-shaped accessory function for $0, 7 \leq x$.

4 Calculating the Quality Level Using a Two-Level Optimization Model

The example of a task is listed in Table 2.

Table 2. The example of a task.

№	Q	x_1	$x_2(y_2)$	z_2	y_3	z_3	b	№	Q	x_1	$x_2(y_2)$	z_2	y_3	z_3	b
1	E1	0,8	0.3	0,1	0,8	0,8	$\geq$ 0,68	1	E3		1	–		–	$\leq$ 1,0
2	E2	0,3	0,1	0,6	0,9	1	$\geq$ 0,63	2	E4			–	1	–	$\leq$ 1,0
3	E3	0.7	0.9	1	0,7	1	$\geq$ 0,84	3	E5			–	0,9	–	$\leq$ 0,9
4	E4	0,1	0.9	1	1	0,5	$\geq$ 0,68	4	E6	0,9	0,7	–		–	$\geq$ 0,8
5	E5	1	1	1	1	1	$\leq$ 1,00	5	F_2	x_1	$x_2(y_2)$	z_2	y_3	z_3	$b^{//}$
6	F_1	x_1	$x_2(y_2)$	z_2	y_3	z_3	$b^{/}$	6	E1	0.3	–	0,5	–	0,8	$\geq$ 0,53
7	E1	0,1		–		–	$\leq$ 0,1	7	E2	1	–	1	–	1	$\leq$ 1
8	E2		0,5	–		–	$\leq$ 0,5	8							

More detailed information about two-level programming could be found in [8]. Based on the results of the implementation of the simplex method, the quality level of the flaw detector at the production stage was determined as $Q(x, y, z) = 0, 67$.

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