Application of Differential Geometry to Calculations of Stress–Strain State of Nuclear Reactors Covers of WWER Type

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Abstract. The problem of determining the stress–strain state (SSS) of the covers of nuclear reactors of the WWER type is considered, which is reduced to calculating an inexact two-layer shell trapped along the contour in the form of a truncated circular rotation ellipsoid under the combined action of internal pressure and temperature. To construct the equations of thermoelasticity of such shells, it is necessary to have expressions describing the internal and external geometry of the surface of the junction of the two-layer shell. In this paper, the basic relations of differential geometry are constructed for an inexact ellipsoidal bimetallic shell of revolution simulating the cover of a WWER-type nuclear reactor. As a result of the analysis, based on the study of quadratic forms of the ellipsoid surface, expressions are obtained for the Lame coefficients and the main radii of curvature of the ellipsoidal bilayer shell of revolution, while the first quadratic form allows the determination of the Lame coefficients, and the main radii of curvature were obtained from their second quadratic form. Their validity is confirmed on the basis of the fulfillment of the Codazzi–Gauss relations.

Keywords: Reactor cover · Ellipsoidal rotation shells · Differential geometry

1 Introduction

The cover of a nuclear reactor, for example, of the WWER type [1–3] is one of the main units of the upper block (Fig. 1) of the reactor. The calculation of the reactor lid is reduced to the calculation of the stress–strain state of a thin, inexact, two-layer shell pinched along the contour in the form of a truncated circular rotation ellipsoid under the combined action of internal pressure and temperature [4–8]. To construct the equations of thermoelasticity of such shells, it is necessary, first of all, to have expressions describing the geometry of the junction surface.

However, ellipsoidal non-pathological shells are studied much less than shells of revolution with constant curvature of the meridian (cylinder, cone, or sphere), and it is very difficult to find a complete system of defining relations for them in the literature. Suffice it to say that in such a fundamental and becoming a reference book for shell experts, the monograph by AS Volmira "Stability of deformable systems" [9] with a

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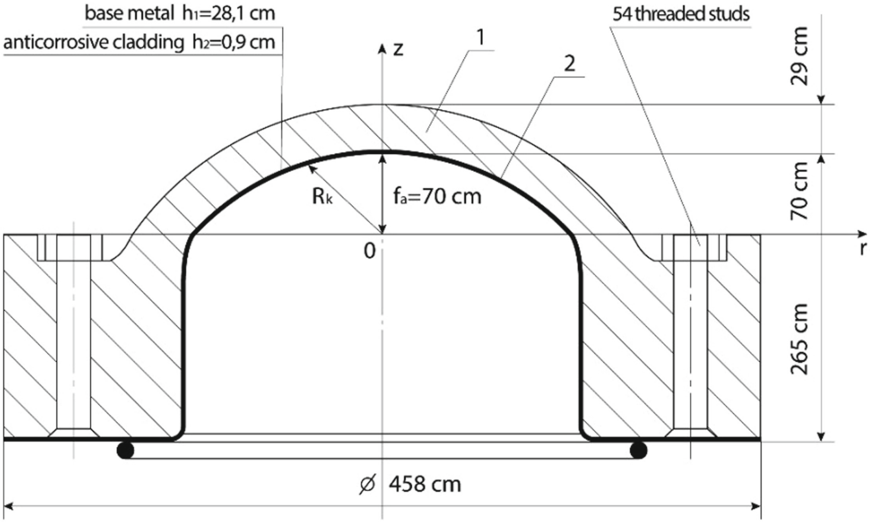

Fig. 1. Reactor cover

volume of more than 900 pages, less than two pages is devoted to ellipsoidal shells; basically formulas for the upper critical pressure are given.

2 Setting Goals and Objectives of Scientific Research

The purpose of the scientific work is to build the basic relations of differential geometry for a non-blank ellipsoidal bimetallic geometry for a non-blank ellipsoidal bimetallic shell of revolution of rotation simulating the cover of a nuclear reactor, for example, of the WWER type.

3 Geometrical Parameters of the Surface of the Reactor Cover

Below, using [10–12], the basic relations of the differential geometry of the non-blank ellipsoidal bimetallic shell of revolution are obtained: expressions for the coefficients of the first and second quadratic forms of the junction surface, for the Lame coefficients, the main radii of curvature, their validity is confirmed based on the fulfillment of the Codazzi relations–Gauss.

To describe the internal geometry and construction of the first quadratic shape of the surface of an ellipsoid (Fig. 2), it is convenient to use the terms, and most importantly, the coordinates adopted in geodesy, since the differential geometry of the ellipsoid has been studied in detail precisely in spheroidal geodesy studying the terrestrial ellipsoid (note that it studies methods for determining the relative position of points located on the surface of the earth's ellipsoid in a system of geodetic coordinates.

In foreign literature, spheroidal geodesy is sometimes called mathematical or geometric geodesy [6]).

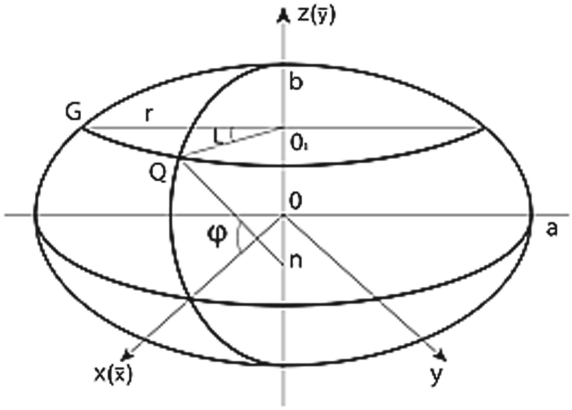

Fig. 2. On the geometry of the ellipsoid cover

The relative parameters of the ellipsoid of revolution, which are associated with the major and minor axes and characterize its shape, are described by the formulas:

$$a = 1 - \sqrt{1 - e^2}, \quad b = a(1 - a) = a\sqrt{1 - e^2}$$

where a is the semi-major axis of the ellipsoid, eccentricity $e = \frac{\sqrt{a^2 - b^2}}{a}$.

The system of spatial rectangular coordinates is established as follows. The origin coincides with the center of the ellipsoid O, the axis coincides with the vertical axis of symmetry (axis of rotation), the axis is defined by the intersection of the horizontal (equatorial) and vertical (meridional) planes, and the axis complements the three coordinate vectors to the right. Angular coordinates are accepted for meridians and parallels: L, B—geodesic longitude latitude, Qn—normal to the surface of the ellipsoid at the point Q.

Here L is the geodesic longitude—this is the angle between the plane of the initial meridian and the plane of the given meridian,—geodesic latitude is the acute angle between the normal to the surface of the ellipsoid and the plane of the equator (or the plane of any parallel).

The meridian ellipse resulting from the cross section of an ellipsoid by a plane passing through its minor axis is shown in Fig. 3. Its consideration together with Fig. 2 allows you to record the position of an arbitrary point Q in various coordinate systems on the plane and establish a relationship between the planes and spatial coordinates of an arbitrary point. Let be:

- $\bar{x}$, $\bar{y}$ Cartesian rectangular coordinates of Q on the plane;
- φ the angle formed by the normal to the tangent to the ellipse at the point;
- χ the angle formed by the radius vector OQ with the equator plane (called geocentric latitude);

u—is the so-called reduced latitude is the acute angle formed by the straight line Q_1Q_2O with the equatorial plane, where Q_1 and Q_2 are the projections of the point Q on

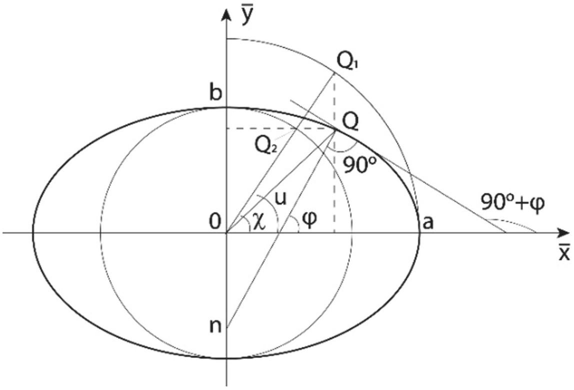

Fig. 3. Meridian ellipse

the circle of radii a and b described around the point O as a center (If we use this concept, we get a simple formula for calculating the radius of an arbitrary parallel $r = a\cos u$).

$$tg\chi = \frac{\bar{y}}{\bar{x}}, \quad \bar{x} = a\cos u, \quad \bar{y} = b\sin u \tag{1}$$

here $a = OQ_1, b = OQ_2$,

$$\mathrm{tg}u = \frac{a}{b}\frac{\bar{y}}{\bar{x}} = \frac{1}{\sqrt{1-e^2}}\frac{\bar{y}}{\bar{x}}. \tag{2}$$

Comparing (1) and (2), we establish the relationship of "geocentric" and "reduced" latitudes

$$\mathrm{tg}u = \frac{1}{\sqrt{1-e^2}}\mathrm{tg}\chi. \tag{3}$$

To establish the connection of flat Cartesian rectangular coordinates $\bar{x}$, $\bar{y}$ with geodetic latitude ϕ, we use the fact that as follows from Fig. 3

$$\mathrm{tg}(90+\varphi) = -\mathrm{ctg}\varphi = \frac{\mathrm{d}\bar{y}}{\mathrm{d}\bar{x}} \Rightarrow \mathrm{tg}\varphi = -1/(\mathrm{d}\bar{y}/d\bar{x}). \tag{4}$$

Differentiating further the equation of the "meridian" ellipse $\frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2} = 1$ with respect to $\bar{x}$, we find:

$$\left(\frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2}\right)' = \frac{2\bar{x}}{a^2} + \frac{2\bar{y}\bar{y}'}{b^2} = 0, \quad \Rightarrow \frac{\mathrm{d}\bar{y}}{\mathrm{d}x}\cdot\frac{y}{b^2} = -\frac{\bar{x}}{a^2}, \quad \Rightarrow \frac{\mathrm{d}\bar{y}}{\mathrm{d}\bar{x}} = -\frac{b^2}{a^2}\frac{\bar{x}}{\bar{y}}.$$

Equating the resulting expression to Equality (4), we obtain

$$\mathrm{tg}\varphi = \frac{a^2}{b^2}\frac{\bar{y}}{\bar{x}} = \frac{1}{1-e^2}\frac{\bar{y}}{\bar{x}}$$

Given (2) and (3), we can write the relations connecting the latitudes of an arbitrary point of the ellipsoid Q:

$$\mathrm{tg}\chi = \sqrt{1-e^2}\mathrm{tg}u = (1-e^2)\mathrm{tg}\varphi, \quad \mathrm{tg}u = \sqrt{1-e^2}\mathrm{tg}\varphi.$$

Using the obtained equalities, we find further the expressions for the flat Cartesian coordinates $\bar{x}$, $\bar{y}$ in the latitude function:

$$\bar{x} = \frac{a\cos\varphi}{\sqrt{1-e^2\sin^2\varphi}}, \quad \bar{y} = \frac{a(1-e^2)\sin\varphi}{\sqrt{1-e^2\sin^2\varphi}} \tag{5}$$

and

$$\bar{x} = \frac{a\sqrt{1-e^2}\cos\chi}{\sqrt{1-e^2\cos^2\chi}}, \quad \bar{y} = \frac{a\sqrt{1-e^2}\sin\chi}{\sqrt{1-e^2\cos^2\chi}}. \tag{6}$$

To determine the position of an arbitrary point $Q(x, y, z)$ on the surface of an ellipsoid, we will use, as is customary in geodesy, spatial angular coordinate systems: geodesic (φ), reduced (u) and geocentric (χ) latitudes and geodesic longitudes (L).

The coordinates u, φ and L, are curved, since the corresponding coordinate lines—meridians and parallels—are curved lines.

The equations of coupling of rectangular spatial coordinates x, y, z and coordinates $\bar{x}$, $\bar{y}$ in the plane of the meridional ellipse, as follows from a comparison of Figs. 2 and 3, can be written in the form

$$x = \bar{x}\cos L, \quad y = \bar{x}\sin L, \quad z = \bar{y}.$$

In view of (5) and (6), we obtain the communication equations in the form:

$$\begin{aligned} &x = a\cos u\cdot\cos L, \quad y = a\cos u\cdot\sin L, \\ &z = b\sin u = a\sqrt{1-e^2}\cdot\sin u \end{aligned} \tag{7}$$

$$x = \frac{a\cos\varphi\cdot\cos L}{W}, \quad y = \frac{a\cos\varphi\cdot\sin L}{W}, \quad z = \frac{a(1-e^2)\sin\varphi}{W}. \tag{8}$$

Here $W = \sqrt{1-e^2\sin^2\varphi}$—is the so-called (in geodesy) the first main function of latitude. Equalities (7) are the parametric equations of the surface of an ellipsoid. On the other hand, the equation of the surface of the lid (ellipsoid) can be written in vector form

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \tag{9}$$

where $\vec{r}$—is the radius vector drawn from the center of the ellipsoid to an arbitrary point on its surface, $\vec{i}, \vec{j}, \vec{k}$—are unit vectors, the coordinates of the vector $\vec{r}$ are determined by Formulas (7) or (8).

Substituting them in (9), we obtain a parametric representation of the equation of the surface of the ellipsoid in the form:

$$\vec{r} = a\Big(\cos u\cdot\cos L\cdot\vec{i} + \cos u\cdot\sin L\cdot\vec{j} + \sqrt{1-e^2}\sin u\cdot\vec{k}\Big)$$

or

$$\vec{r} = \frac{a}{W}\left[\cos\varphi \cdot \cos L \cdot \vec{i} + \cos\varphi \cdot \sin L \cdot \vec{j} + (1 - e^2)\sin\varphi \cdot \vec{k}\right] \tag{10}$$

As is known, for an arbitrary curve $\bar{r} = \bar{r}(t)$, where t—is a parameter (this equation expressing the radius vector of an arbitrary point of the curve as a function of the scalar argument is called the vector parametric representation of the curve [4]), arc differential $\mathrm{d}s = s'(t)\mathrm{d}t = \left|\bar{r}'(t)\right|\mathrm{d}t$ whence it follows that the differential modulus of the arc is equal to the differential modulus of the radius—vector [10]

$$|\mathrm{d}s| = |\mathrm{d}\vec{r}|.$$

But the full differential of the radius vector as a function of two independent variables u, L is determined by the expression

$$\mathrm{d}\vec{r} = \frac{\partial \bar{r}}{\partial u}\mathrm{d}u + \frac{\partial \bar{r}}{\partial L}\mathrm{d}L,$$

Therefore, $|\mathrm{d}s| = |\mathrm{d}\vec{r}| = \left|\frac{\partial \bar{r}}{\partial u}\mathrm{d}u + \frac{\partial \bar{r}}{\partial L}\mathrm{d}L\right|$.

Therefore, using the well-known formula for the modulus of the vector below, we find the expression in parametric coordinates for the first quadratic form

$$\begin{aligned}\mathrm{d}s^2 = \mathrm{d}\vec{r}^2 &= \left(\frac{\partial \bar{r}}{\partial u}\mathrm{d}u + \frac{\partial \bar{r}}{\partial L}\mathrm{d}L\right)^2 = \left(\frac{\partial \bar{r}}{\partial u}\right)^2\mathrm{d}u^2 + 2\left(\frac{\partial \bar{r}}{\partial u}\right)\left(\frac{\partial \bar{r}}{\partial L}\right)\mathrm{d}u\mathrm{d}L + \left(\frac{\partial \bar{r}}{\partial L}\right)^2\mathrm{d}L^2\\ &= E(u, L)\mathrm{d}u^2 + 2F(u, L)\mathrm{d}u\mathrm{d}L + G(u, L)\mathrm{d}L^2\end{aligned} \tag{11}$$

Here E—is the scalar square of the derivative of the radius — the vector is

$$\begin{aligned}E = \left(\frac{\partial \vec{r}}{\partial u}\right)^2 &= \left(\left(\frac{\partial \vec{r}}{\partial u}\right), \left(\frac{\partial \vec{r}}{\partial u}\right)\right) = \left[(-a\sin u \cdot \cos L)\,\bar{i} + (-a\sin u \cdot \sin L)\bar{j}\right.\\ &\quad \left.+ (b\cos u)\bar{k}\right] \cdot \left[(-a\sin u \cdot \cos L)\,\bar{i} + (-a\sin u \cdot \sin L)\bar{j} + (b\cos u)\bar{k}\right]\\ &= (a\sin u \cdot \cos L)^2(\bar{i}, \bar{i}) + (a\sin u \cdot \sin L)^2(\bar{j}, \bar{j})\\ &\quad + (b\cos u)^2(\bar{k}, \bar{k}) = a^2\sin^2 u \cdot \cos^2 L + a^2\sin^2 u \cdot \sin^2 L\\ &\quad + b^2\cos^2 u = a^2\sin^2 u + b^2\cos^2 u = a^2(1 - e^2\cos^2 u),\end{aligned}$$

since all mixed scalar products of unit vectors $(i, j) = (i, k) = (j, k) = 0$ and scalar squares of unit vectors $(i, i) = (j, j) = (k, k) = 1$.

The first quadratic shape defines the metric (or, as they say, the internal geometry) of the surface, because, knowing the first quadratic shape, you can find the angle between any lines passing through an arbitrary point on the surface (that is, the angle between the tangents to these lines), you can calculate the area areas on the surface, the full length of the arc curve. The definition of "internal" emphasizes the fact that we mean the proper properties of the surface, which are inherent in it and remain unchanged during its bending.

The expression for E can also be represented as

$$E = \frac{a^2(1 - e^2)^2}{W^6}.$$

It is easy to show that for an orthogonal coordinate grid (meridians and parallels) on the surface of an ellipsoid $F(u, L) = 0$.

Indeed, since $z_L = 0$, then

$$F = \left(\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial L}\right) = \frac{\partial x}{\partial u}\frac{\partial x}{\partial L} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial L} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial L} = (-a \sin u \cdot \cos L)\left(-\frac{a \cos\phi \cdot \sin L}{W}\right) + (-a \sin u \cdot \sin L)\frac{a \cos\phi \cdot \cos L}{W} = 0. \quad (12)$$

$$G = \left(\frac{\partial \vec{r}}{\partial L}\right)^2 = \left(\frac{\partial x}{\partial L}\right)^2 + \left(\frac{\partial y}{\partial L}\right)^2 + \left(\frac{\partial z}{\partial L}\right)^2 = \frac{a^2 \cos^2\phi \cdot \sin^2 L}{W^2} + \frac{a^2 \cos^2\phi \cdot \cos^2 L}{W^2} = \frac{a^2 \cos^2\phi}{W^2}.$$

In view of (12), the linear element of the ellipsoid surface will be determined by the formula

$$\mathrm{d}s = \sqrt{E\mathrm{d}u^2 + G\mathrm{d}L^2} = \sqrt{M^2\mathrm{d}u^2 + r^2\mathrm{d}L^2}, \quad (13)$$

where $E = A_1 = M^2,\ G = A_2 = r^2$.

Here $A_1,\ A_2$ are the so-called Lame coefficients (they have the meaning of proportionality coefficients in formulas expressing the differentials of the arcs of coordinate lines through the differentials of curvilinear coordinates: $\mathrm{d}\alpha_1 = A_1\mathrm{d}u$, $\mathrm{d}\alpha_2 = A_2\mathrm{d}L$).

When $L =$ const, $\mathrm{d}L = 0$, we obtain the equation of the meridian as a function of geodesic and reduced latitudes

$$\mathrm{d}s_{мер} = M\mathrm{d}u = a\sqrt{1 - e^2\cos^2 u}\,\mathrm{d}u = \frac{a}{V}\mathrm{d}u,$$

where the function $V = \frac{1}{\sqrt{1-e^2\cos^2 u}} = \frac{W}{\sqrt{1-e^2}}$ is called the second main latitude function.

Similarly, for a parallel from the conditions $\phi =$ const, $\mathrm{d}\phi = 0$ it follows $\mathrm{d}s_{\mathrm{nap}} = r\mathrm{d}L = a\cos u \cdot \mathrm{d}L$.

As you know, on any surface you can always choose two such tangents to the surface and directions that are orthogonal to each other along which the radii of curvature are the largest and smallest. Such directions are called principal, and the radii of curvature of the normal sections passing along these directions (*that is, sections whose planes contain the normal to the surface at a given point*) are called the principal radii of curvature. The curvature of an arbitrary normal surface section is expressed by the Euler formula [10]

$$k_{\mathrm{A}} = \frac{1}{R_{\mathrm{A}}} = k_1\cos^2 A + k_2\sin^2 A = \frac{\cos^2 A}{M} + \frac{\sin^2 A}{N},$$

where A—is the angle formed by the tangent of this normal section with the first main direction (azimuth), k_1 and k_2—are the main curvatures at a given point on the surface.

Since $\sin^2 A$ it reaches the largest value of 1 for $A = \pm\pi/2$, and the smallest 0 for $A = 0$ or $A = \pi$, k_A it takes the largest and smallest values for the same values A. Therefore, with $A = 0$ or $A = \pi$ $k = k_1$, with $A = \pm\pi/2$ $k = k_2$.

It can be shown that one main direction is characterized by an equation $\mathrm{d}u = 0$, and the other main direction is characterized by an equation $\mathrm{d}L = 0$.

For an ellipsoid, one of the main directions will be the direction along the meridian. The curvature of this direction will be equal to the reciprocal of the radius of the meridian

$$k_1 = K_{\text{mer}} = \frac{1}{M} = \frac{W^3}{a(1-e^2)}$$

or taking into account the expression for W the radius of curvature of the meridian is

$$M = \frac{a(1-e^2)}{W^3} = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}. \tag{14}$$

The second main direction on the surface of the ellipsoid is a direction characterized by a normal section extending at an angle $\pm\pi/2$ to the meridional ellipse, this direction along the so-called first vertical.

Therefore, by Meunier's theorem $r = N\cos\varphi$ [11, 12]:

$$k_2 = K_{\text{vert}} = \frac{1}{N} = \frac{W}{a}. \tag{15}$$

Substituting expressions for eccentricity $e = \frac{\sqrt{a^2-b^2}}{a}$ and for $W = \sqrt{1-e^2\sin^2\varphi}$ in (14) and (15), after elementary transformations, we obtain the final expressions for the principal radii of curvature of the ellipsoid in the following form:

$$M = \frac{a^2b^2}{(a^2\cos^2\varphi + b^2\sin^2\varphi)^{3/2}}, \quad N = \frac{a^2}{(a^2\cos^2\phi + b^2\sin^2\phi)^{1/2}}.$$

As can be seen from the formulas obtained, the magnitude of the principal radii of curvature depends on latitude. If, in axisymmetric deformation, in the shell equations operate with an angle—an angle between the normal to the junction surface and the axis of rotation, then $\alpha = \pi/2 - \varphi$.

Then, when designating the small radius of the elliptic cover, for the main radii of curvature of the ellipsoid, we finally obtain the formulas that coincide with those given in the monograph by S. P. Timoshenko and S. Voinowski–Krieger [13]:

$$R_1 = \frac{a^2c^2}{\left(a^2\sin^2\alpha + c^2\cos^2\alpha\right)^{3/2}}, \quad R_2 = \frac{a^2}{\left(a^2\sin^2\alpha + c^2\cos^2\alpha\right)^{1/2}}.$$

For what follows, it will be convenient to use the relations adopted in the theory of shells.

Then, the first quadratic surface shape can be set in the form:

$$I = a_{11}\mathrm{d}u^2 + 2a_{12}\mathrm{d}u\mathrm{d}L + a_{22}\mathrm{d}L^2,$$

where the coefficients of the first quadratic form, which are functions of curvilinear coordinates, correspond to the so-called Lame coefficients:

$$a_{11} = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2 = A_1^2, \quad a_{22} = \left(\frac{\partial x}{\partial L}\right)^2 + \left(\frac{\partial y}{\partial L}\right)^2 + \left(\frac{\partial z}{\partial L}\right)^2 = A_2^2.$$

For shells in a triorthogonal coordinate system, $a_{12} = 0$. We substitute Formula (7) into the last expressions. Then,

$$a_{11} = A_1^2 = (-a \sin u \cdot \cos L)^2 + (-a \sin u \cdot \sin L)^2 + (b \cos u)^2 = a^2 \sin^2 u + a^2(1-e^2)\cos^2 u = a^2(1 - e^2\cos^2 u) = a^2\left(1 - e^2\frac{\cos^2\varphi}{W^2}\right) = \frac{a^2(1-e^2)}{W^2}$$

Therefore,

$$A_1 = \sqrt{a_{11}} = \frac{a\sqrt{1-e^2}}{W} = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2\sin^2\varphi}} = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\alpha}}.$$

Similarly, we find

$$a_{22} = \left(\frac{\partial x}{\partial L}\right)^2 + \left(\frac{\partial y}{\partial L}\right)^2 + \left(\frac{\partial z}{\partial L}\right)^2 = (-a\cos u \sin L)^2 + (a \cos u \cos L)^2 = a^2\cos^2 u = \frac{a^2\cos^2\varphi}{W^2} = \frac{a^2\sin^2\alpha}{1-e^2\cos^2\alpha} = A_2^2$$

Therefore, $A_2 = \frac{a\sin\alpha}{\sqrt{1-e^2\cos^2\alpha}}$.

So, the Lame coefficients for an ellipsoid are equal

$$A_1 = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\alpha}}, \quad A_2 = \frac{a\sin\alpha}{\sqrt{1-e^2\cos^2\alpha}}.$$

The construction of the first quadratic form and expression for the principal radii of curvature are well known in the literature, since our planet is an ellipsoid of revolution. But the Earth is a solid body, and therefore, the deformation of the surface during its curvature in spheroidal geodesy is not studied.

To study the surface of a thin shell, which is a reactor cover, it is necessary to know the curvature of the surface and obtain relations characterizing the deviation of the surface from the tangent plane in the vicinity of the point of contact.

These relations are determined by the so-called second quadratic surface shape [10, 11]

$$II = b_{11}\mathrm{d}u^2 + 2b_{12}\mathrm{d}u\mathrm{d}L + b_{22}\mathrm{d}L^2$$

the coefficients of which b_{11} and b_{22} characterize the normal curvature of the coordinate lines, and b_{12} is the surface torsion parameter.

According to the ratios of differential geometry [10], they are determined by the formulas:

$$b_{11}(u,L) = (\bar{r}_{uu}\bar{n}) = \frac{(\bar{r}_{uu}\bar{r}_u\bar{r}_L)}{|[\bar{r}_u\bar{r}_L]|} = \frac{1}{\sqrt{EG-F^2}}\frac{\partial^2\vec{r}}{\partial u^2}\cdot\frac{\partial\vec{r}}{\partial u}\cdot\frac{\partial\vec{r}}{\partial L},$$

$$b_{12}(u,L) = (\bar{r}_{uL}\bar{n}) = \frac{(\bar{r}_{uL}\bar{r}_u\bar{r}_L)}{|[\bar{r}_u\bar{r}_L]|} = \frac{1}{\sqrt{EG-F^2}}\frac{\partial^2\vec{r}}{\partial u\partial L}\cdot\frac{\partial\vec{r}}{\partial u}\cdot\frac{\partial\vec{r}}{\partial L},$$

$$b_{22} = (\bar{r}_{LL}\bar{n}) = \frac{(\bar{r}_{LL}\bar{r}_u\bar{r}_L)}{|[\bar{r}_u\bar{r}_L]|} = \frac{1}{\sqrt{EG-F^2}}\frac{\partial^2\vec{r}}{\partial L^2}\cdot\frac{\partial\vec{r}}{\partial u}\cdot\frac{\partial\vec{r}}{\partial L}.$$

Here $\bar{n} = \frac{\frac{\partial\bar{r}}{\partial u}\times\frac{\partial\bar{r}}{\partial L}}{\left|\frac{\partial\bar{r}}{\partial u}\times\frac{\partial\bar{r}}{\partial L}\right|}$ is the unit normal vector to the surface, determined by the vector product of the vectors and $\frac{\partial\bar{r}}{\partial u}$, a $\left|\frac{\partial\bar{r}}{\partial u}\times\frac{\partial\bar{r}}{\partial L}\right| = \sqrt{EG-F^2}$ is its modulus.

Subscripts mean differentiation by the corresponding coordinate. In the generally accepted notation of the theory of shells, $\sqrt{EG-F^2} = \sqrt{a_{11}a_{22}-a_{12}^2}$. For orthogonal grid,

$$F = a_{12} = 0 \quad \text{and} \quad \sqrt{a_{11}a_{22}} = A_1A_2 = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\alpha}}\cdot\frac{a\sin\alpha}{\sqrt{1-e^2\cos^2\alpha}}$$
$$= \frac{a^2\sqrt{1-e^2}\sin\alpha}{1-e^2\cos^2\alpha}.$$

If expressed $\vec{r}$ in Cartesian coordinates, then the coefficients of the second quadratic form will be calculated through differential determinants [11]:

$$b_{11} = \frac{1}{\sqrt{a_{11}a_{22}}}\begin{vmatrix}\frac{\partial^2x}{\partial u^2} & \frac{\partial^2y}{\partial u^2} & \frac{\partial^2z}{\partial u^2}\\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u}\\ \frac{\partial x}{\partial L} & \frac{\partial y}{\partial L} & \frac{\partial z}{\partial L}\end{vmatrix}, \quad b_{12} = \frac{1}{\sqrt{a_{11}a_{22}}}\begin{vmatrix}\frac{\partial^2x}{\partial u\partial L} & \frac{\partial^2y}{\partial u\partial L} & \frac{\partial^2z}{\partial u\partial L}\\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u}\\ \frac{\partial x}{\partial L} & \frac{\partial y}{\partial L} & \frac{\partial z}{\partial L}\end{vmatrix}$$

$$b_{22} = \frac{1}{\sqrt{a_{11}a_{22}}}\begin{vmatrix}\frac{\partial^2x}{\partial L^2} & \frac{\partial^2y}{\partial L^2} & \frac{\partial^2z}{\partial L^2}\\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u}\\ \frac{\partial x}{\partial L} & \frac{\partial y}{\partial L} & \frac{\partial z}{\partial L}\end{vmatrix}$$

Given Formulas (10) and (13), we obtain

$$b_{11} = \frac{1}{\sqrt{a_{11}a_{22}}}\begin{vmatrix}-a\cos u\cdot\cos L & -a\cos u\cdot\sin L & -b\sin u\\ -a\sin u\cdot\cos L & -a\sin u\cdot\sin L & b\cos u\\ -a\cos u\cdot\sin L & -a\cos u\cdot\cos L & 0\end{vmatrix} = \frac{a^2b}{\sqrt{a_{11}a_{22}}}\cos u$$
$$= \frac{a^2b\cos\phi}{W\sqrt{a_{11}a_{22}}} = \frac{a^2b\sin\alpha}{\sqrt{1-e^2\cos^2\alpha}}\cdot\frac{1-e^2\cos^2\alpha}{a^2\sqrt{1-e^2}\sin\alpha} = a\sqrt{1-e^2\cos^2\alpha},$$

$$b_{12} = 0,$$

$$b_{22} = \frac{a^2b}{\sqrt{a_{11}a_{22}}}\cos^3 u = \frac{a^2\cdot a\sqrt{1-e^2}\cos^3\phi\cdot W^2}{W^3a^2\sqrt{1-e^2}\cos\phi} = \frac{a\cos^2\phi}{W} = \frac{a\sin^2\alpha}{\sqrt{1-e^2\cos^2\alpha}}.$$

Since $b_{12} = 0$, the second quadratic form for the ellipsoid of revolution will be determined by the expression $II = b_{11}\mathrm{d}u^2 + b_{22}\mathrm{d}L^2$.

The first and second quadratic forms define a surface up to its position in space. The coefficients of both quadratic forms cannot be arbitrary; they are connected by certain differential relationships; namely, for all lines on the surface that have a common tangent, the curvature of the normal section is the same and is expressed by the well-known formula [9]:

$$k_0 = \frac{II}{I} = \frac{b_{11}\mathrm{d}u^2 + 2b_{12}\mathrm{d}u\mathrm{d}L + b_{22}\mathrm{d}L^2}{a_{11}\mathrm{d}u^2 + 2a_{12}\mathrm{d}u\mathrm{d}L + a_{22}\mathrm{d}L^2}. \tag{16}$$

If we put in (16) $\mathrm{d}u = 0$, and then $\mathrm{d}L = 0$, we obtain, respectively,

$$k_{11} = \frac{1}{R_1} = \frac{b_{11}}{a_{11}} = \frac{b_{11}}{A_1^2}, \quad k_{22} = \frac{1}{R_2} = \frac{b_{22}}{a_{22}} = \frac{b_{22}}{A_2^2}. \tag{17}$$

Substituting the expressions found above for the coefficients of quadratic forms and the Lamé coefficients, we see that relations (17) are satisfied.

4 Conclusion

A complete system of the basic relations of the differential geometry of an inexact ellipsoidal bimetallic shell of revolution simulating the cover of a nuclear reactor is obtained.

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