2

Portfolio Theory and International Diversification

2.1 Introduction

Modern portfolio theory is concerned with the characteristics and analysis of individual securities as well as portfolios whose characteristics are significantly different from the individual assets from which they are built. Every investor should be aware of the basics of portfolio theory, from the relationship of portfolio characteristics to security characteristics and a desirable set of portfolios. It is crucial to understand how investors might choose the optimal portfolio from among a set of different portfolios meeting his objectives. According to Markowitz, an optimal portfolio minimizes the risk for a given level of return or maximizes return at a given level of risk. Implementing the above portfolio risk-return formula, any investor would find the capital market theory and capital asset pricing model (CAPM) useful, as it focuses on the appropriate measure of risk, which is the beta coefficient. Portfolio theory generates a number of benefits, i.e., proper asset selection and risk reduction for a properly selected set of investments. One possible way to achieve above-average returns is international diversification, which gives significant benefits, including market risk reduction far beyond the national level.

The main aim of this chapter is to familiarize the reader with portfolio theory and international diversification, which, in fully integrated and efficient capital markets, are the best and most natural strategy. A proper understanding of the risk-return characteristics of an investment portfolio will provide investors with future support for international investments.

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2.2 Risk and Rate of Return

An investment could be defined as the current commitment of funds for a certain period to derive a future flow of funds that will compensate the investing unit for the time the funds are committed, for the expected rate of inflation, and the uncertainty involved in the future flow of funds (Reilly 1986). The primary purpose of investing is to consume more in the future, so the increase in wealth results from the investment. It means that an investment generates a return, and this return is influenced by many different factors. Return is measured in terms of the relationship between the amount invested and the amount returned. This relation is expressed as the **rate of return** and can be written as follows:

$$R_{i,t} = \frac{\text{Ending value} - \text{Begining value}}{\text{Begining value}} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} = \frac{P_{i,t}}{P_{i,t-1}} - 1;$$

where:

$R_{i,t}$—The rate of return of the *i*-th asset at time *t*,

$P_{i,t}$—The price of the *i*-th asset at the end of the period,

P_{it-1}—The price of the *i*-th asset at the beginning of the period.

Many investments provide a cash flow (income received) in addition to changing value while the funds are invested. If we consider a stock investment, it could be a dividend. If we consider bonds, it could be interests. If cash flow is considered, the above relation can be written as follows:

$$R_{i,t} = \frac{(P_{i,t} - P_{i,t-1}) + C_{i,t}}{P_{i,t-1}};$$

where:

$C_{i,t}$—Cash flow of the *i*-th asset at time *t*.

This formula indicates the rate of increase in wealth, and it could be split into two parts: capital appreciation or capital gain (the change in price) and cash flow (income received). When the rate of return is positive, it is considered a gain; when it is negative, it reflects a loss. The rate of return is a relative measure usually expressed in the form of a percentage.

The second factor is **risk**. Investment risk is defined as uncertainty regarding the expected rate of return from an investment. The terms risk and uncertainty are usually used interchangeably, but formally, there is a difference between them. The distinction was explained by Knight (1921), who used *risk* to mean that there is a situation in which the decision-maker assigns probabilities to events based on "known chances." By contrast, *uncertainty* means there are situations in which the decision-maker is unable to assign probabilities to events because it is not possible to calculate chances.

From the investor's perspective, the expected return can be defined under certain economic conditions. The return could be high or low, negative or positive. The most important thing is that the wider the range of possible returns, the more uncertain the actual return is, and the greater the risk. To determine the investor's level of certainty, the probability distribution of expected returns must be analyzed. The probability distribution indicates the possible returns and assigns probabilities to each of them. The probabilities of return range from zero (no chance of this particular return) to one (complete certainty of this particular return). Those probabilities could be subjective estimates or based on past frequencies. The expected rate of return is calculated by multiplying the potential outcomes by the chances of them occurring, and it could be written as follows:

$$E(R_i) = \sum (Probability\, of\, return)(Possible\, return)$$
$$= \sum_{t=1}^{T} P_{i,t} R_{i,t} = P_1 R_{i,1} + P_2 R_{i,2} + \ldots + P_T R_{i,T};$$

where:

$E(R_i)$—The expected return of the i-th asset,

P_{it}—The probability of the i-th asset (chances),

R_{it}—The particular rate of return of the i-th asset (potential outcomes),

T—The number of events.

The expected rate of return is usually based on historical data, and it cannot be guaranteed. Making an investment decision on expected rates of return could be dangerous because it does not contain risk. Thus, investors need one more characteristic, a measure of the dispersion of returns. Of the many different measures of risk, the most important one is the variance of the estimated distribution of expected returns, or the square root of variance standard deviation. The investor must know how much the outcomes differ from the average. In a literal meaning, variance is a measure of dispersion, and it shows how far from the expected return the actual outcome might be. The variance of return can be written as follows:

$$\sigma^2(R_i) = \sum_{t=1}^{T} P_{i,t}[R_{i,t} - E(R_i)]^2 = P_{i,1}[R_{i,1} - E(R_i)]^2 + P_{i,2}[R_{i,2} - E(R_i)]^2$$
$$+ \ldots + P_{i,T}[R_{i,T} - E(R_i)]^2;$$

where:

$\sigma^2(R_i)$—Variance.

The larger the variance, while everything else remains constant, the greater the dispersion and risk. In the event of perfect certainty, there is no variance and no risk.

The standard deviation is calculated by taking the square root of the variance.

$$\sigma(R_i) = \sqrt{\sigma^2(R_i)};$$

where:

$\sigma(R_i)$—The standard deviation.

Generally, it is assumed that investors are risk-averse. It means that if they are given an investment with a smaller standard deviation, i.e., a smaller risk, they will choose it.

To better illustrate the previous discussion, it is crucial to explain the relationship between risk and return and emphasize what causes changes in the required returns over the investment period. This basic relationship between risk and return is positive and linear, as can be seen in Fig. 2.1.

Looking at Fig. 2.1, we can see that investors select investments that are consistent with their risk preferences. Some will consider low-risk investments, whereas others will consider high-risk ones. Figure 2.5 also indicates the risk-free rate (RFR) point. This basic rate indicates no uncertainty of

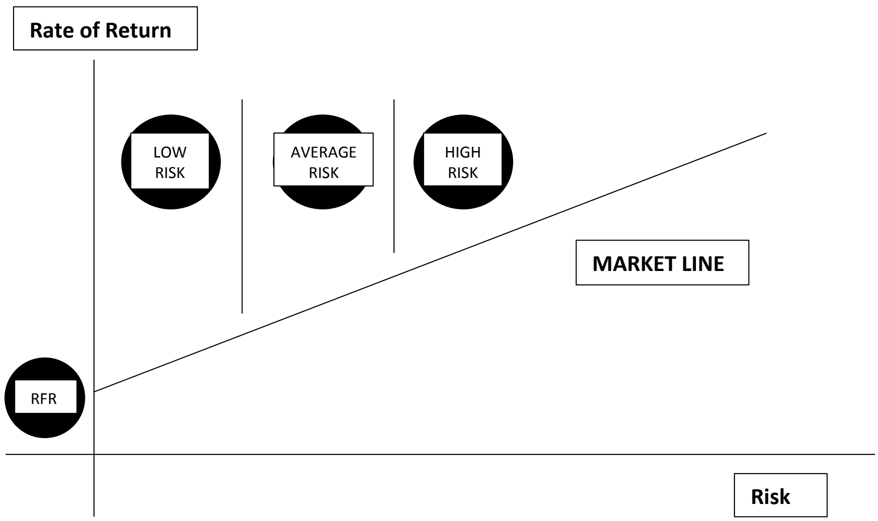

Fig. 2.1 Relationship between risk and rate of the return (*Source* Reilly 1986, p. 18)

future flows, meaning that investors know what cash flow they will receive and when. Additionally, there is no probability of default. The graph shows that investors want the risk-free rate on riskless investments and that they increase the required rate of return as perceived uncertainty increases. A crucial issue is also the slope of the market line, which indicates the composite return per unit of risk required by all market participants (Reilly 1986, pp. 17–18).

2.3 Markowitz's Portfolio Theory

After the comprehensive discussion of all issues connected with the rate of return and risk, it is time to combine individual assets into a portfolio that reflects risk and return preferences. The basic portfolio theory was developed by Harry Markowitz (1952). He was pondering how investors should combine assets into a portfolio that would provide the best possible combination of risk and return, i.e., the highest potential rate of return for a given level of risk or that would minimize the amount of risk for a given level of return.

Firstly, investors should consider the relationship between different investment opportunities, including all types of assets and liabilities, not only stocks. It is vital to consider the whole spectrum of investments because the returns from all these investments interact, and this relationship is important. Secondly, portfolio theory assumes that investors are **risk-averse**, meaning that given a choice between two assets with equal rates of the return, they will choose the one with the lower level of risk. Therefore, it is expected that the relationship between the return and risk is positive. Hence, investors require a higher rate of return to accept the higher risk (Reilly and Brown 1997).

As previously stated, the basic portfolio model was proposed by Markowitz, who showed that the variance of the rate of the return was a significant measure of portfolio risk. He derived the formula for the portfolio risk using the variance of the portfolio, and this formula indicates the importance of diversification in reducing the total portfolio's risk. This model is based on assumptions regarding investor behavior:

- Investors consider each investment alternative to be represented by a probability distribution of expected returns over the holding period.
- Investors maximize one-period expected utility, and the utility curves show a declining marginal utility of wealth.

- Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
- Investors make decisions regarding expected return and risk alone.
- For a given risk level, investors prefer higher returns to lower returns, and for the given level of expected return, less risk to more risk (Reilly and Brown 1997, p. 253).

The first most important factor for each investment is the rate of return. The expected rate of return for the portfolio of assets is simply the weighted average of the expected rates of return for the individual assets in the portfolio. The weights are the proportion of the total value of the assets. This relation can be written as follows:

$$E\left(R_{\text{portfolio}}\right) = \sum_{i=1}^{n} W_i R_i;$$

where:

$E_{\text{portfolio}}$—The expected return of the portfolio,
W_i—The percent of the portfolio in asset i,
R_i—The expected rate of return for asset i.

The second important characteristic is risk. As previously stated, the variance and standard deviation of the return are used as the measure of risk. To present the formula for the standard deviation of the portfolio, we must recall two basic concepts in statistics: covariance and correlation. Covariance is a measure of how returns of assets move together, as they have positive and negative deviations at similar times or dissimilar times, or if they are unrelated (Elton and Gruber 1995, p. 56). A positive covariance means that the rates of return for two investments move in the same direction relative to their individual means during the same period. In contrast, negative covariance means that the rates of return for two investments move in different directions relative to their individual means during the same period (Reilly and Brown 1997, p. 256).

In order to simplify the whole concept, it is useful to standardize the covariance. Dividing the covariance between two investments by the product of the standard deviation of each one, the formula produces a measure called the correlation coefficient with a range of −1 to 1. This formula can be written as follows:

$$r_{i,j} = \frac{\mathrm{Cov}_{i,j}}{\sigma_i \sigma_j};$$

where:

$r_{i,j}$—The correlation coefficient of returns,

σ_i—The standard deviation of $R_{i,t}$,

σ_j—The standard deviation of $R_{i,t}$.

A value of 1 indicates a perfect positive linear relationship, meaning that two returns of investments move together in a completely linear manner. A value of -1 indicates that there is a perfect negative linear relationship between two return series; when one investment rate of return is above its mean, the other is below by a comparable amount. A value of 0 indicates that the returns have no linear relationship, and they are uncorrelated statistically, but it does not indicate that they are independent.

According to portfolio risk, it is now possible to present the basic formula of the standard deviation of returns for a portfolio of assets. Markowitz derived the general formula for portfolio risk using the standard deviation. This formula can be written as follows:

$$\sigma_{\text{portfolio}} = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2} + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{Cov}_{i,j};$$

where:

$\sigma_{\text{portfolio}}$—The standard deviation of the portfolio,

w_i—The weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio,

σ_i^2—The variance of rates of return asset for i,

$\mathrm{Cov}_{i,j}$—Covariance between the rates of return for assets i and j.

The above formula shows that the standard deviation for a portfolio is a function of the weighted average of the individual variances plus the weighted covariances between all assets. What is also shown is that the standard deviation for a portfolio indicates not only the variance but also the covariance between pairs of individual securities in the portfolio. Further, it can be proved that in a portfolio with a large number of assets, this formula reduces to the sum of weighted covariances (Reilly and Brown 1997, p. 261).

Now it is time to consider what happens to the portfolio risk when you add a new security to such a combination. According to the above formula, there are two effects. The first is the assets' variance of returns, and the second is the covariance between the new asset and every other asset that is already in the portfolio. The relative weight of these covariances is substantially greater than the asset's variance; hence, the more securities in the portfolio, the more this is true (Reilly and Brown 1997, p. 262). This means that the contribution to the portfolio variance of the variance of the individual assets goes to zero as the number of securities in a portfolio gets very large. It means that the individual risk of an asset can be fully diversified, but the contribution to the total risk caused by the covariance terms cannot be diversified (Elton and Gruber 1995, p. 60). So, the bottom line is that the most critical factor is not a single security's own variance, but the average covariance with all the other securities in the portfolio. Still, what is important is that in most international markets, the correlation coefficient and the covariance between assets are positive; therefore, the risk of the portfolio cannot be made to go to zero, but it can be much less than the variance of individual assets in a portfolio.

Markowitz showed that the variance (standard deviation) of a portfolio is a function not only of the variance (standard deviation) for the individual assets but also of the covariance between the return for all pairs of assets that are part of the portfolio (Reilly and Brown 1997, p. 272).

To visualize all conceivable combinations of risky assets in the return standard deviation space, it is possible to derive different curves that assume different possibilities. In theory, it is also possible to plot an infinite number of possibilities that group risky and non-risky assets in all possible percentage comparisons. However, we must remember that investors are risk-averse, and they would prefer more return to less, and less risk to more. Thus, it is desirable to find a portfolio that offers a greater return for the same risk, or a lower risk for the same return. That is why there is an efficient set that consists of an envelope curve of all portfolios that lie between the global minimum variance portfolio and the maximum return portfolio. This specific set of portfolios is called the **efficient frontier** (Elton and Gruber 1995, pp. 82–83). The efficient frontier contains the best of all possible combinations. It represents the set of portfolios that has the maximum rate of return for every given level of risk or the minimum risk for every level of return. Figure 2.2 depicts the graph of the efficient frontier.

As can be seen in Fig. 2.2, each portfolio that lies on the efficient frontier has either a higher rate of return for an equal risk or a lower risk for an equal rate of return. We can observe that portfolio A is better than portfolio C because it has an equal return but substantially less risk. The same rule is adjustable to portfolio B, which is better than C because it has equal risk but a higher expected return. The slope of the efficient frontier curve

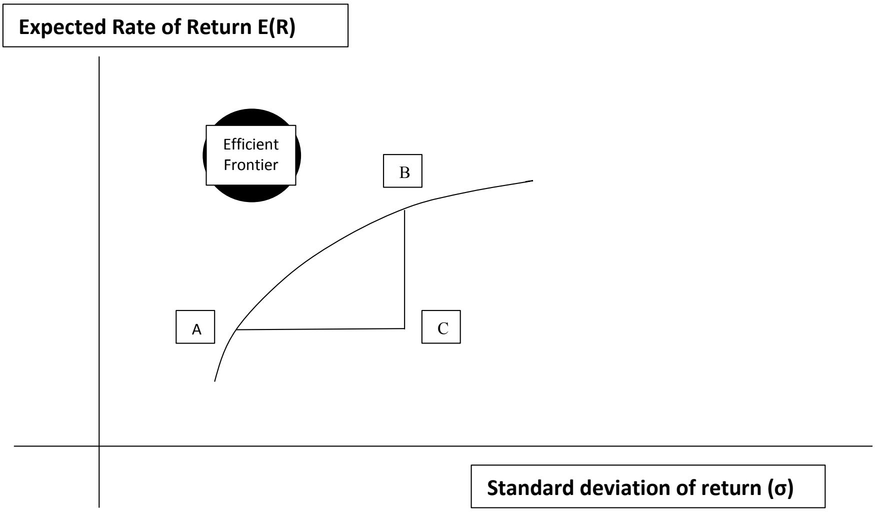

Fig. 2.2 The efficient frontier for alternative portfolios (*Source* Reilly and Brown 1997, p. 271)

steadily decreases as you move upward. This shows that adding equal increments of risk as the investor moves up the curve gives diminishing increments of expected return. It means that the efficient frontier is a concave function in the expected return standard deviation space that extends from the minimum variance portfolio to the maximum return one (Elton and Gruber 1995, p. 84).

Every investor can choose a point along the efficient frontier based on his or her utility function and risk awareness. What is important is that no portfolio on the efficient frontier can dominate any other portfolio on the efficient frontier; thus, all of them have different rates of return and risk characteristics. Because each investor's risk-return utility function differs, an individual investor's portfolio choice will be different from others.

2.4 The Single-Index Model

After outlining the basis of Markowitz's portfolio theory, it is important to keep in mind that the results of asset allocation entirely depend on the data being implemented. The first problem is the simplification of the amount and type of input data required to perform the portfolio analysis. The second problem is the simplification of the computational procedure because, for each security, the expected return and standard deviation have to be estimated, not to mention the correlation coefficients among the entire set of assets. The most widely used simplification of portfolio theory is the **single-index model**[1] proposed by William Sharpe (1963).

Sharpe invented a practical application of Markowitz's portfolio analysis technique after casually observing stock prices. He noticed that when the market goes up (as measured by the stock market index), most stocks tend to increase in price, and when the market goes down, most stocks decrease in price. Those movements reveal that one reason asset returns might be correlated is the common response to market changes, which could be shown by the return of the stock market index. Consequently, it is possible to reduce the number of correlation coefficients by assuming that stock returns can be described by a single-index market model. According to this model, returns on a security can be represented by the performance of a single-factor-market index. The formula of the model can be written as follows (Elton and Gruber 1995, pp. 130–131):

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i;$$

where:

R_i—The rate of return for asset i.

α_i—The component of the i-th security return that is independent of the market index,

β_i—The slope coefficient that relates the return of the i-th security to the return of the market index,

R_m—The rate of return for the aggregate stock market index,

$\mathcal{E}_i$—Random variable, $E(\mathcal{E}_i) = 0$.

The new, crucial measure is **beta**, and it is a measure of the sensitivity of a stock to market movements. The use of a single-index market model calls for estimates of the beta parameter for individual stocks that could potentially be included in a portfolio. The single-index market model is mostly used to estimate historical beta parameters, which can be used as an estimate of a future beta. There is evidence that historical betas provide useful information for future investments. To estimate the risk measured by beta, investors use the regression model. The procedure is to plot R_i versus R_m to obtain a scatter of points; each one represents the return on a particular stock and the return on the market. The next step is to fit the straight line to the data that minimized the sum of the squared deviation from the line in the vertical direction. The slope of the line is the best estimate of beta over the period to which the line was fit, and the intercept is the estimate of alpha. This regression line is called

[1]There is a distinction between the single-index model and the market model. The market model is identical to the single-index model except the assumption that cov(e_i,e_j) = 0 is not made.

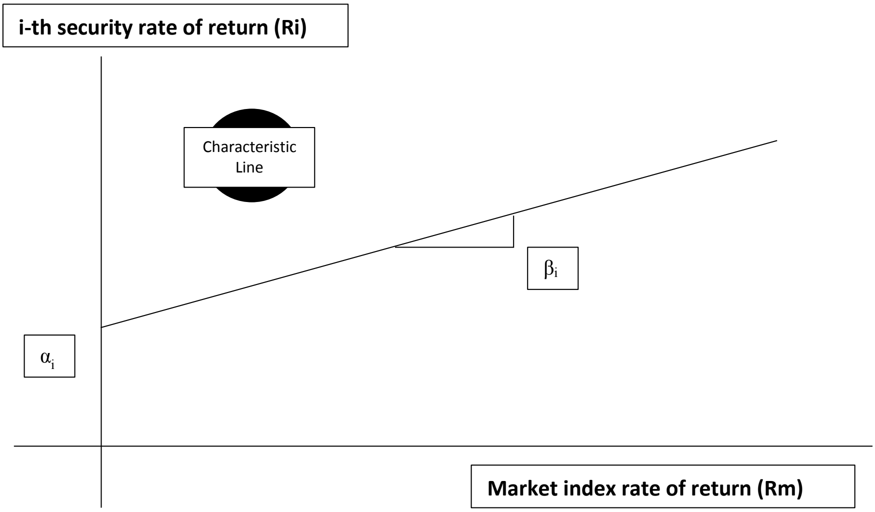

Fig. 2.3 Security characteristic line (*Source* Elton and Gruber 1995, p. 138)

the **security characteristic line**. It is defined as the regression line of best fit through a scatter plot of rates of return for individual risky stock and for the market portfolio over a designated period (see Fig. 2.3).

As can be seen from Fig. 2.3, beta is a measure of a stock's volatility relative to the overall market. The beta parameter is treated as an indicator of risk, and the value of the beta could be interpreted as a measure of single stock risk:

$0 < \beta < 1$—a beta of less than one indicates that the stock return moves less than the market return; there is a lower systematic risk than the market. Defensive stocks have a beta of less than one.

B = 1—a beta equal to one indicates that the stock return is the same as the market return.

B > 1—a beta greater than one indicates stock return moves larger than the market return; there is a higher systematic risk than the market. Aggressive stocks have a beta greater than one.

Beta is a measure of risk because it relates the covariance of any asset with the variance of the market portfolio. Another basic formula to calculate the beta parameter can be given as (Periasamy 2009, p. 7.33):

$$\beta_i = \frac{\mathrm{cov}_{i,m}}{\sigma_m^2};$$

where:

β_i—The i-th stock beta parameter,

$\mathrm{cov}_{i,m}$—Covariance of the i-th stock with the market,

σ_m^2—Variance of the market returns.

The beta that measures relative risk in finance, which most investors estimate, is subject to errors. Furthermore, the entire process of estimation is complicated by the fact that betas are not perfectly stationary over time. Numerous studies have examined the stability of beta and reached similar conclusions; beta is not stable for individual stocks, but for portfolios, its stability increases dramatically (Levy 1971). Marshall Blume (1971) similarly indicated that beta coefficients were highly stable for portfolios containing a large number of securities but unstable for individual stocks. The beta parameter is made to measure the stock's risk, which is related to many economic factors that vary over the cycle, so it is vulnerable to change. Blume proposed a scheme to correct the estimated beta parameters by directly measuring the adjustment toward one and assuming that adjustment in one period is a good estimate for the adjacent one.

In practice, there are several issues that can influence the beta estimates, and each investor should be aware that they exist.[2] The first problem is the selection of a market index. In fact, there are no indices that measure the market portfolio. Many equity market indices measure domestic or international stock market performance, but they are not comprehensive. The most widely used indices for beta estimation are the S&P500 or EURO STOXX, but they include only a subset of stocks that are traded in the USA or European stock exchanges.

The second problem is the choice of period. In choosing a period for beta estimation, it is vital to be aware of the trade-off effect. By going further back in time, an investor gets more observations, but this might be offset by changes in the company's characteristics. The best solution is to select a period that is relatively stable in terms of a firm's business and financial development.

The third problem is the choice of the return interval, which can affect the beta estimates. Stock returns can be measured daily, weekly, monthly, quarterly, or even annually, depending on data availability. Using short time intervals increases the number of observations, but when there is non-trading, the beta estimates could be affected. By contrast, longer return intervals result in few observations, and the information from the market is incomplete. A consequence of different choices in the above-mentioned market index, period, and return interval is that the individual investors can obtain different

[2] Read more: Dębski et al. (2018, pp. 5–16).

beta coefficient estimates for the same companies and make other investment choices (Damodaran 1999).

Many attempts have been made to incorporate more data than only returns to estimate beta coefficients. One idea is to relate the beta parameter to fundamental company variables, such as dividend payout, asset growth, liquidity, and many more (Beaver et al. 1970). In addition, another idea was to combine the historical beta and the fundamental beta (Rosenberg and Guy 1976), and implement a dummy variable in the regression model to capture differences in the beta parameters in different industries (Rosenberg and Marathe 1975).

2.5 The Capital Asset Pricing Model (CAPM)

Earlier, we explained how an individual investor should act to select the optimum portfolio following Markowitz's theory. If we assume that all investors behave according to portfolio theory rules, it is possible to determine how the aggregate of investors will behave and how the prices of securities are set. One major theory that explains the valuation of risky assets is capital market theory, which extends portfolio theory and proposes a model for pricing all risky assets. The main idea of this theory is the **Capital Asset Pricing Model (CAPM)**, which enables investors to determine the required rate of return for any risky asset in efficient markets (Sharpe 1964; Lintner 1965; Mossin 1966).[3]

The CAPM is built under a set of assumptions to better explain the valuation of risky securities. It is also built on the Markowitz portfolio model, so it requires the same assumptions and some additional ones (Elton and Gruber 1995, p. 295):

- There are no transaction costs.
- Assets are infinitely divisible.
- There are no taxes.
- Individual investors cannot affect the price of a stock through their buying or selling actions.
- Investors make decisions solely in terms of expected values and standard deviation on their returns on their portfolio.
- Unlimited short sales are allowed.
- Investors can lend or borrow any amount of funds at a risk-free rate.

[3]CAPM was invented by William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966) independently.

- Investors have homogeneous expectations regarding necessary inputs—expected returns and standard deviation of returns.
- There is no inflation, or inflation is fully anticipated.
- All assets are markable.

Some of these assumptions may be considered unrealistic, but relaxing many of them would have only a minor impact on the model and its conclusions. This theory is regarded as very useful in explaining the rates of return on a wide variety of risky assets.

One of the above-mentioned factors that is very important, and that allowed portfolio theory to develop, is the concept of risk-free rate (see Fig. 2.5). Following to Markowitz's model, several authors considered the assumption of a risk-free asset with no variance. This asset provides a risk-free rate of return, which lies on the vertical axis of the portfolio graph. We assume that the risk-free asset expected return is entirely certain, so the variance or standard deviation of return is zero. This return is a risk-free rate of return, and it should be equal to the expected long-run growth of the economy (Reilly and Brown 1997, p. 280).

Combining a risk-free asset with the Markowitz portfolio model has important implications for the whole capital market theory. Because the return of a risk-free asset is certain, the covariance of a risk-free asset with any risky asset will always equal zero, like the correlation (see equation on page 39). At this point, it is essential to consider what happens to the average rate of the return and risk (the variance or standard deviation of return) when you join a risk-free asset to a risky asset portfolio. Like the expected return of two risky assets, the expected rate of return for a portfolio is the weighted average of two returns, written as follows:

$$E\left(R_{\text{portfolio}}\right) = w_{\text{RF}}(\text{RFR}) + (1 - w_{\text{RF}})E(R_i);$$

where:

W_{RF}—The proportion of the portfolio invested in the risk-free asset,

$E(R_i)$—The expected rate of return on risky portfolio i.

Risk for a two-asset portfolio, expressed by variance according to the formula present on page 39, is:

$$E\left(\sigma^2_{\text{portfolio}}\right) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_1 w_2 \text{cov}_{1,2};$$

Substituting the risk-free rate for the first security and the risky asset portfolio for the second, the formula is as follows:

$$E\left(\sigma^2_{\text{portfolio}}\right) = w^2_{\text{RF}}\sigma^2_{\text{RF}} + (1 - w_{\text{RF}})^2\sigma^2_2 + 2w_{\text{RF}}(1 - w_{\text{RF}})\text{cov}_{\text{RF},2};$$

As stated before, the variance of the risk-free asset is zero. The correlation between the risk-free asset and the risky portfolio is also zero, and the covariance is also zero. After the adjustments, the formula for variance is:

$$E\left(\sigma^2_{\text{portfolio}}\right) = (1 - w_{\text{RF}})^2\sigma^2_2;$$

The standard deviation is:

$$E\left(\sigma_{\text{portfolio}}\right) = (1 - w_{\text{RF}})\sigma_2;$$

Therefore, the standard deviation of such a portfolio with risk-free assets and risky assets is the linear proportion of the standard deviation of the risky asset portfolio (Reilly and Brown 1997, p. 281). Because the expected return and the standard deviation of return are linear combinations, the graph of possible returns and risk looks like a straight line (see Fig. 2.4).

Figure 2.4 shows a graph with portfolio possibilities when a risk-free asset is combined with risky portfolios on the Markowitz efficient frontier. An

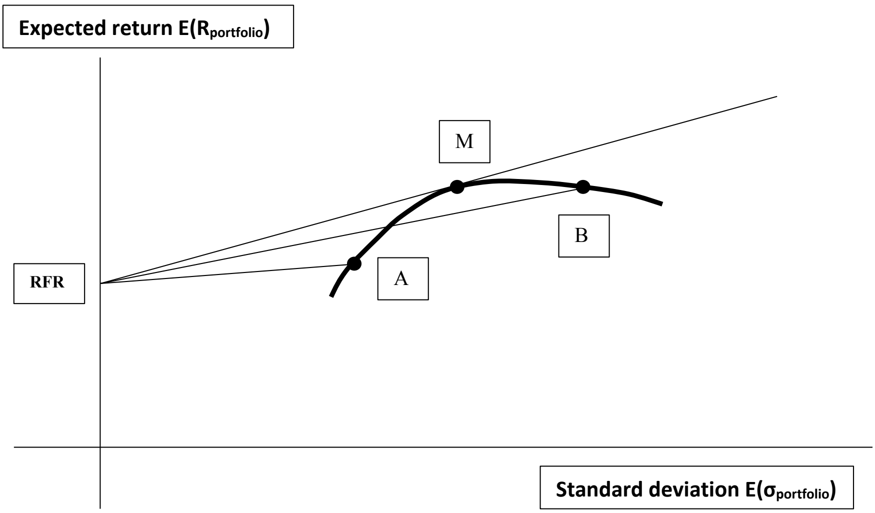

Fig. 2.4 Portfolio possibilities combining a risk-free asset and risky portfolios on the efficient frontier (*Source* Reilly and Brown 1997, p. 282)

investor may attain any point along the straight line between RFR and A by investing money in the risk-free asset W_{RF} and the risky asset portfolio $(1-W_{\mathrm{RF}})$ at point A on the efficient frontier. This portfolio set dominates all the risky asset portfolios on the efficient frontier below point A because some portfolios along the line have equal variance with a higher rate of return than the portfolio on the original efficient frontier. Similarly, an investor can attain any point along RFR and B, and again this combination dominates all portfolio possibilities on the original efficient frontier below point B. The investor can draw a line from the RFR point to the efficient frontier until he reaches the point where the line is a tangent to the frontier at point M. The set of portfolio possibilities along the RFR and M line dominates all portfolios below point M (Reilly and Brown 1997, p. 282).

We can imagine that an investor would like to attain a higher expected return than that available at point M, while accepting a higher risk. One possible way to do it is to add (leverage) to the portfolio by borrowing money at the risk-free rate and investing it in a risky assets portfolio. Consequently, both risk and return increase in a linear fashion along the RFR and M line. This means that an investor can have a new efficient frontier—from the RFR tangent to point M—and it is known as the **capital market line (CML)** (see Fig. 2.5).

As can be seen from Fig. 2.5, the capital market line is straight, implying that all portfolios lying on the CML are perfectly positively correlated. All of

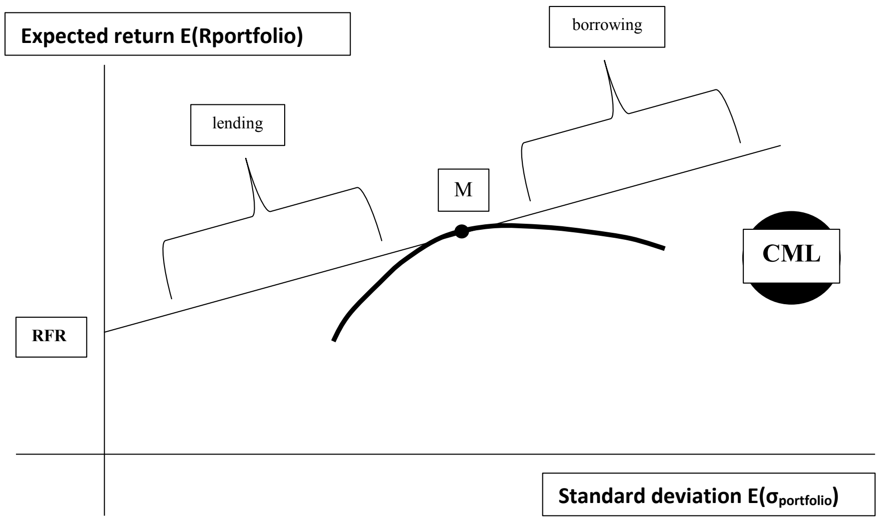

Fig. 2.5 Derivation of the capital market line with lending and borrowing at RFR (*Source* Reilly and Brown 1997, p. 283)

them consist of the risky asset portfolio M and a risk-free asset. Investors have a portfolio partly built on the risk-free rate asset and the risky portfolio M, or they borrow at a risk-free rate and invest those funds in the risky portfolio (Reilly and Brown 1997, p. 283).

Portfolio M lies on the tangent point, and it means that it has the highest portfolio possibility line. Thus, all investors would like to invest their money in portfolio M, borrow, or lend to be somewhere on the capital market line. We can assume that this M portfolio contains all risky assets in proportion to their market value because the whole market is in equilibrium. If all investors hold the same risky portfolio, then, in equilibrium, it must be the **market portfolio**. The market portfolio includes not only stocks but also bonds, derivatives, commodities, and real estate. Each asset is held in the proportion that the market value of that asset represents of the total market value of all assets. The market portfolio contains all risky assets, which may imply that it is completely diversified; each unique risk of any asset is offset by the unique variability of other assets that are part of this portfolio. This unique risk is called an **unsystematic risk** (it is specific to a particular security, sometimes called idiosyncratic risk),[4] and it is fully diversifiable. For every well-diversified portfolio, the unsystematic risk tends toward zero. This means that only **systematic risk**, which is caused by macroeconomic variables, remains in the market portfolio and it is not diversifiable. Systematic risk arises from changes in macro-level factors, like national income, or monetary and fiscal policy, which affect the overall market. This systematic risk, measured by the standard deviation of the returns of the market portfolio, changes over time with macroeconomic variables that affect the valuation of all risk assets. To sum up, the total risk of each security can be broken down into two parts: market risk (systematic), which is proportional to the risk of the market portfolio, and specific risk (unsystematic), which is uncorrelated with the market risk, which is fully diversifiable (see Fig. 2.6).

The capital market line concept leads all investors to build the same risky asset portfolio, called the market portfolio. With different risk preferences, individual investors have a different position on the CML based on financing decisions. If the investor is relatively risk-averse, he will lend part of the portfolio at the RFR by buying some risk-free assets and investing the rest in the market portfolio. In contrast, if the investor is less risk-averse, he can borrow funds at the RFR and invest everything in the market portfolio. As proven earlier, portfolios on the CML dominate other portfolios, and the CML is

[4] For example, labor strike or technological breakthrough.

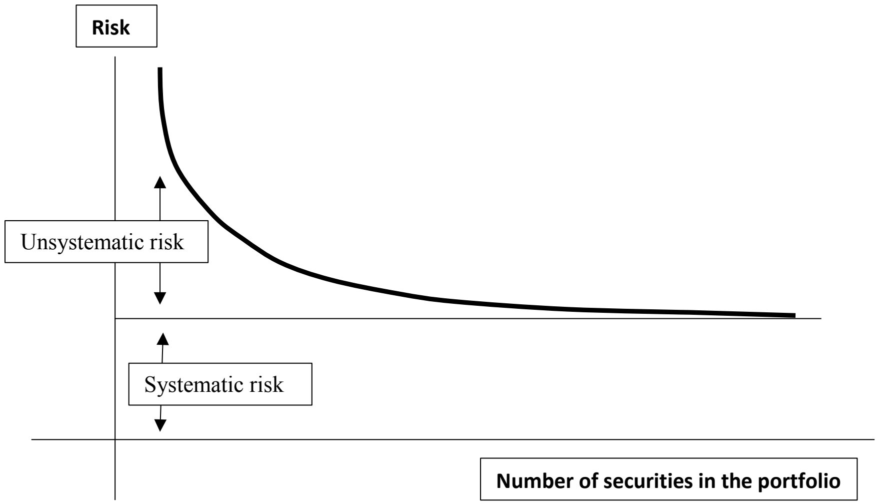

Fig. 2.6 Systematic and unsystematic risk (*Source* Siddaiah 2009, p. 392)

the efficient frontier. James Tobin (1958) defined this division of the investment decision as the **separation theorem**. The separation theorem claims that everyone should hold a portfolio of risky assets—the market portfolio—made up of all the assets traded, and adjust their risk preferences by putting some of the funds in risk-free assets (Solnik 1988).

Now it is time to consider what the measure of risk is for the capital market line. As stated earlier, the relevant risk measure for a risky asset is the covariance with the market portfolio. It was first discussed in Markowitz's portfolio model, where it was noted that the relevant risk for an investor who adds securities to a portfolio is their average covariance with all other assets in the portfolio. Later, it was proven that the only relevant portfolio is the market portfolio. Consequently, these two findings show that the only consideration for an individual risky asset is its average covariance with all the risky assets in the market portfolio or the asset's covariance with the market portfolio (Reilly and Brown 1997, p. 286).

As previously stated, an asset's covariance with the market portfolio emerged as a relevant risk measure; therefore, now it is time to determine an appropriate expected rate of return on a risky asset. This measure is critical because it enables you to value an asset and compare this estimated rate of return to the required rate of return implied by the Capital Asset Pricing Model and stipulate whether it is undervalued or overvalued. The visual representation of the relation between risk and the required rate of return of an asset is the **security market line (SML)** (see Fig. 2.7).

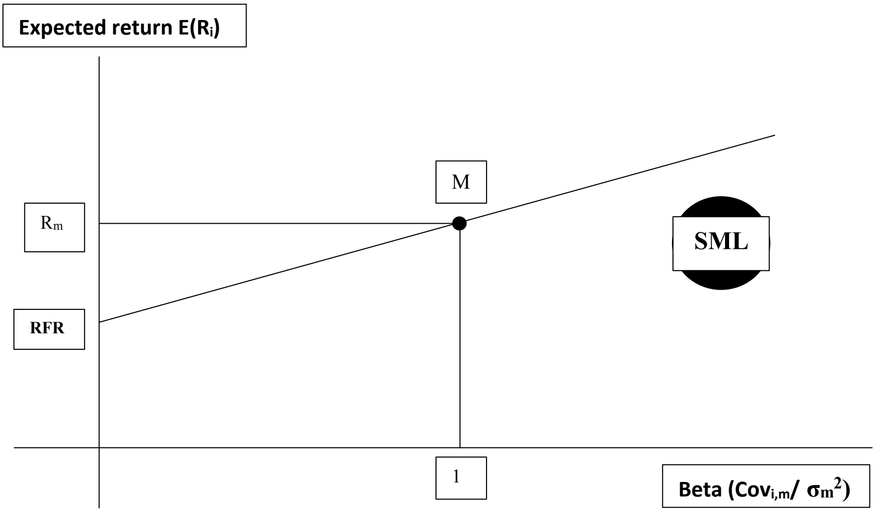

Fig. 2.7 Security market line (*Source* Reilly and Brown 1997, p. 283)

We already know that the relevant measure of risk for an individual risky asset is its covariance with the market portfolio ($\mathrm{Cov}_{i,m}$). The return for the market portfolio should be consistent with its own risk (R_m), which is the covariance of the market with itself, the covariance for any asset with itself its variance σ_m^2. In turn, the equation for the risk-return line is as follows:

$$E(R_i) = \mathrm{RFR} + \frac{R_m - \mathrm{RFR}}{\sigma_m^2}\left(\mathrm{cov}_{i,m}\right) = \mathrm{RFR} + \frac{\mathrm{cov}_{i,m}}{\sigma_m^2}(R_m - \mathrm{RFR});$$

If we define $\mathrm{Cov}_{i,m}/\sigma_m^2$ as the beta parameter, the equation can be written as:

$$E(R_i) = \mathrm{RFR} + \beta_i(R_m - \mathrm{RFR});$$

The equation of SML explains that the expected rate of return for a risky asset is determined by the RFR plus a risk premium for the individual asset. The risk premium is defined as a product of the systematic risk of an asset (beta) and the prevailing market risk premium (Reilly and Brown 1997, p. 288).

In market equilibrium, all assets and portfolios should be plotted on the security market line (SML). This means that their estimated rates of return are consistent with their level of systematic risk. If an asset with an estimated rate of return that plots above the SML is perceived as **underpriced**, it means that an investor would receive a rate of return that is above its required rate of return based on its systematic risk. In contrast, if an asset with an estimated rate of return that plots below the SML is perceived as **overpriced**, it means that an investor would receive a rate of return that is below its required rate of return based on its systematic risk. In an efficient market in equilibrium, an investor cannot expect any asset to plot off the SML because all securities should provide returns that are equal to their required rates of return (Reilly and Brown 1997, p. 290). A direct implication of CAPM is that the equilibrium expected return of an asset should be equal to the risk-free rate plus a risk premium that is proportional to the covariance of the asset return with the return on the market portfolio, which is the famous measure of the systematic risk beta coefficient (Solnik 1988).

2.6 International Diversification and the Reduction of Risk

Earlier, we showed that the risk of a portfolio is measured by the ratio of the variance of the portfolio's return relative to the variance of the market return.[5] This ratio is the beta coefficient. When the number of securities in a portfolio increases, the portfolio risk declines rapidly and then asymptotically approaches the level of systematic risk. As a result, the total risk of the portfolio is composed of a systematic risk and an unsystematic risk, which could be fully diversifiable. A fully diversified domestic portfolio has a beta parameter equal to one, which is the market risk.

Now, it is time to explain what happens when we attempt to reduce risk by investing in more than one country. The opportunity set of possible investments is growing extensively. Internationally, more assets and more kinds of financial products are available. The indication of the gain of including foreign stocks in a portfolio was presented by Bruno Solnik (1974a). He computed the risk of randomly selected international portfolios and showed that an international portfolio of stocks has about half of as much risk as a portfolio of the same size containing only US stocks (see Fig. 2.8).

As we can see from Fig. 2.8, there are incremental gains from diversifying both domestically and internationally. The risk of a US portfolio is 27% of the risk of a typical security; the risk of an internationally diversified portfolio (the lower line) is 12% of the risk of a typical security. This means that, for an American investor, the international portfolio's risk is lower than the domestic one. This relation arises because the returns from international markets are

[5] More formally, the covariance between portfolio's return and the variance of market portfolio return.

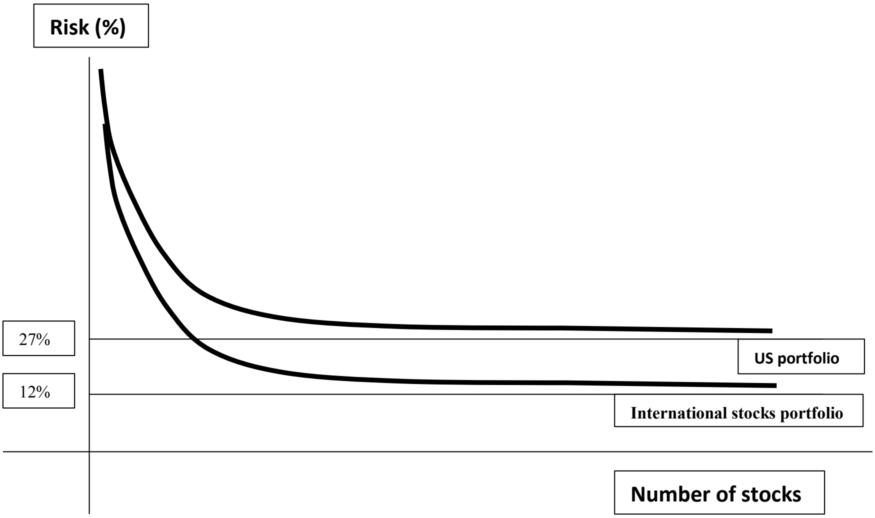

Fig. 2.8 International diversification gain (*Source* Solnik 1974a, pp. 45–54)

not perfectly correlated. Internationally diversified portfolios are the same in principle because the idea is to find stocks that are not perfectly correlated in order to reduce the portfolio's risk (Eiteman et al. 2016, pp. 381–382).

While there is a gain from **international diversification** because of the independent returns between domestic and international assets, there is a possibility of added risk from unanticipated changes in exchange rates. The investor has to acquire an additional asset: currency. In principle, it is one asset, but it is two in the expected return and risk. The risk associated with international diversification, including currency risk, is more complex than domestic diversification. However, when measured in terms of the local currency, it is crucial to decide whether the gains from imperfect correlations between stock returns more than compensate for the exchange rate risk. This additional risk factor depends on both the volatility of exchange rates and the correlation of exchange rates and security prices. It is also important whether the stocks come from one foreign country or more (Levi 2009).

Some investors may observe that, due to international economic integration and the globalization of the financial markets, the benefits of international diversification have declined in recent years. Nowadays, national economies are closely linked due to transnational companies and organizations, informational technology, cross-border investments, and the convertibility of major currencies. This closeness of the world's economies is strengthened by their interdependence, and the benefits of international diversification may decrease. This observation was verified by Kevin Chang and Christian Leonhard (2007), who showed that the benefits of international diversification have remained stable in recent years. This interesting implication was explained by the fact that the globalization process has a mainly regional effect, like the EU, and when the variance composition is examined at a global scale, the gain still exists. It is more likely that much closer economic links within an economic region reduce the gains of international diversification, and it has been proven that regional diversification within the EU has become less effective.

Consequently, the high degree of independence between regions, not countries, is the source of diversification opportunities for internationally oriented investors. However, we have to remember that even closely linked countries may not be closely correlated because of different business cycles and levels of economic development, e.g., advanced, emerging, or frontier. Nowadays, international diversification is still effective at the regional level. To conclude, it is vital to identify international stock markets or correlation coefficients of economic regions to determine the countries and regions whose stock prices move together and those who move in opposite and unrelated directions.

2.7 The Efficient Frontier for the International Investor

Based on the above, international investors can possibly obtain a better risk-return trade-off in comparison with domestic investors. Expanding the universe of assets should lead to higher returns for the same level of risk or less risk for the same level of expected return. For the presentation of the international portfolio's diversification, the efficient frontier has to be mentioned to explain the gains from building the portfolio on the global marketplace.

Herbert Grubel (1968) was one of the first to propose a model consisting of two countries that can both interchangeably invest in their bonds, showing gains on the international efficient frontier. Further studies have extended this research, showing a positive diversification impact on a portfolio's risk (Levy and Sarnat 1970).

We already know that the efficient frontier represents portfolios that have a minimum expected risk for each level of expected return. However, with the international environment, the efficient frontier shifts to the left of the purely domestic environment (see Fig. 2.9).

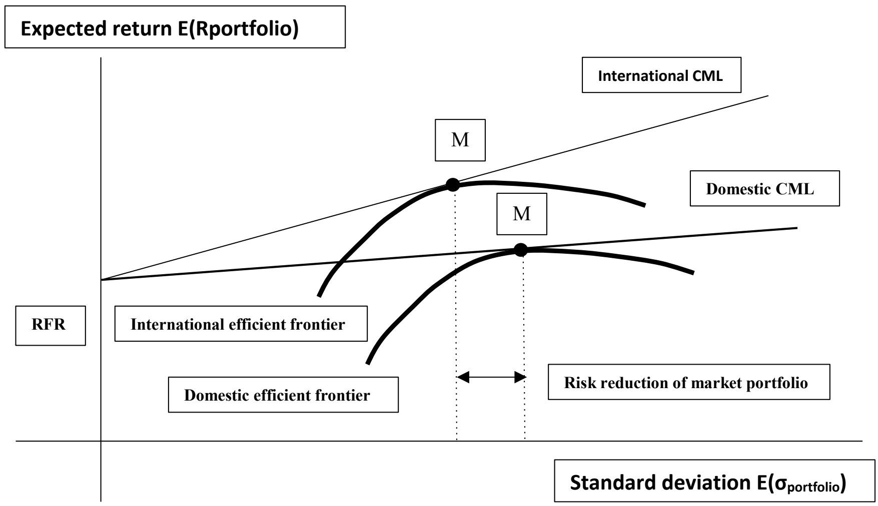

Fig. 2.9 International and domestic efficient frontiers (*Source* Siddaiah 2009, p. 395)

As can be seen from Fig. 2.9, the curvature of the international efficient frontier increases; the greater the curvature, the greater the risk reduction for the given level of return. We can assume that an internationally diversified portfolio provides a lower risk for each level of the expected return. The new CML with the steeper slope starts from the same risk-free rate and goes through the tangent point along the internationally diversified efficient frontier. So, we can assume that the international market portfolio is superior to the domestic market portfolio, giving a higher expected return and lower risk (Siddaiah 2009, p. 395).

From an application perspective, it is useful to know which countries' portfolios lie on the efficient frontier and provide better diversification alternatives, and if such relationships are time-invariant. This research question was explicitly explained by Abuaf et al. (2019). Their study was made from the US perspective, so they stated that countries that are more economically independent from the USA provide better diversification for American investors. Securities issued by Mexican and Chinese companies appeared to be the best diversifiers of US portfolios.

2.8 International Capital Asset Pricing Model (ICAPM)

The traditional CAPM discussed earlier proposes a clear theory of asset pricing in the domestic environment. This analysis is related to the return and risk of the market and portfolio under a single currency. The theoretical framework of the CAPM can be easily extended to the international market. Recently, investors have witnessed the extension of investments in foreign countries, and they are aware that they need explanations for factors that affect expected returns. At present, investors build portfolios in different financial markets all over the world. It is important to assume that the assets are priced in an internationally integrated capital market because the expected returns on foreign stock are appropriate for the risk of these stocks in an internationally diversified portfolio. The **International CAPM** could be a single-factor ICAPM or a multiple-factor ICAPM (Solnik 1974b). The equation of a single-factor ICAPM for the risk-return line is as follows:

$$E(R_w) = RFR_w + \beta_w(R_{\mathrm{wm}} - RFR_w);$$

where:

$E(R_w)$—Expected return for the risky asset as part of the international portfolio,

RFR_w—World risk-free interest rate,

β_w—International or global beta parameter,

R_{wm}—World market index.

The first problem is to define the risk-free element in the model, which is usually the risk-free rate in the currency in which the overall returns are being measured. The next step is to evaluate the beta parameter, in which case it is advisable to use the world index. There is a whole range of world indices calculated by MSCI, S&P, the FTSE, and EURO STOXX. The beta parameter for this particular model indicates the world risk premium regarding the world index (Madura and Fox 2017, p. 589).

The single-factor International Capital Pricing Model is based on the following assumptions:

- The world market portfolio is stable.
- Purchasing Power Parity holds all over the whole period.
- All investors have the same consumption basket.
- Investors hold a portfolio of risk-free assets in their own currency and the unchanged world market portfolio,

- All investors are homogeneous, and they hold every security in the market portfolio (Siddaiah 2009, p. 396).

The above single-factor CAPM variation does not capture foreign exchange risk and it is called the global CAPM applied by Stulz (1995a, b). If Purchasing Power Parity holds, a percentage depreciation of the domestic currency is offset by the same increase in domestic prices. In that case, the return of foreign assets is not exposed to exchange risk, meaning that the returns are subject only to the global market factor, and all assets are priced correctly (Siddaiah 2009, p. 396). Single-factor ICAPM has the same structure that domestic CAPM with the global market index and it is simpler to use than the multi-factor model.

The next step is the violation of the Purchasing Power Parity. It means that investors in different countries realize different real returns for a given asset when PPP does not hold and it is connected with exchange rate risk exposure. International CAPM implies that investing in foreign assets, measured in the home currency, is exposed to two different kinds of risk: the sensitivity of the domestic country index to a global market portfolio and the performance of a domestic currency against foreign currency. When the domestic market portfolio does not move in line with the world market, the beta coefficient evaluated on the domestic CAPM will be different from the beta evaluated on the International CAPM. It means that the International CAPM established the condition under which integrated financial markets are in equilibrium. If the world's financial markets were not integrated, the world market portfolio would not exist. The markets are considered integrated if all assets with the same risk are priced equally; if not, those world's markets are segmented.

The second issue is currency risk. If an investor holds foreign assets, the return in domestic currency is influenced by the exchange rate. From the investor's perspective when the Purchasing Power Parity does not hold, it means that investors from different countries expect different returns for the same assets.[6] If the market is in equilibrium, the expected return on any security, denominated in the domestic currency, is equal to the risk-free domestic return plus the risk premium for the exposure to the global market and the exchange rate risk. The formula can be written as follows:

$$E(R_w) = RFR_w + \beta_w(R_{\mathrm{wm}} - RFR_w) + \beta_w(FCRP);$$

FCRP—Foreign currency risk premium.

[6]The offset mechanism does not work.

This formula depicts a simple multi-factor international CAPM but this model could be relatively complex in terms of estimating risk coefficients and risk premium, see Dumas and Solnik (1995).

Another solution, if Purchasing Power Parity does not hold, is to hedge foreign assets against exchange the rate risk using available derivatives. When an investor cannot hedge against the currency risk, the return on international investments is influenced by changes in exchange rates. Foreign currencies might be used for financing investments and investments per se.[7] If the foreign currency depreciates against the domestic currency, the cost of financing the investment will be low, and if the foreign currency appreciates, the cost will be high. The same rule applies to foreign investments. Appreciation of the foreign currency would yield high effective returns for the investor, and depreciation would yield low effective returns.

As previously stated, investors can use a portfolio of currencies to reduce exchange rate risk aimed at financing or investments. Foreign financing with a highly diversified portfolio of currencies could be less costly than financing with one or a few currencies. If foreign interest rates are lower, it is unlikely that all currencies appreciate enough to offset the benefits of lower interest rates. Exchange rates do not usually move in the same direction if they are not highly correlated. The same is true with investments in a diversified portfolio of many currencies; it may be more rewarding than investing in a single currency (Siddaiah 2009, p. 398).

To summarize, rather than considering only the domestic market, the International CAPM takes the single global market concept as a market. That idea of the International CAPM was extended by explaining international relations between the prices of securities through a multiple-factor specification that takes into account both national and international factors (Solnik 1974b). The multiple-factor International Asset Pricing Model assumes that investors differ not only regarding risk aversion but also consumption patterns. Regarding the multi-country model, each stock is influenced by the domestic market factor, which in turn is influenced by the single world market factor. This means that all stocks are indirectly influenced by the global factor through the national factor. Hence, a stock risk could be divided into risks caused by the global factor and the internal, country factor. The sensitivity of the stock to the world factor results in many economic relations, like the degree of international trade and investments, monetary policy, and capital flows. Therefore, these led to multiple-factor solutions like the International Capital Asset Pricing Model for the pricing of single assets that are

[7] Investment per se means buying and selling currencies.

Table 2.1 Comparison between the international and domestic capital asset pricing models

International CAPM	Domestic CAPM
Risk and return are influenced by different currencies	Risk and return are influenced by one country's currency
Investors have homogeneous expectations toward return and risk	Investors have different expectations toward return and risk
Portfolio efficiency is influenced by different currencies	Portfolio efficiency is influenced by one currency
The market is considered a whole, and it has linked with other countries	The market has segments within a country

Source Naderi et al. (2012, p. 5)

part of international portfolios, and which take into consideration inflation, exchange rates, and forward premiums.

In theory, the primary distinction between the standard CAPM and the ICAPM is the definition of the market (the global portfolio index) and the calculation of the beta parameter. There are also numerous model variations that take into account different factors. To compare the standard CAPM model and International CAPM, it is possible to point out four major differences (see Table 2.1).

Another problem could be the practical application of the International multiple-factor CAPM. It requires defining the world's risk-free rate and making assumptions about the preferences of investors from different countries. Many studies have been made, and the empirical results of international portfolio diversification can be concluded in three propositions: (1) country (region) selection is better than security selection; (2) do not hedge against currency risk when investing in emerging markets; and (3) the degree of segmentation of international markets is still considerable. Presently, the degree of national market integration with the global market is difficult and subjective. To use those solutions in practical investing, the best option is to select the most segmented country markets and not hedge against currency risk (Thalassinos and Kiriazidis 2003).

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