Variational Identification of the Underwater Pollution Source Power

Sergey Vladimirovich Kochergin and Vladimir Vladimirovich Fomin

Abstract This article is devoted to the development of a variational procedure for identifying the rate of water discharge at the outlet of an underwater source, as well as the analysis of the sensitivity of the algorithm to the level of random noise in the measurement data. The estimation of the input parameters of the problem was carried out on the basis of the iterative procedure of minimization of the quadratic functional. As a result of numerical experiments, the efficiency of the linearization algorithm is shown.

Keywords Functional minimization · Parameter identification · Linearization method · Numerical simulation · Problem in variations · Measurement data assimilation

1 Introduction

The ecology situation in marine coastal zones is highly dependent on the damping of pollutants from underwater sources. Data about the state in the water area of source can be obtained on the basis of mathematical modeling and assimilation of contact and remote measurement data (Bondur and Grebenyuk 2001; Bondur 2005, 2011).

Due to implementing numerical models, the problem of identifying model parameters from measurement data naturally arises. Known methods of searching for optimal parameters consist of minimization of cost functions, which determine the residuals between values of concentration and measurement data. One of the effective procedures for finding optimal parameters is the estimation algorithm based on the linearization method (Alifanov et al. 1988; Gorsky 1984).

S. V. Kochergin (✉) · V. V. Fomin
Marine Hydrophysical Institute RAS, Sevastopol 299029, Russian Federation
e-mail: vskocher@gmail.com

T. Chaplina (ed.), *Processes in GeoMedia—Volume II*, Springer Geology,
https://doi.org/10.1007/978-3-030-53521-6_8

The purpose of this work is to build and test the variational procedure for finding the input parameters of the transfer model. The paper uses a dynamic model (Ivanov and Fomin 2016), in which the parameter to be identified is the flow rate (w_p) at the vent of the underwater source. In work (Kochergin and Fomin 2019) identification of pollution concentration (C_p) for the investigated model is made.

2 Dynamic Model

With the help of baroclinic nonlinear model (Ivanov and Fomin 2008, 2016; Bondur et al. 2018) fields of velocity, impurity concentration, temperature and salinity was calculated. The three-dimensional equations of ocean dynamics in σ-coordinate with the Boussinesque approximation and hydrostatics approximations have following form (summation is assumed by repeated indices α and β from 1 to 2):

$$\frac{\partial}{\partial t}(Du_\alpha) + \Lambda u_\alpha + \varepsilon_{\alpha\beta} f D u_\beta + gD\frac{\partial \eta}{\partial x_\alpha} + DB_\alpha = \frac{\partial}{\partial x_\beta}\left(D\tau_{\alpha\beta}\right) + \frac{\partial}{\partial \sigma}\left(\frac{K_M}{D}\frac{\partial u_\alpha}{\partial \sigma}\right), \tag{1}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x_\alpha}(Du_\alpha) + \frac{\partial w_*}{\partial \sigma} = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(DT) + \Lambda T = \frac{\partial}{\partial x_\beta}\left(A_T\frac{\partial T}{\partial x_\beta}\right) + \frac{\partial}{\partial \sigma}\left(\frac{K_T}{D}\frac{\partial T}{\partial \sigma}\right), \tag{3}$$

$$\frac{\partial}{\partial t}(DS) + \Lambda S = \frac{\partial}{\partial x_\beta}\left(A_S\frac{\partial S}{\partial x_\beta}\right) + \frac{\partial}{\partial \sigma}\left(\frac{K_S}{D}\frac{\partial S}{\partial \sigma}\right), \tag{4}$$

$$\frac{\partial}{\partial t}(DC) + \Lambda C = \frac{\partial}{\partial x_\beta}\left(A_C\frac{\partial C}{\partial x_\beta}\right) + \frac{\partial}{\partial \sigma}\left(\frac{K_C}{D}\frac{\partial C}{\partial \sigma}\right), \tag{5}$$

$$\rho = \rho(T, S), \tag{6}$$

$$\Lambda\varphi = \frac{\partial}{\partial x_\beta}\left(Du_\beta\varphi\right) + \frac{\partial}{\partial \sigma}(w_*\varphi),\ B_\alpha = \frac{g}{\rho_0}\left(\frac{\partial}{\partial x_\alpha}D\int_\sigma^0 \rho d\sigma' + \sigma\frac{\partial D}{\partial x_\alpha}\rho\right), \tag{7}$$

$$\tau_{\alpha\alpha} = 2A_M\frac{\partial u_\alpha}{\partial x_\alpha},\ \tau_{\alpha\beta} = \tau_{\beta\alpha} = A_M\left(\frac{\partial u_\beta}{\partial x_\alpha} + \frac{\partial u_\alpha}{\partial x_\beta}\right), \tag{8}$$

where $(x_1, x_2) = (x, y)$; σ—vertical coordinate, varying from -1 to 0; $D = h_0 + \eta$—dynamic depth; A_M, K_M, A_T, K_T, A_S, K_S, A_C, K_C—coefficients of turbulent viscosity and diffusion; $\tau_{\alpha\beta}$—components of the turbulent stress tensor; f—Coriolis parameter; g—acceleration of free fall; ρ_0—average water density; $\varepsilon_{\alpha\beta} = 0$ at $\alpha = \beta$; $\varepsilon_{12} = -1$; $\varepsilon_{21} = 1$.

The required variables of the system (1)–(8) are: $(u_1, u_2) = (u, v)$—horizontal velocity components, w_*—normal to the surface $\sigma = const$ of the flow velocity component,fields T, S, C and the field of seawater density ρ.

On a free surface $\sigma = 0$, the boundary conditions have the form:

$$w_* = 0, \frac{K_M}{D}\frac{\partial u_\alpha}{\partial \sigma} = 0, \frac{K_T}{D}\frac{\partial T}{\partial \sigma} = 0, \frac{K_S}{D}\frac{\partial S}{\partial \sigma} = 0, \frac{K_C}{D}\frac{\partial C}{\partial \sigma} = 0. \tag{9}$$

Boundary conditions at the bottom outside the source ($\sigma = -1, x \neq x_p, y \neq y_p$) are written as follows:

$$w_* = 0, \frac{K_M}{D}\frac{\partial u_\alpha}{\partial \sigma} = \mu|u|u_\alpha, \frac{K_T}{D}\frac{\partial T}{\partial \sigma} = 0, \frac{K_S}{D}\frac{\partial S}{\partial \sigma} = 0, \frac{K_C}{D}\frac{\partial C}{\partial \sigma} = 0, \tag{10}$$

where $|u| = \sqrt{u_1^2 + u_2^2}$, μ—the bottom friction coefficient.

At the bottom in the outlet area ($\sigma = -1, x = x_p, y = y_p$) boundary conditions can be written as (Bondur et al. 2018):

$$w_* = w_p, \frac{K_M}{D}\frac{\partial u_\alpha}{\partial \sigma} = \mu|u|u_\alpha, \tag{11}$$

$$w_pT - \frac{K_T}{D}\frac{\partial u_\alpha}{\partial \sigma} = w_pT_p, w_pS - \frac{K_S}{D}\frac{\partial S}{\partial \sigma} = w_pS_p, w_pC - \frac{K_C}{D}\frac{\partial C}{\partial \sigma} = w_pC_p. \tag{12}$$

Initial conditions have the following form:

$$u = U_0, v = w = 0, \eta = 0, T = T_0(\sigma), S = S_0(\sigma), C = 0. \tag{13}$$

Here u, v, w—components of the velocities along x, y, σ respectively; U_0 is a constant depth rate of background currents; T, S—the temperature and salinity of water; $T_0(\sigma)$—background temperature distribution; $S_0(\sigma)$—background distribution of salinity; C—concentration of contaminants Q_p—water consumption; d—the horizontal size of the source; $w_p = Q_p/d^2$—the rate of outflow of water; T_p, S_p—the temperature and salinity of the inflowing water; C_p—impurity concentration at the exit of the source.

The numerical procedure for solving the system of equations is described in detail in Ivanov and Fomin (2008). The Smagorinsky formula is used to calculate the coefficients of horizontal turbulent diffusion (Smagorinsky 1963). Mellor-Yamada model (Mellor and Yamada 1982) is used for determined the coefficients of vertical turbulent viscosity and diffusion. Advective terms approximation in the model is done on the basis of TVD schemes (Harten 1984; Fomin 2006).

The model is implemented on a time period $[0, t_0]$ for a rectangular area $\Omega = \{0 \le x \le L, 0 \le y \le L, 0 \le \sigma \le L\}$; with a free surface and open lateral border. By $t > 0$ at the bottom of the pool ($\sigma = -1$) in area Ω_p begins to operate the source of contamination with the following parameters: w_p—the rate of outflow of water from the source; T_p,S_p—the temperature and salinity of the flowing water; C_p—concentration of impurity at the exit of the source.

3 Algorithm for Identifying the Rate of Flow from the Outlet

Let we need to identify w_p—the velocity of source flow. Method of linearization (Alifanov et al. 1988) can be used for solving of this task. Following (Kochergin and Kochergin 2017), we have identified in accordance with the model (1)–(13) the problem in variations with constant A_M, K_T, K_S, K_C:

$$\frac{\partial}{\partial t}(D\delta u_\alpha) + \Lambda\delta u_\alpha + \delta\Lambda(u_\alpha) + \varepsilon_{\alpha\beta} f D\delta u_\beta + gD\frac{\partial\delta\eta}{\partial x_\alpha} + D\delta B_\alpha$$
$$= \frac{\partial}{\partial x_\beta}\left(D\delta\tau_{\alpha\beta}\right) + \frac{\partial}{\partial\sigma}\left(\frac{K_M}{D}\frac{\partial\delta u_\alpha}{\partial\sigma}\right),$$
$$\frac{\partial\delta\eta}{\partial t} + \frac{\partial}{\partial x_\alpha}(D\delta u_\alpha) + \frac{\partial\delta w_*}{\partial\sigma} = 0,$$
$$\frac{\partial}{\partial t}(D\delta T) + \Lambda(\delta T) + \delta\Lambda(T) - \frac{\partial}{\partial x_\beta}\left(A_T\frac{\partial\delta T}{\partial x_\beta}\right) - \frac{\partial}{\partial\sigma}\left(\frac{K_T}{D}\frac{\partial\delta T}{\partial\sigma}\right) = 0,$$
$$\frac{\partial}{\partial t}(D\delta S) + \Lambda(\delta S) + \delta\Lambda(S) - \frac{\partial}{\partial x_\beta}\left(A_S\frac{\partial\delta S}{\partial x_\beta}\right) - \frac{\partial}{\partial\sigma}\left(\frac{K_S}{D}\frac{\partial\delta S}{\partial\sigma}\right) = 0,$$
$$\frac{\partial}{\partial t}(D\delta C) + \Lambda(\delta C) + \delta\Lambda(C) - \frac{\partial}{\partial x_\beta}\left(A_C\frac{\partial\delta C}{\partial x_\beta}\right) - \frac{\partial}{\partial\sigma}\left(\frac{K_C}{D}\frac{\partial\delta C}{\partial\sigma}\right) = 0. \quad (14)$$

in (14) the following designations are used:

$$\delta\rho = \delta\rho(T, S), \text{ depends on the type of function } \rho = \rho(T, S)$$
$$\delta\Lambda\varphi = \frac{\partial}{\partial x_\beta}\left(D\delta u_\beta\varphi\right) + \frac{\partial}{\partial\sigma}(\delta w_*\varphi),$$
$$\delta\Lambda\varphi = \frac{\partial}{\partial x_\beta}\left(Du_\beta\varphi\right) + \frac{\partial}{\partial\sigma}(\delta w_*\varphi),$$
$$\delta B_\alpha = \frac{g}{\rho_0}\left(\frac{\partial}{\partial x_\alpha}D\int_\sigma^0 \delta\rho d\sigma' + \sigma\frac{\partial D\delta\rho}{\partial x_\alpha}\right),$$
$$\delta\tau_{\alpha\alpha} = 2A_M\frac{\partial\delta u_\alpha}{\partial x_\alpha}, \delta\tau_{\alpha\beta} = \delta\tau_{\beta\alpha} = A_M\left(\frac{\partial\delta u_\beta}{\partial x_\alpha} + \frac{\partial\delta u_\alpha}{\partial x_\beta}\right). \quad (15)$$

The model uses the boundary conditions, on a free surface:

$$\delta w_* = 0,\ \frac{K_M}{D}\frac{\partial \delta u_\alpha}{\partial \sigma} = 0,\ \frac{K_T}{D}\frac{\partial \delta T}{\partial \sigma} = 0,\ \frac{K_S}{D}\frac{\partial \delta S}{\partial \sigma} = 0,\ \frac{K_C}{D}\frac{\partial \delta C}{\partial \sigma} = 0, \quad (16)$$

outside the source at the bottom ($\sigma = -1,\ x \neq x_p,\ y \neq y_p$):

$$\begin{aligned} &\delta w_* = 0,\ \frac{K_M}{D}\frac{\partial \delta u_\alpha}{\partial \sigma} = \mu|u|\delta u_\alpha + \mu\delta(|u|)u_\alpha, \\ &\frac{K_T}{D}\frac{\partial \delta T}{\partial \sigma} = 0,\ \frac{K_S}{D}\frac{\partial \delta S}{\partial \sigma} = 0,\ \frac{K_C}{D}\frac{\partial \delta C}{\partial \sigma} = 0 \\ &\delta|u| = \frac{1}{2}\left(u_1^2 + u_2^2\right)^{-1/2} \cdot (2u_1\delta u_1 + 2u_2\delta u_2), \end{aligned}$$

at the bottom in the source area ($\sigma = -1,\ x = x_p,\ y = y_p$):

$$\begin{aligned} &\delta w_* = 1,\ \frac{K_M}{D}\frac{\partial \delta u_\alpha}{\partial \sigma} = \mu|u|\delta u_\alpha + \mu\delta(|u|)u_\alpha, \\ &w_p\delta T + T - \frac{K_T}{D}\frac{\partial \delta u_\alpha}{\partial \sigma} = w_p\delta T_p + T_p, \\ &w_p\delta S + \delta S - \frac{K_S}{D}\frac{\partial \delta S}{\partial \sigma} = w_p\delta S_p + S_p, \\ &w_p\delta C - \frac{K_C}{D}\frac{\partial \delta C}{\partial \sigma} = w_p\delta C_p + C_p. \end{aligned} \quad (17)$$

Let the model (14)–(17) decided for period $[0, t_0]$ inside domain Ω. To fined the optimal parameters of model we can find the minimum of cost function:

$$J = \frac{1}{2}\langle P(RC - C_{obs}),\ P(C - C_{obs})\rangle, \quad (18)$$

where $\langle a, b\rangle = \int_0^{t_0}\iiint_\Omega abd\Omega \mathrm{dt}$—the scalar product; C_{obs}—the measured values of C at the given points of the domain Ω at time moment t_0; R—a projection operator to the points of observations; P—the fill operator the residuals field with zeros in the case of lack data measurements.

Let's write the variable as follows:

$$C = \overline{C} + \delta C_{w_p}\left(w_p - w_p^*\right), \quad (19)$$

where $\overline{C}$—some value of the concentration, and w_p^*—its value, to be determined. From (19) in (18) we obtain:

$$J = \frac{1}{2}\left\langle P\left(R\left(\overline{C} + V\left(w_p - w_p^*\right)\right) - C_{obs}\right),\ P\left(R\left(\overline{C} + V\left(w_p - w_p^*\right)\right) - C_{obs}\right)\right\rangle. \quad (20)$$

And due to condition $\frac{\partial J}{\partial C_p} = 0$, we have:

$$w_p^* = w_p + \frac{\langle P(RC - C_{obs}), PRV \rangle}{\langle PRV, PRV \rangle} \tag{21}$$

If data is received from sea surface, formula (21) is also correct. In this case you mast select necessary operators P and R.

The problem of numerical realization of the problem in variations even in the approximation of constant coefficients A_M, K_T, K_S, K_C is quite difficult to solve. Therefore, a simplified identification algorithm for w_p based on the direct modeling method is implemented. Variation δC_p is determined by solving a series of basic problems at different values w_p near the true value $w_p = w_p^*$ by averaging over the ensemble of realizations. After solving the main problem, the required value w_p is specified iteratively.

4 Numerical Experiments and Results

Numerical experiments were carried out for the coastal zone of the Sevastopol city in the area of the Blue Bay, where urban sewage is mainly located. The computational domain had a horizontal size $L = 2$ km, depth $h_0 = 30$ m Fig. 1. Horizontally, uniform rectangular grid was used with a step $d = 20$ m and vertically step equal 1 m was selected. The time step time was 5 s. On the western boundary of the

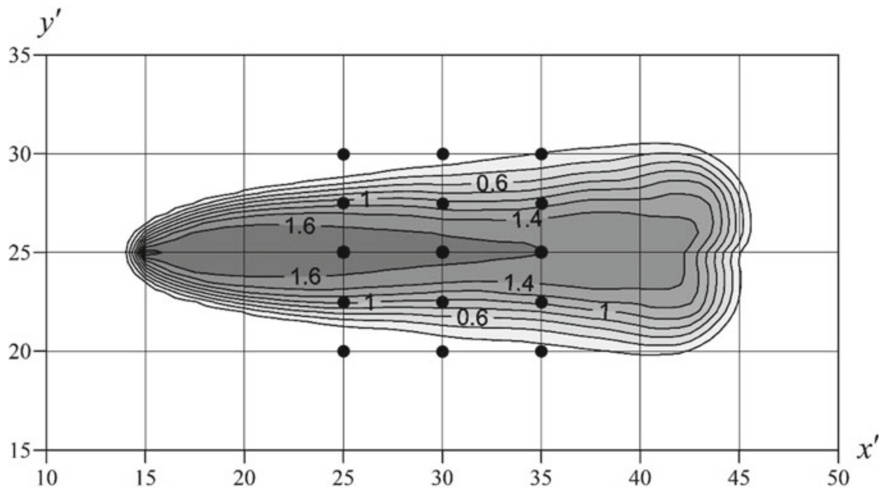

Fig. 1 Relative concentration of the mixture C (%) in the area of the underwater source at $\sigma = -0.3166$ ($z = -9.5$ m)

estimated domain ($x = 0$) was used conditions of the form 13. In the remaining fluid boundaries conditions were used to smooth continue.

The center of the outlet was located at the point with coordinates: $x_p = 600$ m; $y_p = 1000$ m. Horizontal size of the source in x, y axes was equal to the step of the calculated grid d. It was meant that the water flowing from the source is zero salinity ($S_p = 0$), and its temperature is equal to the ambient temperature, i.e. $T_p = T_0(-1)$. The results of sounding in the area of underwater release were used as T_0 and S_0. Due to Morozov et al. (2016), the value of U_0 was taken to be 0.05 m/s.

Initially, a model calculation of the characteristics of the model with a given value C_p^* is 0.05 kg/m^3 was done. The spatial distribution of the polluted water field on a fixed horizon is shown in Fig. 1. Here are the contours of the relative concentration $c = 100\% \cdot C/C_p^*$ for a finite moment in time. Dimensionless horizontal coordinates have the form $x' = x/2d$, $y' = y/2d$. The horizon corresponds to the depth of the layer of the jump in which the impurity is mainly concentrated. A detailed description of the effect of sea water density stratification and background flow velocity on the impurity field is given in Fomin (2006), Kochergin and Kochergin (2017).

Some numerical experiments was implemented to test the parameter w_p identification algorithm. It was believed that the measurements of all fifteen vertical profiles C, at points arranged according to the scheme in Fig. 1, performed simultaneously at t is 4 h. the results of recovery w_p at different noise levels in the input data are shown in Table 1. As you can see, at a noise level of 1.25% and below, the value is restored almost exactly. The greatest decrease of the functional (18) occurs at the first iterations. In this numerical experiment, information was absorbed from all stations, including uninformative ones, located on the periphery of the pollution spot. At assimilation of data from the stations located in the area of maximum values of conditionality of the solved problem is improved that leads to improvement of the received results.

Table 1 Results of recovery of the rate of water outflow from an underwater source at different noise levels in the input data

r, kg/m^3	Noise level $100\% + r/C_p^*$, %	Restored w_p, m/s	Error recovery $100 \cdot \left\|1 - w_p/w_p^*\right\|$, %
0.0000	0.0	$34{,}991{,}985 \times 10^{-3}$	0.0
0.00675	1.25	$35{,}180{,}813 \times 10^{-3}$	0.5
0.0125	2.5	$38{,}568{,}601 \times 10^{-3}$	10.2
0.0250	5.0	$46{,}490{,}142 \times 10^{-3}$	32.8

5 Conclusions

Numerical experiments have shown reliable operation of the linearization method when searching for the values of the identifiable parameters. In continuation of the work (Kochergin and Fomin 2019), in which the concentration of pollution in the source is determined, a variational algorithm for identifying the rate of entry of polluted water into the basin is constructed and tested. The problem in variations is constructed and the search for the solution of the problem in variations in the simplest case is implemented. The proposed algorithm has a good convergence rate and accuracy of recovery of the required parameter. The realized procedure can be used for solving different ecological.

The work was carried out within the framework of the state task on the topic 0827-2018-0004 “Complex interdisciplinary studies of Oceanological processes that determine the functioning and evolution of ecosystems of the coastal zones of the Black and Azov seas” (“Coastal studies”) and supported by grant RFFI 18-45-920035 p_a.

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