Underground Density-Driven Convection of Saline Fluids with Constant and Variable Viscosity

E. B. Soboleva

Abstract Numerical simulation of density-driven convection in a porous medium modeling underground flows in crusts is conducted. Convective flows are initiated by an admixture source of finite length in a semi-infinite domain. An influence of the Rayleigh-Darcy number and variable viscosity on the structure and the number of convection regimes is discussed.

Keywords Density-driven convection · Porous medium · Darcy's filtration · Dissolved admixture · Variable viscosity · Numerical simulation

1 Introduction

Investigations of groundwater flows containing dissolved admixture and filling a porous medium are important for environmental management, mining, alternative energy engineering. Filtration problems are considered for carbon dioxide sequestration, underground repositories for nuclear waste, displacement of oil by water, extraction of hot fluids transporting geothermal energy.

One can specify a class of problems in which the fluid motion is driven by density inhomogeneities induced by dissolved admixture. If the system is under gravity, then natural haline convection develops. Such flows are generally called density-driven convection in the groundwater literature. Along with variable density, other factors can be essential for the hydrodynamic behavior of fluid phase, for example, temperature inhomogeneities, nonuniform solid matrix, variable physical properties of fluid.

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In this work, the onset and development of density-driven convective flows and mass transfer in a fluid phase with constant and variable viscosity is investigated. We consider a semi-infinite porous domain bounded by the upper horizontal surface. The part of this surface is the source of admixture triggering convective flows.

2 Theoretical Background

2.1 Solution Viscosity

The viscosity of groundwater depends on the concentration and composition of dissolved salts and other matters. The temperature is considered to be constant. The data (Aleksandrov et al. 2012) for water with sodium chloride based on numerous experiments are used to estimate the viscosity change. In theoretical studies, the Frank-Kamenetskii law is often used, i.e., the viscosity of solution is fitted to an exponential function of admixture density. We have

$$\mu = \mu_0 \exp(\Gamma' \rho_c). \tag{1}$$

Here, μ and μ_0 are the viscosity of solution and pure water, ρ_c is the density of dissolved admixture, Γ' is the viscosity variation parameter. We transform Eq. (1) introducing the dimensionless density $S = \rho_c/\rho_{c(sat)}$ and dimensionless viscosity variation parameter $\Gamma = \Gamma' \rho_{c(sat)}$, i.e., replacing the product $\Gamma' \rho_c$ by the equality: $\Gamma' \rho_c = \Gamma S$. Here, $\rho_{c(sat)}$ is the density of admixture in saturated solution. Taking the logarithm of the left-hand and right-hand sides of the result equation, one can obtain the linear dependence of $ln(\mu)$ on S: $ln(\mu) = ln(\mu_0) + \Gamma S$. We fit the data on μ (Aleksandrov et al. 2012) depending on S to the last linear relation and find out μ_0 and Γ. In Fig. 1, the markers coincide with data (Aleksandrov et al. 2012) corresponding to the pressure $P = 10$ MPa and the temperature $T = 423$ (a), 473 K (b). The selected thermodynamic conditions are typical for geothermal systems. The theoretical straight lines drawn for each temperature T are obtained by the method of least squares. According to our estimations, $\mu_0 = 190.7$ μPa s and $\Gamma = 1.022$ (a), $\mu_0 = 143.5$ μPa s and $\Gamma = 1.004$ (b). Note that the found viscosity of pure water is close to that in the open database NIST (National Institute of standards and Technology, USA) (NIST Chemistry WebBook 2016). We have in NIST for thermodynamic conditions indicated above: $\mu_0 = 184.9$ μPa s (a) and $\mu_0 = 136.4$ μPa s (b). Obviously, that the estimated parameter Γ does not change significantly if T varies, therefore we take $\Gamma = 1$ in numerical simulation.

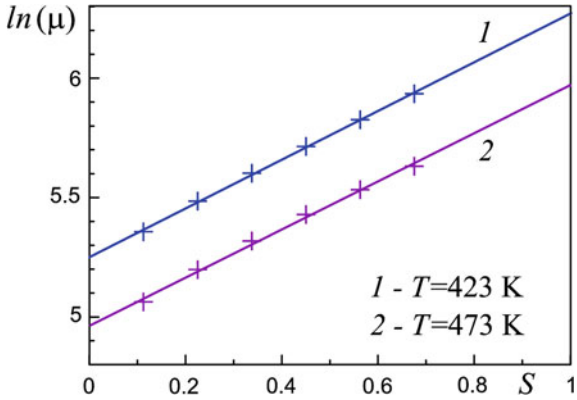

Fig. 1 Dependence of solution viscosity μ (in μPa s) on the dimensionless density of dissolved admixture S. Markers denote the data (Aleksandrov et al. 2012), solid lines are the theoretical straight lines

2.2 Governing Equations and Numerical Method

Flows and mass transfer in a solution filling a porous medium is described by the hydrodynamic model that includes the equations of continuity, motion (in the form of the Darcy equation) and admixture transport. The model is added by the linear equation of state relating the density of solution ρ to the density of dissolved admixture ρ_c. The equations are as follows.

$$\nabla \cdot \mathbf{U} = 0 \tag{2}$$

$$\mathbf{U} = -\frac{k}{\mu}(\nabla P - \rho g \cdot \mathbf{e}) \tag{3}$$

$$\phi \frac{\partial \rho_c}{\partial t} + \mathbf{U} \cdot \nabla \rho_c = \nabla(\phi D \nabla \rho_c) \tag{4}$$

$$\rho = \rho_0 + \alpha \rho_c \tag{5}$$

The values $\mathbf{U}$, P, ρ_0, k, ϕ, D, g, $\mathbf{e}$ are the filtration velocity, pressure, density of pure water, permeability, porosity, admixture diffusion coefficient, modulus of gravity acceleration and unit vector co-directional with the gravity. The constant $\alpha = 0.815$ is actual for water.

The porosity ϕ, permeability k, diffusion coefficient D and viscosity μ can be variable. The density ρ in Eq. (3) is substituted by Eq. (5) to exclude ρ from our consideration. Then, Eqs. (2)–(4) are reduced to a dimensionless form. The diffusion velocity and diffusion time are used as scales. Next, the differential equations are approximated by the finite difference equations. The last ones are solved numerically

using the numerical code designed for density-driven convective problems and effectively employed during several years (Soboleva and Tsypkin 2014, 2016; Soboleva 2017a, b, 2018, 2019a). The code is based on the finite-difference method applied in the case of straggled nonuniform grid. The algebraic linear equations are soled by the tridiagonal matrix algorithm called also as the Thomas algorithm (Aziz and Settari 1979). At the present stage of research, the code is modified by applying a more efficient numerical scheme for the convective term (QUICK scheme instead of the central difference scheme) and extended to fluids with variable viscosity (Soboleva 2019a).

3 Numerical Simulation

3.1 Problem Under Study

We consider a semi-infinite porous domain under the Earth's gravity bounded by the upper horizontal surface. Initially, the domain is filled with pure water at rest. There is a source of admixture at the upper boundary with length l which is held at the constant admixture concentration c equal to one in saturation solution c_{sat}. The sketch of the problem is given in Fig. 2.

The admixture diffuses into the domain dissolving in water. Thus, a zone of concentrated solution is formed below the source. Solution is heavier than pure water and therefore tends to fall down. Gravitational density-driven convection develops.

The density of dissolved admixture ρ_c is related to c by the relation:

$$\rho_c = \rho_0 \frac{c}{1 - \alpha c} \tag{6}$$

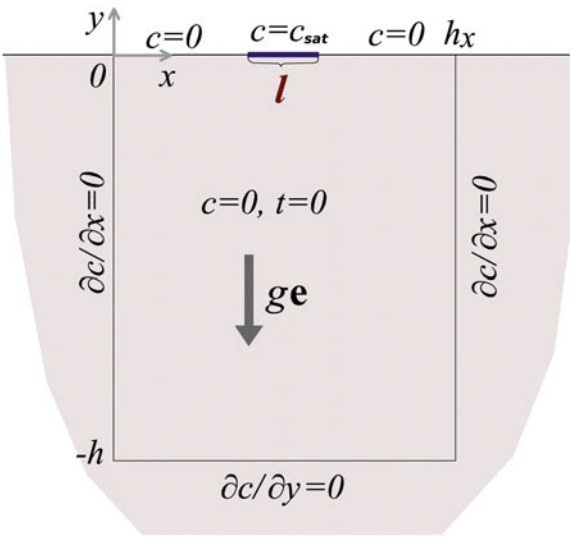

Fig. 2 Sketch of the problem

The dimensionless system of the governing equations includes the Rayleigh-Darcy Rd number which is the criterion of similarity of convection observed. We have

$$Rd = \frac{(\rho_{sat} - \rho_0) g l k_0}{\mu_0 D_0} \tag{7}$$

Here, ρ_{sat} is the density of saturated solution, subscript "0" denotes some reference values. Note, that the length scale in Eq. (7) is the source length l.

Before beginning numerical simulation, we have analyzed the Elder problem that is on thermal convection induced by a finite source in the Hele-Shaw cell (Elder 1967). Later, they started to apply this problem to haline convection as well. As shown in (Van Reeuwijk et al. 2009), the problem can have several solutions depending on the Rayleigh-Darcy Rd_h number. A single regime is induced if $Rd_h < 76$. Note, Rd_h in the Elder problem includes the cell height h being twice shorter than the source length. Recalculating the Rayleigh-Darcy number with the use of the source length, we obtain the twice bigger threshold magnitude and condition ensuring a single solution in the form: $Rd < 152$. One can believe, that the system passes through the bifurcation points in the beginning of convection when small disturbances are

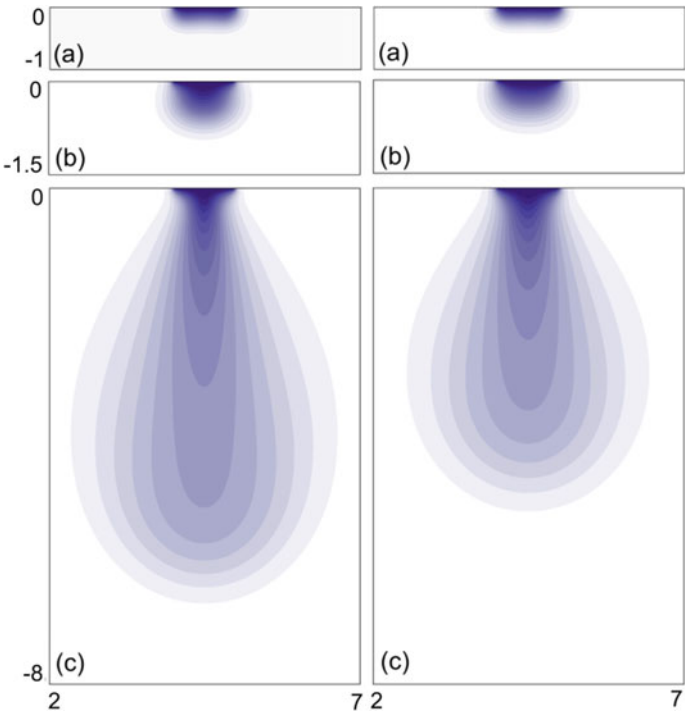

Fig. 3 Fields of admixture concentration at $Rd = 60$ and at times $t = 2.0 \times 10^{-2}$ (**a**), $t = 7.0 \times 10^{-2}$ (**b**), $t = 7.7 \times 10^{-1}$ (**c**). Viscosity is constant (left) and variable (right)

self-organising in the boundary layer under the source. It means, that the condition on the Rayleigh-Darcy number can be applied in the case of a semi-infinite domain and we expect to obtain several solutions if $Rd > 152$.

3.2 Results of Simulation

We conducted numerical simulation at $Rd = 60$ when regime of convection is single and analyzed the role of variable viscosity. The porous domain 9×10 is considered. The space grid is uniform and includes 900×1500 cells. The time step is $\tau = 3.3 \times 10^{-6}$. The fields of admixture concentration are shown in Fig. 3; the part of considered domain is presented. As obvious, convective structure is very simple. A single convective plum is developed. In the case of variable viscosity, the depth of penetration is shorter, as the velocity of flows near the source becomes lower. As a result, a smaller amount of admixture moves into the domain.

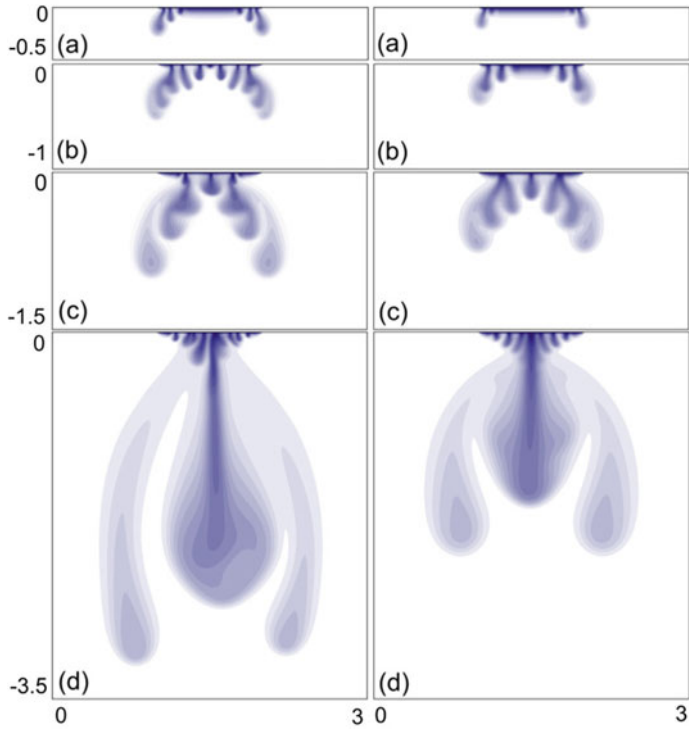

Fig. 4 Fields of admixture concentration at $Rd = 2000$ and at times $t = 1.0 \times 10^{-3}$ (**a**), $t = 2.0 \times 10^{-3}$ (**b**), $t = 3.0 \times 10^{-3}$ (**c**), $t = 1.3 \times 10^{-2}$ (**d**). Viscosity is constant (left) and variable (right)

If $Rd > 152$, we expect to obtain not a single solution. However, only one of possible regimes at $Rd = 2000$ is exhibited in Fig. 4. The porous domain is 3×5. The space grid with steps refining near the source is 1500×2000. The time step is $\tau = 1.25 \times 10^{-7}$. We see, that motion arises near the edges of source there the horizontal gradient of density is present. Initially, several small fingers occur. Then, motion is rearranged and a central plum is formed. Motion and mass transfer are concentrated in the central plum transporting admixture down. When variable viscosity is taken into consideration, convective structure is qualitatively similar to that observed at constant viscosity. However, convection develops slower and the depth of penetration of central plum is shorter.

4 Conclusion

We presented here some results on numerical simulation of density-driven convection triggered by an admixture source of finite length in semi-infinite porous domain. As discussed, the regime of convection strongly depends on the Rayleigh-Darcy Rd number. When $Rd > 152$ we expect to obtain several solutions. The variable viscosity taken into consideration can influence the structure and the number of convection regimes.

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