OntoMath^{Edu}: A Linguistically Grounded Educational Mathematical Ontology

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Abstract. We present the first release of OntoMath^{Edu}, a new educational mathematical ontology. The ontology is intended to be used as a Linked Open Data hub for mathematical education, a linguistic resource for intelligent mathematical language processing and an end-user reference educational database. The ontology is organized in three layers: a foundational ontology layer, a domain ontology layer and a linguistic layer. The domain ontology layer contains language-independent concepts, covering secondary school mathematics curriculum. The linguistic layer provides linguistic grounding for these concepts, and the foundation ontology layer provides them with meta-ontological an-notations. The concepts are organized in two main hierarchies: the hierarchy of objects and the hierarchy of reified relationships. For our knowledge, OntoMath^{Edu} is the first Linked Open Data mathematical ontology, that respects ontological distinctions provided by a foundational ontology; represents mathematical relationships as first-oder entities; and provides strong linguistic grounding for the represented mathematical concepts.

Keywords: Ontology · Mathematics education · Linked Open Data · Natural language processing · Mathematical knowledge management · OntoMath^{Edu}

1 Introduction

We present the first release of OntoMath^{Edu}, a new educational mathematical ontology. This ontology is intended to be:

- A Linked Open Data hub for mathematical education. In this respect, the ontology lies at the intersection of two long-established trends of using LOD for educational purposes [1–4] and for mathematical knowledge management [5,6].
- A linguistic resource for common mathematical language processing. In this respect, the ontology can complement mathematical linguistic resources, such as SMGloM [7,8], and serve as an interface between raw natural language texts and mathematical knowledge management applications.

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– An end-user reference educational database, and play the same role in secondary school math, that PlanetMath or MathWorld play in professional mathematics.

This ontology is a central component of the digital educational platform under development, which is intended for solving such tasks as: (1) automatic questions generation; (2) automatic recommendation of educational materials according to an individual study plan; (3) semantic annotation of educational materials.

In the development of OntoMath^{Edu} we would rely on our experience of the development of OntoMath^{PRO} (http://ontomathpro.org/) [9], an ontology of professional mathematics. This ontology underlies a semantic publishing platform [10,11], that takes as an input a collection of mathematical papers in LaTeX format and builds their ontology-based Linked Open Data representation. The semantic publishing platform, in turn, is a central component of OntoMath digital ecosystem [12,13], an ecosystem of ontologies, text analytics tools, and applications for mathematical knowledge management, including semantic search for mathematical formulas [14] and a recommender system for mathematical papers [15].

Despite the fact that OntoMath^{PRO} has proved to be effective in several educational applications, such as assessment of the competence of students [9] and recommendation of educational materials in Virtual Learning Communities [16–19], its focus on professional mathematics rather than on education prevents it to be a strong foundation for the digital educational platform. The main differences between OntoMath^{PRO} and a required educational ontology are the following:

– *Conceptualization*. OntoMath^{PRO} ontology specifies a conceptualization of professional mathematics, whilst the required educational ontology must specify a conceptualization of school mathematics. These conceptualizations are noticeably different, for example, in school conceptualization, *Number* is a primitive notion, while in professional conceptualization it is defined as a subclass of *Set*.
– *Selection of concepts*. The required educational ontology must contain concepts from a school mathematics curriculum.
– *Terminology*. Concepts of OntoMath^{PRO} ontology are denoted by professional terms, whilst concepts of the required educational ontology must be denoted by school math terms. There isn't so much difference between professional and educational terminology in English, but this difference is more salient in such languages as Russian or Tatar. For example, the term 'mnogochlen' (the native word for 'polynom') should be used instead of the professional term 'polinom' (the Greek loan word with the same meaning) in educational environment.
– *Prerequisite relations*. In the required educational ontology, logical relations between concepts must be complemented with prerequisite ones. The concept A is called a prerequisite for the concept B, if a learner must study the concept A before approaching the concept B. For example, comprehension of

the *Addition* concept is required to grasp the concept of *Multiplication*, and, more interesting, to grasp the very concept of *Function*, even though, from the logical point of view the later concept is more fundamental and is used in the definitions of the first two.

– *Points of view*. In addition to universal statements, the required educational ontology must contain statements relativized to particular points of view, such as different educational levels. For example, a concept can be defined differently on different educational stages; and a statement can be considered as an axiom according to one axiomatization, and as a theorem according to another.

Concerning to common mathematical language processing, OntoMath^{PRO} is suitable for extraction of separate mathematical objects, but not for extraction facts about them. The same fact can be linguistically manifested in many different ways. For example, the incidence relation between point a and line l can be represented by a transitive verb ("l contains a"), a verb with a preposition ("a lies on l"), an adjective with a preposition ("a is incident with l") and an adjective with a collective subject ("a and l are incident") [20]. So, the required ontology should define concepts for representing mathematical facts as well as mappings to their natural language manifestations.

With regard to the foregoing, we have lunched a project for developing a new educational ontology OntoMath^{Edu}. The project was presented at the work-in-progress track of CICM 2019 *cicm2019* and was recommended by PC to be re-submitted to the main track after the release of the first stable version. In this paper, we describe the overall project as well as the first release, consisting in the domain ontology layer for Euclidean plane geometry domain.

2 Ontology Structure

According to the project, OntoMath^{Edu} ontology is organized in three layers:

1. **Foundational ontology layer**, where a chosen foundational ontology is UFO [22].
2. **Domain ontology layer**, which contains language-independent math concepts from the secondary school mathematics curriculum. The concepts are grouped into several modules, including the general concepts module and modules for disciplines of mathematics, e.g. Arithmetic, Algebra and Plane Geometry. The concepts will be interlinked with external LOD resources, such as DBpedia [23], ScienceWISE [24] and OntoMath^{PRO}. Additionally, relaying on the MMT URIs scheme [25], the concepts can be aligned with MitM ontology [26], and through it with the concepts of several computer algebra systems.
3. **Linguistic layer**, containing multilingual lexicons, that provide linguistic grounding of the concepts from the domain ontology layer. The lexicons will be interlinked with the external lexical resources from the Linguistic Linked Open Data (LLOD) cloud [27,28], first of all in English [29,30], Russian [31] and Tatar [32] (Fig. 1).

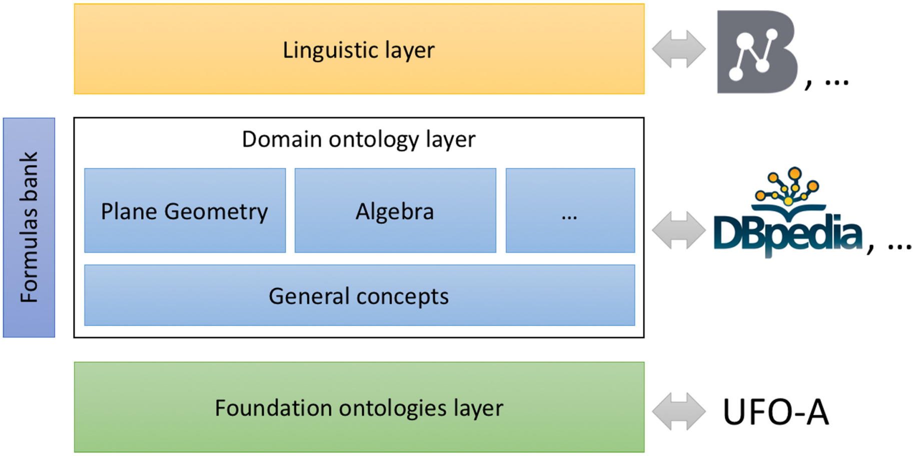

Fig. 1. OntoMath^{PRO} ontology structure.

3 Domain Ontology Layer

The domain ontology layer of OntoMath^{Edu} is being developed according to the following modelling principles:

1. Common mathematical language conceptualization. OntoMathEdu reflects the conceptualization of the Common mathematical language (CML) [33], not that of the language of fully formalized mathematics. These conceptualizations are very different. For example, according to the fully formalized mathematics conceptualization, the *Set* concept subsumes the *Vector* concept, but in the CML conceptualization *Vector* is represented by *Set*, and is not subsumed by it. More important, in contrast to the fully formalized mathematics conceptualization, according to the CML conceptualization, mathematical objects are neither necessary nor timeless, and the domain of discourse can expand in a process of problem-solving.
2. Strict adherence to ontological distinctions provided by the foundational ontology. For example, we explicitly mark concepts as Kinds or Roles.
3. Reification of domain relations. Mathematical relations are represented as concepts, not as object properties. Thus, the mathematical relationships between concepts are first-order entities, and can be a subject of a statement.
4. Multilinguality. Concepts of ontology contains labels in English, Russian and Tatar.
5. Educational literature warrant. The ontology contains only those concepts, that are represented in actual education literature.

Current version of OntoMath^{Edu} contains 823 concepts from the secondary school Euclidean plane geometry curriculum (5th–9th grades), manually developed by experts relying on mathematical textbooks. The description of a concept contains its name in English, Russian and Tatar, axioms, relations with other concepts, and links to external resources of the LOD cloud and educational reference databases.

The concepts are organized in two main hierarchies: the hierarchy of objects and the hierarchy of reified relationships.

3.1 Hierarchy of Objects

The top level of the hierarchy of objects consists of the following classes:

1. *Plane Figure*, with subclasses such as *Line*, *Polygon*, *Ellipse*, *Angle*, *Median of a Triangle* or *Circumscribed Circle*.
2. *Plane Geometry Statement*, with subclasses such as *Axiom of construction of a circle with a given center and radius* or *Pythagorean Theorem*.
3. *Plane Geometry Problem* with subclasses such as *Problem of straightedge and compass construction* or *Heron's problem*.
4. *Plane Geometry Method* with subclasses such as *Constructing an additional line for solving plane geometry problem*.
5. *Unit of Measurement*, with subclasses such as *Centimeter*, *Radian*, or *Square meter*.
6. *Measurement and Construction Tool*, with subclasses such as *Protractor*, *Astrolabe*, *T-square*, *Sliding T bevel*, or *Marking gauge*.

A fragment of the hierarchy of objects is represented at the Fig. 2.

There are two meta-ontological types of the concepts: kinds and roles.

A kind is a concept that is rigid and ontologically independent [22,34]. So, for example, the *Triangle* concept is a kind, because any triangle is always a triangle, regardless of its relationship with other figures.

A role is a concept that is anti-rigid and ontologically dependent [22,34]. An object can be an instance of a role class only by virtue of its relationship with another object. So, for example, the *Median* concept is a role, since any line segment is a median not by itself, but only in relation to a certain triangle. Any role concept is a subclass of some kind concept. For example, the *Median* role concept is a subclass of *Line* segment kind concept.

Figure 3 represents the *Median* role concept and one of its instances, namely median AO, related to triangle ABC.

3.2 Hierarchy of Reified Relationships

Relations between concepts are represented in ontology in a reified form, i.e. as concepts, not as object properties (such representation fits the standard ontological pattern for representing N-ary relation with no distinguished participant [35], but is applied to binary relations too). Thus, the relationships between concepts are first-order entities, and can be a subject of a statement.

The top level of the hierarchy of reified relationships consists of the following classes:

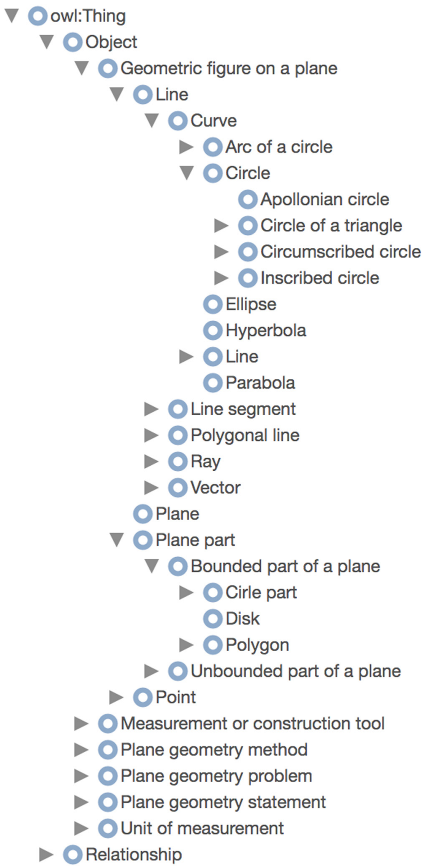

Fig. 2. A fragment of the hierarchy of objects.

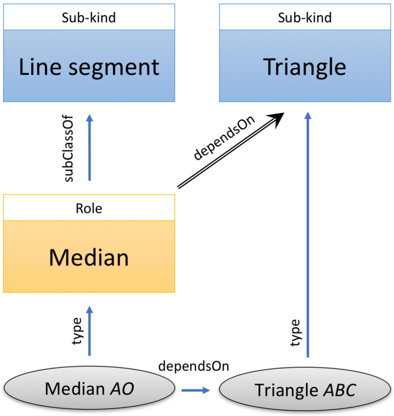

Fig. 3. A role example.

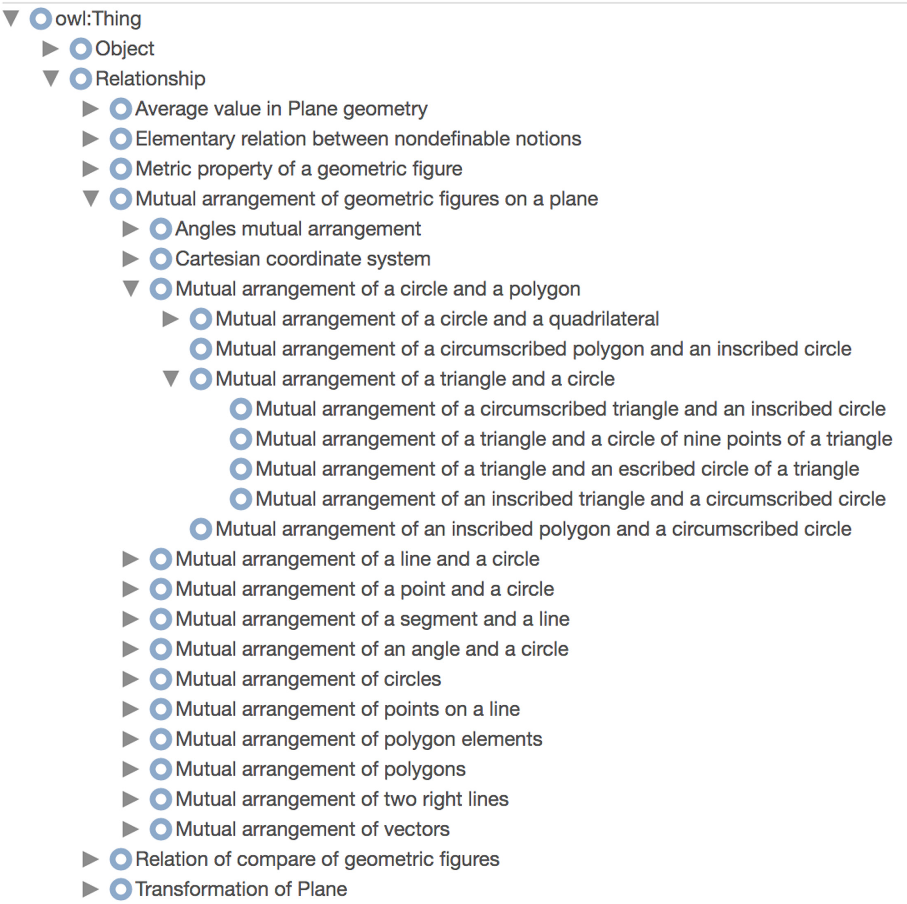

Fig. 4. A fragment of the hierarchy of reified relationships.

1. *Mutual arrangement of geometric figures on a plane*, with subclasses such as *Inscribed polygon* or *Triangle with vertices at Euler points.*
2. *Comparison relation between plane figures*, with subclasses such as *Congruent Triangles* or *Similar Polygons.*
3. *Plane Transformation*, with subclasses such as *Translation* or *Axial Symmetry.*
4. *Metric property of a plane figure*, with subclasses such as *Length of a circle*, *Tangent of acute angle in right triangle*, or *Eccentricity of an ellipse.*

A fragment of this hierarchy is represented at the Fig. 4.

Reified relationships are linked to their participants by has argument object properties and their subproperties.

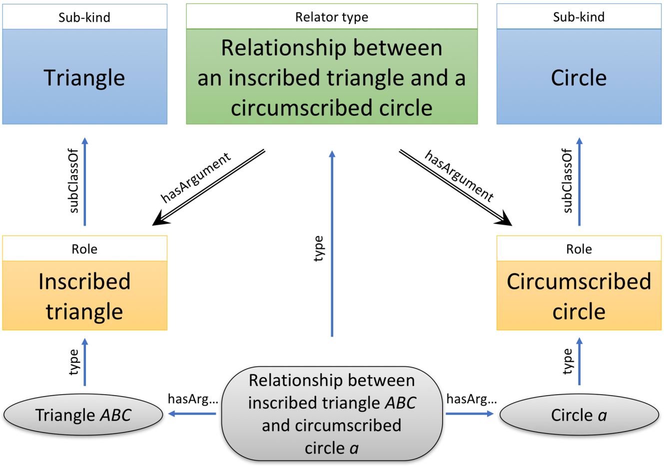

Fig. 5. An example of a reified relationship, and its instance corresponding to the "Triangle *ABC* is inscribed in circle *a*" statement.

Figure 5 shows one of the relations, represented by the *Relationship between an inscribed triangle and a circumscribed circle concept*. This relation is linked to its participants, represented by *Inscribed triangle* and *Circumscribed circle* role concepts. These roles, in turn, are defined as subclasses of the *Triangle* and the *Circle* kind concepts respectively. The bottom of the figure depicts an instance of this relation, namely the *Relationship between inscribed triangle ABC and circumscribed circle a*, that binds triangle *ABC* and circle *a*.

This relationship is a representation of natural language statement "Triangle *ABC* is inscribed in circle *a*". The mappings between ontology concepts and corresponding natural language statements are defined at the linguistic level of the ontology.

3.3 Network of Points of View

Points of view are represented using the "Descriptions and Situations" design pattern, and are based on the top-level ontology DOLCE + DnS Ultralite [36–38]. The network of points of view is under development now and is not included in the first release of the ontology.

3.4 Object and Annotation Properties

The ontology defines the following relations, represented by the object and annotation properties as well as their subproperties:

1. *Has argument* relation, that binds a reified relationship and its participants.
2. Relation of *Ontological dependence* that binds a role concept to its dependee concept.

3. *Has part* relation. For example, any *Vertex of a Triangle* is a part of a *Triangle*.
4. *Aboutness* relation that holds between a *Statement* and the subject matter of this statement. For example, *Heron's formula* is related to the *Area of a polygon* concept.
5. *Prerequisite relation.* The concept A is called a prerequisite for the concept B, if a learner must study the concept A before approaching the concept B. In the first release of the ontology, these relations are introduced only indirectly in coarse-grained manner by arrangement of the concepts by successive educational levels.
6. *Belongs to educational level*, that binds a concept and an educational level (such as an age of leaning) at which the concept is firstly introduced.
7. *External resource*, that interlinks a concept and an external Linked Open Data or reference educational resource describing this concept.

3.5 External Links

Currently, OntoMath^{Edu} ontology has been interlinked with the following external resources:

DBpedia. The mapping was constructed semi-automatically on the base of the method proposed in [41] and then manually verified. This mapping contains 154 connections, expressed by the skos:closeMatch properties.

External Reference Educational Resources. The mapping was constructed manually and contains 71 connections, expressed by the ome:eduRef annotation properties and its subproperties.

4 Linguistic Layer

The linguistic layer contains multilingual lexicons, that provide linguistic grounding of the concepts from the domain ontology layer.

Currently we are developing Russian and English lexicons and are going to develop the lexicon for Tatar.

A lexicon consists in:

- Lexical entries, denoting mathematical concepts. Examples of lexical entries are "triangle", "right triangle", "side of a polygon", "Riemann integral of f over x from a to b", "to intersect", "to touch", etc.
- Forms of lexical entries (in different numbers, cases, tenses, etc).
- Syntactic trees of multi-word lexical entries.
- Syntactic frames of lexical entries. A syntactic frame represents the syntactic behavior of a predicate, defining the set of syntactic arguments this predicate requires and their mappings to ontological entities. For example, a syntactic frame of the "to touch" verb determines that in "X touches Y at Z" phrase, subject X represents a tangent line to a curve, direct object Y represents the curve, and prepositional adjunct Z represents the point of tangency.

```
@prefix : <http://ontomathpro.org/ontomathedu/lexicons/>.
@prefix ome: <http://ontomathpro.org/ontomathedu#>.
@prefix ontolex: <http://www.w3.org/ns/lemon/ontolex#>.
@prefix synsem: <http://www.w3.org/ns/lemon/synsem#>.
@prefix lexinfo: <http://www.lexinfo.net/ontology/2.0/lexinfo#>.

#The "to touch" verb
:EN-v-touch
  a ontolex:LexicalEntry;
  lexinfo:partOfSpeech lexinfo:verb;
  ontolex:canonicalForm :EN-v-touch-form0;
  synsem:synBehavior :EN-v-touch-frame1;
  ontolex:sense :EN-v-touch-sense1.

#The canonical form of this verb
:EN-v-touch-form0
  a ontolex:Form;
  ontolex:writtenRep "touch"@en.

#A syntactic frame "X touches Y at Z"
:EN-v-touch-frame1
  a lexinfo:TransitivePPFrame;
  lexinfo:subject :EN-v-touch-frame1-subj;
  lexinfo:directObject :EN-v-touch-frame1-obj;
  lexinfo:prepositionalArg :EN-v-touch-frame1-pp_at.

#The subject of the verb
:EN-v-touch-frame1-subj
  a lexinfo:Subject.

#The direct object of the verb
:EN-v-touch-frame1-obj
  a lexinfo:DirectObject.

#A prepositional adjunct of the verb
:EN-v-touch-frame1-pp_at
  a lexinfo:PrepositionalAdjunct;
  synsem:marker :EN-prep-at;
  synsem:optional "true"^^xsd:boolean.

#A lexical sense, expressing the mapping
#of the verb to the ontology
:EN-v-touch-sense1
  a ontolex:LexicalSense, synsem:OntoMap;
  synsem:ontoMapping :EN-v-touch-sense1;
```

Fig. 6. "To touch" lexical entry

```
ontolex:reference
  ome:Relationship_between_a_tangent_line_and_a_curve;
synsem:submap
  [
  a synsem:OntoMap;
  ontolex:reference
    ome:Relationship_between_a_tangent_line_and_a_curve;
  synsem:isA _:Relationship1
  ],
  [
  a synsem:OntoMap;
  ontolex:reference ome:tangent_line;
  synsem:subjOfProp _:Relationship1;
  synsem:objOfProp :EN-v-touch-frame1-subj
  ],
  [
  a synsem:OntoMap;
  ontolex:reference ome:curve;
  synsem:subjOfProp _:Relationship1;
  synsem:objOfProp :EN-v-touch-frame1-obj
  ],
  [
   a synsem:OntoMap;
   ontolex:reference ome:tangent_point;
   synsem:subjOfProp _:Relationship1;
   synsem:objOfProp :EN-v-touch-frame1-pp_at
   ],
   [
   a synsem:OntoMap;
   ontolex:reference ome:Tangent_line;
   synsem:isA :EN-v-touch-frame1-subj
   ],
   [
   a synsem:OntoMap;
   ontolex:reference ome:Tangent_point;
   synsem:isA :EN-v-touch-frame1-pp_at
   ].
```

Fig. 6. (*continued*)

The lexicons are expressed in terms of Lemon [43,44], LexInfo, OLiA [45] and PreMOn [46] ontologies.

Figure 6 represents an example of the "to touch" verb, its canonical form, syntactic frame and lexical sense. The syntactic frame defines three arguments of this verb: a subject, a direct object and an optional prepositional adjunct, marked by the "at" preposition. The lexical sense defines a mapping of the verb and its syntactic arguments to the corresponding ontological concepts. According to the mapping, the verb denotes the reified relationship between a tangent line and a curve, while the syntactic arguments express the participants of this relationship: the subject expresses a tangent line to a curve, the direct object expresses the curve, and the prepositional adjunct expresses the tangent point.

5 Conclusions

In this paper, we present the first release of OntoMath^{Edu}, a new educational mathematical ontology.

While there are many educational ontologies on the one hand, and several mathematical ontologies on the other, to our knowledge, OntoMath^{Edu} is the first general-purpose educational mathematical ontology. Additionally, it is the first Linked Open Data mathematical ontology, intended to: (1) respect ontological distinctions provided by a foundational ontology; (2) represent mathematical relationships as first-order entities; and (3) provide strong linguistic grounding for the represented mathematical concepts.

Currently, our first priority is to release the linguistic layer of the ontology that is still under development and hasn't been published yet. After that, we will extend the ontology to other fields of secondary school mathematics curriculum, such as Arithmetic, Algebra and Trigonometry.

Finally, we are going to apply the modeling principles, drafted on this project, in the development of the new revised version of the ontology of professional mathematics OntoMath^{PRO}.

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