

Chapter 2

Going Beyond the Standard Model



B. G. Sidharth

Abstract In this communication we had argued that we could account for the shortcomings of the standard model by including noncommutative geometry which could lead to a non-zero (electron) neutrino mass.

At that point in time it was widely accepted that the standard model of particle physics is the most complete theory and yet there have been frantic efforts to go beyond the standard model to overcome its shortcomings. Some of these are:

1. In the theory prevalent at that time, it was stated that it fails to deliver the mass to the neutrino which thus remains a massless particle.
2. This apart, it did not include gravity, which is otherwise one of the four fundamental interactions.
3. We had to keep in mind the hierarchy problem viz., the wide range of masses for the elementary particles or even for the quarks.
4. It appears that other as of yet undiscovered particles exist which could change the picture, for example, in supersymmetry in which the particles have their supersymmetric counterparts.
5. The standard model has no place for dark matter, which on the other hand has not yet been definitely found. Nor is there place for dark energy.
6. Finally, the 18 odd arbitrary constants which creep into the theory need to be explained.

There are however obvious shortcomings which could be addressed in a relatively simple manner which could enable us to go beyond the standard model. Let us start with the standard model Lagrangian

B. G. Sidharth (✉)
Birla Science Centre, Hyderabad, India

$$\begin{aligned}
L_{\text{GWS}} = & \sum_f (\bar{\Psi}_f (i\gamma^\mu \partial_\mu - m_f) \Psi_f - e Q_f \Psi_f \gamma^\mu \Psi_f A_\mu) + \\
& + \frac{g}{\sqrt{2}} \sum_i \left(a_L^{-1} \gamma^\mu b_L^i W_\mu^+ + \bar{b}_L^i \gamma^\mu a_L^i W_\mu^- \right) + \frac{g}{2C_\omega} \sum_f \bar{\Psi}_f \gamma^\mu \left(I_f^3 - 2S_\omega^2 Q_f - I_f^3 \gamma_5 \right) \Psi_f Z_\mu + \\
& - \frac{1}{4} \left| \partial_\mu A_\nu - \partial_\nu A_\mu - ie \left(W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^- \right) \right|^2 - \frac{1}{2} \left| \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \right. \\
& \left. -i.e. \left(W_\mu^+ + A_\nu - W_\nu^+ + A_\mu \right) + ig' c_\omega \left(W_\mu^+ Z_\nu - W_\mu^+ W_\nu^- \right) \right|^2 + \\
& - \frac{1}{4} \left| \partial_\mu Z_\nu - \partial_\nu Z_\mu + ig' c_\omega \left(W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^- \right) \right|^2 + \\
& - \frac{1}{2} M_\eta^2 \eta^2 - \frac{g M_\eta^2}{8M_W} \eta^3 - \frac{g'^2 M_\eta^2}{32M_W} \eta^4 + \left| M_W W_\mu^+ + \frac{g}{2} \eta W_\mu^+ \right|^2 + \\
& + \frac{1}{2} \left| \partial_\mu \eta + iM_Z Z_\mu + \frac{ig}{2C_\omega} \eta Z_\mu \right|^2 - \sum_f \frac{g m_F}{2M_W} \bar{\Psi}_f \Psi_f \eta
\end{aligned} \tag{2.1}$$

which includes the Dirac Lagrangian amongst other things.

We pointed out that all these have been on the basis of the usual point spacetime which is what may be called commutative. But in recent years several authors including in particular the present author has worked with a noncommutative spacetime which originates back to Snyder in the late forties itself. (This was an attempt to overcome the divergences.)

We first observed that it was Dirac (1958) who pointed out two intriguing features of his equation: (1) The Compton wavelength and (2) Zitterbewegung.

For the former, his intuition was that we can never make measurements at space or time points. We need to observe over an interval to get a meaningful definition of momentum for example. This interval was the Compton region (Sidharth and Das 2017). Next, his solution was rapidly oscillatory, what is called Zitterbewegung. This oscillatory behaviour disappears on averaging over spacetime intervals over the Compton region. Once this is done while meaningful physics appears, we are left with not points but minimum intervals.

This leads to a noncommutative geometry. One model for this is that of Snyder (1947). Applied at the Compton wavelength this leads to the so-called Snyder–Sidharth dispersion relation, the geometry being given by Sidharth (2008)

$$[x_i, x_j] = \beta_{ij} \cdot l^2 \tag{2.2}$$

As described in detail in Sidharth (2010), this leads to a modification in the Dirac and also the Klein–Gordon equation. This is because Eq. (2.2) in particular leads to the following energy momentum relation (cf. Sidharth 2008)

$$E^2 - p^2 - m^2 + \alpha l^2 p^4 = 0 \quad (2.3)$$

where α is a scalar constant, $|\alpha l| \approx 10^{-3}$ (Sidharth et al. 2015, 2016). Though the value of α is of no consequence for the present work, it may be mentioned that α gives the Schwinger term. If we work with this energy momentum relation (2.3) and follow the usual process, we get as in the usual Dirac theory

$$\{\gamma^\mu p_\mu - m\}\psi \equiv \{\gamma^o p^o + \Gamma\}\psi = 0 \quad (2.4)$$

We now include the extra term in the energy momentum relation (2.3). It can be easily shown that this leads to

$$p_o^2 - (\Gamma\Gamma + \{\Gamma\beta + \beta\Gamma\} + \beta^2\alpha l^2 p^4)\psi = 0 \quad (2.5)$$

Whence the modified Dirac equation

$$\{\gamma^o p^o + \Gamma + \gamma^5 \alpha^2\}\psi = 0 \quad (2.6)$$

The modified Dirac equation contains an extra term. The extra term gives a slight mass for the neutrino which is roughly of the correct order viz., $10^{-8}m_e$, m_e being the mass of the electron. The behaviour too is that of the neutrino (Sidharth 2010, 2017).

To sum up the introduction of the noncommutative geometry described in Eq. (2.2) leads to a Dirac like Eq. (2.6) and a Lagrangian that leads to the electron neutrino mass.

It must be pointed out that the modified Lagrangian differs from the usual Lagrangian in that the γ^o matrix is now replaced by a new matrix

$$\gamma^{o'} = \gamma^o + \gamma^o \cdot \gamma^5 l p^2$$

that includes the term giving rise to the neutrino mass. We could verify that the modified Lagrangian gives back the modified Dirac equation (2.6). Further as has been discussed in detail, the extra term arising out of the noncommutative geometry is the direct result of the dark energy which thus also features in the modified standard model Lagrangian. This apart, this argument has been shown to lead to a mass spectrum for elementary particles that includes all the elementary particles, giving the masses with about 5% or less error (Sidharth 2008).

References

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