Chapter 4
Vector-Tensor Gravities and Problem of Lorentz Symmetry Breaking in Gravity

4.1 Introduction and Motivations

The interest to vector-tensor gravity models strongly increased in recent years. One of the main motivations to studying these models arises from the idea of the Lorentz symmetry breaking. Indeed, as it is well known, in the flat space the explicit Lorentz symmetry breaking is implemented through introduction of a constant vector (tensor) generating a space-time anisotropy (see f.e. [73, 74]). As we already noted in the previous chapter, this methodology allowed to define, for example, the Carroll–Field–Jackiw term (3.7) as well as many other terms discussed in [73]. However, in the curved space the explicit Lorentz symmetry breaking faces serious problems. First of all, the definition of the constant vector (tensor) itself in this case becomes highly controversial: for example, while in the flat space the constant vector k^μ is defined to satisfy the condition $\partial_\nu k^\mu = 0$, this condition cannot be applied in a curved space since it breaks the general covariance. A possible "covariant extension" of this condition like $\nabla_\nu k^\mu = 0$ would imply in extra restrictions for the space-time geometry (and, moreover, nobody could guarantee these restrictions to be satisfied for a general choice of the vector k_μ). In principle, one can also deal with derivative expansions of corresponding effective actions, where various orders of derivatives of "constant" tensors can be obtained (see f.e. [75]), however, it is clear that in this case the definition of a constant vector (or tensor) simply loses its sense, and such a vector becomes an extra field. Moreover, in many cases such possible new terms are not gauge invariant which means that together with the Lorentz symmetry, the general covariance for such terms is broken as well (the problem of breaking the general covariance in modified gravity is discussed in details in [76]; in principle, it should be noted that breaking of general covariance occurs for the term $u^\mu u^\nu R_{\mu\nu}$ proposed in [77] as a possible example of a CPT-even Lorentz-breaking term for gravity, as well as for the one-derivative linearized term discussed in [47]).

Therefore, the most appropriate method for implementing the Lorentz symmetry breaking into a curved space-time turns out to be based on the spontaneous symmetry

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breaking. Its essence is as follows. One considers the action of the metric tensor coupled to the vector field (again, similarly to the previous chapter, this vector field is treated as an ingredient of gravity model itself but not a matter, thus, we have the vector-tensor gravity) so that the purely metric sector is presented by the usual Einstein–Hilbert action, and the dynamics of the vector field is described by the Maxwell-like term, plus a potential whose minimum yields a vector implementing the Lorentz symmetry breaking, and maybe also some extra terms responsible for a vector-gravity coupling. The paradigmatic example is the bumblebee action [78] (the name "bumblebee" itself was introduced in [79]), looking like

$$S=\int d^4x\sqrt{|g|}\left(\frac{1}{16\pi G}(R+\xi B^{\mu}B^{\nu}R_{\mu\nu})-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-V(B^{\mu}B_{\mu}\pm b^2)\right). \quad (4.1)$$

Here ξ is a dimensionless constant, $B_{\mu\nu}=\partial_{\mu}B_{\nu}-\partial_{\nu}B_{\mu}$ is the stress tensor for the bumblebee field B_{μ}, and V is the potential possessing an infinite set of minima $B_{0\mu}$ satisfying the condition $B_0^{\mu}B_{0\mu}=\pm b^2$ (the difference of signs reflects that the vector $B_{0\mu}$ can be either time-like or space-like, while $b^2>0$). So, actually choosing of one of the vacua $B_{0\mu}$ allows to introduce the privileged direction. The potential is usually chosen to be quartic in the field B_{μ} by renormalizability reasons. Alternatively, one can deal with Einstein-aether theory where, instead of this, the minima arise due to a constraint multiplied by a Lagrange multiplier σ, so that one has $V=\sigma(B^{\mu}B_{\mu}\pm b^2)$, but the kinetic term is not Maxwell-like being a more generic quadratic function of covariant derivatives of the vector B_{μ}. In principle, one can consider the vector-tensor gravity models without any potential [80], however, in this case the spontaneous Lorentz symmetry breaking cannot occur. Such theories are considered mostly within the cosmological context (see f.e. [80]).

Within this chapter, we discuss some interesting classical results for the Einstein-aether gravity and for the bumblebee gravity. At the end of the chapter, we also will review some terms proposed in [73, 74] as possible extensions of the Einstein gravity allowing to break the Lorentz symmetry explicitly. As for the Horava-Lifshitz gravity, although it represents itself as an example of non-Lorentz-invariant gravity model, it is described in terms of the essentially distinct methodology and will be discussed in the next chapter.

4.2 Einstein-aether Gravity

So, let us implement the spontaneous Lorentz symmetry breaking in a curved space-time. To justify importance of this approach, one can remind that namely the spontaneous breaking mechanism has been initially proposed to explain the origin of the Lorentz symmetry breaking in the low-energy limit of the string theory [81]. Following this concept, one considers a vector field B_{μ} with a constant square, i.e. $B^{\mu}B_{\mu}=\pm b^2$, which is implemented via introducing the constraint with use of the Lagrange multiplier σ, adding to the Lagrangian the potential $V=\sigma(B^{\mu}B_{\mu}\pm b^2)$.

Alternatively, as we already noted above, one can introduce the quartic potential. The approach based on the Lagrange multiplier has been adopted within gravity studies performed in the paper [82]. In this case, the above constraint is generalized to a curved space-time as $g^{\mu\nu}u_\mu u_\nu - 1 = 0$, where u_μ is the aether vector field.

Our starting point is the action [82]

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R + \lambda(u^\mu u_\mu - 1) + K^{\alpha\beta}_{\mu\nu}\nabla_\alpha u^\mu \nabla_\beta u^\nu\Big], \tag{4.2}$$

where

$$K^{\alpha\beta}_{\mu\nu} = c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta^\alpha_\mu\delta^\beta_\nu + c_3\delta^\alpha_\nu\delta^\beta_\mu + c_4 u^\alpha u^\beta g_{\mu\nu}. \tag{4.3}$$

This action involves an above-mentioned constraint introduced with use of the Lagrange multiplier λ. The c_1, c_2, c_3, c_4 are some dimensionless constants. It is interesting to note that the term $R_{\alpha\beta}u^\alpha u^\beta$ proposed as the aether term in [77] arises in this theory (together with some other terms) for the particular case $c_3 = -c_2$ when the commutator of covariant derivatives yielding a curvature tensor emerges [83].

The corresponding equations of motion look like [83]:

$$\begin{aligned} g_{\alpha\beta}u^\alpha u^\beta = 1;\quad \nabla_\alpha J^\alpha_{\ \mu} - c_4\dot{u}_\alpha\nabla_\mu u^\alpha = \lambda u^\mu;\\ T_{\alpha\beta} = -\frac{1}{2}g_{\alpha\beta}\mathcal{L}_u + \nabla_\mu\Big(J^\alpha_{\ (\mu}u_{\beta)} - J^\mu_{\ (\alpha}u_{\beta)} - J_{(\alpha\beta)}u^\mu\Big) + \\ + c_1[(\nabla_\mu u_\alpha)(\nabla^\mu u_\nu) - (\nabla_\alpha u_{mu})(\nabla_\beta u^\mu)] + c_4\dot{u}_\alpha\dot{u}_\beta + [u_\nu\nabla_\mu J^{\mu\nu} - c_4\dot{u}^2]u_\alpha u_\beta. \end{aligned} \tag{4.4}$$

Here $\dot{u}^\mu = u^\alpha\nabla_\alpha u^\mu$, $J^\alpha_{\ \mu} = K^{\alpha\beta}_{\mu\nu}\nabla_\beta u^\nu$, and $\mathcal{L}_u$ is u-dependent part of the Lagrangian. We note again that the vector u_μ has nothing to do with the usual matter, so, the Einstein-aether theory is an example of a vector-tensor gravity.

So, now our task will consist in finding some solutions for these equations, or, to be more precise, in checking the consistency of known GR solutions within the Einstein-aether gravity.

As the simplest example we choose the spherically symmetric static metric, which is consistent since the vector u_μ is time-like, in order to satisfy the constraint. In our case, it is convenient to choose this metric in the form slightly different from (3.20), namely,

$$ds^2 = N(r)dt^2 - B(r)(dr^2 + r^2 d\Omega^2). \tag{4.5}$$

The consistency of this metric within the Einstein-aether gravity has been verified within perturbative methodology for various relations between the parameters c_1, c_2, c_3, f.e. $c_1 + c_2 + c_3 = 0$, and c_4 can be chosen to be zero without any problems since it can be removed through a simple change of variables (see details in [83]) so that the $N(r)$ and $B(r)$ turn out to be represented as power series in $x = 1/r$ providing that they tend to 1 at infinity as it must be, with some lower coefficients in these power series, up to $1/r^3$ terms in large r limit have been explicitly found in certain cases.

For example, treating the black holes solutions, one can show [83] that the metric

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{2\beta M^2}{r^2}\right) dt^2 - \left(1 - \frac{2\gamma M}{r}\right)(dr^2 + r^2 d\Omega^2). \tag{4.6}$$

is consistent in this theory, with $\gamma = 1$ (the usual value characteristic for Schwarzschild metric) and β expressed in terms of coefficients c_1, c_2, c_3. Actually this solution is the Schwarzschild metric modified by the additive term.

Similarly, much more solutions for the Einstein-aether gravity can be obtained, in particular, the cosmological ones. In this context, the detailed study of various cosmological aspects of this theory has been performed in [84] where the model involving two scalar fields coupled to Einstein-aether gravity was considered, and it has been explicitly demonstrated on the base of the numerical analysis of solutions that the consistent potential for these fields is the exponential one, and the de Sitter-like solutions can arise both in the past (inflationary Universe) and in the future (de Sitter attractor). Earlier the idea of using the Einstein-aether model in order to explain the cosmic acceleration has been claimed in [85]. All this allows to conclude that the Einstein-aether gravity can be considered as an acceptable solution of the dark energy problem. Besides of this, a detailed discussion of various aspects of Einstein-aether gravity, including discussion of plane wave solutions and observational constraints on parameters of the theory, can be found in [86]. Also, we note that the Einstein-aether gravity also displays some similarity to the Einstein–Maxwell theory, see [82].

However, it is clear that the Einstein-aether model is problematic from the quantum viewpoint. Indeed, its action involves a constraint. As it is well known (see f.e. [87]), a theory with constraints, being considered at the perturbative level, requires special methodologies like $1/N$ expansion which clearly cannot be applied to the Einstein-aether gravity since it involves only four fields u^μ. Moreover, in principle such a theory, when treated in an improper manner, can display various instabilities. Therefore, the natural idea consists in introducing the spontaneous Lorentz symmetry breaking not through constraints but through introducing some potential of the B_μ field displaying a set of minima. This idea gave origin to the bumblebee gravity [78, 79] which we begin to discuss now.

4.3 Bumblebee Gravity

So, let us start with considering the bumblebee gravity. Our initial point will be the action (4.1). The key features of this action, in comparison with the Einstein-aether theory, are the following ones.

First, this action is characterized by a generic potential, instead of the constraint, which makes it better for quantum studies since the usual perturbative methodology can be applied. Second, the kinetic term is Maxwell-like which is essential to avoid arising of ghost modes [88]. Again, the $\pm$ sign reflect the fact that $b^2 > 0$. We note again that the vacua $B_{0\mu}$ are given by the condition $B_0^{\mu} B_{0\mu} = \pm b^2$, and these vacua are not required to be constants, in a curved space-time, which avoids the difficulties connected with definition of the constant vectors in this case.

First effect to note here is that after Lorentz symmetry breaking, we will have Nambu–Goldstone modes: if we introduce the vector b_μ corresponding to one of the vacua, i.e. $b^\mu b_\mu = \pm b^2$, define $B_\mu = b_\mu + A_\mu$, and rewrite the action (4.1) in terms of b_μ and A_μ, the resulting form of the action will be given by the Maxwell term, plus the axial gauge term proportional to $(b^\mu A_\mu)^2$, plus new couplings of the vector A_μ with the curvature, like $A^\mu A^\nu R_{\mu\nu}$, plus the Carroll-like term $b^\mu b^\nu R_{\mu\nu}$ [77].

Let us discuss some exact solutions for this theory. First, we consider the static spherically symmetric metric, following the lines of [79]. For the reasons of convenience, we rewrite the metric (3.20) as:

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\rho(r)}dr^2 + r^2 d\Omega^2. \tag{4.7}$$

Then, we choose the vacuum vector to be purely radial, i.e. $b_\mu = (0, b(r), 0, 0)$, thus one has $\nabla_\mu b_\nu = 0$ if $b(r) = \xi^{-1/2} b_0 e^{\rho(r)}$, ξ is a constant, and the variable $\phi(r)$ becomes irrelevant within modified Einstein equations.

For this metric we find the only non-zero component of the Ricci tensor and the corresponding scalar curvature to be

$$R_{rr} = \frac{2\rho'}{r}; \quad R = \frac{2[1 + 2(r\rho' - 1)e^{-2\rho}]}{r^2}. \tag{4.8}$$

It is convenient to introduce a new dynamical variable $\Psi = \frac{1-e^{-2\rho}}{r^2}$. Its action will look like:

$$S = \frac{2}{\kappa} \int dtdrr^2 e^{\rho+\phi} \left[(3 + b_0^2)\Psi + \left(1 + \frac{b_0^2}{2} r\Psi' \right) \right], \tag{4.9}$$

where b_0 was defined above.

The equation of motion, after varying with respect to ϕ, is

$$(3 + b_0^2)\Psi + \left(1 + \frac{b_0^2}{2} r\Psi' \right) = 0. \tag{4.10}$$

Its solution is $\Psi(r) = \Psi_0 r^{L-3}$, with $3 - L = (3 + b_0^2)/(1 + b_0^2/2)$, and

$$g_{rr} = e^{2\rho} = (1 - \Psi_0 r^{L-1})^{-1}, \tag{4.11}$$

so, this component is similar to g_{rr} of the Schwarzschild metric, therefore our solution is characterized by the event horizon. In principle, more results for this metric can be obtained, f.e. the Hawking temperature [79]. The case when the b_μ vacuum vector possesses not only the radial component but also the temporal one has been also discussed in [79], as a result, the Schwarzschild-like solution will carry extra factor $e^{\pm 2K_i r^\alpha}$, where α is a constant, the sign $+$ is for the temporal component, and the sign $-$ for the radial one, with the values of K_i are different for these two components. Therefore, we conclude that the Lorentz symmetry breaking generates the black hole solutions.

Another important example is the cosmological FRW metric. Here we review its description within the bumblebee context presented in [89]. Explicitly, as a first attempt, we suggest the vector B_μ to be directed along the time axis, $B_\mu = (B(t), 0, 0, 0)$. Evidently, in this case the stress tensor for the bumblebee field vanishes, and the only nontrivial component of the equations of motion for the B_μ is

$$\left(V' - \frac{3}{2\kappa^2}\frac{\ddot{a}}{a}\right) B = 0. \tag{4.12}$$

Thus, the bumblebee field either vanishes or, at $\xi = 0$, stays at one of the minima of the potential. In this case, it is possible to show numerically that one has the de Sitter-like expansion of the Universe.

More generic solutions can be obtained for $B_{\mu\nu} \neq 0$. However, in this case the numerical analysis is necessary. Explicit studies carried out in [89] show that in this case, de Sitter-like solutions arise for many values of parameters of the theory confirming this a possibility to have a cosmic acceleration due to the bumblebee field, therefore, one can conclude that the spontaneous Lorentz symmetry breaking can explain the dark energy problem.

Finally, we consider also the Gödel solution (1.8). Within the bumblebee context it has been considered in [90]. In this case, the energy-momentum tensor is suggested to be a sum of that one for the relativistic fluid (we note that namely this form has been employed in [3]):

$$T^M_{\mu\nu} = \rho v_\mu v_\nu + \Lambda g_{\mu\nu}, \tag{4.13}$$

and that one for the bumblebee:

$$T^B_{\mu\nu} = B_{\mu\alpha} B_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} B_{\lambda\rho} B^{\lambda\rho} - V g_{\mu\nu} + 2V' B_\mu B_\nu, \tag{4.14}$$

where V' is a derivative of the potential with respect to its argument. Therefore, the modified Einstein equation (in an appropriate system of units where $\kappa = 1$) looks like

$$G_{\mu\nu} = T^M_{\mu\nu} + T^B_{\mu\nu}. \tag{4.15}$$

The Einstein tensor $G_{\mu\nu}$ and the matter energy-momentum tensor $T^M_{\mu\nu}$ (4.13) in the bumblebee gravity are the same as in the usual Einstein gravity with the cosmological term. Therefore, the Gödel metric continues to be solution in our theory if and only if the energy-momentum tensor of the bumblebee field will vanish. To achieve this situation, we suggest that the field B_μ is one of the vacua which, for the quartic potential $V = \frac{\lambda}{2}(B^\mu B_\mu \pm b^2)^2$, will yield vanishing of the potential and its derivative. So, it remains to find the vacuum for which the stress tensor $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ would vanish as well (the part proportional to Christoffel symbols vanishes identically). It is clear that the case of the constant B_μ is an excellent example. Some interesting cases of such vacua, for the metric in the form (1.8), are: $B_\mu = (ab, 0, 0, 0)$, $B_\mu = (0, ab, 0, 0)$, $B_\mu = (0, 0, 0, ab)$ (we note that the Gödel metric is characterized by the constant parameter a).

It remains to check consistency of these solutions with the equation of motion for the bumblebee field:

$$\nabla_\mu B^{\mu\nu} = 2V'(B^2)B^\nu. \tag{4.16}$$

These equations are satisfied immediately. Indeed, the l.h.s. is zero since $B^{\mu\nu} = 0$ for these solutions, and its covariant derivative is also zero, and the r.h.s. is zero for the quartic potential, if B_μ is one of the vacua. Therefore, we conclude that the Gödel solution is consistent in the bumblebee gravity. More detailed discussion on this solution can be found in [90]. It is clear that a more generic Gödel-type solution (2.26) can be analyzed along the same lines.

An interesting discussion of the bumblebee field is presented also in [91]. The starting point is the generalized bumblebee Lagrangian

$$\mathcal{L} = R - \zeta \bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta}B_{\alpha\beta}B_{\gamma\delta} - V(B^2), \tag{4.17}$$

where V is a some potential of the bumblebee field, ζ is a coupling constant, and $\bar{g}^{\alpha\gamma} = g^{\alpha\gamma} + \beta B^\alpha B^\gamma$ is the effective metric.

Then, we carry out background-quantum splitting for gravitational and bumblebee fields by the formulas $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ and $B_\alpha = \bar{B}_\alpha + A_\alpha$, where $\bar{B}_\alpha$ is one of vacua, i.e. $V(\bar{B}^2) = V'(\bar{B}^2) = 0$.

As a result, we arrive at the linearized equations of motion for the fluctuations $h_{\alpha\beta}$, A_α:

$$\begin{aligned} G_{\alpha\beta}| &= V''(\bar{B}^2)\bar{B}_\alpha\bar{B}_\beta B^2|, \\ \bar{\eta}^{\alpha\delta}\bar{\eta}^{\beta\gamma}\partial_\beta F_{\gamma\delta}[A] &= \frac{1}{2\zeta}V''(\bar{B}^2)\bar{B}^\alpha B^2|. \end{aligned} \tag{4.18}$$

where $|$ symbol is for a part linear in fluctuations $h_{\alpha\beta}$, A_α, f.e. $B^2| = 2\bar{B}^\alpha A_\alpha - \bar{B}^\alpha\bar{B}^\beta h_{\alpha\beta}$, and $\bar{\eta}^{\alpha\delta} = \eta^{\alpha\delta} + \beta\bar{B}^\alpha\bar{B}^\delta$. The $F_{\gamma\delta}[A] = \partial_\gamma A_\delta - \partial_\delta A_\gamma$ as usual.

We can introduce background-dependent densities

$$\begin{aligned} \rho_m &= -V''(\bar{B}^2)\bar{B}^2 B^2|, \\ \rho_e &= \pm\frac{V''(\bar{B}^2)\sqrt{|\bar{B}^2|}}{2\zeta}B^2| \end{aligned} \tag{4.19}$$

and a 4-velocity $u_\alpha = \pm\frac{\bar{B}_\alpha}{\sqrt{|\bar{B}^2|}}$, as a result the equations of motion become

$$G_{\alpha\beta}| = \rho_m u_\alpha u_\beta,$$
$$\partial^\beta F_{\beta\alpha}[A] = \rho_e u_\alpha, \tag{4.20}$$

replaying thus the Einstein and Maxwell equations respectively. Effectively we showed that our background field B_μ plays the role of the charged dust. We note that in principle, the $\bar{B}_\alpha$ and A_α fields can be coupled to usual matter in various manners being treated either as a usual photon or as a some extra particle.

To conclude, we see that the bumblebee gravity can be treated as a sound candidate, first, to implement the Lorentz symmetry breaking within the gravity context, second, to display consistency with astronomical observations, due to validity of most important general relativity solutions. Among other results one can mention study of dispersion relations in a linearized bumblebee gravity where the constant bumblebee field triggers deviations from the standard dispersion relations [92]. However, much more aspects of the bumblebee gravity, especially problem of validity and consistency of many other solutions, are still to be studied. In this context, one of the most important issues is the study of perturbative aspects of the bumblebee gravity, and only first steps along this line are done now.

4.4 Conclusions

We discussed vector-tensor gravity models. Just as in the previous chapter, the additional field, in this case the vector one, is treated not as a matter field but as an ingredient of the complete description of the gravity itself. The most important aspect of these models consists in the fact that some of them, namely those ones involving potential terms for the vector field, can be extremely useful within the context of the spontaneous Lorentz symmetry breaking. The known examples of these theories are the Einstein-aether gravity and the bumblebee gravity.

The Einstein-aether theory has been formulated earlier. Within it, the potential term generating the spontaneous Lorentz symmetry breaking is implemented through the constraint with the corresponding Lagrange multiplier field. From one side, this action is rather simple, but from another side, the presence of the constraint generates essential difficulties for the perturbative description. Therefore, the bumblebee model is certainly much more promising. Moreover, the bumblebee approach displays an advantage in comparison with the naive application of the QFT approach suggesting to couple dynamical fields with the constant vectors (tensors) which, as we already noted, cannot be consistently defined in a curved space-time.

The bumblebee approach allows to introduce many Lorentz-breaking vector-tensor terms. The term $B^\mu B^\nu R_{\mu\nu}$ from (4.1) is effectively nothing more that the gravitational aether term proposed in [77]. We note that treating of the B_μ as one of the bumblebee vacua rather than the usual constant vector allows to avoid breaking of the general covariance. In a similar manner, other Lorentz-breaking gravitational terms introduced in [74] can be treated. As a result, relaxing the condition for the Lorentz-breaking vector to be constant, we have a theory consistent with the general covariance requirement.

We note that the term $B^{\mu}B^{\nu}R_{\mu\nu}$ is the particular case of the term $s^{\mu\nu}R_{\mu\nu}$ discussed in [74]. Actually, in [74], two terms are presented, so, the possible Lorentz-breaking extension of gravity is introduced through adding the term

$$\delta S = \int d^4x\sqrt{|g|}(s^{\mu\nu}R_{\mu\nu} + t^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}), \tag{4.21}$$

where $s^{\mu\nu}$, $t^{\mu\nu\lambda\rho}$ are coefficients of explicit Lorentz symmetry breaking (in this review, we consider only the zero torsion case). However, up to now the main attention (see f.e. [92]) was paid to the $s^{\mu\nu}$ term while the $t^{\mu\nu\lambda\rho} = 0$ condition was applied.

To close the discussion of the Lorentz symmetry breaking in gravity, let us say some words about the weak (linearized) gravity. We have noted already that, for the specific form of the Chern–Simons coefficient, the gravitational CS term (3.6) displays Lorentz symmetry breaking. In [47], another, one-derivative Lorentz-breaking term in the linearized gravity has been studied. In principle, much more Lorentz-breaking terms in the linearized gravity can be introduced. However, it is clear that many studies of Lorentz symmetry breaking in gravity are still to be carried out, and it is natural to expect that such studies will be performed in the next years.