Chiara Andrà
Domenico Brunetto
Francesca Martignone *Editors*

Theorizing and Measuring Affect in Mathematics Teaching and Learning

Insights from the 25th International Conference on Mathematical Views

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Preface

The International Conference on MAthematical VIews (MAVI) has come to its 25th edition in 2019 and it has been an honor and a pleasure to host it in Intra (Italy), in the beautiful surroundings of Maggiore Lake from June 6 to June 9. As for MAVI tradition, the number of participants was small, young and expert scholars had the opportunity to share their ongoing research on affect-related issues in Mathematics Education. Many newcomers to the MAVI conference joined us for the first time, according to MAVI's spirit and aims. Newcomers bring new perspectives and new research foci; hence, the following chapters in this book can be read in terms of a balance between two binary classifications: traditional themes and methods versus innovative ones and perspectives "from inside" versus "from outside" the MAVI community.

The table of contents partly reflects this special feature of MAVI25: Part I of the book is a collection of invited chapters from colleagues who have published research results that are of utmost interest for the themes that are at stake within MAVI. In particular, emotions and identity are emerging themes in affect-related research, which have been dedicated little attention in the past (if compared to beliefs). Furthermore, sociocultural approaches to teaching and learning suggest new ways of considering the role of affective variables in mathematics classrooms, as well as embodiment invites us to consider the body as an integral part of thinking. New theoretical perspectives and new constructs require also a reflection on the methods of research: three participants to MAVI25 try to summarize methodological issues emerged during the conference in the last chapter of Part I. This is not to be intended as an exhaustive and comprehensive overview of all the methodologies employed in affect-related research, but rather as a window into the lovely discussions that took place during the conference.

The subsequent two parts of the book, namely Parts II and III, are dedicated to beliefs, which are under study within the MAVI community since its start in 1995 and with no doubt constitute a tradition. During the conference, we had the opportunity to reflect and discuss about beliefs within new theoretical perspectives, on one's side, and on the other's side new methodologies for capturing and examining beliefs allow us as researchers to understand teachers' and students' beliefs about mathematics not only in terms of "what an individual declares" in words, but through pictures, drawings, gestures, and the like. The reader will find examples of different kinds with respect to the potentialities and limitations of methodologies that rely on images and on body movements (e.g., gestures) to study a person's beliefs and more generally her affective state in a mathematical activity.

Part IV is dedicated to a theme that is among the traditional foci of friends in MAVI, namely the transition from one school level to another one, while Part V is dedicated to an emerging issue in affect-related research, that is: cultural aspects in the affective dimensions of mathematics teaching and learning. These last two parts are shorter, if compared to the first three ones, but collect: (1) perspectives from stakeholders on the theme of transition, giving voice to teachers and lecturers who "live" the transitions through the eyes of their students, and (2) reflections on cultural issues which are gaining increasing attention in educational research in general. Both themes are linked back to traditions within MAVI studies, but for sure they point to original and emerging issues. In our wishes, they represent a first step toward new research interests for the MAVI community and for Mathematics Education in general.

This book is dedicated to young researchers and PhD students who approach affect-related research for the first time and it complements recently published books on mathematical affect by blending traditional perspectives (which are recapped to a significant extent) with innovative and sometimes out-of-field ones, in order to give to the reader a broad sense of "affect" in mathematics education. It also represents an invitation for our colleagues to join the next MAVI conferences, to discuss affect-related themes in a friendly and open-minded group of scholars.

Torino, Italy Chiara Andrà
December 2019

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Part I
Emerging Theories in Affect-Related Research, and Related Methodologies

Chapter 1
Emotions and Learning

Peter Liljedahl and Chiara Andrà

1.1 Research Traditions: Emotions and Anxiety

A class of pre-service elementary teachers was attending a course on mathematics method, lectured by the first author. Although seeing the merit of the many alternative methods he was modelling, the teachers seemed not ready to abandon the traditional "drill" method (direct instruction, repetitive exercises, and timed tests) of teaching fluency of basic facts. In fact, many of the prospective teachers mentioned that they regularly used *Around the World*, which is a basic facts activity during which the students stand in a circle and the teacher points at one of them and asks them a basic multiplication question (e.g., 3×4, 6×8, etc.). Each student has 2 s to respond. If the student responds correctly in that time she would be allowed to sit down. If she fails to give response, or the response is incorrect, she would remain standing and the teacher would come back to her after having gone all the way around the class. This would continue until all the students were sitting. This game is often used as a way for students to practice their basic facts. It is traditionally given once a week at school. The possible negative consequences of this method are many, yet it continues to be practiced for its efficiency, simplicity, tradition . . . and parents like it.

To emphasise the potentially negative consequences of this method, the first author began a lesson by gathering the prospective teachers around him and told them that they would be playing *Around the World*. He noticed that they were, as a group, visibly uneasy. There were a few who seemed excited at the prospect of

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playing a "game" and the thrill of competition. But the vast majority were horrified at what was about to happen. When the tension had built to a crescendo, he pointed at the first prospective teacher and, instead of asking a basic multiplication question, asked, "How are you feeling right now?" And then, to the whole group, "How are all of you feeling right now?" The relief in the room was tremendous, and for over an hour they talked about their past experiences, sharing the negative impact these types of "games" had on them as learners. Lots of emotions arose. Most of the evoked emotions were related to anxiety. A few of them shared their positive experiences with these types of activities, but even then quickly acknowledged that their enjoyment was not worth the price of misery that the rest of the students had to pay. Not only emotions emerged, however. The soon-to-be teachers expressed their beliefs and showed a shift of attitudes towards teaching mathematics, becoming aware of the negative consequences of rather traditional test-and-drill methods. They discussed why parents liked these "games" and ways, as future teachers, to deal with that. In the end they vowed, individually and as a group, that they would never do this to their future students.

At first glance, a conclusion can be drawn from this episode: prospective teachers hold central and hard-to-change beliefs (see, e.g., Philipp 2007), which have been developed during school years, and they had powerful emotional experiences and those emotions are brought forward at university and in the transition from undergraduate courses to teaching practices. One of the emotions that emerges the most is anxiety (as it is also noted by Zan et al. (2006)). After the activity proposed by the first author, Beth described her experience as "heart racing *anxiety*! The thought of being picked on and not knowing gives me the *heebie jeebies*, especially in a subject that is probably my weakest. Being that it is multiplication and is something that I probably would get right doesn't really help shake the feeling you get when you know that there is pressure to perform. [. . .] If I feel like this at 23 how would a kid feel?" (Liljedahl 2014, pp. 24–25). And Jocelyn adds "I am feeling really *anxious* and *nervous*. I am worried about being embarrassed about not being able to answer the multiplication question in front of the class and I am also really worried about being the last person standing" (Liljedahl 2014, p. 25).

Mathematics-related *anxiety* has a long tradition of research (Zan et al. 2006) and it has been understood as a by-product of "traditional" mathematical learning activities like *Around the World*. In general, emotions have been traditionally understood as a reaction to an interpretation of an experience (Mandler 1984). For Radford (2015), the term "emotion" entails a cognitive reflection on one's own physiological reaction to a stimulus from the environment, namely an interpretation about how one "feels" with respect to the actions and reactions undertaken. If we take fear as an example, we can refer to Massoumi's (2005) understanding of the way perception of fear is transformed into reflection about fearful feeling, until perception and reflection overlap: "The perception has been wrapped in reflection, and the reflection, in turn, has been taken up in memory" [p. 38]. As a consequence, emotions have a state nature and are fleeting, but they can fix and assume a trait nature (Hannula 2011). For these reasons, emotions are acknowledged to affect learning in general (Zan et al. 2006) and cognitive processing in particular (Hannula 2002).

From what Liljedahl (2014) calls an "acquisitionist perspective", the role of emotions in learning and teaching is understood in these terms. However, such a view fails, in Liljedahl's (2014) claim, to answer the question as to *what psychological mechanisms link emotions to behaviour*. In order to answer to this question, we suggest to shift to a Vygotskian, or participationist (Liljedahl 2014), view as we briefly recall in what follows.

Activity theory (Leont'ev 1978) maintains that any human activity, including mathematics, problem-solving, etc., has a goal. For example, Liljedahl (2014) notes

> All of these teachers wanted to be good teachers. This was one of their many motives. But they also wanted to please parents, have their students be good at basic multiplication facts, and to not make their students anxious or fearful, to name a few. These many goals were organized into hierarchies, unique to each prospective teacher. For the most part, these prospective teachers were not aware of many of their motives. Instead, they were fixated on their current goals of learning how to teach mathematics, getting good grades, and/or having their knowledge experience acknowledge. The "emotional residue" left from their experience playing *Around the world* helped them to see some of these motives. And it helped them to re-orient them. (p. 29)

It is not only negative feeling that prevent the teachers to propose this game to their future students, but it is mostly an emotional sense that learning can take place under better conditions if alternative teaching methods are taken into consideration. It is not merely the goal of avoiding negative emotions in elementary mathematics classrooms, but a sense of likelihood of more effective learning, attached to alternative teaching methods, that is surfacing, even if not all the teachers are aware of such a change in their motives. A sense of "better" success as teachers, indeed, is unfolding from the teachers' narratives, as it can be exemplified from this quote from Khaly: "I'm not afraid of mathematics any more, to learn or to teach. I also think that mathematics can actually be fun. I am excited to teach my new students (when I get my first class). Show them that math is not as scary as it seems" (Liljedahl 2014, p. 26). The goal has been refined: from "teaching mathematics" to "teach mathematics that is fun". In a participationist approach to mathematics-related emotions, these goals are seen not only as a consequence of learning experiences, but a comprehensive view that entails emotions, motives, goals, and actions of teacher is taken. Within this episode, and with Liljedahl (2018), we see emotions intertwined with beliefs in a unique and connected affect system.

As also Radford (2015) notes, the aforementioned shift of perspective entails a radical leap from considering emotions as inner, subjective, and physiological experiences, usually of an irrational nature, to a sociocultural conception of emotions. Radford (2015) argues that, rather than momentarily subjective phenomena, emotions (for instance, anger, frustration, love) are socially and historically constituted. In order to understand this shift of perspective, and the potentialities for the researcher, we now present another example, from a longitudinal classroom observation undertaken by the second author within the Italian *BetOnMath* project, a research project aimed at developing problem-solving activities to model betting games and to prevent gambling abuse by secondary school students (see Andrà et al. 2016).

1.2 Challenging the Tradition: Emotions as Orienting Experiences

In this episode, three students are dealing with a task regarding the Italian Lotto game. In the Lotto game, a player bets on 5 numbers, selected from 1 to 90 (with no repetition). In 10 different Italian cities, 5 numbers are randomly and independently extracted from the set of 90 numbers (from 1 to 90). The extracted number is not put again with the others, so that there is no repetition of numbers in the same city. An example of extraction is presented in Fig 1.1. Mathematically speaking, a "cinquina" is a combination without repetition of five different numbers selected from 1 to 90. The player wins if he guesses all the five numbers, or at least two of them, extracted in the same city (namely, the same row in Fig. 1.1).

The students have to compute the number of all possible cinquinas that can be extracted in a city. Carlo, Elisa, and Giulia are three grade-12 students, who have been already given the formulas of combinatorics. The teacher has also provided them with a schema that organises the use of the different formulas according to the different situations they model. Namely, combinations, permutations, dispositions are understood in the schema according to two criteria: (1) whether the order is important and (2) whether the elements can be repeated. On Giulia's notebook we read that in permutations, the order is important and the elements cannot be repeated. In combinations, the order does not count and the elements cannot be repeated (like in cinquinas). The episode begins with Elisa reading the text of the task out loud. She has a sceptical facial expression and the tone of her voice reveals that she is a little bored.

Giulia, who conversely seems involved and interested, fixes her glance to Elisa and, after having listened to Elisa reading the question, invites her classmates to use the schema. She then looks at the schema and concentrates on reading it. Having the schema in front of her, at her disposal, seems to provide her with a sense of confidence that she will be able to actively contribute to the group work. Carlo and Elisa, who are more self-confident as we can infer by their posture, follow Giulia and turn their eyes to the schema, then Giulia reads it: "Is the order important?".

Ruota	1^ estr.	2^ estr.	3^ estr.	4^ estr.	5^ estr.
Bari	81	43	31	90	56
Cagliari	35	47	31	67	74
Firenze	4	64	56	32	16
Genova	52	17	13	38	89
Milano	35	3	34	16	28

Fig. 1.1 March 6, 2014 extraction (taken from the Italian official Lotto website)

Both Carlo and Elisa say “yes”, looking at her, but Giulia keeps looking at the schema and doubts this: “Why is it important?”. She glances rapidly to Carlo and Elisa, then Giulia closes her eyes. Carlo replies before she has ended her question, and Giulia looks at him when he starts saying: “The first extraction is this one, the second extraction is this one ... ” The three students end up looking at the paper where Carlo is pointing.

On the outcomes of the Lotto game, in fact, it is written: “1st extraction”, “2nd extraction”, and so on until “5th extraction” (see Fig. 1.1, first row). Carlo, hence, is mistaken by the presentation of the Lotto game, namely he misinterprets the fact that extractions are made in a certain order with the fact that a better wins whichever the order of the extraction of the numbers. Elisa takes for granted Carlo’s proposal and nods. Carlo’s answer to Giulia’s question about why do they consider the order does not convince her, but she goes on reading the schema: “disposition or permutation?”. Elisa proposes: “disposition”. In dispositions, “the same element can be repeated”, reads Giulia, that echoes Elisa. Giulia is confident to have understood this feature and Elisa follows her. It is not disposition. Carlo is silent, and doubtful. He curls his hair with a perplexed facial expression. Giulia says out loud the conclusion that can be drawn from the path they have followed: “the number of possible cinquinas is 905, this is what we can read from the schema”.

Carlo is still doubtful, he has not changed his posture and facial expression, and repeats “90 to the power of 5” and adds: “No, wait. Does it repeat?”. Elisa echoes Carlo’s facial expression and stares at Giulia as if she is “guilty” of having proposed repetition. Giulia takes the floor: “Yes, it is possible that the numbers are repeated”. She is confident. Carlo reacts: “For example, here there is 68, can there be another 68 here?”. Giulia looks at the paper, and notices: “Ah no!” She gets at the same point at which Carlo is. Also Elisa is with them: they confirm to each other that this is right. The relief given by feeling that they all agree about “no repetition” has short life, since they also feel that there is something wrong with their conclusions, but they do not know how to fix it.

Carlo goes back to the first question written on the schema, and asks: “Is the order important?”. Giulia, whose first idea was that it is not important, silently looks at Carlo, waiting for him. Her glance to him is intense, as if she is expecting that Carlo changes his mind. Carlo, instead, asks for the teacher’s help. The teacher approaches the group, she listens to Carlo’s question but she actually does not reply to their doubts directly. She clarifies the meaning of 1st extraction, 2nd extraction, etc. Carlo, then, concludes “so, the order is not important!” and Giulia stays silent, but her face tells her satisfaction. She now knows that she was right from the beginning.

From Radford (2015) we notice that Carlo’s, Elisa’s, and Giulia’s emotions are entrenched in both physiological processes and conceptual categories through which the three students perceive, understand, reflect, and act. In other words, it is not because Carlo, or Giulia, became emotional that they fail to think and calculate in an appropriate way. Although unpredictable in fine-grain details, the emotional–cognitive process, that the students underwent, unfolded shaped by the manner in which each student perceives him/herself in his/her relationship to both mathematical knowledge and to the others. In other words, these students’ emotions are the emotions of concrete and unique persons but, on the other hand, they relate to a sociocultural and historical world that transcends the individual. Giulia's emotions relate to the way her peers react to her proposal, and to the way the teacher responds to their doubts. Emotions are personal, and at the same time they are social. Furthermore, emotions and thinking are strictly inter-related, to the point that

> Only computers can "think" without feeling anything. They do not even feel the heat of their chips. They feel nothing. They display pure mechanical calculations of which humans are definitely incapable. We can make some calculations, and we can do it while feeling boredom, thrill, excitement, challenge or something else; what we cannot do is simply feel nothing (Radford 2015, p. 27).

There are other, several considerations that can be made from this episode. The first one has a relatively methodological nature. In the first episode, even if the first author is able to notice his teachers-to-be emotional burden directly, in the room, at the time the activity took place, he relies mostly on their narratives to report their emotional experience. In the second episode, we infer the emotions from the students' bodies. This shift of attention from reported emotions to emotions expressed through the body is possible because, as Roth (2000) notes, "the human body maintains an essential rationality and provides others with the interpretive resources they need for building common ground and mutual intelligibility" (Roth 2000, p. 1685). In other words, human beings are well equipped for correctly inferring the emotions felt by other individuals. More specifically, simulation theories (e.g., Goldman 2006) refer to mirror neuronal circuits to suggest that, in order to recognise an interlocutor's actions, the perceived action is simulated in one's own motor system. The idea extends to emotions (e.g., Gallese et al. 2007), so that in understanding how others feel we experience that emotion ourselves. By extension, then, when we are engaged with others in social interaction it seems that one aspect of the interaction should be such simulation to the point that interlocutors mimic each other's actions, including gestures, and share similar emotions. Vertegaal et al. (2000) make a strong link between the amount of eye contact people give and receive to their degree of participation in group communications. The occurrence of mimicry, or echoing, in co-speech gesturing has been examined by Kimbara (2008) and in face-to-face communication by Holler and Wilkin (2011) , who concluded that "mimicked gestures play an important role in creating mutually shared understanding" [p. 148]. Non-verbal gestures were also found to be important in signalling incremental understanding, something the authors paraphrased as "I am following what you are saying" [p. 145]. The potentialities of such a prioritisation of focus to the body, as well as to movement of learners, in line with a participationist vision, are well discussed in Ferrari and Ferrara's chapter (this book, Chap. 2).

The second consideration we can make is that we can confirm that (students') emotions are related to motives in a time-projection manner: they relate to the possibility to succeed (or to fail) in reaching the object of the activity. Giulia, for example, lives an emotional sense of unlikelihood of success when the group follows Carlo's idea that "the order of extractions is important". We can comment that this emotion can be counted as "negative", and related to frustration. However, it is this particular emotion, which surfaces also for Carlo later, that contributes for the activity to take another, possibly correct, direction. An understanding of emotions as orienting the actions allows us to overcome a traditional, binary distinction between "positive" and "negative" emotions. The former, like joy and pleasure, are traditionally deemed to be elicited and promoted in mathematics classroom, whilst the latter, among which we count boredom, anxiety, fear, discomfort, have to be avoided. Hannula (2011) notes that early studies on emotions and problem-solving point out that good achieving students are more able to control their emotions. Within a sociocultural perspective, however, emotions are not to be avoided or controlled in mathematical activities, but they have to be provoked, made visible and talked about, and above all emotions can be exploited by students to solve mathematical problems since, being pre-verbal and pre-conceptual, emotions provide the learner with a sense of likelihood of success of the actions she is undertaking in order to solve the problem (see Radford 2015). This role of emotions also surfaced in the first episode involving pre-service teachers.

We can also note that traditional approaches to teaching try their best to avoid negative emotions but, as it emerged from the first episode, they rather promote them and their sedimentation in learners' experiences. Conversely, it seems that an alternative approach to teaching, where "negative" emotions are welcome and somehow deliberately promoted (see also Andrà et al. 2016) allow the learners to have positive learning experiences and provoke a change in pre-service teachers' views of mathematics as having fun (Liljedahl 2014).

1.3 Conclusions and Further Perspectives

As Liljedahl (2018) notes, affect-related research in Mathematics Education tends to focus on a single affective variable at time. This allows the researcher to deeply investigate a single affective dimension, but at the same time impedes us to investigate how affective variables tend to cluster (Green 1971, theorises belief clusters and we believe that this idea can be extended to all affective variables). Liljedahl (2018) offers a justification for this phenomenon by claiming that, perhaps, we lack of theories for "affective clusters" and proposes a more comprehensive view of affect. In this way, we can understand teacher change in terms of a change in their beliefs, attitudes, motives, values, and emotional disposition as a whole and somehow coherent system. Not necessarily "logically coherent", but psychologically coherent and central. A network instead of a fragmentation of affective variables can be, thus, investigated. Hannula's (2011) meta-affective framework for research on affect also points to this direction and we wish that future research in affect-related mathematics education takes this approach further.

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Chapter 2
Affective Bonds and Mathematical Concepts: Speaking of Affect Through Sympathy

Giulia Ferrari and Francesca Ferrara

2.1 Filippo and the Pattern

In a grade 3 classroom, the (8-year-old) children solved a patterning activity, in the context of a long-term intervention aimed to develop early algebraic thinking at primary school. For this chapter we draw attention to a single child, Filippo, who had just worked in pair with Lara on a written task. The task involves the figural pattern shown in Fig. 2.1, which captures terms of the form $6n - 2$ (or $6(n - 1) + 4$), being n the position or figure number.

The children already encountered the pattern in grade 2 when focus was put on the recursive way of passing from one term to the next, in order to complete the pattern with figure 2 and to find the missing figures 5 and 6. The task in grade 3 represents the same pattern but asks the children to shift attention to the mathematical structure that captures the pattern.

Right after pair work, Filippo approaches the teacher. He proudly wants to share what he and Lara have been discovering while facing the task. In particular, the two students have disclosed a direct relationship between the number of circles on the bottom row of each figure and the corresponding figure number. The relationship is captured by the division by 2 as follows: *the figure number is equal to the number of bottom circles divided by 2*. We report the experience of Filippo after he tells this story when the teacher challenges him to a new task: to find the position of the twenty-two circle-term.

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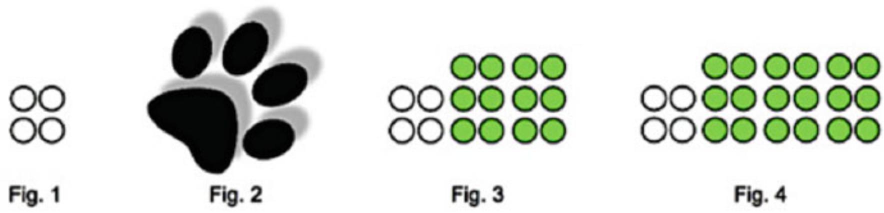

Fig. 2.1 The figural pattern at hand

Excerpt

Teacher: I have a position, which I don't know [*Filippo jumps, excited*], which has twenty-two circles, how can I discover which position is?

Filippo: Twenty-two circles? [*Puzzled*]

Teacher: As a whole [*Performs a grouping gesture in the air*]

Filippo: As a whole? [*Amazed and curious*]

Teacher: Yes!

Filippo: Twenty-two, oh, you take away four from twenty-two [*Performs the taking away by moving his hands together in front of his torso from right to left*. Figure 2.2 (1i). *Looks at the teacher*] and you get eighteen, and it's the first group [*Mimes a grouping*. Figure 2.2 (1ii)] of four [circles], eighteen. Then, you take away six from eighteen [*Mimes a block, with a top-down movement of his right hand, then looks at the teacher*. Figure 2.2 (2i)] and you get twelve [*Shifts right hand to the right, moves closer to the camera, sliding with his body to his right hand side*] that, so, are four and a row of six [*Mimes the grouping again and a new block with a vertical gesture*. Figure 2.2 (2ii–2iii), *looks at the teacher*]. Then [*Moves closer again with both hands open to mime the remaining circles, bends his head forward*], take away six from twelve and it gives six [Fig. 2.2 (3i)], so they are four [*Marks a grouping with left hand*, Fig. 2.2 (3ii)] and two rows of six [*Mimes a block with right hand*, Fig. 2.2 (3iii), *left hand still in the same position, marking the group of four*]. Then [*Pauses*], you do six minus six [Fig. 2.2 (3iv)], it gives three rows of six [*Keeps still left hand, mimes the three blocks with right hand, looks at the teacher*], plus four [*marks the grouping again with left hand, look at the teacher, smiles*], and then, oh, there are no more [*Turns towards the teacher*. Figure 2.2 (3v)], you do, they are four plus three rows of six [*Mimes the grouping with left hand, the blocks with right hand unfolding from left to right*. Figure 2.2 (3vi) left]

Teacher: Ok, so which position is? [*Filippo still keeps the left and right hand suspended*, Fig. 2.2 (3vi) right]

(continued)

Filippo: So, well, [*moves to his left, left hand keeping the group of four*] one, two . . . [*Slides to his right, points with left and right index fingers, looks at the teacher*. Figure 2.3] three, four . . . [*Slides to his right, points with left and right index fingers*] five, six . . . [*repeats the movement and gets out of the camera's sight*] four! [Fig. 2.3]
Teacher: Position four! [*Filippo smiles*]

In the short dialogue, Filippo focuses on the pattern through an inverse relationship: indeed, he is asked for the first time to find the figure number by knowing only the total number of circles that constitute the figure. We observe that the child moves in coordination with the pattern while recreating it, enmeshing the arithmetic relationships that emerge from the structural appearance of any term. We also see how the use of space and directionality in his movements actualises a creative (productive) engagement with the pattern, in a way that mobilises both the student's body and the linear (recursive) relationship that the sequence embeds.

Our aim in this chapter is to unfold these aspects in order to present a discussion of the episode in terms of *sympathetic agreement* between Filippo and the pattern. In doing so, we will use the concepts of affectivity and sympathy to propose a theoretical perspective in conversation with current theories of affect.

In particular, we will draw on these concepts to shed light on the ways in which an affective, sympathetic movement can illuminate mathematics teaching and learning.

2.2 The Sensuous Body

In the 1-min interaction with the teacher, Filippo moves around, shifting his body, tilting his head, gesticulating with his hands, gazing from one place to another, stepping back and forth. We dwell into the very *sensuous* nature of Filippo's experience of the pattern.

Initially, Filippo jumps, dangles with his shoulders then little with his torso, widely gesticulates to grasp the totality ("As a whole?") of circles to which the teacher refers. As he digs into the problematic of finding the position of the twenty-two circle-term, he moves about actions through which to operate on the number of circles and re-construct the corresponding figure. He gestures operations: the taking away; he gestures differently specific parts of the figure: the first group, and the rows of six; he gestures the width of the entire figure—twice—as if to underline its massiveness; he moves from left to right (farther to closer to the teacher), as if to follow the recursive growth of the pattern; he moves farther again, to the left, as to keep the ordinal unfolding of the figure, and the usual positioning of the bottom circles. At the end, once the figure is entirely there, with the circles spatially imagined in the same way as in the pattern, he bends over, towards the

Fig. 2.2 Filippo's movements and the pattern

little bottom circles as if to significantly isolate them from the rest, he points at each of the imagined green circles to count them and disclose the position of the

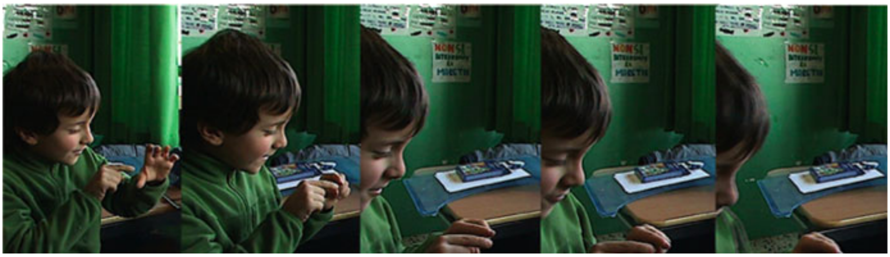

Fig. 2.3 Filippo counts the bottom circles

figure. The head and the eyes move, among the teacher, the outside (in pausing instants), and the figure. Imagination is visceral to the moving body, and to the thinking process. The figure is brought forth by imagination: in this sense, it is there and not there, past and future, the was and the yet-to-be, at the same time. It is actualised by the bodily movements, the gripping gazes, the repentine turns and changes, bigger than it was on paper, in front of Filippo's eyes. At the same time, it is not there as a physical coloured diagram, but the different colours emerge from the degree of different gesturing (performed with left or right hand, groupings vs. miming blocks). In recreating the figure for naming it, Filippo's gestures are entangled with imagination of the figure itself. We can see how this also speaks to the imaginative dimension of diagramming, which is nourished by the specificity of the diagram proposed in the task (circles' disposition, different colours, and so forth).

We refer to Filippo's body as the *sensuous body* to stress the ways in which the body, through the senses, is constitutive of the understanding of the pattern and meaning making is pregnant with multimodal engagement. We are inspired in the use of the word by the perspective of a sensuous, or multimodal, cognition, as introduced first in Radford (2009) and further discussed in Radford (2013). According to this approach, gestures and bodily actions (the tactile and the kinaesthetic) are not ephemeral and epiphenomenal aspects of the cognitive process that announce the imminent arrival of abstract mathematical thinking, but genuine constituents of it as they unfold against the background of social praxes, like those of the mathematics classroom. The discussion of movement and learning in concurrent years brought forth similar visions of how thinking occurs in the gestures and the bodily movements themselves rather than behind them (e.g., Nemirovsky et al. 2012; Sheets-Johnstone 2012; Stevens 2012). In the sensuous cognition perspective, sensation is considered as the substrate of mind and all psychic activity (cognitive, affective, volitional, etc.). While moving away from a vision of mathematical knowledge as external to and separate from the sensory-motor to one that sees it as emerging out of the bodily, we want to contribute to the recent line of research on embodiment and the body specifically in relation to the affective dimension of mathematical activity.

Recovering the somatic and embodied nature of Filippo's experience of the pattern allows us to focus on the role of the human body at a first extent. We have looked at diverse nuances of bodily movement in the short video, but we still need to account for the qualitative experience of movement and the way that it sustains knowledge of the pattern in the event. We need to shift attention to the forces that populate Filippo's engagement with the task and his positioning toward the pattern. Therefore, we turn the discussion to the *emotional nature* of Filippo's experience of the pattern.

2.3 The Affective Body

Filippo's bodily movements can be analysed at different scales. We can treat the body as that which permits identifying traits of embodied, sensuous cognition that are visible to the observer. Investigators can easily see these bodily movements as perceptuo-motor-imaginary activity (Nemirovsky and Ferrara 2009), or ways of working with a number of circles and re-organising/re-presenting them in a spatial array that is constitutive of a specific figure. These might be too quickly interpreted as evidence, in the particular situation, of how the child uses his (bodily) resources to successfully solve the task posed by the teacher, overlooking the fact that Filippo affects and is affected by the pattern as he grapples with the mathematical concepts. Indeed, we can observe his movements as the ways in which Filippo and the pattern respond to each other. The concept of number is not just embodied, but also affected and dispersed as regards its force to form a relational assemblage with the child. Movement here is change in position but has also to be intended as a way to live the event, a way in which Filippo is creatively and responsively plugged into the mathematics of the event, an *attunement* to the pattern.

Filippo smiles, his facial expression changes quite a lot, his eyes shine and escape here and there, sometimes he seems curious, sometimes surprised, enchanted by the challenging question. He ends up smiling, with amusement. His little fingers impatiently move almost in the attempt of grabbing something in the proximity. He rhythmically walks from left to right, then from right to left and back again. He rhythmically repeats the specific gestures, he rhythmically speaks of the specific parts or refers to the specific operations, he rhythmically counts the bottom circles. We can realise that the auditory is one of the modalities to take into account in studying movement. Researchers have shown that rhythm partakes in affective dimensions of mathematical learning (Ingold 2011; Bautista and Roth 2012; Sinclair et al. 2016; Roth and Walshaw 2019). Rhythm is in general repeated movement. Sinclair et al. (2016) in particular stress that we absorb rhythm rather than synthesise separate sensory stimuli into a perception of rhythm, therefore rhythm is first and foremost physical. For Ingold (2011), one significant aspect of rhythm is that it is continued movement but also conveys differences and nuances. Following Bautista and Roth (2012), it is in rhythm that we find the inseparability of affect and cognition. Roth and Walshaw (2019) also argue that affect is never external to intellect and thought is affective through and through. With these researchers we share a vision of affect that goes beyond a cause–effect logic in descriptions of the performance of affected individuals. We therefore avoid to refer to psychological or social constructs of traditional research on mathematics-related affect, like attitudes, beliefs, motivation, and norms, as explanatory factors of behaviour and positive achievement (Zan et al. 2006; Hannula 2012), while we follow scholars who emphasise the intertwinement of embodiment, cognition, and affect (like in the case of Sheets-Johnstone (2009, 2011) and de Freitas and Sinclair (2014)). Rather than focusing on performance, which might easily fade into the individual, we are not interested in whether Filippo is successful or not (as advanced above), but much more in the fluid and variable attunement of Filippo and the pattern, and in the way that affect is impersonal and dispersed in the assembling of concept and child. The potentialities of a vision of affect as distributed and entangled in mathematical experience are also discussed in Sinclair and Coles' chapter (this book, Chap. 8) in relation to ritualisation activity.

In the current literature in mathematics education research, regardless of its theoretical source, we find that the impersonal nature of affect is not yet adequately treated. This may sound counterintuitive, especially when thinking of emotions such as love, hate, etc., that are considered to be deeply personal and belonging to the individual. Our approach just questions the conventional use of emotion and affect, as assumed to be a human trait or expressive behaviour that is ultimately at source individual. We follow Radford (2015) in pushing past a theory of emotion that rests on individualism, but we pursue a radically different approach than cultural-historical activity theory. We also shift away from a focus on affect as individual, positive and negative judgements of value, in line with Liljedahl and Andrà's attempt (this book, Chap. 1). In line with Sinclair et al. (2016) we see assemblages as social and affective by nature. Drawing from the inclusive materialism of de Freitas and Sinclair (2014), in previous work we have investigated the idea of how affect circulates in learning assemblages (de Freitas et al. 2017, 2019). In that work, we have been offering a vision of affect to investigate how human bodies come together with technology in mathematical activity. Emphasis was on the coordinated movement and entanglement of the technology, the students, and the mathematical concepts. Here we want to expand our vision to specifically shed light on the *responsive* nature of bodies in the different situation, which involves only one child (Filippo) and the patterning task on paper (and the teacher, of course).

To better tap into this responsive nature as a qualitative dimension of movement, we turn to the theoretical notion of *affectivity* drawing on the work of Sheets-Johnstone (2009, 2011). In a long line of phenomenology, Sheets-Johnstone (2009) introduces affectivity as the fundamental "responsivity" of life, which characterises how bodies turn away or lean in, and at the same time how they join with other bodies in coordinated movements. In Sheets-Johnstone (2011), the idea of affectivity is further conceptualised in terms of movement and aliveness, through the two complementary aspects of receptivity and responsivity. Animate forms of life enjoy the fact that their surrounding world is not simply clearly present to them, but just as clearly interests, excites, or disturbs them to move in some way. They enjoy a dynamic congruency between affect and bodily motion, precisely because affect is lived through bodily movement. The two aspects are constitutive of an *affective*/tactile–kinaesthetic body. As we emphasise in de Freitas et al. (2019), the dynamics of feelings coincide with micro-facial expression, minute changes in bodily posture, foot-tapping rhythms, changes in heart rate, and so on. Following Sheets-Johnstone, receptivity is an awakeness, a turning toward, and refers to the natural tendency or capacity of bodies to be (attracted and) affected. This tendency to turn toward may also propel the reverse tendency to turn away or against. Short of awakeness and movement, knowledge and meaning would never emerge, since the essential dynamic and active character of life. Responsivity is, on the other hand, the capacity to affect. Animate forms could hardly be responsive if they were not receptive to begin with.

Filippo *affects and is affected* by the pattern. We see how the child and the pattern respond to each other, how they move each other to move, how they change one the other and together, producing a coordinated effort of creatively tapping into the regular structure of the figures and the mathematics of the pattern. Affect is impersonal in this exact sense, of not being statically or intentionally possessed by the human body, but rather of mobilising the mathematics while sustaining the assemblage. To further explode the potentiality of the body in relation to how it informs the flow of affect and feelings (or, the affective bonds of Filippo and the pattern) as they circulate across any assemblage, in the last section we turn to the notion of *sympathy*.

2.4 Sympathetic Bonds

Filippo seems to move as if there is something in becoming in front of him, namely the twenty-two circle-term of the pattern. On the one hand, we might speculate that the different ways of gesturing and moving around the figure he is creating relate to the different parts of the term, the rows and the group of four white circles, for example, which get to be *differentiated* while he is actualising the figure. On the other hand, there is a structure that his movements follow, as if the figure was imposing constraints on the unfolding (e.g., moving from left to right, recursively). We see this as speaking directly to the *sympathetic bonds* with the mathematical concepts.

Even though there is a tendency to psychologise sympathy and make it part of a personality, misleadingly confusing it with empathy, sympathy is *not* a property of the individual, but rather traverses people and materials. The ancient history of the concept of sympathy shows its emergence in relation to the coaffection of mind and body, or the forces of attraction that operated in the physical world (Brouwer 2015) at micro and macro scale. According to Spuybroek (2016), sympathy revolves around a certain elastic immediacy of feelings that makes possible to things to understand (in an extremely wide sense) each other. It is as much a feeling as a form of thinking and is responsible for the reciprocity that exists between us and things.

Bergson (1911/1998) discussed intuition as a form of sympathy, a way of feeling-knowing, operating at the interior of things, an unspecified feeling (an instinct that has become disinterested). *Sympathy is what things feel when they shape each other*. Sympathetic relationships, therefore, do not result in the erasure, or complete identification or passive alignment of one or the other. Sympathy is much more a matter of modulating related movements, a resonance of two different sounds.

The specific task that Filippo faces allows sympathy to circulate beneath consciousness in the minute interactions, as we have detailed in the previous sections, at a micro scale. There is a productive tension towards the structure that emerges in the bimanuality and transcends the pattern and the child in their individuality. There is a dynamic effort of synchronisation and coordination with the recursive relationship, which is populated by changing rhythms, excitement, little variations that reverberate through the bodies and unfold in the creative act.

The notion of sympathy, pointing to the ways in which the pattern, number, and the child shape each other in the doing, also directs our attention to the *specificity* of the mathematical concepts at play in and through the task. We are not, therefore, proposing that mathematical concepts have transcendent affect attached to them, independent of place or context, but that *affects* (or emotions or feelings) are immanent to distinctive kinds of activity, and thus research must attend to that specificity. In other words, we are trying to pay attention to the relational dimension, but not to simply abandon the terms of the relation, as thought they had nothing to do with the creative flux of activity. The *concepts matter*. For instance, the concept of number partakes of mathematical assemblages in crucial ways, and colours and flavours an event with particular emotions, when it is at play. The pattern and Filippo provide a noteworthy example. Similarly, other mathematical concepts, if considered as dynamic and variable, commingle with other factors to produce particular events, each of which is deeply emotional. Rather than reduce all experiences of mathematics to the same emotional note, our approach proposes to attend to the nuanced or tonal differences between one experience and another, highlighting how the specific mathematical concept at play is fuelling affective forces. Meanwhile, the episode we have discussed throughout the chapter is not reducible to simply noticing that Filippo answers happily and correctly to the teacher's question. That would reduce the argument to a vision of bodily movements as representations of a fix affect. Bodily movement is instead an actualisation of the flow and tonality of affect as it circulates across the activity.

Sympathy offers us a way of thinking about the power of affect in the classroom (as an historical and material context), but also a way of thinking about affect and emotion as creative and onto-genetic in that they reconfigure the relationship between the human and the non-human. But the relational, impersonal dimension of affect needs further inquiry and investigation in mathematics education research to better understand the complex nature of mathematics teaching and learning.

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Chapter 3
Identity Research in Mathematics Education: Reifications Within Multiple Theoretical Viewpoints

Einat Heyd-Metzuyanim and Mellony Graven

3.1 Introduction

Identity research in mathematics education has seen an explosion of studies in the past two decades (Darragh 2016; Radovic et al. 2018). Researchers with diverse interests, including learning in all grades, pre- and in-service teacher development, issues of social equity and opportunities to learn, classroom practices—all have found the term "identity" to be helpful for describing some aspects of the phenomena of interest. With this proliferation has also come a profusion of theoretical viewpoints, definitions of identity, and operationalizations of this construct. Although multiple theoretical viewpoints are often productive for understanding phenomena in the social sciences, they also carry with them the danger of fragmentation (Brubaker and Cooper 2000). Calls for standardizing definitions of identity (e.g., Sfard and Prusak 2005) in the field are almost as old as the field itself. Although standardization has not been achieved, and perhaps it is even unwanted, advancements towards clarifications of definitions and theoretical viewpoints in the field have been made (e.g., Radovic et al. 2018; Rø, this book, Chap. 4). What remains to be more deeply inquired are the gains and losses of each theoretical viewpoint. In this paper, we focus on three studies that exemplify different theoretical lenses on identity: the psychological, performance/practice, and critical realism. We ask: what are the implications of each viewpoint for the reifications entailed in the concept of identity? We do this by using a discursive lens on identity research.

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3.2 Background

Historically, identity research in education stems from two distinct lineages (Graven and Lerman 2014): the sociologically oriented work of Mead (1934) and the psychologically oriented work of Erikson (1968). Mead conceptualized identity as developing in interaction with the environment. As such, it was multiple, sometimes contradictory. For Erikson (1968), identity was in essence a mental entity that belonged to the individual. As reviewed by Darragh (2016) and reinforced by our own review of identity research in the past 5 years (Graven and Heyd-Metzuyanim 2019), the Meadian, socio-cultural view of identity has been almost unanimously adopted in mathematics education research. This is as a result of researchers mostly aligning their research on identity with other socio-culturally oriented frameworks in the field such as positioning theory (Harré and van Langenhove 1999), discourse (Sfard 2008), participation in communities of practice (Wenger 1998), and socio-political approaches to the learning and teaching of mathematics (Gutiérrez 2013; Martin 2009). One of the earliest works in our young field, which has been especially influential in terms of promoting a Meadian view of identity, is that of Sfard and Prusak (2005).

Sfard and Prusak (2005) defined identity as a collection of reified, significant, and endorsable narratives about a person. They stressed that such narratives can be authored not just by the person about him/herself (1st person narratives) but also by others to her (2nd person) and about her (3rd P). To signify this, they suggested a triplet aBc indicating a particular identity story, told by a about B to c. Many have adopted Sfard & Prusak's definition, not just because of its socio-cultural roots, but also because of its operationality. Defining identity as a narrative enables the researcher to find identity in precisely those places where researchers usually look for it: interviews and interviewee's talk. Yet this adoption relates only to the "narrative" aspect of Sfard and Prusak's definition. It ignores the more nuanced and complex aspects of it: reification and significance.

To understand what Sfard and Prusak mean by the term "reification", one needs to turn to Sfard's (2008) commognitive theory, which stems from discursive theories of human activity (e.g., Harré and Gillett 1994). Sfard (2008) explains reification as the process "which consists in substituting talk about actions with talk about objects" (p. 44). She gives as an example of such reifying actions in turning the statement "In the majority of school tests and tasks dealing with function she regularly *did well* and *attained* above average scores" into "She *has acquired (constructed, developed) a conception* of function" (p. 44, italics in original). This reification turns the observations of actions into mental objects that are assumed to impact behaviour. Sfard (2008) however warns that reification of actions into mental constructs entails multiple pitfalls, a main one being that researchers may assume (and imply) that the relationships existing between objects in the physical world (such as cause and effect relations) exist also between mental objects and actions.

One thing that such a discursive lens shows is the prevalence of ontological collapses in the domain of research on learning and teaching. The term "ontological collapse", offered by Sfard (2008) and adopted by others (Heyd-Metzuyanim 2019; Skott 2015), refers to instances where stories about events in the world are talked about as the same as stories about stories about the world. These two stories are then collapsed into one story told by the researcher to the readers. Heyd-Metzuyanim (2019) gives several examples of such collapses from studies on teachers' beliefs. There, stories about what the teacher (or participant) believes (stories about the world) are collapsed with the teachers' own account of "what she believes" (story about a story about the world). Heyd-Metzuyanim (2019) signifies these collapses as treating *participant-World-participant* (pWp) stories (i.e., what the participant tells themselves about the world) as identical to *participant-World-researcher* (pWr) stories (i.e., what the participant tells the researcher about the world). Similarly, instances in beliefs research where participants' account of what they believe are treated as their "true" inner beliefs are signified as collapses between *participant-Participant-participant* (pPp), namely, private stories told to oneself, and pPr stories (stories told by the participant to the researcher). All these stories are then collapsed into one story of the type ${}_{R}pPp_{O}$. That is, the researchers' reports about the participants' stories told to themselves (privately, i.e., pPp), told to others—or the readers.

In this chapter, we adopt this discursive view for the purpose of comparing three different identity studies, which exemplify different strands and approaches for the study of this phenomenon. For this comparison we adopt Sfard and Prusak's (2005) definition of identity, particularly looking at how researchers reify certain actions into stories about identity and how they give them significance.

3.3 Three Illustrative Examples of Identity Research

In this section we analyse three studies of identity that are useful for revealing ontological collapses in relation to authoring stories across three different theoretical frames. These are: Boaler and Selling's (2017), paper *Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives*; Grootenboer and Edwards-Groves' (2019), *Learning mathematics as being stirred into mathematical practices: an alternative perspective on identity formation*; and Westaway's (2019), *The role of reflexivity in the emergence and expression of teachers' identities in teaching primary school mathematics*. We chose these papers from a data-set of more than 50 papers reviewed by us for a recent Special Issue on identity published in ZDM (Graven and Heyd-Metzuyanim 2019). We examine how different types of ontological collapses or ontological blurriness are related to the significance given to different excerpts of data.

In choosing these papers for our analysis we, by no means, undermine their important contributions to our field. Indeed, the Boaler and Selling (2017) paper shows strong empirical evidence about how different instructional practices in mathematics have long-term effects on students' lives; the Grootenboer and Edwards-

Groves (2019) paper contributes a means to understanding micro level identity processes as they play out in a single lesson to reveal student learning as being stirred into mathematical and learning practices; and the Westaway (2019) study contributes to connecting life histories of teachers with much broader social, political, and historical processes. We acknowledge these contributions and draw on them to analyse the way in which the reifications of identity in these papers, along with their related frameworks, give rise to various ontological challenges. In particular we look at how various identity narratives become combined and transformed into quintet-type stories told by the researcher to the reader. Our choice of these papers was based on the way they provide recent illustrative examples of contrasting theoretical frameworks on identity in mathematics education.

Boaler and Selling (2017)—Psychological Viewpoint: Identity as a Dependent Variable

In this study, Boaler and Selling set out to examine how a group of young adults who had been studied by Boaler as high-schoolers 8 years beforehand, continued on in their lives, specifically with regard to their relationship with mathematics. The initial study (Boaler 2002) compared between 290 students studying in two different schools where the mathematics instruction was distinctly different: one was project-based and involved group problem-solving as its main activity, the other taught math in traditional teacher-centred ways. The goal of Boaler and Selling was to investigate "whether the differing forms of identity and expertise that were identified in the first study had persisted and influenced the students in their adult lives and work" (p. 84). The fact that the authors use the term identity as something that can "influence the student" already hints at a division between an inner, mental construct (identity) which influences behaviour. This is a reification of stories about people's actions into stories about the mental objects that influence these actions. Such objectification is most common in psychological studies, which main theories are built upon such mental objects.

Common to such psychologically oriented studies is the use of surveys and interviews as "windows" to inner-mental objects, in this case—to identity (see Maffia et al. (2020) for an overview of such quantitative measures in research related to affect). The first study of the students in the two high schools (Boaler 2002) made use of surveys, structured interviews, and observations and concluded that the "Phoenix Park students' higher achievement resulted from an adaptive approach to their mathematics learning" (p. 84). They exemplify this "adaptive approach to learning" with a quote from one of the Phoenix Park students who says

> Well, sometimes I suppose they put it in a way which throws you, but if there's stuff I haven't done before, I'll try and make as much sense as I can, try and understand it, and answer it as best as I can, and if it's wrong, it's wrong. (p. 84)

In this way, the student's story to the researcher (e.g., "I'll try and make as much sense as I can") is equated with the students' private inaccessible stories of the pPp type (e.g., the student telling herself "I can make sense of this if I try"). These in turn are collapsed into a simple reified ${}_{R}pPp_{O}$ story of the type "the student had an adaptive approach to learning".

Another example of such an ontological collapse can be found in the following quote: "In describing their enjoyment of school mathematics, many of the Phoenix Park adults referred to the openness of the approach" (Boaler and Selling, p. 90). They then share the following quote, from one of the graduates of Phoenix Park to illustrate this "openness of the approach":

> I think it was definitely more creative. We were never too much said like "this is going to be on your exam, you need to memorize this." That's another thing that I've had a problem with the education system-just the whole regurgitation just for the exams. I don't know, they might have been prepping us, but it didn't feel like it. (Neil, Phoenix Park) (p. 90).

In the ontological collapse here between the researchers' claims and interviewee words, some nuances are missed. For example, the fact that Neil (the interviewee) says "That's another thing that I've had a problem with the education system" (our emphasis) positions this statement as a continuation of critique about the current "system", yet we (the readers) do not gain access to what the former statements were. Moreover, this statement hints that the interviewee and interviewer are collaborating here in some form of dichotomizing between Phoenix Park and "other" schools, which may show that this is not a solely authored story of the interviewee, but rather a collaborative story influenced also by the researchers' stories (which the interviewee may have had access to before this conversational episode). All these meanings of the interviewee's words are absent in the report as the quote is used only to exemplify the researchers' story that learners at Phoenix Park enjoyed an "openness of approach" to teaching mathematics.

An approach such as exemplified in this study gives much significance to the 1st person stories of participants, as told to the researcher. At the same time, it backgrounds the possibility that the particular identity story told to the researcher, at the moment of interviewing, may be different than what the interviewee may tell others, or even what they would tell themselves at other points in time.

We acknowledge that such foregrounding of certain types of narratives while backgrounding others may be appealing and needed for making strong, causal claims in the field of education. Indeed, the findings reported by Boaler and Selling (ibidem) lead them to such conclusions:

> Phoenix Park participants talked with confidence about tackling any problem that they encountered and seeing mathematics as knowledge that they could adapt and use. ... Their words seem to reflect actions and beliefs that combine in the development of more active and capable mathematical identities, growth mindsets [...] and adaptive expertise. These differences seem likely to explain, at least in part, the Phoenix Park participants' greater advancement in life with jobs that were significantly higher on the social class scale. (p. 97).

We see here a distinct form of use of "mathematical identity" as a causal factor for long-term and significant processes such as climbing higher on the social class scale and adapting more successfully to mathematical demands in real life.

Grootenboer and Edwards-Groves (2019)—Practice Viewpoint: Identity as Performance

Contrasting this approach to identity that relies on self-reports, Grootenboer and Edwards-Groves (2019) take a practice perspective on identity, based mostly on observing people in action:

> we suggest that our identities include: how we talk and think about ourselves (sayings) which is shaped by how others (including systems and discourses) talk and think about us (cultural-discursive arrangements); how we act and what we do (doings) which is shaped by the materials and resources in our practice site (material-economic arrangements); and, how we relate to others within and beyond the community of practice which is shaped by norms of behavior and site-based hierarchies and structures (social-political arrangements). (p. 436)

Grootenboer and Edwards-Groves emphasize the reciprocal relations between identities and the surrounding practices. They seek to "delineate the ways that learning arises from being stirred into the characteristic site-based practices" while stressing that "mathematical identities are expressed in and, at the same time, exert pressure on the sayings, doings, and relatings encountered in lessons." (p. 433).

In line with their definition of identity, Grootenboer and Edward-Groves' study focuses on processes of identity formation in action. They illustrate this process through reporting on a single mathematical lesson and examining how we might interpret classroom sayings, doings, and relatings as identity forming processes. Their focus is thus on enacted identities. These enacted identities are communicable to the readers via the transcript of a lesson. Excerpt 1 is given as an example below:

Excerpt 1

Revising doing and learning in maths (Grootenboer and Edwards-Groves 2019, p. 438) (Tch=teacher; ˆ =raised voice; (0.1) pause in seconds)

1. Tch: Okay, readyˆ (0.1) we're going to get straight into maths today. So, let's revise what we're doing in maths and what we've learnt about so far through the last half of last week and the first part of this week, particularly for people like Jace who weren't here yesterday to see what we did yesterday, I'm sure you'll get up to speed pretty quickly ((Kiri and other students raise their hands as offers to respond)) (0.2) Kiri?

The observed and transcribed action of Kiri putting up her hand is noted by the researchers as indicating two aspects related to substantive practices and practices of learning:

> Kiri's identity as knower is reinforced by her nomination as responder . . . hand raising for these students indicates they know how to participate as a student in this class (regardless of whether or not they know the 'answer' deemed correct by the teacher) because participation is expected. It could be argued then, that Kiri's hand raising can at best be seen as

an indicator of Kiri as 'willing participator' knows how student learning practices are displayed. (p. 438)

Here we see a very different approach to the problem of ontological collapses of different identity stories. Rather than saying anything about a pPp story (such as "Kiri believes she knows the answer"), Grootenboer and Edwards-Grove are careful not to commit to any particular triplet-story. For example, "Kiri's identity as knower is reinforced by her nomination as responder" avoids any commitment to who precisely authors a story of Kiri as a "knower". Is it the teacher who nominated her to answer? Is it Kiri herself? Or is it her peers? Thus we see here a blurring of alternative aBc stories, rather than treating them as congruent. This hides the fact that the alternative aBc stories may be different. For example, the teacher may call on Kiri because he wants to encourage her to answer a relatively simple question (about what has been happening yesterday), identifying her as a struggling student rather than a "mathematical knower". Some of Kiri's peers may be authoring a story of her as a "teacher pleaser" while others may indeed author a story of her as "confident knower".

A similar analysis is made in the next excerpt from the same lesson:

Excerpt 2

Decimal fractions and whole numbers (p. 439) ([indicates parallel speech)

2. Kiri: decimal fractions =
3. Tch: = okay, thanks Kiri, so [we're looking at decimal fractions so far in the last three lessons or so?
4. Kiri: [we learnt parts that you do, a whole number then you have parts of a number, small parts of a whole number.
5. Tch: okay Kiri, yep you got this (0.1) they're to do with parts of a whole number ˆ is that what you're saying?
6. Kiri: yep
7. Tch: that's right.

Here, the researchers analyse Kiri's hand raising and subsequent response (turn 2) as showing that Kiri "identifies: (1) as a student (by demonstrating compliance with an established school-type pedagogical routine) responding to a teacher question; and (2) as a knower of mathematics by confidently producing the response "decimal fractions"." (p. 439). They continue interpreting the transcript by turning the readers' attention to turn 4, where "Kiri self-initiated an extended turn by continuing her yet-unfinished response to overlap with (or interrupt) Mr. Brayshaw's confirmation of her correct response" (ibidem). They conclude that "This action provides evidence of how the student Kiri displays her agency (by interrupting the teacher to continue her answer) and her identity as a confident mathematics practitioner at this given moment." (ibidem).

Again, the authors here are careful not to talk about Kiri's private thoughts and feelings, thus avoiding stories about pPp narratives. For example, they do not say anywhere "Kiri feels confident she knows the answer". Rather, they describe the situation with words such as "Kiri displays her agency" and "her identity as confident mathematics practitioner" which does not commit to any particular identifying triplet. It could be Kiri who feels confident and agentive, but it could also be others who identify her as such.

In terms of significance, the participation/performance lens privileges local actions in short episodes of time. As such, it reifies momentary actions into identity labels such as "knower of math", "confident math participator," and "compliant participator in the social structure of the class". With this reification, two factors are backgrounded: one is that different stories about the meaning of particular actions could be told by different authors. The other is that actions observed in other points of time may lead to different stories of actions in the moment.

We wish to stress that the theorizing of identity as being expressed in local actions is a very promising avenue for advancing our understandings of how identities are constructed in actual mathematical interactions. Our aim is to raise awareness of the risk of telling identity stories (that imply longer term significance) based on local (at one point in time) actions, and blurring possible alternative triplet stories into a single quintet story of the researcher about the participant.

Westaway (2019)—Social Realism Viewpoint: Identity as Part of Social Structures

A rather extreme contrast to the above identity in practice in one moment in time is Westaway's use of a "social realist" perspective on identity in which she examines full life stories of teachers describing periods of 20–30 years. In her study, Westaway attempts to theorize one 5th grade teacher's (pseudonamed Buhle) in South Africa in terms of her identity as it relates to social and historical processes of the Apartheid and post-Apartheid era. She defines teacher identity as "the way teachers express their roles as teachers, that is, how they articulate and enact their roles" (p. 482). Drawing on Archer (2000), Westaway explains that the "social identity", which is the expression of our social roles, is related to the personal identity, which is "what one cares about in the world" (p. 483). Already in these definitions we see a blurring of the differences between 1st person stories ("what one cares about") and 3rd person stories (enactment of social roles, as perceived by others).

Data, in Westaway's study consists of multiple lengthy interviews with Buhle, as well as observations of her teaching. Both sources of data are used to form stories about Buhle's identity as it changed over the years. This, despite the researcher having no access to Buhle in the past, besides through her 1st person stories. The following excerpt shows how these identity stories are told by the researcher:

> Buhle managed to identify her ultimate concerns easily as a young adult. She was independent, goal directed and committed to moving beyond her context of economic hard ship. When nursing and high school teaching were not viable career options, she decided to become a primary school teacher. This was the only way for her to realise her

> ultimate concerns, that is her aspirations to have financial stability and "be somebody" (LHI, t.10). As explained earlier in this paper, Archer (2007) maintains that individuals who experience contextual discontinuity in their lives learn to become independent and goal directed. It appears that Buhle's dominant mode of reflexivity, at that time, indicated that of an autonomous reflexive. (p. 487).

Here we note multiple collapses occurring. First, the researcher collapsed Buhle's current story of her past (assuming it was something like "I knew well what I wanted back then") into her reports of the actual happenings in the past. This is a collapse of pPp stories at different points of time into one $_{\mathrm{R}}$pPp$_{\mathrm{O}}$, which is especially conspicuous since there are good reasons to believe people change their narratives about the world and themselves over the course of 20 years. Next, the story of Buhle's actions ("She was independent, goal directed, and committed") is collapsed with Buhle's stories in the interview. We do not gain access in this paper to the original transcripts of the interviews, thus we can only hypothesize that these 1st person stories were not told in the form of characteristics (e.g., "I was independent") but rather interpreted from how Buhle described her actions in the past. This is a collapse of different pPr stories into one unified $_{\mathrm{R}}$pPp$_{\mathrm{O}}$ story, yet of a more complicated kind. It is not based on current observations of the researcher, but rather on imagined observations as elicited from the interviewee's stories. As such, it implies not just $_{\mathrm{R}}$pPp$_{\mathrm{O}}$ stories (about how Buhle thought or felt in the past), but also how the researcher would have observed her in the past, that is, $_{\mathrm{R}}$rPr$_{\mathrm{O}}$ stories.

In terms of reification and significance, we see in Westaway's study two main types of reification, that result in significance given mainly to 1st person stories of the past and present. First, the researcher forms reifications of what Buhle said in her interviews into several distinct attributes characterizing her as a "communicative-autonomous reflexive" (p. 490). Communicative reflexives, according to Archer (2007, cited in Westaway 2019) are individuals who "experience much contextual continuity in their lives" and who "primarily like to share their deliberations with others before pursuing their projects" thus prioritizing social order (Westaway 2019, p. 484). Autonomous reflexives, on the other hand, are those people who "Having experienced a life of contextual discontinuity . . . tend to be independent and self-reliant" (p. 485). Buhle's stories of her history in the past 20 years are thus reified by Westaway as locating her first towards the autonomous type (when she was young and goal-driven) and later on in life as turning into a "communicative reflexive" (when she became more concerned with keeping her job to get a pension so that she could support her children financially). The second type of reification is done by Buhle herself, in telling a 20-years long set of events in one or two interviews of several hours. Both types of reifications are needed for Westaway to fit Buhle's stories into Archer's theory of how individuals progress through life in terms of aligning themselves with or against the social order.

3.4 Discussion and Conclusions

Our goal in this chapter was to examine how different perspectives on identity in mathematics education may involve ontological collapses in their telling of coherent, accessible, and relatable stories to their readers. We have focused on some of the implications of these ontological collapses in terms of reifications and significance. This, in line with adopting Sfard and Prusak's (2005) suggestion to see identity as collections of stories that are reifying and significant, as well as authored by different people and to different audiences. Acknowledging the fact that one can never include in one's examination all stories about a person, and that determining significance of a story is a highly complex task, we set out to examine how recent works stemming from different traditions in identity studies choose to reify and assign significance to identity stories. As described in the analyses of the three papers, we identified several ontological collapses in the studies examined as well as blurriness with regard to different possible triplet stories that eventually collapse into a quintet story (see Fig. 3.1). This is not surprising, since such quintet-stories are the "bread and butter" of any research on human actions, thoughts, and feelings. Indeed, such reifications are necessary for telling a coherent and recognizable story to the readers. Nevertheless, our analysis also pointed to distinct traditions (psychological, performance/practice, and sociological) being susceptible to different types of ontological challenges. We claim that this is mainly a function of whether the study focuses on narrated identities (as obtained by 1st person stories in the form of interviews and surveys) or enacted identities (as obtained through observations and discourse analysis). Boaler and Selling (2017) is an example of a study focusing mostly on narrated identities; Grootenboer and Edwards-Groves' (2019) deal mainly with enacted identities, and Westaway (2019) relates to both enacted and narrated.

What is important to observe in Fig. 3.1 is that studies of narrated identities seem to be different from studies of enacted identities in both the significance they give to different stories, and the ways they cope with possible ontological collapses. Studies of narrated identities give significance to 1st P stories spanning a considerable length of time. They are thus in a better position to capture reified and significant stories, yet at the same time background alternative 3rd P stories and are susceptible to collapses between different 1st P stories (told to oneself or to the researcher).

Studies of enacted identities give significance to actions. Yet since actions are always a matter of interpretation, theories that conceptualize identity as forming in action necessarily must background possible alternative interpretations that may form of any particular action. We thus saw Grootenboer and Edwards-Grove avoiding committing to any one particular triplet story. This blurriness makes sense when actions are viewed over time, assuming that consistent actions give rise to coalescing triplet stories about people. For example, if a student consistently raises her hand and produces correct answers, and if the teacher consistently calls upon students that she identifies as "knowers", then the particular action where a teacher asks this student to respond at a particular point in time, might convincingly be

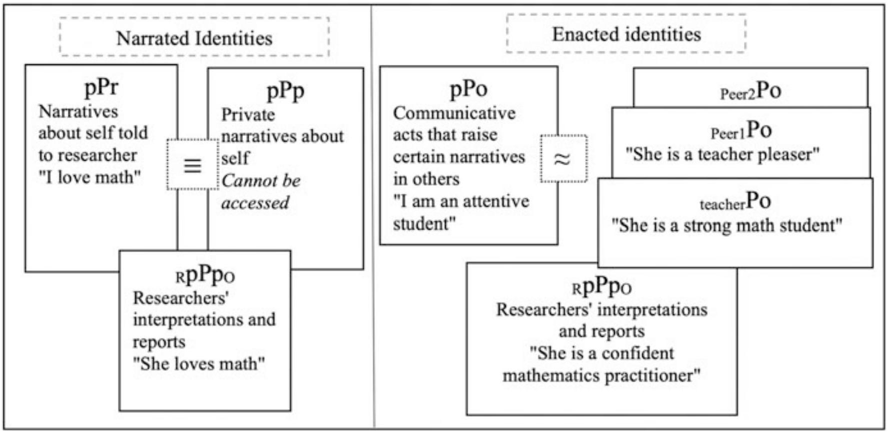

Fig. 3.1 Common ontological collapses in studies of identity. ≡ means assumed congruence; ≈ means blurred with

interpreted by all the participants involved as indicating "she is a knower". However, the shorter the time of observation, and the lesser the consistency of actions across time, the more one may expect the possibility of different identity triplets forming by different authors about a particular person.

We see the different forms of significance given in these studies not only indicative of the field as a whole, but also echoing the major theoretical perspectives underlying identity research. One of these is Wenger's (1998) perspective that powerfully connects identity and learning in showing how identity is a form of "becoming" a participant in certain practices. Rø's study of early-career teachers (2018; this book, Chap. 4) presents an example of the usefulness of this perspective for understanding processes of teachers' transition between pre-service education settings to the school settings. Performance/participation perspectives are consonant with Wenger's theory of "learning as becoming". However, when such performance/participation approaches take a micro-perspective on identity formation, what may remain overlooked is Sfard and Prusak's (2005) dimension of narratives needing to be "significant" to be an indicator of identity. In this respect, the supplementing of observational data with 1st person stories may be useful to give significance to the identity stories told by researchers based on observations.

The challenges of examining identity as it relates to different levels of activity were treated by Gresalfi and Hand (2019; Hand and Gresalfi 2015), who located identity as existing in three spheres: (1) local interactions among people and tools which are governed by norms (2) frames, which are available storylines by which people make sense of what is going on in a situation, and (3) broader narratives and social constructs such as societal ideologies and stereotypes. Their conceptualization brings to the fore the fact that no one particular study can relate to all these levels at once. The foregrounding and backgrounding of certain narratives and the over-reliance on certain types of identity stories, may thus be inevitable. Our argument however, is that in order to move forward, the field should reflect on the challenges entailed in certain ontological collapses, as well as to acknowledge the tentativeness, or fragility, of the stories produced from certain data. Our recent review (Graven and Heyd-Metzuyanim 2019) has shown us that the young field of identity studies is already moving forward rapidly, both in terms of volume of publications as well as in its clarification of definitions and theoretical underpinnings (see also Darragh (2016), Radovic et al. (2018), Rø (2020), for reviews advancing these goals). We argue that additional maturity could be achieved if researchers became more explicit about the aspects of identity that are highlighted under their theoretical lenses, as well as acknowledge those aspects that may remain hidden by them.

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Chapter 4
Researching Mathematics Teacher Identity: A Theoretical Consideration

Kirsti Rø

4.1 Introduction

As also highlighted by Heyd-Metzuyanim and Graven (this book, Chap. 3), the concept of identity has gained an increasingly prominent role in mathematics education research over the past two decades (Darragh 2016; Graven and Heyd-Metzuyanim 2019; Losano and Cyrino 2017; Lutovac and Kaasila 2018a). One reason for its prosperity is its adjustable lens through which to study mathematics teacher learning (Lerman 2000). As described by Darragh (2016), the researcher can zoom out on the wider socio-political context of mathematics teacher's developing identities. Also, he or she can zoom in to the level of interactions between individuals or to an individual's relationship with mathematics and the reasons he or she chooses to continue or discontinue the study of mathematics or mathematics teaching. Moreover (Grootenboer and Zevenbergen 2008, p. 243) argue that identity is "a unifying and connective concept that brings together elements such as life histories, affective qualities, and cognitive dimensions". Although identity is considered across the literature as dynamic and a constantly evolving phenomenon, being under the influence of a range of individual and external factors (see, e.g., Beauchamp and Thomas 2009), there are considerable differences in how to conceptualise identity. These differences have been explained by Darragh (2016), who draws a distinction between studies perceiving identity as an action and those perceiving it as acquisition. In this chapter, I account for these two main paradigms of identity research, from the viewpoint of previous research on mathematics teacher identity. In addition to building on recent reviews and commentaries on the topics mathematics identity research and mathematics teacher identity (Darragh 2016;

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Graven and Heyd-Metzuyanim 2019; Heyd-Metzuyanim et al. 2016; Lutovac and Kaasila 2018a), I supplement my overview of theoretical perspectives taken with a literature search. The search criteria in the databases MathEduc and ERIC were “mathematics” AND “education” in any field AND “teacher identity” in either title, abstract, or keywords, from the five last years. The reference list can be obtained upon request.

Moreover, I report on the theoretical considerations made for a study situated within the action paradigm, of secondary school mathematics teacher identity in the transition from university teacher education to employment in school (Rø 2018). In addition to providing a framework for analysing mathematics teachers’ developing identities when entering the profession, the study treats mathematics teachers as highly active participants who negotiate their experiences, and from that, create images of themselves and their challenging tasks and dilemmas in mathematics teaching. Guiding questions for this chapter are:

- What are the trends of theoretical perspectives taken in recent research on mathematics teacher identity?
- What are the challenges and what are possible future directions for the research field?
- What theoretical considerations can be made in a study of developing identities of secondary school mathematics teachers entering the profession?

4.2 Mathematics Teacher Identity as Action or Acquisition

Identity in mathematics teaching has been explored from a range of theoretical approaches. It spans from categorising aspects of teacher identity in order to describe it and better understand the possible influences on teachers’ practice, to view identity as a function of participation in different communities of practice. With her attempt to “clear otherwise muddy waters”, Darragh (2016, p. 29) makes a distinction between two main paradigms of research on identity: a psychological frame related to the work of Erikson (1968), in which identity is seen as acquisition; and a sociological frame stemming from Mead (1913/2011), in which identity is perceived as action. The psychological frame represents an understanding of identity as something that individuals acquire and have inside themselves or something that becomes coherent and consistent. One example, as noted by Darragh (2016), is (Anderson’s 2007) elaboration of “four faces” of identity, in which the “nature” face leads to a discussion of identity towards attributes. Further, affect research is a strand sometimes being labelled as identity research, where beliefs about or dispositions towards mathematics teaching and learning are treated as something inside people and brought across contexts. For instance, van Putten et al. (2014) take into account the affective aspect of mathematics teachers’ identities by addressing the component of caring in their study of possible incongruities between pre-service mathematics teachers’ self-perceived and actualised identities. Moreover, Bosse and

Törner (2015) study out-of-field teaching mathematics teachers, where identity is perceived as a unifying concept connecting cognitive and affective-motivational perspectives. They claim, on the basis of Grootenboer and Zevenbergen (2008) and Grootenboer et al. (2006), that "the identity concept lets us explore many different facets of out-of-field teaching mathematics teachers' activities, challenges, and needs", and further, that "the different identity facets are not detached anymore. They are brought together and allow us to gain a holistic view of our research objects" (Bosse and Törner 2015, p. 4). Another recent example of labelling affect research as identity research is (Erens and Eichler's 2019) study of prospective and trainee teachers' belief change when undergoing school practice. Although Erens and Eichler do not mention the term identity once, their work is placed under the topic "Identity" in what is a presentation of the latest trends in research in the area of affect and mathematics education (see also the commentary of Heyd-Metzuyanim 2019).

In contrast to an acquisition perspective on identity, the paradigm related to the work of Mead (1913/2011) treats identity as something a person does, and which is multiple, contradictory, and socially constituted. Influential theorists drawn on by mathematics education researchers are Lave and Wenger (1991), Wenger (1998), and Holland et al. (1998). Examples of recent studies drawing on (Wenger's 1998) perspectives are (Crisan and Rodd's 2017) investigation of non-specialist teachers of mathematics in professional development and their identification with school mathematics and negotiability in mathematics teaching, and (Essien and Adler's 2016) account of operationalising theory for studying the preparation of pre-service teachers teaching mathematics in multilingual contexts. Jong (2016) takes on the notion of identity as defined by Holland et al. (1998) when studying the factors that support one novice teacher to enact a reform-oriented practice in mathematics as she transitions from being a pre-service to becoming an in-service teacher. Similarly, (Losano et al. 2017) focus on a newly educated mathematics teacher and her development of a professional identity and agency specific to the practice of teaching mathematics situated in the culturally constructed world of her school of employment.

Treating identity as action is also done by relating the concept of identity to the activity of communicating. According to this tradition, teacher identities are represented by stories they tell about themselves as learners in and teachers of mathematics (Beauchamp and Thomas 2009; Beijaard et al. 2004; Darragh 2016). Such narrative research was first applied to educational research by Connelly and Clandinin (1990), who recognised the importance of narrative inquiry as a methodology when focusing on lived educational experiences. In a later work (Connelly and Clandinin 1999), they report on interconnectedness of knowledge, context, and identity in the stories of teachers and administrators. These stories are considered as personal, as they are shaped by the storytellers' knowledge, values, feelings, and purposes. However, they are also collective, in the sense of being shaped by the broader social, cultural, and historical contexts where the stories are lived out.

In their elaboration of an operational definition of identity, (Sfard and Prusak 2005) delineate the concept of identity as collections of stories about persons that are "reifying, endorsable, and significant" (p. 16). Hence, identifying oneself as a mathematics teacher is by Sfard and Prusak (2005) seen as a discursive activity, in which the stories themselves constitute a mathematics teacher identity. Further, identity development is connected to stories about the current state (current identity) and about states expected to be (designated identity). The teachers' stories give the reader a sense of what the narrators care about, and the conditions in which they carry out their work. They are, therefore, considered to represent teachers' growing understanding of their professional identities within changing contexts. One example of recent research using (Sfard and Prusak's 2005) operational definition of identity is (Mosvold and Bjuland's 2016) study of identity development in mathematics teacher education. They investigate how two pre-service mathematics teachers position themselves, and how they are positioned by a mentor teacher, in mentoring conversations during field practice. The importance of talk in research on teacher learning is also highlighted by Lutovac and Kaasila (2018b), in their narrative study of one elementary teacher's identity work in the context of teaching mathematics at two points in time—the present, as an experienced teacher, and two decades prior, as a pre-service teacher. They subscribe to (Ricoeur's 1991) framework of narrative identity, claiming that the framework acknowledges that people become aware of their identities by narrating their experiences, and that it allows for simultaneously understanding the continuity of and changes to teachers' identities over time.

Moreover, researchers have extended the theoretical landscape on mathematics teacher identity by combining theories within the Meadian tradition. For instance, Palmér (2015, 2016, 2013) draws on a participatory framework in her studies of novice primary school teachers' identity development, in which (Skott's 2010 and Skott et al.'s 2011) concept of patterns of participation and (Wenger's 1998) theorisation of identity are coordinated. Conducting a case study with an ethnographic direction from their graduation and 2 years onwards, Palmér claims to make both the individual and social part of the teachers' identity development process visible. Placing the individual mathematics teacher in the foreground, she investigates the teachers' patterns of participation regarding mathematics teaching and the sense of becoming as a teacher. This participation takes place in communities of practice during teacher education and the professional debut. Moreover, by placing the social dimension of identity development in the foreground, her analyses of communities of practice lead to interpretation of how memberships affect patterns of participation of individual teachers. Similarly, Skott (2019) makes use of the concept of patterns of participation in his longitudinal study of a novice teacher and her changing identity over the first 4 years of her career. The study of Skott (2019) enables interpretations of how a teacher's engagement with a multitude of different practices play a role for professional experiences, as opposed to prioritising teacher engagement in one particular practice (e.g., as promoted by teacher education or professional development).

There are also examples of studies combining theoretical constructs across the Eriksonian and Meadian paradigms. One is the study of van Zoest and Bohl (2008), who take (Wenger's 1998) social learning theory as the basis of their analytic framework. However, they argue that Wenger's conception of identity leaves something to be desired in terms of concrete reference to individual cognition and to the *mathematics* teacher profession. In line with (Lerman's 1998) suggestion of changing the analytic focus from cognitive development to social participation, their mathematics teacher identity framework contains two interacting and overlapping components called aspects of "self-in-mind" and aspects of "self-in-community". While the latter aspects relate to (Wenger's 1998) communities of practice, the aspects of self-in-mind include teachers' knowledge, beliefs, commitments, and intentions in mathematics that the teachers "carry with themselves as they move from context to context" (van Zoest and Bohl 2008, p. 338). Here, the notion of teacher knowledge proceeds from the work of Shulman (1986). Inspired by the framework of van Zoest and Bohl (2008); Bennison (2015a) has developed a framework for researching mathematics teacher identity as an *embedder-of-numeracy*, being organised around five domains of influence (knowledge, affective, social, life history, and context). In contrast to the comprehensive theoretical work made by van Zoest and Bohl (2008), Bennison's focus is on "the very specific situated identity a teacher has that enables or limits their capacity to embed numeracy into the subjects they teach" (Bennison 2015a, p. 9).

4.3 Challenges and Possibilities in Research on Mathematics Teacher Identity

In continuation of the given overview, I summarise the trends of theoretical perspectives taken by pointing at challenges and possible future directions for the field of mathematics teacher identity research. This is done with support from recent reviews and commentaries within the research field (Darragh 2016; Graven and Heyd-Metzuyanim 2019; Heyd-Metzuyanim et al. 2016; Lutovac and Kaasila 2018a). Common for these reviews is the observation of a dominance of sociocultural perspectives taken. Graven and Heyd-Metzuyanim (2019) even claim that "there is unanimous reliance on social ("Meadian") views of identity rather than psychological ones" (p. 370). This reliance might be due to the critique that has been raised against acquisitionist perspectives on teachers' learning, as these refer to individual, relatively stable, mental constructs to be enacted by the teacher in the mathematics classroom (Ponte and Chapman 2008; Skott et al. 2013). According to Ponte and Chapman (2008), teaching should instead be considered a holistic, participative activity, as mathematics teachers are engaged in practice not just with their mental dispositions such as knowledge and beliefs, but with all their being. Beyond epistemological processes of coming to know content, learning in terms of identity development is thus what (Wortham 2006) denotes as an ontological process of learning that changes who the learner is. A student's case in this direction might be seen from Filippo's interaction with his teacher in Ferrari and Ferrara (this book, Chap. 2).

The dominance of sociocultural perspectives is exemplified in my overview by recent studies building on social theorists, either self-contained, or in combination with other social perspectives. Lutovac and Kaasila (2018a), on their hand, point to the need of balancing individual and social perspectives in future studies and linking the research with studies on cognition and affect. For the research field to be successful in informing how to assist pre- and in-service teachers in their identity development, they claim it is crucial to also consider the inner world of mathematics teachers. Examples of attempts to balance individual and social perspectives in identity research are shown in the studies adopting a unifying or holistic definition on identity by including affect and cognition (e.g., Bennison 2015a; Bosse and Törner 2015; van Zoest and Bohl 2008). Yet, Darragh (2016) warns against treating identity as a "catch-all term for affect", as it "muddies water already filled with a variety of definitions" (p. 28). A re-branding of affect as identity also brings with it the challenge of discussing identity as if it is acquired, while defining it within a theoretical frame that views identity as an action. According to Darragh (2016), many researchers seem to draw from the broad theories within the Meadian paradigm when defining identity, yet, using a psychological frame when analysing the individual. A similar challenge is highlighted by Graven and Heyd-Metzuyanim (2019) of veiling the dynamic and situated nature of identity by essentialising or objectifying it into a mental stable entity that has a causal effect on behaviour. In Heyd-Metzuyanim and Graven (this book, Chap. 3), they claim such essentialising to often involve *ontological collapses* between the stories told by observers and those told by the participants themselves. Consequently, researchers might reinforce stereotypes about the sorts of identities that excel in the mathematics teaching profession.

Another way of balancing individual and social perspectives is to consider mathematics teacher identity as teacher-in-the-learning-community-in-the-teacher (Graven and Lerman 2003; Lerman 2000). The term refers to (Wenger's 1998) social theory on learning as participation. Focusing on the first part (teacher-in-the-learning-community), the researcher can consider teacher identity from a community perspective. It involves accounting for possible communities of practice or social constellations for mathematics teachers to be engaged in, their possible negotiated meanings of mathematics, its teaching and learning, related mathematics practices, and their development over time. Considering this first half from the individual mathematics teacher's perspective (as (teacher-in-the-learning-community)-in-the-teacher), the researcher can as well study identity formation based on individual aspects (such as mathematical background, expressed perspectives on the nature of mathematics, expressed considerations about being a mathematics learner and teacher), yet, assuming them being continuously negotiated and reconsidered through participation in communities of practice. In line with (Lerman's 2000) account are the studies of Palmér (2013, 2015, 2016) and Skott (2019), the latter stating the need to re-centre the individual to investigate how a teacher "draws on a multiplicity of prior and present practices and figured worlds when facing the challenges of her new profession" (p. 472). The emerging trend of combining theories within the Meadian paradigm appears promising for highlighting both individual and social processes, without yielding to the temptation of essentialising or objectifying identity while avoiding ontological collapses.

Moreover, a challenge addressed when theorising mathematics teacher identity is the tendency to emphasise the personal side of becoming a mathematics teacher in preference of the disciplinary side of mathematics teaching (Beauchamp and Thomas 2009). This is highlighted by Graven and Heyd-Metzuyanim (2019), who state the need of making identity relevant for learning processes in mathematics and in specific domains of mathematics, thus, not rending the discipline invisible. Accordingly, Adler et al. (2005) have called for a greater understanding of how mathematics and teaching combine in teachers' development and identities. They claim that "we do not understand well enough how mathematics and teaching, as inter-related objects, come to produce and constitute each other in teacher education practice" (Adler et al. 2005, p. 378). The work of Bennison (2015a,b) responds to this challenge, as she makes topical the concept of identity in teachers' mathematics learning when investigating the situated and dynamic nature of a teacher's identity in the mathematical context of numeracy. However, Lutovac and Kaasila (2018a) point out that research has demonstrated great differences between the identities of specialist and non-specialist mathematics teachers. Elementary teachers, they claim, often do not personally relate with the mathematics discipline, and thus, they identify themselves as "teachers of mathematics" rather than "mathematics teachers". This is supported by the study of Palmér (2013): none of the seven novice primary school teachers she followed from graduation into their professional debut developed a professional identity as a primary school *mathematics* teacher 2 years after graduation. Specialist mathematics teachers, on the contrary, hold a university degree in mathematics and need somehow to relate to and cope with mathematics both during university studies in mathematics and during teacher education. Hence, the images of what constitutes a mathematics teacher have different conditions for growth for these two cohorts of teachers. In the upcoming section, I continue along the path of secondary school mathematics teachers' learning by taking an action perspective on identity and assuming mathematics to be a prominent part of the teachers' developing identities.

4.4 An Action Perspective on Secondary School Mathematics Teachers' Identities

In the remaining of this chapter, I present the theoretical considerations made for a study of secondary school mathematics teachers' learning when entering the teacher profession (Rø 2018). Here, I adopt identity as a concept for investigating the participative experiences of prospective secondary school mathematics teachers, as they undergo the transition from subject studies in mathematics and university teacher education to a professional practice in school. Identity in transition is, therefore, understood as negotiated experiences of self when participating within and at the boundaries of *communities of practice* (Wenger 1998). The research reported stems from a larger narrative case study of three secondary school mathematics teachers as they move from university teacher education to employment in school (Rø 2018). Based on analyses of the prospective teachers' accounts of participating in mathematics practices at university and school, I presented in Rø (2018) their learning trajectories when undergoing the transition into their professional career. The longitudinal research design included a series of interviews distributed across the three prospective mathematics teachers' last year in university teacher education and their first year as mathematics teachers in secondary school. By methods from narrative analysis, the prospective teachers' accounts constituted evolving stories of becoming mathematics teachers in secondary school. The stories were further thematically compared through a cross-case analysis, to describe more generally the development of an *identity* as a secondary school mathematics teacher.

Wenger (1998) provides a general theorisation of learning and a superior framework for my study. However, to enable a mathematical profile to the prospective teachers' developing identities, I have in Rø (2018, 2019) applied (Ernest's 1991) accounts of educational ideologies in mathematics, and (Belenky et al.'s 1986 and Povey's 1995; 1997) accounts of authoritative knowing to the theoretical framework. This provide helpful terminologies for portraying the communities of practice in which the teachers participate and for describing the prospective mathematics teachers' negotiability regarding mathematics, its teaching and learning within the given communities. Responding to the abovementioned challenges and suggestions for future directions, I seek to contribute to a further theorisation of mathematics teacher identity, by presenting a framework with a descriptive power regarding the process of becoming a secondary school mathematics teacher. Here, I give a brief presentation, yet, a comprehensive framework is available in Rø (2018).

Mathematics Teacher Identity in Communities of Practice: Identification and Negotiability

Within the frames of Wenger (1998), a prospective teacher's movement between university teacher education and employment in school implies various forms of participation in communities of practice, and the work of reconciling memberships across the community boundaries (Akkerman and Bakker 2011). The concept of *community of practice* refers here to a set of relationships between people, who share competence through interaction, communication, and negotiation of meaning (Wenger 1998). When moving between practices, the prospective mathematics teacher is negotiating ways of being a person in a community, e.g. being a student teacher in mathematics, being a schoolteacher in mathematics, and a teacher colleague. Hence, the existence of a community of practice concerned with mathematics teaching and learning is at the same time a negotiation of related identities. On this basis, Wenger (1998) defines *identity* as negotiated experience of self when participating within and between communities of practice. Developing an identity as a mathematics teacher can thus be characterised as "increasing participation in the practice of teaching, and through this participation, [. . .] becoming knowledgeable in and about teaching" (Adler 2000).

A gradual change in participation, from the periphery towards the centre of a community of practice, is by Wenger (1998) described in terms of three modes of belonging: engagement, imagination, and alignment. For instance, a prospective mathematics teacher might *engage* with ideas of inquiry-based mathematics teaching through involvement in communicative practices with other community members (e.g., teacher educators, tutors, fellow student teachers, students in the classroom, teacher colleagues) during teacher education or in school. Consequently, he or she takes part in ideas of mathematics teaching through *imagination*, by envisioning himself as a teacher in a future classroom who is implementing the community's practice. Doing what it takes to play part in the community, the prospective teacher also *aligns* with the conditions or characteristics of the community's practice. This can take place through reading and sharing relevant literature with other actors, implementing inquiry-based activities in own teaching and getting involved in professional development projects. Further, each of the three presented modes of belonging is a source of the dual process of identification and negotiability (Wenger 1998). *Identification* concerns a person's investment in various forms of belonging to communities of practice that are both participative ("identifying with") and reificative ("identifying as") (Wenger 1998). Hence, identifying *with* a mathematics teaching practice or a group of practitioners is simultaneously a process of being identified *as* a special kind of mathematics teacher. Processes of identification with community practices thus define which meanings matter to the mathematics teacher regarding mathematics teaching and learning.

To characterise the social practices in which a prospective mathematics teacher exercises identification, I refer to Ernest's (1991) model of educational ideologies in mathematics. Here, he describes a range of ideologies regarding the nature of mathematics, spanning from *dualist absolutist* to *relativistic fallibilist* views. The former extremity represents the discipline of mathematics as certain, made up of absolute truths and structured into simple dichotomies such as right or wrong, true or false, while the latter combines an acceptance of multiple intellectual and moral perspectives with an understanding of knowledge as a social construction. Along this spectrum, Ernest (1991) places objectives for the teaching and learning of mathematics in school and related theories of the teaching and learning of children. I further assume that the practices of a mathematics community mirror one or several of the ideologies in Ernest's model, to which a prospective mathematics teacher might exercise identification. For instance, a mathematics teacher can identify him/herself with absolutist ideologies in mathematics, when aligning with a mathematics teacher community's emphasis on students' work on routine tasks and use of learned procedures to develop their applicable mathematical skills for future employment. In contrast, a teacher can exercise identification with a community of inquiry-based mathematics teaching, by imagining becoming a teacher who facilitates questioning and critical thinking and aims for students' mathematical confidence and social empowerment. The latter is related to relativistic-fallible perspectives, where mathematics is considered a social construction: "tentative, growing by means of human creation and decision-making and connected with other realms of knowledge, culture, and social life" (Ernest 1991, p. 209).

However, processes of identification do not determine the teacher's ability to negotiate the meanings of the community. *Negotiability*, then, refers to the "ability, facility, and legitimacy to contribute to, take responsibility for, and shape the meanings that matter within social configurations" (Wenger 1998, p. 197). For instance, the meanings that a mathematics classroom community (teacher and students) produce of mathematics, its teaching and learning, are not only local meanings. They are also part of a broader economy of meaning in which different meanings are produced in different locations and compete for the definition of what mathematics teaching and learning is or should be. Consequently, some meanings of mathematics teaching and learning achieve special status. Wenger's (1998) notion of *ownership of meaning* refers here to the teacher's ability to take responsibility for negotiating the meanings of mathematics within the classroom community. The interplay between the mathematics teacher's expressed identification and his voiced negotiability displays, therefore, possible tensions between how he or she desires his mathematics teaching to be and what he or she experiences as possible in practice.

To describe negotiability in the context of a prospective mathematics teacher's identity development, I find it useful to draw on the work of Povey (1995, 1997). Her studies are a continuation of (Belenky et al.'s 1986) account on women's ways of knowing, however, based on data from both male and female beginning mathematics teachers. The framework consists of the following categories: silence, external authority, internal authority, and the author/ity of self and reason. The state of *silence* is recognised by a teacher's disconnection with mathematics and its teaching, in which he or she is feeling "deaf" in terms of not learning from others and "dumb" because he or she lacks a voice. This state is in accordance with (Wenger's 1998) description of marginalisation and absent negotiability: since the mathematics teacher is struggling to maintain and develop a sense of self as a legitimate member of the profession, his or her negotiation of a current community's practice is also absent. It can, for instance, take place in unpleasant classroom situations, when being under judgement of what appear as hostile mathematics students. Further, the state of *external authority* is characterised by absolute and fixed knowledge given by "experts". The voice of an external authority is thus heard; however, the mathematics teacher lacks an inner voice to challenge the authority. One example can be the mathematics teacher's possible reliance on external resources for his or her teaching, by acting in accordance with a tutor's, the colleagues' or teacher educator's expectations. Also, the discipline of mathematics is commonly perceived as absolute and being based upon the authority of others, who will make judgements about right or wrong (Povey 1995). Lecturers at the university are examples of such authorities. Hence, received knowing can be a possible state of knowing for student teachers when undergoing mathematics studies at the university. On the contrary, the state of *internal authority* is recognised by the teacher's abstain from external authorities: the voice of authority is still heard and provides absolute answers but now the authority is the self. Within this position, the mathematics teacher might find the purpose of teacher education to be an examination of a range of mathematics teaching styles and approaches, to see what fits him or her. Common for both external and internal authority is lacking negotiation and uneven ownership of meaning regarding mathematics, its teaching and learning. However, critical judgement and joint negotiation of meaning are present in the state of *author/ity of self and reason*. Here, both external sources of authority and one's internal voice are listened to and evaluated. A way of exercising this kind of authorship is approaching mathematics as fallible, socially constructed, and a subject for critique.

Povey (1995, 1997) emphasises that ways of knowing in her work is not developmental, meaning that there is no clear sequential ordering of epistemological positions. Hence, becoming a mathematics teacher is not about undergoing a universal developmental pathway of knowing in mathematics and mathematics teaching. Instead, the categories are meant to illuminate the thinking of beginning teachers of mathematics, and further, their differences in action regarding the teaching of mathematics in school. For my study and within the frames of Wenger (1998), I assume that the way a mathematics teacher negotiates truth and reality related to mathematics, its teaching and learning, shapes the ways he or she negotiates ways of being a mathematics teacher in it. Consequently, the ways of knowing or the sense of negotiability in mathematics communities is part of the mathematics teacher's identity and development.

4.5 Closing Remarks

In this chapter, I have accounted for the trends of theoretical perspectives taken in recent research on mathematics teacher identity, and I have pointed at some challenges and possible future directions for the research field on mathematics teacher identity. Several issues came up:

- The concept of identity brings with it a possibility of taking a holistic perspective on mathematics teacher learning, by combining elements such as life histories, affective qualities, and cognitive dimensions; yet, there is a danger of muddying waters already filled with various definitions.
- The concept of identity provides an adjustable lens for looking at individual and social perspectives on teachers' learning; yet, researchers should be careful about combining theoretical perspectives across the Meadian and Eriksonian paradigms and yielding to the temptation of essentialising or objectifying identity involving ontological collapses.
- Identity gives the opportunity to moving beyond mathematics teacher learning as acquisition of knowledge and beliefs; yet, there is a need for making identity relevant for learning processes in *mathematics*, thus, not rending the discipline invisible.

I have further presented the theoretical considerations made in a study of developing identities of secondary school mathematics teachers entering the profession. Following a Meadian tradition, it is the mathematics teacher's activity of identifying oneself as a mathematics teacher, rather than to single out the one, true mathematics teacher identity, that is of my interest (Darragh 2016; Sfard and Prusak 2005). In line with an action perspective, Sfard and Prusak (2005) claim that "human beings are active agents who play decisive roles in determining the dynamics of social life and in shaping individual activities" (p. 15). Developing a teacher identity thus brings with it "a sense of agency, of empowerment to move ideas forward, to reach goals, or even to transform the context" (Beauchamp and Thomas 2009, p. 183), in which mathematics teaching takes place. Although agency is not a term used by Wenger (1998), his theorisation of identity as identification and negotiability provide a conceptual tool for gaining insight into prospective mathematics teachers' room for manoeuvre regarding mathematics teaching and learning when entering the profession. I have presented (Ernest's 1991) model of educational ideologies and (Povey's 1995; 1997) categorisation of ways of knowing as helpful terminologies for describing identity formation in terms of identification and negotiability in mathematics practices, thus, not rendering the mathematics discipline invisible. However, there is a need to make some final remarks regarding the combination of theoretical elements for studying the phenomenon of mathematics teacher identity.

According to Ernest (1991), perspectives on the nature of mathematics constitute the primary component of a person's philosophy of mathematics and underpin espoused theories of teaching and learning mathematics. These in turn have an impact on the mathematics teacher's practice, mediated however by the opportunities and constraints provided by the social context. Such a system of beliefs, understood as mental objects acquired by a mathematics teacher, is initially not compatible with (Wenger's 1998) framework. I am therefore in danger of talking about identity in theoretically inconsistent ways (see the second bullet point). However, by perceiving identity as action, I understand here a teacher's expressed identification with perspectives of mathematics and mathematics teaching and learning to be continuously negotiated through interaction with other participants within various communities of practice. Becoming a participant in a community is thus about growing into the practice in which one engages, including the community's artefacts and negotiated abstractions, symbols, concepts, stories, and discourses on which the members can act. In my work, (Ernest's 1991) model has been helpful for characterising possible mathematical discourses to which a prospective teacher exercises identification. This is done without treating system of beliefs as properties of the individual mathematics teacher and underlying reasons for his or her teaching. Instead, such attributes are turned into representative characteristics of the social practices in which prospective teachers negotiate their mathematics teacher identities, and thus, are considered active agents in creating images of themselves and their challenging tasks and dilemmas in mathematics teaching. Hence, I claim there is a potential within the Meadian tradition to make visible both social and individual aspects of the being and becoming of mathematics teachers, yet, without providing inconsistent theoretical accounts or yielding to the temptation of making ontological collapses.

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Chapter 5
Methodological Approaches in Research on Affect in Mathematics Education

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5.1 Introduction

Research on affect in learning and teaching mathematics has been growing steadily over the past decades. Linked to this growth, the terminology in this field of research has become differentiated and is now based on different but interconnected basic concepts (Hannula et al. 2019). Older publications tend to regard the concept of beliefs as central and distinguish it from related terms, such as emotions or attitudes (cf. Thompson 1992; Pehkonen 1994; Philipp 2007, especially p. 259). A more recent meta-theoretical classification by Hannula (2012), on the other hand, starts from the concept of affect and forms the following classification system on that basis:

> Hannula has suggested three dimensions to categorize theories related to affect. The first dimension identifies three different types of affect: cognitive (e.g. beliefs), motivational (e.g. values), and emotional (e.g. feelings). The second dimension distinguishes between theories that focus on the relatively stable aspects of affect (i.e. traits) from the theories that focus on the dynamically changing aspects of affect (i.e. states). The third dimension identifies three different traditions for theorizing affect: physiological theories, psychological theories, and social theories (Hannula et al. 2019, p. 3).

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Particular research on affect-related issues can be located at different points of this classification and uses different research methods, depending on the research question and basic concepts. Within the research on beliefs, as an example of affect-related research, a variety of methods has already been used:

> Mathematics education researchers have typically approached the study of [. . .] beliefs in one of two ways: by using case-study methodology or by using beliefs-assessment instruments [. . .] that include some combination of classroom observations, interviews, surveys, stimulated recall interviews, concept mapping, responses to vignettes or videotapes, and linguistic analyses. These data are often collected over a period of time and triangulated (Philipp 2007, p. 268).

With the further development and the increasing integration of emotions and attitudes to the research that was primarily related to (teachers') beliefs, the range of research methods has become more widespread and differentiated—considering only empirically oriented research. Philosophical, theoretical, and historical topics, in turn, have different research questions and require other approaches to the research subject (e.g. see the chapter by Suriakumaran, Vollstedt & Hannula, this book). In the following, we give an overview of different empirical methods—in particular with regard to the contributions that are presented in these proceedings and previous editions of the MAVI conference.

5.2 Overview of Methodological Approaches

Besides philosophical, theoretical, and historical approaches, the field of empirical research is often seen as divided into the two areas of qualitative and quantitative methodologies. A central idea of many qualitative studies is the starting point, that the object of research is already involved in a type of 'understanding' or 'meaning', that individuals or groups have attached to this object. The goal is to reconstruct and clarify this meaning or understanding:

> Qualitative research is an approach for exploring and understanding the meaning individuals or groups ascribe to a social or human problem. The process of research involves emerging questions and procedures, data typically collected in the participant's setting, data analysis inductively building from particulars to general themes, and the researcher making interpretations of the meaning of the data (Creswell and Creswell 2018, p. 4).

While qualitative methods are often associated with a detailed interpretative focus on smaller samples, quantitative research, on contrary, is often understood to approach the subject of research more from an outside perspective by analysing data from larger samples by means of statistical methods. In this view, theories and hypotheses are often seen as the starting point of quantitative research, which are evaluated based on data:

> Quantitative research is an approach for testing objective theories by examining the relationship among variables. These variables, in turn, can be measured, typically on instruments, so that numbered data can be analysed using statistical procedures (Creswell and Creswell 2018, p. 4).

While some researchers conceptualize qualitative and quantitative research as insurmountably distinct from each other, the field of mixed methods has been constantly growing in the last decades trying to combine qualitative and quantitative approaches in order to use the advantages of both. Before we give examples of such approaches with a focus on mathematical views, we give examples of studies which mainly focus on quantitative or qualitative aspects.

5.3 Examples of Quantitative Research

In this book, the article of Girnat is an example of a quite traditional use of quantitative methods in research on affect: It uses seven Likert scales of a questionnaire and the score of a mathematics performance test for first-year university students to examine the relationships between the underlying variables and some co-variates (like gender and school marks). In this kind of questionnaires, rating scales are often used to measure to what extent participants are concerned with a specific phenomenon, for instance, anxiety related to mathematics, in a standardized way so that it is possible to compare the measurements among different participants.

Among research on affect in mathematics education, there is a broad variety in the usage of this format (like Guttman scaling or Osgood's semantic scale, see Leder and Forgasz 2006). For instance, respondents are often asked to express their degree of agreement to particular statements in order to get insight into individual manifestations of affect-related aspects, such as self-efficacy, in a standardized and, therefore, comparable way. When choosing such a scale format, participants often respond by selecting integer values between 1 and the maximum of the scale (4, 5 or 7 in some cases). Such Likert scales have also been used in different ways; e.g. Soro (2000) investigates teachers' beliefs about boys and girls, providing a list of possible students' behaviour and then, she asks the teachers to select if the proposition refers to something that 'usually a girl' will do, or 'usually a boy' or 'a girl as often as a boy'. Another variation consists in using a continuous rather than a discrete interval of values. An example can be found in the chapter by Peters-Dasdemir and Barzel (this book).

Likert scales are, obviously, not the only type of questions adopted in standardized questionnaire instruments. In research on affect in mathematics education, we also have productive examples of the use of dichotomic (yes/no) questions (e.g. Ambrus (1996), ranking of statements according to the level of agreement (Törner and Kalesse 1996), or multiple-answer questions. The latter format consists of asking to select more than one answer among a list of given ones; for instance, Kasten (1998) asks future teachers to select three important aspects for children's learning.

Data coming from standardized questionnaires are analysed by means of descriptive and inferential statistics, such as correlation analysis, factor analysis (e.g. Risnes 1998), frequencies of found categories (e.g. Krummenauer & Kuntze, this book), and much more. In his chapter, Girnat (this book) examines the group differences on these scales and the correlations between the variables. In addition, linear models are used to explain the performance in the mathematics test depending on the other variables. The main results of the latter can be described as follows: The self-efficacy scales can explain a considerable amount of the students' mathematics performance and self-concept scale, revealing that the self-concept is mostly determined by the students' self-efficacy related to applied mathematics and calculus, whereas their performance could be best explained by their algebraic self-efficacy. This is a typical outcome of a quantitative research project: It describes the relationship between the variables measured by different survey instruments in a statistical sense. Compared to qualitative approaches, it provides different insights into characteristics of students' subjective thinking, feeling or understanding.

Finally, we want to mention, that data for quantitative studies on affective aspects may also come from national (e.g. Nevanlinna 1997) and international institutions (e.g. Torner 1997). In the chapter of Caponera, Palmerio, and Pozio (this book), TIMSS data are analysed to identify personal and context factors predicting mathematics achievement of students.

5.4 Examples of Qualitative Research

Within the wide body of research on affect in mathematics education, a variety of qualitative research methods has emerged (e.g. Bikner-Ahsbahs et al. 2015). For instance, there are several examples of investigations with a focus on single students' (e.g. Furinghetti and Morselli 2011; Viitala 2012) or teachers' (Larsen 2014) beliefs, emotions, or attitudes. As exemplified in the chapter by Andrá and Brunetto (this book), focusing on qualitative data from an individual allows to study individual processes in-depth, so case studies can provide deeper insight into individual forms of phenomena, as *empowerment*, *self-efficacy* or *identity*.

Data in qualitative research are often collected by means of interviews. Structured and semi-structured interviews can be realized in presence (e.g. Viitala 2012), in a written format (as in the chapter by Ferretti et al, this book), by phone (e.g. Lindgren 2000) or online communication.

Besides interview studies, other methods of data collection are applied. For instance, in the study by Wong (2000), students are asked to judge whether 'doing mathematics' is involved in each case of a list of hypothetical situations. Leder and Forgasz (2006) call *projective* techniques a set of stimuli which are not based on a scale or closed questions and are aimed at provoking a reaction in the respondent. According to them 'this technique [. . .] is often adopted by those who favour qualitative approaches to the measurement of beliefs and by those concerned that respondents to Likert items may not express the beliefs they actually hold' (p. 409). A particular example is given by *pictorial tests* (as they are named by Hannula 2007): In the work by Perkkilä and Aarnos (2007), children were asked to evaluate the mathematics they saw in a set of pictures to express their feelings about the picture and to write down their mathematical ideas regarding the image. 'This test was able to initiate enough richness and variability in responses in order to be useful for analysing childrens' authentic responses. Furthermore, their experiences were that the students enjoyed doing the test' (Hannula 2007, p. 199).

In this book there are two examples of pictorial tests: Hatisaru (this book) uses the *draw a mathematician test* by Picker and Berry (2000), which asks students to draw a mathematician at work. The students were also asked to write down an explanation of their drawing. Another example of an innovative qualitative methodological approach based on pictures is the study by Blum (this book), in which students from different countries were asked to send photos about mathematics. Blum analyses the pictures in terms of the involved mathematical contents in order to get insights into the mathematical views students might have.

Projective techniques open the problem of big amounts of qualitative data. This issue may be dealt with pre-established coding techniques (as in Krummenauer and Kuntze, this book) or adopting a grounded theory approach. An example is given by Huhtala (2000) who studies the views about mathematics of nurse-students. She realizes a coding in three steps: first, phenomena are labelled according to an open coding; second, the emerging concepts are grouped under different categories like 'mathematics has always been difficult for me', 'I hate mathematics', 'math anxiety', and so on. This is an axial coding. Finally, three core categories are drawn from the storyline of the theory of student's own mathematics. In her case, these core categories are experiences, emotions, and encounters.

In the last three studies mentioned above also statistical methods were applied to analyse the qualitative data. This fact shows that the separation between quantitative and qualitative methods is in many cases neither clear nor sufficient for classifying particular research. In the following, we focus on methodological approaches which bridge the gap between qualitative and quantitative research in the domain of research on mathematics-related affect.

5.5 Bridging the Gap

During the last years, important efforts have been made in combining different methods to get a more precise picture of affect in mathematics education by addressing both qualitative and quantitative questions. For instance, there are several studies in which questionnaire studies with large samples are combined with in-depth analyses focusing on smaller samples (cf. Hannula 2007, p. 199). Such an approach is used in the above-mentioned study by Soro (2000), in which teachers had to choose in a questionnaire whether a given statement may usually apply to a boy or a girl, more often to one than the other, or as often to both genders. In the qualitative part of this study, a number of teachers were interviewed to get a deeper understanding of the thinking behind different types of questionnaire responses. An approach which combines different methods is also realized by Pehkonen (1997), who investigated teachers' beliefs using both a questionnaire and class observations, and by Lewis (2012), who combined a test with interviews based on pictorial stimuli and a card-sort technique to gain deeper understanding of motivational and emotional aspects of disaffection. Lindgren (1997) uses pre- and post-test results based on Likert scales to select prospective teachers which had a great shifting in their views of mathematics during an intervention study. With this smaller sub-group, a qualitative study was conducted afterwards based on interviews. A similar approach is used by Philippou et al. (2000) to select relevant participants for their task-based interviews. A shared characteristic of these studies is that questionnaire data were used to identify a relevant sub-sample for qualitative in-depth analysis. Besides such studies, there are approaches that use different kinds of data collection in parallel: For instance, an open questionnaire is used along with observations in class (as in Hoskonen 1998) or in combination with a focus group (e.g. Martinez-Sierra 2012). Besides such studies combining different methods, which each could be located within the categories of quantitative and qualitative methods, there are methodological approaches focussing simultaneously on both qualitative and quantitative aspects of affect. Such an approach, for instance, is proposed in the chapter by Maffia and colleagues (this book, Chap. 13), where a clustering method is adopted to identify different belief profiles of future teachers about influencing factors of students' mathematical abilities. Answers to a single multiple-answer question are used to detect answering patterns which provide insight in whether the participants pay attention more to natural abilities, learned ones, creativity or affective aspects. This shows that analyses often deemed as 'quantitative' can also be consistent with an understanding of 'qualitative' aspects of the phenomenon under investigation. This is particularly (but not only) evident when statistical techniques are not used to analyse structured questionnaires but open questions or self-reports. An example is given in the chapter of Krummenauer and Kuntze (this book, Chap. 15), in which teachers' non-standardized written answers to open questions were subjected to a two-step analysis, which combines a theory-driven top-down coding with a bottom-up identification of distinct categories representing different types of teachers' answers. The reliability of the coding is ensured by conducting a second rating and measuring the inter-rater-reliability. In literature, there are also studies using word clouds, computations of word association and word counting as means of statistical analysis of open questions (e.g. Hoskonen 1996).

5.6 A Reflecting View on the Methods

Although the examples mentioned above are only a small extract of the wide body of research on affect in mathematics education, they nevertheless illustrate that there is a variety of methodological approaches and a continuous (further-)development addressing the multi-faceted field.

Besides the outlined strengths of qualitative and quantitative methods in this field, both approaches also have their constraints in their own right. Discussions in MAVI conferences often pivot around such issues. For instance, single case interview studies might provide deep insight into individuals' views, but they leave many questions open concerning to what extent the findings do apply to other people. However, when it is intended to develop scientific knowledge which goes beyond particular cases, research has to deal with the question of to what extent findings from case studies can be generalized, which, in many cases, makes it necessary to take into account larger groups of people and to consider quantities.

While methods traditionally considered as quantitative are able to provide information on larger samples, for instance, when using standardized questionnaires, they can be limited concerning grasping individuals' thinking and meaning, as, for instance, standardized questionnaires provide already reified narratives (see Heyd-Metzuyanim & Graven, this book, Chap. 3) to which respondents must compare their personal ones.

Hannula (2007), therefore, considers research which uses both qualitative and quantitative methods as most productive (cf. Hannula 2007, p. 199). The above-mentioned examples illustrate that there are various ways to combine different methods to focus simultaneously both on quantities and qualitative aspects of affect. For instance, interview studies can be used for reconstructing individual views and for developing hypotheses, which has a range beyond the interview sample; subsequently, these hypotheses can be evaluated by means of questionnaire studies with a larger amount of people. Additionally, interview studies, even single case studies, can benefit from quantitative analyses. For instance, measuring the inter-rater-reliability when subjecting the analysed data to a second rating, can substantially increase the objectivity of a study. Vice versa, the same applies to quantitative approaches, which should also take issues, such as dialogue consensus or issues on meaning into account.

However, there are many recent approaches which go beyond the distinct categories of qualitative and quantitative methods. As the examples have shown, research often deemed as quantitative is not only testing hypotheses on the basis of data. As outlined in this chapter, statistical analyses, such as cluster analysis, can provide to deepen the understanding of 'qualitative' aspects, for instance, when reconstructing different types among participants. In our view, a strict binary distinction between qualitative and quantitative methods is often neither appropriate nor productive for research as it unnecessarily narrows the researcher's view when focussing only at one paradigm and might by this also impede the (further) development of advanced methods. Research results need to be connected considering the questions, theories, and methods that are behind the results to acquire a consistent interpretation of the same phenomena (see Suriakumaran, Vollstedt & Hannula, this book, Chap. 7). Furthermore, as choosing methods should depend on the aim of research and the requirements of the research object, not on a pre-established paradigm, it should be questioned critically whether using the label of *mixed methods* as a third paradigm has a benefit for research on affect in mathematics education (cf. e.g. Gorard 2010).

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Part II
Capturing, Understanding and Changing Students' Affect: Theories and Methods

Chapter 6
Introduction

Domenico Brunetto

This part of the book focuses on students beliefs and learning, whilst the third, following part focuses on teachers beliefs and teaching. More precisely, on how the students' views impact their own learning of mathematics. The term "views" is meant as general as possible, but the reader can find more nuanced definitions in each chapter. The relevance of such a topic is highlighted by several scholars in the last decades: from McLeod (1989) to Hannula's (2018) plenary lecture at the 42nd Conference of the International Group for the Psychology of Mathematics Education. The former discusses the role of affect factors, such as beliefs and attitude, in learning mathematics, reconceptualizing the affective domain; the latter sheds new light on a long researched dimension, namely anxiety, in Mathematics Education and its role in learning. What emerges in these findings is the need of identifying affective dimensions that play a crucial role in the learning process and better understanding the dialectical relationship with teachers' beliefs (Hannula et al. 2016).

In such a direction, the work of Di Martino and Zan (2010) can be considered a milestone, where the researchers provide a multidimensional characterization of a student's attitude towards mathematics. They point out a lack of theoretical clarity (Di Martino and Zan 2010) in characterizing student attitudes, in particular, three main types are reported. The peculiarity of such types is the different numbers of dimensions used, for instance, Haladyna et al. (1983) define attitude as positive and negative affect towards mathematics, so that they adopt a unidimensional approach, whilst a bi-dimensional definition is provided by Daskalogianni and Simpson (2000), where emotion and beliefs characterize the student attitudes. However, different types of approaches are not meant as ambiguity by large, but as evidence of complexity in providing a single universal definition. Moreover, once a

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theoretical characterization is adopted, the attitude measurement is all but easy, and it remains an issue (Di Martino and Zan 2001). However, a narrative approach may contribute to achieving the aim as many researchers claim in the teacher context (e.g., Chapman 2002). Therefore, Di Martino and Zan (2010) exploit the narratives for both characterizing and measuring the student attitudes towards mathematics. Three dimensions emerge from their work: emotional disposition, the image of mathematics, and perceived competence.

As said above, in the following chapters, authors deal with “views” contributing to characterize them better and to a deeper understanding of the relation between them and other dimensions. In Chap. 7, Suriakumaran, Hannula, and Vollstedt focus on the relationship between motivational constructs, personal meaning and value, using networking strategies. The merit of the chapter is twofold: on one’s side, the authors bring to the fore a rather under-researched construct in the affect-related domain, that is: values. A literature review on values is provided, briefly, and its connections to other affective constructs are discussed. On the other side, resorting to networking theories, the authors elaborate on the relations among different affective variables at theoretical level. A plea for attempts of this sort emerges from researches within the MAVI community, as Liljedahl (2018) argues: “Research in the affective domain has often been restricted to focused attention on a single affective variable. This is ironic given that we know that affective variables tend to cluster. Perhaps the reason for this is that we lack theories for thinking about affective clusters” (p. 21). An affective cluster comprises beliefs, emotions, motivations, etc., related to an experience, or to a context, that is relevant for the individual. They are activated in a connected (rather than an isolated) manner, and hence it is necessary that the researcher considers them as a whole, rather than trying to isolate one variable or the other from the network of their (strong) relations.

Also, the Chap. 8 by Sinclair and Coles brings to the fore an under-researched (but emerging in the very last years) construct, focusing on ritualization: the role of the body in learning. The authors provide evidence that affect is not just a property of the student: affect does not reside “inside” an individual person, but it is thought of as a distributed and relational flow among students and objects. With Ferrari and Ferrara’s (this book, Chap. 2) contribution, this represents an innovative approach to affect, since embodied theories seem to be overcome by more radical approaches with respect to the role of the body in learning. More specifically, insight into the role of rhythm and rituals in mathematics classrooms is offered.

As concerns the characterization of students’ views, in Chap. 9, Hatisaru, exploiting the Draw a Mathematician Test (Picker and Berry 2001), investigates how multiple-choice test effects on student views about mathematics. Interestingly, at methodological level, Hatisaru employs draws to let students’ views emerge. This poses some issues when it comes the time to be interpreted, but seems to be a promising way to investigate students’ views without the need for explicit verbalization. In the MAVI tradition, such a methodology has been employed also by Erkki Pehkonen and his research group, as it is testified in Hatisaru’s chapter.

Østergaard, in Chap. 8, explores how the role of “real-life math” can influence student beliefs about mathematics, reporting an example of teacher activity in such direction. Interestingly, History of Mathematics is the context within the research is conducted.

Finally, Blum, in the last chapter of this part, argues about how multimedia data produced by students from different countries can help researchers in a deeper understanding of the nature of students' beliefs towards mathematics. The use of pictures taken by students as a way to capture their beliefs about mathematics represents a novelty in affect-related research.

The majority of chapters in this part of the book adopt methodologies for collecting data that represent a shift from a rather declarative approach to affect, to observational and interpretative ones, by resorting to gestures, draws, pictures as source of evidence. Declared beliefs, as they emerge in interviews and questionnaires, which represent a tradition for MAVI, are replaced by somehow observed beliefs, as if a more recent approach to affect deems it as "enacted", instead of being deemed as "declared".

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Chapter 7
Comparing and Contrasting Personal Meaning and Value

Neruja Suriakumaran, Markku S. Hannula, and Maike Vollstedt

7.1 Introduction

The field of mathematics-related affective constructs includes a wide subdomain about learner's *motivation*. To name but a few, motivation (Ryan and Deci 2017; Hannula 2006), personal meaning (Vollstedt and Duchhardt 2019), and value (DeBellis and Goldin 2006; Eccles et al. 1983) highlight different aspects of learners' inducement to deal with mathematics. A challenge is that there are no clear conceptual boundaries between the concepts and the question arises how far these concepts overlap. A profound comparison between different theoretical approaches is challenging. Several aspects should be considered to guarantee a fine-granulated, analytical comparison.

In this spirit, we compare the two constructs *personal meaning* (Vollstedt and Duchhardt 2019) and *value* (Eccles et al. 1983). We chose these constructs, as both elaborate personal motives that direct learner's behaviour. In this article, we highlight similarities of these constructs, while also specifying their crucial differences, and individual research foci. Our analysis is motivated by two main reasons: First, it is difficult to make the construct you use understandable for other researchers unless you thoroughly understand how the construct relates to similar affective constructs. Establishing such connections may help to enhance the concept's, *descriptive/explanatory power*, depict its focus in research, locate the questions it follows, and thereby foresee the answerable questions within its theoretical frame (Schoenfeld 2002). Secondly, comparing a concept with similar

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ones may help to identify gaps in theory, highlighting ways to develop them. Through such comparison, researchers get an overview of the related constructs, find profound reasons to develop their own theory in a specific focus to locate or, rather, to define the scope of their own research (Mason and Waywood 1996).

To conduct a criteria-guided dialogue between personal meaning and value, we first network the constructs in the *semiosphere* that (Radford 2008) describes as a space where theories are distinguished by their individuality and theoretical limitations. Radford characterizes dialogue as "the door for entering the semiosphere" (p. 318). Thereby, he suggests that theories can be understood "as a way of producing understandings and ways of action based on (P, M, Q)" (p. 320). P contains *basic principles*, M is a set of *methodologies* facilitating P producing relevant data, and Q is a set of *research questions*. These three components are deeply related to each other. Second, we use the *networking strategies comparing* and *contrasting,* to highlight the constructs' key elements and explicitly pointing out their different structures and functions (Prediger et al. 2008). In this paper, we present a brief introduction of the two affective constructs in the context of mathematics education, followed by networking the two approaches in triplets: basic principles, methodologies, and research questions (Radford 2008).

Our major aim is to examine the ambiguity in the definitions for these motivational constructs. Therefore, we explore which core ideas are explicitly considered to shape their theoretical approach. At the same time, we point out which components are consciously left aside to specify their certain definition on learner's individual motivation. In this connection, we intend to examine the benefits and obstacles we meet by networking with comparing and contrasting.

Research Background: Quality of Motivation in Mathematics Education
Before one decides to use a theoretical approach—which are always fundamentally connected to their methodology and research questions—one needs to clarify what empirical phenomena to study and why to study them. In our case, we aim to understand learners' subject-related motivation in mathematics education. Clearly, learner's motivation differs from individual to individual and intraindividual variation can be seen during the learner's lifetime. In the educational context, particularly the psychological needs for autonomy, competence, and relatedness have a significant influence on learner's goal choices (Hannula 2006). Therefore, we also need to consider the learner's psychological basic needs when examining their subject-related motivation. The basic needs are a foundation of motivation and at the same time, they function as a feedbacksystem for optimal biological/psychological development and wellbeing (Ryan and Deci 2017). The examination of the specific subject-related personal motivation may be a promising direction for elaborating the different personal motivations and their regulatory styles (ibidem). In line with that, we focus on the quality of motivation. Thus, we investigate what it is that the student wants, rather than measuring how strongly the student wants something that we researchers have determined beforehand (Hannula 2006). Correspondingly, we need to study a construct that has relations to the basic needs and a potential to describe the quality of motivation in mathematics.

As we are comparing similar theoretical constructs in the field of affect in mathematics education, we want to highlight what construct we why want to study further. Moreover, we may formulate an epistemological contribution to the domain of mathematics affect through the reflections on networking. In that sense, the following research questions deal with the metaquestion of handling similar affective constructs.

Networking strategies in the field mathematics-related affect. The first (methodological) question is what we can learn in this case using networking strategies.

Choice of affective constructs. Secondly, we discuss which motivational construct comes with the convenient components to understand the quality of learner's personal motivation in the subject mathematics.

7.2 Motivational Constructs Considered

The following sections introduce the affective constructs personal meaning and value, aiming at an immanent analysis of how the concepts were constructed in the initial studies. After that, their theoretical basic principles, methodologies, and research questions will be tentatively networked using the networking strategies comparing and contrasting. Finally, they will be discussed comparatively.

Personal Meaning

Teaching (mathematics) is not just telling the contents to the learners. In the mathematics classroom, the realm of possible activities and teacher's impulses should preferably overlap with the learner's individual "world" so that what is said can become meaningful and thus accessible to the learner. Learners are active interpreters and act towards the meaning mathematics has for their individual world. In that sense, personal meanings (Vollstedt and Duchhardt 2019) depict learners' perceived self-relevance that they connect to (learning) mathematics by asking: "What is relevant for me when I learn mathematics?". Based on constructivism, personal meaning focusses on the individual while also involving social aspects. The construction of personal meanings is influenced by two main preliminaries, namely the learner's personal background (aspects that one cannot influence, e.g. migration background) and personal characteristics (aspects that one can influence, e.g. self-concept (Marsh 1986) and basic needs (Ryan and Deci 2017) from educational psychology, beliefs (Törner 2002) from mathematics education, and developmental tasks (Havighurst 1972) from educational science). The construction of personal meaning takes place within a mathematics learning environment where learners decide to act in ways that are either conducive or obstructive with respect to learning. Based on interview data with students from Germany and Hong Kong, Vollstedt reconstructed 17 different kinds of personal meaning. The role of (learning) culture and its relation to the subject mathematics were explicitly considered. The personal meanings vary between *Obligation* ("I mainly deal with mathematics because I have to"), *Purism of mathematics* ("The structure of mathematics fascinates me"), and

Experience of relatedness among the fellow students ("I prefer to do mathematics when I collaborate with others in a group"). Personal meanings are not always conscious (Vollstedt and Duchhardt 2019).

Value

In mathematics education, value is another construct that plays a crucial role in investigating learner's individual attribution of personal motivation to learn mathematics. The conceptualization of value by Eccles et al. (1983) is based on ideas of constructivism. Based on a long ancestry in achievement motivation (cf. Gaspard 2015), Eccles et al. (1983) linked the expectancy and value concepts with several psychological (e.g. child's affective reactions and memories; ibidem), social (e.g. socializer's beliefs and behaviours, ibidem), and cultural factors (e.g. cultural milieu, ibidem) within specific subject domains (e.g. mathematics) in education as expectancies and values vary across different subject domains. Values can be defined as a relative worth of an object or an activity accompanied with the psychological experience of attraction (or averse) regarding this object or activity (ibidem). Wigfield and Eccles (2000, p. 68) argue that "individuals' choice, persistence, and performance can be explained by their beliefs about how well they will do on the activity and the extent to which they value the activity". Thereby, achievement-related motivation is conceptualized as an interplay between expectancy and value. Value depicts the extent to which a goal or an activity is desirable. They describe four subjective task value components: *Attainment value* is characterized as the personal importance of doing well on a task for self-enhancing reasons ("Being good at math means a lot to me"; Gaspard 2015). *Intrinsic value* is identified as the sense of pleasure or enjoyment in an activity ("Math is fun to me"; ibidem). *Utility value* is defined as how a task represents one's perceived future plans or goals ("Learning math is worthwhile, because it improves my job and career chances"; ibidem). *Cost* is concerned with the negative consequences about how the decision of putting effort into an activity contains additional effort and negative emotions ("I'd have to sacrifice a lot of free time to be good at math"; ibidem). The subjective task values serve the estimation of effort, the likelihood of task achievement, and emotional cost (Wigfield and Eccles 2000). The modern expectancy-value theory is very influential in education, as it contains a broad conceptualization of various components. Empirical studies have confirmed a strong relation between these multiple components (ibidem).

7.3 Method: Networking Strategies

To counter the richness of theoretical approaches in mathematics education, networking strategies provide a methodological framework in which different theoretical approaches can be served without overshadowing important conflicts. We follow the ideas of comparing and contrasting as a method to investigate the typical characteristics of personal meaning and value. According to the Networking Theories Group (Bikner-Ahsbahs and Prediger 2014), comparing and contrasting only show subtle differences: “Whereas comparing refers to similarities and differences in a more general way of perceiving theoretical components, contrasting is more focused on extracting typical differences. [. . .] strong similarities are points for linking and strong differences can make the individual strengths of the theories visible” (Prediger and Bikner-Ahsbahs 2014, p. 119). For a profound use of comparing and contrasting, the theoretical approaches should analyse the exact same data set. Only then a direct comparison of results is possible. Each approach contains scholar’s implicit notions. By analysing the same data we may grasp more precisely the theoretical and empirical considerations of each approach (Bikner-Ahsbahs and Prediger 2014).

To the best of our knowledge, there has not yet been a study that used the theoretical approaches of personal meaning and values to analyse the same data set. Every data set is individual and, thus, in our case, the datasets that were used to interpret and develop the theories of personal meaning and value, respectively, are seen in isolation. Each data set has a different focus on the respective research direction and thus represents a difficult undertaking to consider all data. For this reason, we intend a tentative analysis of the constructs presented in the following section. For this reason, the analysis presented cannot meet criteria-guided empirical methods as Networking Theories Group (ibidem) recommended. Nevertheless, it gives a first impression of the construct’s key factors and reflects this attempt with its limits if the analysable data set is *not* the same.

7.4 Results

The different constructs presented here are products of various lengths of scholarship. This does not mean that one is better than another is, rather that some are young and some are matured. In our study, a profound comparison of the constructs personal meaning and value was not possible. Still, as a result we point out why the comparison has failed, so that future research may take these reasons into account.

Basic Principles (P)
Personal meaning is a quite young construct (Vollstedt and Duchhardt 2019). Vollstedt has worked out several components that have explicitly proven to be relevant for the construction of learner’s personal meaning like psychological needs (Ryan and Deci 2017) and (learning) culture. The personal meanings are influenced by the certain cultural context of the learner and focuses on mathematics-related aspects.

In contrast, *values* (Eccles et al. 1983) have a long history and include a conceptualization of various components. The expectancy-value model of achievement-related motivation was initially developed and tested to explain mathematics-related gender differences. Therefore, the focus was on various social-psychological influences like choice, persistence, performance, and learner’s expectancies and values. This approach also focuses on the inclusion of cost as an important factor of making choices. Both components, expectancies and values, are affected by task-specific beliefs, e.g. perceptions of competence, individual's goals, and self-schema accompanied with their former achievement-related affective experiences. Individual's perception and interpretation are affected by a broad array of social-psychological determinants. Eccles et al. explore how these links vary within the expectancy-value model and across culture (Wigfield et al. 2004).

Comparing and Contrasting Looking at the "ingredients" within the basic principles, both concepts are aligned with a constructivist approach. The learner is conceptualized as an individual, whose development is influenced by several psychological, social, and cultural aspects. In addition, both constructs consider similar theoretical concepts of constructivism, such as developmental tasks (personal meaning), child's goals, and general self-schemata (value). Within the frame of personal meaning, the basic psychological needs are considered implicitly and explicitly. Value is conceptualized with multiple components and the role of the basic needs is left aside. Both constructs have different foci with respect to culture: Vollstedt worked out the different personal meanings for the subject mathematics, with a specific focus on culture-specific aspects in Germany and Hong Kong. Thus, the developed constructs account for both cultures, providing different patterns of preferences among the kinds of personal meanings in each place. In contrast, Eccles et al. (1983) tested how much the expectancy-value model varies across cultures. Furthermore, the relations to the phenomenon motivation seem to be different. Vollstedt conceptualizes action that can be either conducive or obstructive, depending on the personal meaning constructed. Personal meaning is a function of learner's perceived self-relevance and it shows links to the quality aspect of motivation. The consideration of basic needs as a key element of personal meaning may be a promising direction to find out connections between personal meaning and learning motivation (Ryan and Deci 2017). In contrast, Eccles et al. (1983) study the subjective task values in order to infer effort, the likelihood of task achievement, and cost. These aspects highlight that the construct of value is clearly rooted within the research of achievement motivation.

In terms of the basic principles, we could refer to the key elements of both constructs alone. Construct's identities *cannot* be understood by just referring to their key factors (Bikner-Ahsbahs and Prediger 2014). These results are giving a little insight into the characteristics of each construct on the level of basic principles, yet they are eventually insufficient, as it was impossible to make a fair comparison on both constructs' strengths and gaps. For this reason, we could only compare and contrast both approaches on a more general level by considering their key constructs.

Methodologies (M)
Vollstedt conducted 17 interviews each in Germany and Hong Kong and developed a grounded model of personal meaning from the data. Students were aged 15–17. Latest studies confirm the assessment of personal meanings with reliable scales (Vollstedt and Duchhardt 2019).

In contrast, Eccles et al. studied value, within their framework, by means of longitudinal studies consisted of student record data, parent/teacher questionnaires, classroom observations, and multiple scales of students' ability beliefs, expectancy, and value. The participant age varied as the links within the model were tested with primary as well as higher secondary school students.

Comparing and Contrasting These constructs were studied using qualitative and quantitative approaches. Vollstedt's initial study intended to reconstruct individual meanings constructed by learner's from two cultures. A qualitative approach was inevitable as there were no earlier studies on this topic. Eccles et al. aimed to understand the relations between expectancy-value and a broad array of psychological, social, and cultural determinants. For this reason, correlational methods were used to assess the relations across cultures.

In general, we see that each data set is individual and that the scholars are only able to come to conclusions with respect to what can be found in the specific data sets. For a fine-granulated comparison, looking only at the theoretical concept is insufficient for understanding the exact interpretation of results as each coding paradigm and manual have their individual definition. These coding paradigms also need to be comparatively analysed to understand the specific articulation of each theoretical concept. For this reason, the results constantly contain the interwoven roots of their particular method and the data set from which they result. In our case, a comparison according to the Group of Networking Theories on the level of methodologies is *not* possible.

Research Questions (Q)
Vollstedt's study on personal meaning has the major focus on the effects of the different cultural context and includes the subject-related features. Therefore, she wants to understand what meanings learners relate to learning mathematics in an educational context and which role the cultural background does play.

Eccles et al. explore the interrelation between values and expectancies to explain gender differences in choice of mathematics courses and majors. They want to understand achievement-related behaviour that has a wide range of influences and at the same time, the various links among these determinants.

Comparing and Contrasting In both theoretical approaches, the pragmatic research questions are consistent with the initial research approach. Thus, each construct is used with the intention to understand particular problems related to the phenomenon motivation in mathematics. Interestingly, Eccles et al. focus on extending and testing the stability of their model regarding achievement motivation. In contrast, Vollstedt intends to enhance the understanding of learner's perceived self-relevance depending on their individual biography. The two constructs seem to have different foci. With a common database we could see more clearly how far the systematic and epistemological perspectives and their research questions contrast, overlap, or may even be complementary. At the moment, a comparison in terms of the research questions is *not* possible. Yet, personal meaning and value seem to be individually marked with a research perspective, which makes the distinction relatively clear for a reflection.

7.5 Reflections on the Application of Networking Strategies

We examined personal meaning and value, two similar constructs related to motivation in mathematics education. Our major aim was to address the ambiguity in the definitions for motivational constructs that learners assign to deal with mathematics.

When looking at networking personal meaning and value, it was not possible to analyse the same set of data from these two theoretical perspectives as originally recommended by the Networking Strategies Group. Each approach has their individual data sets, which have been analysed and interpreted through different methodological approaches and within different theoretical frameworks. Results contain interwoven roots to their particular background theory, method, data set, and thereby give meanings to their foreground theory (Mason and Waywood 1996). Nonetheless, the use of the strategies provided an opportunity to conduct a tentative analysis. Thus, we could formulate an epistemological contribution in view of comparing similar affective constructs and reflect this attempt with its limits if the analysable data set is not the same. Thereby, we met some benefits and limits for the use of networking strategies.

One interesting issue is the role of cultural aspects for these constructs. According to constructivism, "knowledge is not passively received but actively built up by the cognizing subject" (von Glasersfeld 1989, p. 162). The constructivist components of these constructs (value: e.g. child's goals/general self-schemata; personal meaning: e.g. developmental tasks) belong to the individual learner. Thus, these components would affect the process of learning independently from culture. However, cultural aspects matter in another direction. If we look at completely different cultures, we assume that the importance of certain personal meanings or values directing learner's behaviour might vary. Additionally, we expect to find certain culture-specific links between the constructivist components of personal meaning and value.

Another important aspect concerns how learners make choices. Learners may construct multiple personal meanings occurring simultaneously when dealing with mathematics and making choices within a certain situation. Therefore, we gain insight into learner's different preferences and their individual emphasis of importance. In contrast, the expectancy-value model additionally discusses learner's cost that accompanies his/her decision of putting effort into an activity (e.g. solving a mathematical task) restraining opportunities for other activities (e.g. meeting friends).

For an optimal comparative analysis of similar affective constructs we agree with the advice to study the same set of data from different theoretical approaches. Countering the terminological ambiguity is certainly a challenge (e.g. deliberate handling of coding manuals from the different theoretical approaches), but not impossible. In this case, we suggest that it is necessary to assess the different components of both constructs with a common quantitative data set when analysing the connections of personal meaning and value. Therefore, we recommend conducting a longitudinal study to meet both construct's methodological requirements.

This would simultaneously enhance our understanding of both quality and quantity aspect of motivation. Thereby, we could elaborate their different research foci and might be able to deliver a more coherent insight into the phenomenon motivation in mathematics. For instance, personal meaning and value both differentiate on learner's personal motivation to learn mathematics as they study differently on what directs certain behaviour in class (personal meaning : *Vocational precondition* "I deal with mathematics as I need it for my desired profession") and how much the task is valued by the student (value: *utility value-job* "Good grades in math can be of great value to me later on"). Both constructs include different cultural flavours. For this reason, validated questionnaires and methods that do not oversee the cultural differences are indispensable for this attempt.

With respect to the second research question, the construct of personal meaning depicts learner's self-relevance towards the subject mathematics. Additionally, the basic needs within educational settings are considered for the construction of personal meaning. Furthermore, the relevance of mathematical procedure is another example that plays a crucial role and thus shows a focus on the subject. This approach may be a promising step in order to elaborate how self-relevance is motivationally regulated (Ryan and Deci 2017). In this case, the expectancy-value model offered useful insight into learner's achievement-related motivation. The use of this model might be a promising step towards enhancing our understanding in learner's achievement motivation in different mathematical contents (e.g. algebra) or tasks (e.g. real-world problems).

To avoid ambiguity in the future, scholars should consider neighbouring work of their colleagues. This may minimize the overlap between constructs and lend their study a distinctive taste. A helpful way might be Schoenfeld's standards (2002) to identify the individual degree of one's own theory and develop it further to enrich a dense connection within the mathematically affected domain.

From a holistic perspective, we deny a mind-body dualism and state that affect "is never external to intellect" ((Roth and Walshaw 2019, p. 2)) as "separation of affect from intellect" implies "the separation of body from mind" (ibidem). The different motivational constructs within the field of affect aim to study distinctive aspects of learner's motivation that simultaneously affect body and mind in certain ways. Respecting each concept's special feature (i.e. identity), we cannot easily encapsulate these concepts within one "meta-concept". If so, one "meta-concept" may not be able to study the motivational links between body and mind in a coherent and consistent manner. To counter the terminological ambiguity in a sustainable way, we suggest elaborating the connections between the constructs.

Even though the analyses presented here are not accurately following the guidelines of the Networking Theories Group, this contribution is intended to encourage to test and thereby to profit from the amenity of other fields in view of extending the own domain by networking with other fields from mathematics education.

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Chapter 8
Affect and Ritualisation in Early Number Work

Nathalie Sinclair and Alf Coles

8.1 Introduction

In prior work (see Coles and Sinclair 2019), we have used the concept of ritualisation to perturb the long-standing dichotomy between action and thought that can be found in the mathematics education literature. This dichotomising tends to associate thought with language and discourse, and action with bodily movements that may (or may not) give rise to thought, but that are often seen as being *merely* imitative or physical. Our notion of ritualisation draws on the work of anthropologist Catherine Bell (1991), who critiques the way that ritual has been put in opposition to the conceptual aspects of religion and culture. It also resonates with the ontological assumptions of monist philosophies that refuse to see matter as passive and insert, and thus do not make a priori distinctions between the physical and the mental or the individual and the social.

In this chapter, we extend this habit of troubling dualisms by folding affect into our notion of ritualisation. As explained in the next section, we distinguish affect from emotions, where the latter refer to identified states such as happiness, fear, anxiety or boredom. Affect emerges from the way different bodies *affect* and are *affected* by each other, which guide perceptions and actions. It is often not consciously experienced, but is *felt* in the way that you might feel someone is looking at you or is near you, without seeing or hearing or touching or smelling that person. In the section that follows, we elaborate our notion of ritualisation and also describe how we see affect functioning in a mathematics education teaching and

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learning context. We then use episodes taken from a one-to-one teaching–learning session involving a 6-year-old boy, one of the authors and the Gattegno Tens Chart, to show how ritualisation, and particularly its affective tonality, plays a powerful role in the teaching and learning of number. We argue against any bifurcation of affect and thought, underscoring the fundamental role of affect in mathematical experience, and its distributed, material and temporal nature.

8.2 Doing and Thinking

Rituals are central aspects of society and have been studied by scholars of anthropology, sociology and history of religion. Their function has been described in many ways, such as maintaining certain institutional structures, allaying fears, enabling non-verbal forms of communication and regulating human environmental interactions (Rappaport 1999). Researchers have also studied rituals as they relate to mathematics education at a variety of scales including the larger cultural scale of school mathematics, as well as the much smaller scale of individual student or teacher activity, which is the focus of our research. We briefly consider how Sfard (2008) has written about ritual—which has gained much traction in the past decade– since it provides a counterpoint to our own approach. For Sfard, rituals are seen as acts of solidarity with those with whom they are performed. For her, ritual activity is thus "strictly defined and followed with accuracy and precision so that different people can perform it in identical ways (possibly together)" (p. 244). Additionally, rituals are "about performing, not about knowing" (ibid.).

This framing of ritual produces a clear separation between doing and thinking. In Bell's work, such bifurcation is avoided, in part by her focus on ritual as performance, for which she uses the word *ritualisation*:

> Ritualisation is a way of acting that is designed and orchestrated to distinguish and privilege what is being done in comparison to other, usually more quotidian, activities. As such, ritualisation is a matter of various culturally specific strategies for setting some activities off from others, for creating and privileging a qualitative distinction between the 'sacred' and the 'profane,' and for ascribing such distinctions to realities thought to transcend the powers of human actors (1991, p. 74).

In this characterisation, ritualisation does not distinguish action from thought, but instead points to what is sacred or special. Further, while some ritualisations might be formal and punctilious, Bell refuses to take this as a defining characteristic:

> That is to say, formalizing a gathering, following a fixed agenda and repeating that activity at periodic intervals, and so on, reveal potential strategies of ritualisation because these ways of acting are the means by which one group of activities is set off as distinct and privileged vis-à-vis other activities (p. 92).

The function and purpose of ritualisation is to privilege certain activities over others. This might be done through formality and punctiliousness; but other strategies involve repetition. For example, Manning (2016) emphasises the variation within repetition that arises in rituals related to art-making. She evokes the way in which a repetitive practice initiates change, both in the practitioner and the event. In contrast to characterising rituals as unthinking or procedural or formalised, therefore, they are "capable of shifting the field of experience" (p. 67).

Following Bell, we also use the word ritualisation, and see it as a particular form of thinking. For Bell, it is non-discursive in nature; it is a "mute" form of activity that is "designed to do what it does without bringing what it is doing across the threshold of discourse or systematic thinking" (p. 93). Meanings emerged "within the dynamics of the body defined within a symbolically structured environment" (ibid.). As a means of disrupting the boundary between acting/performing and knowing in mathematics education research, we have thus defined ritualisation as:

> those practices in the mathematics classroom that: (a) set themselves apart as distinct and privileged compared to other activities (e.g., by following a fixed agenda or occurring at periodic intervals); (b) are embedded in a symbolically structured environment; and, (c) do not bring what is being done across the threshold of discourse or systematic thinking (Coles and Sinclair 2019, p. 182).

As practices, and particularly as practices tending towards the sacred or the special, ritualisations will involve not only thinking and doing, but also—and perhaps most intensely—feeling. The "structure" of a symbolically structured environment arises through interaction, it is actions and relations that are symbolised. In the next section, we elaborate on a non-dualistic approach to feeling as it concerns ritualisation.

8.3 Doing, Thinking and Feeling

There is a long tradition of research on the role of affect in mathematics education, dating to the 1980s, that has drawn heavily on psychological theories and focused on affect as a kind of property of a person, such as her beliefs, attitudes and emotions (see McLeod 1992; Zan et al. 2006). More recently, sociocultural perspectives have conceptualised affect in terms of socially organised phenomena that are constituted in discourse and shaped by relations of power (e.g. Evans et al. 2006; Op't Eynde and Hannula 2006). Both early and current researchers working in this domain tend to separate thinking and feeling, and then assume a cause–effect relationship between affective states and students' (or teachers') concomitant behaviours or capacities.

In contrast, Drodge and Reid (2000) use enactivism (Maturana and Varela 1987) to study the bodily basis of affect, and its relation to "emotional orientation", which Drodge and Reid see as a characteristic feature of mathematical activity. Their non-dualistic perspective contrasts with the early research on emotions, while also attempting to grapple with the complex interaction of the body and the social in studying affect. There are several other theoretical approaches that can support the study of affect in a similarly entangled and distributed fashion, emerging in the humanities as part of “the affect turn” (Clough and Haley 2007; Hannula 2012). As part of shifting away from psychological and dualistic approaches that take affect to be about individual judgements of value, a new focus is on “the collectively dispersed nature of affect across a material ecology” (de Freitas et al. 2019, p. 306). Following Massumi (2002), such affect is not seen as belonging to an individual, but is instead taken as impersonal intensive flows. As with the work of Drodge and Reid, the somatic and embodied expressions of affect are not merely subordinate to ideational enactments of interior states.

In addition, there is a move away from the individual. By framing affect in an entangled and distributed manner, recent research has moved towards studying the flow of affect in classrooms. For example, de Freitas et al. (2019) focus specifically on the concept of sympathy, or “feeling together”, in collaborative mathematical tasks, which they frame as being both social and bodily. Rather than speak of emotions and their tendency to be seen as individualised inner states, the authors draw on contemporary work on affective ecologies where affect is treated as fundamentally relational and sourced at the micro-scale of the somatic body. They show how affect imports the individual body into the trans-individual ecology, and fuels the making of mathematical concepts. Similarly, Chronaki’s (2019) study of the body’s capacity to affect and be affected—and, importantly, to affect in different ways than mandated by norms and assumptions—takes the body as an agential, vital force that cannot be entirely determined by sociopolitical forces. Using the concept of intercorporeality and the affective negotiation that it requires, Vogelstein et al. (2019) are also working with affect in a distributed, relational way.

In this chapter, we aim to develop ways of attending to affective flow in the teaching and learning of mathematics. It is possible to observe, for example, the extent to which there is *alignment* between participants and objects, or *misalignment*. We do not mean that there is alignment (or misalignment) in the emotional states of participants and objects. Instead, the forces of affect can be thought of as vectors (that are both affected and affecting), so that alignment is about going, at least partly, in the same direction, or with a similar intensity. This aligning resonates with the “feeling together” of de Freitas et al. (2019), but in using the word “aligning”, we can also consider its counterpart, misaligning, which we think is of particular interest in any teaching–learning situation. Further, we are interested in aligning and misaligning in the context of ritualisation. We take one role of ritualisation practices to be co-ordination of actions and the possibility of a concomitant affective alignment.

8.4 Early Number Teaching and Learning

In attending to affective flows and the role of ritualisation, we now turn to some empirical data. Given the complexity of the challenge of attending to affect, we have

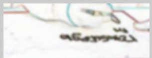
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000

Fig. 8.1 Gattegno's whole number tens chart

simplified the task by taking data from a one-to-one teaching situation, in which Alf was working with a 6-year-old boy who had been identified by his teacher as the lowest achieving in mathematics in his year group in the school. In order to support his work with number, Alf used a Gattegno Tens Chart as well as the multitouch application *TouchCounts* (Jackiw and Sinclair 2014; Sinclair and de Freitas 2014). We took video recordings of all the sessions. There were thirteen in total, each one lasting around 20 min. Through an interrogation of the videos, we offer two examples of what we take to be quite different kinds of affective flow that recurred; these were sequences of aligning and misaligning. We have chosen examples from one video, which is largely representative of the data set. We are far from claiming these are the only affective arrangements present, but rather offer them as exemplars of what attention to affect might afford for analysis. Given the important role that the Gattegno Tens Chart (GC) plays in this excerpt, we provide an image of it here (Fig. 8.1). It makes available the structure of our written number system via an ordinal, relational sense of making links among numerals themselves (see Coles 2014, for further possibilities with the chart).

Aligning When the video recording begins, Alf is talking and Aidan (a pseudonym) is sitting in his chair, with his head in his hands, staring straight forward, seemingly refusing any interaction with Alf. Alf says that he has a video from the last session a week previously and asks Aidan, "I wonder if you can remember this. You were writing out some numbers and then you suddenly spotted a pattern. Shall I play it to you and see if you can remember?" There is the faintest nod from Aidan, but no change in body position or eye gaze. On the video, Aidan is writing out the numbers 1 to 10 and then starts on 11 under the 1, 12 under the 2. At this point, Aidan smiles and moves his arms. He says, "You just can copy it. Like this [taps pencil on chart, starting at 1, then on 3 and moving right]". As he is saying "sixteen", Aidan moves back in his chair, away from the screen, and looks at Alf, who asks, "What pattern did you spot? Do you remember?"

Excerpt 1

1.12	Aidan	Yeah [*touching the GC*]. You can just [*placing his index finger on the left side of the chart*] you can just go a long way [*sweeps with hand left to right on the chart*].	
1.17	Alf	Mm, mm [looks at Aidan].	
1.19	Aidan	It goes like this.	
1.20	Alf	Shall I pause it? [*Aidan stands up*] Yeah go on. Can I get you one of these [*passes mini whiteboard and Aidan sits down*].	
1.30	Aidan	Give me the pen [*lifts the pen up high in the air*]. It goes, it goes like this [*starts writing 1, then looking at the GC, then writing 2, then looking at the GC, then writing 3, then writing 4 and then 5, then looking at the GC and writing 6, then 7 (backwards), 8, 9 and 10 along a row*]	
2.20	Aidan	[*writes 11 below 1, then writes 1, and adds marks to the top and bottom to make it look like a 2 and then puts a 1 to the left of the 2, then writes 3*]	

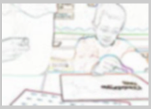
At the start of the video, it is possible to focus on Aidan's lack of interest, or perhaps his fatigue or boredom. This seems to change over the course of a couple of minutes, where he is making eye contact with Alf, writing numerals down and proposing patterns. This would be a reading of the (changing) emotional states that we can infer, from Aidan's posture and tone of voice and actions. However, to study the distribution of affect, we shifted our focus to a more relational and impersonal point of view–the material, embodied ecology that involves not only Alf and Aidan, but also the video from the previous lesson, the GC and the mini whiteboard.

After watching himself write numerals, Aidan engages with the chart, sweeping his hand along it from left to right, a direction of travel that matches the counting of numbers from 1 to 10. He is thus affected by the chart, as his hand moves, and he says "it goes". The manner in which Aidan states what he wants (e.g. "Give me the pen") is unusual across the video recordings in terms of the sureness of his speech. He knows what to do, and perhaps knows that he knows, and when Alf offers the mini whiteboard (at 1.20), Aidan begins writing right away, in a slow and careful way, without prompting. For 45 s, it is calm, quiet, concentrated, as the numerals are formed.

Aidan and Alf are both silent, as Aidan writes. Aidan's gaze is on his own writing, with some glances at the GC. Alf is looking at what Aidan is writing. Aidan's actions bring into being a ritualisation around the writing of number symbols, in the sense of this activity being privileged by silence and mutual attention, by the writing having a structure (i.e. a known order), by the repetition of elements and through the fact that there is no discourse about what Aidan is doing. The GC is present in this ritualisation through the way its own visual structure serves as a resource for Aidan. We interpret in this 2-min clip an affective aligning—both of humans, and of humans and objects. Alf's and Aidan's bodies are more similarly positioned in relation to each other (see image at 1.30) and more *still* in this clip than during any other stretch of time on the video.

Misaligning Immediately following on from the transcript above, the following interaction occurs. (In the transcription, we have attempted to use spellings which convey the sounds made, even when these are not full words)

Excerpt 2

2.33	Alf	Can you say them as you are doing them?	
2:39	Aidan	[*Writes the 1 to the left of the 3*] It is easy [*rolls back in chair away from the table*].	
2.40	Alf	Go on, so what did you notice, what was the pattern you spotted?	
2.43	Aidan	You just go like [*still sitting back, with writing hand holding the pen, but palm extended*], you just go like [*comes forward and points to the 11 with the pen*], whenev, look [*puts the pen back in the cap*], you know when you done one.	
2.50	Alf	Uh hum.	
2.55	Aidan	Eleven [*pointing to the 11 with the pen*], two-twelve [*points to the 12 with the pen*], four [*points to the 13 with his pen*], three-three-ty.	
3.00	Alf	Thirteen.	
3.05	Aidan	Fourteen [*sits back and takes the cap off*]. Yeah, thir, th, it just goes in, it's copy [*points to the top row than the bottom row and looks at Alf, then at the pen*].	

Following Alf’s question (2.33), Aidan stops writing 5 s later. Alf’s question (2.40) “what was the pattern you spotted?” is an attempt to bring the activity across the threshold of discourse. Both the break in writing and the question about pattern stop the ritualisation aspect of the previous 2 min. However, a different kind of patterned activity develops, as Aidan (2.43) points to numerals in the top row (1, 2, 3 . . .) and the second row (11, 12, 13). With this structure, of the 11 below the 1, there is a kind of automatic potential: “you just” (2.43). As he returns to his writing, he begins with the “2” then places the “1” to his left to make 12, repeating the same process for 13 and 14. It is not the “1” that matters, but the numeral that is being taken from the row above: “it’s copy”.

In the ritualisation around the writing of numbers, which Aidan himself initiated (he could have explained the pattern to Alf, but instead started to make it), there is a privileging of the order of number. He does not simply write the 1 and the 11 then the 2 and the 12, which would break with the sequential unfolding of number over time, the ordinal enactment that takes time. This ritualisation practice occurs again when Aidan is pointing at 11, 12 and 13, tapping them like Alf had done before, setting them out as discrete, nameable entities. The pen moves from being the writing instrument to the pointing stick, from maker to namer. The transcript continues.

Excerpt 3

3.10	Alf	I think I understand.	
3.15	Aidan	Look like this [*writes 4 then 1 to the left*].	
3.20	Aidan	Like when you want to learn properly [*writes 5 and 1 to the left*].	
3.25	Alf	I would write them with the one first, because that is kind of how, yeah, yeah, anyway, so it copies, so what would it be and how would it carry on the row underneath [*pointing to the space below 11*].	
3.30	Aidan	It little bit changes.	
3.35	Alf	Go on.	
3.40	Aidan	Look [*writes 1 and then 2 to the right, making 12*].	

(continued)

3.50	Alf	Ahh, but don't we write it with the two first. I think we want that with the two and the one there do not we [*Alf rubs out 1 and re-writes it to the right of the 2, 21. Aidan moves back in his chair.*]	
3.55	Aidan	Uh un [shakes head].	
4.00	Alf	Don't we do it like that? [*Aidan pulls his knees up*]	
4.05	Aidan	No oh.	

Aidan takes up the ritualisation activity in (3.15) by continuing to write his sequence of numbers, from 14. He had spoken (3.05) about a "-teen" number and "it's copy" in the ones row and it appears as though his attention is taken by the pattern of numerals hence there is a logic in writing the units numeral before the tens numeral. Alf (3.25) prompts him to write in a more standard manner and the comment breaks the *ritualisation* and, perhaps, recognising a halt in the flow of Aidan's activity, Alf suggests a new prompt of looking at what would be a row starting 21, 22, etc. This is a new pattern, not one Aidan has been attending to and it is approached with explicit instructions. The GC structure has 20 to the right of 10; in Aidan's writing 20 would come below 10 and 21 below 11, so the chart is not of direct use. There is a flow of aligning and misaligning intentions and affect through the episode. From 3.50, the structure of numeral naming seems unavailable; there is disagreement about conventions and a physical misaligning of bodies.

8.5 Conclusion

In this chapter, we have set out our thinking about the nature of affect and the role of ritualisation in teaching and learning mathematics. We presented data from a video recording in which one author (Alf) worked one-to-one with a 6-year-old student on number naming and reading/writing. We pointed to moments in the video recording in which there appeared to be an affective aligning, characterised by shared attention and resources providing support. The example we analysed of affective aligning was during a time of ritualisation activity fitting our three criteria: (1) that the activity (in this case writing numbers) was privileged; (2) that there was a symbolic structure to the actions (i.e., the writing followed the standard ordering of number symbols);

(3) that the activity took place without crossing the threshold of discourse about what was being done. We also pointed to moments of misaligning, when there was a cessation of ritualisation, that is, an attempt to make explicit what was being done. (There may of course be times when making something explicit is helpful in learning.) We are not proposing a necessary or causal link between ritualisation and affective aligning; however, we do see the data and theoretical considerations in this chapter as pointing to possibilities for ritualisation activity as productive in teaching and learning mathematics, in allowing an opening into a space of shared attention, shared doings, shared knowings. This is a space in which mathematical knowing might not be articulated or articulable. Ritualisation potentially offers a route into expert behaviour, from which further sense making and reasoning can follow (rather than the other way around).

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Chapter 9
"[He] Has Impaired Vision Due to Overworking": Students' Views About Mathematicians

Vesife Hatisaru

9.1 Introduction

Globally, there has been a growing interest in STEM both educational and workforce perspectives for over a decade, and significant activities have been taking in education systems to encourage students stay in STEM to contribute their nation's productivity (see Marginson et al. (2013), for an international comparison). In Turkey, one of the fundamental concerns of business organizations is the need for an adequate number of qualified employers (e.g., The Turkish Industry and Business Association). The business sector calls for STEM skilled workforce, 'in order to stay in the race in the global economy, which is led by technology, innovation, and digitalization' (PwC 2017, p. 9). Nevertheless, within the country, the participation in tertiary mathematics courses has declined (Nesin 2014), as has the interest in science related careers (Narayan et al. 2013). Mathematics is 'an enabling discipline for Science, Technology, Engineering, and Mathematics (STEM)-based university studies and related careers' (Forgasz et al. 2014, p. 369). Unpacking students' images of mathematicians is a useful first step in getting students to think about future careers in mathematics (Latterell and Wilson 2012) or in mathematics-related fields (Piatek-Jimenez et al. 2018).

A review of literature showed that many students hold narrow, limited, or erroneous perceptions about mathematicians and their work. When students think of a mathematician, generally mathematics teachers come to their mind (e.g., Aguilar et al. 2016; Picker and Berry 2000), and they associate the need for a mathematician with teaching or doing calculations (Picker and Berry 2000; Rock and Shaw 2000). The work of mathematicians is essentially invisible to most of the students (e.g.,

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Latterell and Wilson 2012; Rock and Shaw 2000). Many students tend to view that mathematicians do the same mathematics in their work as students themselves do in their mathematics classrooms (Aguilar et al. 2016; Rock and Shaw 2000). Clearly, most students associate mathematics mostly with numbers and arithmetic (Martin and Gourley-Delaney 2014) and the work of mathematicians with performing calculations (Gadanidis and Scucuglia 2010). Moreover, among students there is a dominant male perception of mathematicians (e.g., Aguilar et al. 2016; Picker and Berry 2000), and students sometimes associate negative or aggressive behaviours to mathematicians such as being large authority figures, crazy men, or having some special power (Picker and Berry 2000).

Relatively little research has been done on students' images of mathematicians in Turkey. Ucar et al. (2010) examined the image of mathematicians held by a small group of 19 elementary students attending a supplementary school and observed that some students described mathematicians as 'unsocial, lonely, angry, quite who always work with numbers'. (p. 131). Yazlik and Erdogan (2016) researched secondary students' perceptions of mathematicians and found that out of 150 participants, 146 described their mathematics teacher. Since the images of mathematicians in that study derived from the mathematics teacher depictions, it is not possible to know the results reflect respondents' perceptions of mathematicians, nor are the results representative. This study is part of a previous research sought the image of mathematics and views about mathematicians held by a large group of lower secondary students. The study not only responds to the current gap in the literature regarding Turkish students' views about mathematicians but also identifies key issues that might be turning students away from mathematics in order to make recommendations for improvement.

9.2 Perspectives on the Image of Mathematics and Views About Mathematicians

Some scholars claimed that the views about mathematicians is the consequent image of mathematics that is conditioned by it (Furinghetti 1993). In the literature no universal definition has been built of the image of mathematics. Brown (1992) defined the image of mathematics as the feelings, expectations, experiences, and confidences individuals hold about mathematics. Sam and Ernest (2000) operationalized the image of mathematics to include 11 components including students' stated attitudes; feelings; descriptions or metaphors for mathematics; beliefs about the nature of mathematics, mathematical ability, or sex differences in mathematical ability; and views about mathematicians and their work. Wilson (2011) proposed an operational construct to define the factors that might influence individuals' engagement in mathematical activity which coincides with the image of mathematics construct. He used the term 'disposition' composing of beliefs/values/identities, affect/emotions, behavioural intent/motivation, and needs. Combining the definitions of Wilson

(2011) and other research in the affective domain, Lane et al. (2014) defined the image of mathematics as 'a mental representation or view of mathematics, presumably constructed as a result of past experiences, mediated through school, parents, peers, or society'. (p. 881). According to Lane et al. (2014), the term image of mathematics composed of three domains: the affective domain, the cognitive domain, and the conative domain.

The previous study focused on the students' stated attitudes (Lane et al. 2014; Sam and Ernest 2000; Wilson 2011), perceived needs for mathematics, and views about mathematicians and their work (Sam and Ernest 2000). In this article, I present students' views about mathematicians and their work. The research question asked is: *What views do lower secondary students have about mathematicians and their work?* Whether gender or grade level differences were evident in the students' views of mathematicians is also of interest.

9.3 The Study

This study was primarily qualitative in which the Draw a Mathematician Test (DAMT) (Picker and Berry 2001) was used to collect data. The DAMT was patterned from the 'Draw a Scientist Test (DAST)' (Chambers 1983) which itself had been patterned from Goodenough's (1926) 'Draw a Man Test' (see Piatek-Jimenez et al. (2018), for a comprehensive review). Over time, the use of instruments such as DAST or DAMT as perception type measures among young students who might avoid completing many written questionnaires has been found valid (Losh et al. 2008) and a less expensive alternative to systematic classroom observations (Haney et al. 2004). Through the years, the DAMT has been used in researching students' images of mathematics or views about mathematicians in many countries including in Europe, the Middle East, Asia, and the United States (e.g., Aguilar et al. 2016; Picker and Berry 2000; Ucar et al. 2010).

DAMT combines drawings with written responses. The front page provides a rectangular area in which participants are asked to draw a mathematician at work. Open-ended items eliciting written responses are provided on the back of the sheet. Relevant to this article is the item: 'Look back at the drawing you made of a mathematician at work and write an explanation of the drawing so that anyone looking at it will understand what your drawing means, and who the persons are in it'.

Before being used in this study, the DAMT was piloted with 130 grade 6–8 level students at three schools not participating in the actual study to ensure the clarity of the instrument and determine the time necessary for completing. After the pilot, DAMT was sent to schools by the respective district Directorate of National Education to maximize the response rate. In schools, teachers other than mathematics teachers provided directions to and collected data from the students. We chose to survey students in classes other than mathematics to eliminate a possible mathematics teacher effect. It took students ~30 min to complete the

DAMT. The schools sent the data in a sealed envelope to protect participant confidentiality.

Participants
A convenience sample of 1284 students in grades 6–8 who were enrolled in twenty different lower secondary schools in Ankara, Turkey participated in data collection under the auspices of the Ministry of National Education. The schools were co-educational metropolitan schools located in the centre of the city, with a relatively middle or high socioeconomic population based on family income. Students' ages ranged from 12 to 15 years.

In the study, as previously described by Picker and Berry (2000), students' DAMT drawings fell into two distinct groups: drawings that depicted their view of what a mathematician at work would look like (19.8%), and drawings that depicted a mathematician who was clearly a mathematics teacher (70.5%). In the latter group, the character was depicted in a classroom environment; main activity of the character was teaching; there were a whiteboard, teacher desk, and/or student desks; and/or the creator called the character as a mathematics teacher and/or their mathematics teacher and mostly wrote the teacher's name. In the former group, the creator called the main character as a mathematician (including some famous mathematicians) and described a mathematician but not a mathematics teacher or their former/current teacher. The famous mathematicians that appeared in student drawings were Ali Kuşçu (14 mention), Cahit Arf (13 mention), Pisagor (3 mention), and John Nash (1 mention). Cahit Arf (1910–1997) was a Turkish mathematician known for, e.g., Hasse–Arf theorem. He is famous in Turkey and is frequently represented as a role model in mathematics curriculum materials (in textbooks, educational videos). Ali Kuşçu (fifteenth century) was an astronomer, a mathematician, and a linguist in the period of Ottoman Empire, and also appears in school textbooks. In this article, I present the data that emerged from this former group involving 114 boys and 139 girls, 254 students total. For the results from the latter group, mathematicians depicted as a mathematics teacher, see Hatisaru (2019) and Hatisaru and Murphy (2019).

9.4 Data Analysis

The data analysis focused on identifying the patterns in drawings, instead of seeking the meaning behind each of the drawings (Haney et al. 2004). Spreadsheets were used for data analysis. Chi-square test was used to compare perceived gender difference by student gender and grade level.

By focusing on the elements that emerged in the students' drawings and narrative descriptions particular to this study and drawing on the prior research (e.g., Aguilar et al. 2016; Blake et al. 2004, Losh et al. 2008), I focused on six elements in the analysis of the drawings and associated written words, each of those comprised

Table 9.1 Associated codes of the elements in student depictions

Elements	Associated codes
Gender of figure	Female; Male; Undefined
Physical environment	Office/room; Classroom; Outdoor; Library; No indication
Activity of the figure	Studying math; Teaching; Creating math; In the field; Undefined; No indication
Content area	Algebra; Numbers and operations; Geometry; Arf theorem; Undefined; No indication
Tools of the profession	Whiteboard; Books; Concrete materials; Pinboard; Technological tools; No indication
Attractiveness	Smiley; Serious, Focused, Dedicated; Mad, Angry, Silly; Undefined

several associated codes (Table 9.1). Table 9.2 presents descriptions of these codes, and Fig. 9.1 gives typical examples of student drawings to illustrate them.

9.5 Results

A summary of the elements and respective associated codes (%) that emerged in the depictions that students created to describe a mathematician at work are presented in Table 9.3. Some responses were coded in more than one category (e.g., across the books and concrete materials categories), and, therefore, the responses might not align perfectly. Below, I present the results in four sections around these elements and use students' own words to illustrate them. Except for the perceived gender, the results did not vary across grade level or student gender; therefore, the results have been presented for the whole group.

Gender

Overall, students showed a greater tendency to depict mathematicians as male. In total, 70.8% of students pictured a male mathematician, while only 23.3% of students drew a female. Compared to girls, boys showed statistically significant tendency to depict a male mathematician ($\chi^2 = 19$, $p < 0.05$). That is, 61.2% of girls depicted mathematicians as male and 33.8% of girls as females, whereas 82.5% of boys pictured male and 10.5% of boys did female mathematicians.

The percentage of depicted male mathematician decreased from grade 6 to 8, whereas the percentage of depicted female mathematicians increased; 71.8% of grade 6, 73.6% of grade 7, and 64.3% of grade 8 students depicted male mathematicians, while 20.9% of grade 6, 25.3% of grade 7, and 25.0% of grade 8 students depicted female mathematicians. Nevertheless, the difference was statistically in significant ($\chi^2 = 6.78$, $p < 0.148$); between the grades, at each grade level, students largely depicted mathematicians as male.

Activity

The mathematician was portrayed at a desk or in front of a whiteboard in a room or in an office in 85.82% drawings. In 7.08% of the drawings, s/he was depicted

Table 9.2 Description of the associated codes of the elements in student depictions

Element	Description
Gender	The gender of figure as female or male considering the physical appearance of the figure (e.g., hair, clothes, moustache) or student writing (e.g., the figure's name indicated by the student). If the drawing or writing is not clear enough to decide the figure's gender (e.g., the stick figure had no clothes or details), it is coded as 'undefined'.
Physical environment	The setting or context in where the figure is depicted. The 'classroom' code is used when the drawing consists of elements typically found inside classrooms such as a whiteboard, desks, or students and the figure teaches; whereas the 'office' code is used when the figure is depicted in an office or a room at a table, working/studying alone, and the typical classroom elements are not included. When there is no indication to a context, the 'no indication' code is used.
Activity	The figure's action. When the figure is in an office environment alone and studying math or solving questions, it is coded as 'studying math', and is coded as 'creating math' when drawing or writing includes reference to working for an invention or for proving a theorem; researching; or writing books. When the figure is depicted in a classroom, at the whiteboard or desk, instructing, demonstrating, or explaining the content area to the students, it is coded as 'teaching'. When the drawing or writing includes no reference to the action of the figure, the 'no indication' code is used.
Content area	The mathematical expressions and symbols that appear in the drawings such as algebraic statements (e.g., $a + b = 3$, $3x + 1 = ?$) (Algebra), four operations (e.g., $4 \times 4 = ?$ $4/3 + 7/2 = ?$) (Numbers and operations), geometric shapes (e.g., squares, triangles) (Geometry), or statements such as $(g) = \sum_{i=1}^{n} q(a_i)q(b_i) \in Z_2$ (Arf theorem). In some drawings, there may be no indication to a content area, or the content area may be ambiguous (e.g., doodles on a piece of paper). When there is ambiguity the 'undefined' code is used.
Tools of the profession	The occupational materials represented on drawings such as whiteboards, books, and concrete materials (e.g., rulers, geometrical objects, compasses, protractors, etc.) used in mathematics teaching. When there is no indication to occupational materials, the 'no indication' code is used.
Attractiveness	The perceived image of the figure. It is coded whether the student's drawing appears to be some kind of Positive rendering such as a 'smiley' figure, Neutral rendering such as a 'serious', 'focused', or 'dedicated' figure, and Negative rendering such as an 'angry', 'silly', or a 'mad' figure. When the figure is turned away thereby obscuring its face and there is no any other evidence either in the drawing or writing, it is coded as 'undefined'.

in a classroom environment with students. In two drawings, the mathematician was pictured at the library, and in four in outdoor like in a construction site or in nature. Most of the students had a perception of the mathematicians sitting in an office/room and working in isolation for hours usually practicing math questions. Although some of the students depicted famous mathematicians, they viewed that the main activity of even those mathematicians is studying to solve questions. In 72.83% of depictions, the mathematician was pictured when working. In one of

[The Study Room of Mathematician] [No Entry] [There Are Secret Formulas]
In the picture there is a female mathematician; there is pinboard of the female mathematician. On the pinboard there are secret formulas. She wants to make discovery by these formulas. She even has written a book. The name of the book is Math Formulas. The formulas are so secret that she always keeps the book with her. (Grade 7, girl)
[Codes: Female; Office/room; Creating math; Undefined; Book, ruler, pinboard; Smiley.]

Here there is a mathematician with long hair. He solves a problem onto a paper. He does not find it enough but becomes thinking calculations. Even when sleeping his quilt becomes full of numbers. He dreams calculations. (Grade 8, girl)

[Codes: Male; No indication; Studying math; Algebra, Numbers; Book, ruler, protractor; Smiley, Dedicated.]

My picture shows a mathematician. S/he remains undaunted to solve a struggled problem. (Grade 7, boy)

[Codes: Undefined; No indication; Studying math; Algebra, Numbers; Book, whiteboard; Focused.]

Fig. 9.1 Examples illustrate the associated codes in Table 9.2, respectively

Table 9.3 A summary of results (%) by elements in student depictions ($N = 254$)

Environment	Activity	Content area	Attractiveness	Tools used
		Algebra 31.50		Books 47.36
Office/room 85.82	Studying math 72.83	Numbers 27.73	Positive 30.93	Whiteboard 25.26
Classroom 7.08	Creating math 13.77	Geometry 13.35	Neutral 55.93	Concrete materials 16.57
Outdoor 1.57	Teaching 7.87	Arf theorem 0.68	Negative 7.5	Technological tools 2.63
Library 0.78	In the field 1.57	Undefined 24.65	Undefined 5.62	Pin board 2.10
No indication 4.72	No indication 3.93	No indication 2.05		No indication 6.05
Total 254	254	292	320	380

these, s/he prepares mathematics tests, and in another prepares a project. In the remaining depictions, s/he studies mathematics. These depictions contained fuzzy calculations, illegible scribbles, doodles on a piece of paper, or piles of books and papers that the drawer intends to show that the mathematician works hard:

> Works day and night. (Grade 6, girl)
> Formulates a problem and solves it; works very hard. (Grade 7, girl)
> Delves into the base of mathematics; preoccupied with so many things, amid papers; has a room in mess. (Grade 6, girl)
> [He] Has impaired vision due to overworking. (Grade 8, boy)

Variance in the activity of the mathematician could only be observed in 13.77% of depictions. In these depictions, the mathematician was described as creating math; mostly conducting research, writing books, working for an invention, or proving theorems, but there were little hints about what the research or inventions might be about. Within this group, in eight depictions, a famous mathematician is represented, and in two of them, the mathematician explains the theory that he has found to professors or introduces his/her invention to other scientists. Below are some related quotations:

> Einstein, Posteur! I have made it! I have solved the theory that has been unsolved for years. (Grade 7, boy)
> Proves Tales Theorem. Reads about history of trigonometry. (Grade 8, girl)
> Finding the theory of relativity [E = mc2]. (Grade 6, girl)
> I [the mathematician] must announce this to the world. (Grade 7, girl)

In 7.87% of drawings, the mathematician was depicted teaching. In four drawings (1.57%), a mathematician was pictured outdoor measuring the length of household objects or length of plants. There was no hint about the activity of the mathematician in the remaining ten drawings (3.93%). It was observed that, in the depictions, no matter what type of activity the mathematician engaged in, problems or calculations appeared to be into three areas: Numbers (27.73%), Algebra (31.50%), or Geometry (13.35%). In some drawings, there was no indication to a content area (2.05%), or the content area was ambiguous (e.g., doodles on a piece of paper) (24.65%). Some representative statements included:

> A mathematician is solving a problem [The problem: 2 + 2 = 4]. (Grade 6, boy)
> Do calculations to find the unknown variable: if x.y = 20, x and y =?
> (Grade 7, girl) Tackles with problems [1535687 x 1538763] (Grade 7, girl)
> Works, with mathematical thoughts in his head [addition, subtraction, square root, and inequations]; [his] left brain has been activated. (Grade 8, girl)

Attractiveness

In 30.93% of the drawings, the references in the pictures or writing reflected a positive mathematician image; they were pictured with smiley faces. In most of the other drawings (55.93%), mathematicians were pictured as tending to think seriously about mathematics, i.e. mathematics is always on their mind, even sometimes they see mathematics in their dreams. These depictions were grouped as 'Neutral'. In this group, the mathematician was depicted as serious (69 mention), concentrated on mathematics (53 mention), or contemplating associated with thought balloons full of calculations, numbers, or formulas (57 mention). Indeed, in some of these depictions, the mathematician was somewhat detached from the life around, totally dedicated to mathematics:

> Work-oriented; has devoted his/her life to work. (Grade 8, boy)
> Busy with mathematics and only work. (Grade 6, boy)
> An introvert; his/her only focus is mathematics and work! (Grade 6, boy)
> Thoughtful, indulged in the solution of a problem. (Grade 7, girl)

Although more commonly there was evidence of positive or neutral views for the mathematicians, negative views of the mathematician were presented in some students' depictions (24 drawings, 7.5%). Within this group, three depictions show a frowning, angry mathematician. In the other 21 drawings, the mathematician is depicted as silly or mad. It is such that, in some of these depictions, the mathematician is driven insane by studying math a lot or is exhausted or mad. Some wordings associated with these depictions are as follows:

> His/her brain is exhausted. The pencils scattered over the floor. (Grade 7, girl)
> Gone mad with over-studying. (Grade 8, boy)
> Has lost his/her mind, solving problems all the time. (Grade 7, boy)

The Tools of the Profession
In the depictions, the most commonly observed occupational tools were books (47.36%) or a whiteboard (25.26%). In some (63 mention, 16.57%), concrete materials such as rulers (44 mention), geometrical objects (5 mention), compasses (12 mention), and protractors (2 mention) could be seen. Technological tools were observed just a few times (2.63%), namely computers (7 mention) and calculators (3 mention).

9.6 Discussion and Concluding Words

The most common patterns that emerged in participant students' the Draw a Mathematician Test (DAMT) drawings and associated descriptions were that mathematicians predominantly: are male; work in an office/room; engage in solving Numbers, Algebra, or Geometry questions; are quite serious, focused, or dedicated; and use a whiteboard or books as the tools of the profession.

These results may be relevant in Turkey where transition from lower secondary to secondary schools (and later years from secondary education to the university) depends on a large degree on students' scores on a nationwide (multiple-choice) standardized test, and mathematics (for Numbers and operations, Algebra, Geometry and Measurement, Data and Chance content domains) makes up a significant percent of this exam. To be placed in relatively good schools, students work very hard, namely practice hundreds of questions in those content areas, either at the school or after-school times. It is clearly observed that one of the (negative) consequences of this exam is that students associate mathematics narrowly with mainly numbers and arithmetic (Martin and Gourley-Delaney 2014; Ucar et al. 2010) and the work of mathematicians with performing calculations (Gadanidis and Scucuglia 2010) or practicing textbook questions (Ucar et al. 2010) for hours like students themselves do to be prepared for exams.

The dominant male perception of mathematicians has been reported in previous research (e.g., Aguilar et al. 2016; Picker and Berry 2000), but what is further indicated in this study is that boys showed implicit 'gender of mathematician' stereotype, which is needed to be researched more. Generally, students' perception of the gender of mathematicians might be influenced by the society as a whole. In Turkey, although they are also sought-after career paths for females, mathematics or mathematics-related professions are viewed more as male professions in Turkish society, and it is not uncommon to have more male than female mathematicians at universities, for instance. This worries some cohorts such as the Association for Turkish Women in Maths, a European Women in Mathematics (EWM) affiliate, missioning to encourage women to study mathematics and to empower Turkish women in mathematics.

The study confirmed earlier findings that the work of mathematicians was essentially invisible to most of the students (e.g., Latterell and Wilson 2012; Rock and Shaw 2000; Ucar et al. 2010). Although more research is needed, the results also showed that curriculum materials may have a more significant influence on students' views about mathematics or mathematicians than the popular media (e.g., Latterell and Wilson 2004). Whereas famous mathematicians like Ali Kuşçu and Cahit Arf who appear in school materials were depicted by some students, Ali Nesin, a Turkish mathematician, academic, author of many popular mathematical journals and books, founder of well-recognized Mathematics, Art and Philosophy Villages, and often appearing on TV and newspapers did not appear on a single drawing. John Nash, whose story told in the movie, *The Beautiful Mind*, was seen in only one depiction.

The reader should bear in mind that the data for this study was collected from students at 20 schools in a city within Turkey. The sample may not be representative of the entire population of lower secondary students within the country or in other countries. Nevertheless, the study contains several implications. Below I would make three recommendations.

9.7 Recommendations

Research shows that distribution of career information, the presentation of role models (Piatek-Jimenez et al. 2018), classroom experiences and textbooks (Aguilar et al. 2016), and the media (Latterell and Wilson 2004) can influence students' views of mathematics and mathematicians. My first recommendation is that it is critical to provide students with a variety of quality experiences in their mathematics education programmes so that students reduce the stereotypic gender view of mathematicians and have a more equal gender view.

The good news is that only a small number of participant students associated negative feelings to mathematicians which appeared in Picker and Berry's (2000) study and also found in Hatisaru (2019) associated to mathematics teachers. Similarly, few students appeared to be of the view that mathematics drives people insane which generally provided by popular culture (e.g., Latterell and Wilson 2004). Nevertheless, what is further indicated in this study is that more than half of the students viewed mathematicians to be overwork practicing mathematics questions. While its dedication and hard work might not appear negative, students with this idea of mathematicians may not be attracted to the mathematics (Latterell and Wilson 2012) or math-related careers (Piatek-Jimenez et al. 2018). If we want students to keep doing mathematics, we need them to see it as an attractive or relevant career option. My second recommendation is that we need to show students that mathematicians work on exciting projects relevant for the modern world (Gadanidis and Scucuglia 2010; Rock and Shaw 2000) and touch us in our everyday life such as developing ATM machines, cell phones, and space stations (Rock and Shaw 2000). Changing the way of learning mathematics from drill and practice of number facts and computational manipulations to a more problem and experiential way of learning may change students’ current views about mathematicians and their work. In relation to this, lastly, I suggest that an interesting follow-up research would be to test the effectiveness of learning mathematics in those ways on students’ views about mathematicians. Another possible area of future research would be to investigate the differences among students with different profiles for the image of mathematics and views about mathematicians.

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Chapter 10
How to Design an Activity That Influences Middle School Students' Beliefs About Mathematics as a Discipline

Maria Kirstine Østergaard

10.1 Introduction

It is generally agreed that students' beliefs about mathematics have a significant influence on their learning, motivation, and approach to the subject (e.g. Furinghetti and Pehkonen 2002; McDonough and Sullivan 2014; Schoenfeld 1988). Unfortunately, it seems that students' attitudes towards mathematics become more negative with age (Blomqvist et al. 2012), particularly regarding the relevance of the subject. Furthermore, the negative relation to mathematics apparently applies more to girls, especially when it comes to their self-perception as learners (Markovits and Forgasz 2017). According to Boaler (1997), girls are often more context-seeking than boys in their mathematical understanding and therefore do not find that a solution-focused and performance-oriented teaching supports their thinking. In addition, mathematics is traditionally viewed as a masculine domain (Brandell and Staberg 2008; Sánchez Aguilar et al. 2016), which can affect the appeal of the subject to girls at puberty, where they mainly tend to connect with gender-specific areas as part of establishing their gender identity (Bowd and Brady 2003).

A large part of the students' beliefs about mathematics is developed from experiences in mathematics classrooms (Grootenboer and Marshman 2016; Schoenfeld 1992). Thus, it might be possible to counter these tendencies within the teaching of mathematics by offering the students experiences and evidence that support the development of availing beliefs. It is my hypothesis that a *targeted* focus on making the role of mathematics in the world visible to the students from an early age will have a positive effect on their beliefs about mathematics. Having the above-mentioned gender aspects of mathematics learning in mind, working

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with mathematics in a context based on mathematics as a *discipline*, could have a beneficial effect on particularly girls' relationship to the subject. Mathematics as a discipline exceeds school mathematics by also focusing on, e.g. mathematics as a science, its application in society, and characteristic mathematical methods, as well as epistemological and philosophical aspects of the subject. As a result of societal and environmental factors, girls often prefer small group discussions and exploring ideas in depth. These traits mean that girls generally benefit from the use of real experiences (Geist and King 2008), and by focusing on mathematics as a discipline, they may experience an enhanced ability to relate their thinking to the subject's scientific methods.

During the next couple of years, I will test this hypothesis in a longitudinal intervention study in two middle school classes (age 10–13). By initiating this work in middle school, the students' beliefs about mathematics might not yet be too influenced by the development of their gender identity, but they will be old enough to be able to express their thoughts and reflections about mathematics. Furthermore, research indicates that negative attitudes towards mathematics often emerge during middle school (Di Martino and Zan 2011; Grootenboer and Marshman 2016), which makes it an interesting age to focus on in the perspective of the present research.

As beliefs can be both difficult and time consuming to change (Green 1971; Lake and Kelly 2014), the intervention proceeds over 2 years, starting when the students are in fifth grade (age 11). Principles of teaching that focus on the development of the students' beliefs on mathematics as a discipline are constructed in cooperation with the mathematics teachers. As argued later, this means that the activities in the classroom must offer concrete examples placing mathematics in a societal, cultural, and historical context, and illustrate how mathematics differs from other disciplines.

For now, I will concentrate on the actual implementation of the suggested focus by examining the following research question: *How to design an activity that foster the development of middle school students' beliefs about mathematics as a discipline?* Below, I will present arguments for the suggested approach based on existing research about students' beliefs and then situate my hypothesis in the Danish context. Finally, I will describe an example of a teaching activity that has the potential of developing the students' beliefs about mathematics as a discipline.

10.2 Developing Beliefs About Mathematics as a Discipline

In 2002, Op't Eynde, de Corte and Verschaffel developed a framework on students' mathematics-related beliefs based on prevalent models. Here, the students' belief system was categorized into three interdependent dimensions: beliefs about mathematics education (mathematics as a subject, mathematical learning and problem solving, and mathematics teaching in general), social context (social and socio-mathematical norms in class), and self (self-efficacy, control, task-value, and goal-orientation). Jankvist (2015) expanded this framework by adding the dimension of "mathematics as a discipline," which exceeds the content of "mathematics as a

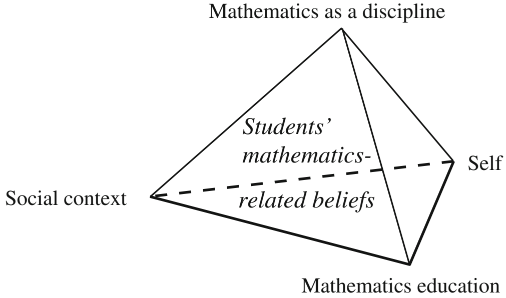

Fig. 10.1 Students' mathematics-related belief system. Beliefs about mathematics as a discipline are developed in the context of mathematics education and therefore in interplay with the three other dimensions. Thus, the fourth dimension is placed "outside" the original triangle, which then forms the "base" of the tetrahedron. (Jankvist 2015, p. 45)

subject" in the original model by including perspectives of mathematics that are not educational. This dimension concerns beliefs about mathematics as "a pure science," "an applied science," "a system of tools for societal practices," and "beliefs about the philosophical and epistemological nature of mathematical concepts, theories, etc." (Jankvist 2015). More specifically for middle school students, the fourth dimension might include questions about, e.g. where mathematics is applied, how, why, and when it came into being, what mathematicians do, etc. This addition to the framework clarifies how mathematics is more than a school subject. It places the school subject in the context of the real world and of the world of mathematics itself. The relevance, application, origin, and esthetics of mathematics are all included in this dimension of the belief system (Fig. 10.1).

Considering Philipp's (2007) definition of beliefs as "lenses through which one looks when interpreting the world" (p. 258) and the filtering role that beliefs play for new experiences and information (Pajares 1992), it seems highly relevant to include the students' beliefs about mathematics as a discipline in their belief system—even if the belief system only concerns mathematics education. As the dimensions in the belief system are interdependent (Op't Eynde et al. 2002), the students' beliefs about mathematics as a subject influence their beliefs about mathematics as a discipline—and vice versa. This means that the approach to mathematics in the classroom will affect how students' perceive mathematics as a whole, as will socio-mathematical norms, criteria for success, tasks, dialogue, organization, etc.

A so-called traditional approach to mathematics education that focuses on accuracy, speed, procedural work, and memorization and that is organized as individual work with disconnected concepts seems to be related to beliefs about mathematics as rule-based, unrelated to the real world, and only understandable for geniuses (Schoenfeld 1988). Beliefs of this kind is similar to what Grigutsch (1998) calls a schema- and algorithm-oriented view of mathematics. It is also comparable to a conception of mathematics that Ernest (1989) categorizes as a Platonist view, where mathematics is viewed as a structured, unchanging body of knowledge. The teacher is the source of this knowledge, and the student is the receiver, who must learn (or even memorize) the appropriate procedures for solving certain problems. Sadly, several studies have shown that such beliefs are common among students (e.g. Di Martino and Zan 2011; Grootenboer and Marshman 2016; Jäder et al. 2017; Kelly 2004; Sumpter 2013; Szydlik 2013).

Conversely, there are well-documented indications that an enhanced focus on the process rather than product of learning mathematics can contribute to more availing beliefs (Higgins 1997; Liljedahl 2018; Verschaffel et al. 1999; Yackel and Cobb 1996). Inquiry-based teaching, working with realistic, complex and open-ended problems, redefining the teacher's role towards facilitator rather than a source of knowledge and focusing on more conceptual understanding than on efficient computational skills, seem to correlate with the beliefs that mathematics is a way of thinking rather than just rules and procedures, and that it is an applicable tool in other areas of life as well. Students even become more persistent and autonomous in their problem solving (Yackel and Cobb 1996) and their mathematical achievements improve (Verschaffel et al. 1999). This is what Grigutsch (1998) categorizes as a process- and application-oriented view of mathematics.

The development of students' beliefs about mathematics is a continuous process that has begun even before starting school. If the belief dimension concerning mathematics as a discipline is not considered, there might be a risk that students primarily perceive mathematics as something that only belongs in school and with no important part in the real world. For example, if the teaching in the first school years signals that mathematics is mostly about rules and formulas, that the criteria for success is to find the correct answer quickly, and that mathematics mostly is about working individually with training tasks, then this image is likely to form the basis for the students' central beliefs about mathematics. And since central beliefs can be quite difficult to change (Green 1971), it is essential to address mathematics in a bigger picture than as a school subject from the beginning of school.

Of even greater importance is that the students form their beliefs on the basis of evidence (Green 1971). As opposed to beliefs derived from others, beliefs developed from experience tend to be more central and therefore more powerful and influential. Teaching methods based on the teacher transferring knowledge to the students do not support that. On the contrary, such an approach can endorse non-evidentially held beliefs, which make the beholder reject evidence that does not support them, and so they cannot be changed with reason. Hence, it is crucial that the activities in the mathematics education offer the necessary evidence to form the base of students' beliefs. This will develop their critical thinking skills and enable them to consider and reflect on new evidence by using reason.

10.3 The Danish Context

The above-mentioned arguments show how setting the development of students' beliefs about mathematics as a discipline as a stated goal for the mathematics education can potentially contribute to the process- and application-oriented approach to the teaching of mathematics. Unfortunately, there is little indication of this kind of prioritizing in the Danish elementary school. A study from 2014 among fourth grade students showed that most tasks given had an explicit procedure or were simply filing in numbers (Bremholm et al. 2016), which implies a somewhat traditional approach. This is in spite of the fact that the development of students' beliefs about mathematics as a discipline is actually already included in the Danish curriculum.

The Danish mathematics programs are partly based on a competency framework described in the report "Competencies and Mathematical Learning" (Niss and Høgaard 2011). The authors' definition of mathematical competence has recently been updated and is now described as "someone's insightful readiness to act appropriately in response to all kinds of *mathematical* challenges pertaining to given situations" (Niss and Højgaard 2019, italic in original). In addition to eight action oriented mathematical competencies, the framework presents three forms of overview and judgement concerning mathematics as a discipline. Unlike the mathematical competencies, they are not behavioral, but aim to develop "insight into the character of mathematics and its role in the world" (p. 50). They are based on knowledge as well as beliefs about mathematics as a discipline, and they are described as a *set of views* regarding "the relations between mathematics and conditions and chances in nature, society and culture."

The three forms of overview and judgment concern (p. 74): (a) the actual application of mathematics in other subject and practice areas, (b) the historical development of mathematics, both internally and from a social point of view, and (c) the nature of mathematics as a subject area.

The authors of the report exemplify these by questions. The first form of overview and judgment concerning the actual application of mathematics is characterized by questions such as (p. 74): "Who, outside mathematics itself, actually uses it for anything? What for? Why? How? By what means? On what conditions? With what consequences? What is required to be able to use it?, etc."

Examples of questions related to the second form of overview and judgment concerning the historical development of mathematics are (p. 75): "How has mathematics developed through the ages? What were the internal and external forces and motives for development? What types of actors were involved in the development? In which social situations did it take place? What has the interplay with other fields been like?, etc." It is emphasized that concrete historical examples are needed to build a solid understanding of how mathematics has developed culturally and socially.

The nature of mathematics as a subject could be addressed through the following questions (pp. 75–76): "What is characteristic of mathematical problem formulation, thought and methods? What types of results are produced and what are they used for? What science philosophical status does its concepts and results have? How is mathematics constructed? What is its connection to other disciplines? In what ways does it distinguish itself scientifically from other disciplines?, etc."

Working with these three forms of overview and judgment is thus a way to develop not only students' knowledge and insight of the nature of mathematics and its role in the world but also their beliefs about mathematics as a discipline. This work is of course a complex matter, but considering the importance of building students' beliefs on evidence (Green 1971), it must necessarily involve concrete examples of at least the application and the historical development of mathematics, but perhaps also of the nature of mathematics.

The Danish curriculum recommends introducing and implementing mathematical overview and judgment in the mathematics teaching from lower secondary classes. Yet, they seem to be quite overlooked in the mathematics education at this level (and lower) compared to the eight mathematical competencies, and they are only sparsely mentioned in the curriculum's learning objectives. Jankvist (2015) has successfully worked with the development of beliefs about mathematics as a discipline in upper secondary school by implementing teaching activities concerning the three forms of overview and judgment. However, as I have stated previously, there are good reasons for this work to be initiated earlier.

10.4 Designing an Activity

In the following, I suggest a teaching activity designed to enhance middle school students' awareness of (1) the historical development of mathematics, (2) the application of a specific subject area (probability), and (3) characteristic mathematical problem formulation, thought, and methods. The activity thereby addresses elements of the three forms of overview and judgment. It is inspired by Renaud Chorlay's work with 17-year-old students reading of a historical text on probability (Chorlay 2018). By giving a historical example on how mathematical ideas and knowledge are developed, the students are offered evidence of the nature of mathematics while at the same time given the opportunity to make their own experiences, which can serve as evidential basis of their beliefs.

First of all, this activity presents the students with an historical source. It is based on an unpublished manuscript from 1678 by Gottfried Wilhelm Leibniz called "The game of Quinguenove." In this text, Leibniz wishes to examine if the rules of a dice game is fair. Since the rules are quite complicated, he uses the strategy of studying a similar, but much simpler game of chance, which is the problem that the students will explore. The text is not easy to read, so it will not be presented in its entirety for the students. Instead, they will receive a short extract of Leibniz' reasoning concerning the simplified problem (Chorlay's translation):

> Two people are playing dice: one will win if he scores eight points [...], the other if he gets five. It is a question of knowing which of the two it would be best to bet on. I say that it should be the one who needs eight points, and even that his advantage compared with the hope that the other must have, is three to two. [...] I suppose they are playing with two dice [...] There are only two ways to reach five points; one is 1 and 4, the other 2 and 3. However, there are three ways to score eight points, i.e. 2 and 6, 3 and 5 and also 4 and 4. [...] So if five points can only be made in two ways, but eight points can be made in three ways, it is clear that there are two chances of getting five and three chances of getting eight (Chorlay 2018, p. 118).

In his considerations of the problem, Leibniz gives us an example of both mathematical application, method, and reasoning. Furthermore, this example shows us how Leibniz performs a process of mathematical modelling: In the beginning, his motives for developing a solution to his problem are external in relation to mathematics (he wants to know if the game rules are fair). He then mathematizes it to solve it, thereby shifting the motive to be of a more mathematically internal character.

When investigating this problem, one might discover that the law of large numbers reveals that the chance of getting five rather than eight is *almost*, but not *exactly* two to three. Leibniz of course oversees the fact that there are four ways of reaching five points: $1 + 4$ and $2 + 3$, but also $4 + 1$ and $3 + 2$! This conclusion might be reachable to 17-year-old students, but fifth grade students most likely need some scaffolding. Therefore, we copy the strategy of Leibniz and simplify the problem even more. The question that the students first must consider is: "When rolling two dice, are the chances of getting 9 and 10 points equal?" It might appear so, since there are two ways of getting 9 ($3 + 6$ and $4 + 5$) and two ways of getting 10 ($4 + 6$ and $5 + 5$). After considering and sharing reflections on this question, the students are urged to perform experiments and simulations of the situation. Experiments with dice might lead to discussions concerning how many throws are needed to be convinced of an argument, which might again lead to the need of digital simulation. The result of experiments may be confirmed by throwing two dice of different color. The experiments and simulation will link statistics and probability as well as qualify the students' reasoning and it will give them an opportunity to verify or revise their understanding of the situation. This process is expected to enable them to work with Leibniz' problem by transferring their experiences to the more complex situation, and even detect his error.

10.5 Concluding Discussion

As stated in the introduction, it is my hypothesis that a targeted focus on making the role of mathematics in the world visible to the students from an early age will have a positive effect on their beliefs about mathematics. By explicitly implementing elements of mathematical overview and judgment and providing concrete examples, we can create a solid foundation for both mathematical knowledge and beliefs.

First of all, the use of a historical source in the teaching activity provides the middle school students with a concrete example that contextualizes the mathematics. The correlation between statistics and probability becomes clear by using the law of large numbers as a natural argument in a real situation. As mentioned, this could particularly benefit the girls, as could the included room for reflection and discussion.

Secondly, Leibniz' approach to the problem illustrates that simplification is an approved mathematical method and that systemization can be used to make mathematical arguments. By using his modelling approach as an inspiration for the students' own work, they acquire personal experiences that come to serve as solid evidence of the characteristics of a scientific method. Likewise, the experiments and simulations provide evidence for their understanding of probability.

Thirdly, Leibniz' manuscript and his considerations about the game rules may serve as a starting point for approaching some epistemological questions about mathematics. For one, Leibniz' error signals that even famous mathematicians sometimes make mistakes and that mathematics is not a structured, unchanging body of knowledge. The case can also open a discussion about what kind of motives and driving forces there can be for the development of mathematics—both in this particular case as well as historically and within the field of mathematics. This might involve questions such as: What external and internal reasons might there be for developing mathematical theories? Can such theories change? Historically speaking, what has caused the field of mathematics to develop? Moreover, it might address justification issues of mathematics education—why do we need to learn mathematics?

This teaching activity is an example of how middle school students can be confronted with evidence that may provide them with a reflected view and multifaceted image of mathematics as a discipline. As I have shown, it addresses all of the three forms of overview and judgment: the application, the historical development, and the nature of mathematics. Implementing this sort of activities in a middle school classroom is a teaching approach that determinedly offers the students the evidence needed as a foundation for their beliefs about mathematics as a discipline to change or be established. And by beginning this implementation early, the greater the chances are that the students will develop beliefs that allow them to think critically, to reason and to reflect.

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Chapter 11
Students' Beliefs about Mathematical Content Based Thinking Represented in Photos from Everyday Life all over the World

Sabrina Blum

11.1 Introduction

In 1996 Grigutsch investigated German students' beliefs from Grade 6, 9, and 12 with regard to the nature of mathematics. He was interested in the so-called mathematical world views, which could be classified in the following four aspects: formalism aspect, schema aspect, process aspect, and application aspect. In addition, he distinguished between a static view of mathematics as a system (formalism and schema aspect) and a dynamic view of mathematics as a process (process and application aspect). Most of the respondents especially perceived mathematics within the schema aspect, which turned into a more dynamic view by starting university. TIMSS-III underlined that a dynamic view led to a better understanding, higher learning motivation, and a higher mathematical self-concept (Felbrich et al. 2010). Rolka and Halverscheid (2006) examined students' world views by mainly using pictures, but also short written explanations and oral interviews. The task was to feel like a painter or author and to present on a paper what mathematics meant for them personally. This creative methodical decision to use pictures instead of a long questionnaire enabled younger students with reading problems to participate. Students' beliefs may have been represented in an adequate way: On the one hand, the additional explanations facilitated an understanding of the students' intentions towards their pictures. On the other hand, students could express what they were not able to draw. Following Ernest (1989), Rolka and Halverscheid (2006)

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differentiated between instrumentalist, Platonist, and problem-solving view. Except for one person, all of the investigated students of Grade 5 described mathematics as a science and a useful tool for everyday life, whereas the focus for respondents of Grade 9 was on the reflection about their own mathematical learning process. Some of them described explicitly their frustrating learning experiences and their general frustration concerning mathematics at school. Several studies (e.g. Rolka and Halverscheid 2006) could show that students' attitude towards mathematics tend to develop in a negative way between Grade 5 and Grade 9. Although pictures are a creative way to access students' beliefs, there might be a problem of artistic or oral expression. For example, a 10-year-old girl gave the following explanation about her self-painted picture:

> In my picture, mathematics is found in forms, numbers [sic] and calculation signs. I have opted for this presentation because for me maths has something to do with forms and numbers. Forms have something to do with geometry. [. . .] What I would have liked to sketch: a grocery store with a cashier who cheats a client because he is not able to calculate (Rolka and Halverscheid 2011, p. 523).

On the one hand, we can sense that the girl has difficulties in her artistic expression. She would have liked to draw another situation, but obviously felt not capable of doing so. A solution might be to use photos instead. On the other hand, the girl wrote in a general way about numbers and forms instead of giving concrete examples (such as pyramids or cardinal numbers). As a consequence, it is not clear how deep her beliefs about mathematics are (cf. degree of conviction). In this case, like in many others, it is worth investigating mathematical content based beliefs as another approach in contrast to beliefs about the nature of mathematics in terms of Grigutsch (1996). Where exactly in the picture do students recognise mathematical content and of which kind? Which aspect of mathematical content is most frequent in students' minds and at which age? May there be changes? All of these considerations resulted in an analysis of students' explanations about their photos from everyday life. In the last few years, social media has increasingly become important to students and especially photos represent a means of expression in their everyday lives. That is another reason why this research uses photos instead of pictures.

With regard to an international framework, studies underline the differences between the beliefs of learning and teaching of mathematics (e.g. Felbrich et al. 2010). There seem to be country specific as well as cultural distinctions. Felbrich et al. (2010) found out that there are constructivist beliefs in rather individualistic orientated countries like Germany, whereas in rather collectivistic orientated countries like Russia there are transmission orientated beliefs. A working field, which should be deeper investigated, is students' beliefs about mathematical content based thinking in relation to everyday life in different countries.

11.2 Theoretical Background

Belief: Definition

The use of the notion *belief* is not standardised and other terms like *conceptions*, *philosophy*, *perception*, *world views*, *image*, *attitude*, *conviction*, etc., are used as synonyms (cf. Törner 2002, p. 75). These words may have cognitive as well as affective aspects. This paper focuses on Pehkonen's understanding of beliefs:

> as one's stable subjective knowledge of a certain object or concern to which tenable ground may not always be found in objective considerations. The reasons why a belief is adopted are defined by the individual self—usually unconsciously (Pehkonen 1994, p. 180).

In his view, the selection and evaluation of experiences and perceptions in the world around individuals happen subjectively so that a belief does not only consist of a cognitive dimension (on which lies the focus) but also of an affective one. There are several researchers (e.g. Philipp 2007), who pointed out three core differences between *beliefs* and *knowledge*: Firstly, in contrast to knowledge, one does not have to be convicted 100% of a belief (degree of conviction). Secondly, there does not have to be a consensus about a thing or an object, so people may discuss about a belief (degree of consensus). Thirdly, beliefs depend on personal experiences and propositions, whereas knowledge is by evidence shown as in fact true (degree of validity).

Mathematical Thinking

Ulm (2010) classifies mathematical thinking in three dimensions: content based thinking, process based thinking, and mathematics based information processing. The focus of this paper lies on the first dimension, *content based thinking*, which includes numerical, geometric, algebraic, stochastic, and functional thinking. These components can be seen as analogous to the German educational standards for the general higher education entrance qualification (2012) and for the general education school leaving certificate (2003).

Numerical thinking deals with the development and the use of number concepts (e.g. natural numbers, real numbers) as well as different aspects of numbers like cardinal numbers or ratios (Ulm 2010; Padberg and Büchter 2015). Geometric thinking concerns mathematics in plane but also the spatial domain. Mental representations of notions of figures and fields emerge and students are operating with it. In addition, geometric thinking implies conversion between different representation forms. *Algebraic thinking* is about the understanding and the application of properties (e.g. commutative property, associative property, rules for fractions) as well as the solving of equations and terms. Also, algebraic thinking means making estimations. With *stochastic thinking*, Ulm (2010) refers to stochastic and combinatorics, including probabilities, statistical analysis, and finding the number of possibilities. This includes systematically collecting data and representing it by tables or diagrams. Also the interpretation has an essential role. *Functional thinking* is about the relation between reason and consequence. One quantity depends on another, i.e., functions analyse which consequence a change of one quantity may have on another depending quantity. You can also distinguish between different forms of representation: function term, graph, table, and verbal explanation.

In addition to content based thinking, *process based thinking* is according to Ulm (2010) about algorithmic, formal, deductive, problem-solving, modelling, experimental, or notion building thinking. *Mathematics based information processing* includes mathematical sensitivity, thinking in mathematical patterns, overcoming complexity, flexibility concerning thoughts, mathematical creativity, mathematical literacy, and a mathematical memory (Ulm 2010).

11.3 Research Questions

The research focuses on students from different countries, school types, and all grades. The following questions are investigated in this study:

- Which beliefs can be found about mathematical content based thinking with regard to photos from everyday life?
- How many different aspects of mathematical content based thinking are mentioned in the explanations?
- Which differences in beliefs of the five aspects of mathematical content based thinking can be found between students of different ages and cultures?

11.4 Methodology

This paper sheds a light on an international photo competition, which was announced from April to May 2018 for all students of every type of school worldwide. Several schools were contacted via email in March 2018. They were either partner schools of the International Office of the Martin Luther University Halle-Wittenberg (cf. Hussner 2019) or foreign schools on the PASCH website (cf. PASCH Initiative 2019). Hundred out of 1800 schools were chosen randomly so that a number of states of the continents America, Europe, Africa, Asia, and Australia were represented. They were informed in the hope that those schools could have a broader interest to take part in a research project rendered by a German university. In addition, for the German part, it was fallen back on experienced announcement procedures: On the one hand, schools in the surroundings of the city Halle were informed by the university marketing and, on the other hand, mathematics teachers nationwide got the information by the DMV (= German mathematical society) newsletter. In total, worldwide $N = 91$ students from all ages (7–20 years; $\bar{x} = 14.44$; $\sigma = 3.38$) and grades (1–13; $\bar{x} = 8.55$; $\sigma = 3.198$) participated voluntarily. The following tasks, which aimed at mathematics in everyday life, were available in German, English, French, Spanish, Russian, and Chinese:

> Send us your photos (max. 3), on which you can see something mathematical. Tell us where exactly mathematics is on your photo and why you feel that the photo is interesting in a mathematical way.

Table 11.1 Example for clustering a photo

Photo	Student's explanation	Associated codes
 Eight-year-old girl, Grade 2, Pretoria [In this case, the student's mother wrote the explanation of the photo, but it is assumed that Yana's beliefs are described in an authentic way.]	"[Yana] herself had the idea when we were eating our oranges in the afternoon. We were talking about school and while peeling the fruits, she noticed that they are symmetrical and also that they consist of 10–11 pieces (slices) most of the time. Then of course we had to check if that's true. We haven't eaten so much fruits for a long time ☺" (translated by the author)	Numerical thinking (Ratios): "10–11 pieces" Geometric thinking (Symmetry): "symmetrical" Stochastic thinking (Activities: collecting, analysing, and evaluating data): "while peeling the fruits, she noticed that [. . .] they consist of 10–11 pieces (slices) most of the time. Then of course we had to check if that's true".

By using six of the most spoken languages worldwide, the students could feel free to answer in their mother tongue in order to avoid language hurdles in expressing their beliefs. However, German was mainly used as communication language. All of the replies included photos and written explanations from students from Mexico City (34.1%), from different cities in Germany (22.0%), Pretoria (11.0%), Cairo (9.9%), Quito (9.9%), Washington D.C. (9.9%), and from Ho-Chi-Minh-City (3.3%). They were categorised following the model of mathematical content based thinking by Ulm (2010). The full coding scheme can be found on the following website: https://didaktik.mathematik.uni-halle.de/lehrende/blum/ (including core examples and associated codes for each aspect of mathematical content based thinking). In Table 11.1, an example for the coding process is given.

For this qualitative content analysis, only the respondents' explanations were taken into account. The photos were not interpreted, as there are several different views on a photo, which might not represent the students' beliefs. Multiple references were possible. The software SPSS 25 was used for the data analysis. If a group of two or three students sent their photos together, it was assumed that they discussed their beliefs, so that each student was regarded to express the same beliefs and in the research the same photo was taken two or three times into account.

11.5 Results

Table 11.2 shows the representation frequency of each code of mathematical content based thinking in the students' explanations of mathematics in their everyday life. It underlines a clear order of mentioning mathematical content based thinking: (1) geometric (84.6%), (2) numerical (51.6%), (3) algebraic (30.8%), (4) functional (25.3%), and (5) stochastic thinking (14.3%).

Table 11.2 Frequencies of all codes for mathematical content based thinking that were mentioned at least once in the students' explanation texts

	Geometric thinking (84.6%)	Numerical thinking (51.6%)	Algebraic thinking (30.8%)	Functional thinking (25.3%)	Stochastic thinking (14.3%)
Activities	5.5%	2.2%	Calculating (18.7%), measuring (1.1%)		Collecting, analysing, and evaluating data (8.8%)
General notions	23.1%	5.5%	2.2%	6.6%	
Other aspects	Figures and shapes (42.9%)	Cardinal numbers (indefinite 29.7%, definite 26.4%)	Estimations (12.1%)	Properties of functions and tools for their description (11.0%)	Probability (3.3%)
	Relative positions (27.5%)	Ratios (22.0%)	Proportion (5.5%)	Sequences (6.6%)	Combinatorics (1.1%)
	Symmetry (27.5%)	Constants (4.4%)	Fundamental rules of arithmetic (4.4%)	Types of functional equations (4.4%)	Diagram (1.1%)
	Lines (15.4%)	Number ranges (3.3%)	Matrix (1.1%)		
	Fields (13.2%)	Ordinal number (3.3%)			
	Golden ratio and Fibonacci (4.4%)	Properties of natural numbers (3.3%)			
	Similarity (1.1%)				

In order to go more into detail, each aspect of mathematical content based thinking was analysed with regard to the codes included. Firstly, within *geometric thinking*, nearly one quarter of the respondents (23.1%) used general geometric notions like "geometry", "construction", or "shape". Figures and shapes were the most significant part (42.9%) of geometric thinking as well as within all content based thinking aspects. This code occurred three times more often than fields (13.2%). Relative positions (27.5%) and symmetry (27.5%) also played an important role for the respondents. Secondly, with respect to *numerical thinking*, both the cardinal number (definite 26.4%, indefinite 29.7%) and the ratios (22.0%) appeared most frequently, whereas other codes were rather of secondary importance (5.5% or less). Activities of *algebraic thinking* mentioned in the explanations were calculating (18.7%) as well as measuring (1.1%). Besides calculating, the second most important part of algebraic thinking was estimations (12.1%). Other codes were individually mentioned (less than 5.5%). Properties of functions and tools for their description (11.0%) made up the biggest share among the explanations containing elements of *functional thinking*. *Stochastic thinking*, which was least named (14.3% in total), was mostly represented by activities, which were collecting, analysing, and evaluating data (8.8%).

The frequency of the different described aspects of mathematical content based thinking (numerical, geometric, algebraic, stochastic, functional), which were used to describe the photos, was measured. The total amount of different aspects mentioned ranged from zero up to four ($\bar{x} = 2.07$; $\sigma = 1.143$). The majority mentioned one (33.0%) or two (29.7%) different aspects of mathematical content based thinking. Compared to that, 4.4% of the respondents described one aspect of mathematical content based thinking, whereas 17.6% of the students showed three and 15.4% four different aspects.

11.6 Discussion

The research concerned students' beliefs about mathematical content based thinking by using photos from everyday life.

Firstly, the data analysis shows that the investigated students mostly mentioned geometric (84.6%) and numerical thinking aspects (51.6%), which implies that students particularly connect numerical and geometric aspects to real-life. This finding is in line with the research by Martin and Gourley-Delaney (2014), who described that numerical and geometric thinking aspects could be seen as key factors for students considering if a photo or an activity contained mathematics. Furthermore, the PISA 2021 Mathematics Framework (OECD 2018) gives an explanation for the high number of numerical thinking aspects: Numbers represent a key concept in all of the mathematical areas as the students' concepts of other mathematical areas depend on first experiences and work with numbers. In respect of this outcome, the numerical and geometric thinking aspects were analysed in detail in this paper: Cardinal numbers (definite 26.4% or indefinite 29.7%) played the most important role in everyday life within numerical thinking, as well as figures and shapes (42.9%) followed by relative positions and symmetry (each 27.5%) in relation to geometric thinking. With regard to the geometric thinking results, there was a high frequency of general geometric notions like “geometry” or “figures” instead of clearly naming geometric aspects like “rectangle” or “congruent”, which might underline the tendency of imprecise formulation. It is also possible that students might have possessed in fact more complex beliefs than they expressed. In other words: There might have been the problem of written explanations in contrast to real beliefs. Interestingly, three times more students mentioned figures and shapes compared to fields. It has to be further investigated if figures and shapes may be better or faster perceived than fields. Another explanation could be that students were not exact enough in their explanations and mixed up notions like “cube” and “square”. That is why further research is necessary. In the students’ explanations, algebraic (30.8%) and functional thinking (25.3%) were less mentioned than numerical and geometric thinking. This result is supported by Baki et al. (2009) focussing on students’ beliefs about the connection between mathematics and real-life. Stochastic thinking is rarely represented which may be explained by the fact that “probability classrooms often fail to develop sustainable conceptions of probability as strategic tools that can be activated for decisions in everyday random situations” (Prediger 2008, p. 126).

Secondly, the paper investigated the frequency of different aspects of mathematical content based thinking. One the one hand, the study shows, that students are able to recognise at least one aspect of mathematical content based thinking. On the other hand, students are capable of developing different lines of thought for a single photo from everyday life. In this study, the respondents mostly mentioned one or two different aspects for each photo. A reason for this might be that students have problems in connecting different area topics. Interestingly, the PISA 2021 Mathematics Framework (OECD 2018) will examine these frequencies of connections in the mathematics classroom. This outcome of the study might have been a problem of the task itself, which did not necessarily require the use of different lines of thought. To respond to the research question, it seems essential to conduct further studies with a more specific focus on the different aspects of content based thinking. Furthermore, the data analysis made clear that it can be difficult to interpret the focus or the intention of the students in some sentences as there might be more than one aspect of mathematical content based thinking within an explanation.

It is clear that there are some limits of the study: Due to the voluntary participation in the photo competition, we expect the respondents to be rather mathematically interested students, even though there was no survey about their mathematical interest or their results at school. Therefore, in addition to the age, further studies should take the students’ interest in mathematics into account as well as the type of school. Furthermore, symmetry but also geometry in general seems to be a mathematical aspect of content based thinking which is easily recognised as mathematically interesting and of which a photograph might be taken easily. In contrast to that, algebraic as well as stochastic thinking topics might be difficult to photograph. As a consequence, it has to be reconsidered to what extent a photo as a medium of research sheds the light on geometry.

Thirdly, the research aimed at differences in beliefs of the five aspects of mathematical content based thinking between students of different ages and cultures. Because of the limits mentioned above, it could not be shown how far the students' age influences the five aspects of content based thinking. Furthermore, cultural differences could not be determined as the sample of respondents was too small for a comparative study.

11.7 Conclusion

The photos from the competition could show that for some students, mathematics is a vivid science appearing in their everyday lives. To say it with the 15-year-old South African girl's words about a building: "The round ball has different shapes of triangles which fit together in a circular shape. [. . .] It is interesting because it is a building that everyone goes to but do not realise the math in it". Several connections can be seen between the five dimensions of mathematical content based thinking. The international photo competition can be regarded as a good starting point to get a broader insight in students' beliefs about mathematical content based thinking, especially, if it was supported by questionnaires and interviews. In the end, an international comparison should follow.

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Part III
Understanding, Measuring and Changing Teachers' Beliefs

Chapter 12
Introduction

Annette Rouleau and Chiara Andrà

This part of the book comprises four contributions concerning teachers' beliefs. While the tripartite model of affect (McLeod, 1992) encompasses beliefs, motivation, and emotion, the first component has received far more attention in MAVI's history, and so it does also here. Foundational to this research is the premise that there is a strong relationship between beliefs and behaviour, as the former influence and shape the latter (Beswick 2005). From a variety of perspectives, in fact, researchers have offered experimental evidence that beliefs play a crucial role when teachers are invited to abandon traditional teaching practices and adopt new ones (e.g., Liljedahl, 2012; Skott, 2009). That is surely one of the reasons why issues concerning understanding and measuring beliefs have received special attention in affect-related research in Mathematics Education (e.g., Hannula, 2012).

Teacher change is a complex process as it is firmly rooted in what teachers believe about the teaching and learning of mathematics. For teachers, changing their practice is not only a question of transmitting knowledge but also involves changing their beliefs (Beswick, 2005). Traditionally, these beliefs have been seen as a barrier to change (Richardson & Placier 2001; Cooney et al., 1998). As teachers try to move from familiar practices to new ones, their beliefs in the veracity of their previous practices may hinder the process or result in the imposition of new ideas within traditional practices to which they still strongly adhere.

Significant change in practice, then, should be accompanied by a shift in beliefs. This suggests a dialectical relationship between change and belief in that one influences the other (Buehl & Beck, 2015; Sullivan & Mousley, 2001). This stance

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avoids the conundrum of whether change in practice precedes a belief change (e.g., Guskey, 1986) or follows it (e.g., Cooney, 1994). Furthermore, determining which changes first seems to be less important than acknowledging the tension that is often apparent when teachers are involved in changing up practice, for instance, when implementing new forms of assessment or a new curriculum (Dietiker & Riling, 2018).

Tension arises as teachers are faced with having to decide between two or more equally important choices (Sparrow & Frid, 2001). In our own work, we have found tension to be a powerful lens to better understand change at the affective level (e.g., Andrà et al., 2019). As Chapman and Heater (2010) suggest, "Meaningful change can occur when the process is initiated and rooted in the teacher's experience based on a tension in self and/or practice that is personal and real to him or her" (p. 456). We agree and suggest that tension research can offer insight into the frustrations and needs of the classroom and the changes that result. Furthermore, recognition of the tension inherent in teaching can help us as researchers in better understanding those apparently inconsistent behaviours we observe. If we think of teachers as "dilemma managers" (Lampert, 1985), what might be construed as minimal or no change could be recast as a rational decision that weighed the practicality of the change against its potential consequences.

The four chapters in this part are interested in teacher beliefs and/or change. We can see a strong connection among the four contributions, if we consider the papers through the lens of tension.

Maffia, Rossi Tisbeni, Ferretti, and Lemmo present us with methodological tension regarding the analysis of teachers' beliefs about mathematics. They suggest that researchers engaged in qualitative analysis reap a rich data set but are faced with a time-consuming analysis. On the other hand, having respondents select from a restricted range of possible answers makes quantitative analysis easier, but the formulation of the questions and answers can be influenced by the researchers' ideas. Like Sparrow and Frid (2001) suggest, having to choose between the two sacrifices the advantages of the other. To manage this tension, the authors propose an alternative: a clustering technique to analyse the answers to a multiple-answer question which, they suggest, offers more insight than does frequency calculations. In Maffia et al.'s chapter we, thus, find a way to manage the tension by finding a third, alternative way with respect to the two, traditionally existing ones. We can further comment that it is the existence of the tension between fine-grain, qualitative analysis and a broader quantitative approach for identifying and collecting data on teachers' beliefs that justifies the employment of the new methodology presented in their chapter. Thus, another important characteristic of tensions emerges, namely, whether they can be overcome. Sometimes it is not possible to do so, and teachers and researchers might live with tensions, and manage them, while in other cases a choice is necessary.

Researchers are not only interested in measuring teachers' beliefs; however, they also want to understand how beliefs change. And, just as tension surfaces in the measurement of beliefs, it is also inherent in change (Chapman & Heater, 2010). In their paper, Ferretti, Funghi, and Martignone suggest an epistemological tension with standardized assessment. They are particularly interested in how the presence of these tests has influenced teachers' beliefs towards mathematics teaching. The authors note that, although teachers are prone to negative emotions regarding the implementation of these assessments, their study identifies different types of change in both practice and belief that resulted from the implementation. We suggest Lampert's (1985) description of a dilemma manager as a "broker of contradictory interests" (p. 178) may be an apt description for the teachers in this study. As the authors note, the teachers fear being judged by the results of standardized assessments yet find value in adopting its question types into their teaching practice. In this chapter, the initial tension appears unresolvable, and its presence in the teachers' lives propels their reflections on their practices, provoking dialogue, thoughts, and actions in an attempt to find a balance between two competing forces.

The impetus for Krummenauer and Kuntze's study was tension with teacher competence in statistical variation. Specifically, they examined how teachers analyse learning situations in the classroom, tasks, or textbook material. They suggest this competence is closely related to the quality of the learning opportunities teachers are able to provide their students. In their study, only a minority of their primary teacher participants showed an awareness of the learning potential related to statistical variation, a finding they attributed to both lack of knowledge of, and a lack of interest in, statistical variation. They conclude that a focus on the cognitive and affective dimensions of statistical variation is needed to affect change in competence. It is as if Krummenauer and Kuntze's research aims at provoking a tension in teachers under their study: the fact that teachers do not see troubles in not grasping the deep meaning of some crucial statistical concepts and practices is the core of their research. In general, teachers' professional development sometimes provokes new tensions, which are relevant for the teachers' professional evolution towards new and more effective practices.

Peters-Dasdemir and Barzel address the tension faced by teachers in integrating technology in their teaching. As the authors suggest, teachers have to consider how to include technology in order to support learning, yet many do not have the skills for its meaningful use. Looking for deeper insight into changes in teaching practice regarding digital technology, their study focuses on investigating teacher beliefs about its use in the classroom. They noted inconsistency with the teachers professed beliefs and their actual usage of technology in two groups: those with a positive belief regarding technology who seldom used it and those with a negative belief who were frequent users. If we think of these teachers as dilemma managers, these apparent inconsistencies can be explained as rational responses to tension. As the authors suggest, the first group do not feel supported in their technology use, so they made the choice not to implement it in their classrooms despite their belief in its value. The second group made the choice to incorporate technology solely to satisfy curriculum demands despite their disbelief in its effectiveness. Both groups of teachers made a choice, and the existence of a relevant tension makes them so interesting for the research. Digging deeper into the intertwining of beliefs and practice by focusing on the tensions and on the ways teachers deal with tension opens up opportunities for further investigation.

Teacher tension may result in difficult choices, no choice, a block, a surprise, a search for new strategies, or other nuances in their practice. To analyse the beliefs that give life to the tension and that may guide the change is, in our opinion, an alternative theoretical lens for the researcher, who is invited not to consider beliefs as sole barriers to change. Many times, indeed, beliefs promote change, but other factors impede it (e.g., Andrà et al., 2019). For researchers, therefore, who are interested in designing authentic opportunities for teachers' professional growth, to understand beliefs and the tension therein is a crucial research aim. In what follows, some source of inspiration is provided from a variety of both theoretical perspectives and practical needs.

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Chapter 13
A Clustering Method for Multiple-Answer Questions on Pre-service Primary Teachers' Views of Mathematics

Andrea Maffia, Simone Rossi Tisbeni, Federica Ferretti, and Alice Lemmo

13.1 Methods for Studying Pre-service Teachers' Views of Mathematics

It is recognized internationally how the teachers' beliefs are decisive in the process of teaching–learning mathematics; since several years, research in mathematics education follows this direction of investigation (e.g. Schoenfeld 1989). As pointed out by Pajares (1992), teachers' beliefs are in most cases already developed during pre-service university courses and for this reason many studies in literature focus on the affective sphere of prospective mathematics teachers (e.g. Hannula et al. 2007; Brady and Bowd 2005). In line with this perspective, our research focuses on pre-service primary teachers' views of mathematics.

Due to the epistemological and cultural nature of the research on affect, many studies employ qualitative methods mainly using ethnographical and linguistic methods (e.g Ebbelind 2015) or using open questionnaires (see Hart 2002). As highlighted by Kloosterman and Stage (1992), closed instruments (like Likert-scale) can suggest the researcher's ideas, thus influencing the respondent in a social desirability perspective. To overcome this, some authors use open questionnaires

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in which respondents are free to express their emotions, beliefs and memories by using their own words and they are not forced to align their opinion on a ready-made list chosen by the researcher (Di Martino and Sabena 2011; Di Martino et al. 2013). Again, even if the respondents are free to use their own words, we could argue that the formulation of a question may still influence the answers, especially in the case of inverse formulated items. The data gathered by the questionnaire are often analysed through an inductive content analysis (Patton 2002); in some cases, descriptive statistics are also used (Di Martino and Sabena 2011).

Other authors use closed-ended tools that may be adapted or inspired by already validated scales. For example, Zollman and Mason (1992) created the Standard Beliefs Instrument (SBI). Such instrument consists of a battery of Likert-scale questions aimed at determining the consistency of teachers' beliefs with NCTM Standards (1989). Obviously, that one is not the only study on the topic of teachers' believes using Likert-scale (another well-known example is given by the Problem-Solving Project, Schoenfeld 1989).

In this paper, we present a clustering method for analysing multiple-answer questions. Cluster analysis was also used by Hannula et al. (2005) in the case of a different format of questions: they clustered the respondents of two questionnaires about pre-service teachers' views of mathematics (see also Roesken et al. 2011). Such analysis produced three main types of belief profiles: positive, neutral and negative view, each one then divided into subclasses (Hannula et al. 2005). Similarly, we use cluster analysis to obtain different profiles of respondents, but we cluster the answers to only one multiple-answer question.

13.2 Clustering of Multiple Answers

Multiple-Answer Questions

As we have seen in the previous section, research about pre-service teachers' views of mathematics often favours qualitative methods over quantitative ones and open questions rather than multiple-choice questions. Open questions give to the respondents the possibility to express freely their own opinion, using personal wording and so giving thicker descriptions (Geertz 1973). However, the analysis becomes more time-consuming limiting the amount of processable data. In contrast, when administering a multiple-choice question, the designer of the questionnaire forces the respondent to select just one among a limited number of given answers (so rising many possible critiques, see Cohen et al. 2002), but making it easier to perform quantitative analysis. We get just a restricted image of human behaviour when "social scientists concentrate on the repetitive, predictable and invariant aspects of the person; on 'visible externalities' to the exclusion of the subjective world; and on the parts of the person in their endeavours to understand the whole" (Cohen et al. 2002, p. 18).

Several solutions could be found "in between" the fully open-question and the multiple-choice. One possibility is the use of rating scales (like Likert-scales). This kind of scales allow to know more than just a "yes/no" agreement on a statement giving some shadows to an otherwise black/white picture of the results. As also seen before, Likert-scales are largely used in psychological and educational research and many researchers developed this kinds of method (ibidem). It is much rarer, especially in research on affect in mathematics education, to see the use of *multiple-answer questions*. A multiple-answer question is similar to a multiple-choice one, with the only difference that the respondent can select more than one answer option. Research has paid attention on identifying the optimal number of options in a multiple-choice question (e.g. Baghaei and Amrahi 2011), but less is known about the optimal number of choices for a multiple-answer question. When the respondents are forced to select between a yes/no answer (or even an agree/disagree choice) they tend to express more positive opinions rather when they have a "select-all-that-apply" option (Dillman et al. 2003); hence, multiple-answer questions allow the respondents to express better their opinion. Rasinski et al. (1994) show that it is possible that respondents do not really mark "all-that-apply" when answering to this kind of question. Furthermore, there is evidence that the first given answers receive more selections than the other ones (Dillman et al. 2003; Rasinski et al. 1994). We conjecture that, by giving a fixed amount of answers to select, we induce the respondents to read all the possible answers. Thus they will opt just for those that mostly represent their own opinion. It could be argued that, in this way, we restrict the possibility of expression of the respondent, getting closer to the case of multiple-choice format. This is true, but a following interpretative process may help in getting still a not-superficial picture of the respondents' opinions.

Usually, multiple-answer questions are analysed computing the relative frequency of each answer. In this way, the obtained results are not different from those obtainable by a multiple-choice question. Much more insight is given when some of the answers are "linked." This could be realized by a correspondence analysis (Greenacre 2017) of the answers or by clustering techniques. In this paper we take the latter option, as it is presented in the following sections.

Clustering

The answers are encoded in a binary vector of size n, with n the number of possible options. The i-th position of the vector has a value of 1 if the i-th option was selected, 0 otherwise. For example, if a respondent in a 6-options multiple-answer question selected options A, D and F, the resulting vector would be (1, 0, 0, 1, 0, 1). The vectors are concatenated in matrix $M_{p \times n}$ with p rows, one for each participant.

A similarity matrix is computed as $A_{p \times p} = M \cdot M^T$, where the element $a_{i,j}$ corresponds to the number of options selected by participant i also selected by participant j. The resulting matrix is a symmetric matrix, with 3 in every element of its diagonal (each participant has 3 answers in common with herself).

A new matrix is obtained by calculating the pair-wise Euclidean distance between each row of the matrix A. We then use classical *multidimensional scaling* to map the points into a three-dimensional space (e.g. Borg and Groenen 1997).

The resulting points are then clustered using an agglomerative hierarchical clustering algorithm, meaning that at each step the clusters with the *shortest distance* are merged (Fig. 13.2). The distance D between two clusters X, Y can be computed in multiple ways (e.g. Backhaus et al. 2016). Single linkage may function to determine the outliers in the data, and then performing the Ward algorithm classifies the remaining elements. While this algorithm can result in a valid clustering, in this work its performance was reduced, due to the absence of isolated data points (a more detailed discussion is reported in the final section).

The complete linkage rule was then chosen since it tends to find compact clusters of similar diameters, avoiding chaining phenomena (Everitt et al. 2011). This distance is defined as:

$$D(X, Y) = \max_{x \in X, y \in Y} d(x, y)$$

where $d(\cdot, \cdot)$ is the Euclidean distance. In this way, cluster are joined where the distance between the furthest members of the clusters is the lowest. The chosen number of clusters is the one that minimizes the absolute maximum deviation from the median of the number of particpants per cluster.

13.3 An Example: Pre-service Teachers' View of Mathematical Ability

In this paper we are presenting a method for analysing multiple-answer questions by means of clustering. Rather than presenting a huge amount of results, we prefer to show clearly how the method is applied and what kind of results is possible to get. For this reason, we will analyse the results of just one question that was inserted in a questionnaire administered to students attending the course of "Mathematics Education" within the last year of the master-degree in Primary Education in the University of Bologna, Italy. The sample consists of all the students enrolled in the last year of the master-degree ($N = 207$). They all answered to a questionnaire containing some mathematical tasks, some Likert-scale items about their emotions toward mathematics and mathematics teaching, and two multiple-answer items; one about their views of mathematics ability and one on their beliefs about teaching mathematics. These questions were always administered in the same order. In the following, we refer to one of these questions:

> Select among the following options the three features that, according to you, are fundamental for having success in mathematics.

Twelve options were available (Fig. 13.1).

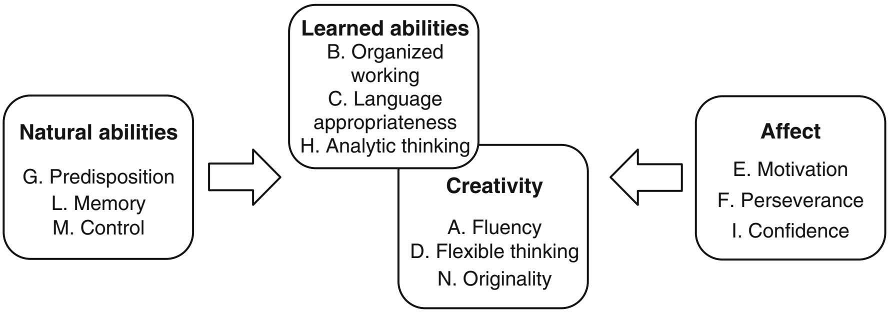

Fig. 13.1 Classification of options within to the adopted theoretical framework

We selected the options according to the model of mathematical giftedness described by Pitta-Pantazi et al. (2011). Following these authors, mathematical ability is the result of *learned mathematical abilities* (like verbal, spatial, quantitative abilities and other) and *creativity* (defined as a combination of fluency, flexibility and originality). Both learned abilities and creativity are supported by *natural abilities* (comprehending working memory, control and speed of processing). We decided to integrate this model adding the dimension of affect. Indeed, research has shown strong evidence of the influence of emotions and motivation on mathematical performances (e.g. Zan et al. 2006).

Figure 13.1 shows how each of the available options for our question is related to one of these dimensions. The capital letter before each option corresponds to the order in which the 12 options were given in the questionnaire: A is the first option and N is the last one (the letters J and K are not commonly used in the Italian alphabet). As explained above, we asked the respondents to select only a fixed number of options. The choice of the number "three" makes possible to select only the options belonging to one dimension. Pre-service teachers responding to the questionnaire do not know the framework and so their interpretation of the terms could differ from the formal definition. For this reason, we changed some terms into more colloquial synonyms or phrases. However, the problem is not so easily solved and so an interpretative process of the given answers is necessary. In the following, we will discuss those cases in which a word (in the original language of administration of the questionnaire, Italian) may be interpreted in more than one way.

The questionnaires were administered by means of paper and pencil, dividing the participants in groups of a maximum size of 33. Three researchers (among them the second and third authors of this contribution) administered the questionnaire to each group. A time limit was given, and the question analysed was in the central part of the questionnaire. We analyse only the complete answers, so we are not considering blank answers and those cases in which less than three options were selected. In this way, the amount of analysed answers is $p = 178$.

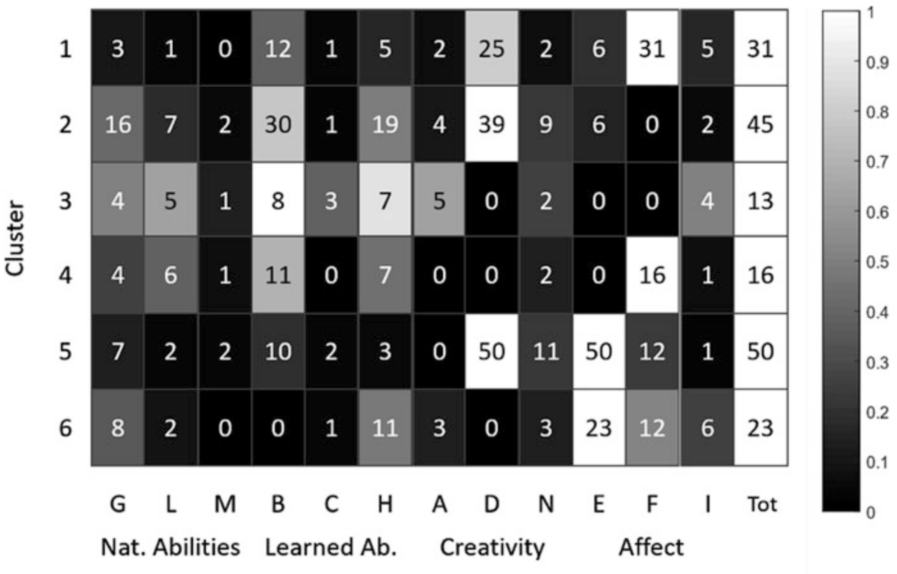

Fig. 13.2 Frequency of choices of options (A–N) in the six clusters. Colours represent the relative frequency compared to the maximum frequency (tot) within the cluster

Figure 13.2 shows a dendrogram representing the process of clustering described in the previous section. Starting from the bottom, elements with a distance smaller than the value reported on the ordinate axis are merged. We decided to take 6 clusters even if the distance from the median value is minimized also in the cases of 14 or 15 clusters. This is because a high number of clusters creates difficulties in the interpretative process. Figure 13.3 shows how the choice of the options is distributed in the six clusters; the last column reports the number of respondents in each of the obtained clusters

The *first cluster* comprehends pre-service teachers who selected mainly the answers D and F (flexible thinking and perseverance). 12 over the 30 members of this cluster (40%) selected also option B (organized working). This cluster contains those pre-service teachers giving relevance to the relation between creativity and affect, taking also in consideration learned abilities. They do not give importance to natural abilities.

According to the members of the *second cluster*, mathematical ability is related mainly to learned abilities and creativity: they give relevance to the relation between organized working (option B) and flexible thinking (option D). There are also high percentages dedicated to analytic thinking (42%) and predisposition (36%). Apparently, members of this cluster see flexible thinking as the main component of mathematical ability, supported by a certain behaviour and, maybe, a predisposition. They give less relevance to the dimension of affect.

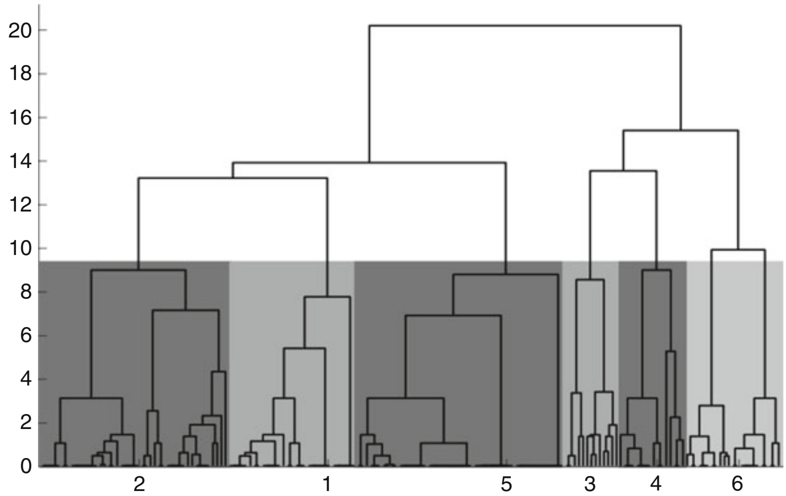

Fig. 13.3 Dendrogram representing the clustering process. The obtained clusters are highlighted in grey. The *y*-axis reports the distance at which clustering occurs

The *third cluster* is less populated; most of its members (62%) select the option B (organized working) often paired with analytic thinking (54%), predisposition (31%), memory (38%) or fluency (38%). Pre-service teachers in this cluster give strong importance to innate abilities and learned behaviour. Indeed, they often select one between the options G, L or A. In our model, we related option A (fluency) to the dimension of creativity, but the Italian translation could also be interpreted by the participant as "quickness", so relating to the speed of processing that is a natural ability in the model of Pitta-Pantazi et al. (2011).

The dimension of creativity is ignored also by the *fourth cluster*. They give relevance to keeping perseverance (100%) on an organized work (69%), eventually with the support of natural abilities. The other aspects of affect are not relevant for this group.

The *fifth cluster* is clearly characterized by the choice of option D and E, respectively, flexible thinking (100%) and motivation (100%). This is not the only group giving relevance the role of motivation, indeed also the *sixth cluster* does (100%). The difference between these two last clusters is that the former puts affect mainly in relation to the creative dimension, while the latter gives more relevance to the other dimensions.

Table 13.1 summarizes the relevance given to the different dimensions of mathematical ability by the members of each cluster according to the interpretation we gave in this section. The positive sign indicates a high relevance, while the negative sign represents a low relevance. No sign is indicated when we did not see a strong tendency in the selection of options by respondents.

Table 13.1 Numerosity of respondents in each cluster and relevance given to the four dimensions of mathematical ability

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6
Natural abilities	−		+		−	
Learned abilities		+	+	+	+	
Creativity	+	+		−		
Affect	+	−		+	+	+

13.4 Discussion

Large scale studies about pre-service teachers' views of mathematics provide a huge amount of data that is unrealistically analysable only by means of qualitative methods. Quantitative analysis has its limits, as many researchers noticed (Cohen et al. 2002). The availability of several different instruments allowing the respondents to express their opinion as freely as possible appears relevant. In this paper we have studied the use of multiple-answer questions within a questionnaire about pre-service primary students' views of mathematics. We used a clustering technique to analyse the answers to a multiple-answer question about respondents' opinions on mathematical ability. In our literature review we did not find other works using these methods in our field of research.

The used hierarchical clustering algorithm does not explicitly consider the existence of links (common options) between two answers. To verify that this information is not lost, for each answer we measured the average number of common options with other answers in the same cluster. The selected cluster for an answer is considered valid if the maximum average of common option is within the selected cluster while, for the other clusters, the average is lower. In our results, using a complete linkage, 161 answers over 178 belonged to clusters with highest average number of shared options, and 9 of the 17 remaining where paired to the cluster with the second highest average. We tested also single linkage (obtaining a ratio of 63/178) and Ward linkage (153/178), so concluding that the chosen algorithm was the best option in this case. As shown in the previous section, some limits of closed questions still remain, in particular the possible different interpretation of the terms used in the answer options (for instance, in the analysis of the third cluster). However, we can observe that the choice of allowing the selection of only three options resulted in avoiding one of the usual limits of multiple-answer question: differently than observed in previous research (Dillman et al. 2003; Rasinski et al. 1994), the respondents did not show a preference for the first options in the list, indeed options A and C were among the less selected. This observation suggests that while answering to this kind of questions, respondents pay more attention to the selection of those options that mainly represent their opinion. As it was already done with multiple-choice questions (Baghaei and Amrahi 2011), further research is needed to identify the effects of changing the number of options to list and to select.

The example of analysis in the previous section shows that it is possible to cluster respondents according to their answers to a question about their views of mathematical ability. Such clustering revealed different groups that we were able to describe according to an interpretative process based on a reference theory (Pitta-Pantazi et al. 2011). We can see that the cluster of pre-service teachers giving more importance to natural abilities is the smaller while much attention is paid to learned abilities and affect. Concerning the affect dimension, we have two clusters giving more relevance to perseverance and two clusters paying more attention to motivation.

Clustering appears as a suitable method in analysing multiple-answer questions giving more insights than just frequency calculation. Furthermore, assigning each respondent to a cluster, we can deepen the analysis comparing the results of a multiple-answer question with those of different type of questions. For instance, in our questionnaire we can compare the membership to a specific cluster with the emotions that the respondent associates with mathematics and/or mathematics teaching. It will be possible to look for interdependence between emotions and a certain views of mathematics.

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Chapter 14
How Standardised Tests Impact on Teacher Practices: An Exploratory Study of Teachers' Beliefs

Federica Ferretti, Silvia Funghi, and Francesca Martignone

14.1 Background

Italian National Standardised Tests

The National Institute for the Evaluation of the Educational System and Training (INVALSI) is an Italian research institute with the status of legal entity governed by public law. This Institute runs the National Evaluation System to assess levels of learning of Italian primary and secondary school students in Mathematics, Italian, and English. Since the school year 2007/2008, INVALSI has carried out surveys nation-wide for all students in the second and fifth classes of primary school (grades 2 and 5), in the third class of middle school (grade 8), and in high school (grade 10 and, from 2019, grade 13). These tests are designed by expert teachers, educational and disciplinary researchers, statisticians, and experts of the school system, and their primary purpose is evaluation of Italian educational system. The processing of statistical data and the measurement of students' learning levels are carried out through sampling, but the standardised assessment procedures are carried out in census mode, so that each student has the opportunity to try INVALSI tests even if his/her answers will not be included in the sample for statistical analysis. Each school receives its own data and each teacher can analyse the data of his/her students.

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INVALSI mathematics tests are part of the external assessment of Italian students' learning outcomes as expected according to Italian National Guidelines (NG). INVALSI tests try to combine, as far as possible, the needs of a summative evaluation system with the requests and perspectives given by the NG and by current paradigms on the teaching–learning of mathematics. For this reason, in INVALSI tests there are both multiple-choice tasks and open-ended tasks with a request for argumentation or of showing the work (Garuti and Martignone 2015, 2019).

The data collected by INVALSI can be useful for teachers because they can compare the statistical data about their own classes or educational institution with the overall test results, interpreting these in light of the specific context in which their own school operates. Moreover, these tests can provide examples of tasks in line with the goals and objectives stated in the NG. In order to be able to better interpret and use the data of INVALSI tests, INVALSI offers various resources such as the "INVALSI Guides". The analysis of INVALSI tasks and results may lead teachers to reflect on student performance and the validity of educational development in the implemented curriculum (Ferretti et al. 2018). Nevertheless, standardised tests, like all summative tests, can only provide certain information and assess specific aspects of student competence (Osta 2014). This is one of the reasons why INVALSI tests have often been rejected by teachers and, more generally, by school system. In fact, teachers view student assessment as a long and complex process involving different educational activities; moreover, they often feel negative emotions towards INVALSI tests, such as anxiety or fear of being judged as teachers (Signorini 2017).

Teachers' Beliefs

Teachers' beliefs, emotions, and attitudes in mathematics are held to strongly affect teachers' practices and the quality of teaching, and consequently to have an impact on students' learning processes (Burton 1979). However, several studies have shown that the relationship between behaviour and affective constructs (such as attitudes or beliefs) cannot be described simply as a cause–effect relationship, but rather as a sort of continuous and complex mutual influence (Richardson 1996; Di Martino 2016). Within the research on teachers and their professional development, in particular, contrasting results were found regarding the consistency between teachers' *espoused beliefs* and *enacted beliefs* (Cross 2009): "the beliefs that teachers declare are, in the end, definitely different from those that guide their solving processes and their behaviour in general" (Malara and Zan 2002, p. 559). In Italy, studies on primary and middle school mathematics teachers—who often do not have a degree in mathematics—show that many of them have a problematic relationship with mathematics: their memories are frequently marked by difficult school experiences, and they often feel negative emotions towards mathematics and the idea of teaching it (Di Martino and Sabena 2011). In these cases, in order to cope with the lack of self-confidence, it may happen that teachers choose to carry out frontal lectures or "drill and practice" teaching instead of innovative teaching practices suggested by educational studies or presented in national curriculum reforms.

14.2 Aim of the Study and Research Questions

In recent years, large-scale assessment projects—such as PISA—have played an increasingly important role both in the educational field and in terms of evaluation and monitoring of educational systems. While 10 years ago research using the results of OECD-PISA surveys in the field of mathematics education was still limited (Sfard 2005), now many researchers have started to use large-scale assessment analysis for their studies. There is a growing international interest in how to integrate results, methods, theoretical frameworks, and tools of large-scale assessment projects, designed and implemented in order to impact at a systemic level into the teachers' and schools' activities and programme (Looney 2011). In a previous study, we reflected on how the INVALSI tests can influence the implemented curriculum (Ferretti et al. 2018). However, Signorini (2017) raises also the issue that INVALSI tests can be perceived by teachers as a means to assess their efficacy.

Following these considerations, we now develop a study to gather information about mathematics teachers' perception of the impact that large-scale INVALSI assessment tests have on their teaching practices. This is a complex issue that we intend to investigate by means of a large-scale study: the present study constitutes a partial preparatory work toward this ultimate goal. In particular, our research questions are:

1. Do INVALSI data influence teachers' beliefs about teaching in any way? If so, how do they perceive that these tests affect their teaching practices?
2. Do teachers perceive any changes in their teaching practices? If so, which are these changes?
3. In the institutional documents it is underlined that each item of the INVALSI tests refers to the National Guidelines, but is this clear to teachers?

As a first step we carried out an exploratory study consisting principally of a qualitative analysis of the data. In this article we present the first results obtained. The categories of changes identified in this study will be the starting point for the design of a large-scale survey.

14.3 Methodology

In order to generate hypotheses starting from teachers' statements, we chose to create a brief questionnaire with mostly open-ended questions, since "It is the open-ended responses that might contain the 'germs' of information that otherwise might not be caught in the questionnaire" (Cohen et al. 2007, p. 330). With this kind of questions, in fact, the respondents are left free to express their thoughts in their own words, focusing on the aspects they believe more relevant (Di Martino 2016). In our questionnaire, we asked teachers the school grade they teach, the number of years of teaching mathematics, and the following open-ended questions:

1. Think about the INVALSI tests and your teaching practices. Did these tests influence or somehow change your teaching practices? For both yes and no answers, try to explain how/why.
2. If you answered yes, what aspects of your teaching practice did the INVALSI tests influence most? In what way?
3. In your opinion, how consistent are the INVALSI tests with the goals of the National Guidelines? Explain your answer.

The analysis of the answers was carried out by means of a content-categorical approach to the analysis of narrative data (Lieblich et al. 1998), since this approach is considered suitable for the study of a phenomenon common to a group of people (Kaasila 2007). We sent the questionnaire by e-mail to about 40 teachers of primary school (grades 1–5) and middle school (grades 6–8). Participation in the study was voluntary. We asked teachers who wished to participate to reply to us within 2 weeks, and we received 22 completed questionnaires in this period. Among the 22 completed questionnaires, 12 were from primary teachers, 7 from middle school, and 3 questionnaires did not reveal this information. In the following section, we will indicate the questionnaires with P followed by serial numbers between 1 and 22 (e.g. P1 as acronym of "participant 1"). For reasons of space, in this paper we focus just on the analysis of the open-ended questions.

14.4 Results of the Analysis of Open-Ended Questions

Regarding the first two questions, 14 teachers affirm that INVALSI tests influenced their own didactic practice. Just three of them state that this influence was little, while three other teachers state that the impact of INVALSI tests on their practice has been scarce. Furthermore, there are two particular cases, related to P17—a young teacher, who has not yet taught in the school grades in which INVALSI tests are administered—and P21 who seems to be very critical about the usefulness of INVALSI tests:

> I try every year not to be influenced by INVALSI tests. But the truth is that I am eventually forced to make my students learn the "INVALSI mode", and it seems to me a great, huge waste of time that I could more fruitfully use in other ways".

Even though it is not clear from the answer what the "INVALSI mode" is, we can hypothesise that P21 is contrary to the standardised test features.

Taking together answers to open-ended questions 1 and 2, we can find a variety of ways that teachers believe their practice is influenced by INVALSI tests (at least according to their statements), and not all of them to the same extent. We identified at least four categories in which the changes declared by teachers can be classified.

Changes to Task Aims and Learning Activities In this case, teachers say that they enhanced activities aimed at improving students' competencies in problem solving and logical thinking (e.g. "*I have learned to give more space to solving strategies and to the logical thought processes and connections of each student, trying to activate, as much as possible, meaningful learning [. . .]*", P1).

Changes to Task Format and Assessment In this case, teachers write that they used tasks taken from past INVALSI tests, or similar tasks, or introduce standardised tests in classroom assessment (e.g. "*[. . .] I improve preparatory exercises by rewriting them in INVALSI format*", P5; "*I pay more attention to the formulation of tasks*", P22). Some teachers also organise simulations of INVALSI tests ("*I have periodically carried out lessons in which I make the students do simulation tests, and then correction*", P9).

Changes to Teaching Method In this case, teachers state that they began to use more examples of application of mathematics to reality to explain mathematical concepts (e.g. "*Mathematics applied to realistic situations has become an important part of my teaching. Every time I explain a topic I always look for examples of problems and exercises that can be found as INVALSI questions*", P7). P12 explicitly talks about the link between INVALSI tasks and his/her changes in the methodology developed in the lesson: "*work on real problems in cooperative learning or flipped classroom*". P17 talks of "*differentiated didactical methods*" but it is not clear if s/he is referring to the inclusiveness of Italian schools, or to the need to propose different kinds of activities—so we did not include this answer in any of the categories.

Changes to Curriculum Implementation In this case, teachers state that they changed the order, the selection or the level of consolidation of some contents. They focus on the fact they have to explain the topics included in the INVALSI tests or to revise them in order to prepare students adequately ("*I try to briefly explain all the topics that are required in the tests. For example, I didn't use to explain probability, now I try at least to mention it [. . .]*", P22). They may also prefer to change the order of the topics covered in order to anticipate those included in the INVALSI tests ("*[INVALSI tests had an influence] On the choice of topics to be addressed (for example the clock, which was anticipated because it could be a topic of the INVALSI tests [. . .]*", P16).

The most frequent categories of declared changes are linked to the tasks proposed to students: the exact number of answers per category is illustrated in Table 14.1. In some answers we found different types of declared changes. P21 cannot be classified in any of the categories quoted previously because s/he does not explain the meaning of "INVALSI mode". Moreover, P2 states basically that he/she did not change his/her previous practices.

The choice to use INVALSI tasks in classroom activities seems to be motivated by different reasons for teachers: some teachers say that they introduced this kind of

Table 14.1 Distribution of the changes indicated by the teachers of our sample

Declared changes in . . .	Frequency
Task aims and learning activities	7
Task formats and assessment	11
Teaching methods	3
Curriculum implementation	3

practice simply to train students (e.g. P6), whereas others claim that they believe INVALSI style tasks useful for working on students' argumentative and logical skills (e.g. P11 and P10). In particular, teachers stated that:

- students have to be trained to tackle INVALSI tests in terms of time management or the way questions are posed (e.g. "*[. . .] it is important to "train" the students to INVALSI-type questions, in order to prepare them adequately*", P6);
- the use of INVALSI tasks could enhance students' capability of logical reasoning and of argumentation, or foster deeper reflection by students about contents and activities (e.g. *"[. . .] I also appreciated the fact that INVALSI tests invite students to motivate and argue their solutions, so that they are explicitly centred on the "process" and not only on the "product".*", P10; "*I find many INVALSI tasks of interest in developing problem solving strategies and overcoming the mechanisms of the discipline, in fact I make regular use of them*", P11);
- INVALSI tests made them more aware of the need to link mathematics to realistic problems and examples (e.g. "*INVALSI tests led me to reflect on my way of teaching mathematics, and I had to change a didactic approach that was still linked to a traditional methodology which mainly consisted in theoretical explanation without any connection to everyday life*", P7).

Regarding the answers to the third open question, almost all teachers in our sample find INVALSI tests consistent or quite consistent with the NG (18 teachers out of 22); only P21 seems doubtful about their consistency ("*In my opinion there is little consistency. For example, I think INVALSI tests are not at all adequate for assessing students' competencies*") but he/she does not support this opinion with examples. Only P22 and P12 state explicitly that INVALSI tests are created starting from the objectives of the NG ("*I attended courses that included simulating INVALSI style items. For this draft we refer to the goals [. . .] present in the NG*", P22); the other teachers state simply that they find INVALSI tests a good tool for the assessment of competencies indicated as final goals in the NG ("*I believe that INVALSI tests are consistent with the goals of the NG because they test students' mathematical skills and competencies at the end of primary school*", P7), or that they believe INVALSI tests are good for showing how mathematics tools are useful for facing real-life problem situations ("*In my opinion, INVALSI tests are consistent [with NG] because they aim to assess how students use the knowledge they have learned when they have to face realistic contexts*", P3). A significant group of teachers (8 out of 22), however, claim there is consistency but—instead of explaining this belief—they focus on the aspects that make INVALSI tests not completely suitable for student assessment. Some of them simply claim that the time available is not sufficient to prepare students adequately, while others express negative opinions about standardised tests and external evaluation because they do not take into account students' efforts, their progress from their individual starting point and the difficulties they have managed to overcome (Signorini 2017).

14.5 Summary and Further Perspectives

Teachers' answers provided us with useful data to start drawing hypotheses about how teachers perceive that INVALSI tests affect their teaching practices. The results presented involved a small sample that is not representative of the population of Italian teachers, but it allowed us to collect some information. In fact, qualitative analysis of teachers' answers let us identify different types of changes. Among these, we believe that the changes concerning task aims and learning activities are significant, both in terms of task design and of their implementation in the classroom.

The introduction of tasks similar to INVALSI tests in order to prepare students for standardised tests could be predictable (a practice that obviously should not be exaggerated), but teachers also argue that they propose INVALSI tasks because they recognise in them an emphasis on problem solving and realistic situations. Teachers aim to develop skills related to problem solving in mathematics, not only because these skills are assessed in a summative evaluation system (like INVALSI), but because they consider them fundamental in the teaching–learning of mathematics: in our data, declared practice changes are often linked to the recognition of aspects that teachers consider important in mathematics teaching–learning processes, as the production of argumentation. Therefore, teachers say that they analyse the tests and then select problems that can be used for a classroom assessment because they consider these tasks in line with the NG. In fact, the majority of teachers interviewed recognised the consistency of the tests with the requirements of the NG. Criticism about INVALSI tests seems to concern mainly the form of administration (these are standardised tests used for external evaluation), not the type of exercises set. In conclusion, this study confirms the possible influence of INVALSI tests on teachers' choices about the implemented curriculum (Ferretti et al. 2018). Moreover, it also shows possible links with teachers' beliefs about mathematics teaching.

There is one aspect that was unexpected for us: teachers' statements about changes in teaching methods, such as the development of laboratory activities or flipped classroom activities. These changes are obviously not directly related to INVALSI tests (which contain tasks suitable for standardised and summative assessment), but the analysis of these tests somehow led some teachers to reflect on how to carry out methodologies that foster the development of students' argumentative skills and, in general, communication and transversal competencies. These issues call for further investigation and will form part of our future research, along with the design of a large-scale questionnaire to gather more specific data about the teachers' declared changes in teaching practices as identified in this preliminary study.

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Chapter 15
Primary School Teachers' Awareness of Learning Opportunities Related to Statistical Variation

Jens Krummenauer and Sebastian Kuntze

15.1 Introduction

Fostering students' statistical literacy is today considered as an important goal of the mathematics classroom (e.g. Watson and Callingham 2003) and is implemented in the curriculum of many countries in K-12 education (e.g. KMK 2004, p.11). Within approaches of statistical literacy, it is often highlighted, that dealing with statistical variation is crucial for students to develop statistical literacy, so that a focus on statistical variation is key when fostering students in this regard (e.g. Watson and Callingham 2003; Wild and Pfannkuch 1999; Kuntze et al. 2008). In several studies it has been found that already primary students can be able to deal with statistical variation intuitively and that it is possible to foster primary students with this respect (e.g. Watson 2018; Reading and Shaughnessy 2004). So, when teachers design learning opportunities for the statistics classroom, they should be aware of statistical variation and corresponding learning opportunities in order to make phenomena related to statistical variation accessible to their learning. As research on this topic is scarce—especially with focus on primary school teachers—we carried out a study with $N = 44$ in-service primary school teachers and investigated to what extent the teachers were aware of statistical variation and the related learning potentials (LPs) when they analysed textbook material and a classroom situation. The results indicate that only a minority of the participants were aware of possible learning opportunities related to statistical variation. This might result from affective dispositions, which could explain the teachers' restricted awareness in this domain.

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15.2 Theoretical Background

Key domains of mathematics teachers' expertise can be described by their competences to analyse in profession-related contexts (e.g. Kuntze et al. 2015). In line with Weinert (2001), we understand *competence* as "the cognitive abilities and skills that individuals possess or can learn for solving specific problems, as well as the associated motivational, volitional, and social readiness and abilities for using these problem solutions responsibly and successfully in variable situations." (Weinert, 2001, p. 27–28, translated from German)

The way how teachers *analyse* learning situations in the classroom, tasks or textbook material is such a competence, which is related to requirements of the teaching profession. This competence can be expected to be closely related to the quality of the learning opportunities teachers are able to provide their students with.

When teachers analyse learning situations in the classroom, tasks for students, or textbook material, parts of the teachers' analysis may take place *unconsciously*. In the perspective of Hannula's (2012) meta-theoretical framework on affect, as it is described in more detail in the chapter on *Methodological approaches for research on mathematical views* (Maffia et al., this book, Chap. 5), such unconscious processes can be seen as driven by affect. While *affect* is often connotated primarily with motivational and emotional aspects, Hannula highlights in his framework also the importance of cognitive aspects and integrates a cognitive dimension in his model (cf. ibid.). This paper mainly contributes to evidence related to this cognitive dimension of affect. However, the notion of competence affords a holistic approach to both affective and cognitive aspects: The notion of competence thus does not aim at "separating the non-separable".

With the competence of analysing we refer to an "*awareness-driven, knowledge-based process which connects the subject of analysis with relevant criterion knowledge and is marked by criteria-based explanation and argumentation*" (Kuntze et al. 2015, p. 3214). Analysing is thus considered as driven by the teachers' awareness for specific criteria, so that criteria a teacher may be aware of can be expected to shape her or his analysis process. We have described criterion awareness as "*a part of professional knowledge which influences the readiness and ability of teachers to use [. . .] professional knowledge element[s] in instruction-related contexts*" (Kuntze and Dreher 2015, p. 298). Criterion awareness, therefore, affords access to the use of related professional knowledge in the process of analysing classroom situations. With the term *professional knowledge* (see multi-layer model in Kuntze 2012), we refer to both knowledge as well as individual views, beliefs, and convictions of the teachers with relevance for their profession. Consequently, we would like to underline that elements of professional knowledge are, therefore, not to be meant only as "cold cognition" (Hannula 2012, p. 138).

Figure 15.1 shows a model of the process of analysing, which uses a circular structure comparable to the modelling cycle (e.g., Blum and Leiss 2005). Figure 15.1 focuses on analysing classroom situations, but it can easily be transferred if

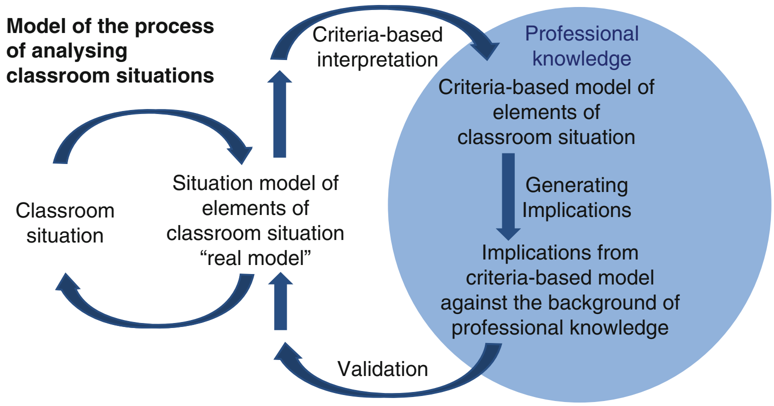


Fig. 15.1 Model of the process of analysing (Kuntze and Friesen 2018, p. 277)

the object of analysis is, e.g. a task, or a piece of content, e.g. when the teacher is preparing a lesson. The process consists of generating (1) a situation model ("real model") of the classroom situation which is then interpreted against specific criteria (2) derived from professional knowledge (see model described in Kuntze 2012). These knowledge-based criteria provide the analysing teacher with models for describing observations—and on this base, the criteria afford drawing conclusions (3). The explanatory power of these conclusions can then be validated against the situation model of the classroom situation (4). As it has been shown to be the case for the modelling cycle (Borromeo Ferri 2006), we assert that jumps between the phases shown in Fig. 15.1 can occur. Parts of the analysis process may take place unconsciously and can be influenced by individual affective dispositions. These unconscious or affective elements may shape the intensity of the knowledge-based analysis process in which repeated cycles are likely to occur.

We assert that (possibly simultaneous) awareness of (possibly different) criteria continuously supports the possible criteria-based interpretation, connection with professional knowledge and validation. To put it into a nutshell, criterion awareness keeps the cycle moving, comparable to a computer stand-by, which fully activates the corresponding explicit knowledge-based analysis cycle in case a criterion appears as useful for describing a relevant situation aspect. This paradigm has shown to be useful for explaining prior findings in which different awareness aspects have been observed to have played competing, antagonist roles for the teachers' analysis (e.g. Kuntze and Dreher 2015).

Teachers' Awareness of Learning Opportunities Connected to Statistical Variation As introduced already above, teachers should be aware of learning opportunities related to statistical variation, given the importance of dealing with statistical variation for developing competence in the area of statistical literacy. For being able to foster students in statistics classroom with focus on statistical variation, teachers have to create learning opportunities which activate students in dealing with statistical variation. Therefore, teachers have to be aware of potential learning opportunities which arise from statistical variation, such as making and discussing predictions or distinguishing relevant from irrelevant variation in data.

We expect such related awareness as strongly dependent on corresponding professional knowledge, including teachers' views (see model described in Kuntze 2012) and convictions. For example, teachers who are convinced that confronting students with statistical variation should be avoided as it could confuse students, might prefer homogeneous data sets in order to avoid statistical variation when conceiving learning opportunities for their students.

In a prior study (Kuntze 2014; Kuntze and Kurz-Milcke 2010), 65 academic-track secondary teachers were asked about their views with relevance for fostering statistical literacy in their classrooms. The findings related to a questionnaire survey on the teachers' views suggested "that there are hardly any obstacles in the examined views of the teachers inhibiting them to design rich learning opportunities for developing and fostering statistical literacy" (Kuntze and Kurz-Milcke 2010, p. 199). However, in addition, the sample was also asked to assess the LP of tasks for the statistics classroom. The results show that the teachers had a preference for tasks with rather unrealistic and uniform data sets in which statistical variation did not play a significant role.

From the awareness perspective introduced above, these findings show an ambiguous picture: On the one hand, the positive views related to statistical literacy stated by the teachers suggest that the teachers' awareness had not been impeded by their views. On the other hand, in their evaluation of the LP of the tasks, the teachers were on average less aware of the LPs related to statistical variation than one might have expected according to the above-mentioned questionnaire data. This ambiguous picture raises the need for follow-up research. In particular, different contexts of teachers' profession-related analysis should afford deeper insight and a corresponding methodological approach promises to be closer to their professional practice. Still, relatively little is known to what extent teachers are aware of potential learning opportunities related to statistical variation, in particular, as far as teachers in primary school are concerned. Moreover, evidence related to potential inhibiting factors for teachers' awareness in this domain is needed. The study presented in this paper addresses these research needs.

15.3 Research Aim

Correspondingly, this study aims at investigating to what extent primary school teachers show an awareness of the LPs related to statistical variation when they analyse given learning material (in particular data sets and learning tasks related to data) and classroom situations. This leads to the following research questions:

1. Do the teachers show an awareness of the LPs related to statistical variation when they analyse data sets, learning tasks, and classroom situations?
2. Is it possible to assign those answers, which do not indicate an awareness of statistical variation, to distinct sub-categories to identify potential obstacles for teachers' analysis?

15.4 Methods and Sample

For investigating teachers' awareness of the LPs related to statistical variation, in-service teachers were asked to answer a paper-and-pencil-questionnaire, which requires analysing three different item formats: data sets (two items), learning tasks (four items), and a classroom situation (one item). In all items of the questionnaire, statistical variation played a central role in the material to be analysed, as it can be seen in the sample data set in Fig. 15.2. The data set shows measurement results of Paul's height from January to June. The values do not increase constantly, as they decrease from February to March and stay constant from May to June. In the second format type of the questionnaire (the learning tasks), a data set is combined with questions. The third format type—a classroom situation which is to analyse by the participants—was presented as a comic. In the classroom situation, statistical variation is connected to a learning context, in which a teacher does not take notice of the statistical variation.

In the questionnaire, the teachers were asked through open questions to analyse the textbook material and classroom situation (e.g. in case of the data sets: "Would you use this data set in the mathematics classroom? If yes: What questions would you ask the students? If no: What would you change?"). Through these open questions, we intended to initiate a process of analysing as is it described by the model in Fig. 15.1. The open questions did not aim at focusing the teachers on statistical variation in any way so that the process of analysing is only driven by teachers' individual criterion awareness, not by externally suggested criteria.

Data set 1:
Paul measures his height on every first Sunday of the month.

Month	***Height***
January	*1m 26cm*
February	*1m 27cm*
March	*1m 26cm*
April	*1m 28cm*
May	*1m 29cm*
June	*1m 29cm*

Fig. 15.2 Example of a data set as part of a test item

Consequently, the participants had to identify statistical variation as a potentially meaningful criterion for students' learning in the context of the items by their own. As laid out in the following, the teachers' answers could thus be used as indicating teachers' awareness of the LPs related to statistical variation.

Data Analysis

To get insight into whether the teachers were aware of statistical variation and related LPs, we subjected the written products of teachers' analysis to a two-step analysis: In a first step, we analysed in a dichotomous top-down rating whether the teachers' answers indicate an awareness of the LPs related to the statistical variation. For assigning an answer to the category *awareness of the learning potentials related to the statistical variation is shown* (Code A-LPSV), the answer had to fulfil two criterions: they had to contain (1) at least one reference to the statistical variation in the corresponding task and (2) a possible LP related to the statistical variation has to be mentioned. For example, when analysing the data set in Fig. 15.2, the decrease or stagnation of Paul's height has to be mentioned to fulfil the first criterion. As a corresponding LP (criterion 2), the statistical variation in this case could be used as a starting point to reflect with students about the origin of the decrease in the data set. When analysing the classroom situation, the participants had to identify that the teacher did not take notice of the LP related to statistical variation.

In a second step, we carried out a bottom-up analysis to identify possible subcategories within those answers which did not fulfil the requirements of answers rated with the Code A-LPSV. Afterwards, we combined the codes of the top-down coding with those found in the bottom-up analysis to a joint coding and subjected all answers of the participants to a second coding by instructed raters. For the instruction of the raters, fictional answers were used which have not been part of the data material to be analysed. The over-all inter-rater-reliability was $k = 0.84$.

Sample

The sample consists of $N = 44$ in-service teachers (95% female, 5% male) from southern Germany who teach mathematics in primary schools. About 32% of the sample had studied mathematics as a major subject, 29% as a minor subject, and about 40% of the sample had not studied mathematics at all during their initial teacher education at university. The amount of the teachers' professional experience ranges from 1 to 36 years with an average experience of 8 years (SD = 7).

15.5 Results

The frequency of answers which were rated in the top-down rating with the code A-LPSV has a range from 0% to 30.8% among the teachers. Figure 15.3 presents the average rates of answers assigned to code A-LPSV according to the three item categories (data sets, learning tasks, and the classroom situation).

In Fig. 15.4, an example of an answer which was rated with the code A-LPSV is given, which refers to the data set shown in Fig. 15.2.

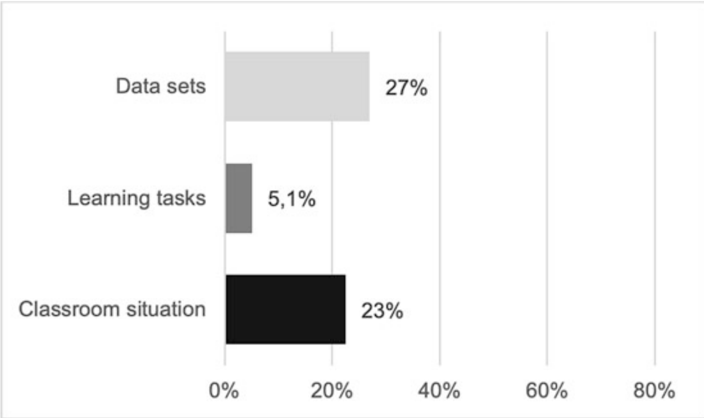
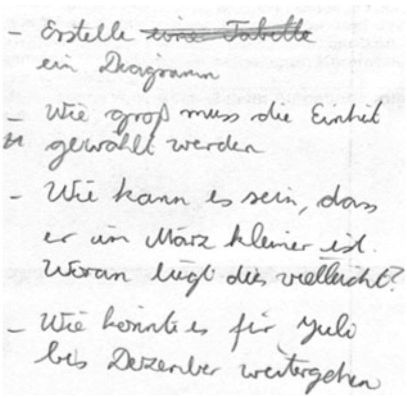


Fig. 15.3 Mean frequencies of answers rated with the code A-LPSV according to the different item formats



Translation:

- Create a diagram
- How big does the unit [for the scale] has to be chosen?
- How is it possible, that he is smaller in March. What might be a reason for that?
- How could it continue for July up to December?

Fig. 15.4 Example of an answer rated with code A-LPSV

Responding to the questionnaire question, the teacher gives questions s/he would ask her or his students for the data set. The first two questions focus on creating a diagram on the basis of the given data set. For a response to these questions it is not necessary for the students to deal with the statistical variation, so these questions alone would not lead to a rating with the A-LPSV code. The subsequent questions, though, appear to focus on the statistical variation given in the data set, so the first requirement of the A-LPSV code is fulfilled (reference to the statistical variation). With the subsequent two interconnected questions "How is it possible, that he is smaller in March? What might be a reason for that?" the focus is directed to the decreasing height values of Paul, and students have to find out about possible explanations for this. The following question "How could it continue from July to

December?” requires the students to make a prediction, which requires not only dealing with the variation given in the data set but also taking into account possible statistical variation beyond the given data, as the use of the term *could* indicates several possible solutions. Therefore, by raising these questions, the teacher uses the statistical variation as a LP, as the questions would encourage students to deal with the statistical variation, which would in this case mean to consider different possibilities concerning the further development of Paul’s height based on the given data. The teacher’s answer, therefore, indicates an awareness of the LPs related to the statistical variation.

In the bottom-up analysis of those answers, which were not rated with the code A-LPSV, we found four distinct sub-categories. Combined with the code A-LPSV, we developed the following joint coding consisting of five categories:

Code 0: No answer (blank response field). Code A-LPSV: The answer has to contain at least one reference to the given statistical variation, and a LP related to the statistical variation has to be mentioned (for example, in form of a corresponding question for students or a description of how to deal better with the statistical variation in classroom).

Code A: No awareness of statistical variation and the related LPs is shown, as there is no mentioning of statistical variation and a related LP in the answer. The answer only focuses on didactical or pedagogical aspects. For instance, some participants mentioned that the teacher in the presented classroom situation would not act according to common didactical principles.

Code B: No reference to the given statistical variation and a LP related to the statistical variation is mentioned. The answer suggests an unspecific opportunity for some form of discussion, but without connecting it explicitly to a LP related to statistical variation. Some teachers, for instance, wrote that they would ask the question “What do you notice?” when using the data set in Fig. 15.2. This question could be a starting point for some form of discussion with students (possibly also with focus on the statistical variation), but there is no clear reference to the given statistical variation and a related LP.

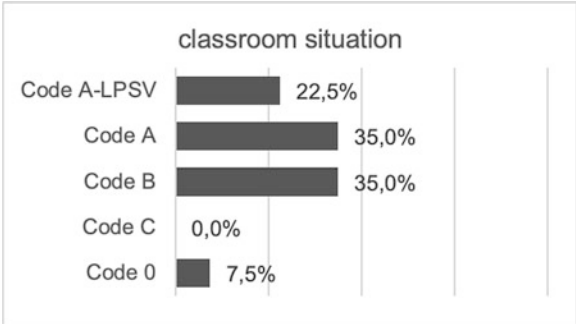
Code C: The answer contains a reference to the given statistical variation, but it is suggested within the answer to remove statistical variation from the task or data set; a LP related to the statistical variation is not mentioned. As an example, some teachers suggested to remove the decrease of Paul’s height in March in the data set in Fig. 15.2.

Figure 15.5 shows the distribution of the mean code frequencies of the items focusing on the data sets (left diagram) and learning tasks (right diagram).

The coding of teachers’ comments to the classroom situation (see Fig. 15.6) shows that around a quarter of the in-service teachers showed an awareness of the LP related to statistical variation.

Fig. 15.5 Mean frequencies of the codes assigned to teachers' answers related to the data sets (left diagram); mean frequencies of codes assigned to the teachers' answers related to the learning tasks (right diagram)

Fig. 15.6 Frequencies of codes assigned to teachers' answers corresponding to the classroom situation



15.6 Discussion and Conclusions

When interpreting the teachers' answers, which we consider as a product of an analysing process, we found that only a minority of the answers contained elements indicating an awareness of LPs related to statistical variation. For all item formats, the rate of answers assigned to the code A-LPSV remains below one third of the sample, so that it is to conclude that the majority of the participants showed low awareness of statistical variation and the related LPs when analysing data sets, learning tasks, and classroom situations. However, we still found answers indicating an awareness of the LPs related to statistical variation (e.g. example in Fig. 15.4).

The rate of answers with code A-LPSV differs between the different item formats: The mean rate of A-LPSV answers to items focusing on learning tasks was only 5.1%, whereas the same codes related to the data sets and classroom situations were more frequent (around 25%). The context in which statistical variation is embedded might thus have an impact on whether or not teachers take statistical variation into account for their analysis. In case of the data sets, for example, focusing on phenomena linked with statistical variation might have been easier for the teachers, compared with the learning tasks, for instance. Task-related views (e.g. Kuntze and Dreher 2015) with emphasis on different criteria might have impeded the analysis based on the awareness of the LP related to statistical variation.

The codes found in the bottom-up analysis indicate potential obstacles for teachers' analysis, which might play an inhibiting role for teachers' awareness of the LP related to statistical variation: Most of the answers, which did not fulfil the requirements of the code A-LPSV, were rated with code A or B, indicating that the answers do not contain any reference to statistical variation and a related LP. These findings suggest that the teachers' awareness of LPs related to statistical variation should be strengthened, e.g. through corresponding professional development activities (see below).

In answers rated with code C, the teachers even suggest to remove the variation from the data contained in the task, so these teachers noticed the statistical variation as a relevant criterion for their analysis but were not aware of related LPs. In contrast, as the teachers suggest to remove the variation, they might expect that the variation would have a negative impact on students' learning. Further reasons could also be associated with affective dispositions, such as low interest for statistical phenomena or a preference for homogenous pattern in data. At this point, further investigations with focus both on cognitive and affective dimensions are needed in order to explore the role of such affective aspects for the competence of analysing learning opportunities related to statistical variation in more detail. Results of such investigation could inform the design of related professional development activities.

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Chapter 16
Beliefs of Teachers Concerning Teaching and Learning with Digital Technology in Upper Secondary Level in Mathematics

Joyce Peters-Dasdemir and Bärbel Barzel

16.1 Theoretical Framework

In this paper, digital technologies will refer to digital mathematics tools (cf. Barzel et al. 2005). For example, Heintz et al. (2014) summarize dynamic geometry software, spreadsheet software, function plotters, computer algebra systems (CAS), as well as multi-representation programs or systems as digital mathematics tools. The term will also refer to graphic calculators, which is still a relevant topic, especially in North Rhine Westphalia. For meaningful integration, teachers have to consider how to include technology to complement their non-technology practice, in order to support learning processes and specific needs of the students (Barzel 2012; Artigue 2013), e.g. by using digital technologies to create cognitive activating learning surroundings. Thus, the potential of digital technology can unfold and an added value for mathematical learning can be generated (Barzel 2012). Benefits of using digital technology have been widely described, for example, problem-solving abilities can be strengthened (Ellington 2006), different representations can be used for the same mathematical situation (Laakmann 2013), and technology-based learning environments can be constructed (Barzel and Möller 2001). Teachers who do not have skills in the operation and concepts for meaningful use in mathematics lessons, express an appropriate training need (Schmid et al. 2017).

PD programs have an influence at various levels (Lipowsky and Rzejak 2012), all of them need to be considered in an analysis of influence. Lipowsky and Rzejak (2012) distinguish between four different levels of efficacy of PD programs: (1) participant's acceptance and reactions, (2) effects on the professional competence of participant, (3) consequences for the practice in classroom, and (4) changes in

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the achievements or the motivation of the students. Schulz (2010) points out the necessity and aim that all four levels should be integrated in research on the effects of PD programs. However, research studies often focus on a single level or even only on subranges, especially on the first and second level. In addition, there are merely a few results for the third and fourth level. Transfer between all levels must therefore be given greater consideration because they are connected to each other.

Therefore, this study focuses on questions concerning the second and third level: Changes in the teachers' teaching and their beliefs: Which ideas from the training do teachers adapt and use in their own mathematical lessons? What changes can be identified in their beliefs? With this focus, the study is complementing other studies, in particular (Thurm et al. 2017), who has examined technology-related beliefs (level 2) and participants' self-assessment of the type and frequency of digital use (level 3).

In this paper, however, we will concentrate on the second research question and consider the actual state of the participants' beliefs, since—together with their knowledge—teachers' beliefs about digital technology and mathematics are relevant to understand their teaching activities (e.g. Baumert and Kunter 2006; Reusser et al. 2011).

Beliefs About Mathematics and Technology

According to Baumert and Kunter (2006), the values and beliefs of teachers play an important role alongside knowledge in terms of their professional competence to act. Further differentiation is made within the beliefs (cf. Baumert and Kunter 2006). In this paper the epistemological beliefs (cf. Grigutsch et al. 1998) and specially the beliefs on subjective theories about teaching and learning (Peterson et al. 1989; Staub and Stern 2002) are set more into focus. It is assumed that the beliefs of teachers about mathematics and the teaching and learning of mathematics has a high impact on their instructional practice (cf. Philipp 2007).

Beliefs are assigned an orienting and action-guiding function for the application of knowledge (cf. Leder et al. 2002). They are, thus, of outstanding importance for the teaching activities of teachers, since they can be seen as a bridge between knowledge and action. There are already numerous studies on epistemological beliefs, which can be divided into the *structure of mathematics* (dynamic vs. static aspects) and for the *acquisition of mathematical knowledge* (transmission vs. construction orientation).

From a content-related perspective, epistemological beliefs on the structure of mathematics are among those that have been studied more frequently (Grigutsch et al. 1998; Schoenfeld 1998). The following aspects are distinguished: the formalism aspect, which considers mathematics as an abstract system of axioms and relations, the schema aspect, treating the mathematical knowledge as a collection of rules, facts, and procedures (embody *static aspects*), and the application aspect, which focuses on mathematics as a tool for solving everyday problems (*dynamic aspects*).

In addition to epistemological beliefs on the structure of mathematics, beliefs on the acquisition of mathematical knowledge or on the teaching and learning of mathematics represent another important dimension (cf. Hofer and Pintrich

2002). Overall, empirical findings show that beliefs in teaching and learning can be distinguished in two fundamental perspectives (see Fennema et al. 1990; Staub and Stern 2002; Voss et al. 2011). Transmission orientation means learning content in the sense of a directed mediation process by the teacher. In contrast, the constructivist perspective is characterized by a more student-oriented understanding of teaching and learning. Here, the learning process is seen as a self-controlled active construction process. Kaput had already stated in 1992:

> Major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures (Kaput 1992, p. 515).

This statement is already over 20 years old and Kaput makes it sound somewhat disrespectful. But the core statement remains that beliefs will be a central influencing factor in the area of professional competence.

This paper, thus, has the same understanding of technology-related beliefs as the contribution of Klinger et al. (2018). Accordingly, technology-related beliefs are beliefs, which refer to the use of technology as an object of beliefs (cf. Klinger et al. 2018; Goldin et al. 2009). In the area of technology-related beliefs, there have been numerous qualitative studies and case studies as yet (e.g. Doerr and Zangor 2000; Pierce et al. 2009; Drijvers et al. 2010). On the other hand, there is a lack of quantitative studies and of proven measurement tools to capture technology-related beliefs. The study by Klinger et al. (2018) quantitatively measures five dimensions of beliefs for teachers that reflect the technology-related beliefs. For example, the dimension *principle of shifting* which means that technology is used to execute certain procedures in the mathematics classroom, so that they are not done by hand anymore. Or the dimension that technology supports the connection of multiple representations (*support of multiple representations*). In addition to positive beliefs, there are also negative ones such as time consuming. An overview of the items of technology-related beliefs can be found in the MAVI23 paper (Klinger et al. 2018).

There is also a link between technology-related beliefs and frequency of technology use (cf. Thurm 2018). In this quantitative study with 160 secondary school teachers from Germany, the relationship between technology-related beliefs and the benefits in teaching practice was examined. An analysis of the latent profile shows four subgroups of teachers in terms of the relationship between beliefs and practice, with the number of groups being determined by Likelihood tests. Group 1 '*positive beliefs—frequent users*' and group 3 '*negative beliefs—rare users*' are consistent in their beliefs about technology. The first contains more positive ideas about technology integration and therefore often uses technology. In contrast, the two groups 2 and 4, referred to as '*positive beliefs—rare users*' and '*negative beliefs—frequent users*', seem to contradict their beliefs and the question remains what causes this apparent inconsistency. This discrepancy will be illustrated later in our results.

So, in addition to the beliefs about mathematics, the beliefs about technology are also relevant.

16.2 Research Questions

Based on these considerations, the following research questions regarding this paper are considered for the presented excerpt from the PD program study:

Q1: What beliefs do teachers have about using digital technology?
Q2: What are the connections between beliefs about the use of digital technology and the teaching–learning orientation or the mathematical worldviews?
Q3: Do teachers assess the frequency of digital technology in their classroom lessons as high or low?

16.3 Research Design and Methodology

The aim of the study is to examine the influence of a PD program with the DZLM PD module Teaching and Learning with Digital Technology in upper secondary level in mathematics on the specific teaching situation and thus also on the beliefs. It evaluates how teachers are supported with regard to curricular requirements. The advanced PD program was developed taking into account the DZLM design principles (Barzel and Selter 2015; Barzel and Biehler 2017) and is available as disseminatable material (www.dzlm.de). The central aim is to get to know tasks with digital technology on the basis of examples for teaching and examinations and to reflect on them on the basis of the added value of the subject didactics (for more detail see Fig. 16.1). The modules are targeted at teachers at upper secondary level (grade 11 till 13) for whom teaching and learning with digital technologies is new and at those who have already gained experience in this field.

As a main study a PD program at the University of Duisburg-Essen with 20 teachers was investigated. Methodologically, PD program and selected classroom lessons are observed and videotaped, classroom features are collected on observational sheets and guided interviews are conducted (Fig. 16.1). Through this, it is aimed to explain the influence of the PD program considering the teaching of lessons in more detail. For deeper insight into teachers' views and changes in their teaching, it

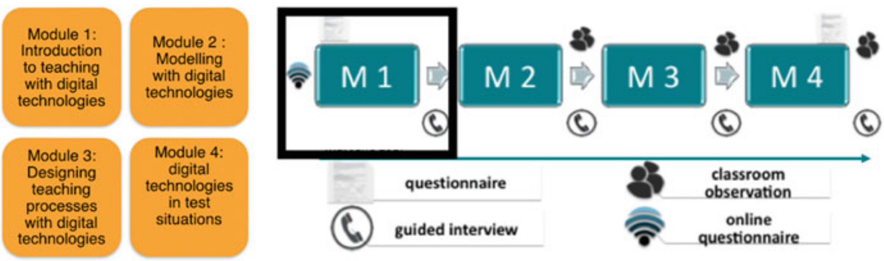

Fig. 16.1 Learning content of modules and research design

is important to investigate their beliefs about the use of digital technology. These are recorded by means of mixed-method design, quantitatively through questionnaires, and qualitatively by evaluation of statements during the PD program. They serve as the basis for the subsequent lesson observations. So far in this paper, the focus has been on the evaluation of the first phase of the PD program (see Fig. 16.1). The first phase (black box in Fig. 16.1) is useful to answer the research questions mentioned above, as we first record general data, values, and beliefs with the help of questionnaires and, through the first sequence in the first module, we can capture the more precise backgrounds of the values and beliefs.

In the online *questionnaire* before the first PD module, the basic data (professional years, textbook, school equipment, etc.) as well as previous knowledge of PD programs already attended or by collegial exchange are collected. In addition to the online questionnaire, the *first guided interview* examines the school situation in more detail, and, after that, the acceptance of the participating teachers is recorded in the first module. There is also another *questionnaire* to investigate beliefs about the use of digital technology and the nature of mathematics (quantitatively). This is based on items by Klinger et al. (2018) and the TEDS-M study 2008.

Thurm (2018) has identified four teacher profiles that were presented in the section above, which differ according to the positive or negative assessment of beliefs and frequency of use of digital technology. For this reason, an *introduction method for self-assessment* was integrated in the first module: the two criteria represented as coordinate axes in the room along which the participants were to position themselves (Fig. 16.2 is a replica of the position of the participants in the room). As a result, in addition to the location of the participating teachers, their reasons and causes can also be determined by a *plenary discussion* (qualitatively).

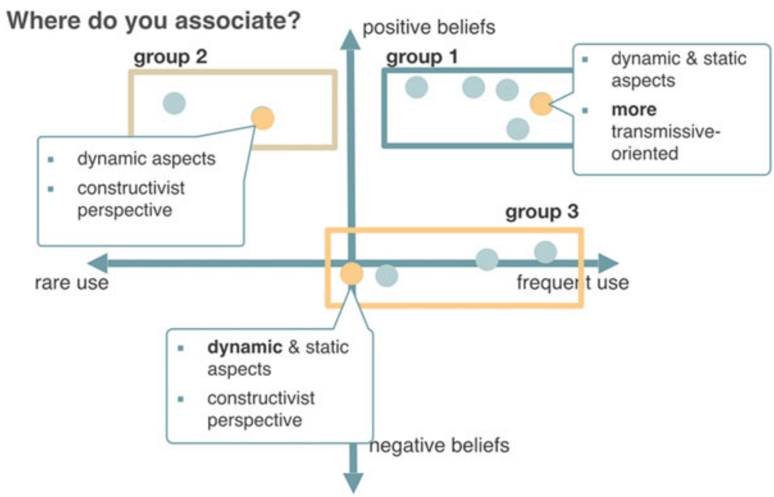

Fig. 16.2 Position of the participating teachers during the PD module 1

16.4 Results

The results could only be evaluated for eleven participants. This is due to the fact that one part refused to participate in the interviews, did not fill in any questionnaires or aborted participation after the first module. The acceptance was given (level (1) of efficacy of PD programs), but there were still reasons like 'too little time for further training' or 'too many other things like exams' at the moment. Therefore, nine of the initially 20 participants had to be excluded from the evaluation. The guided interviews showed that the eleven teachers, aged 27–62, express acceptance of the PD program (level (1) of Lipowsky and Rzejak 2012).

Overall, three out of the four profiles could be identified (Fig. 16.2). Therefore, compared to the quantitative study of Thurm (2018), no participants of the group 'negative beliefs—rare users' participated in our PD program. The three yellow dots marked in yellow show examples of the evaluation results of their questionnaires.

Teachers of group 1 (*positive beliefs—frequent use*) mention in the plenary discussion substantial advantages of the assignment, such as stronger reflection phases with learners and a deeper understanding through a change of multiple representations. Digital technologies are called 'discoverers' (support of discovery learning). Taking the questionnaire into account, it becomes apparent that teachers are more transmissive-oriented, even if a relationship between mathematics and creative problem-solving is identified by digital dynamic visualizations.

Teachers of group 2 (*positive beliefs—rare use*) support the commitment, but do not know how this is done sensibly. They have little experience with digital technology and do not feel supported at school. Uncertainty in the operation is mentioned as the reason of the rare use. Both participating teachers of group 2 show a constructivist perspective in the questionnaire results and see mathematics as a tool for solving everyday problems.

Teachers of group 3 (*negative beliefs—frequent use*) seem to have neutral beliefs. But they mention more or less just disadvantages of using digital technology, such as time consuming and the risk of cognitive decline. Digital technology is mainly used as 'calculating servants', with teachers recognizing no added value in mathematics learning (e.g. one teacher of group 3: 'Of course it does not help if you calculate 300.000 stupid matrices. But I noticed again that they do not see that. They just want the result'). As in Group 2, the constructivist perspective becomes clear. But they do not see the use of digital technology for support. For them, the use is rather contradictory to their perspective.

The videotaped and evaluated statements during the first PD module correspond to the items on the dimensions of the base model (cf. Klinger et al. 2018).

16.5 Discussion and Outlook

In the study of Thurm (2018) the two groups, referred to as 'positive beliefs—rare users' and 'negative beliefs—frequent users', seem to contradict their beliefs and the question remains what causes this apparent inconsistency. By positioning the participants in the room during the PD module one, we could assume which of the groups they should be assigned to with regard to the profiles of the questionnaire. In addition, we were also able to find reasons through the open plenary discussion. Even though this is only a small group, assumptions raised by the quantitative study can be confirmed for our individual participants.

For group 1, beliefs and usage notices are not considered. In general, it can be assumed that these participants internalize the concepts and suggestions for reflection of the PD program in the case of the different activities in module one till four (see Fig. 16.1) and transfer these to other topics. It remains interesting to see whether the stronger transmission orientation prevents using digital technology for discovering learning and whether the view of the learning process can be steered as a self-directed active construction process.

Group 2 needs support regarding the PD concepts. Factors such as self-efficacy beliefs can influence technology-based beliefs and prevent the use of digital technology. Self-efficacy beliefs describe here the beliefs about being able to cope with a task or situation in a targeted manner (cf. Bandura 1997). Some studies have already shown that this can be strengthened by PD programs (e.g. Bennison and Goos 2010). The more pronounced constructivist perspective may be due to the temporal proximity to the university.

Group 3 shows an apparent contradiction between beliefs and usage. Due to their statements during the discussion phase in the first module, it is obvious that they use them only because of the curricular requirements. Teachers cannot understand why the digital technology adds value to the classroom lessons. With regard to their constructivist perspective, they do not see sufficient potential for support through the use of digital technology.

In the last two groups, this apparent inconsistency of use, which contradicts their beliefs, can thus be explained. In Germany, teachers, who want to participate in a voluntary PD program, have to approved and released it by the school management. Therefore, it is not surprising that none of the participants has assigned themselves to the fourth profile due to their voluntary participation, since their negative beliefs do not interest them in the use of digital technologies either. But it could also be that they were influenced by doing it during a PD program about technologies and in front of other teachers and researchers.

Regarding research question one, several positive beliefs about digital technology can be found. In group 1 and 2, positive aspects such as support of discovery learning, encouragement to reflect more strongly on mathematical results, and reference to everyday contexts can be mentioned. Through the mixed-method design (plenary discussion and questionnaire), the belief principle of shifting can be determined qualitatively as well as quantitatively such as negative beliefs like danger to thinking and understanding.

In summary, we can answer questions 2 and 3 as follows: The participants use digital technologies differently often in certain teaching phases. This is how they expressed themself in the plenary discussion and gave examples as described above. This can be related to their general form of teaching, but also to their operating competence. Nevertheless, they are able to classify themselves generally and distinguish digital technologies from other forms of teaching. In the discussion during the PD program they could differentiate themselves from each other. As this was a voluntary PD program, the larger number of people belonging to group 1 and 2 is not unexpected. Group 1 is looking for further ideas and concepts for use, group 2 needs even more intensive support, especially in connection with their constructivist perspective, in order to use technology sensibly. Since group 3 cannot reconcile the frequency of use with their own beliefs, they seek justification for themselves or for other concepts, since they often use digital technologies, but not in accordance with their perspective on teaching. It will be interesting to see whether the added value can be conveyed through the PD program and result in a more positive attitude towards technology. However, this can only be achieved through complementary practical experience.

Beliefs of mathematics and the use of digital technology are an important basis as a bridge between professional knowledge and practice. Therefore, in the first step, the beliefs were recorded. In the further course of study, the changes in the implicit action are focused. Through classroom observations and evaluations of lesson plans, teaching activities shall be recorded and possible changes identified. It is of interest to see which changes arise in beliefs as a result of a PD program and the newly gained practical experience.

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Part IV
Transition(s)

Chapter 17
Introduction

Chiara Andrà

The International Conference *Mathematical Views* has a long and rich research tradition on transitions, focusing especially on the secondary-tertiary one and on the one from undergraduate studies to teaching (for mathematics teachers). The former line of research overlaps with research, within and outside Mathematics Education, on post-secondary teaching and learning, while the latter overlaps with research on mathematics teacher training. In this part of the book, contributes on primary-to-secondary and on secondary-to-university transitions can be found.

When they start the first year at university, undergraduate students in mathematics, engineering, and sciences all around the world face several difficulties with mathematics, as it has arisen in UK, Canada, Australia, and Ireland (Rylands and Coady 2009), Spain (Gòmez-Chacòn et al. 2012), Germany (Griese et al. 2012), and Italy (Andrà et al. 2011, 2012). The problem is interpreted by Rylands and Coady as a consequence of the undergraduate students' increasingly diverse backgrounds. As a matter of fact, indeed, the variety of high schools, from which the students enrolling in mathematics come from, is broad and a multifaceted situation is depicted. As observed by Bozzi et al. (this book, Chap. 21) students seem to face same problems with physics: it is of no doubt that to investigate students' difficulties with the secondary-tertiary transition needs to take into account cognitive aspects, as well as didactical ones, and cannot be reduced to a pure affective phenomenon, as it conversely happens in other areas of affect-related research. This represents an opportunity, for the researcher, to investigate the interplay of cognitive and affective variables (see, e.g. Gòmez-Chacòn et al. 2012), specifically considering affect as a predictor of cognitive performance and behaviour.

Gueudet's (2008) review of studies on the secondary-tertiary transition suggests to distinguish two streams of research: (a) observation and analysis of students'

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difficulties; (b) discussion of teaching interventions aimed at supporting students' difficulties and preventing dropout. As regards the first stream, Gueudet notes that researchers may adopt different perspectives, focus on different aspects of the transition issue and, consequently, draw different conclusions, in terms of didactical actions. Researchers may focus: on the different thinking modes that are required at university, as evidenced by all the studies on Advanced Mathematical Thinking (Tall 1991); on the different organisation of knowledge and on the intrinsic complexity of the new contents to be learnt (see for instance Robert 1998); on the different processes and activities that are at issue, proof for one (Moore 1994); on the different didactical contract (Bosch et al. 2004) and, more generally, on institutional issues, such as university courses organisation (Hoyles et al. 2001). A common feature of these studies is the focus on the differences between secondary school and university: in terms of content, organisation, teaching methods, and so on. Generally, difficulty in the transition is read in terms of a difficulty for students to adapt to the new context. Clark and Lovric (2008) propose to look at the transition as a modern-day rite of passage, which encompasses a sort of shock. Along the second stream of research, Clark and Lovric (2008) suggest that transition should be smooth, and communication between the two institutions (school and university) should be improved.

We note that the majority of the aforementioned studies focus mainly on the cognitive aspects in relation to students' difficulties. This is not surprising, since from a cognitive point of view lack of students' knowledge or weak mathematical background cause troubles to students. This is also the reason why many universities organise bridge mathematics courses for freshmen, in order to recapitulate the main mathematical topics that can cause difficulties to students (see Girnat, this book; Zani, this book; Andrà et al. 2020). However, affect-related research has revealed that beliefs and other affective factors, such as motivation, self-efficacy, anxiety, have a key role in determining difficulties and reaction to them. The crucial role of affect in mathematics learning is evidenced by a large amount of studies that focus on the situation of undergraduate students. For instance, Furinghetti and Morselli (2009) discuss the intertwining of affective and cognitive factors in the proving processes of university students in mathematics. The authors underline that also fourth-year students in mathematics, who, in principle, should have a good relationship with the discipline, may have beliefs about themselves and about mathematics that can hinder the proving process and affect the process of causal attribution in front of difficulties. Causal attributions in general are processes that people activate when looking for explanation of either their own behaviour or the one of other persons, by inferring causes beside specific actions and feelings. Attributions and perceptions of success/failure are categorised along three main dimensions: locus (internal vs. external), stability (stable vs. unstable), and controllability (controllable vs. uncontrollable) of the causal agent. From a methodological point of view, causal attribution is observed when students are interviewed about their coping with first day at university, or more generally in the new context.

Interestingly, Daskalogianni and Simpson (2001) discuss the concept of "beliefs overhang": some beliefs, developed during schooldays, are carried forward in university, and this fact may cause difficulties. The study points out the crucial role of beliefs (about mathematics) in determining university success or failure. Beliefs about mathematics, about oneself, and about university may affect the way university courses are lived. Mathematical self-confidence and other beliefs may affect the performance during the university examinations.

There is also a long tradition of research about motivation. From a psychological perspective, Geisler (2018), for example, investigates the psychological resources of the students at the beginning of university studies as predictor of dropout. Research shows that a major aspect influencing persistence at university is learning motivation. In particular, that students who dropped out show less motivation during their studies than students who go on with their studies. Moreover, learning can be influenced by intrinsic and extrinsic motivation. If intrinsic motivation is combined with having fun and enjoy experiences at university, likelihood of success is higher if compared with high intrinsic motivation alone (Geisler 2018).

Griese (2018) addresses another central issue in transition research, namely the gender gap. She observes that mathematics is traditionally regarded as a male domain, although there have been numerous attempts to challenge this stereotype, many of them successful. Looking at OECD results, Griese (2018) notes that males often outperform females in mathematics and other subjects from the STEM range. Griese's (2018) results seem to confirm that females fulfil the *cliché* of being more diligent (more highlighting, more making lists, more summarising, more memorising) and more socially minded, whereas males tend to elaborate on concepts and to get to the bottom of things. This is just an example of recent research developed in affect-related domain about a possible relation between gender gap and transitions.

In this book, Girnat's chapter (Chap. 18) focuses on first-year university students who attend a bridge course in mathematics in Germany and investigates the role of (rated) self-efficacy and (declared) anxiety as a predictor of success. Addressing the gender issue, an interesting finding is that female students show the same performance of males at math tests, but they perceive themselves as less good than males. Addressing the dropout issue, another interesting finding is that those who later abandon their studies obtain lower scores at entrance test, but they perceive themselves as good as those who continue their studies until the degree. As regards beliefs, finally, Girnat concludes that also for first-year students beliefs are stable and hard to change, but self-concept increases during the bridge course.

Caponera et al.'s chapter (Chap. 19 in this book) deals with a rather under-researched kind of transition, namely the one from primary to secondary studies that focuses not only on students' difficulties. One of the reasons for being less researched may reside in the fact that transitions of this sort take place at different ages in different countries and, thus, they are less homogeneous if compared to the transition from secondary education to university. As stressed before, also in earlier transitions, often students' difficulties linked to cognitive or didactical factors are the main aspects investigated in vertical studies.

The context of Caponera and colleagues' research is an Italian national assessment test for mathematics and the mutual relationship among achievement, self-concept, and identity is investigated. A result of the research is that correlations among these three dimensions increase over time, as if experiences at school support this process.

Four commentaries follow the aforementioned chapters by Girnat and Caponera, Pozio, and Palmerio, and they serve the purpose of offering four different perspectives on possible relations between affect-related research and other fields. Two commentaries have been written by researchers in Mathematics Education and in Physics Education, respectively, and offer the perspective of those who design and develop cognitive assessment tests for freshmen at STEM studies. A third commentary is contributed by a secondary math teacher who has also a long experience of lecturing mathematics courses at first-year STEM studies. We have asked her to tell us what a teacher can get from these researches in affect-related mathematics education. The third commentary focuses on methodological issues, and in particular on anonymous versus non-anonymous investigations.

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Chapter 18
Inventing Scales for a Multidimensional Model of Mathematics Self-efficacy to Analyse First-Year Students' Mathematical Self-assessment, Performance, and Beliefs Change

Boris Girnat

18.1 Introduction

Beliefs have become an important field of research in mathematics education (Philipp 2007). They are research objects of their own, but they are also used as background variables to explain psychological or behavioural aspects of the teaching and learning of mathematics—especially with regard to mathematics performance. One important question is the relationship between performance and self-assessment. Nearly every international large-scale study is combined with some scales to measure students' mathematical self-assessment mostly to predict and to explain their performance (OECD 2005; OECD 2013). Research has shown that the relationship among self-concept, self-efficacy, and mathematics performance is strong and that mathematics self-concept and self-efficacy are in general powerful predictors to students' mathematics performance (Multon et al. 1991). The aim of the study presented here is to extend the application of scales related to mathematics self-concept and self-efficacy from lower secondary schools to upper secondary schools and universities. New scales have been developed to cover the mathematical content and skills of the transition phase from school to university. This paper presents the results of a first application of these scales in the context of a two-week bridging course that was designed to repeat secondary school mathematics. All participants of this course had to take part in a mathematics pre-test and a post-test and both tests were combined with a context questionnaire including the new scales. The research questions are as follows: (1) Do the new scales possess satisfactory psychometric properties? (2) Can these scales reveal group differences concerning relevant co-variates? (3) How are these scales related to each other

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and to other attributes of the students (e.g. performance, mathematics self-concept, and mathematics anxiety)? (4) Are these scales good predictors for the students' performance? (5) Can the scales be used to detect belief changes between the pre-test and the post-test and—if so—how are these changes related to the change of the students' general mathematics self-concept and mathematics performance?

18.2 Theoretical Background

There are two different ways to conceptualise students' *mathematical self-assessment*. The first one is related to a person's *mathematics self-concept*, measured by general statements like "I have always believed that mathematics is one of my best subjects" (Marsh 1990). The second approach is called *mathematics self-efficacy*. It was introduced by Bandura's idea of measuring a person's self-assessment by his level of confidence about feeling able to solve specific problems that are relevant to the domain of interest (Bandura 1977, 1986). Bandura defined self-efficacy beliefs as "people's judgments of their capabilities to organise and execute courses of action required to attain designated types of performances" (Bandura 1986, p. 391). Related to this idea, Shavelson et al. (1976) invented a hierarchical model starting with the "general self-concept" on the most abstract level, going down on different steps like the "academic self-concept" and the "mathematical academic self-concept", and finally arriving at the "evaluation of behaviour in specific situations" that is very close to Bandura's concept of self-efficacy. Two theoretical claims of this hierarchical model are most relevant to our study: (1) the higher and more abstract the level, the more stable the associated concept and (2) the lower and less abstract the level is, the more the associated concept can be represented by several different dimensions (Shavelson et al. 1976, pp. 412–414). The second claim was the reason to use a multidimensional model consisting of five scales related to different aspects of mathematics instead of a one-dimensional scale. The first claim culminated in the fifth research question mentioned above, namely whether and how changes in mathematics self-efficacy are related to changes in mathematics self-concept and performance—in particular, whether a different degree of stability is recognisable concerning these three aspects. An examination of the students' mathematics anxiety was added to the questionnaire as the "emotional counterpart" of the mathematics self-concept: according to Hannula's distinction between mathematics affects as states and traits, mathematics anxiety as a trait is stable and typically highly negatively correlated to mathematics performance and self-concept (Hannula et al. 2019).

18.3 Self-efficacy, Self-concepts, and Anxiety

The new scales are based on the PISA self-efficacy scale used in 2003 and 2012 (OECD 2005, pp. 291–294, OECD 2014, pp. 322–323). The PISA scale is short and closely related to tasks typically used at lower secondary schools. But there were three obstacles to adopt this scale to university courses without changes: (1) the PISA scale was designed to measure mathematics self-efficacy only on lower secondary school level; (2) an exploratory factor analysis of this scale indicated that it cannot be considered as one-dimensional, but as multidimensional with respect to mathematical subdomains: algebra, elementary geometry, and applied mathematics/word problems (Girnat 2018); (3) Shavelson et al.'s model mentioned above is based on different subdomains on the more basic and less abstract levels. Insofar, the idea was to invent a bundle of self-efficacy scales that are (a) as short as possible, (b) related additionally to upper secondary school mathematics, and (c) multidimensional with respect to its different subdomains. To match the demands, it was necessary to define subdomains of upper secondary mathematics in a similar manner the re-analysis of the PISA scale suggested (Girnat 2018). To do so, some subdomains could remain, some had to be changed: algebra and applied mathematics are also relevant to upper secondary schools. These subdomains could remain and even some of the items of the PISA scale could be reused (indicated by "PISA"). Elementary geometry was replaced by analytic geometry; and calculus and probability theory were introduced as new subdomains. Since every subdomain should be represented by a scale that could stand for its own, four items per subdomain ought to be regarded as the minimum. Hence, the solution contains 20 items organised in five scales to represent a multidimensional model of mathematics self-efficacy (Table 18.1). The items were introduced with the following question: "How confident do you feel about having to do the following mathematics tasks (using a simple, non-graphing calculator)?"

The items on mathematics anxiety were directly reused from the PISA study in 2012 (OECD 2014, p. 323). The mathematics self-concept has to be adapted from the PISA scale (OECD 2014, p. 323), since some items with a direct reference to school contexts had to be removed or reformulated to be applicable at university level.

18.4 A Performance Test for a Bridging Course

The primary focus of this paper is the scales described above. Since they were used in the context of a bridging course that was evaluated by a mathematics performance test with a pre-/post-test design, it is also necessary to describe the bridging course and the performance tests. The teaching and learning of mathematics at university level have become a prominent part of mathematics education in recent years—especially the discrepancy between the requests of university mathematics and the skills students have achieved at school (Di Martino and Gregorio 2018). There are several proposals how to deal with this challenge. One of them is a bridging course to enable first-year students to university mathematics. In 2013, the University of Hildesheim decided to establish a voluntary bridging course for all courses of study including a substantial amount of mathematics (Hamann

Table 18.1 The items of the five mathematics self-efficacy scales

Scale	Items
(a) Self-efficacy applied mathematics (self.app)	1. Calculating how much cheaper a TV would be after a 30% discount. (PISA) 2. Calculating the petrol consumption rate of a car. (PISA) 3. Calculating how much interest is given on a savings plan within 10 years. 4. Calculating how long it takes to completely fill a swimming pool.
(b) Self-efficacy algebra (self.alg)	1. Solving an equation like $3x+5=17$. (PISA) 2. Solving an equation like $2(x+3)=(x+3)(x-3)$. (PISA) 3. Multiply and simplify an algebraic expression like $2a(5a-3b)^2$ 4. Solving an equation like $6x^2+5=29$
(c) Self-efficacy calculus (self.calc)	1. Determining the derivative of a function such as $f(x)=x \cdot e^x$. 2. Determining the maxima and minima of a function such as $f(x)=x^3-2x^2+1$. 3. Specifying the primitive of a function such as $f(x) = \frac{1}{2} \sin(x)$. 4. Calculating a definitive integral such as $\int_1^2 (x^2 - 2x)dx$.
(d) Self-efficacy analytical geometry (self.ageo)	1. Calculating the length of a vector such as $v=(4, -2, 3)$. 2. Calculating the scalar product of two vectors such as $v=(-2, 1, 3)$ and $w=(2, -3, 2)$. 3. Calculating the intersection of a straight line such as $g: x=(1, -2, 0)+t \cdot (-1, 0, 1)$ with a plane. 4. Calculating the distance of a point such as $P(3\|-1\|4)$ from a plane (for example, $E: x-3y+2z=3$).
(e) Self-efficacy probability theory (self.prob)	1. Calculating the probability of throwing a 6 twice in a row. 2. Calculating the probability of winning the jackpot in the lottery. 3. Calculating how likely it is to draw two sweets of the same colour from a sweet jar. 4. Calculating how likely it is that two students in a class have their birthday on the same day.

et al. 2014): computer science, business informatics, and elementary and secondary education with mathematics as a major. Although there are different views on which mathematical skills first-year students should possess, it is common sense that the skills students should have learnt at secondary schools are regarded as crucial for being successful at university (Nicholas et al. 2015). The department of mathematics decided to restrict the content to the following topics, since the bridging course had to be limited to 2 weeks and a preceding study had shown that the students demanded a special need of algebra, arithmetic, functions, graphs, and calculus and their applications in real-world situations (Kreuzkam 2013). These five topics have become the core area of the bridging course and the items of the test had to match these circumstances. The bridging course took place in October 2018. 312 students attended the course; 271 of them took part in the pre-test, 224 in the post-test, and 194 in both tests. The participants were distributed as shown in Table 18.2.

Table 18.2 Properties and sub-groups of the sample

Gender	Study course	Mathematics at school	Course left before end
Female: 136	Primary education: 138	Basic level: 152	Not left earlier: 224
Male: 168	Secondary education: 73	Advanced level: 125	Left earlier: 88
	Computer science: 48		
	Business informatics: 41		
Others/no answer: 8	Others: 12	No answer: 35	

Table 18.3 Reliability and fit indices of the scales in the pre-test

Scale	Cronbach's alpha	RMSEA	CFI	SRMR
Self.app	0.75	0.029	0.998	0.031
Self.alg	0.76	0.000	1.000	0.024
Self.calc	0.78	0.043	0.997	0.034
Self.ageo	0.80	0.046	0.997	0.038
Self.prob	0.83	0.000	1.000	0.024
Matcon	0.74	0.000	1.000	0.007
Matanx	0.75	0.000	1.000	0.012

The labels "basic level" and "advanced level" refer to a characteristic of the German school system: students have to choose in grade 10 if they want to be taught in mathematics on a basic or on an advanced level during grade 11 and 12.

18.5 Psychometric Properties of the Scales and the Test

The psychometric properties of the scales and the performance test are reported now. The reliabilities of the scales were estimated by Cronbach's alpha (Cronbach 1951). Since we will later use confirmatory factor analyses (CFA) and structural equation modelling (SEM), the typical fit indices used within this paradigm are reported additionally (Beaujean 2014, pp. 153–166). All calculations were done using R (R Core Team 2018) and the R package "lavaan" (Rosseel 2012) with a DWLS estimator (Beaujean 2014, pp. 92–113).

Table 18.3 contains the values of the pre-test (the values of the post-test are quite similar and are omitted to save space). According to the usual criteria, all scales have good properties. For the performance test, a unidimensional Rasch model was used to obtain a scale that expresses the "overall" mathematics performance of the participants (Linden 2016). The analysis was done using the R package "TAM" (Robitzsch et al. 2018). The Rasch model had an excellent EAP reliability (0.904).

18.6 Results

The results are presented in four steps: (1) correlations between all scales; (2) group differences with respect to background variables mentioned in Table 18.2; (3) Table 18.3 shows the changes between the pre-test and the post-test; (4) linear models to explain students' mathematics performance and beliefs. The asterisks stand for the usual significance levels: * for $p < 0.05$, ** for $p < 0.01$, and *** for $p < 0.001$.

Correlations
Table 18.4 shows the latent correlations (Beaujean 2014, pp. 100–103) between the performance test, the scales of the context questionnaire, and the final school exam mark with respect to the data of the pre-test—estimated using a structural equation model with good fit indices (RMSEA 0.013, CFI 1.000, SRMR 0.052). The correlations are mostly as expected: there are substantial correlations between the five scales of the multidimensional self-efficacy model, but they are not that high that they could not be empirically differentiated. This is different in case of the mathematics self-concept (matcon) and mathematics anxiety (matanx). This finding supports the hypothesis that both scales are (positive and negative) indicators of the same underlying concept (Hannula et al. 2019, mentioned above).

Group Differences
The focus of the analysis is now set to mean differences related to different sub-groups of the sample. Here, we omit the co-variate "choice of the study course", since there are no significant differences observable. This is very surprising, since the most relevant study to this topic (Betz and Hackett 1983) indicates that mathematics self-efficacy is a good predictor for the students' choices of their study courses. The reason may be the fact that the possibilities to choose a study course is rather limited at the University of Hildesheim and, therefore, remarkable differences cannot occur. To make the differences on different scales comparable, we report the differences in terms of Cohen's d (Cohen 1988), i.e. the mean of one group (the "reference group") is set to zero and the mean of the other group is given as the difference to zero on a standardised metric. Cohen's d is usually interpreted as follows (Cohen 1988): $d = 0.2$ indicates a small effect, $d = 0.5$ a medium effect, and $d = 0.8$ a strong effect (Table 18.5).

There is no significant gender difference concerning the performance test ($d = -0.120$), but the situation is quite diverse with respect to the different self-efficacy scales. There is a small to medium difference ($d = 0.339$*) in favour to the female group in case of analytic geometry; and there is nearly a large ($d = -0.678$***) difference to the detriment of the female group in case of applied mathematics. Both cases are remarkable aberration in perception compared to the measured (insignificant) performance difference.

Table 18.4 Latent correlation between all scales (pre-test)

	Mark	Seff.app	Seff.alg	Seff.calc	Seff.ageo	Seff.prob	Matcon	Matanx
Pre-test	0.387***	0.192*	0.552***	0.496***	0.397***	0.243**	0.328***	–0.286***
Mark		0.103	0.297***	0.232**	0.018	0.114	0.397***	–0.345***
Seff.app			0.370**	0.263**	0.048	0.542***	0.429***	–0.418***
Seff.alg				0.618***	0.347**	0.318**	0.453***	–0.420***
Seff.calc					0.725***	0.329***	0.504***	–0.429***
Seff.ageo						0.290**	0.333***	–0.230***
Seff.prob							0.409***	–0.354***
Matcon								–0.833***

Table 18.5 Mean differences between sub-groups of the sample (pre-test)

Variable	Gender (ref. gr.: male)	Level in mathematics (ref. gr.: basic level)	Course left before end (ref. gr.: not left)
Pre-test	$d = -0.120$	$d = 1.041^{***}$	$d = -0.661^{***}$
Seff.app	$d = -0.678^{***}$	$d = 0.087$	$d = 0.168$
Seff.alg	$d = 0.106$	$d = 0.953^{***}$	$d = -0.319^{*}$
Seff.calc	$d = 0.134$	$d = 1.020^{***}$	$d = -0.180$
Seff.ageo	$d = 0.339^{*}$	$d = 0.763^{***}$	$d = -0.337^{*}$
Seff.prob	$d = -0.209$	$d = 0.414^{**}$	$d = -0.090$
Matcon	$d = 0.074$	$d = 0.501^{***}$	$d = -0.243$
Matanx	$d = 0.340^{*}$	$d = -0.370^{*}$	$d = -0.043$

Table 18.6 Mean differences between pre-test and post-test

Variable	Difference
Performance test	$d = 0.978^{***}$
Seff.app	$d = 0.581^{***}$
Seff.alg	$d = 0.848^{***}$
Seff.calc	$d = 0.887^{***}$
Seff.ageo	$d = 0.210^{*}$
Seff.prob	$d = 0.026$
Matcon	$d = 0.144$
Matanx	$d = -0.105$

Differences Between Pre-test and Post-test

Now, we compare the results of the pre-test and the post-test. The two tests were connected using the linking method according to Stocking and Lord (1983). Again, the differences are expressed in terms of Cohen's *d* (Table 18.6).

The difference concerning the performance tests is huge ($d = 0.978^{***}$). However, it is remarkable that this successful development is reflected very diversely with respect to the beliefs and emotions measured by the scales of the context questionnaires: both mathematics self-concept ($d = 0.144$) and mathematics anxiety ($d = -0.105$) did not change significantly. That may be an evidence for the conjecture that these beliefs (or emotions, respectively) belong to the set of central beliefs within a beliefs system that do not change rapidly—especially not on the basis of a relatively short experience of a two-week bridging course (Philipp 2007, p. 260). Therefore, they seem to be no good indicators for detecting short-term changes. It is exactly the opposite concerning the scales of mathematics self-efficacy: these scales reflect the achievement change almost in the same size that is observed in the performance tests.

Linear Models

Finally, we come back to the pre-test. We suspected that the scales of the context questionnaire may be good predictors for the students' performance. We analyse this hypothesis using linear models (Searle and Gruber 2016) that include scales of the questionnaire as predictors (independent variables) and the results of the pre-test as

Table 18.7 Linear models explaining students' mathematics performance (pre-test)

Predictors	b_i	SE(b_i)	β_i	Total R^2
Seff.app	−0.117	0.232	−0.031	$R^2 = 0.357$
Seff.alg	1.532***	0.267	0.417***	
Seff.calc	0.188	0.172	0.103	
Seff.ageo	0.508	0.270	0.159	
Seff.prob	0.069	0.112	0.033	
Matcon	0.076	0.121	0.022	
Matanx	−0.035	0.080	−0.014	
Seff.alg	2.087***	0.307	0.579***	$R^2 = 0.335$

Table 18.8 Linear models explaining students' mathematics self-concept (pre-test)

Predictors	b_i	SE(b_i)	β_i	Total R^2
Seff.app	0.248*	0.112	0.215*	$R^2 = 0.369$
Seff.alg	0.159	0.098	0.137	
Seff.calc	0.178*	0.072	0.307*	
Seff.ageo	0.012	0.094	0.013	
Seff.prob	0.092	0.054	0.140	
Seff.app	0.261**	0.054	0.422***	$R^2 = 0.353$
Seff.calc	0.397**	0.119	0.311**	

the dependent variable to explain. The model was defined in two steps (Table 18.7): firstly, all variables were used as predictors (model above the line, RMSEA 0.36, CFI 0.963, SRMR 0.53); then, all insignificant predictors were removed leading to the result that only one predictor has a significant and substantial explorative value: the self-efficacy concerning algebra (model below the line, RMSEA 0.32, CFI 0.976, SRMR 0.43). We used b_i to denote the unstandardised regression coefficients, SE(b_i) for their standard errors, and β_i for the standardised regression coefficients.

Since we already observed that the mathematics self-concept scale has different properties than the self-efficacy scales, we analyse the relationship between these concepts, defining linear models in the same way as above, but now using the mathematics self-concept as depending variable (Table 18.8).

The result is remarkable. The self-concept is explained by only two significant predictors: the self-efficacy concerning applied mathematics and calculus. That may be an explanation why the mathematics self-concept acts in different way than the self-efficacy scales: it is mostly determined by two self-efficacy scales that are irrelevant for explaining the students' performance (Table 18.7).

18.7 Conclusion

The first application of the new scales on mathematics self-efficacy in the context of a bridging course shows the following results: firstly, the five scales of this model (related to algebra, applied mathematics, calculus, analytic geometry, and probability) are short and easy to apply, have good statistical properties, and are substantially, but not too highly correlated, so that they describe different facets of interrelated concepts. Secondly, they allow analysing group differences in details. For example, the self-concept did not allow detecting gender differences; the self-efficacy scales, on the contrary, showed a large difference specifically related to applied mathematics and—not having a similar large counterpart in mathematics performance—suggesting that this difference is "exaggerated". Thirdly, the scales are useful tools to describe and to explain changes related to beliefs and to mathematics performance. Whereas the mathematics self-concept is more stable and obviously a part of a student's central beliefs system, the scales of self-efficacy are more peripheral, so that they are appropriate to detect short-term changes as shown by the analyse of the two-week bridging course. Lastly, the self-efficacy scales can explain a considerable amount of both students' mathematics performance and self-concept, revealing that the self-concept is mostly determined by the students' *self-efficacy* related to *applied mathematics* and to *calculus*, whereas their *performance* could be explained best by their *algebraic self-efficacy*. Insofar, the scales of self-efficacy can clarify relationships between different mathematically relevant beliefs and concepts.

All results have to be understood against to the background that the two-week bridging course was the first and very limited opportunity to apply and to test the scales. This fact induces some limitations: until now, there is no information about the marks and the drop out of the participants during their upcoming academic courses; there are also no data about the long-term effects on the participants' performance, beliefs, and emotions; and there was no possibility to control the tasks given during the bridging course. Especially the latter would be desirable to check if the predominant role of the algebraic self-efficacy as a predictor of the students' performance is a general fact or if this role is limited to the specific content of the bridging course at Hildesheim.

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Chapter 19
Does the Relationship Between Mathematics Achievement and Students' Self-Concept Change from Primary to Secondary School?

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19.1 Introduction

In recent years, the improvement of students' mathematics achievement has become central across countries. Previous research has investigated the factors that contribute to predicting mathematics achievement. In those studies, the socioeconomic and cultural background is often regarded as an important factor, since marked differences in mathematics achievement are usually observed between students with an advantaged background and student with a disadvantaged background (Sirin 2005; Chiu and Xihua 2008; Levpušček et al. 2013). Beyond the differences in performance between different socioeconomic groups, some studies have recently tried to verify whether or not the type of task used can facilitate students from a disadvantaged or advantaged background. For example, Piel and Schuchart (2014), using TIMSS-2007 data, found that class differences were more likely to occur in the group of items based on a real context than in the group of pure items, where the item is not embedded in a daily context.

Besides, different measures of students' cognitive abilities are also used in predicting students' academic performance (Deary et al. 2007; Levpušček et al. 2013).

Next to the socioeconomic and cultural background and cognitive abilities that are relevant predictors of student performance, previous research indicated that other factors, related to affect, cognition, and social interactions have been found to be related to mathematical learning (e.g., Heyd-Metzuyanim 2013).

Among these factors, the concept of identity has become more relevant in education research in the last decades (e.g., Grootenboer et al. 2006). The concept

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of identity encompasses together beliefs, attitudes, emotions, cognitive ability, and life history. Different conceptualizations of the term identity were made. Among these, a socio-cultural perspective includes the concept of participation, where the mathematics learning contributes not only to the acquisition of fundamental knowledge, but also influences what the person becomes, determining the development and identity in the community of "mathematics learners" (e.g., Boaler and Greeno 2000; Nasir 2002; Solomon 2007; Cobb et al. 2009).

Some authors have included in the concept of identity not only the narrative aspect, but also "beliefs," "perceptions of self," "perceptions of mathematics," and "ways of being" (e.g., Bishop 2012), and students' self-concept has been shown to be one of the most remarkable factors. Marsh and Craven (2006) carried out one of the most comprehensive narrative reviews of the relationship between self-conception and academic achievement. They found that the relationship between specific self-concept and related academic outcomes is much stronger than that of the global self-concept and non-specific academic domain of self-conception with academic outcomes. Second, the relationship between self-conception and academic achievement was reciprocal. Pajares and Miller (1994) evidenced that self-concept can be domain-specific but not task-specific.

With specific reference to mathematics, the review of the literature offers evidence of a coherent pattern in finding concerning the positive association between self-concept in mathematics and achievement in this subject (e.g., House 2009; Yoshino 2012; Suárez-Álvarez et al. 2014).

Self-concept in mathematics could be defined as the self-perceptions and beliefs one has about his/her capacity to do well and his/her self-confidence in learning mathematical topics, which is based on previous experiences (Felson 1984; Wilkins 2004).

Self-concept is formed through experiences with the environment and is influenced especially by environmental reinforcements and significant others (Shavelson et al. 1976) and influences performance through motivational processes: students with a stronger self-concept are more likely to engage in mathematical tasks than those with a poorer self-concept (Skinner et al. 2008; Skinner et al. 2009; Green et al. 2012).

There is now strong evidence that academic self-concept also affects achievement. Indeed, considerable self-concept research in recent years has been devoted to the reciprocal effects model (REM; see Marsh and Yeung 1997; Guay et al.; 2003; Marsh 2007): students' perceptions of their capacity is based on previous experiences in a specific subject and, at the same time, exerts an influence on future performance; for example, students with a higher self-concept devote more time to the study of mathematics.

Research on international surveys such as OECD PISA and IEA TIMSS has consistently shown the strong relationship between achievement and self-concept across countries (e.g., Lee 2009; Marsh and Hau 2004; Marsh et al. 2013).

In literature, another key factor related to achievement, intrinsic motivation was defined as the enjoyment one gains from doing the task (Deci and Ryan 1985; Harter 1981; Deci and Ryan 2010). Intrinsic motivation and academic achievement are seen as developmentally interconnected; intrinsic motivation lies at the core of self-determined activity (Ryan and Deci 2000) and is expected to be reciprocally associated with achievement. However, the relationship between intrinsic motivation and mathematics achievement is not clear: some studies have found that intrinsic motivation is positively correlated with achievement (e.g., Walker et al. 2006; Ayub 2010; Garon-Carrier et al. 2016), while other studies have found a moderate or no relationship (Bouffard et al. 2003; Marsh et al. 2005; Spinath et al. 2006).

Moreover, different research found a decline in intrinsic motivation for mathematics with age (e.g., Gottfried et al. 2001; Gottfried et al. 2007). Concerning Italy, boys have been shown to outperform girls both in international and national standardized mathematics tests at all school levels (Mullis et al. 2016; OECD 2016; INVALSI 2018), and the results also evidenced that girls have a lower mathematics self-concept than boys.

In recent decades, studies have adopted more sophisticated data analysis approaches—such as structural equational modeling and path analysis—to investigate whether a variable, such as SES, affects mathematics achievement directly or through the mediation of students' self-concept (e.g., Fin and Ishak 2013; Suárez-Álvarez et al. 2014).

The present study aims to identify personal and contextual factors predicting the mathematics achievement of Italian students in the fourth and eighth grades by using data derived from TIMSS (Trends in International Mathematics and Science Study). The main aim of TIMSS is to measure trends in the mathematics and science achievement of fourth- and eighth-grade students across countries to provide comparative information about educational achievement that is useful for the improvement of teaching and learning in mathematics and science. TIMSS is a cross-sectional survey conducted every 4 years; thus, the cohort of students assessed in mathematics in 2011 in the fourth grade reached the eighth grade in 2015, and the TIMSS 2015 provided information about the relative progress of the same cohort of students across grades.

In this study, structural equation modeling was used to assess the effects of socioeconomic and cultural background and students' self-concept and attitude towards mathematics on achievement utilizing Mplus (Muthén and Muthen 2017). In particular, we tested a specific causal model to verify the following:

1. Whether mathematics achievement is related both to students' self-concept, intrinsic motivation (namely, students' like learning mathematics), after taking into account other characteristics, such as their socioeconomic status and gender.
2. Whether the relation of achievement with self-concept and attitudes towards mathematics changes from the fourth to the eighth grade.

19.2 Method

Participants

In this study, the fourth-grade students who participated in TIMSS 2011 and the eighth-grade students who participated in TIMSS 2015 were considered, as they are two samples from the same population selected in two different points in time (2011 and 2015).

Even though TIMSS does not test the same students, the 2011 sample is representative of Italian fourth-grade students, and the 2015 sample is representative of Italian eight-grade students, thus we have two representative samples of Italian students born in 2001. The following table shows the sample and population distribution (Table 19.1).

Measures

Students from both grades answered the same questions as those on the international questionnaires (for detailed information, see Martin et al. 2016).

Mathematics Achievement Scale Developed by the TIMSS working group at the TIMSS & PIRLS International Study Center, Boston College, the scale consists of multiple-choice and open-ended questions (that required either short answer or extended response). TIMSS survey uses matrix sampling that involves packaging the entire pool of mathematics and science assessment items into a set of different student achievement booklets, with each student completing just one booklet. In this way, it is possible to avoid each student answering all the questions to ensure that the duration of the test is sustainable for the students. TIMSS uses item response theory (IRT) scaling methods to assemble a comprehensive picture of the achievement of the entire student population from the combined responses of individual students to the booklets they are assigned. Based on the IRT estimates, a total score is calculated for each student, which is then converted into a standardized score. The TIMSS results refer to a common metric: the TIMSS performance scales were defined in 1995 to have a scale average of 500 and a standard deviation of 100, corresponding to the mean and the international standard deviation calculated for all countries that participated in TIMSS 1995 for the fourth year of schooling. For both levels of schooling involved in the TIMSS survey, the mathematics test is divided into two different dimensions: content domains and cognitive domains. The following variables derived from the student questionnaire were used in the analyses. Using

Table 19.1 Sample and population estimate by gender and grade

	Fourth grade (2011)		Eighth grade (2015)	
	Sample	Population estimate	Sample	Population estimate
Male	2082	260,202	2257	259,980
Female	2118	260,616	2224	268,859
All students	4200	520,818	4481	528,839

IRT partial credit scaling, the student responses were placed on a scale with a mean scale score of 10 across all countries and a standard deviation of 2.

Socioeconomic and Cultural Status (SES) Based on the answers from students and parents (only for the fourth grade), a general index of each student's SES was created based on the following: (1) students' home environments, including the parents' educational levels and parents' occupational status (parents' occupational status was used only for fourth-grade students and was derived from the home questionnaires), (2) the number of study resources available at home, and (3) the number of books at home (for a detailed description of the index, see Martin et al. 2016).

Student Self-Concept in Mathematics This scale includes 7 items (e.g., "Mathematics is harder for me than any other subject" [reversed], "I learn things quickly in mathematics," and "I am good at working out difficult mathematics problems") from TIMSS student questionnaire to measure how students think about their abilities in mathematics. Students indicated their answers on a 4-point Likert scale (from 1 = completely disagree to 4 = completely agree). Higher scores indicate a higher self-concept in mathematics. In the TIMSS framework (Mullis et al. 2009; Mullis and Martin 2013), the self-concept was defined as a multidimensional construct and is often estimated relative to students' peers or experiences.

Intrinsic Motivation Towards Mathematics This scale includes 6 items related to the like mathematics (e.g., "Mathematics is boring" [reversed], "I enjoy learning mathematics," and "I learn many interesting things in mathematics"). As on the previous scale, students indicated their answers on a 4-point Likert scale (from 1 = completely disagree to 4 = completely agree). Higher scores indicate a higher intrinsic motivation for mathematics. In the TIMSS framework (Mullis et al. 2009; Mullis and Martin 2013), the authors stated that Students' motivation to learn can be influenced by the fact that they find the topic pleasant, value the topic, and think it is important in the present and for future careers. Personal interest in a subject motivates the learner and facilitates learning.

19.3 Data Analysis

The descriptive analyses were conducted using the software IEA IDB Analyzer (IEA 2012). A mediation analysis with structural equation modeling using Mplus assessed the effects of socioeconomic and cultural background and student characteristic factors on mathematics achievement. A structural equation model (SEM) was used to perform a path analysis where:

- Family SES was used as an independent variable;
- Students' self-concept and intrinsic motivation towards mathematics were also used as independent variables;

- The TIMSS mathematics test score (from TIMSS 2011 for the fourth grade and TIMSS 2015 for the eighth grade) was considered the dependent variable.

19.4 Results

Table 19.2 shows the descriptive statistics for all variables considered in this study.

Males outperform females in mathematics achievement in both the fourth and eighth grades. Males have better perceptions of their ability in mathematics than females and have higher levels of intrinsic motivation. Italian students perform better in mathematics in the fourth grade and worse in the eighth grade.

Table 19.3 shows correlation analyses. The results do not show relevant differences between males and females. Intrinsic motivation and self-concept in mathematics are highly correlated, and the correlation increases over time. The correlations between self-concept, intrinsic motivation, and SES, on the one hand, and mathematics achievement, on the other hand, increase over time.

Path Analysis

A structural equation model (SEM) was used to perform a path analysis, Fig. 19.1 shows the model tested.

The SEM has good indices according to the recommended cut-off values (Byrne 2001): RMSEA = 0.03, CFI = 0.99 and 34% of the variance in mathematics achievement explained.

Achievement in mathematics is predicted by all factors considered in the path model, except for intrinsic motivation.

Mathematics achievement is found to be strongly and positively associated with the socioeconomic and cultural index ($\beta = 0.25$. $p < 0.01$) and self-concept in mathematics ($\beta = 0.30$. $p < 0.01$ for males and $\beta = 0.29$. $p < 0.01$ for females).

The model explains mathematics performance better for the eighth grade (34% of the variance in mathematics achievement explained both for male and female students) than for the fourth grade (16% of the variance explained both for male and female students).

More specifically, in the fourth grade, in addition to SES ($\beta = 0.25$. $p < 0.01$ for males and $\beta = 0.29$. $p < 0.01$ for females), students' self-concept about their ability in mathematics contributes to explaining performance ($\beta = 0.30$. $p < 0.01$ for males and $\beta = 0.25$. $p < 0.01$ female students), while in eighth grade the β coefficient is 0.56. ($p < 0.01$ for both male and female students).

In both the fourth and eighth grades, intrinsic motivation does not seem to have a relationship with mathematics performance after controlling for self-concept in mathematics: there is no significant association between achievement and intrinsic motivation in students with the same level of self-concept.

With the transition from the fourth grade to the eighth grade, the relationship between self-concept and mathematics performance becomes much higher. The

Table 19.2 Descriptive statistics (means and standard errors) by grade and gender

	All students		Girls		Boys	
	4th grade	8th grade	4th grade	8th grade	4th grade	8th grade
	Mean (SE)					
Math achievement	507.82 (2.65)	494.73 (2.56)	503.34 (3.20)	491.00 (3.01)	512.31 (2.92)	497.68 (2.79)
Intrinsic motivation	10.05 (0.05)	9.42 (0.05)	9.91 (0.06)	9.25 (0.07)	10.19 (0.06)	9.58 (0.06)
Self-concept in mathematics	9.93 (0.04)	10.01 (0.05)	9.69 (0.05)	9.70 (0.07)	10.16 (0.05)	10.31 (0.07)
SES	9.66 (0.05)	10.20 (0.05)	9.65 (0.06)	10.24 (0.07)	9.67 (0.05)	10.16 (0.05)

Table 19.3 Correlation[a] analysis by gender and grade

	Self-concept		Intrinsic motivation		SES	
	4th grade	8th grade	4th grade	8th grade	4th grade	8th grade
Girl						
Self-concept	1.00 (0.00)	1.00 (0.00)	0.59 (0.02)	0.79 (0.01)	0.11 (0.02)	0.19 (0.02)
Intrinsic motivation	0.59 (0.02)	0.79 (0.01)	1.00 (0.00)	1.00 (0.00)	–	–
SES	0.11 (0.02)	0.19 (0.02)	–	0.11 (0.03)	1.00 (0.00)	1.00 (0.00)
Math achievement	0.27 (0.02)	0.49 (0.02)	0.16 (0.02)	0.33 (0.03)	0.29 (0.03)	0.40 (0.03)
Boys						
Self-concept	1.00 (0.00)	1.00 (0.00)	0.58 (0.02)	0.77 (0.01)	0.09 (0.03)	0.23 (0.03)
Intrinsic motivation	0.58 (0.02)	0.77 (0.01)	1.00 (0.00)	1.00 (0.00)	–	–
SES	0.09 (0.03)	0.23 (0.03)	–	0.11 (0.03)	1.00 (0.00)	1.00 (0.00)
Math achievement	0.31 (0.03)	0.51 (0.02)	0.19 (0.03)	0.34 (0.03)	0.25 (0.03)	0.39 (0.02)

[a]Correlations are statistically significant ($p < 0.01$). The standard errors are in parentheses

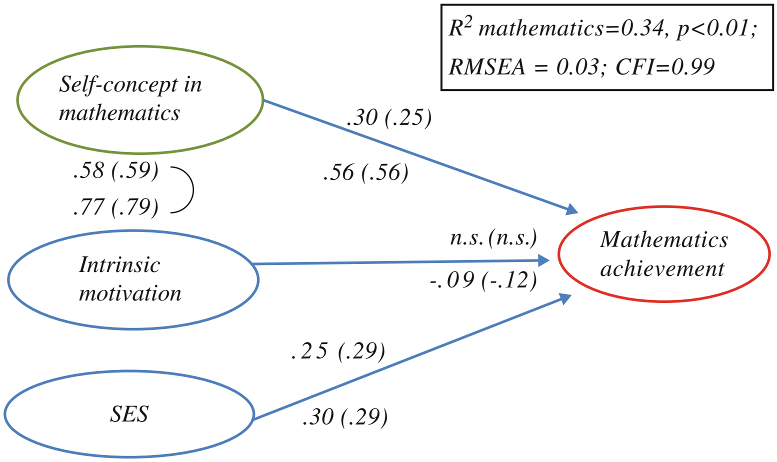

Fig. 19.1 Relationships between self-concept, intrinsic motivation, and mathematics achievement on TIMSS for the fourth and eighth grades. To improve the readability of Fig. 2.1, the measurement errors considered in this investigation are not depicted. The values for the fourth grade are above the lines, and the values for the eighth grade are below the lines. The values for males are in parentheses, and the values for females are not in parentheses

effect of intrinsic motivation towards mathematics was not significant after controlling by self-concept.

19.5 Discussion

The present study investigated the role of self-concept and intrinsic motivation in predicting student mathematics achievement over time, after controlling for student SES.

The model was successful in explaining TIMSS scores in mathematics, with 34% of the variance explained.

According to the literature, the socioeconomic and cultural background plays a significant role in determining students' performance (Sirin 2005; Deary et al. 2007; Levpuščcek et al. 2013).

Moreover, in both the fourth and eighth grades, intrinsic motivation does not seem to have a relationship with mathematics performance after taking into account self-concept in mathematics, according to some studies that found a moderate or no relation (Bouffard et al. 2003; Marsh et al. 2005; Spinath et al. 2006). The advantage of using a more sophisticated approach to analyze data, such as path model, instead a simple correlation is to evidence the unique contribution of a variable, after controlling for other relevant variables. The apparent contradiction between the results of correlation analysis and path model regarding the association between intrinsic motivation and self-concept reveals that intrinsic motivation is completed affected by self-concept both at primary and secondary school. Also, the literature evidenced a decline in intrinsic motivation for mathematics with age (Gottfried et al. 2001; Gottfried et al. 2007; Wigfield et al. 2006; 2007).

Self-concept seems to play a relevant role and contributes significantly to explaining individual differences in mathematics performance. The role of self-concept becomes more evident from the fourth grade of primary school to the last year of lower secondary school (e.g., House 2009; Yoshino 2012; Suárez-Álvarez et al. 2014): the relation between students' self-concept with achievement is stronger associated at grade eight and less at fourth grade. Our finding was consistent with the results of previous research (Marsh et al. 1998; Skaalvik and Valås 1999; Chen et al. 2013) finding that the reciprocal effects were weaker at the elementary school because students' academic self-concepts were still developing and not yet fully established. Moreover, the increased pressure to perform in mathematics, combined with an improved capacity to self-evaluate their competence with age and the changes in children's processing of the evaluative feedback they receive for teachers, could increase the likelihood of reciprocal associations between self-concept and achievement in mathematics over time.

The literature evidenced that the self-concept, and more generally the sense of identity is influenced especially by environmental reinforcements and significant others (Shavelson et al. 1976), such as peer and teacher. Furthermore, Vygotsky (1978) highlighted that cognitive development is a process that is promoted and supported primarily by the processes of negotiation and interaction with (more competent) persons and learners acquire concepts, ways of thinking and strategies through interaction with more competent people by gradually internalizing them.

Different studies from the past evidenced that changes in self-concept attributable to some treatment they often provide important insights into the factors that motivate students in and out of school and into alternative courses of action that may enhance students' self-concepts (e.g., Häussler, P. and Hoffmann, L. 2002; Gest et al. 2005; O'Mara et al. 2006; Molloy et al. 2011). In this line, as suggested by Vygotskij (1990), a teacher might propose a slightly more complex task within the area to be developed next, a task that students could not undertake and solve on their own and support them in the search for a solution. In so doing, these students could acquire new skills while avoiding the frustrating experience of failure.

In Italy, teachers often care a lot about teaching, disregarding the development of an adequate system of self-regulation of learning, this study seems to suggest that self-concept development activities in students would be necessary, and teachers should be adequately trained to deal with this aspect. One of the teacher's goals should be to help students strengthen their self-concept so that they can improve their performance, especially in the primary school where students' academic self-concepts were still developing and not yet fully established. Further studies are necessary to investigate whether and to what extent different strategies during classroom work, could be used by teachers to strengthen a student's self-concept.

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Chapter 20
Commentary on Cognitive and Non-cognitive Factors in a Vertical Perspective: The Case of Percentages

Pier Luigi Ferrari and Francesca Martignone

20.1 Introduction

In this contribution we analyze data from standardized tests carried out at the beginning of some Italian scientific undergraduate courses (cognitive assessment tests for freshmen at STEM studies). In particular, the data analyzed concern basic knowledge and skills that should have been already acquired from primary and middle school. As pointed out also in other contributions of this book section, it is important to understand in depth the nature of students' difficulties both to better organize the recovery activities and to dialogue effectively with previous school grades, by means of a vertical point of view. The aspects that we have taken into consideration are both cognitive and didactic. We chose the topic of percentages because on the one hand it proved to be one of the most problematic in the tests we analyzed, on the other hand it is proposed, in the Italian National Guidelines (MIUR 2010a; 2010b; 2010c; 2012), as a learning goal already at the end of primary school. Moreover, this topic is relevant also because different factors (cognitive, non-cognitive, linguistic) affect students' performance.

20.2 Background

In our research we analyze word problems in which we evoke realistic situations related to changes in the price of goods. Word problems are usually defined as word descriptions of problem situations where, in order to find an answer,

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schemes are applied and mathematical operations are performed using data from the problem statement (Verschaffel et al. 2014). The focus of our study is on both students' externally observable performance and the cognitive schemes and thinking processes possibly underlying the different actions.

According to Verschaffel et al. (2014), word problems have been widely included in mathematics curricula to accomplish several goals, among which is to offer practice for everyday situations in which mathematical tools and concepts are to be applied. Other goals are motivating students to study mathematics, to train students to think creatively, to develop their problem-solving abilities, and to develop new mathematical concepts and skills. We could add also the need for making practice with language as it is used not just in mathematics but also in everyday-life applications of some simple mathematical ideas. For example, if the price of a good was P_1 at the beginning of the year and P_2 at the end, by "percentage increase" is meant the value given by the formula $100 \cdot \frac{P_2 - P_1}{P_1}$ and not by $100 \cdot \frac{P_2 - P_1}{P_2}$. This is obvious for anybody who uses percentages in his/her job, but not for a good share of freshman students.

As remarked by Jacobs Danan and Gelman (2017), there is very little research work on percentages. They find this surprising as, in their opinion, percentage changes are the numerical format most commonly encountered in adult life in numerate cultures and are used on a daily basis to describe and advertise changes in the prices of goods of all kinds. It also describes voting preferences, taxation rates, interest rates, fractions of populations, and much else.

These authors interpret difficulties with percentages in terms of conceptual structure; in particular, they underline the contrast between the relatively rapid progress of learning when the material to be learned maps readily into an existing conceptual structure and the much slower and uncertain progress when it does not. In other words, the early arithmetic schemes seem more suitable for additive structures than for multiplicative ones, and this may explain difficulties with other multiplicative concepts too, such as fractions. This interpretation may be fully satisfactory in lower secondary school but it is not enough in order to explain the behavior of undergraduates, who have had plenty of time to interiorize multiplicative concepts. The teaching strategies proposed by Jacobs Danan and Gelman are reasonable, as they suggest graduality and care for the development of the meanings involved. They also suggest to work on vocabulary, for example, to clarify that increase does not always imply addition. The difficulties we are describing below seem to require consideration for some other factors, such as language as a whole (not just vocabulary) and non-cognitive factors such as self-concept and intrinsic motivation (as studied by Caponera, Pozio, and Palmerio) and self-efficacy, as studied by Girnat.

We will analyze data coming from the different editions of the "togliere spazio Initial skills Test" carried out for the freshmen of the Biology, Chemistry, and Computer Science courses of the University of Eastern Piedmont in Italy. This standardized test is conducted at universities and consists of 20 closed-ended, mostly multiple-choice items. 10 items concern the understanding and use of scientific language in general, another 10 items are more focused on mathematical notations. Students take the test individually via a Moodle platform and have 40 min to submit their answers.

20.3 Data and Analysis

Let us see some examples from the test administered in Alessandria in October 2016. All the translations from Italian are by the authors.

Example 20.1 A promotional sale is underway in a shop: all appliances are discounted by 30%. A microwave oven is now sold for 199 €. How much did that microwave oven cost before the discount? If one of the indicated values is acceptable, choose it, otherwise choose the item, "None of the values given is acceptable."

a) About 284 € **b**) About 259 € **c**) About 229 € **d**) About 139 € **e**) None of the values given is acceptable

The distribution of responses is shown in the following table. The sample was 360 units. 3 students did not provide any answer (Table 20.1).

The choice of the option "About 259 €" probably comes from the application of a 30% increase to the final price, while "About 229 €," which is obtained by adding 30 € to the final price, could depend both on some miscalculation or from an incorrect interpretation of the meaning of the expression "30%." "About € 139" results from the application of a 30% reduction to the final price.

The answer "None of the values given is acceptable" could depend on the lack, among the distractors, of values such as "30 €" or "60 €" which correspond to the extent of the reduction (or increase) in the cases of the most popular answers. Another possibility is that some students have calculated the correct numerical result (284.28 . . .) or, more likely, an incorrect one (258.7) and have been misled by the presence of significant figures to the right of the comma. Possibly they did not carefully read the statement and the options. Variants of this problem, with the same conceptual and textual structure, have been administered in October 2018 to about 1000 freshman students. The percentage of correct answers varied, in the various samples and versions, from 46 to 56%, with an overall average of 49%.

Let us see an example:

Example 20.2 In a shop there is a 20% discount on all appliances. Moreover, on blenders an additional 20% discount is made on the discounted price. If the blenders are sold for 16 €, what was their initial price?

Table 20.1 Percentage of answers to Example 20.1. In bold the correct answer

Answer	About 284 €	About 259 €	About 229 €	About 139 €	None of the values given is acceptable
%	**46%**	26%	13%	<1%	14%

Table 20.2 Percentange of answers to Example 20.2. In bold the correct answer

Answer	25 €	22.4 €	12.8 €	56 €	19.2€	No answer
%	**51.11%**	18.89%	1.11%	4.41%	15.56	8.89%

Table 20.3 Percentange of answers to Example 20.3. In bold the correct answer

Answer	200%	300%	20%	30%	3%	No answer
%	**18%**	55%	2%	17%	8%	1%

a) 25 € **b**) 22.4 € **c**) 12.8€ **d**) 56 € **e**) 19.2 €

This time the option "None of the values given is acceptable" was not provided. The choices and distribution of responses are shown in the following table. The sample was of 90 units (Table 20.2).

We note that a not negligible share of students choose the distractor 22.4 €; seemingly they added the 20% of 16 twice to the final price. The 19.2 € distractor is obtained by calculating 20% of 16 and then subtracting from 16. Possibly these students directly used the numbers they found in the statement and calculated 20% only once probably because the two discounts are by the same percentage. Finally, it should be noted that some students chose 56 perhaps because they added 40–16.

Let us see another problem.

Example 20.3 The value of a property has tripled over the course of a year. How much did it increase in percentage?

a) 200% **b**) 300% **c**) 20% **d**) 30% **e**) 3%

This is the distribution of the answers of a sample of 301 freshman students (Table 20.3).

Distractors that include "3" as a digit have been chosen by 80% of the sample.

The outcomes of this problem (which has been proposed with some variations in many occasions to thousands of students, with similar results) underline the relevance of linguistic factors or, at least, of students' attitudes towards language. It seems that a good number of students focus on some word in isolation (such as "tripled") and do not try to interpret the statement of the problem as a whole.

20.4 A Vertical View

Given the results of the tests described above, one may wonder what results could be obtained by administering the same problems to secondary school students. We have not yet carried out this kind of survey, but as a first step in this direction we analyzed some data collected by the Italian National Assessment System concerning similar problems. In Italy the National external assessment is carried out by means of standardized tests. These tests aim at assessing students' learning outcomes according to the expectations stated in the Italian National Guidelines.

A quantitative vertical analysis between the results on the university samples presented above (certainly not a faithful representation of all freshman students) and the results of the national surveys for secondary school students is not possible: as a matter of fact, these samples are not comparable and the problems are different. Therefore, it does not make sense to carry out quantitative comparisons, but we can develop a qualitative analysis of the different answers and reflect on the possible motivations of the students' behaviors.

In these perspective let us analyze two examples of percentage problems, selected from the Italian National standardized tests for the grade10 and for grade 8.

Problem administrated to grade10 students (our translation).

Problem 20.1 To the members of a supermarket, a detergent is sold at a discount of 20% at the price of 1.40 €. How much does that detergent cost to customers who are not members of the supermarket and therefore are not entitled to the discount?

A. 1.68 €
B. 1.75 €
C. 2.80 €
D. 1.12 €

This problem and the different options of choice investigate some of the typical errors underlined above in the analysis of the wrong options in the tests for freshman students. As we have already remarked, we cannot make comparisons on the correct (and incorrect) answer percentages because we are talking about samples with very different characteristics and nature, but we can see that there are some specific options that are more widely chosen by students. Option A (which is obtained by calculating 20% of 1.4 and adding it to 1.4) has the same response rate as the correct option. This choice could be related to the application of a sort of pseudo-rule so that if x plus y % of x makes z, then z minus y % of z makes x; this pseudo-rule may be taken as an example of what argued by Jacobs Danan and Gelman (2018) about the contrast of different conceptual structures; in this case an additive interpretation of percentages (suggested by the current vocabulary) is an obstacle to learning. Anyway, this is not the whole story, as these behaviors may be amplified by the practice of some students who try to apply algorithms to the numbers they are given before representing the problem in some way. This seems to derive more from school habits than from cognitive difficulties related to the different conceptual structures (Table 20.4).

Table 20.4 Percentange of answers to Problem 20.1. In bold the correct answer

Option	A	B	C	D	Missing
%	38.6%	**38.6%**	15.8%	4.5%	2.5%

Table 20.5 Percentange of answers to Problem 20.2. In bold the correct answer.

Option	A	B	C	D	Missing
%	**2.2%**	**15.2%**	18.8%	59.5%	4.3%

Option D can be chosen by those who work on the numbers given in the text, with in addition the calculation of 0.8 (as difference between 1 and 0.2) and with little semantic control on the final result, i.e. on the fact that the price found is lower than that obtained after the discount.

Option C (which is obtained by doubling 1.4) could also be chosen by students who focus on the digit"2" of "20%" and then multiply by two or interpret 20% as a doubling factor. In this case, as in the third example shown above, language aspects could have a strong influence.

If we continue to go backwards in the Italian standardized assessment tests of previous school grades, we find problems on percentages like the next one (administered to grade 8 students—our translation). The percentage of correct answers is very low: the correct answer was chosen by only 15.2% of students!

Problem 20.2 In October a sweater costs 100 €. Before Christmas its price increased by 20%. In January, with the sales, the cost of the sweater was reduced by 10% compared to the Christmas price. Which statement is true?

A. A. The sweater in January costs the same as in October.
B. The sweater in January is 8% more expensive than in October.
C. The sweater in January costs 10% less than in October
D. The sweater from October to January has a 10% increase in price

In this problem it is clear that the intention is to assess whether students actually choose to calculate percentages or manipulate only the values written in the text without identifying to which prices percentages are referred. What is remarkable is that in grade 8 the choice that could be the result of the subtraction between 20% and 10% (option D) is chosen by the majority of the students! (59.5% of the students). This behavior may be somewhat close, even if it does not reach such a high percentage, to that highlighted in the freshmen or in grade 10 students who calculate the percentage of the number they see written in the text. Option A is chosen by very few students because it is not supported by the manipulation alone of the percentage values given in the text and therefore should be the result of a calculation: but those who do the actual calculation of the percentages, unless errors, can find the correct value. On the other hand, option B is probably chosen by those who always focus on percentages, but the fact that it is written "sweater was reduced by 10%" probably directs them to choose the option with "10% less" (Table 20.5).

20.5 Final Reflections

Even if the use of percentage is supposed to be customary in daily life, the problems involving percentages seem to be hard to students even at undergraduate level. There is very little research work on percentages and the studies deal above all with lower secondary school. A central issue seems to be the difficulty in working on the meaning of the text. One gets the impression that some students do not interpret the problem situation but look in the text for some words and numbers in order to activate solution processes that often lead to incorrect choices. A good deal of research highlights the difficulties students encounter in solving problems when they have to choose the elements to build a mathematical model. In the past (and unfortunately still today) some teachers believed that in order to overcome these difficulties, students could be taught to identify " key words": i.e. to quickly identify numbers or terms within the problem, perhaps highlighting them and listing them in some way. This is a very limited and limiting aid, which can create serious difficulties for students who focus only on specific numbers or terms in the text without understanding the relationship between them (Nesher 1980; Sowder 1988; Ferrari 2004).

In this contribution we presented some examples of word problems on percentages administrated by means of standardized tests at the beginning of some Italian undergraduate courses concerning scientific disciplines or in secondary school. In a vertical perspective, we showed some students' recurring errors: they are linked to the practice of searching for possible key words and applying algorithms to the numbers found in the text before interpreting the relationships among the data. In our analysis we tried to give some explanations of the processes that might have led to the students' behaviors. Our analysis takes into account above all cognitive and language aspects. A more complete study should take into account the affective-related aspects that may influence the choice processes in assessment situations. For many years now, a good amount of educational research has been focusing on the non-cognitive aspects involved in the management of problem-solving processes: these studies have long been concerned with the role of metacognitive and affective aspects in mathematical problem-solving activities (e.g. Schoenfeld 1985; Hannula 2020). As underlined by Caponera et al. and Girnat (in this book) there are non-cognitive factors, such as self-concept, intrinsic motivation and self-efficacy, that have to be more deeply investigated. It would be also interesting to carry out interviews about the students' background and then try to understand what were the influences of previous teaching methods on the students' interpretation of the problems.

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Chapter 21
Misconceptions in Physics at Politecnico di Milano: Preliminary Results

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21.1 Introduction and Background

Constructivist science education research has pointed out that university learners do not start with a clean slate on the physical world (Hammer 2000). Indeed, students who begin their academic career in a scientific programme, like engineering, science, chemistry and biology, may generally reveal some erroneous viewpoints and incorrect interpretative schemas of a broad spectrum of Physics topics (Bozzi et al. 2019; Planinic et al. 2006). These are, broadly speaking, related to intuitive thinking in sciences. Dewey (1938) states that intuitions are not "part of the theories of logical forms" (p.103). Intuition is a form of thinking that provides the learner with a sense of certainty (Fischbein 1987): it is perceived as global (rather than analytical), coercive and self-evident. Sometimes intuitions from everyday experience contrast with mathematical knowledge and can impede learning. Andrà and Santi (2013) underline that intuitions are a way of establishing a relationship between the learning subject and the object of knowledge, they are a mode of existence of the consciousness which intertwines with perception, sensorimotor activity, emotions and scientific generalisation.

Students' wrong ideas and lines of reasoning on a considerable number of physical phenomena, accumulated over the years from their previous learning and experience, were christened "misconceptions" for the first time in 1972 (Doran 1972) in an article that illustrated a study focused on elementary school children. Thoroughly investigated, misconceptions have assumed a more and more paramount

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role in the didactics of Physics, being classified into five different classes or categories: preconceived notions, non-scientific beliefs, conceptual misunderstanding, vernacular misconceptions and factual misconceptions (Committee on Undergraduate Science Education 1997). From all accounts, they are essentially defined as ideas at variance with recognised views (Fisher and Lipson 1983) or dissimilar from the ones generally acknowledged by scientists (Odom and Barrow 1995). Needless to say, not only are these alternative conceptions considered as inconsistent with physicists' accepted views, but at the same time they could represent an obstacle to the process of maturation of a correct canonical students' understanding. In fact, intuitions can start in a private, individual moment, but it is in the moment during which they are shared that they develop towards mathematical generalisations (Andrà and Santi 2013) and may contribute to determine a learners' identity.

In this context, the Experimental Teaching Lab ST2 of Politecnico di Milano and another Italian institution, Università degli Studi di Trento, which provides pedagogic support, have developed a case study aimed at answering to the following research questions:

1. do university students enrolled for Engineering at Politecnico di Milano reveal any misconceptions in Physics, related to notable topics addressed in their academic basic Physics courses and studied at high school?
2. How widespread are these incorrect ideas?
3. Given the important role of social interaction in framing intuitive thinking, does attending university for some months reduce these misconceptions?

21.2 Methodology

Our research aimed at identifying and analysing some significant and widespread misconceptions shown by both first-year and second-year university students enrolled for engineering, in relation to their knowledge of Physics. These incorrect viewpoints were chosen on the basis of the following conditions:

- to be concerned in some topics taught in the academic Physics courses of Politecnico di Milano, with specific reference to Mechanics, Thermodynamics and Electromagnetism;
- to pertain to some issues that these undergraduates studied at high school.

The study target consisted in undergraduates enrolled for four different engineering branches, i.e. Chemical Engineering, Materials and Nanotechnology Engineering, Mathematical Engineering and Physics Engineering in the academic year 2018–2019. With regard to the purpose of our research, the students were grouped in three sections named 1, 2 and 3, respectively, taking into account the Physics course they were about to attend (Experimental Physics A+B, Experimental Physics I and Experimental Physics II) and their engineering branches, as well as their year and

Table 21.1 Data about the basic Physics courses involved in the research

Section	Students number	Physics course	Engineering study course	Students year	Course term
1	449	Experimental Physics A+B	Chemical, Materials and Nanotechnology	1	1
2	370	Experimental Physics II	Mathematical, Physics	2	1
3	170	Experimental Physics I	Physics	1	2

the term in which they attended their Physics course. Table 21.1 synthesises the data collected in relation to the basic Physics courses involved in our study.

It is appropriate to point out that the students included in section 1 (S1) were at the beginning of their academic career; consequently, they had never attended a university course and their knowledge along with understanding of Physics phenomena was related to their own previous experience and education. Differently, even though section 3 (S3) consisted of first-year university students like S1, these freshmen were at the start of the second academic term, hence they had already taken some university courses, among which Chemistry. Notwithstanding that they had not studied Physics in a previous academic course, some issues related to Thermodynamics and Electromagnetism still had been addressed in their Chemistry classes.

The second-year university students included in section 2 (S2) were at the beginning of their second academic year; needless to say, they had already attended a good deal of university courses, including Chemistry and, more importantly, Experimental Physics I which was focused on Mechanics and Thermodynamics.

To investigate the possible undergraduates' misconceptions in Physics, researchers have adopted various techniques over the years, for instance, interviews (Park and Han 2002), open-ended tests (Colin et al. 2002) and multiple-choice tests (Martìn-Blas et al. 2010). Since the number of overall students potentially involved in this study had been estimated to be massive when the research was planned—on balance they were 989—a multiple-choice test appeared to be an appropriate option to carry out our study. Consequently, the Experimental Teaching Lab ST2 of Politecnico di Milano created an authentic ad hoc multiple-choice test, identical for S1, S2 and S3, on the basis of the students' most recurrent mistakes in their Physics courses final examination, their more frequent questions during lessons or drills as well as researchers own teaching experience and the literature on misconceptions in Physics. Università degli Studi di Trento corroborated the educational and didactic suitability of how this trial was created.

This test, administered at the beginning of every university Physics course involved in our study, consisted of twelve quizzes, divided equally among Mechanics, Thermodynamics and Electromagnetism. The overall number of quizzes was set taking into account some priorities; on the one hand, the trial could not last too much, on the other hand it was essential to have an adequate number of quizzes. Every question was characterised by four possible answers: three alternatives were incorrect and focused on different misconceptions related to the issue investigated in that quiz and only one was correct. The trial was administered to all the students through the online portal Socrative (Tretinjak et al. 2015) and their own electronic devices, like smartphones, tablets and laptops, aligning with the Bring Your Own Device (BYOD) strategy (Afreen 2014).

21.3 Results and Discussion

Notwithstanding the abundance of gathered data would allow to explore more thoroughly our research theme, in this context we will illustrate only some preliminary results.

Particularly, the first step of our analysis consisted in evaluating the average percentage of correct answers given by every section involved in the research as a function of each macro-area previously identified. These data are synthesised in Table 21.2 where it is shown that misconceptions were broadly highlighted by undergraduates of all the three groups involved in our study.

Firstly, the worst performances were achieved by all the learners in Mechanics. This is startling owing to the fact that this branch of physics is commonly considered easier than others as is widely studied by students at high school. Moreover, it should be emphasised that the S2 undergraduates had already learnt Mechanics and Thermodynamics in their first university Physics course, attended in the previous academic year. That course had consisted of traditional lectures: the low rate of correct answers reached by the second-year students in Mechanics and partly in Thermodynamics confirms that this type of lectures might not be so effective in terms of learning and its persistence.

Secondly, taking into account that S2 and S3 students had already attended a university Chemistry course, where they had studied some preliminary issues about Thermodynamics and Electrostatics, all the three sections attained substantially equivalent outcomes in Electromagnetism. As a consequence, although to attend university for 6 months (S3) or 1 year (S2) probably contributes to developing learners' general and integral processes of constant growth, it could be argued

Table 21.2 Mean rate of success achieved by each section involved in the research as a function of every macro-area identified

Section	Right answers (%)		
	Mechanics	Thermodynamics	Electromagnetism
1	21.8 %	44.5 %	27.0 %
2	27.8 %	62.6 %	32.1 %
3	19.7 %	56.0 %	33.5 %

that this attendance does not appear to change some incorrect frames of mind that students employ to interpret the physical reality. Comparing S1 and S3 results in Mechanics seems to confirm this claim.

21.4 Conclusion

Our research corroborates the awareness that both Politecnico di Milano freshmen and second-year students frequently highlight some incorrect ideas and erroneous interpretative schemas on many Physics phenomena: only 35.7% of the answers in the multiple-choice test administered to the learners was correct. It is worth emphasising that the investigated misconceptions were related to some issues that undergraduates would have subsequently addressed in their academic career; as a result, these incorrect viewpoints could constitute a significant obstacle that may cause their failure at academic final exams, thus increasing the number of students who take longer than expected to graduate, as well as the number of dropouts (Oldfield et al. 2018). This latter phenomenon depends on both cognitive and affective aspects and our research can be regarded as an initial move to tackle this problem. Actually, highlighting some important misconceptions on classical Physics can represent a first step towards the redesign of some high school and academic Physics lectures. A learning experience attentive to these misconceptions and based on active learning may not only enhance the scientific background of the students (Bozzi et al. 2018; T. Vickrey et al. 2015), helping them to reduce their future learning difficulties, but also increase their self-efficacy and self-esteem. Indeed, this renewed didactic strategy should consider the issue of fostering students' mastery experiences, observational experiences, social persuasions and positive physiological and psychological mood states, i.e. the four main sources of creating self-efficacy according to social cognitive theory (Dinther et al. 2011).

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Chapter 22
Commentary on the Secondary-Tertiary Transition from a Teacher's Perspective

Paola Landra

22.1 Introduction

Self-study research is an emerging area that lies between biography and history, between the way private experience provides insight and solution for public issues and troubles and the way public theory provides insight and solution for private trial (Bullough and Pinnegar 2014). Even if self-study refers to direct personal experiences of the researcher, it transcends the mere experience. Data collected from personal experiences are analysed through the theoretical lenses that are provided (and accepted) by the community of practice (Wenger 1998), to which the teller belongs. As such, these experiences become a case for a more general phenomenon, thereby informing the community's knowledge about it and providing a special perspective from which to observe it. Self-study research does not focus on the self per se, but rather on the space between the self and practice (Mooney 1957), and between the self and the others who share the practice setting (Hamilton 1998). In other words, because data from personal experience are not just mere data from an experiment, but belong to the researcher's private sphere, they prompt the researcher herself to reflect on her research practice, and this may lead her colleagues to focus on the practices of their community.

Bullough and Pinnegar (2014) suggest some guidelines for self-study researcher. When self-study takes the form of an autobiographical narrative, it should enable connection, promote insight and interaction, portray character development, and include dramatic action. Clearly, biographical and autobiographical self-studies

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in mathematics education concern problems and issues of teaching and learning mathematics. Autobiographical self-studies attend carefully to persons in context and offer fresh perspectives on established truths (Bullough and Pinnegar 2014). This is the aim of what follows in the commentary. In the author's teaching experience, in fact, the transitions from one school level to the subsequent one represents a critical point for mathematics teaching and learning. A student has to change not only his habits, but also his beliefs towards mathematics (Clark and Lovric 2008).

The teacher, on the other hand, finds herself having to intervene on the didactic level: not only on the contents to be taught, which certainly undergo transformations both in the way they are dealt with and in the specific goals attached to them, but also on motivations and method of study.

I divide the reflection on my experiences into the two different transitions that I have extensively observed and cared upon, namely the one from middle (grades 6–8 in Italy) to secondary school (grades 9–12 in Italy), and the one from secondary school to university.

22.2 Mathematics from Middle to Secondary School

My experience is above all the one of the students who come from middle schools and who are low achievers. For example, in the calculation with fractions, which occupies a large part of the middle school curriculum, it is very recurrent that my students have serious difficulties in using almost all the rules for calculating fractions and, above all, in determining the common denominator. The teacher immediately realizes that the gaps, and the uncertainties that derive from them, are structural. We, as a team of math teachers in my school, try to overcome these difficulties by suggesting to the student, for example, for the calculation of the common denominator of several fractions, techniques that bypass all the calculation processes (from the factoring of a number to the search for the least common multiple), and by trying to engage the student in a more active and responsible attitude.

Back to my particular standpoint and experience with low achievers in mathematics, I maintain that this very particular point of view has allowed me to observe many cases where difficulties and memories of bad experiences with mathematics prevail. The students who enroll in my school can be said to be fearful and uncertain, they tend to show a very low interest in mathematics itself, and they tend to adjust to and please the teacher's requests and attitudes in order to minimize their effort in doing mathematics. However, I can say that these students are well aware of the differences, in terms of methods and requests, between middle and secondary school. They feel these differences as enormous.

Do middle school teachers build their teaching paths on the basis of the skills and motivations of the part of the class who has no problem understanding the contents of mathematics and tend to overlook those who struggle? Perhaps that teachers are very concerned about teaching, ignoring the development of an adequate system of self-regulation of learning (Caponera, Pozio and Palmerio, this book, Chap. 19).

At the beginning of high school, those who feel unworthy of math have the perception that there is nothing more they can do to improve their learning in mathematics: they see themselves destined to never be able to access the "secrets" of math and therefore they somehow get by exploiting more or less lawful stratagems to "survive" to math lessons. In fact, those who have proved weaker in learning mathematics enter the secondary school without having developed their own method of personal study. A student is, thus, afraid of making a mistake and this blocks him at any constructive opportunity, when facing new mathematical problems; a student continually evades the possibility of approaching contexts other than those already explored. The student tends to slow down her learning processes, because, faced with the risk of failure, she seeks for all possible opportunities for distraction, among which, in particular, those offered by her classmates. Moreover, the following prejudices frequently occur in low achieving students: numerical calculation is preferred to literal calculation; it is preferred to solve expressions rather than to face problems that presuppose the reading and interpretation of a text; algebra is preferred to geometry. The elements underlying these prejudices are: weakness in abstract thought, low ability in interpreting a text and arguing about it, low ability in seeing the "abstract" properties of forms present in the real world. The elements that, in my opinion, are most lacking in students who show little interest in the study of mathematics are the ability to listen and concentrate.

Faced with groups of students who present the above described problems in mathematics, the teacher struggles to promote a method of work and study that is effective and efficient, both individually and at the class level. She must be very determined to constantly intervene, with continuous reinforcement actions, that is, to enhance all the positive elements offered by the students in their, albeit uncertain, proceeding towards the math contents presented. And, often, she finds herself intervening with strongly persuasive actions, in order to convince the students to start making an effort to overcome their prejudices against mathematics and towards their ability to be successful in mathematics. When the teacher manages to convince the student of the need to "immerse herself" in the reasoning and the steps necessary to understand mathematical contents, and to keep trying, accepting herself making mistakes, then she begins to see tentative positive results. And the student begins to take math lessons more willingly, because she finds satisfaction in the results she obtains and starts to believe that her effort to commit herself is paying off.

As a final remark, I would like to add that, in my experience, when there is difficulty in tackling mathematics, there is a demotivated and demotivating family environment, often with poor cultural background. The stimulus to do better, to overcome oneself comes above all from the family context.

22.3 Mathematics from Secondary School to University

In my long (more than 10 years) experience as a lecturer for first-year university students, I have had the opportunity to interact with a variety of students from secondary school, from the most excellent to the weakest ones. I noticed that the quality of preparation is independent from the type of high school attended, while it is the type of experience with math that seems to count more (i.e., in which educational context the student lived and which teachers she met). I was also able to note that passing the final math exam depends more on the motivation and the "tenacity", with which the student faces the mathematics course (within the chosen university course), rather than from the preparation received during high school. This partly confirms Girnat's (this book, Chap. 18) findings.

I have observed many different attitudes of students in approaching the first-year mathematics course at university. Some already feel well prepared and capable of learning without the teacher's need for mediation and accompany this feeling with a sense of superiority towards both the new context and their classmates. For others, conversely, there is a great respect that leads to a constructive participation and adaptation to the demands and objectives of the university course. Usually these students, if they also have good preparation from secondary grades, are the ones who manage to achieve the best results at the exams. For students with a more fragile preparation (fragility can be either actual or felt), an attitude of fear prevails. Different types of students belong to this group. The uncertainties can depend both on a real poor preparation, but also on uncertainties related to the student's identity and her current or past experience. It can also happen that there are students who show off with a critical attitude and a constant request for attention towards them. Unlike insecure students, on the contrary, these ones seem to have clear and certain expectations. They are very ready to ask to repeat an explanation several times or a clarification or an example or, vice versa, to immediately get to the point, and so on. The latter students do not necessarily succeed in achieving positive results, and often get lost on the street.

In conducting, over the years, bridge courses, tutoring, review courses, OFA (Obblighi Formativi Aggiuntivi, that is Additional Training Obligations) courses, I have experimented with different ways in which students face their specific difficulties in mathematics in the transition from high school to university. Unfortunately, only a small part of students immediately become aware of their difficulties and have the courage to manage them promptly, as early as they enter the university environment. Usually, it is much more frequent that awareness occurs after some time, perhaps after having failed the math exam a significant number of times! Overcoming one's difficulties and, therefore, the ability to achieve a positive outcome in mathematics is very tied to the student's awareness of her condition and the ability to ask for help adequately.

What are the previous mathematical contents that cause more difficulties to students with poor and incomplete preparation? In the first place, I would put the second degree inequalities, followed by calculation with letters and properties of elementary geometric figures.

In conducting the first-year mathematics course, I notice that it is much easier to deal with those topics that are completely new to the students. I would say that the student's previous experience with the topic, even if on the one hand offers her points of reference, on the other hand this provides her with self-confidence about those concepts she feels already acquainted, preventing her from widening her perspectives and consider new approaches towards the topic.

Even in a university course, I find it important to act on students' motivation and study method by offering a precise, though not binding, method for study. I find it important not to assume that all students are well versed in math and have an effective and efficient study method. It is important, in my opinion, in this orienting endeavour, also to offer the opportunity to make personal insights by giving a wide bibliography and interesting material downloadable from the web.

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Chapter 23
Commentary on Anonymous Versus Non-anonymous Surveys

Chiara Andrà, Domenico Brunetto, and Alessia Pini

23.1 Literature Review

In this part of the book, within the previous chapters, affective and cognitive factors that shape and influence transition(s) have been highlighted, with a special attention paid to the mutual influence of affective variables and cognitive achievement.

In order to investigate the role and mutual relationships of these variables, both questionnaires and tests are useful. Usually, the former allow the researcher to get an insight on affective and social aspects, and the latter can give information about mathematical knowledge: Girnat (this book, Chap. 18) used a questionnaire to collect data on self-efficacy and anxiety with respect to mathematics, inspired by OECD-PISA tests, and two local tests for mathematical achievement; while Caponera, Pozio, and Palmerio (this book, Chap. 19) referred both to OECD-PISA and to IEA TIMMS studies for social and affective variables, while they relied on the Italian national evaluation system for achievement.

Within Girnat's context, if the students attend a Bridge Course (at the beginning of university studies) that lasts some days, it can be interesting to administer a questionnaire in the first day and another questionnaire in the last day, plus some tests to measure also the students' understanding of the mathematical concepts delivered during the course. Within Caponera et al.'s context, the national evaluation

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test can be accompanied by a questionnaire, administered at the same time. At this point, it comes the research question that frames our contribution: *should the questionnaires and tests be anonymous or identifiable?*

The question has methodological implications, as the example provided in the next section highlights, but it has also foundational premises, which we illustrate in what follows.

The categories of *public* and *private* seem to correspond to various important aspects and activities of mathematical learning. Among the private aspects of mathematical learning, for example, may be counted reflection, internalization, visualization, and the creation of mathematical meanings, which have been given considerable weight in mathematics education and mathematics education reform (e.g. Clarke 2001; Fried and Amit 2003; NCTM 2000; Schoenfeld 1992; Skemp 1987). When learning is private, the students feel free to make mistakes, to express their doubts, and to be creative (Fried and Amit 2003). To note, in learning activities based on social construction of meanings, also in public learning the students are free to make mistakes, to express, and share their doubts with their mates and/or the teacher, and this is valued. However, the distinction between 'private' and 'public' is rooted on a different stance.

In our use of the words 'public' and 'private', we have primarily in mind the way they are employed in political theory, where these terms refer to 'access' and, especially, 'accountability' (Benn and Gaus 1983). By *public activities*, then, we mean those activities with regard to which one is accountable to teachers, peers, or co-workers; these are activities, therefore, in which one is bond by common practices and by the necessity of formal communication. Usually, questionnaires and tests, where to show one's identity is requested, belong to a public sphere; while tests for self-evaluation belong to a private one.

By *private activities*, on the other hand, we mean those with regard to which one is not accountable to teachers, peers, or co-workers; these are activities in which one is free from the expectations and constraints of common practice, activities that, as one writer puts it, take place in 'a zone of immunity' (Duby 1985, p. 10); here, one is free to explore, backtrack, and reflect. Anonymous tests and questionnaires belong to a private sphere.

The distinction between identifiable and anonymous questionnaires and tests is tied to a more general consideration that different pedagogical practices or assessment regimes may cause a given mathematical activity to be termed as either private or public. Students' homework, for example, might in one pedagogical setting be discussed, collected, and marked, and, therefore, take on a public character; in a different pedagogical setting, homework might be given only to reinforce classroom material and never be seen by anyone except the students themselves. Some aspects of mathematical activity, on the other hand, seem to resist relocation from one sphere to another; for example, mathematical papers or projects as public and individual preliminary reflections as private would be hard to reclassify as private and public, respectively (Fried and Amit 2003). Moreover, students can cheat in both settings, for different reasons, but this issue is out of the scopes of our research.

Both ‘cognitive’ tests and ‘affective’ questionnaires can take a private or a public form, and these differences can play an important role in the ways respondents participate to the survey conducted by the researcher. Let us consider an example that regards a research conducted under very similar methodological demands with respect to Girnat’s one.

23.2 An Example

A Bridge Course for mathematics is delivered every year at the Polytechnic of Milan (Italy): it is a preparatory course before the beginning of the first semester. The funding source of the course is the Polytechnic of Milan, which had no involvement in any phase of this research. Data analysis had been partly funded by the Italian National Project ‘Lauree Scientifiche’ (STEM degrees), which aims at enhancing interest towards scientific career in young people. As such, the national project aims also at understanding students’ difficulties with science and mathematics at different school levels, since these difficulties may impede learning and even lead a student to leave university (drop out). The Bridge Course recapitulates the basic math knowledge learned at school (i.e., arithmetic, algebra, 2D Euclidean geometry, calculus) and is made of an e-Learning part (i.e., a Pre-Calculus MOOC on Polimi Open Knowledge platform, www.pok.polimi.it), and an attendance part.

The data for our study come from two questionnaires, which investigate *affective* factors, and four tests, which assess the students’ *knowledge* on algebra, geometry and logics, calculus, and probability and statistics. The first questionnaire (label: Q1) has been given to the students at the very beginning of the attendance part, on the first day, while the second one (Q2) was administered at the end of it, on the last day.

Every year, since 2014, around 1200 students attend the Bridge Course. Table 23.1 shows the number of questionnaires and tests answered by students in the 2 years under study, namely 2016 and 2017. Data collected in 2016 were anonymous, while data collected in 2017 were not. This difference between 2016 and 2017 significantly impacts the number of respondents, as Table 23.1 displays. In 2016, when questionnaires and tests were anonymous, more students answered than in 2017, when the identity of the respondents was asked.

Interestingly, in the non-anonymous setting, the dropout regarding Q2 is huge: only 38 students responded, which corresponds to 16% of the students who

Table 23.1 Number of questionnaires and tests answered by students in the 2 years under study

Time of investigation	Q1	T1	T2	T3	T4	Q2
September 2016	589	535	505	500	331	369
September 2017	231	193	163	181	136	38

answered to Q1 in the same edition. In the anonymous setting (i.e., September 2016), there is a dropout as well, but the percentage of respondents is 63% of the ones who participated to Q1 in the same edition.

A first, empirical conclusion that we can draw is that, when the students are asked to provide a feedback about the course (i.e., answering an affective questionnaire), being anonymous encourages them to respond. When they are asked to answer to math tests, instead, being anonymous or not seems to affect their will to respond to a smaller extent.

From the perspective of the researchers, in an anonymous setting we cannot link students' answers to affective questionnaires with students' answers to mathematical tests. In our specific research, however, since we identified different communities of students with respect to gender, school type, and views about online learning formats, and since these information were collected in all tests and questionnaires, we were able to establish a (weak) connection between the four questionnaires and the two tests, and to see if similar affective features influence the test scores, as an indicator of mathematical achievement. On the one hand, this is a limitation since the communities are not a partition of the students in terms of gender, school type, and e-learning views. On the other hand, we claim that, even in a totally anonymous setting, it is possible to identify four overarching, general trends that at a gross grain give a representative picture of well-known phenomena related to dropout.

Data collected in 2017 were not anonymous and, in this case, it was possible to link all the students' answers. Interestingly, we were able to identify the same communities and general trends that emerged in the anonymous setting, validating the results that we found in the former case (see Andrà et al. 2020). Three different communities have been outlined: the first one is populated by students with a strong mathematical background, who perform well at the math tests and prefer rather traditional teaching formats. The second community also values frontal lessons, but the students are weaker in mathematics and they are more likely to drop out the university studies. The third community is populated by the weakest students in mathematics, with low background and poor performance in math tests, but with positive attitudes towards innovative teaching formats like online learning. The students in the first and in the third communities have highest probability to succeed in their STEM careers.

23.3 A Final Remark

This commentary proposes a comparison between two ways of collecting data: anonymously, which leaves space for the students to think and to feel free to 'be themselves', but also encourages them to respond, or identifiably, which allows the researcher to establish stronger connections among data. This is an element of novelty for our research: questioning about better settings for respondents to provide data for the researcher seems to be an under-researched area—which may deserve some attention from a researcher' perspective. To our knowledge, some attempt has been made only by the OCSE-PISA test. In the identifiable setting the number of analyses that can be performed on data of course increases, and thus the amount of information that can be possibly gained from such an analysis also increases. Nonetheless, the number of students that are willing to answer to a non-anonymous questionnaire is lower, and results from this setting might be less strong due to the smaller sample size that negatively influences the power of the statistical procedures. In addition, nothing can be said about the selection bias in the identifiable setting: it is natural to assume that students willing to answer to a non-anonymous questionnaire are not a random sample of all students, but they are likely to be the most interested to the course and the most comfortable about answering. This fact can be relevant for affect-related research, specifically.

In the case of the data analysed in this paper, taking on an overarching perspective of the phenomenon, we can comment that data collected in the two settings are consistent with each other. Hence, we are prone to conclude that—especially in contexts when a finer grain analysis is not at hand—the anonymous setting is slightly better than the other one. In other words, to know the students' names did not add richness to the data analysis. This is particularly important in an educational setting, since anonymous data on learners also fit within ethical requirements about the accessibility to sensitive data concerning their knowledge, abilities, and skills.

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Part V
Cultural Issues in Affect-Related Research in Mathematics Education

Chapter 24
Introduction

Francesca Martignone

Nowadays we have the possibility to have contact with educational research and practices developed in different cultural contexts. Culture permeates both mathematical practices and mathematics education. Recent plenaries at CERME and ICME addressed cultural aspects in educational studies in mathematics (Jaworski et al. 2015; Barton 2017). Dealing with these aspects is crucial in the communication of research in Mathematics Education, therefore this is an issue that cannot be considered as secondary when presenting educational studies (Andrews 2010; Bartolini Bussi and Martignone 2013).

Bishop (1988) argued that mathematics practices are social phenomena embedded in those cultures and those societies that generated them. Ethnomathematics studies (e.g., D'Ambrosio 2006) show that taking into account cultural issues contribute to better understand mathematics itself. The different cultural backgrounds generate also different mathematics education perspectives and then different school mathematics practices (Mellone and Ramploud 2015). Teaching and learning are culturally determined activities. Also the affective domain is related to a sociocultural and historical world (Radford 2015). When we reflect on educational activities carried out in a country, its own mathematics history, school curricula, political, social and philosophical features have to be investigated. Andrews (2010) argued how historical and cultural forces have shaped the development of the curricula of different countries. A recent approach suggests to observe and consider the meanings embedded in the educational practices of other cultural in order to rethink those that are rooted in one's own educational practices (Ramploud and Di Paola 2013; Bartolini Bussi et al. 2013). Therefore, reflecting on cultural diversity and the ways to benefit from it is also a topic of the research about mathematics teacher education. Recent approaches, such as a study about "cultural transposition"

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(Mellone et al. 2019) analyse the process of decentralizing the educational practice of one's own cultural context through the contact with educational practices of other cultural contexts.

Knowing and trying to understand other cultural contexts can help us better understand ours, even though it is always very difficult to be aware and to analyse the features of the culture in which we are embedded (Jullien 1993). International conferences on mathematics education are a suitable room for comparison and meeting between different researchers who bring their experiences linked to the context in which they work and to their cultural beliefs. For example, in the last CERMEs the topic of cultural aspects has been explicitly addressed (e.g., in the TWG10 "Diversity and mathematics education: Social, cultural and political challenges" and in other CERME Theoretical Working Groups contributions). Also in the recent ICMI study conferences cultural aspects emerged clearly: e.g., in the discussion about school mathematics curriculum reforms developed in different countries around the world (ICMI study 24: http://www.human.tsukuba.ac.jp/textasciitildeicmi24/). MAVI25 was a fruitful opportunity to discuss these aspects in studies about affect and beliefs and then to open up new research perspectives. In particular, even if, as previously stressed, in recent years there has been an increasing attention to cultural aspects in mathematical education research, it emerged that there are still many strands of research in which this interpretative lens has not yet been widely adopted. For this reason, three researchers have chosen to investigate a topic that emerged during the conference: the contribution of Ferretti, Funghi and Blum (this book, Chap. 25) aims at focusing on cultural beliefs and national standardized assessment features. The authors present some characteristics of external assessments in four European countries (England, Germany, Italy and Spain) in order to point out specific questions about possible cultural beliefs underlying choices on assessment. In particular, they underline the influence of cultural beliefs on choices made at the institutional level to conduct standardized assessment. The countries considered are all Western countries, therefore it is possible that in countries where there is a different conception of learning—for example, Far East countries—the purpose and the use of feedback of the outcomes of external assessment could be conceived in a different way. As a matter of fact, the choices of national institutions about standardized assessment are linked to beliefs about what assessment means and what kind of outcomes are desired.

Finally, in this part of the volume dedicated to contributions that somehow address the theme of cultural aspects related to beliefs, we find the chapter written by Andrà and Brunetto. Their research is developed within the Italian project "Teenagers Experience Empowerment by Numbers" (TEEN). This pilot research presents some activities proposed by means of an app for teen-immigrants who need to learn basic mathematics to cope with European lifestyle. This contribution describes the choices on the type of problems proposed and how they are presented taking into account the needs of the subjects for whom the app was conceived. The *empowerment* interpretative lens is used to analyse the cognitive and affective aspects identified in the activities carried out with the app.

Both the researches presented in this part of the volume are initial studies, but they open interesting and new scenarios that could be discussed and developed also in the next MAVI conferences.

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Chapter 25
Issues About Culture, Affect and Standardized Assessment

Federica Ferretti, Silvia Funghi, and Sabrina Blum

25.1 Introduction: Cultural Issues on Affective Aspects

In one of his famous papers, Hannula (2011) proposed to systematize the affective domain distinguishing three dimensions: "(1) cognitive, motivational and emotional aspects of the affect, (2) rapidly changing affective states vs. relatively stable affective traits, and (3) the social, the psychological and the physiological nature of affect" (p. 34). These dimensions are included and intersected within an ideal 3 $\times$ 2 $\times$ 3 matrix that should serve as a theoretical framework for the domain of affect. Nevertheless, in this schema the cultural variable seems to be missing—or at least, it seems to remain hidden in the overall structure. In fact, even if it can be argued that cultural issues can be included in studies on the sociological perspective, there are studies illustrating that cultural influence regards not only the social level, but also the psychological one—see the discussion on Radford's words in the following paragraphs.

If we focus on the first dimension—the one distinguishing the focus of the research: cognition, motivation, emotion—we can see that there are studies showing cultural issues for each of them. On the side of cognition, many studies illustrate that there is the need to take into account the cultural context in relation with beliefs underlying teaching and learning practices (e.g. Bryan et al. 2007; Li 2012; Stigler and Perry 1988; Tobin et al. 1999). On the side of motivation and emotions, Radford

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(2015) emphasized that they are both deeply connected with the way humans see the self in relation with the world.

> [. . .] the affective domain in general and motives and motivation in particular are not only subjective but also sociocultural phenomena. They are subjective and sociocultural in the sense that on the one hand motives are the *motives of a concrete and unique person* but, on the other hand, they relate to a sociocultural and historical world that *transcends the individual*. [. . .] Emotions do not only drive our affective life; they also shape the manner in which we understand the world and ourselves [. . .] They entail a range of cultural conceptual categories that are instantiated differently by different people (e.g. moral and ethical categories; notions of privacy, responsibility, autonomy, etc.). (Radford 2015, p. 29; italics in original)

In this perspective, motivation and emotions can be seen as culturally determined, because they depend on what is thought to be right, moral, etc. in a certain culture. If we recall that Schoenfeld (1985) described beliefs systems as "one's mathematical world view" (pp. 43–45), we can recognize a sort of *trait d'union* regarding all the parts of the first dimension detected by Hannula: affect in his whole can be seen as a sort of mirror of the way humans conceive themselves in the world. But this conception, as any conception a person has, cannot be taken as isolated from the cultural context in which the person has grown up. As Bruner (1996) emphasizes, "culture shapes mind . . . provides us with the toolkit by which we construct not only our world but our very conceptions of ourselves and our powers" (p. X). Thus, culture should be considered as one of the main factors influencing the way people think about what is moral, what is legal, what is "better" for the community and for the individual, and also how a person should feel in certain circumstances.

We believe that in a world that is more and more globalized, where the world of research reflects this trend, there is a growing need to become more and more explicit about the implicit beliefs, assumptions and meanings hold by different cultures. In the following, we will recall some results of research on cultural beliefs, in order to address the lack of studies exploring the influence of cultural beliefs on the process of construction, administration and realization of standardized assessment.

25.2 Cultural Issues on Beliefs

Despite the growing body of literature regarding cross-cultural studies on beliefs about teaching and learning in different countries, we believe that it is still missing in literature a clear acknowledgement of the influence of cultural beliefs on the way the school is organized, on the way teachers teach and on the way children are supposed to learn—what Bruner (1996) calls "folk pedagogy". According to Andrews (2010), in fact, culture "permeates all aspects of educational endeavour and should be acknowledged more explicitly than it is" (p. 3).

The impact and the consequences of different beliefs about education on the real school can be grasped thanks to many studies highlighting profound differences between Western culture and Eastern culture. Li (2012) analyses in detail the gap dividing Eastern and Western perspectives on education, on learning, starting from the very meaning of the verb *to learn*. She emphasizes that, while in the West learning is seen as a process driven by feelings of interest and curiosity through which the learner discovers the external world, in the East learning has a completely different meaning: “learning is the most important thing in life; it is the life’s purpose. Learning enables one to become a better, not just smarter, person. The ultimate purpose of learning is to self-perfect and to contribute to others at the same time” (p. 14). On the other hand, also teaching has a different meaning: “[In the East] learning has more to do with forming attitudes than cognitive work … teaching is not so much about knowledge but a way of life” (Lee and Sriraman 2013, pp. 151–152). East and West differ also in the way society see failure: in the West failure is attributed to a lack of inner qualities (Tobias 1993), in the East to a lack of effort by the learner Li (2012).

Moreover, some studies highlight that teachers tend to have different beliefs depending on their culture. One of the main differences concerns, for example, the right of the learner to talk during the lessons. Lee and Sriraman (2013) highlight that in Confucian societies teachers believe that the learner must elaborate his/her own thoughts in silence before talking; in the West, instead, talking is seen as an indispensable part of the entire process of learning. In some cultures, young people are even not allowed to ask question to elder ones, so that pupils are even prevented to make questions to teachers (Jaworski et al. 2015). Bryan et al. (2007) emphasize also that in Mainland China teachers often believe that true learning has to be necessarily anticipated by a phase of knowledge memorization, which is seen as a first step to be followed by variated exercise in order to achieve a real understanding (Wong 2006). Yang and Cobb (1995) highlight that Chinese and U.S. people have different beliefs regarding the “natural” development of arithmetical abilities in children, which affect the different kind of arithmetical learning activities proposed to children in the two contexts.

These aspects we have briefly illustrated, without pretending to be exhaustive, are just some examples to show the extent of cultural influence on education and teaching practices. But also at the institutional level the choices made about education are implicitly based on cultural assumption about how children are supposed to learn and teachers are supposed to teach. Andrews (2010) shows that different beliefs on education are reflected in the curricula proposed in some countries of Europe: for example, he underlines that loosely structured curriculums—as the English and Finnish ones—reflect cultures where “dissent and deviation are tolerated and people are willing to take risks” (p. 8), whereas cultures that are less tolerant toward dissent are characterized by structured curriculums. Similarly, Xie and Carspecken (2008), analysing the differences between the Chinese and one of the U.S. curricula for mathematics, underline that choices made by policy makers are deeply rooted in the beliefs shared in the two cultures.

> The construction of a curriculum and its recommended instructional methods are largely based on beliefs about how student learn and how knowledge develops. These beliefs in turn entail more deep-seated beliefs . . . pertaining to philosophical anthropology, developmental theories, mathematics philosophy and philosophy in general. (Xie and Carspecken 2008, p. 20)

25.3 Culture and Assessment

According to a Vygotskian perspective, knowledge is inextricably linked to the activities in which the subjects engage and this must be considered in close relationship with the cultural institutions of the social context from time to time considered (Radford 1997, 2003). A strong epistemological assumption underlying the socio-cultural perspective is that knowledge is built up socially; so cultural institutions influence pupils. This inevitably has repercussions on the whole process of learning and teaching mathematics, therefore also in the sphere of assessment. In a perspective where knowledge is not produced in an exclusive relationship established between the individual and the problem to be solved, the social dimension of knowledge must be taken into account by assessment tools and techniques. In this dynamic socio-cultural approach, the learner's understanding of mathematics is seen as a process of intellectual cultural appropriation of meanings and concepts (Radford et al. 2000). The class context assumes a fundamental role and it is considered a specific society of individuals. It is socially constituted as a unit, because it has defined and shared practices to carry out (Godino and Batanero 1994). As a consequence, social and individual practices are framed by the context, i.e. the environment. From this point of view, classroom—and assessment—practices are part of a system of adaptation of individuals to society, and to its culture. Cai and Wang (2010), for example, show that teachers' assessment practices are influenced by cultural aspects. They compare teachers' evaluation criteria for representations used by students to solve a set of items, interviewing a group of Chinese and a group of U.S. teachers: it emerges that Chinese teachers tended to give higher scores to strategies involving symbolic representation and discouraged the use of concrete representations because of their lack of generalizability; U.S. teachers, instead, tended to give equal value to both concrete and symbolic representations.

On the other hand, also the way large scale assessment is conceived depending on culture. Dealing with assessment both from the large scale viewpoint and from a classroom viewpoint, Gipps (1999) underlines that—if assessment in its origin was born with the aim to select and certificate those who had a competence to aspire to some professions—over the last 20 years the spread of constructivist and socio-culturalist theories of learning has produced a shift in the way assessments is conceived: "the focus has shifted toward a broader assessment of learning, enhancement of learning for the individual, engagement with the student during assessment, and involvement of teachers in the assessment process" (p. 367). She describes this approach to assessment as alternative to the traditional, "psychometric" one:

"Fifteen years ago ... Testing (and examining in the United Kingdom) was seen as a technological activity based in psychometric theory with its emphasis on replicability and generalizability" (p. 367).

Nowadays, large scale assessments are increasingly playing a crucial role in assessing students' learning internationally. Whether international or national surveys, the situation that is created involves several actors also at institutional level. In fact, external assessment is permeated by decisions (including political ones), for example:

Assessment objectives. Is the main objective to perform an assessment of the school system at what levels? At the entrance and exit of a particular school segment or only at the beginning or end of a course? Is the aim to have a system photograph or to focus more on the individual?

Construction of the assessment. Are the standardized tests built with the aim of identifying knowledge, skills or both? Is the assessment aimed at investigating all areas of mathematics content or just some? Does the framework refer to the country's National Guidelines? Does it refer to the frameworks of international standardized assessments (such as the OECD-Pisa or IEA-TIMMS surveys)? Do the tests consist of open-ended or closed-ended questions or both?

Assessment administration. Are the tests administered at the sample level or at the census level? Are they administered by external agents or by class teachers? In anonymous mode or not? Are there any predefined times? Is the time taken by each student to perform the test or is it not an important information? Is the administration Paper & Pencil or Computer Based Testing?

Assessment feedback. Who corrects the tests? The class teachers or an external evaluator? Is there a centralized correction? How is the feedback made? Is there only a system-level return or is it also returned to individual schools? Does the feedback refer to a statistically representative sample of the population?

We believe that the answer to each of the above issues is closely related to the cultural context in which they are immersed. In fact, both the assessment of the learning—understood as the choice of content to be investigated and the way in which the attainment of knowledge and competencies is investigated—and the way in which the surveys are administered and returned, should be strictly intertwined with the mathematics curriculum, and therefore also with the cultural beliefs underlying it. In this paper we compare some features of the standardized assessment carried out in some European countries, in order to point out some questions about possible cultural beliefs underlying choices on assessment.

Standardized Assessment of Some European Countries: Are There Hidden Cultural Beliefs?

We present a comparison on some aspects of standardized assessment conducted in four different countries: England, Spain, Germany and Italy (see Table 25.1). We choose these countries depending on the possibility for us to collect information about national assessment practices—thus it can be considered a "convenience sample".

Table 25.1 Characteristics of standardized assessment conducted in England, Spain, Germany and Italy

	England	Spain	Germany	Italy
Objectives				
– *Main objective*	Evaluation of the educational system	Evaluation of the educational system	Improvement of teaching/school development	Evaluation of the educational system
– *Focus (system/individual)*	System	System	System	System
– *Grades*	Grades 1 and 5 (Primary School)	Grades 3 and 6 (Primary School); Grade 10 (High Secondary School)	Grade 3 (Primary School); Grade 8 (Secondary School)	Grades 2 and 5 (Primary School); Grade 8 (Low Secondary School); Grades 10 and 13 (High Secondary School)
Construction				
– *Subjects*	English, Mathematics and Science	Spanish and Mathematics	German and Mathematics; in Grade 8 are included also foreign languages and sciences (biology, physics and chemistry)	Italian and Mathematics; English (since 2018)
Administration				
– *Respondents*	Whole population of students	Whole population of students	Whole population of students	Whole population of students
– *Role of external agents/teachers*	Administration by teachers	Administration by teachers	Administration by teachers	Administration by teachers

– *Paper&Pencil (P&P) /* Computer Based Testing (CBT)	P&P	P&P	P&P	P&P until 2016–2017; CBT for students older than 12 from 2017 to 2018
Feedback				
– *Correction*	Decentralized teachers	Decentralized teachers	Decentralized teachers and public institutions	Centralized correction for CBT tests; decentralized correction for P&P tests
– *Addressee*	Feedback both at the national level and at the individual level (Families are not directly informed of student's result for test in Grade 1, but they are informed for the test in Grade 5)	Feedback both at the national level and at the individual level (Families are informed of student's result)	Feedback both at the national level and at the individual level (Families are not directly informed of student's result)	Feedback both at the national level and at the individual level (Families are not directly informed of student's result)

As it can be seen, administration modality and subjects are quite similar in all the countries considered: tasks are administered by teachers and tests involve the whole population of students; the focus is on mother tongue and mathematics, adding sometimes other subjects. There are differences instead regarding the age at which tests are administered: all countries present the first compulsory test between 6 and 8 years, but there are considerable differences in the number of standardized tests conducted during compulsory education in its wholeness—in particular, Italy is the only country establishing compulsory standardized tests *throughout the whole students' compulsory education*, and this could be indicative of an aim to evaluate the educational system *on the long term*. Italy is also the only country that introduced CBT testing in higher levels of education—which could be related to a change of the kind of skills and competencies that standardized tests aim to assess.

There are also differences in the way correction is carried out and feedback is given—for example, in Spain and England families are explicitly supposed to be one of the addressee of the feedback, whereas in Germany and Italy tests seem to be mainly addressed to teachers and school directors. This difference could be connected with the purpose of assessment. In fact, even if in all countries but Germany national assessment has the main aim to evaluate the educational system, there is also the will to give a feedback useful to grasp information about the attainment and progress made by each student, in Germany and Italy there could be the implicit belief that the communication of this information to families should be mediated by teachers and school institution. Moreover, the fact that correction in Italy has become centralized for CBT tests could be linked with a different conception of the kind of correction and of feedback that is needed.

In order to go a little bit further than the overall structure of standardized assessment, we chose four official documents explaining the framework of the national standardized tests for the first test carried out in primary school (age 7 for UK and Italy, age 8 for Spain and Germany) and we checked if and how *mathematical competence* is defined. We found out that Spain and Italy refer to the same definition—the one proposed by the European Council in 2006—whereas in the English framework the word *competence* is not even used—it appears just twice and it is not defined. It talks more frequently of mathematical *skills*—but even this term is not defined. The German document is entirely focused on "mathematical competence"—beginning from the title—but also in this case there is no definition of its meaning, and there is no reference to the work of the European Council previously mentioned. It seems to be taken for granted. Moreover, differently from the three other countries, Germany makes an explicit distinction between competencies related to specific content domains and general, transversal competencies (i.e., solving problems mathematically, communicating, arguing, modelling and representing). In relation to this consideration, we can observe that the aspect of communication and arguing seems to remain implicit in the definition proposed by the European Council.

25.4 Remarks and Further Directions

In this paper we attempt to address some issues regarding cultural influence on affective aspects. In the introduction we argued about the need to acknowledge the cultural dimension of affective aspects, and we used this issue to get to another problem, namely the lack of studies highlighting the influence of cultural beliefs on choices made at the institutional level to conduct standardized assessment. Since the aim of this work was to point out this issues and to address them as research perspectives to be investigated, we do not draw conclusions, but we offer some hypotheses and some hints that we hope will be developed in further studies.

The standardized tests in the countries we have compared seem really similar in the way they are conducted, for example, they all involve the entire population of students and—at least for the tests carried out in the first years of primary school—they are addressed to pupils of similar age. However, what can be more significant in our perspective is that the presence of a double kind of feedback—at both national and individual level—could be linked both to the need to "measure" the effectiveness of the school system, and the need to translate the outcomes of such assessment in a constructive feedback for teachers, school directors and (in some cases) families and students. This could be a consequence of the Western shift in the view of assessment described by Gipps (1999), and in particular of the coexistence in Western countries of both the purposes of the traditional assessment—aiming to measure constructs and to predict—and the purposes of the formative assessment—which should be focused on supporting students' educational development. In this perspective, it is possible that in countries where there is a different conception of learning—for example Far East countries—the purpose and the use of feedback of the outcomes of such tests could be conceived in a different way.

Moreover, the quick lexical analysis we have carried out within the official documents related to the frameworks of standardized assessment in four different European countries made us think that the meaning of words like *competence* or *skills* has not to be taken as shared, even within the European context. In fact, the differences found in the official documents analysed could be due to differences among European countries about the way being "competent" in mathematics is conceived and, thus, what the standardized tests should assess. Germany explicitly distinguishes from transversal competencies and domain-specific ones and this, for example, could be due to a different way of conceiving mathematical, with respect to the Spanish or Italian one, were transversal competencies are given more importance with respect to domain-specific ones.

We believe that, as it is for curriculum design, the choices of national institutions about standardized assessment are linked to cultural beliefs about what assessment means and what kind of outcomes are desired. In this paper we give just some suggestions concerning aspects that deserve further analysis and study—for example: (1) a deeper comparison between national reference frameworks, (2) a detailed analysis about tools of correction and on the way feedback is given, (3) an accurate comparison between the kind and the goal of items. We hope that further studies will shed a new light on this issues about assessment, also extending the comparison between non-European countries.

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Chapter 26
Experiences of Empowerment in Mathematics

Chiara Andrà and Domenico Brunetto

26.1 Introduction

The research project TEEN (Teenagers Experience Empowerment by Numbers, http://www.teen.polimi.it) deals with the phenomenon of young immigration, from Africa to Europe, which has received increasing attention given the very large number of young immigrants who leave their country (most of them without parents). Once they arrive in Italy, the teen-immigrants are accommodated in communities for minors. Institutional protection plans provide teen-immigrants for accommodation, food, health services and a language course. This turns out to be insufficient to deal with the requirements of the "real world" that they need to face early, considering that the protection guaranteed by the Italian legislation to unaccompanied minors ends on the day of their eighteenth birthday. The TEEN project aims at promoting basic mathematical literacy as another fundamental right that may significantly increase the level of autonomy of teen-immigrants (age 13–19), helping them to manage their monthly budget, to buy a train ticket, to read the pay slip, and even to manage their time. All these activities have a common root, that is the understanding of elementary mathematics, such as operating with integer numbers (sum, subtraction, multiplication and division), computing percentages, number sense, and understanding numerical information concerning time, cost, distance, equivalence.

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The idea that mathematics can play a role in *social integration* of disadvantaged students has been proven successful in other contexts. Civil (2008) reports a growing attention towards developing educational methodologies which improve young immigrants' learning, exploring and pointing out the social and ethical implications of teaching mathematics to a "minority". Powell and Brantlinger (2008) shed a light on the actual tension between "academic" and "everyday" mathematics, underlying the centrality, for mathematics teachers, to create alternative curricula with the purpose of showing mathematics as an accessible (and useful) discipline. To achieve this goal, it is crucial that teachers promote a new idea of mathematics as valuable skills and social integration. Stathopoulou and Kalabasis (2007) and Chronaki (2005) have provided evidence that Romani students learn better when mathematics is related to their identity and their attitudes. Gutstein (2003) shows that if teachers consider the students' language and the way they interact when they design the lesson, the students turn out to be more aware of the role of mathematics, and tend to appreciate it.

Drawing on these research findings, the TEEN project aims at developing a mobile app that can be used by teen-immigrants, outside the context of the classroom and without the interaction with a teacher, to develop their basic mathematical skills. In order to make the discipline more accessible and attractive, the app is designed to deal with everyday mathematics, with non-academic language and, to be inclusive, with the least possible amount of written words. The activities proposed by the app address situations that are familiar for teen-immigrants and are connected teenagers' identities. In the pilot phase of the project, we worked with small groups of teen-immigrants who volunteered to spend some time with us. We collected data on their becoming acquainted with the app, and on any related process that emerged. In particular, we tried to understand the way(s) in which empowerment increased. In the next section, we recall the main research findings concerning empowerment in Mathematics Education and Psychology, which provide us with a lens of analysis of the episode subsequently presented.

26.2 Theoretical Background

The pivotal term of the TEEN project in its acronyms is "empowerment", a construct that had been discussed since the first half of the last century, from a psychological point of view. In order to understand its meaning, firstly we focus on "powerlessness", namely the condition from which empowerment evolves. In Psychology, Freire (1973) argues that an individual becomes powerless in assuming the role of the "object" acted upon by the environment, rather than the "subject" acting in and on the world. Thence, the sense of powerlessness is due to the interaction between the individual and the environment, combining negative feelings such as self-blame, a sense of distrust and a sense of hopelessness (Kieffer 1984). The concept of empowerment emerges with the need to clear this condition of powerlessness, in particular it is seen as a process of becoming and as an ordered and progressive development of particular skills and political understanding. Empowerment, then, assumes a dual meaning: longitudinal dynamic of development and attainment of a set of insights, and abilities best characterised as "participatory competence" (Kieffer 1984).

Kieffer (1984) identifies four distinct and progressive phases, or "eras", for the developmental process: "entry", "advancement", "incorporation" and "commitment". Moving through all phases, at least two pervasive themes are identified. The first one is the function of continuing internal "constructive dialogue": in other terms, people need to feel confrontation to respond, that is an issue that touches them in gut, which is referred to as "gut issues". The second theme is the dynamic of the "praxis", which refers to the circular relationship of experience and reflection: actions evoke new understandings, which in turn provoke new and more effective actions. These two themes suggest that empowerment is not a commodity to be acquired, but a transforming process constructed through action, and the experience is the way through which this learning process occurs.

Furthermore, Zimmerman (1990) argues that another theme related to empowerment is the "learned hopefulness", which is the process of learning and utilising problem-solving skills and the achievement of perceived or actual control. The learned hopefulness can be seen as the opposite paradigm of learned helplessness, which occurs when individuals are exposed to uncontrollable conditions. Zimmerman (1990) claims that experience of some events can lead to either a feeling of lack of control, or successful control. Outcomes of the former are symptoms of helplessness, whilst in the second case the individual feels a sense hopefulness and psychological empowerment, as shown in Fig. 26.1. Hence, learned hopefulness suggests that experiences that provide opportunities to enhance perceived control help individuals to cope with stress and solve problems in their personal lives.

In the mathematical context, empowerment concerns the role of mathematics in daily activities and its impact both on the learning process at school and in social life (Ernest 2002). Three different domains of empowerment have been identified: *mathematical*, *social* and *epistemological*.

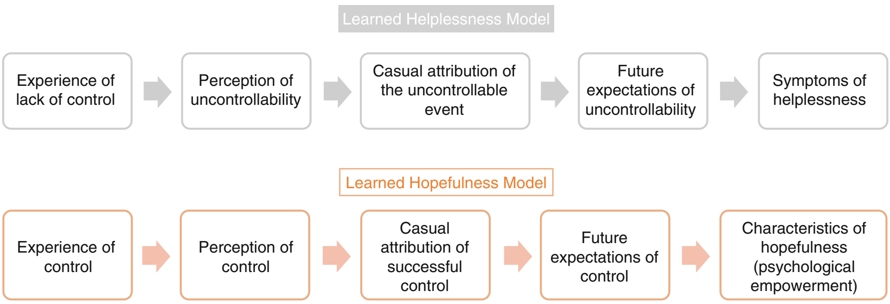

Fig. 26.1 Comparison of learned helplessness and learned hopefulness models (Zimmerman 1990)

Mathematical empowerment consists of power over the language, symbols, knowledge and skills of mathematics and the ability to confidently apply them in mathematical applications within the context of schooling, and possibly to a lesser extent, outside of this context. In such a domain, two different perspectives are distinguished: (1) the cognitive, which refers to the acquisitions of the facts, skills and the general strategies of problem solving (Bell et al. 1983) and (2) the semiotic, which consists of the development of power over the signs of mathematics. From this second perspective, mathematical empowerment comprises the ability to understand and solve problems (both routine and non-routine), to pose problems, to set tasks, and to judge the correctness of mathematical solutions.

Social empowerment ranges from the straightforwardly utilitarian to the more radical "critical mathematical citizenship". Briefly, the former extreme is related to the use of mathematical qualification as a "critical filter" controlling entry into higher education and higher paid occupations. The latter extreme regards the development of mathematically literate or socially numerate citizens who are able to exercise independent critical judgements and political decision-making. The idea of being critical is meant by making careful judgements, using all available evidence, reasoning and balanced arguments to evaluate claims and to reach conclusions. Within this perspective, mathematics becomes a "thinking tool" for viewing the world critically, it contributes to both political and social empowerment of the learner, and hopefully to the promotion of social justice and a better life for all.

Epistemological empowerment concerns the individual's growth of confidence not only in using mathematics, but also a personal sense of power over the creation and validation of knowledge. In particular, according to the model of the stage of empowerment of the knower (Belenky et al. 1986), the ultimate goal of epistemological empowerment is to achieve the stage of being a "constructing knower", who combines intuition, procedures and skills of mathematics to make sense of the world and confidently apply mathematical thinking to it.

26.3 Methodology

We aim at answering the following research question:

what are the cognitive and affective features that characterise the process of empowerment (at all)?

To address such question, we report data from an activity that took place on November 2018 in one community for minors that hosts teen-immigrants. It involved two teenagers who come from Africa, which are fictitiously named Drissa and Ibra. Drissa comes from Mali, where he attended elementary school; he has been in Italy for 2 years and he is attending middle school (grade 6). Ibra comes from Ivory Coast and has been in Italy for 1 year. Ibra did not attend school in Africa and he is attending an Italian language course (level A1). Both of them do not work and they live in a large community of 20 people.

This activity is part of a three-months-long pilot phase of the project TEEN. The research project follows a design-based research methodology, which focuses on examining a particular intervention by continuous iteration of design, enactment, analysis, and redesign (Cobb et al. 2003).

The *pilot phase* of TEEN consists of meetings with groups of teen-immigrants in different communities, which provided us with important information about the functioning of the app. In this phase, the interviewer was a participant observer, posing questions and prompting the teenagers in the use of the app (for example, he selected the tasks to work with).

In subsequent phases, not reported here, its role was less active because the app reached a more mature level (and the teenagers chose the tasks). Following Powell and Brantlinger (2008), we aim at designing an app that proposes "everyday" mathematical problems, given the importance of showing mathematics as an accessible discipline. Following Stathopoulou and Kalabasis (2007) and Chronaki (2005), the problems presented in the app relate to teen-immigrants' identities and attitudes. Along this line, following also Gutstein (2003), we consider the teen-immigrants' language, and in particular we tried to reduce the *verbosity* of the tasks given their difficulties with the Italian language, supporting the problem statements with *graphical content*. Four meetings in three different communities took place in the pilot phase. Each meeting lasted between 60 and 90 min. We selected the case of Drissa and Ibra, because it illuminates important aspects of the interaction with the app. In some meetings, we observed difficulties mostly related to the Italian language. In other meetings, the emotional burden provoked by our presence was overwhelming. We agree with Civil (2008) that the difficulties related to data collection within this special sample of teenagers are part of the ethical issues concerning working with a minority.

The first version of the app is structured as follows (Fig. 26.2a): it proposes some practice problems (pink button in Fig. 26.2a), which involve basic mathematical facts, to allow the user to become confident with the instrument (an example is shown in Fig. 26.2b), in term of Báguin and Rabardel (2000) these activities are design to aid the instrumental genesis, namely the process that underlies the construction and evolution of the instrument (the app). For example, a calculator is embedded in the app and it is activated once the user clicks on the blank space to submit her answer (Fig. 26.2c). After the practice section, the users are engaged in a budgeting activity (green button in Fig. 26.2a), in which they have to make their lifestyle compatible with a given salary (as an 18-years-old boy in Italy). Then, more complex scenarios are proposed and problem-solving tasks are shown. One of the "complex" scenarios is the focus of this paper: the user has to deal with problems concerning construction sites, and the tasks are tied to realistic situations at the workplace.

The notion of empowerment, and the understanding proposed by Ernest, seems to be particularly relevant for this purpose, since students' engagement seems to be related to their sense of being able to cope with the tasks, and thus a motivating element for not leaving the app. In such a way, the app provides for an experience to empower the mathematical domain which enacts the transforming process at

Fig. 26.2 Screenshot from the app used for the pilot phase. (**a**) The dashboard. (**b**) Task 1. (**c**) The calculator. (**d**) Task 4

psychological domain. We want to highlight that the app experience is meant as whole interaction with the digital environment composed of the tasks supported by the related image, the feedback system and the built-in calculator. We recall that the transforming process requires an action, which is, in our context, the action of answering to the task. As result, the experience is intertwining of the familiar scenario the user is exposed, the thoughts of the user, the action of answering and the feedback system.

We analyse an excerpt that concerns the task 4 in Fig. 26.2d. It reads: "The 80% of concrete is gravel. How many cubic meters of gravel are necessary to obtain 1.8 cubic meters of concrete?" In Italy, the percentage topic is introduced at school at the end of primary school (grade-5) but it is deeper taught in the middle school (grade-7). During the meeting, some difficulties related to the Italian language occurs, but they were overcome in two ways: (1) talking in English and Bambara language thank the mediation of a peer who speaks Italian, (2) using mathematics (symbols and number).

26.4 Data Analysis

Ibra and Drissa read the task on the screen. Ibra, in particular, reads it out loud. When he reads "the 80%", then the researcher asks "what is it meant by 80%? Provide an example!" and

Excerpt 1

Ibra: Yes. Yes. It is the discount! [he starts to talk in Bambara for 3 minutes]
R: 80% of 1.8. How to compute the 80% of a quantity? What does it mean?
Ibra: 80 minus 100.

From the first minutes of the episode, we can notice that Ibra seems not to have any idea of the meaning of 80%. He recalls the concept of discount, but he thinks about a subtraction, since the concept of discount entails a subtraction somehow. Even if, at the level of cognitive empowerment, there is not a mathematical knowledge that supports Ibra's understanding, in the realm of epistemological empowerment we can see that Ibra feels some confidence about his ability to compute the percentage, since there is a context that is meaningful for him ("yes, yes, it is the discount") and this also sustains his will to engage in the assigned task.

In the sequel of the episode, the researcher proposes two (simpler, according to him) examples of percentage: 50% of 100 and 120. Both Ibra and Drissa compute the percentages correctly, then the researcher proposes to compute 80% of 100 and 120. Ibra and Drissa compute the difference: $100-80=20$ and $120-80=40$. Without commenting on the results, the researcher invites to compute 80% of 200€.

Excerpt 2

R: How many hundreds are in 200€?
Ibra: Two.
R: Well, I have 100 here and 100 here. How much do you take off here? [points to the first 100]
Ibra: 80.
R: And here? [points to the second 100]
Ibra: 80.
R: Good. Then, how much do you pay?
Ibra: 20 and 20, 40.

We can notice that the researcher is guiding Drissa's and Ibra's attention to some details of the task, and that he is suggesting to split 200 into two 100s in order to get the correct solution. From the answers of Drissa and Ibra, however, we cannot infer anything about their understanding. In other words, we do not know whether they are performing algorithms that belong to the range of mathematical capabilities they possess. With respect to epistemological empowerment, indeed, looking at the model suggested by Ernest (2002), we can say that Drissa and Ibra seem to belong to the case of subjective knowledge in their relationship with "the authority", namely that they are responding intuitively. Drissa and Ibra, in fact, are not merely repeating the pronouncements of the authority, as it is for previous stages of the process of empowerment (Ernest 2002, p.11). Some skills that they possess seem to emerge: the skills related to the computation of a percentage value of 100. In order to understand more about their process of both knowing and solving the task, we need to consider another piece of dialogue between Ibra, Drissa and the researcher.

Excerpt 3

R: Now, more tricky. How many hundreds are here? [points to 120]
Ibra: Only one 100.
R: Do you have a remainder? I: Yes. 20€.
R: How can I write it using a hundred?
Drissa: With the point 100.
R: Fine. I need to use the point. How? Let us try to get what we did here . . . We have 200€, and we split them in two of 100€. Then we multiplied by 80. Thus, what do we do now?
Ibra: 120 divided by 100, that is 12
R: No, attention.
Ibra: 1.2, yes 1 point 2.
R: and now?
Ibra: Times 80.
R: What is the discount? 1.2 times 80, equal to 96. And now, 120 minus 96
Ibra: 24.
R: 24 right!
Ibra: Haha! I get it! This is 80%. This is the result of 80 percent.

Ibra has an illumination, which is both cognitive and affective. He recognises that the result is correct, and this provides him with satisfaction. There is a cognitive empowerment, both procedural and semiotic: procedural empowerment is gained from Ibra's ability to use an algorithm to compute the percentage (that is, divide by 100 and then multiply by 80), while semiotical empowerment relates to his ability to manage the representation of the percentage, making sense of it. By seeking a form of objective knowledge, embedded in the algorithm that he is using, Ibra is reaching procedural knowledge as regards epistemological empowerment, which entails to be no longer overpowered by an unquestioned "authority". The episode goes on:

Excerpt 4

R: What did you do?
Ibra: 120 divided by 100 times 80.
R: Let us do with 200.
Ibra: 200 divided by 100 times 80, 160.
R: Right.
Ibra: But . . . It is the same!

From this statement made by Ibra, we can confirm that he is able to recognise that two different procedures lead to the same result. After this statement, Ibra takes a sheet of paper to write and compute the percentage of gravel required by the task.

Excerpt 5

Ibra: We have to do 1.8 divided by 100, then multiply by 80.
R: This is the quantity of gravel you need. Let us use the app!
Ibra: 1.8 divided by 100 times 80 [the feedback from the App is ‘right’]
Ibra: This is a fundamental computation. We must know it!

From the first time, also a form of social empowerment emerges. Ibra, in fact, uses the “we”, as this kind of skill needs to be possessed by all the teen-immigrants he knows. He recognises its importance for life, not only for cognitive reasons that pertain some unclear practical usefulness. Ibra now takes the sheet of paper again and silently starts to write. This is not the “silence” of helplessness described by Ernest (2002), since Ibra is not passive, but he is actively writing computations. In order to understand what he is doing, the researcher asks

Excerpt 6

R: What are you doing? Tell us!
Ibra: If the shoes cost 125, with 70% off . . . [to note, no task with shoes is presented by the App]
R: How much do you pay them?
Ibra: I am doing it!
R: so . . .
Ibra: Write on the paper, then I use the app to compute the discount.

Ibra seems to talk to himself, repeating out loud the “instructions” to compute the quantity he wants to get. He is computing the discount of a pair of shoes. We do not know whether Ibra bought the shoes, or he just saw them in a shop. We know that the discount caught his attention and that he wanted to be able to compute it. Now, Ibra is able to perform this, and he shows his satisfaction. Ibra’s process of empowerment has come to a significant stage.

26.5 Discussion and Conclusions

In this paper, we aim at characterising the process of empowerment as both cognitive and affective. To this end, we draw on previous researches on empowerment and we present an excerpt from an activity with teen-immigrants interacting with an app. Ernest's tripartite model for mathematical empowerment allows us to unfold insightful aspects of the interaction between the young immigrants and the app. For example, whilst cognitive empowerment is weak, this is not the case for epistemological one: in particular, it emerges that Ibra develops a sense of confidence and control over the percentages, and satisfaction in finding himself able to apply the concept in the context of discount. Also social empowerment emerges, as Ibra feels that "all" them need to know percentages, since "this is important". It also emerges that all the three components of empowerment need to be present for learning to take place, but it also emerges that epistemological empowerment, which is linked to affective dimensions of self-efficacy and identity, is a sort of basis upon which social and cognitive empowerment develop and are sustained. In fact, it seems that to feel (enough) confident with the task is the fuel for Ibra to go on with the activity, and to overcome the difficulties that emerge in relation to the language and the mathematical situation. Epistemological empowerment somehow increases firstly his mathematical empowerment, because Ibra finds himself able to solve the task, and then his social empowerment, when he feels that every teen-immigrant needs to experience what that he is experiencing. Namely, he feels that, through these activities, all the teenagers like him have more control over their lives.

To conclude, we comment on the role of the app during the meetings. On the one's hand, to some extent, the app seems to be marginal from the didactical point of view. On the other hand, it had a pivotal role to activate the transforming process, leading to the empowerment.

Indeed, the app, and—more precisely—the possibility to interact with it, was the starting point to activate Ibra and Drissa on the task because of their feeling of "control" over the activities through their actions.

But, in this pilot phase, the two teenagers had the opportunity to extend their actions asking and interacting with the researchers. We can wonder what would have happened if Ibra and Drissa had been working without their intervention. Of course, we cannot answer such a question, but we can claim that the app itself was crucial in this hybrid context. Further follow-up studies are, however, necessary in order to examine in a deeper way the affordances and the potentialities of a lens of this sort in understanding teen-immigrants' learning process with the app.

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