Chapter 5
A Soft Embedding Theorem for Soft Topological Spaces

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Abstract In this paper, based on the researches on soft set theory and soft topology, we introduce the notions of soft separation between soft points and soft closed sets in order to obtain a generalization of the well-known Embedding Theorem to the class of soft topological spaces.

Keywords: Soft set · Soft topology · Soft mapping · Soft slab · Embedding theorem

5.1 Introduction

Almost every branch of sciences and many practical problems in engineering, economics, computer science, physics, meteorology, statistics, medicine, sociology, etc. have its own uncertainties and ambiguities because they depend on the influence of many parameters and, due to the inadequacy of the existing theories of parameterization in dealing with uncertainties, it is not always easy to model such a kind of problems by using classical mathematical methods. In Molodtsov (1999) initiated the novel concept of Soft Sets Theory as a new mathematical tool and a completely different approach for dealing with uncertainties while modelling problems in a large class of applied sciences. Indeed, dealing with uncertainties becomes of the utmost importance, especially when complex systems must be studied. This is particularly true for some mechanical systems, for example, in studying the new conceived materials, so-called metamaterials (Barchiesi et al, 2019; dell'Isola et al, 2019b,a) which are ad hoc designed to provide a specific behaviour, micro-devices that show a "size effect" (Abali et al, 2015), biological applications (Lekszycki and dell'Isola,

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2012; Giorgio et al, 2019; Sheidaei et al, 2019) characterized by an evolution of their mechanical behaviour, energy harvesting and vibration control (Giorgio et al, 2009; Lossouarn et al, 2015; dell'Isola and Vidoli, 1998), and robotics (Giorgio and Del Vescovo, 2018, 2019) and so on.

In the past few years, the fundamentals of soft set theory have been studied by many researchers. Starting from Maji et al (2002, 2003) studied the theory of soft sets initiated by Molodtsov, defining notion as the equality of two soft sets, the subset and super set of a soft set, the complement of a soft set, the union and the intersection of soft sets, the null soft set and absolute soft set, and they gave many examples. In Pei and Miao (2005); Chen et al (2005) improved the work of Maji. Further contributions to the Soft Sets Theory were given by Yang (2008); Ali et al (2009); Li (2011); Qin and Hong (2010); Sezgin and Atagün (2011); Neog and Sut (2011); Ahmad and Kharal (2009); Babitha and Sunil (2010); Ibrahim and Yusuf (2012); Singh and Onyeozili (2012); Feng and Li (2013); Onyeozili and Gwary (2014); Çağman (2014).

In Shabir and Naz (2011) introduced the concept of soft topological spaces, also defining and investigating the notions of soft closed sets, soft closure, soft neighborhood, soft subspace and some separation axioms. Some other properties related to soft topology were studied by Çağman et al (2011). In the same year Hussain and Ahmad (2011) investigated the properties of soft closed sets, soft neighbourhoods, soft interior, soft exterior and soft boundary, while Kharal and Ahmad (2011) defined the notion of a mapping on soft classes and studied several properties of images and inverse images. The notion of soft interior, soft neighborhood and soft continuity were also object of study by Zorlutuna et al (2012). Some other relations between these notions was proved by Ahmad and Hussain (2012). The neighbourhood properties of a soft topological space were investigated in Nazmul and Samanta (2013). The class of soft Hausdorff spaces was extensively studied by Varol and Aygün (2013). In Aygünoğlu and Aygün (2012) defined and studied the notions of soft continuity and soft product topology. Some years later, Zorlutuna and Çaku (2015) gave some new characterizations of soft continuity, soft openness and soft closedness of soft mappings, also generalizing the Pasting Lemma to the soft topological spaces. Soft first countable and soft second countable spaces were instead defined and studied by Rong (2012). Furthermore, the notion of soft continuity between soft topological spaces was independently introduced and investigated by Hazra et al (2012). Soft connectedness was also studied in Al-Khafaj and Mahmood (2014); Hussain (2015). In the same year, Das and Samanta (2013b,a) introduced and extensively studied the soft metric spaces. In Hussain and Ahmad (2015) redefined and explored several properties of soft T_i (with $i = 0, 1, 2, 3, 4$) separation axioms and discuss some soft invariance properties namely soft topological property and soft hereditary property. Xie (2015) introduced the concept of soft points and proved that soft sets can be translated into soft points so that they may conveniently dealt as same as ordinary sets. Tantawy et al (2016) continued the study of soft T_i-spaces (for $i = 0, 1, 2, 3, 4, 5$) also discussing the hereditary and topological properties for such spaces. Fu et al (2017) investigated some basic properties concerning the soft topological product space. Further contributions to the theory of soft sets and that of soft topology were added by Min (2011); Janaki and Sredja (2012); Varol et al (2012); Peyghan (2013); Wardowski (2013); Nazmul and Samanta (2014); Peyghan et al (2014); Georgiou et al (2013); Georgiou and Megaritis (2014); Uluçay et al (2016); Wadkar et al (2016); Matejdes (2016); Fu et al (2017); Bdaiwi (2017), and, more recently, by Bayramov and Aras (2018); El-Shafei et al (2018); Al-Shami et al (2018); Nordo (2018, 2019a).

In the present paper we will present the notions of family of soft mappings soft separating soft points and soft points from soft closed sets in order to give a generalization of the well-known Embedding Theorem for soft topological spaces.

5.2 Preliminaries

In this section we present some basic definitions and results on soft sets and suitably exemplify them. Terms and undefined concepts are used as in Engelking (1989).

Definition 5.1. (Molodtsov, 1999) Let $\mathbb{U}$ be an initial universe set and $\mathbb{E}$ be a nonempty set of parameters (or abstract attributes) under consideration with respect to $\mathbb{U}$ and $A \subseteq \mathbb{E}$, we say that a pair (F, A) is a ***soft set*** over $\mathbb{U}$ if F is a set-valued mapping $F : A \to \mathbb{P}(\mathbb{U})$ which maps every parameter $e \in A$ to a subset $F(e)$ of $\mathbb{U}$.

In other words, a soft set is not a real (crisp) set but a parameterized family $\{F(e)\}_{e \in A}$ of subsets of the universe $\mathbb{U}$. For every parameter $e \in A$, $F(e)$ may be considered as the set of *e-approximate elements* of the soft set (F, A).

Remark 5.1. Ma et al (2010) proved that every soft set (F, A) is equivalent to the soft set $(F, \mathbb{E})$ related to the whole set of parameters $\mathbb{E}$, simply considering empty every approximations of parameters which are missing in A, that is extending in a trivial way its set-valued mapping, i.e. setting $F(e) = \emptyset$, for every $e \in \mathbb{E} \setminus A$.
For such a reason, in this paper we can consider all the soft sets over the same parameter set $\mathbb{E}$ as in Chiney and Samanta (2016) and we will redefine all the basic operations and relations between soft sets originally introduced in Maji et al (2002, 2003); Molodtsov (1999) as in Nazmul and Samanta (2013), that is by considering the same parameter set.

Definition 5.2. (Zorlutuna et al, 2012) The set of all the soft sets over a universe $\mathbb{U}$ with respect to a set of parameters $\mathbb{E}$ will be denoted by $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}$.

Definition 5.3. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be two soft sets over a common universe $\mathbb{U}$ and a common set of parameters $\mathbb{E}$, we say that $(F, \mathbb{E})$ is a ***soft subset*** of $(G, \mathbb{E})$ and we write $(F, \mathbb{E}) \tilde{\subseteq} (G, \mathbb{E})$ if $F(e) \subseteq G(e)$ for every $e \in \mathbb{E}$.

Definition 5.4. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be two soft sets over a common universe $\mathbb{U}$, we say that $(F, \mathbb{E})$ and $(G, \mathbb{E})$ are ***soft equal*** and we write $(F, \mathbb{E}) \tilde{=} (G, \mathbb{E})$ if $(F, \mathbb{E}) \tilde{\subseteq} (G, \mathbb{E})$ and $(G, \mathbb{E}) \tilde{\subseteq} (F, \mathbb{E})$.

Definition 5.5. (Nazmul and Samanta, 2013) A soft set $(F, \mathbb{E})$ over a universe $\mathbb{U}$ is said to be the ***null soft set*** and it is denoted by $(\tilde{\emptyset}, \mathbb{E})$ if $F(e) = \emptyset$ for every $e \in \mathbb{E}$.

Definition 5.6. (Nazmul and Samanta, 2013) A soft set $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ over a universe $\mathbb{U}$ is said to be the ***absolute soft set*** and it is denoted by $(\tilde{\mathbb{U}}, \mathbb{E})$ if $F(e) = \mathbb{U}$ for every $e \in \mathbb{E}$.

Definition 5.7. Let $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be a soft set over a universe $\mathbb{U}$ and V be a nonempty subset of U, the ***constant soft set*** of V, denoted by $(\tilde{V}, \mathbb{E}))$ (or, sometimes, by $\tilde{V}$), is the soft set $(\underline{\mathrm{V}}, \mathbb{E})$, where $\underline{\mathrm{V}} : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the constant set-valued mapping defined by $\underline{\mathrm{V}}(e) = V$, for every $e \in \mathbb{E}$.

Definition 5.8. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be a soft set over a universe $\mathbb{U}$, the ***soft complement*** (or more exactly the *soft relative complement*) of $(F, \mathbb{E})$, denoted by $(F, \mathbb{E})^{\complement}$, is the soft set $\left(F^{\complement}, \mathbb{E}\right)$ where $F^{\complement} : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by $F^{\complement}(e) = F(e)^{\complement} = \mathbb{U} \setminus F(e)$, for every $e \in \mathbb{E}$.

Definition 5.9. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be two soft sets over a common universe $\mathbb{U}$, the ***soft difference*** of $(F, \mathbb{E})$ and $(G, \mathbb{E})$, denoted by $(F, \mathbb{E})\widetilde{\setminus}(G, \mathbb{E})$, is the soft set $(F \setminus G, \mathbb{E})$ where $F \setminus G : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by $(F \setminus G)(e) = F(e) \setminus G(e)$, for every $e \in \mathbb{E}$.

Clearly, for every soft set $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$, it results $(F, \mathbb{E})^{\complement} \tilde{=} (\tilde{\mathbb{U}}, \mathbb{E})\widetilde{\setminus}(F, \mathbb{E})$.

Definition 5.10. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be two soft sets over a universe $\mathbb{U}$, the ***soft union*** of $(F, \mathbb{E})$ and $(G, \mathbb{E})$, denoted by $(F, \mathbb{E})\tilde{\cup}(G, \mathbb{E})$, is the soft set $(F \cup G, \mathbb{E})$ where $F \cup G : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by $(F \cup G)(e) = F(e) \cup G(e)$, for every $e \in \mathbb{E}$.

Definition 5.11. (Nazmul and Samanta, 2013) Let $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be two soft sets over a universe $\mathbb{U}$, the ***soft intersection*** of $(F, \mathbb{E})$ and $(G, \mathbb{E})$, denoted by $(F, \mathbb{E})\tilde{\cap}(G, \mathbb{E})$, is the soft set $(F \cap G, \mathbb{E})$ where $F \cap G : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by $(F \cap G)(e) = F(e) \cap G(e)$, for every $e \in \mathbb{E}$.

Definition 5.12. (Al-Khafaj and Mahmood, 2014) Two soft sets $(F, \mathbb{E})$ and $(G, \mathbb{E})$ over a common universe $\mathbb{U}$ are said to be ***soft disjoint*** if their soft intersection is the soft null set, i.e. if $(F, \mathbb{E})\tilde{\cap}(G, \mathbb{E}) \tilde{=} (\tilde{\emptyset}, \mathbb{E})$. If two soft sets are not soft disjoint, we also say that they ***soft meet*** each other. In particular, if $(F, \mathbb{E})\tilde{\cap}(G, \mathbb{E}) \tilde{\neq} (\tilde{\emptyset}, \mathbb{E})$ we say that $(F, \mathbb{E})$ ***soft meets*** $(G, \mathbb{E})$.

Definition 5.13. (Xie, 2015) A soft set $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ over a universe $\mathbb{U}$ is said to be a ***soft point*** over U if it has only one non-empty approximation which is a singleton, i.e. if there exists some parameter $\alpha \in \mathbb{E}$ and an element $p \in \mathbb{U}$ such that $F(\alpha) = \{p\}$ and $F(e) = \emptyset$ for every $e \in \mathbb{E} \setminus \{\alpha\}$. Such a soft point is usually denoted by $(p_\alpha, \mathbb{E})$. The singleton $\{p\}$ is called the *support set* of the soft point and α is called the *expressive parameter* of $(p_\alpha, \mathbb{E})$.

Definition 5.14. (Xie, 2015) The set of all the soft points over a universe $\mathbb{U}$ with respect to a set of parameters $\mathbb{E}$ will be denoted by $\mathcal{SP}(\mathbb{U})_{\mathbb{E}}$.

Since any soft point is a particular soft set, it is evident that $\mathcal{SP}(\mathbb{U})_{\mathbb{E}} \subseteq \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$.

Definition 5.15. (Xie, 2015) Let $(p_\alpha, \mathbb{E}) \in \mathcal{SP}(\mathbb{U})_{\mathbb{E}}$ and $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be a soft point and a soft set over a common universe $\mathbb{U}$, respectively. We say that ***the soft point*** $(p_\alpha, \mathbb{E})$ ***soft belongs to the soft set*** $(F, \mathbb{E})$ and we write $(p_\alpha, \mathbb{E})\tilde{\in}(F, \mathbb{E})$, if the soft point is a soft subset of the soft set, i.e. if $(p_\alpha, \mathbb{E})\tilde{\subseteq}(F, \mathbb{E})$ and hence if $p \in F(\alpha)$. We also say that ***the soft point*** $(p_\alpha, \mathbb{E})$ ***does not belongs to the soft set*** $(F, \mathbb{E})$ and we write $(p_\alpha, \mathbb{E})\tilde{\notin}(F, \mathbb{E})$, if the soft point is not a soft subset of the soft set, i.e. if $(p_\alpha, \mathbb{E})\tilde{\nsubseteq}(F, \mathbb{E})$ and hence if $p \notin F(\alpha)$.

Definition 5.16. (Das and Samanta, 2013b) Let $(p_\alpha, \mathbb{E}), (q_\beta, \mathbb{E}) \in \mathcal{SP}(\mathbb{U})_{\mathbb{E}}$ be two soft points over a common universe $\mathbb{U}$, we say that $(p_\alpha, \mathbb{E})$ and $(q_\beta, \mathbb{E})$ are ***soft equal***, and we write $(p_\alpha, \mathbb{E})\tilde{=}(q_\beta, \mathbb{E})$, if they are equals as soft sets and hence if $p = q$ and $\alpha = \beta$.

Definition 5.17. (Das and Samanta, 2013b) We say that two soft points $(p_\alpha, \mathbb{E})$ and $(q_\beta, \mathbb{E})$ are ***soft distincts***, and we write $(p_\alpha, \mathbb{E})\tilde{\neq}(q_\beta, \mathbb{E})$, if and only if $p \neq q$ or $\alpha \neq \beta$.

The notion of soft point allows us to express the soft inclusion in a more familiar way.

Proposition 5.1. *Let* $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ *be two soft sets over a common universe* $\mathbb{U}$ *respect to a parameter set* $\mathbb{E}$*, then* $(F, \mathbb{E})\tilde{\subseteq}(G, \mathbb{E})$ *if and only if for every soft point* $(p_\alpha, \mathbb{E})\tilde{\in}(F, \mathbb{E})$ *it follows that* $(p_\alpha, \mathbb{E})\tilde{\in}(G, \mathbb{E})$.

Definition 5.18. (Hussain and Ahmad, 2011) Let $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ be a soft set over a universe $\mathbb{U}$ and V be a nonempty subset of $\mathbb{U}$, the ***sub soft set*** of $(F, \mathbb{E})$ over V, is the soft set $\left({}^V F, \mathbb{E}\right)$, where ${}^V F : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by ${}^V F(e) = F(e) \cap V$, for every $e \in \mathbb{E}$.

Remark 5.2. Using Definitions 5.7 and 5.11, it is a trivial matter to verify that a sub soft set of $(F, \mathbb{E})$ over V can also be expressed as $\left({}^V F, \mathbb{E}\right) \tilde{=} (F, \mathbb{E})\tilde{\cap}(\tilde{V}, \mathbb{E})$. Furthermore, it is evident that the sub soft set $\left({}^V F, \mathbb{E}\right)$ above defined belongs to the set of all the soft sets over V with respect to the set of parameters $\mathbb{E}$, which is contained in the set of all the soft sets over the universe $\mathbb{U}$ with respect to $\mathbb{E}$, that is $\left({}^V F, \mathbb{E}\right) \in \mathcal{SS}(V)_{\mathbb{E}} \subseteq \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$.

Definition 5.19. (Babitha and Sunil, 2010; Kazancı et al, 2010) Let $\{(F_i, \mathbb{E}_i)\}_{i \in I}$ be a family of soft sets over a universe set $\mathbb{U}_i$ with respect to a set of parameters $\mathbb{E}_i$ (with $i \in I$), respectively. Then the ***soft product*** (or, more precisely, the ***soft cartesian product***) of $\{(F_i, \mathbb{E}_i)\}_{i \in I}$, denoted by $\tilde{\prod}_{i \in I}(F_i, \mathbb{E}_i)$, is the soft set $\left(\prod_{i \in I} F_i, \prod_{i \in I} \mathbb{E}_i\right)$ over the (usual) cartesian product $\prod_{i \in I} \mathbb{U}_i$ and with respect to the set of parameters $\prod_{i \in I} \mathbb{E}_i$, where $\prod_{i \in I} F_i : \prod_{i \in I} \mathbb{E}_i \to \mathbb{P}\left(\prod_{i \in I} \mathbb{U}_i\right)$ is the set-valued mapping defined by $\prod_{i \in I} F_i\left(\langle e_i \rangle_{i \in I}\right) = \prod_{i \in I} F_i(e_i)$, for every $\langle e_i \rangle_{i \in I} \in \prod_{i \in I} \mathbb{E}_i$.

Proposition 5.2. (Nordo, 2019b) *Let $\widetilde{\prod}_{i\in I}(F_i,\mathbb{E}_i)$ be the soft product of a family $\{(F_i,\mathbb{E}_i)\}_{i\in I}$ of soft sets over a universe set $\mathbb{U}_i$ with respect to a set of parameters $\mathbb{E}_i$ (with $i\in I$), and let $\left(p_\alpha,\prod_{i\in I}\mathbb{E}_i\right)\in\mathcal{SP}(\prod_{i\in I}\mathbb{U}_i)_{\prod_{i\in I}\mathbb{E}_i}$ be a soft point of the product $\prod_{i\in I}\mathbb{U}_i$, where $p=\langle p_i\rangle_{i\in I}\in\prod_{i\in I}\mathbb{U}_i$ and $\alpha=\langle\alpha_i\rangle_{i\in I}\in\prod_{i\in I}\mathbb{E}_i$, then $\left(p_\alpha,\prod_{i\in I}\mathbb{E}_i\right)\tilde{\in}\widetilde{\prod}_{i\in I}(F_i,\mathbb{E}_i)$ if and only if $((p_i)_{\alpha_i},\mathbb{E}_i)\tilde{\in}(F_i,\mathbb{E}_i)$ for every $i\in I$.*

Corollary 5.1. (Nordo, 2019b) *The soft product of a family $\{(F_i,\mathbb{E}_i)\}_{i\in I}$ of soft sets over a universe set $\mathbb{U}_i$ with respect to a set of parameters $\mathbb{E}_i$ (with $i\in I$) is null if and only if at least one of its soft sets is null, that is $\widetilde{\prod}_{i\in I}(F_i,\mathbb{E}_i)\tilde{=}\left(\tilde{\emptyset},\prod_{i\in I}\mathbb{E}_i\right)$ iff there exists some $j\in I$ such that $(F_j,\mathbb{E}_j)\tilde{=}(\tilde{\emptyset},\mathbb{E})$.*

Proposition 5.3. (Kazancı et al, 2010) *Let $\{(F_i,\mathbb{E}_i)\}_{i\in I}$ and $\{(G_i,\mathbb{E}_i)\}_{i\in I}$ be two families of soft sets over a universe set $\mathbb{U}_i$ with respect to a set of parameters $\mathbb{E}_i$ (with $i\in I$), then it results:*

$$\widetilde{\prod}_{i\in I}\left((F_i,\mathbb{E}_i)\tilde{\cap}(G_i,\mathbb{E}_i)\right)\tilde{=}\widetilde{\prod}_{i\in I}(F_i,\mathbb{E}_i)\,\tilde{\cap}\,\widetilde{\prod}_{i\in I}(G_i,\mathbb{E}_i)\,.$$

According to Remark 5.1 the following notions by Kharal and Ahmad have been simplified and slightly modified for soft sets defined on a common parameter set.

Definition 5.20. (Kharal and Ahmad, 2011) Let $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ and $\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be two sets of soft open sets over the universe sets $\mathbb{U}$ and $\mathbb{U}'$ with respect to the sets of parameters $\mathbb{E}$ and $\mathbb{E}'$, respectively. and consider a mapping $\varphi:\mathbb{U}\to\mathbb{U}'$ between the two universe sets and a mapping $\psi:\mathbb{E}\to\mathbb{E}'$ between the two set of parameters. The mapping $\varphi_\psi:\mathcal{SS}(\mathbb{U})_{\mathbb{E}}\to\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ which maps every soft set $(F,\mathbb{E})$ of $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ to a soft set $(\varphi_\psi(F),\mathbb{E}')$ of $\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$, denoted by $\varphi_\psi(F,\mathbb{E})$, where $\varphi_\psi(F):\mathbb{E}'\to\mathbb{P}(\mathbb{U}')$ is the set-valued mapping defined by $\varphi_\psi(F)(e')=\bigcup\left\{\varphi(F(e)):e\in\psi^{-1}(\{e'\})\right\}$ for every $e'\in\mathbb{E}'$, is called a ***soft mapping*** from $\mathbb{U}$ to $\mathbb{U}'$ induced by the mappings φ and ψ, while the soft set $\varphi_\psi(F,\mathbb{E})\tilde{=}(\varphi_\psi(F),\mathbb{E}')$ is said to be the ***soft image*** of the soft set $(F,\mathbb{E})$ under the soft mapping $\varphi_\psi:\mathcal{SS}(\mathbb{U})_{\mathbb{E}}\to\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$.
The soft mapping $\varphi_\psi:\mathcal{SS}(\mathbb{U})_{\mathbb{E}}\to\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ is said ***injective*** (respectively ***surjective***, ***bijective***) if the mappings $\varphi:\mathbb{U}\to\mathbb{U}'$ and $\psi:\mathbb{E}\to\mathbb{E}'$ are both injective (resp. surjective, bijective).

It is worth noting that soft mappings between soft sets behaves similarly to usual (crisp) mappings in the sense that they maps soft points to soft points, as proved in the following property.

Proposition 5.4. *Let $\varphi_\psi:\mathcal{SS}(\mathbb{U})_{\mathbb{E}}\to\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be a soft mapping induced by the mappings $\varphi:\mathbb{U}\to\mathbb{U}'$ and $\psi:\mathbb{E}\to\mathbb{E}'$ between the two sets $\mathcal{SS}(\mathbb{U})_{\mathbb{E}},\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ of soft sets. and consider a soft point $(p_\alpha,\mathbb{E})$ of $\mathcal{SP}(\mathbb{U})_{\mathbb{E}}$. Then the soft image $\varphi_\psi(p_\alpha,\mathbb{E})$ of the soft point $(p_\alpha,\mathbb{E})$ under the soft mapping φ_ψ is the soft point $\left(\varphi(p)_{\psi(\alpha)},\mathbb{E}'\right)$, i.e. $\varphi_\psi(p_\alpha,\mathbb{E})\tilde{=}\left(\varphi(p)_{\psi(\alpha)},\mathbb{E}'\right)$.*

Corollary 5.2. *Let $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be a soft mapping induced by the mappings $\varphi : \mathbb{U} \to \mathbb{U}'$ and $\psi : \mathbb{E} \to \mathbb{E}'$ between the two sets $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}, \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ of soft sets, then φ_ψ is injective if and only if its soft images of every distinct pair of soft points are distinct too, i.e. if for every $(p_\alpha, \mathbb{E}), (q_\beta, \mathbb{E}) \in \mathcal{SP}(\mathbb{U})_{\mathbb{E}}$ such that $(p_\alpha, \mathbb{E}) \tilde{\neq} (q_\beta, \mathbb{E})$ it follows that $\varphi_\psi(p_\alpha, \mathbb{E}) \tilde{\neq} \varphi_\psi(q_\beta, \mathbb{E})$.*

Definition 5.21. (Kharal and Ahmad, 2011) Let $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be a soft mapping induced by the mappings $\varphi : \mathbb{U} \to \mathbb{U}'$ and $\psi : \mathbb{E} \to \mathbb{E}'$ between the sets $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}, \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ of soft sets and consider a soft set $(G, \mathbb{E}')$ of $\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$. The ***soft inverse image*** of $(G, \mathbb{E}')$ under the soft mapping $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$, denoted by $\varphi_\psi^{-1}(G, \mathbb{E}')$ is the soft set $(\varphi_\psi^{-1}(G), \mathbb{E}')$ of $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ where $\varphi_\psi^{-1}(G) : \mathbb{E} \to \mathbb{P}(\mathbb{U})$ is the set-valued mapping defined by $\varphi_\psi^{-1}(G)(e) = \varphi^{-1}\left(G\left(\psi(e)\right)\right)$ for every $e \in \mathbb{E}$.

Corollary 5.3. *Let $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be a soft mapping induced by the mappings $\varphi : \mathbb{U} \to \mathbb{U}'$ and $\psi : \mathbb{E} \to \mathbb{E}'$. If $(F, \mathbb{E}) \in \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ and $(F', \mathbb{E}') \in \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ are soft sets over $\mathbb{U}$ and $\mathbb{U}'$, respectively and $(p_\alpha, \mathbb{E}) \in \mathcal{SP}(\mathbb{U})_{\mathbb{E}}$ and $(q_\beta, \mathbb{E}') \in \mathcal{SP}(\mathbb{U}')_{\mathbb{E}'}$ are soft points over $\mathbb{U}$ and $\mathbb{U}'$, respectively, then the following hold:*

(1) $(p_\alpha, \mathbb{E}) \tilde{\in} (F, \mathbb{E})$ *implies* $\varphi_\psi(p_\alpha, \mathbb{E}) \tilde{\in} \varphi_\psi(F, \mathbb{E})$.
(2) $(q_\beta, \mathbb{E}') \tilde{\in} (F', \mathbb{E}')$ *implies* $\varphi_\psi^{-1}(q_\beta, \mathbb{E}') \tilde{\subseteq} \varphi_\psi^{-1}(F', \mathbb{E}')$.

Definition 5.22. Let $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ be a bijective soft mapping induced by the mappings $\varphi : \mathbb{U} \to \mathbb{U}'$ and $\psi : \mathbb{E} \to \mathbb{E}'$. The ***soft inverse mapping*** of φ_ψ, denoted by φ_ψ^{-1}, is the soft mapping $\varphi_\psi^{-1} = \left(\varphi^{-1}\right)_{\psi^{-1}} : \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'} \to \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ induced by the inverse mappings $\varphi^{-1} : \mathbb{U}' \to \mathbb{U}$ and $\psi^{-1} : \mathbb{E}' \to \mathbb{E}$ of the mappings φ and ψ, respectively.

Remark 5.3. Evidently, the soft inverse mapping $\varphi_\psi^{-1} : \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'} \to \mathcal{SS}(\mathbb{U})_{\mathbb{E}}$ of a bijective soft mapping $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ is also bijective and its soft image of a soft set in $\mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ coincides with the soft inverse image of the corresponding soft set under the soft mapping φ_ψ.

Definition 5.23. (Aygünoğlu and Aygün, 2012) Let $\mathcal{SS}(\mathbb{U})_{\mathbb{E}}, \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$ and $\mathcal{SS}(\mathbb{U}'')_{\mathbb{E}''}$ be three sets of soft open sets over the universe sets $\mathbb{U}, \mathbb{U}', \mathbb{U}''$ with respect to the sets of parameters $\mathbb{E}, \mathbb{E}', \mathbb{E}''$, respectively, and $\varphi_\psi : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}'}$, $\gamma_\delta : \mathcal{SS}(\mathbb{U})_{\mathbb{E}'} \to \mathcal{SS}(\mathbb{U}')_{\mathbb{E}''}$ be two soft mappings between such sets, then the ***soft composition*** of the soft mappings φ_ψ and γ_δ, denoted by $\gamma_\delta \tilde{\circ} \varphi_\psi$ is the soft mapping $(\gamma \circ \varphi)_{\delta \circ \psi} : \mathcal{SS}(\mathbb{U})_{\mathbb{E}} \to \mathcal{SS}(\mathbb{U}'')_{\mathbb{E}''}$ induced by the compositions $\gamma \circ \varphi : \mathbb{U} \to \mathbb{U}''$ of the mappings φ and γ between the universe sets and $\delta \circ \psi : \mathbb{E} \to \mathbb{E}''$ of the mappings ψ and δ between the parameter sets.

The notion of soft topological spaces as topological spaces defined over a initial universe with a fixed set of parameters was introduced by Shabir and Naz (2011).

Definition 5.24. (Shabir and Naz, 2011) Let X be an initial universe set, $\mathbb{E}$ be a nonempty set of parameters with respect to X and $\mathcal{T} \subseteq \mathcal{SS}(X)_{\mathbb{E}}$ be a family of soft sets over X, we say that $\mathcal{T}$ is a ***soft topology*** on X with respect to $\mathbb{E}$ if the following four conditions are satisfied:

(i) the null soft set belongs to $\mathcal{T}$, i.e. $(\tilde{\emptyset}, \mathbb{E}) \in \mathcal{T}$.
(ii) the absolute soft set belongs to $\mathcal{T}$, i.e. $(\tilde{X}, \mathbb{E}) \in \mathcal{T}$.
(iii) the soft intersection of any two soft sets of $\mathcal{T}$ belongs to $\mathcal{T}$, i.e. for every $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{T}$ then $(F, \mathbb{E}) \tilde{\cap} (G, \mathbb{E}) \in \mathcal{T}$.
(iv) the soft union of any subfamily of soft sets in $\mathcal{T}$ belongs to $\mathcal{T}$, i.e. for every $\{(F_i, \mathbb{E})\}_{i \in I} \subseteq \mathcal{T}$ then $\widetilde{\bigcup}_{i \in I} (F_i, \mathbb{E}) \in \mathcal{T}$.

The triplet $(X, \mathcal{T}, \mathbb{E})$ is called a ***soft topological space*** (or soft space, for short) over X with respect to $\mathbb{E}$.
In some case, when it is necessary to better specify the universal set and the set of parameters, the topology will be denoted by $\mathcal{T}(X, \mathbb{E})$.

Definition 5.25. (Shabir and Naz, 2011) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X with respect to $\mathbb{E}$, then the members of $\mathcal{T}$ are said to be ***soft open set*** in X.

Definition 5.26. (Hazra et al, 2012) Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two soft topologies over a common universe set X with respect to a set of paramters $\mathbb{E}$. We say that $\mathcal{T}_2$ is ***finer*** (or stronger) than $\mathcal{T}_1$ if $\mathcal{T}_1 \subseteq \mathcal{T}_2$ where $\subseteq$ is the usual set-theoretic relation of inclusion between crisp sets. In the same situation, we also say that $\mathcal{T}_1$ is ***coarser*** (or weaker) than $\mathcal{T}_2$.

Definition 5.27. (Shabir and Naz, 2011) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X and $(F, \mathbb{E})$ be a soft set over X. We say that $(F, \mathbb{E})$ is ***soft closed set*** in X if its complement $(F, \mathbb{E})^{\complement}$ is a soft open set, i.e. if $(F, \mathbb{E})^{\complement} \in \mathcal{T}$.

Notation 5.2.1 *The family of all soft closed sets of a soft topological space $(X, \mathcal{T}, \mathbb{E})$ over X with respect to $\mathbb{E}$ will be denoted by σ, or more precisely with $\sigma(X, \mathbb{E})$ when it is necessary to specify the universal set X and the set of parameters $\mathbb{E}$.*

Definition 5.28. (Aygünoğlu and Aygün, 2012) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X and $\mathcal{B} \subseteq \mathcal{T}$ be a non-empty subset of soft open sets. We say that $\mathcal{B}$ is a ***soft open base*** for $(X, \mathcal{T}, \mathbb{E})$ if every soft open set of $\mathcal{T}$ can be expressed as soft union of a subfamily of $\mathcal{B}$, i.e. if for every $(F, \mathbb{E}) \in \mathcal{T}$ there exists some $\mathcal{A} \subset \mathcal{B}$ such that $(F, \mathbb{E}) = \widetilde{\bigcup} \{(A, \mathbb{E}) : \ (A, \mathbb{E}) \in \mathcal{A}\}$.

Proposition 5.5. (Nazmul and Samanta, 2013) *Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X and $\mathcal{B} \subseteq \mathcal{T}$ be a family of soft open sets of X. Then $\mathcal{B}$ is a soft open base for $(X, \mathcal{T}, \mathbb{E})$ if and only if for every soft open set $(F, \mathbb{E}) \in \mathcal{T}$ and any soft point $(x_\alpha, \mathbb{E}) \tilde{\in} (F, \mathbb{E})$ there exists some soft open set $(B, \mathbb{E}) \in \mathcal{B}$ such that $(x_\alpha, \mathbb{E}) \tilde{\in} (B, \mathbb{E}) \tilde{\subseteq} (F, \mathbb{E})$.*

Definition 5.29. (Zorlutuna et al, 2012) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space, $(N, \mathbb{E}) \in \mathcal{SS}(X)_{\mathbb{E}}$ be a soft set and $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$ be a soft point over a common universe X. We say that $(N, \mathbb{E})$ is a ***soft neighbourhood*** of the soft point $(x_\alpha, \mathbb{E})$ if there is some soft open set soft containing the soft point and soft contained in the soft set, that is if there exists some soft open set $(A, \mathbb{E}) \in \mathcal{T}$ such that $(x_\alpha, \mathbb{E}) \tilde{\in} (A, \mathbb{E}) \tilde{\subseteq} (N, \mathbb{E})$.

Notation 5.2.2 *The family of all soft neighbourhoods (sometimes also called soft neighbourhoods system) of a soft point* $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$ *in a soft topological space* $(X, \mathcal{T}, \mathbb{E})$ *will be denoted by* $\mathcal{N}_{(x_\alpha,\mathbb{E})}$ *(or more precisely with* $\mathcal{N}^{\mathcal{T}}_{(x_\alpha,\mathbb{E})}$ *if it is necessary to specify the topology).*

Definition 5.30. (Shabir and Naz, 2011) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X and $(F, \mathbb{E})$ be a soft set over X. Then the ***soft closure*** of the soft set $(F, \mathbb{E})$ with respect to the soft space $(X, \mathcal{T}, \mathbb{E})$, denoted by s-$\mathrm{cl}_X(F, \mathbb{E})$, is the soft intersection of all soft closed set over X soft containing $(F, \mathbb{E})$, that is

$$\text{s-cl}_X(F, \mathbb{E}) \tilde{=} \widetilde{\bigcap} \left\{ (C, \mathbb{E}) \in \sigma(X, \mathbb{E}) : (F, \mathbb{E}) \tilde{\subseteq} (C, \mathbb{E}) \right\}.$$

Proposition 5.6. (Shabir and Naz, 2011) *Let* $(X, \mathcal{T}, \mathbb{E})$ *be a soft topological space over* X*, and* $(F, \mathbb{E})$ *be a soft set over* X*. Then the following hold:*

(1) s-$\mathrm{cl}_X(\tilde{\emptyset}, \mathbb{E}) \tilde{=} (\tilde{\emptyset}, \mathbb{E})$.
(2) s-$\mathrm{cl}_X(\tilde{X}, \mathbb{E}) \tilde{=} (\tilde{X}, \mathbb{E})$.
(3) $(F, \mathbb{E}) \tilde{\subseteq}$ s-$\mathrm{cl}_X(F, \mathbb{E})$.
(4) $(F, \mathbb{E})$ *is a soft closed set over* X *if and only if* s-$\mathrm{cl}_X(F, \mathbb{E}) \tilde{=} (F, \mathbb{E})$.
(5) s-$\mathrm{cl}_X($s-$\mathrm{cl}_X(F, \mathbb{E})) \tilde{=}$ s-$\mathrm{cl}_X(F, \mathbb{E})$.

Proposition 5.7. (Shabir and Naz, 2011) *Let* $(X, \mathcal{T}, \mathbb{E})$ *be a soft topological space and* $(F, \mathbb{E}), (G, \mathbb{E}) \in \mathcal{SS}(X)_{\mathbb{E}}$ *be two soft sets over a common universe* X*. Then the following hold:*

(1) $(F, \mathbb{E}) \tilde{\subseteq} (G, \mathbb{E})$ *implies* s-$\mathrm{cl}_X(F, \mathbb{E}) \tilde{\subseteq}$ s-$\mathrm{cl}_X(G, \mathbb{E})$.
(2) s-$\mathrm{cl}_X((F, \mathbb{E}) \tilde{\cup} (G, \mathbb{E})) \tilde{=}$ s-$\mathrm{cl}_X(F, \mathbb{E}) \tilde{\cup}$ s-$\mathrm{cl}_X(G, \mathbb{E})$.
(3) s-$\mathrm{cl}_X((F, \mathbb{E}) \tilde{\cap} (G, \mathbb{E})) \tilde{\subseteq}$ s-$\mathrm{cl}_X(F, \mathbb{E}) \tilde{\cap}$ s-$\mathrm{cl}_X(G, \mathbb{E})$.

Definition 5.31. (Xie, 2015) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space, $(F, \mathbb{E}) \in \mathcal{SS}(X)_{\mathbb{E}}$ and $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$ be a soft set and a soft point over the common universe X with respect to the sets of parameters $\mathbb{E}$, respectively. We say that $(x_\alpha, \mathbb{E})$ is a ***soft adherent point*** (sometimes also called ***soft closure point***) of $(F, \mathbb{E})$ if it soft meets every soft neighbourhood of the soft point, that is if for every $(N, \mathbb{E}) \in \mathcal{N}_{(x_\alpha,\mathbb{E})}$, $(F, \mathbb{E}) \tilde{\cap} (N, \mathbb{E}) \tilde{\neq} (\tilde{\emptyset}, \mathbb{E})$.

As in the classical topological space, it is possible to prove that the soft closure of a soft set coincides with the set of all its soft adherent points.

Proposition 5.8. (Xie, 2015) *Let* $(X, \mathcal{T}, \mathbb{E})$ *be a soft topological space,* $(F, \mathbb{E}) \in \mathcal{SS}(X)_{\mathbb{E}}$ *and* $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$ *be a soft set and a soft point over the common universe* X *with respect to the sets of parameters* $\mathbb{E}$*, respectively. Then* $(x_\alpha, \mathbb{E}) \tilde{\in}$ s-$\mathrm{cl}_X(F, \mathbb{E})$ *if and only if* $(x_\alpha, \mathbb{E})$ *is a soft adherent point of* $(F, \mathbb{E})$.

Having in mind the Definition 5.18 we can recall the following proposition.

Proposition 5.9. (Hussain and Ahmad, 2011) *Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X, and Y be a nonempty subset of X, then the family $\mathcal{T}_Y$ of all sub soft sets of $\mathcal{T}$ over Y, i.e.*

$$\mathcal{T}_Y = \left\{ \left({}^Y F, \mathbb{E} \right) : (F, \mathbb{E}) \in \mathcal{T} \right\}$$

is a soft topology on Y.

Definition 5.32. (Hussain and Ahmad, 2011) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X, and let Y be a nonempty subset of X, the soft topology $\mathcal{T}_Y = \left\{ \left({}^Y F, \mathbb{E} \right) : (F, \mathbb{E}) \in \mathcal{T} \right\}$ is said to be the ***soft relative topology*** of $\mathcal{T}$ on Y and $(Y, \mathcal{T}_Y, \mathbb{E})$ is called the ***soft topological subspace*** of $(X, \mathcal{T}, \mathbb{E})$ on Y.

Definition 5.33. (Zorlutuna et al, 2012) Let $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ be a soft mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ induced by the mappings $\varphi : X \to X'$ and $\psi : \mathbb{E} \to \mathbb{E}'$ and $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$ be a soft point over X. We say that the soft mapping φ_ψ is ***soft continuous at the soft point*** $(x_\alpha, \mathbb{E})$ if for each soft neighbourhood $(G, \mathbb{E}')$ of $\varphi_\psi(x_\alpha, \mathbb{E})$ in $(X', \mathcal{T}', \mathbb{E}')$ there exists some soft neighbourhood $(F, \mathbb{E})$ of $(x_\alpha, \mathbb{E})$ in $(X, \mathcal{T}, \mathbb{E})$ such that $\varphi_\psi(F, \mathbb{E}) \tilde{\subseteq} (G, \mathbb{E}')$.
If φ_ψ is soft continuous at every soft point $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_{\mathbb{E}}$, then $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is called ***soft continuous*** on X.

Proposition 5.10. (Zorlutuna et al, 2012) *Let $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ be a soft mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ induced by the mappings $\varphi : X \to X'$ and $\psi : \mathbb{E} \to \mathbb{E}'$. Then the soft mapping φ_ψ is soft continuous if and only if every soft inverse image of a soft open set in X' is a soft open set in X, that is, if for each $(G, \mathbb{E}') \in \mathcal{T}'$ we have that $\varphi_\psi^{-1}(G, \mathbb{E}') \in \mathcal{T}$.*

Proposition 5.11. (Zorlutuna et al, 2012) *Let $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ be a soft mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ induced by the mappings $\varphi : X \to X'$ and $\psi : \mathbb{E} \to \mathbb{E}'$. Then the soft mapping φ_ψ is soft continuous if and only if every soft inverse image of a soft closed set in X' is a soft closed set in X, that is, if for each $(C, \mathbb{E}') \in \sigma(X', \mathbb{E}')$ we have that $\varphi_\psi^{-1}(C, \mathbb{E}') \in \sigma(X, \mathbb{E})$.*

Definition 5.34. (Zorlutuna et al, 2012) Let $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ be a soft mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ induced by the mappings $\varphi : X \to X'$ and $\psi : \mathbb{E} \to \mathbb{E}'$, and let Y be a nonempty subset of X, the ***restriction*** of the soft mapping φ_ψ to Y, denoted by ${\varphi_\psi}_{|Y}$, is the soft mapping $\left(\varphi_{|Y}\right)_\psi : \mathcal{SS}(Y)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ induced by the restriction $\varphi_{|Y} : Y \to X'$ of the mapping φ between the universe sets and by the same mapping $\psi : \mathbb{E} \to \mathbb{E}'$ between the parameter sets.

Proposition 5.12. (Zorlutuna et al, 2012) *If $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is a soft continuous mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$, then its restriction ${\varphi_\psi}_{|Y} : \mathcal{SS}(Y)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ to a nonempty subset Y of X is soft continuous too.*

Proposition 5.13. *If $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is a soft continuous mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$, then its corestriction $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \varphi_\psi\left(\mathcal{SS}(X)_{\mathbb{E}}\right)$ is soft continuous too.*

Proof. It easily follows from Definitions 5.20 and 5.21, and Proposition 5.10.

Definition 5.35. (Aygünoğlu and Aygün, 2012) Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over X and $\mathcal{S} \subseteq \mathcal{T}$ be a non-empty subset of soft open sets. We say that $\mathcal{S}$ is a ***soft open subbase*** for $(X, \mathcal{T}, \mathbb{E})$ if the family of all finite soft intersections of members of $\mathcal{S}$ forms a soft open base for $(X, \mathcal{T}, \mathbb{E})$.

Proposition 5.14. (Aygünoğlu and Aygün, 2012) *Let $\mathcal{S} \subseteq \mathcal{SS}(X)_{\mathbb{E}}$ be a family of soft sets over X, containing both the null soft set $(\tilde{\emptyset}, \mathbb{E})$ and the absolute soft set $(\tilde{X}, \mathbb{E})$. Then the family $\mathcal{T}(\mathcal{S})$ of all soft union of finite soft intersections of soft sets in $\mathcal{S}$ is a soft topology having $\mathcal{S}$ as soft open subbase.*

Definition 5.36. (Aygünoğlu and Aygün, 2012) Let $\mathcal{S} \subseteq \mathcal{SS}(X)_{\mathbb{E}}$ be a a family of soft sets over X respect to a set of parameters $\mathbb{E}$ and such that $(\tilde{\emptyset}, \mathbb{E}), (\tilde{X}, \mathbb{E}) \in \mathcal{S}$, then the soft topology $\mathcal{T}(\mathcal{S})$ of the above Proposition 5.14 is called the ***soft topology generated*** by the soft open subbase $\mathcal{S}$ over X and $(X, \mathcal{T}(\mathcal{S}), \mathbb{E})$ is said to be the ***soft topological space generated by*** $\mathcal{S}$ over X.

Definition 5.37. (Aygünoğlu and Aygün, 2012) Let $\mathcal{SS}(X)_{\mathbb{E}}$ be the set of all the soft sets over a universe set X with respect to a set of parameter $\mathbb{E}$ and consider a family of soft topological spaces $\{(Y_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i \in I}$ and a corresponding family $\{(\varphi_\psi)_i\}_{i \in I}$ of soft mappings $(\varphi_\psi)_i = (\varphi_i)_{\psi_i} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(Y_i)_{\mathbb{E}_i}$ induced by the mappings $\varphi_i : X \to Y_i$ and $\psi_i : \mathbb{E} \to \mathbb{E}_i$ (with $i \in I$). Then the soft topology $\mathcal{T}(\mathcal{S})$ generated by the soft open subbase $\mathcal{S} = \left\{(\varphi_\psi)_i^{-1}(G, \mathbb{E}_i) : (G, \mathbb{E}_i) \in \mathcal{T}_i,\ i \in I\right\}$ of all soft inverse images of soft open sets of $\mathcal{T}_i$ under the soft mappings $(\varphi_\psi)_i$ is called the ***initial soft topology*** induced on X by the family of soft mappings $\{(\varphi_\psi)_i\}_{i \in I}$ and it is denoted by $\mathcal{T}_{ini}(X, \mathbb{E}, Y_i, \mathbb{E}_i, (\varphi_\psi)_i; i \in I)$.

Proposition 5.15. (Aygünoğlu and Aygün, 2012) *The initial soft topology $\mathcal{T}_{ini}(X, \mathbb{E}, Y_i, \mathbb{E}_i, (\varphi_\psi)_i; i \in I)$ induced on X by the family of soft mappings $\{(\varphi_\psi)_i\}_{i \in I}$ is the coarsest soft topology on $\mathcal{SS}(X)_{\mathbb{E}}$ for which all the soft mappings $(\varphi_\psi)_i : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(Y_i)_{\mathbb{E}_i}$ (with $i \in I$) are soft continuous.*

Definition 5.38. (Aygünoğlu and Aygün, 2012) Let $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i \in I}$ be a family of soft topological spaces over the universe sets X_i with respect to the sets of parameters $\mathbb{E}_i$, respectively. For every $i \in I$, the soft mapping $(\pi_i)_{\rho_i} : \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ induced by the canonical projections $\pi_i : \prod_{i \in I} X_i \to X_i$ and $\rho_i : \prod_{i \in I} \mathbb{E}_i \to \mathbb{E}_i$ is said the ***i-th soft projection mapping*** and, by setting $(\pi_\rho)_i = (\pi_i)_{\rho_i}$, it will be denoted by $(\pi_\rho)_i : \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$.

Definition 5.39. (Aygünoğlu and Aygün, 2012) Let $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i \in I}$ be a family of soft topological spaces and let $\{(\pi_\rho)_i\}_{i \in I}$ be the corresponding family of soft projection mappings $(\pi_\rho)_i : \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ (with $i \in I$). Then, the

initial soft topology $\mathcal{T}_{ini}\big(\prod_{i\in I} X_i, \mathbb{E}, X_i, \mathbb{E}_i, (\pi_\rho)_i; i \in I\big)$ induced on $\prod_{i\in I} X_i$ by the family of soft projection mappings $\{(\pi_\rho)_i\}_{i\in I}$ is called the ***soft product topology*** of the soft topologies $\mathcal{T}_i$ (with $i \in I$) and denoted by $\mathcal{T}\big(\prod_{i\in I} X_i\big)$.
The triplet $\big(\prod_{i\in I} X_i, \mathcal{T}\big(\prod_{i\in I} X_i\big), \prod_{i\in I} \mathbb{E}_i\big)$ will be said the ***soft topological product space*** of the soft topological spaces $(X_i, \mathcal{T}_i, \mathbb{E}_i)$.

The following statement easily derives from Definition 5.39 and Proposition 5.15.

Corollary 5.4. *The soft product topology $\mathcal{T}\big(\prod_{i\in I} X_i\big)$ is the coarsest soft topology over $\mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I}\mathbb{E}_i}$ for which all the soft projection mappings $(\pi_\rho)_i : \mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I}\mathbb{E}_i} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ (with $i \in I$) are soft continuous.*

Proposition 5.16. (Aygünoğlu and Aygün, 2012) *Let $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i\in I}$ be a family of soft topological spaces, $(X, \mathcal{T}(X), \mathbb{E})$ be the soft topological product of such soft spaces induced on the product $X = \prod_{i\in I} X_i$ of universe sets with respect to the product $\mathbb{E} = \prod_{i\in I} \mathbb{E}_i$ of the sets of parameters, $(Y, \mathcal{T}', \mathbb{E}')$ be a soft topological space and $\varphi_\psi : \mathcal{SS}(Y)_{\mathbb{E}'} \to \mathcal{SS}(X)_{\mathbb{E}}$ be a soft mapping induced by the mappings $\varphi : Y \to X$ and $\psi : \mathbb{E}' \to \mathbb{E}$. Then the soft mappings φ_ψ is soft continuous if and only if, for every $i \in I$, the soft compositions $(\pi_\rho)_i \,\widetilde{\circ}\, \varphi_\psi$ with the soft projection mappings $(\pi_\rho)_i : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ are soft continuous mappings.*

Let us note that the soft cartesian product $\widetilde{\prod}_{i\in I}(F_i, \mathbb{E}_i)$ of a family $\{(F_i, \mathbb{E}_i)\}_{i\in I}$ of soft sets over a set X_i with respect to a set of parameters $\mathbb{E}_i$, respectively, as introduced in Definition 5.19, is a soft set of the soft topological product space $\big(\prod_{i\in I} X_i, \mathcal{T}\big(\prod_{i\in I} X_i\big), \prod_{i\in I} \mathbb{E}_i\big)$ i.e. that $\widetilde{\prod}_{i\in I}(F_i, \mathbb{E}_i) \in \mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I}\mathbb{E}_i}$ and the following statement holds.

Proposition 5.17. (Nordo, 2019b) *Let $\big(\prod_{i\in I} X_i, \mathcal{T}\big(\prod_{i\in I} X_i\big), \prod_{i\in I} \mathbb{E}_i\big)$ be the soft topological product space of a family $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i\in I}$ of soft topological spaces and let $\widetilde{\prod}_{i\in I}(F_i, \mathbb{E}_i)$ be the soft product in $\mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I}\mathbb{E}_i}$ of a family $\{(F_i, \mathbb{E}_i)\}_{i\in I}$ of soft sets of $\mathcal{SS}(X_i)_{\mathbb{E}_i}$, for every $i \in I$. Then the soft closure of $\widetilde{\prod}_{i\in I}(F_i, \mathbb{E}_i)$ in the soft topological product $\big(\prod_{i\in I} X_i, \mathcal{T}\big(\prod_{i\in I} X_i\big), \prod_{i\in I} \mathbb{E}_i\big)$ coincides with the soft product of the corresponding soft closures of the soft sets $(F_i, \mathbb{E}_i)$ in the corresponding soft topological spaces $(X_i, \mathcal{T}_i, \mathbb{E}_i)$, that is:*

$$\text{s-cl}_{\prod_{i\in I} X_i}\left(\widetilde{\prod_{i\in I}}(F_i, \mathbb{E}_i)\right) \widetilde{=} \widetilde{\prod_{i\in I}} \text{s-cl}_{X_i}(F_i, \mathbb{E}_i).$$

5.3 Soft Embedding Theorem

Definition 5.40. (Aras et al, 2013) Let $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ be two soft topological spaces over the universe sets X and X' with respect to the sets of parameters $\mathbb{E}$ and $\mathbb{E}'$, respectively. We say that a soft mapping $\varphi_\psi : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is a

soft homeomorphism if it is soft continuous, bijective and its soft inverse mapping $\varphi_{\psi}^{-1} : \mathcal{SS}(X')_{\mathbb{E}'} \to \mathcal{SS}(X)_{\mathbb{E}}$ is soft continuous too. In such a case, the soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ are said ***soft homeomorphic*** and we write that $(X, \mathcal{T}, \mathbb{E}) \tilde{\approx} (X', \mathcal{T}', \mathbb{E}')$.

Definition 5.41. Let $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ be two soft topological spaces. We say that a soft mapping $\varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is a ***soft embedding*** if its corestriction $\varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \varphi_{\psi}\left(\mathcal{SS}(X)_{\mathbb{E}}\right)$ is a soft homeomorphism.

Definition 5.42. (Aras et al, 2013) Let $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$ be two soft topological spaces. We say that a soft mapping $\varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ is a ***soft closed mapping*** if the soft image of every soft closed set of $(X, \mathcal{T}, \mathbb{E})$ is a soft closed set of $(X', \mathcal{T}', \mathbb{E}')$, that is if for any $(C, \mathbb{E}) \in \sigma(X, \mathbb{E})$, we have $\varphi_{\psi}(C, \mathbb{E}) \in \sigma(X', \mathbb{E}')$.

Proposition 5.18. *Let $\varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X')_{\mathbb{E}'}$ be a soft mapping between two soft topological spaces $(X, \mathcal{T}, \mathbb{E})$ and $(X', \mathcal{T}', \mathbb{E}')$. If φ_{ψ} is a soft continuous, injective and soft closed mapping then it is a soft embedding.*

Proof. If we consider the soft mapping $\varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \varphi_{\psi}\left(\mathcal{SS}(X)_{\mathbb{E}}\right)$, by hypothesis and Proposition 5.13, it immediately follows that it is a soft continuous bijective mapping and so we have only to prove that its soft inverse mapping $\varphi_{\psi}^{-1} = \left(\varphi^{-1}\right)_{\psi^{-1}} : \varphi_{\psi}\left(\mathcal{SS}(X)_{\mathbb{E}}\right) \to \mathcal{SS}(X)_{\mathbb{E}}$ is continuous too. In fact, because the bijectiveness of the corestriction and Remark 5.3, for every soft closed set $(C, \mathbb{E}) \in \sigma(X, \mathbb{E})$, the soft inverse image of the $(C, \mathbb{E})$ under the soft inverse mapping φ_{ψ}^{-1} coincides with the soft image of the same soft set under the soft mapping φ_{ψ}, that is $\left(\varphi_{\psi}^{-1}\right)^{-1}(C, \mathbb{E}) \tilde{=} \varphi_{\psi}(C, \mathbb{E})$ and since by hypothesis φ_{ψ} is soft closed, it follows that $\left(\varphi_{\psi}^{-1}\right)^{-1}(C, \mathbb{E}) \in \sigma(X', \mathbb{E}')$ which, by Proposition 5.11, proves that $\varphi_{\psi}^{-1} : \mathcal{SS}(X')_{\mathbb{E}'} \to \mathcal{SS}(X)_{\mathbb{E}}$ is a soft continuous mapping, and so, by Proposition 5.12, we finally have that $\varphi_{\psi}^{-1} : \varphi_{\psi}\left(\mathcal{SS}(X)_{\mathbb{E}}\right) \to \mathcal{SS}(X)_{\mathbb{E}}$ is a soft continuous mapping.

Definition 5.43. Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space over a universe set X with respect to a set of parameter $\mathbb{E}$, let $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i \in I}$ be a family of soft topological spaces over a universe set X_i with respect to a set of parameters $\mathbb{E}_i$, respectively and consider a family $\{(\varphi_{\psi})_i\}_{i \in I}$ of soft mappings $(\varphi_{\psi})_i = (\varphi_i)_{\psi_i} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ induced by the mappings $\varphi_i : X \to X_i$ and $\psi_i : \mathbb{E} \to \mathbb{E}_i$ (with $i \in I$). Then the soft mapping $\Delta = \varphi_{\psi} : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i}$ induced by the diagonal mappings (in the classical meaning) $\varphi = \Delta_{i \in I} \varphi_i : X \to \prod_{i \in I} X_i$ on the universes sets and $\psi = \Delta_{i \in I} \psi_i : \mathbb{E} \to \prod_{i \in I} \mathbb{E}_i$ on the sets of parameters (respectively defined by $\varphi(x) = \langle \varphi_i(x) \rangle_{i \in I}$ for every $x \in X$ and by $\psi(e) = \langle \psi_i(e) \rangle_{i \in I}$ for every $e \in \mathbb{E}$) is called the ***soft diagonal mapping*** of the soft mappings $(\varphi_{\psi})_i$ (with $i \in I$) and it is denoted by $\Delta = \Delta_{i \in I} (\varphi_{\psi})_i : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i}$.

The following proposition establishes a useful relation about the soft image of a soft diagonal mapping.

Proposition 5.19. (Nordo, 2019b) *Let* $(X, \mathcal{T}, \mathbb{E})$ *be a soft topological space over a universe set* X *with respect to a set of parameter* $\mathbb{E}$*, let* $(F, \mathbb{E}) \in \mathcal{SS}(X)_{\mathbb{E}}$ *be a soft set of* X*, let* $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i \in I}$ *be a family of soft topological spaces over a universe set* X_i *with respect to a set of parameters* $\mathbb{E}_i$*, respectively and let* $\Delta = \Delta_{i \in I}(\varphi_\psi)_i : \mathcal{SS}(X)_{\mathbb{E}} \to \mathcal{SS}(\prod_{i \in I} X_i)_{\prod_{i \in I} \mathbb{E}_i}$ *be the soft diagonal mapping of the soft mappings* $(\varphi_\psi)_i$*, with* $i \in I$*. Then the soft image of the soft set* $(F, \mathbb{E})$ *under the soft diagonal mapping* Δ *is soft contained in the soft product of the soft images of the same soft set under the soft mappings* $(\varphi_\psi)_i$*, that is*

$$\Delta(F, \mathbb{E}) \mathrel{\tilde{\subseteq}} \widetilde{\prod}_{i \in I} (\varphi_\psi)_i(F, \mathbb{E}).$$

Proof. Set $\varphi = \Delta_{i \in I} \varphi_i : X \to \prod_{i \in I} X_i$ and $\psi = \Delta_{i \in I} \psi_i : \mathbb{E} \to \prod_{i \in I} \mathbb{E}_i$, by Definition 5.43, we know that $\Delta = \Delta_{i \in I}(\varphi_\psi)_i = \varphi_\psi$. Suppose, by contradiction, that there exists some soft point $(x_\alpha, \mathbb{E}) \tilde{\in} (F, \mathbb{E})$ such that

$$\Delta(x_\alpha, \mathbb{E}) \mathrel{\tilde{\notin}} \widetilde{\prod}_{i \in I} (\varphi_\psi)_i(F, \mathbb{E}).$$

Set $\left(y_\beta, \prod_{i \in I} \mathbb{E}_i\right) \mathrel{\tilde{=}} \Delta(x_\alpha, \mathbb{E}) \mathrel{\tilde{=}} \varphi_\psi(x_\alpha, \mathbb{E})$, by Proposition 5.4, it follows that

$$\left(y_\beta, \prod_{i \in I} \mathbb{E}_i\right) \mathrel{\tilde{=}} \left(\varphi(x)_{\psi(\alpha)}, \prod_{i \in I} \mathbb{E}_i\right)$$

where

$$y = \langle y_i \rangle_{i \in I} = \varphi(x) = \left(\Delta_{i \in I} \varphi_i\right)(x) = \langle \varphi_i(x) \rangle_{i \in I}$$

and

$$\beta = \langle \beta_i \rangle_{i \in I} = \psi(\alpha) = \left(\Delta_{i \in I} \psi_i\right)(\alpha) = \langle \psi_i(\alpha) \rangle_{i \in I}.$$

So, set $(G_i, \mathbb{E}_i) \mathrel{\tilde{=}} (\varphi_\psi)_i(F, \mathbb{E})$ for every $i \in I$, we have that

$$\left(y_\beta, \prod_{i \in I} \mathbb{E}_i\right) \mathrel{\tilde{\notin}} \widetilde{\prod}_{i \in I} (G_i, \mathbb{E}_i)$$

hence, by Proposition 5.2, it follows that there exists some $j \in I$ such that

$$\left((y_j)_{\beta_j}, \mathbb{E}_j\right) \mathrel{\tilde{\notin}} (G_j, \mathbb{E}_j)$$

that, by Definition 5.15, means

$$y_j \notin G_j(\beta_j)$$

i.e.

$$\varphi_j(x) \notin G_j\left(\psi_j(\alpha)\right)$$

and so, by using again Definition 5.15, we have

$$\left(\varphi_j(x)_{\psi_j(\alpha)}, \mathbb{E}_j\right) \tilde{\notin} (G_j, \mathbb{E}_j)$$

that, by Proposition 5.4, is equivalent to

$$(\varphi_\psi)_j(x_\alpha, \mathbb{E}) \tilde{\notin} (G_j, \mathbb{E}_j)$$

which is a contradiction because we know that $(x_\alpha, \mathbb{E}) \tilde{\in} (F, \mathbb{E})$ and by Corollary 5.3(1) it follows $(\varphi_\psi)_j(x_\alpha, \mathbb{E}) \tilde{\in} (\varphi_\psi)_j(F, \mathbb{E}) \tilde{=} (G_j, \mathbb{E}_j)$.

Definition 5.44. Let $\{(\varphi_\psi)_i\}_{i\in I}$ be a family of soft mappings $(\varphi_\psi)_i : \mathcal{SS}(X)_\mathbb{E} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ between a soft topological space $(X, \mathcal{T}, \mathbb{E})$ and the members of a family of soft topological spaces $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i\in I}$.
We say that the family $\{(\varphi_\psi)_i\}_{i\in I}$ ***soft separates soft points*** of $(X, \mathcal{T}, \mathbb{E})$ if for every $(x_\alpha, \mathbb{E}), (y_\beta, \mathbb{E}) \in \mathcal{SP}(X)_\mathbb{E}$ such that $(x_\alpha, \mathbb{E}) \tilde{\neq} (y_\alpha, \mathbb{E})$ there exists some $j \in I$ such that $(\varphi_\psi)_j(x_\alpha, \mathbb{E}) \tilde{\neq} (\varphi_\psi)_j(y_\beta, \mathbb{E})$.

Definition 5.45. Let $\{(\varphi_\psi)_i\}_{i\in I}$ be a family of soft mappings $(\varphi_\psi)_i : \mathcal{SS}(X)_\mathbb{E} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ between a soft topological space $(X, \mathcal{T}, \mathbb{E})$ and the members of a family of soft topological spaces $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i\in I}$.
We say that the family $\{(\varphi_\psi)_i\}_{i\in I}$ ***soft separates soft points from soft closed sets*** of $(X, \mathcal{T}, \mathbb{E})$ if for every $(C, \mathbb{E}) \in \sigma(X, \mathbb{E})$ and every $(x_\alpha, \mathbb{E}) \in \mathcal{SP}(X)_\mathbb{E}$ such that $(x_\alpha, \mathbb{E}) \tilde{\in} \left(\tilde{X}, \mathbb{E}\right) \tilde{\setminus} (C, \mathbb{E})$ there exists some $j \in I$ such that $(\varphi_\psi)_j(x_\alpha, \mathbb{E}) \tilde{\notin}$ s-cl$_{X_j}((\varphi_\psi)_j(C, \mathbb{E}))$.

Proposition 5.20 (Soft Embedding Theorem). *Let $(X, \mathcal{T}, \mathbb{E})$ be a soft topological space, $\{(X_i, \mathcal{T}_i, \mathbb{E}_i)\}_{i\in I}$ be a family of soft topological spaces and $\{(\varphi_\psi)_i\}_{i\in I}$ be a family of soft continuous mappings $(\varphi_\psi)_i : \mathcal{SS}(X)_\mathbb{E} \to \mathcal{SS}(X_i)_{\mathbb{E}_i}$ that separates both the soft points and the soft points from the soft closed sets of $(X, \mathcal{T}, \mathbb{E})$. Then the soft diagonal mapping $\Delta = \Delta_{i\in I}(\varphi_\psi)_i : \mathcal{SS}(X)_\mathbb{E} \to \mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I} \mathbb{E}_i}$ of the soft mappings $(\varphi_\psi)_i$ is a soft embedding.*

Proof. Let $\varphi = \Delta_{i\in I}\varphi_i$, $\psi = \Delta_{i\in I}\psi_i$ and $\Delta = \Delta_{i\in I}(\varphi_\psi)_i = \varphi_\psi$ as in Definition 5.43, for every $i \in I$, by using Definition 5.23, we have that every corresponding soft composition is given by

$$(\pi_\rho)_i \,\tilde{\circ}\, \Delta = ((\pi_i)_{\rho_i}) \,\tilde{\circ}\, \varphi_\psi = (\pi_i \circ \varphi)_{\rho_i \circ \psi} = (\varphi_i)_{\psi_i} = (\varphi_\psi)_i$$

which, by hypothesis, is a soft continuous mapping. Hence, by Proposition 5.16, it follows that the soft diagonal mapping $\Delta : \mathcal{SS}(X)_\mathbb{E} \to \mathcal{SS}(\prod_{i\in I} X_i)_{\prod_{i\in I} \mathbb{E}_i}$ is a soft continuous mapping.

Now, let $(x_\alpha, \mathbb{E})$ and $(y_\beta, \mathbb{E})$ be two distinct soft points of $\mathcal{SP}(X)_\mathbb{E}$. Since, by hypothesis, the family $\{(\varphi_\psi)_i\}_{i\in I}$ of soft mappings soft separates soft points, by Definition 5.44, we have that there exists some $j \in I$ such that

$$(\varphi_\psi)_j(x_\alpha, \mathbb{E}) \tilde{\neq} (\varphi_\psi)_j(y_\beta, \mathbb{E})$$

that is
$$(\varphi_j)_{\psi_j}(x_\alpha,\mathbb{E})\,\tilde{\neq}\,(\varphi_j)_{\psi_j}(y_\beta,\mathbb{E}).$$
Hence, by Proposition 5.4, we have that:
$$\left(\varphi_j(x)_{\psi_i(\alpha)},\mathbb{E}_j\right)\tilde{\neq}\left(\varphi_j(y)_{\psi_i(\beta)},\mathbb{E}_j\right)$$
and so, by the Definition 5.17 of distinct soft points, it necessarily follows that:
$$\varphi_j(x)\neq\varphi_j(y)\quad\text{or}\quad\psi_j(\alpha)\neq\psi_j(\beta).$$
Since $\varphi=\Delta_{i\in I}\varphi_i : X\to\prod_{i\in I}X_i$ and $\psi=\Delta_{i\in I}\psi_i : \mathbb{E}\to\prod_{i\in I}\mathbb{E}_i$ are usual diagonal mappings, we have that:
$$\varphi(x)\neq\varphi(y)\quad\text{or}\quad\psi(\alpha)\neq\psi(\beta)$$
and, by Definition 5.17, it follows that:
$$\left(\varphi(x)_{\psi(\alpha)},\prod_{i\in I}\mathbb{E}_i\right)\tilde{\neq}\left(\varphi(y)_{\psi(\beta)},\prod_{i\in I}\mathbb{E}_i\right)$$
hence, applying again Proposition 5.4, we get:
$$\varphi_\psi(x_\alpha,\mathbb{E})\,\tilde{\neq}\,\varphi_\psi(y_\beta,\mathbb{E})$$
that is:
$$\Delta_{i\in I}(\varphi_\psi)_i(x_\alpha,\mathbb{E})\,\tilde{\neq}\,\Delta_{i\in I}(\varphi_\psi)_i(y_\beta,\mathbb{E})$$
i.e. that $\Delta(x_\alpha,\mathbb{E})\,\tilde{\neq}\,\Delta(y_\beta,\mathbb{E})$ which, by Corollary 5.2, proves the injectivity of the soft diagonal mapping $\Delta:\mathcal{SS}(X)_\mathbb{E}\to\mathcal{SS}(\prod_{i\in I}X_i)_{\prod_{i\in I}\mathbb{E}_i}$.

Finally, let $(C,\mathbb{E})\in\sigma(X,\mathbb{E})$ be a soft closed set in X and, in order to prove that the soft image $\Delta(C,\mathbb{E})$ is a soft closed set of $\sigma\left(\prod_{i\in I}X_i,\prod_{i\in I}\mathbb{E}_i\right)$, consider a soft point $(x_\alpha,\mathbb{E})\in\mathcal{SP}(X)_\mathbb{E}$ such that $\Delta(x_\alpha,\mathbb{E})\tilde{\notin}\Delta(C,\mathbb{E})$ and, hence, by Corollary 5.3(1), such that $(x_\alpha,\mathbb{E})\tilde{\notin}(C,\mathbb{E})$. Since, by hypothesis, the family $\{(\varphi_\psi)_i\}_{i\in I}$ of soft mappings soft separates soft points from soft closed sets, by Definition 5.45, we have that there exists some $j\in I$ such that
$$(\varphi_\psi)_j(x_\alpha,\mathbb{E})\,\tilde{\notin}\,\text{s-cl}_{X_j}((\varphi_\psi)_j(C,\mathbb{E}))$$
that is:
$$(\varphi_j)_{\psi_j}(x_\alpha,\mathbb{E})\,\tilde{\notin}\,\text{s-cl}_{X_j}((\varphi_\psi)_j(C,\mathbb{E}))$$
that, by Proposition 5.4, corresponds to:
$$\left(\varphi_j(x)_{\psi_j(\alpha)},\mathbb{E}_j\right)\,\tilde{\notin}\,\text{s-cl}_{X_j}((\varphi_\psi)_j(C,\mathbb{E}))\ .$$

So, set $(C_i, \mathbb{E}_i) \tilde{=} \text{s-cl}_{X_i}((\varphi_\psi)_i(C, \mathbb{E}))$ for every $i \in I$, we have in particular for $i = j$ that

$$\left(\varphi_j(x)_{\psi_j(\alpha)}, \mathbb{E}_j\right) \tilde{\notin} (C_j, \mathbb{E}_j)$$

which, by Definition 5.15, is equivalent to say that:

$$\varphi_j(x) \notin C_j\left(\psi_j(\alpha)\right)$$

and since the diagonal mapping $\varphi = \Delta_{i \in I}\varphi_i : X \to \prod_{i \in I} X_i$ on the universes sets is defined by $\varphi(x) = \langle \varphi_i(x) \rangle_{i \in I}$, it follows that:

$$\varphi(x) \notin \prod_{i \in I} C_i\left(\psi_i(\alpha)\right).$$

Now, since the diagonal mapping $\psi = \Delta_{i \in I}\psi_i : X \to \prod_{i \in I} X_i$ on the sets of parameters is defined by $\psi(\alpha) = \Delta_{i \in I}\psi_i(\alpha) = \langle \psi_i(\alpha) \rangle_{i \in I}$, using Definition 5.19, we obtain:

$$\prod_{i \in I} C_i\left(\psi_i(\alpha)\right) = \left(\prod_{i \in I} C_i\right)\left(\psi(\alpha)\right)$$

and hence that

$$\varphi(x) \notin \left(\prod_{i \in I} C_i\right)\left(\psi(\alpha)\right)$$

which, by Definitions 5.15 and 5.19, is equivalent to say that:

$$\left(\varphi(x)_{\psi(\alpha)}, \prod_{i \in I} \mathbb{E}_i\right) \tilde{\notin} \widetilde{\prod}_{i \in I}(C_i, \mathbb{E}_i)$$

that, by Proposition 5.4, means:

$$\varphi_\psi(x_\alpha, \mathbb{E}) \tilde{\notin} \widetilde{\prod}_{i \in I}(C_i, \mathbb{E}_i)$$

i.e.

$$\Delta(x_\alpha, \mathbb{E}) \tilde{\notin} \widetilde{\prod}_{i \in I} \text{s-cl}_{X_i}((\varphi_\psi)_i(C, \mathbb{E})).$$

So, recalling, by Proposition 5.17, that

$$\text{s-cl}_{\prod_{i \in I} X_i}\left(\widetilde{\prod}_{i \in I}(\varphi_\psi)_i(C, \mathbb{E})\right) \tilde{=} \widetilde{\prod}_{i \in I} \text{s-cl}_{X_i}((\varphi_\psi)_i(C, \mathbb{E}))$$

it follows that:

$$\Delta(x_\alpha, \mathbb{E}) \tilde{\notin} \text{s-cl}_{\prod_{i \in I} X_i}\left(\widetilde{\prod}_{i \in I}(\varphi_\psi)_i(C, \mathbb{E})\right).$$

Since, by Propositions 5.19 and 5.6(3) we have

$$\Delta(C,\mathbb{E})\,\tilde{\subseteq}\,\widetilde{\prod}_{i\in I}(\varphi_\psi)_i(C,\mathbb{E})\,\tilde{\subseteq}\,\text{s-cl}_{\prod_{i\in I}X_i}\left(\widetilde{\prod}_{i\in I}(\varphi_\psi)_i(C,\mathbb{E})\right)$$

and, by applying Propositions 5.7(1) and 5.6(5), we obtain

$$\text{s-cl}_{\prod_{i\in I}X_i}(\Delta(C,\mathbb{E}))\;\tilde{\subseteq}\;\text{s-cl}_{\prod_{i\in I}X_i}\left(\widetilde{\prod}_{i\in I}(\varphi_\psi)_i(C,\mathbb{E})\right)$$

it follows, a fortiori, that

$$\Delta(x_\alpha,\mathbb{E})\,\tilde{\notin}\;\text{s-cl}_{\prod_{i\in I}X_i}(\Delta(C,\mathbb{E}))\,.$$

So, it is proved by contradiction that $\text{s-cl}_{\prod_{i\in I}X_i}(\Delta(C,\mathbb{E}))\,\tilde{\subseteq}\,\Delta(C,\mathbb{E})$ and hence, by Proposition 5.6(4) and Definition 5.42, that $\Delta:\mathcal{SS}(X)_{\mathbb{E}}\to\mathcal{SS}(\prod_{i\in I}X_i)_{\prod_{i\in I}\mathbb{E}_i}$ is a soft closed mapping.

Thus, we finally have that the soft diagonal mapping $\Delta=\Delta_{i\in I}(\varphi_\psi)_i$: $\mathcal{SS}(X)_{\mathbb{E}}\to\mathcal{SS}(\prod_{i\in I}X_i)_{\prod_{i\in I}\mathbb{E}_i}$ is a soft continuous, injective and soft closed mapping and so, by Proposition 5.18, it is a soft embedding.

5.4 Conclusion

In this paper we have introduced the notions of family of soft mappings separating points and points from closed sets and that of soft diagonal mapping and we have proved a generalization to soft topological spaces of the well-known Embedding Theorem for classical (crisp) topological spaces. Such a result could be the start point for extending and investigating other important topics such as extension and compactifications theorems, metrization theorems etc. in the context of soft topology.

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