Chapter 12
Nonlinear Phenomena in Granular Solids: Modeling and Experiments

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Abstract The International Conference on Nonlinear Solid Mechanics (ICoNSoM) 2019, held in Rome from 16th to 19th of June 2019, had as main goal to gather together researchers in the field of nonlinear Solid Mechanics in a stimulating research environment. This work is a rational report of activities of one of the several mini-symposia held during the conference titled "Nonlinear Phenomena in Granular Solids: Modeling and Experiments". The main aim is to provide the interesting reader with the main topics treated during the discussions and to furnish all the relevant bibliography. Additional information, such as the abstracts of all the talks, can be found at the official web-site of the conference: http://www.memocsevents.eu/iconsom2019/.

Keywords: Granular solids · Continuum mechanics

12.1 Introduction

The study of granular materials is one of the key topics in the current landscape of Solid Mechanics. It is possible to find applications of results and methods of granular mechanics in several areas of fast and recent development such as biomechanics, geophysics, and material development (dell'Isola et al, 2015, 2017; Alibert et al, 2003; Barchiesi et al, 2019; Abali et al, 2017; Giorgio et al, 2016, 2017; dell'Isola et al, 2019a,b; Yang et al, 2018; Giorgio et al, 2009; Lossouarn et al, 2015; Alessandroni et al, 2004). One of the main reasons of this vast applicability is that several kinds of materials can be considered under the umbrella of granular materials. They span from soft membranes made of aggregations of cells to highly packed and dense solids made of particulate. From both the mathematical and experimental point of view, in

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all the forms that they have been studied, granular material represents a formidable challenge due to fundamental role played by the properties of the grains themselves and their interaction. This has been the main motivation to dedicate a symposium on modeling and experimental challenges in nonlinear granular mechanics within the International Conference of Nonlinear Solid Mechanics 2019 (ICoNSoM 2019), held in Rome from 16th to 19th of June 2019. The International Research Center on Mathematics and Mechanics of Complex Systems (M&MoCS) of L'Aquila University together with the McGill University of Montreal, the Roma Tre University, and the Laboratoire International Associé Coss&Vita have conceived the ICoNSoM 2019 international conference to allow the interaction and exchange of new ideas and points of view between scientists and engineers from all the areas of Solid Mechanics.

Due to the number of topics treated, the conference has been divided into thematic symposia. The main goal of this paper is to report the activities and the discussions of the *Nonlinear Phenomena in Granular Solids: Modeling and Experiments* symposium. In the next section, we will describe the content of the topics treated during the symposium and we will indicate all the relevant references. We aim for providing a comprehensive snapshot of the current state of the art of the research in this fundamental framework.

12.2 Nonlinear Phenomena in Granular Solids: Modeling and Experiments

The symposium *Nonlinear phenomena in granular solids: modeling and experiments* has been the stage of ten contributions. In this section, we will review the content of the talks and provide the relevant bibliography.

The contributions have been presented according to their more applicative or theoretical nature. In the first group we have seen the talks by Tatiana Yuzhina on deformation and deconstruction of hardwood trees, Mikhail Gonov on the dynamic properties of concrete under compression, Ioan Ionescu on damage probing in cemented granular materials, and Nakase Hitoshi on the study of risk assessment of nuclear facilities under severe scenarios. In the second group we have gathered the contributions by Niel Kruyt on the role of fabric in the behavior of granular material, Takashi Matsushima on grain-scale description of higher-order continuum model for granular solid, Luca Placidi on damage and plastic evolution of second gradient effective elastic moduli of heterogeneous granular materials, Payam Poorsolhjouy on micro-macro identification from grain properties, Maurizio Romeo on electromagnetic microcontinuum approaches to granular media, and Antoine Wautier on strain localization from a mesoscale point of view.

12.2.1 Deformation and Destruction at Deformation Rate of Order $10^3\,s^{-1}$ Wood of Hardwood Trees - Tatiana Yuzhina

Tatiana Yuzhina, in her talk "*Deformation and destruction at deformation rate of order* $10^3\,s^{-1}$ *wood of hardwood trees*" presented the results of tests on birch and aspen with different directions, with respect to the fibers location, of cutting samples. The steepness of the load branches and the destructive stresses have been analyzed with respect to different cutting angles and the resulting behavior, including also plasticity, have been represented in terms of stress-strain diagrams. In particular, the greatest destructive stress as well as the greatest steepness of the load branches has been observed at cutting angle of $0°$. On the other hand, the minimum values for such parameters have been observed for $90°$ cut corners. A remarkable behavior of these kinds of wood is that, after exhaustion, they show a relaxation behavior which passes into the area of ideal plasticity until the end of the load. Once the reasons of this behavior have been discussed (Bragov et al, 2018), the results of the analysis of the behavior of the wood batches with respect to the room temperature have been shown. The reader is invited to see the papers by Bragov and Lomunov (1997); Zhao et al (2016); Sciarra et al (2007); dell'Isola et al (2009); Madeo et al (2013); Turco (2018); Turco et al (2019); Bilotta and Turco (2017); Igumnov et al (2019) to gain a deeper insight into the subject, especially from a theoretical viewpoint.

12.2.2 Experimental Study of the Dynamic Properties of Concrete under Compressive Load - Mikhail Gonov

Mikhail Gonov presented a talk titled "*Experimental study of the dynamic properties of concrete under compressive load*". The talk discussed the study of the deformation and fracture of concrete under dynamic loads within the framework of Kolsky method. The main goal of such tests were to characterize the high-speed deformation and destruction of fine concrete. In particular, the role played by copper pulse shapers in the dynamic tests has been discussed. The tests consisted of 35 test shots carried out with copper pulse shapers at 7 different speed. In this kind of tests, the role of the pulse shaper is to enhance the quality of the basic premise of the Kolsky method on the homogeneity of the highly deformed state of the samples. To check on the quality of this assumption, an accurate comparison of the results obtained in dynamic tests with the pulse formers and in absence of them has been performed and the results (Bragov et al, 2013) have been presented. Many efforts are expended to understand the complicated behavior of a very complex material as the concrete do is. In addition to the presentation summarized here, some relevant articles can be provided by Konstantinov et al (2018); Giorgio et al (2019); Giorgio and Scerrato (2017); Scerrato et al (2014, 2016); Chiaia et al (2015); Kezmane et al (2017); Contrafatto and Cuomo (2006); Contrafatto et al (2012, 2017, 2016); Stochino et al (2016).

12.2.3 Damage Probing in Cemented Granular Materials with Ultrasound - Ioan Ionescu

The analysis of fault core sliding or earthquakes in different states of materials has been discussed in the talk "*Damage probing in cemented granular materials with ultrasound*" by Ioan Ionescu. In particular, the results of the experimental and numerical investigation of the transition from cohesive to granular states of synthetic rock under different loadings have been discussed (Langlois and Jia, 2014; Gomez et al, 2020). The rock model used during the investigation is cemented granular material in which it is possible to tune both the nature and amount of cements and the packing density. Damage has been modeled in gedometric configuration by considering an elasto-plastic deformation due to a quasi-static load. In this way the contact stiffness between the grains has been minimized during the damage evolution. The comparison of the results on wave velocity in damaged samples have been, finally, presented and discussed (Khidas and Jia, 2012).

12.2.4 The Role of Fabric in the Behavior of Granular Material - Niels Kruyt

During the talk "*The role of fabric in the behavior of granular material*" by Niels Kruyt, the interplay between the behavior of particles at micro-scale and the continuum model at macro-scale have been discussed. In particular, the speaker went through an overview on the role of fabric in characteristic behavior of granular materials such as shear strength, dilatancy, and elasto-plasticity (Pouragha et al, 2019). The evolution of the fabric of a initially isotropic granular assemblies undergoing small strain has been analyzed by means of a systematic approach in which the fabric changes during the evolution were modeled in terms of two-dimensional isotropic functions by means of a representation theorem. The results on this analysis on the evolution of the fabric has been presented and the crucial role played by contact gains and losses during the evolution has been elucidated (Kruyt, 2012, 2010; Fortin et al, 2005; Bonelli et al, 2012).

12.2.5 Grain-Scale Description of Higher-Order Continuum Model for Granular Solid - Takashi Matsushima

In the talk by Takashi Matsushima, "*Grain-scale description of higher-order continuum model for granular solid*", the results of a series of distinct element method based simulations aimed at exploring the shear band width as a function of packing density and of the grain properties have been presented. In particular, it has been presented the results on the analysis of the relation between the shear band width and

maximum dilation rate within the framework of strain gradient theory (Matsushima and Chang, 2011; Matsushima et al, 2002). Although it is well known that such finite width localization zone cannot be forecasted by classical first order elasticity theories, all the higher-order continuum theories are still phenomenological and the physical nature of the needed higher-order terms has not been clarified. The results of this analysis have been discussed, highlighting the relation between the maximum dilation rate, which is well described by strain-gradient theory, and the shear band width.

12.2.6 Validation of a Simple Model Using the Distinct Element Method for Numerical Simulations of Slope Collapse - Nakase Hitoshi

The contribution "*Validation of a simple model using the distinct element method for numerical simulations of slope collapse*" by Nakase Hitoshi was focusing on the study of risk assessment of nuclear facilities under severe scenarios, like earthquakes. In particular, the results of the investigation, based on the development of a model based on distinct element method simulations, on the reaching distance of fallen rocks and the impacts on nuclear facilities, as well as the risk of slope failure have been presented and deeply discussed (Nakase et al, 2017). In order to reduce the computational cost of distinct element method simulations based on clump elements to model the complex shape of sand or rocks, a new approach focusing on the geo-technical behavior of such complex shaped particles has been proposed. In this method, although the rock mass geometry is simplified, the probabilistic behavior of a single collision is carefully modeled in terms of the roughness of slope. The results of this method and the comparison with the standard approaches have been discussed in the final part of the talk.

12.2.7 Damage and Plastic Evolution of Second Gradient Effective Elastic Moduli of Heterogeneous Granular Materials - Luca Placidi

Luca Placidi, in his talk "*Damage and plastic evolution of second gradient effective elastic moduli of heterogeneous granular materials*", discussed a damage elasto-plastic model aimed at describing the evolution of anisotropy of a heterogeneous granular solid. In particular, the micro-mechanical model has been defined by considering a collection of particles interacting via inter-particle contacts, while to facilitate the derivation of the overall stiffness tensors, the interactions have been considered as continuous in the orientation space (Solyaev et al, 2019; Placidi et al, 2019). In the final part of the talk, some examples involving anisotropy have been discussed.

In particular, in order to entail the anisotropy of the heterogeneous granular solid, a model for elasto-plastic damage, based on spring interaction between the grains, has been outlined. The effectiveness of this model has been analyzed on some relevant examples and compared with standard numerical approaches (see, e.g., Placidi et al, 2018a; Placidi and Barchiesi, 2018; Abali et al, 2015; Placidi et al, 2018b, 2020; Nguyen and Niiranen, 2020).

12.2.8 Micro-Macro Identification: Continuum Parameters from Grain Properties - Payam Poorsolhjouy

In the contribution "*Micro-macro identification: continuum parameters from grain properties*" by Payam Poorsolhjouy a numerical approach to derive average grain-scale properties used in the Granular Micromechanics Approach has been presented. In particular, the results of the analysis under both small and large deformations and the evidences of non-affine movements in the assembly and their relations with higher gradient theories have been precisely discussed (Poorsolhjouy and Misra, 2019). The main advantage in the use of the Granular Micromechanics Approach with respect to other coarse-grained based models is that it is based on a statistical analysis of the directional distribution of both stiffness and geometric properties of the grain pain interactions. Starting with such knowledge of the microstructure, in the final part of the talk it has been discussed a numerical approach to derive the macroscopic properties of the materials. Finally, some considerations on the application of this approach on large deformations problems have been discussed. The literature on micro-macro identification of granular materials is absolutely vast, so it is quite difficult to recall all papers about that subject. However, a selection of papers considered of the utmost esteem by the writer is as follows (Misra and Poorsolhjouy, 2015a; Chang and Misra, 1990; Yang and Misra, 2012; Misra and Poorsolhjouy, 2015b; Misra and Singh, 2015; Misra and Poorsolhjouy, 2017; Nejadsadeghi et al, 2019; Misra et al, 2020; De Angelo et al, 2020; Abali et al, 2019; De Angelo et al, 2019).

12.2.9 Electromagnetic Microcontinuum Approach to Granular Media - Maurizio Romeo

Maurizio Romeo, in his talk "*Electromagnetic microcontinuum approach to granular media*", discussed an approach to the analysis of electro-magneto-elastic interactions in continuum mechanics. In particular, by exploiting the microcontinuum theory framework (Misra and Poorsolhjouy, 2016), an extension has been presented in Romeo (2011) regarding the classical constitutive approach to electromagnetic interactions (Abali and Queiruga, 2019). In this approach, micro-deformations are directly related to electric multipoles and, consequently, the expressions for electric

polarization and magnetization can be explicitly derived. The equations resulting from a variational method have been discussed, highlighting the effects of external electromagnetic dipole and quadrupole fields on granular materials (Romeo, 2016, 2018). The balance equations obtained in this way include Ampère law and Gauss law in the usual formalization in terms of scalar and vector potentials.

12.2.10 Strain Localization from a Mesoscale Point of View - Antoine Wautier

Antoine Wautier, in his contribution "*Strain localization from a mesoscale point of view*", presented the results of his investigation on the micro-mechanical mechanism responsible for the strain localization pattern in granular materials. This response is a peculiar characteristic of granular materials: on loose granular material only a pure hardening behavior happens and it is not possible to observe any strain localization. In particular, by performing an analysis of the mechanical and geometrical characteristic of force chains inside and outside the shear band area, it has been shown that the hardening regime is linked to the concentration of the load on less and less force chains and to the degradation of the contact network. Finally, an explanation of the softening regime in terms of force chain rotations within the shear band domain has been discussed (Wautier et al, 2018a). It is also worth noting the works by Wautier and Guzina (2015); Wautier et al (2017); Nicot et al (2017); Wautier et al (2018b); Eremeyev et al (2019); Eremeyev (2019); Eremeyev and Sharma (2019)

12.3 Conclusions

The centrality of Granular Mechanics in modern Solid Mechanics clearly emerges from the several topics touched during the *Nonlinear Phenomena in Granular Solids: Modeling and Experiments* symposium. Starting from very applicative investigations (such as wood and concrete characterizations, as discussed in the first two contributions) to theoretical discussions on generalized theory of elasticity and electromagnetism (e.g. the talks by Luca Placidi and Maurizio Romeo), Granular Mechanics appears to be an eclectic tool which, if properly understood, will bring promising developments in Solid Mechanics. Of course during the interesting discussions, moderated by the excellent work made by the chairmen, several open problems and questions, such as the role of the grains at different scales of description or the need of generalized theories of elasticity to properly describe the granular behavior, have been highlighted. The role of conferences like International Conference of Nonlinear Solid Mechanics 2019 (and of papers like the present one) is exactly to push this kind of discussions which are catalyst for the advancement of science. To this regard, the symposium *Nonlinear Phenomena in Granular Solids: Modeling and Experiments* can be considered a successful experience. We expect similar reports from other

symposia with the hope of spreading the ideas and creating new collaborations, which are the main and ultimate goals of events like ICoNSoM 2019.

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