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Ethnomathematics in Action

Mathematical Practices in Brazilian Indigenous, Urban and Afro Communities

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Foreword

It was a great pleasure and honor when my colleagues and former students Milton Rosa and Cristiane Coppe invited me to write the preface for this marvelous collection of essays, dealing with the mathematical knowledge and beliefs of Brazilian communities, with special attention to Africa, Afro-Brazilians, and indigenous peoples, and not disregarding those Brazilians of European, and Asiatic ancestry, and mixed populations. Current Brazilian population is a dynamic encounter of several ancestries. The essays address these issues and the consequences in mathematical practices and conceptions.

This book is a very important contribution to the emerging area of ethnomathematics. The summary shows the breadth of this collection. It would be very difficult to comment on each essay. The titles are illustrative of their contents and each one has an extensive bibliography. A special characteristic of the book is its large geographic range. The book is very original in both the choice of contents and the treatment given to all of them. It adds relevant aspects to the area and also to the history of mathematics, to the history of sciences, and to cultural studies in general.

I will use this preface to comment on relevant aspects of ethnomathematics, which are treated with different nuances in this entire collection. As one of the proponents of ethnomathematics, more than four decades ago, I can appreciate the advances of this new research area as reflected in the essays of this collection. In a paper, published in 1977, I referred to ethnosciences and ethnomathematics as the study of scientific and, by extension, technological phenomena in direct relation to their social, economic, and cultural background. I claimed that the great advantage of working in this border zone lies in the possibility of combining academic dynamic treatment with historical studies to improve education. Indeed, this synthesizes the strength of this collection of essays.

Ethnomathematics, as a research area, is relatively new. It is a concept resulting from the recognition of the biological kingdom *Animalia*. With few exceptions, animals consume organic material, breathe oxygen, are able to move, can reproduce sexually, and grow from embryonic development. Through a long evolutionary process, the genus *homo* evolved from animalia and gave origin to our species, *homo sapiens sapiens*, the humanity. From about 50,000, our species occupy the entire planet.

Our species aims at the survival of the individuals, through nourishment, and at the continuation of the species, through procreation. But the species *homo* developed another characteristic, unique to our species, which is to transcend the *pulsion of survival* of the individuals, and continuation of the species, with options that go beyond the anatomo-physiological mechanisms. This *pulsion of transcendence*, unique to the species *homo*, is difficult to explain. It gives rise to sophisticated and creative communication, and language, and to emotions, beliefs, and preferences. Some scholars call it will, others call it consciousness. There are other denominations, some related to myths and religions.

Consciousness is an elusive concept. Many scientists say that terms such as will, mind, consciousness, and other words describing subjective mental experiences cannot be defined. Is it located somewhere in the brain? It allows to predict what will happen in the future and to socially coordinate plans for the future. Consciousness helps to extract meaningful information from the sense organs. There are limitations, such as the recognition of will and of consciousness in anyone except oneself. Self-recognition is the ability to recognize oneself as distinct from another entity, as well as to plan, pay attention, recall memories of specific events, and take the perspective of another creature.

Environmental conditions require also activation of consciousness. Behavior may respond to instructions from the DNA, but also from learning and environmental influences within its own lifetime. To provide for all similar contingencies would require an impossible roster of instructions, wasteful volume of specific directions. In most of the contingencies, decisions must be ad hoc.

It is undeniable that the human species are characterized by the pulsion of survival of the individual and of the species, like all the living species, and by the pulsion of transcendence, unique to the human species. Both, survival and transcendence, are in mutual interaction. The acquisition of the pulsion of transcendence is the focus of mythologies and religions, a fascinating research theme.

Every individual (from birth to death), societies, and the human species in general develop strategies to cope with the ample reality. I clarify that every time I say ample reality I mean everything, the complex of natural and supernatural phenomena and facts, the physiological, sensorial, emotional, and psychic reactions to the environment in the broad sense and the social interactions. Indeed, everything, the *physis* (≈ nature) and the *nomos* (≈ social norms) are in permanent change. *Nomos* is as indispensable ground for *physis* as *physis* is an indispensable ground for *nomos*. They are in a symbiotic relation. *Physis* provides the potentialities, *nomos* the actuality of humanity.

The ensemble of the strategies to cope with ample reality is a complex system of knowledge and behavior, generated by individuals and socialized in a group of individuals with some affinity. This is the scenario discussed in all the essays of this book. The basic questions are how individuals and groups of individuals develop their means for surviving in their own natural and sociocultural environments, satisfying the pulsion of survival [body], and how they go beyond survival, acquiring the pulsion of transcendence [mind].

Survival and transcendence are the quintessence of human life and are dealt with a complex system of knowledge and behavior generated and organized by both each individual (from birth to death) and the affinity group. The way they are generated, and organized, and socialized relies in language and rhetoric in a broad sense.

These questions must be faced with *transcultural* and *transdisciplinarian* strategies, which borrow methods of research from the sciences, from cognition, from mythology, from anthropology, from history, from sociology (politics, economics, and education), and from cultural studies in general. These strategies rely on the analyses of the history of ideas, and of the evolution of behavior, and knowledge of the human species, in every natural and sociocultural environment. All these aspects are present in the essays of this book.

The *Program Ethnomathematics* discusses the corpora of knowledge developed by humans to survive and to transcend. It is a transdisciplinarian and transcultural research program supported by a new historiography. It is conceptually designed as a broad research program of the evolution of ideas, of knowledge, and of practices developed by the human species in different natural and sociocultural environments. Essentially, it implies in regard to the development of an analysis of how groups of humans generated ways, styles, arts, and techniques of *doing* and *knowing*, of learning and explaining, and of dealing with situations, and of solving problems of their natural and sociocultural environments.

I practiced an etymological abuse with the *free* appropriation of Greek roots: *techné* [*tics*] meaning ways, styles, arts, and techniques; *mathema* for *doing* and *knowing*, for learning and explaining, for dealing with situations and solving problems; and *ethno*, as distinct and specific natural and sociocultural environments. Thus, by using these Greek roots, the locution *tics+mathema+ethno* synthesizes the way groups of humans generated ways, styles, arts, and techniques of *doing* and *knowing*, of learning and explaining, and of dealing with situations, and of solving problems of their natural and sociocultural environments. Rearranging this conceptual etymological construction gave origin to the word *ethno+mathema+tics* or simply *Ethnomathematics*.

It is very important to clarify the meaning of the word ethnomathematics as I use it. It may be misleading. I do not use it as *mathematics of an ethnic group*. It is not *ethno+mathematics*. It is important for the readers of this collection of essays to be aware of this important concept. Although the essays of this book are assembled in parts dealing with Africanities, indigenous diversities, urban diversities, and cultural practices, which discuss specific cases, like case studies, they fit into the broader concept of *ethno+mathema+tics*.

Undeniably, each ethnic group, professional, and gremial group have their specific ways and modes of quantitative and qualitative practices, such as counting, weighing and measuring, comparing, sorting and classifying, inferring, and modelling, accumulated through generations in their natural, social, and cultural environments; the program ethnomathematics goes much beyond this.

Repeating what I wrote above, the word ethnomathematics was etymologically constructed as a broad theory of the evolution of ideas, of knowledge, and of practices developed differently by the human species in their different natural and sociocultural environments. Essentially, it implies of an analysis of how groups of humans generated ways, styles, arts, and techniques [*techné*] to *do* and to *know*, to learn and to explain, to deal with situations, and to solve problems [*mathema*] of their natural and sociocultural environments [*ethno*].

Although the words ethnobotany, ethnomusicology, ethnolinguistic, ethnomethodology, and other *ethno+disciplines* are used by anthropologists, by ethnographers, and by sociologists for research of specific disciplines in different ethnic and sociocultural contexts, they base their investigations on the views of an observer of *other* cultures, trying to find commonalities between the culture of the researchers (the observers) and the culture of the researched (the observed).

Obviously, the concept of mathematics of the observers, who studied it as an academic discipline, may have no meaning at all for the members of *other* cultures. Academic mathematics is a European construct springing out of specific styles and ways of counting, weighing and measuring, comparing, sorting and classifying, inferring, and modelling, which were organized by peoples of the Mediterranean Basin, since before the Christian Era, and organized by Euclid (*c.*300 B.C.) and several others academicians and scholars.

Different cultures have their own styles and ways of counting, weighing and measuring, comparing, sorting and classifying, inferring, and modelling, which have been historically generated and organized by their own people, sages, and scholars. Every human community developed cognitive strategies, ideas, language, myths, religions, social stratifications, governance, and other categories of social behavior to serve specific purposes.

The same occurs also for artistic representations and abstractions, figures, numerical tools meaning quantities, as well as ways of inferring. It may not make sense to address different cultural groups asking questions such as: “What is the meaning of a triangle?” or “How would you add 2 and 3?” or “What is the color of this flower?”. The categories: triangle, 2 plus 3, and color may be absolutely senseless in their culture. The conceptual locution *Ethno+mathema+tics* as I conceived it recognizes this important fact.

By drawing on the cultural experiences and practices of individuals and of communities, the pedagogical strand of the program ethnomathematics allows for an easier flow of scientific ideas with children, reducing the effects of cultural blocks that is so damaging in educational systems.

I recognize this in the collection of essays. Well-known specialists address different topics in school practices, which apply to different natural and sociocultural environments. Altogether, the essays mutually complement each other and the readers may draw conclusions that support the theoretical proposal of the program ethnomathematics.

This book is a valuable and necessary contribution to educational scholarship. I congratulate the editors and each individual author for this remarkable accomplishment.

São Paulo, Brazil
February 2020

Ubiratan D'Ambrosio

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Part I
Introduction

The main objective of this introduction is to show that ethnomathematics as a research program has an agenda that offers a broad view of mathematics, which embraces the ideas, processes, methods, and practices that are related to the diverse mathematical practices dealt in the Brazilian context.

Chapter 1
An Overview of Diverse Mathematical Practices in Brazil: An Ethnomathematical Perspective in Action

Milton Rosa

1.1 Initial Considerations

It is necessary to address the interrelations of local mathematical knowledge sources with broader universal forms of mathematics in order for us to understand ideas, procedures, and techniques found in distinct cultural groups. The main objective of this approach is to link or bridge mathematical practices developed by the members of distinct cultures in order to clarify understanding and mutual respect among the members of all communities. In this regard, an ethnomathematics program aims to stimulate broader reflections about the nature of mathematical thinking in cognitive, historical, social, and cultural environments that helps us to understanding the development of *knowing* and *doing* mathematics created worldwide by mankind in distinct cultural contexts.

Thus, there is a need to create a new role in relation to mathematics instruction that empowers people to understand power and oppression more critically by considering the effect of culture and language on mathematical knowledge by working with students to uncover the often distorted and hidden history of mathematical knowledge. In this context, any study of mathematics and its connection to culture represents a powerful means for valuing and validating students' real-life experiences and gives them tools to become critical and reflective participants in society.

This perspective forms the basis for significant contributions of Dambrosian-based ethnomathematical perspectives in re-conceiving the discipline of mathematics and its pedagogical practices. Thus, the use of a Freirean dialogical methodology is essential in developing a pedagogical praxis of ethnomathematics by investigating cultures and languages in order to develop a mathematics curriculum that shows

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the contributions of people from other cultural groups that seeks the enrichment of mathematical knowledge.

In the context of national and international dialogues, it is important to broaden the discussion for the possibilities of the inclusion of cultural polysemy of ethnomathematics in the mathematics curriculum in order to provoke discussions about social justice and respect, which is related to the sociocultural diversity of people in their search for peace. This approach guarantees the development of understanding of differences through dialogical interactions and respect that enables us to avoid domination and oppression.

In this regard, related to ethnomathematical research conducted in the Brazilian context, the authors of the chapters of this books share the necessity of discussing issues regarding mathematics education, classroom practices, and the knowledge of the members of specific cultural groups in order to explore mathematical knowledge, which has a role in helping us to clarify the nature of mathematical knowledge and of knowledge in general.

According to this context, there are hundreds of reasons for teaching mathematics. One of the most relevant motives involves the consideration of mathematics as an expression of human endeavor, development, culture, and thought that forms an integral part of the cultural heritage of humankind (Rosa and Orey 2016a). Contemporary society places great value on a capitalistic scientific Western oriented science and mathematics. Conversely, ethnomathematics demonstrates that mathematics is composed of many diverse and distinct cultural traditions, not just those emerging from the Mediterranean region (D'Ambrosio 1985).

For example, local knowledge inter-relationships are related to varied academic knowledge areas and are important in enabling a more precise understanding about a particular study field and its use of mathematical processes. Here, these interrelations must be clearly addressed, especially, wherever one needs to understand specific concepts acquired through academic knowledge broadly based on biology, ecology, and mathematics, which may be supported by ethnosciences in order to understand its ethnomathematical interactions (Rosa and Orey 2017).

The main goal in this discussion is to further advance the research and understanding on the context of ethnomathematics research programs in diverse contexts in Brazil. As part of this, the authors are invited to discuss how they research ethnomathematical relations with other knowledge fields in their own contexts. In this regard, Rosa and Orey (2015) state that mathematical thinking in regard with science has been influenced by a diversity of human characteristics such as our languages, religions, morals, and economical-social-political activities. In concert with these, humanity has developed logical processes related to our universal need to pattern, quantify, measure, model, and explain, all shaped and operating within different social and historical contexts.

Because each cultural group has its own way of doing mathematics, these connections have come to represent, and are embedded in, a given cultural system, especially in the way that diverse peoples quantify and use numbers, apply geometric forms and relationships, measure or classify objects in their own environments (D'Ambrosio 2011). In concert with these characteristics, humanity has developed

logical processes related to the universal need to pattern, quantify, measure, model, and explain, all shaped and operating within different social and historical contexts.

As part of this approach, the members of each cultural group have developed their own way to *mathematize* their own realities. Western scientific arrogance often presents a disrespect of and outright refusal to acknowledge diverse cultural identities puts processes of understanding and comprehension of the many non-Western cultural systems at risk. Because this aspect gives a sense of confidence and dignity to students when their own *tacit knowledge*[1] is acknowledged, these particularities should not be ignored and they should be respected when people enter school (D'Ambrosio 2011).

In this regard, a search for new methodological approaches is necessary to share, record, and include diverse forms of mathematical thinking, ideas, procedures, and techniques developed in different cultural contexts. These connections have come to represent and are embedded in distinct cultural systems in Brazil, especially in the way that people quantify and use numbers, use geometric forms and relationships, measure or classify objects in their own environment.

For example, Fantinato and Mafra (2017) studied the techniques, processes, and tools involved in the preparation of ornamental patterns on the curved surfaces of vegetable gourds called *cuias,* which are crafted by a group of craftswomen in a riverbank community of the Northern region of Brazil. The results of this investigation are related to the contributions to the research field of ethnomathematics by bringing empirical reflections that aim at the implementation of local mathematical practices for the development of contextualized educational practices.

Even though there are many different perspectives in conducting ethnomathematical research, we agree in regard to the importance of ideas of Brazilian mathematician and philosopher Ubiratan D'Ambrosio in relation to the development and evolution of the research field of ethnomathematics. He is also one of the most important theoreticians in regard to the connections between mathematics and culture and offers encouragement, leadership, and dissemination of new ideas, concepts, and perspectives involved in ethnomathematics around the world and its applications in mathematics education. He is without a doubt our primary leader.

In accordance to Powell and Frankenstein (1997), D'Ambrosio's broader view of ethnomathematics accounts for the dialectical transformation of knowledge within and among societies. Moreover, his epistemology is consistent with Freire's perspective in that D'Ambrosio's standpoint in regard to mathematical knowledge is not static and ordained, but dynamic and the result of human activity.

Because teaching becomes an activity that introduces the creation of knowledge, it is much more than the mere transference of knowledge or information. This

[1] This type of knowledge is embedded in personal experience, it is subjective, contextualized and analogous (Nonaka and Takeuchi 1995). For example, individuals do not learn how to ride a bicycle by reading a manual, as they need personal experimentation and practice to acquire the skills necessary to learn this action. Therefore, this knowledge is acquired and accumulated through individual experience, as it involves intangible factors such as beliefs, perspectives, perceptions, value systems, ideas, emotions, norms, presentiments and intuitions (Rosa and Orey 2012).

approach in mathematics education is the antithesis of turning students into containers to be filled with information (Freire 1972). This perspective forms the basis for significant contributions of a Freirean-based ethnomathematical perspective in reconceiving the discipline of mathematics and in a pedagogical practice.

In this approach, Freire (1998) states that students actively participate in the design of their own education in which the content is developed to strengthen their critical consciousness by searching for experiences that provide meaning to their lives, and gives them direct tools to resolve problems stemming from the conditions of their own communities.

Therefore, these experiences are chosen in a collaboration of both students and teachers in a dialogical relation that fosters the development of a critical consciousness. Learners eventually get to see the beauty of formal or abstract mathematics, but they must first learn by using it to examine and solve problems in their own communities. The use of Freire's (1972) dialogical methodology is an essential pedagogical action in developing the curricular praxis of ethnomathematics by investigating mathematical ideas, procedures, and practices developed by the members of diverse contexts and cultures in constructing curricula with people from other cultures versus imposing norms and regulations on them.

According to this approach, Freire (1998) affirms that the importance of philosophical principles of learning and life allows us to develop notions like freedom, power, individuality, and success from the logic of the dominant society, moving them into the spheres of democratic authority, social justice, and political action in order to emancipate individuals from oppression and colonization. We seek to do this in Brazil by developing mathematical abilities in educators and learners alike by applying an ethnomathematical perspective in the development of mathematics curriculum.

It is necessary to highlight that D'Ambrosio's studies in the area of sociopolitical issues established a strong relationship between mathematics, anthropology, culture, and society. For example, in 1983, D'Ambrosio was honored with the title of *Fellow of the American Association for the Advancement of Science* (AAAS) for his imaginative and effective leadership in *Latin American Mathematics Education* and his efforts towards international cooperation. Because of his development in this research area, Gerdes (1997) and Powel and Frankenstein (1997) have considered D'Ambrosio as the "intellectual father of the ethnomathematics program" (p. 13).

It is also important to point out that D'Ambrosio was also selected as one of the most important mathematicians of the twentieth century in the area of sociopolitical issues and ethnomathematics (Shirley 2000). In 2001, D'Ambrosio was the recipient of the *Kenneth O. May Medal* of *History of Mathematics* granted by the *International Commission of History of Mathematics* (ICHM). According to Andersen (2002), the "ICHM has awarded the May Medal to D'Ambrosio for his never-ending efforts through writing and lectures to promote Ethnomathematics and thereby contributing intensely to make the field established" (p. 1). In 2005, D'Ambrosio was awarded with the second *Felix Klein Medal* of *the International Commission on Mathematical Instruction* (ICMI) that acknowledges his role in the development of mathematics education as a research field.

In Brazil, ethnomathematics has been a research field, important for its own development because of its rich sociocultural environment and history. Many native peoples were surprised by the arrival of European conquerors, then had their cultural roots changed by political repression, which has ranged from a paternalistic reduction into folkloric forms to its own unique forms of suppressive as of late. Some of these cultural roots had their practices excluded from society, even criminalized (D'Ambrosio 2006). Many immigrants, women, LGBTQ's, volunteers, or non-volunteers are currently recipients of the repressive politics. Everyone, minorities, and native and foreign peoples participate in the sociocultural dynamics that is responsible for modern Brazil.

In 1550, Brazil became a major importer of enslaved African peoples. This pattern continued during the Brazilian colonial era. From 1650 to 1850, about 4.5 million enslaved people from Africa were taken to Brazil, which is well over ten times as many as were trafficked to North America and more than the total number of Africans who were transported to all of the Caribbean and North America combined (Curto and Soulodre-La France 2005).

The colonial era in Brazil ended in 1822 with independence from Portugal. During this period, enslaved Africans brought many mathematical, scientific, social, and cultural contributions to the Brazilian culture, indeed they were both bought and sold for their knowledge in these areas. For example, Benjamin (2006) points out that the African presence in the Brazilian culture is shown in the cooking, dance, religiosity, music, handicrafts, children's games and toys, and language.

Similarly, Rosa and Orey (2016b) affirm that an evidence of the development of mathematical knowledge of enslaved African people brought to Brazil is related to the use of their scientific and mathematical knowledge in the construction of gold mines, in the expansion of baroque art, architecture, and in the confection of musical instruments. Investigations in ethnomathematics demystify the Eurocentric view that society developed regarding African and Afro-descendants by revealing the reasons for racism and prejudice and distinguishing the different cultural traits that exist in Brazilian culture.

The enormity of the slave trade's foothold in Brazil up to the end of nineteenth century was so overpowering that the nation failed to develop an effective anti-slavery movement, even while many other nations around the world were making revolutionary reforms to end it. It was not until the late 1800s that Brazilian reformist activities began to foment in institutions of higher learning. Thus, young lawyers, students, and journalists started to urge their fellow Brazilians to follow the example of the liberation of the slaves in North America. The struggle for total abolition kept moving forward, and on May 13, 1888, Brazil was one of the last nations in the world to formally abolish slavery.

These contexts enabled the development of the holistic understanding of the nature of mathematics in the Brazilian context. However, a dilemma related to this issue is to show how to reconstruct mathematical traditions, when, probably, many of them have been, as a consequence of slavery and colonialism, wiped out. Unfortunately, much of the art, and scientific and mathematical knowledge was lost

when colonialism and the slave trade caused the disappearance and/or the *freezing*[2] of many local traditions.

From an ethnomathematical perspective, educators and teachers learn to integrate contributions of diverse African cultures into the school curriculum by defrosting frozen mathematical thinking in order to stimulate a reflection on the impact of colonialism on the historical and political dimensions of mathematics education (Gerdes 2014). This context enables educators, teachers, and learners to study African numbers systems, geometrical thinking, oral history, and the *Orixás*[3], who are the Candomblé deities to Brazil by, especially the Yoruba people (D'Ambrosio and Rosa 2008).

The recognition of these dynamics in relation to the mathematics of *knowing* and the mathematics of *making and doing* contributes to the organization of an educational model that corresponds to the aspirations of its people. This is one of the main objectives of the ethnomathematics program in Brazil. It is crucial to ensure that mathematics is contextualized and grounded in the needs and expectations of the community that utilizes it (Rosa and Orey 2016b). For example, teaching for social justice in Brazil focuses on the context of the understanding of mathematical and scientific ideas, practices, and processes, which force confrontations in relation to assumptions about *truth* and *knowledge*, which can easily be confused with the *right* and *wrong* of science and mathematics (D'Ambrosio and Rosa 2017).

In a similar way, teaching using ethnomathematical perspectives reminds us that information may be meaningless unless is it embedded in appropriate contextual understandings. It relies on relevant political and cultural aspects of mathematics in order to develop its pedagogical action. This perspective encourages exploration, interpretation, and reconsideration regarding to what is understood about mathematics and science.

[2] Probably, the majority of mathematical knowledge of colonized peoples has been lost, hidden, or frozen. There is a necessity to attempt to reconstruct or unfreeze the mathematical thinking that is hidden or frozen in local techniques and strategies developed by the members of distinct cultural groups. Unfreezing frozen mathematical ideas, procedures and practices forces mathematicians and philosophers to reflect on the relationship between geometrical thinking and material production, between doing mathematics and technology (Gerdes 2014).

[3] In Africa, *Orixás* were the kings, queens, mythical heroes and other ancestors rose to the status of gods. In Brazil and other nations of the Americas, as in Cuba with the Santería, African deities were disguised through their association with Catholic saints in order for them to practice their own religion in spite of it being forbidden. In Brazilian Candomblé, for example, Xangô corresponds with Saint Hieronymus and Oxossi with Saint George (D'Ambrosio and Rosa 2008).

1.2 Ethnomathematics as Diverse Ways of *Knowing* and Doing Mathematics

The field of ethnomathematics links students' diverse ways of *knowing*, *doing*, and learning in culturally embedded contexts in order to bridge school mathematics. It explores academic and culturally rich ways to provide inclusive developmental programs for the diverse populations served at educational institutions. Ethnomathematics is a program that includes curricular relevance and builds curricula around the local interests and culture of the students (D'Ambrosio and Rosa 2017).

According to Rosa and Orey (2013), teaching mathematics through cultural relevance and personal experiences helps students to know more about reality, culture, society, environmental issues, and themselves by providing them with mathematical content that enable them to successfully master academic mathematics. An important change in mathematical instruction needs to take place in order to accommodate continuous and ongoing changes in the demographics of students in mathematics classrooms. It is necessary to integrate culturally relevant pedagogies into the existing mathematics curriculum.

This perspective is an essential component of culturally relevant education because it proposes that educators contextualize mathematics learning by relating mathematical content to the real-life experiences of learners (Torres-Velásquez and Lobo 2004). It is also necessary to highlight the importance of building connections between mathematics and students' personal lives and cultures. In accordance to this approach, when practical or culturally based problems are examined in a proper social context, the practical mathematics of social groups is not trivial because they reflect themes that are linked to the daily lives of students.

Ethnomathematics should focus on the role of mathematics in a sociocultural context that involves the ideas and concepts associated with ethnomathematics by using its perspective for solving real-life problems (Rosa and Orey 2008). In this regard, mathematics was for a long time regarded as a neutral and culturally free discipline removed from social values (D'Ambrosio 2006), and it was always taught in schools as a culturally free subject that involve learning supposedly universally accepted facts, concepts, and contents.

Western or academic mathematics consists of a body of knowledge of facts, algorithms, axioms, and theorems. Frequently, mathematics is referred to as a universal language. However, when people speak of universals, it is important to recognize that often something thought of as universal is merely universal to those who share the same cultural and historical backgrounds. In this context, many educators who operate under the assumption that mathematics is acultural, that it is a discipline without cultural significance, fail to see the connection between mathematics and culture (D'Ambrosio 2006).

Because mathematics in any culture has been based upon certain values and needs, students' cultural and linguistic references may interfere in the learning process of mathematical concepts in the classrooms. For example, the results of a study

conducted by Rosa (2010) show that most teachers believe that cultural background of students does not influence their performance on standardized assessments. The data reveal that these teachers do not seem to be aware of the impact of the cultural backgrounds of students on their performance on curricular activities. In their opinion, culture does not play an important role in the mathematical academic success of the students because learning is about their attitude towards mathematics. Thus, mathematics is not influenced by the students' cultural and linguistic backgrounds. It is important that educators use culturally specific contexts in teaching and learning mathematics by exposing students to a variety of cultural contexts and mathematical practices.

The pervasive view of mathematics as Eurocentric and value-free misrepresents the evolution of modern mathematics (Joseph 2000). This perception is also reinforced by students' experiences of the way mathematics is taught in schools. Educators' view of mathematics is transmitted to the students in their instruction and this fact helps to shape students' views about the nature of mathematics (Brown et al. 1990). Even though the universality of mathematical truths is not in question, it is only in the last two decades that the perception of mathematics as culture free has been challenged. In this context, "there is no sense in regarding mathematics learning as abstract and culture free" (Bishop et al. 1993, p. 1). Then, the learning process cannot be decontextualized because it cannot be free of societal influence.

The contextualization of mathematics has been described as the identification of mathematical practices developed in different cultural groups (Orey 2000). In Brazil, if mathematics is considered as a cultural construct, then it is a product of cultural development. For example, Zaslavsky (1996) affirms that this claim of mathematics as a cultural construct contradicts the aims that modern mathematics is universal, objective, and culturally neutral. This leads to the development and inquiries of culturally relevant mathematics (Nasir and Cobb 2007). Frequently teachers are unaware of the norms that govern their behavior until students do not follow their rules because they are unfamiliar with the expectations of the teachers' culture (Rosa 2010).

One goal of the ethnomathematics program in Brazil is to acknowledge that there are different ways of doing mathematics by considering the appropriation of the academic mathematical knowledge developed by different sectors of the society as well as by considering different modes in which different cultures negotiate their own mathematical practices. Hence, ethnomathematics is a program that investigates ways in which the members of distinct cultural groups comprehend, articulate, and apply ideas, procedures, and techniques that can be identified as mathematical practices.

Moreover, ethnomathematics may be described as a way in which the members from a particular culture use mathematical ideas and procedures for dealing with quantitative, relational, and spatial aspects of their lives (Barton 1996). This way of viewing mathematics validates and affirms that the experiences of the members of distinct cultural groups regarding to mathematics demonstrates that mathematical thinking is inherent to their lives. Further evidence of this assertion is offered by Orey (2000) who stated that the "paradigm that diverse cultures use or work within

evolves out of unique interactions between their language, culture, and environment" (p. 248).

Similarly, mathematical thinking as developed by different cultures is in accordance to common phenomena encountered by diverse contexts. Therefore, in order to solve specific problems, *ad hoc*[4] solutions are created, generalized methods are developed from those solutions created to solve similar problems, and theories are developed from these generalized methods (D'Ambrosio 2011). This tendency in the Brazilian context has been to consider *ad hoc* mathematical practices as nonsystematic and nontheoretical constructs. In contrast, the study of ethnomathematics underlies a structure of inquiry in the development of *local* mathematical practices by considering how these practices and problem-solving techniques can be developed into methods and theories.

In the context of ethnomathematics, members of many culturally differentiated groups *know* mathematics in ways that are quite different from academic mathematics as taught in schools. Since different types of problems are common in different cultures, the kinds of solutions, methods, and theories developed in these contexts may differ from culture to culture (Orey 2017). In this regard, it is necessary to recognize that situations that are considered as problems in a specific culture may have no meaning in other cultures. Ethnomathematics refers to mathematical ideas and procedures embedded in cultural practices developed in traditional and nontraditional societies. It recognizes that members of distinct cultural groups develop unique methods and sophisticated explanations to understand, comprehend, and transform their own realities (D'Ambrosio and Rosa 2017).

It also recognizes that the accumulated methods developed in these cultures are engaged in a constant, dynamic, and natural process of evolution and growth through the process of *cultural dynamism*[5]. Ethnomathematics is referred to here as the study of how members of various cultural groups develop techniques and procedures that help them to explain, understand, and comprehend their own world in response to problems, struggles, and endeavors of human survival (D'Ambrosio 1985). This includes material needs, arts, and spirituality through the development of *cultural artifacts*[6], which are objects created by members of a specific cultural group that inherently give cultural clues about the culture of its creator and users.

[4] Ad hoc is a Latin expression that means *for this purpose*. It generally means a solution designed for a specific problem or task, non-generalizable, and which cannot be adapted to other purposes (Rosa 2010).

[5] Cultural dynamism refers to the exchange of systems of knowledge that facilitate members of distinct cultures to exploit or adapt to the world around them. Thus, these cultural dynamics facilitates the incorporation of human invention, which is related to changing the world to create new abilities and institutionalizing these changes that serve as the basis for developing more competencies (Rosa and Orey 2016a).

[6] *Cultural artifacts* are objects created by the members of distinct cultural groups, which inherently give cultural clues and information about the culture of its creators and users (D'Ambrosio 2011). They are also the physical manifestations or expressions of a specific culture and they include but are not limited to food, clothing, tools, art, and architecture (Rosa and Orey 2012).

This perspective "provides an important opportunity for educators to link current events and the importance of these artifacts in the context of ethnomathematics, history, and culture" (Rosa and Orey 2008, p. 33). Cultural artifacts such as art works, language, myths, and literature influence the representational system of different cultures and civilizations and are more often than not related to some form of or use of mathematics. Another presupposition of ethnomathematics is that it values all forms of mathematical explaining and understanding formulated and accumulated by the members of distinct cultural groups (D'Ambrosio 2006).

This knowledge is regarded as part of an evolutionary process of cultural dynamism as the members of each cultural group come into contact with each other (Zaslavsky 1996). This approach is related to diverse forms of mathematics that vary because of being embedded in cultural activities whose purpose is other than doing mathematics. In this regard, "ethnomathematics might be characterized as a tool to act in the world" (Orey 2000, p. 250), and as such it provides insights into the sociocultural role and nature of mathematics in the Brazilian society.

1.3 Ethnomathematics and Mathematical Practices in the Brazilian Context

The field of ethnomathematics links students' diverse ways of knowing and learning and culturally embedded knowledge with school mathematics because it explores academic and culturally rich ways to provide more inclusive developmental programs for the diverse populations served at educational institutions (D'Ambrosio 2006). An ethnomathematics as a program includes curricular relevance and builds a curriculum around the local interests and culture of the learners (Rosa 2010).

Teaching mathematics through cultural relevance helps learners know more about reality, culture, society, environmental issues, and themselves by providing them with mathematics content and approaches that enables them to successfully master mathematics. According to Rosa and Orey (2007), an ethnomathematics approach to the curriculum is considered a pedagogical vehicle for achieving such a goal.

The field of ethnomathematics in Brazil has offered much to mathematics education in that it opposes, and/or advocate for an alternative perspective to the kinds of mathematics taught from a traditional formal school, and test preparation and orientation that is not always related to sociocultural and political aspects of mathematics. As a program, ethnomathematics seeks to understand diverse processes of thinking and ways of explaining, as well creating the awareness to reflect on social and political dimensions of mathematics.

A perspective that offers us important viewpoints for the development of dynamic and *glocalized*[7] communities, which allows us to recognize that all cultures and all people develop unique methods and explanations that allow them to understand, act, and transform their own reality (Rosa and Orey 2019). Looking at mathematical ideas, procedures, and practices from different cultures enables learners to perceive mathematics in ways that are not always valued or distinguished in the traditional curriculum.

In this context, Rosa (2010) argued that educators need to be supportive in constructing understanding between mathematics and the language and culture of students in order to encourage the development of reflective and critical thinking abilities. Currently, both a greater and more sensitive understanding of mathematical ideas, procedures, and practices developed by the members of diverse cultural groups have become available through the growth of and study in the fields of multiculturalism, anthropology, linguistics, and ethnomathematics.

The term ethnomathematics was officially coined by D'Ambrosio (1985) to describe the mathematical practices of identifiable cultural groups and is regarded as the study of mathematical ideas found in any cultural context. The search for solutions for specific problems that help the development of mathematics are always imbedded in a cultural context because in order to understand how mathematics (*tics*) is created, it is necessary to understand the problems (*mathema*) that precipitate it in each diverse context.

In order to understand those problems (*mathema*), it is necessary to consider the cultural context (*ethnos*) that drives them. Ethnomathematics as the study of mathematical ideas developed by different sociocultural groups offers a contextualization of the curriculum that contributes to the elaboration of pedagogical practices in multicultural classrooms (Rosa and Orey 2007) towards social justice. One of the characteristics of ethnomathematics is to help in the development of concepts of what mathematics really is through an understandinf of its deep connection with culture (D'Ambrosio 1985).

The purpose of ethnomathematics is to trace the development and transformation of mathematical ideas by developing research on how ethnomathematical perspectives in the mathematics curriculum contributes to a new approach on mathematics education. In the Brazilian context, an ethnomathematics program offers a very broad view of mathematics, a view that embraces tacit knowledge, ideas, processes, procedures, techniques, methods, and practices related to different cultural environments. This aspect leads to increased evidence of cognitive processes, learning capabilities, and attitudes that may direct learning processes occurring in many mathematics classrooms. For example, according to Masingila and King (1997), ethnomathematics becomes a workable tool that helps students to make connections and develop deeper mathematical understanding.

[7] *Glocalizantion* is the ability of a culture, when it encounters other cultures, to absorb influences that naturally fit into and can enrich that culture, to resist those things that are truly alien and to compartmentalize those things that, while different can nevertheless be enjoyed and celebrated as different (Rosa and Orey 2016a).

In this context, Masingila and King (1997) also argued that ethnomathematics helps students to learn about procedures of *other* peoples as well as develop deeper understandings, indeed come to value, their own practices. For Rosa and Orey (2007), as students learn about the culture of *other* peoples, they can also learn about their mathematical knowledge, since mathematics is an integral part of their own culture.

Ethnomathematics may be used as a tool to motivate disenfranchised students to pursue a study of mathematics (D'Ambrosio 2006). It enables the achievement of two objectives in mathematics teaching:

(a) It can establish a multicultural context for the development of mathematical knowledge and skills.
(b) It can help students in making connections among other disciplines (D'Ambrosio and Rosa 2008).

Consequently, learners begin to maximize their own possibilities for improving their mindset towards mathematics as they are improving their skills. It is very much connected to perspective and self-esteem. In this regard, Rosa (2010) argues that ethnomathematics is a program through which teachers may create lasting positive impact on affective and cognitive domains of students who are underachieving in academic mathematics.

Ethnomathematics investigates the ways in which different cultural groups comprehend, articulate, and apply ideas and concepts that can be identified as mathematical practices (Barton 1996). The very essence of this program is to acknowledge that there are different ways of doing mathematics by taking the time to consider the appropriation of the academic mathematical knowledge developed by different sectors of society. As well, it considers different modes in which diverse cultures negotiated and developed their own unique problem-solving techniques, mathematical ideas, procedures, and practices (D'Ambrosio 2011).

Moreover, ethnomathematics may be described as a way in which people from a particular culture use mathematical ideas and concepts for dealing with the quantitative, relational, and spatial aspects of their lives. In this context, mathematics values the diverse cultural experiences by demonstrating how mathematical thinking is inherent to all our lives, cultures, and places. Mathematics is very much identified in cultural activities and is found in both traditional and nontraditional societies. This refers to the mathematical concepts embedded in cultural practices and it recognizes that all cultures develop unique methods and sophisticated explications to understand, comprehend, and transform their own reality (Orey 2017).

Hence, ethnomathematics means the study of how people within various cultural groups come to develop techniques used to explain and understand the world in response to unique history, location, problems, struggles, and endeavors (D'Ambrosio and Rosa 2008). This approach includes the kinds of mathematics that assists in meeting material needs as well as art and spirituality through the use of the development of artifacts, which are objects created by members of a specific cultural group that inherently give cultural clues about the culture of its creator and users. This perspective provides an important opportunity for teachers to link

current events and the importance of these artifacts in the context of ethnomathematics, history, and culture.

Another presupposition of ethnomathematics is that it validates forms of explaining and understanding the world that are formulated and accumulated by different cultural groups (D'Ambrosio 2011). A study of the different ways in which people resolve problems and the practical algorithms on which they base these mathematical perspectives becomes relevant for any real comprehension of the concepts and the practices in the mathematics that have developed over time for a particular group of people (Rosa and Orey 2008).

Ethnomathematics draws from the cultural experiences and practices of students, their communities, and the society at large in using them as vehicles to make mathematics learning meaningful, but, more importantly, to provide insight of mathematical knowledge as embedded in their unique linguistic and cultural contexts. The main goal of an ethnomathematics program is to accomplish this equity among students by incorporating social justice into mathematics curriculum.

It refers to forms of mathematics that vary as consequence of being embedded in, and reflecting the values and cultural activities. In this perspective, Orey (2000) affirmed that "ethnomathematics might be characterized as a tool to act in the world" (p. 250) and it provides insights into the social role of academic mathematics in the search for social justice. By seeking the development of social justice in society, a culturally relevant pedagogy focuses on the role of mathematics in sociocultural contexts that involve the ideas and concepts for solving daily problems (Rosa and Orey 2008).

An important component of mathematics education should be to reaffirm and restore the cultural dignity of all students and educators. As students experience multicultural activities that reflect the knowledge and behaviors of people from diverse cultural environments, they can learn to see and value the mathematics found therein. Participants in an ethnomathematics program learn to understand and accept the cultural roots of the *dominated* cultural group by coming to understand and value their mathematical ideas, procedures, and practices.

Educators and researchers come to recognize diverse applications of academic mathematics in order to promote the development of mathematical ideas, procedures, and practices that were evolved by *others*. This program also supports the learning of traditional academic mathematics because when dominated cultural groups gain access they become knowledgeable about the mathematics of the dominator (Rosa and Orey 2007). They must critically reflect on their own pedagogical practices in order to avoid a compliant thinking and to foster equity and social justice. Educators who understand historical and cultural variations of mathematical ideas, procedures, and practices that vary across history, time, culture, race, ethnicity, gender, sexual orientation, and other sociocultural characteristics are better equipped to do this (Rosa and Orey 2010).

Another important concept of ethnomathematics is the association of mathematical ideas, procedures, and practices found in diverse cultural contexts. Ethnomathematics as a research paradigm is much wider than traditional concepts of multiculturalism, mathematics, and ethnicity. *Ethno* relates to the many distinct

groups identified by their traditions, codes, symbols, myths, and specific ways of reasoning and inferring (Rosa, 2010). Because it examines how both mathematical ideas and mathematical practices are learned, diffused, and used in daily activities, ethnomathematics is a way to study how various cultural groups mathematize (Orey 2017).

Ethnomathematics can be also described as the arts and techniques developed by students from diverse cultural and linguistic backgrounds to explain, to understand, and to cope with their own social, cultural, environmental, political, and economic environments (D'Ambrosio 1985). It may be considered as the way that various cultural groups mathematize their own reality because it examines how both mathematical ideas and mathematical practices are processed and used in daily activities. In accordance to Barton (1996), ethnomathematics embraces the mathematical ideas, thoughts, and practices as developed by all cultures.

Ethnomathematics embraces the mathematical ideas thoughts and practices as developed by all cultures. From this perspective, a body of anthropological research has come to focus on both intuitive mathematical thinking and cognitive processes largely developed in minority or non-dominate cultural groups (Barton 1996). It is considered as a program that seeks to study how learners understand, comprehend, articulate, process, and ultimately use mathematical ideas, concepts, and practices that solve problems related to daily activity.

According to this context, ethnomathematics is not only the study of mathematical ideas; it is also the study of human interaction using anthropology, pedagogy, and historical context. This means that the study of the history of mathematics attempts to identify the cultural and mathematical contributions of different cultures across the world. Thus, the focus of ethnomathematics consists of a serious and critical analysis of the generation and production of mathematical knowledge and intellectual processes, the social mechanisms, the institutionalization as well as the non-institutionalization of knowledge; and the diffusion of this knowledge (Orey 2017).

The unique cultural background of each student represents a set of values and the unique way of seeing the world as it is transmitted from one generation to another. The principals of anthropology that are relevant to the work of ethnomathematics include the essential elements of culture such as language, economy, politics, religion, art, and the daily mathematical practices of diverse groups of students. Since cultural anthropology gives us tools that increase our understanding of the internal logic of a given society, detailed anthropological studies of the mathematics of distinct cultural groups most certainly allows us to further our understanding of the internal logic and beliefs of diverse group of students by using mathematical modeling (Rosa and Orey 2010).

All individuals and students as well possess and develop both anthropological and mathematical concepts. These concepts are rooted in universal human endowments of curiosity, ability, transcendence, life, and death. They all characterize our very humanness. Awareness and appreciation of cultural diversity that can be seen in our clothing, methods of discourse, our religious views, our morals, and our own unique worldview allow us to understand each aspect of the daily life of humans.

Thus, the essence of an ethnomathematics program is to be aware of the diverse and many different ways of *knowing and doing* mathematics that relate to ideas, procedures, and practices contextualized in the strands of *literacy*, *matheracy*, and *technoracy*, which is a new concept for the mathematics curriculum. In this regard, it is important to recognize the need to consider the appropriation of academic mathematical knowledge in different societal sectors as well as ways in which members of distinct cultural groups negotiate their own mathematical practices (D'Ambrosio 2006).

Literacy is the ability to process information that occurs in their daily life by applying techniques of reading, writing, representing, calculating, and using diverse media (including the internet), which all come together to create modern forms of literacy. It can be understood as the ability to process and create information in our daily routine (D'Ambrosio 1999), which includes actions such as checking prices, times, and schedules; using the units of measurement, and perform mathematical operations. Currently, literacy also includes a sense of numeracy, that is competencies such as the interpretation of graphs and tables as well as the understanding of the condensed language of codes and numbers that are achieved through the use of technology such as calculators and computers (Rosa and Orey 2015).

Matheracy is the capability to interpret and manage signs and codes as well to propose and use models in everyday life. It allows for us to find solutions by using abstract elaboration of problems that represent systems taken from reality (D'Ambrosio 1999). This context allows matheracy to provide symbolic and analytic instruments that help students to develop their creativity, which allows them to understand and solve new problems and situations. In this process, matheracy performs an analysis of the relations between variables, which are considered essential to comprehend phenomena studied through the elaboration of mathematical models by using known and unknown mathematical content (Rosa and Orey 2015).

Technoracy is the capability individuals possess that enables them to critically use and combine different technological tools, from the simplest to the most complex, as well as evaluating their possibilities and limitations to suit both theirs and others individual needs in various everyday situations. Thus, technoracy can be considered as the individuals' critical and reflexive familiarity with technology (D'Ambrosio 1999). In this perspective, the individuals' development of technoracy allows them to mathematically use technological instruments in order to evaluate diverse ways to present and represent mathematical ideas and practices as well as to assess the reasonableness of the results and their contextualization (Rosa and Orey 2015).

The application of this holistic proposed trivium curriculum allows educators to explore the sociocultural roots of their students; there is a need to extend this discussion to the context of mathematics education because the presence of mathematics is noticed in many nuances of reality and often formatting society.

According to D'Ambrosio and Rosa (2017), we face a need for alternative epistemologies if we want to explain alternative forms of knowledge. Although derived from the same natural reality, these knowledges are structured differently. Thus, the time has come for educators to adopt an improved mathematical pedagogy to

include, stimulate, and motivate students from all cultural backgrounds. Mathematicians and educational researchers have begun to establish a new mathematics curriculum which provides a link between practical and abstract mathematics by incorporating the contributions of various cultures.

Ethnomathematicians proposes that different cultures have historically developed a mathematical pedagogy that agrees with facts which the people have known to be true and corresponds to the developments of that culture. Mathematics has been regarded as absolute fact for centuries; however, the *facts* as defined in one culture does not always hold true in another (Orey 2017). Quantities, relationships, and space are defined by different standards throughout history and all over the world. Even in Brazil, there are cultures that use different mathematical standards, yet they have been overshadowed by the dominant Eurocentric paradigm, and therefore are considered obsolete and primitivist, or at best exotic, but irrelevant.

In the Brazilian context, the field of ethnomathematics strives to call attention to the differences in mathematical ideas, procedures, and practices developed across cultural and historical lines and maintain that one mathematical culture cannot be perceived as more accurate or factual than another. According to Barton (1996), fundamentally, ethnomathematics opens up the possibility for mathematical development from several viewpoints by assuming that all ways of thinking mathematically are equally justifiable. Ethnomathematics as a program offers a broader view of mathematics education in Brazil. This is a view that embraces the ideas, processes, methods, and practices that are related to different cultural environments. In addition, by reflecting on the dimensions of ethnomathematics, another important feature of this program is to offer an important perspective for a dynamic and glocalized society, one that recognizes that all cultures and all people develop unique methods and explanations that allow them to understand, act, and transform their own reality**.**

Due to the oversight of alternative explanations, the assumptions have an opposite result towards seeking the truth and are riddled with cultural bias. Therefore, the most effective way to avoid cultural bias is to bring ethnomathematics into the mainstream by exploring the possibility of multiple viewpoints and cosmologies.

1.4 Final Considerations

It is hoped that the awareness of ethnomathematics as raised within the mathematics community exhibits the diversity and wonder found within local practices as the development of humanistic mathematical phenomena in order to offer diverse avenues of investigations related to diverse mathematical practices in the Brazilian according to the perspective of ethnomathematics as a program.

It is important to learn about innovative mathematical ideas, techniques, and procedures that enable educators and researchers to show the value of culturally relevant problems and looking at mathematical practices developed locally through different perspectives and cosmologies in order to question the Eurocentric

worldview. Ethnomathematics is considered as a line of study and research that investigates the roots of mathematical ideas, procedures, and practices in which its starting point is related to the way members of distinct cultural groups share knowledge in their own communities.

According to Sue and Sue (2003), these analyses are culturally specific in regard to the beliefs, thoughts, behaviors, knowledges, and attitudes. It is from their viewpoint that mathematical knowledge is conveyed for the understanding of their cultural context. The ethnomathematical studies in this book adopted diverse qualitative methodologies in order to understand mathematical practices that began in the knowledge of the *others,* in their own rationality, and terms. Usually, research developed in ethnomathematics is a process of strangeness and tension because the quantitative and spatial relations noticed inside the investigated cultural group are not exclusively centered in the explanations of the investigators' realities.

It is important to point out that in the Brazilian context, the conduction of ethnomathematics research is, in general, a process of re-significance of mathematical knowledge that would involve articulations between mathematics and several other areas of knowledge, such as history, linguistics, anthropology, politics, environmental, and economics. It is necessary to develop methodological, theoretical, and pedagogical articulations in a non-disciplinary dimension in order to approach transdisciplinarity trends in education. The preservation of cultural identities is an important issue in the American continent, which is marked by the presence of different Indigenous and African cultures mixing with newly immigrants from across the planet.

For example, in Brazil, research on the culture of these groups has contributed significantly to increasing the understanding of their scientific and mathematical knowledge (D'Ambrosio 2006). However, when the focus of investigations is on the pedagogy of mathematics, attention can center itself around legitimizing students' knowledge that grows from experiences built in their own ways, as well as around the study of the possibilities of working with members of distinct cultural groups *outside* and *inside* of educational institutions. Indeed, a discussion about ethnomathematics and its pedagogical action help educators and investigators to establish cultural models of beliefs, thoughts, and behaviors, in the sense of contemplating the potential of the pedagogic work that takes into account the tacit knowledge of the students, but also a process of teaching and learning mathematics that is more meaningful and empowering.

From his perspective, a body of ethnomathematical research in Brazil has come to focus on both intuitive mathematical thinking and the cognitive processes that are largely developed locally by the members of distinct cultural groups. Ethnomathematics may also be considered as a program that seeks to study how students have come to understand, comprehend, articulate, process, and ultimately use mathematical ideas, concepts, and practices that may solve problems related to their daily activities. Seen in this context, the focus of ethnomathematics consists essentially of a critical analysis of the generation and production of the mathematical knowledge and intellectual processes, the social mechanisms in the institutionalization of knowledge, and the diffusion of this knowledge (Rosa and Orey 2007).

In this much more *holistic context*[8] of mathematics, anthropological perspectives are employed to include diverse perspectives, patterns of thought, and histories, and the study of the *systems*[9] taken from reality are used to help students to reflect on, understand, and comprehend extant relations among all components under study. The unique cultural background of each learners and educators represents a set of values and unique ways of seeing the world as it is transmitted from one generation to another. Detailed studies of mathematical ideas and practices of distinct cultural groups most certainly allow us to further our understanding of the internal logic and beliefs of diverse groups of students.

It is necessary to know and understand the value of the plurality of the nature of the Brazilian sociocultural, economic, environmental, and political aspects of diverse peoples and cultures, in order to take a firm stand against all prejudices based on cultural differences, social classes, beliefs, gender, sexual orientation, ethnics, or other social and individual characteristics. As well, it is important to acknowledge our Africanities and/or Afro-Brazilian and indigenous roots, and immigrant and urban diversities. This is an innovative trend in conducting research in the perspective of ethnomathematics as a program that seeks to investigate diverse cultural mathematical practices in the very distinct and unique Brazilian context.

The chapters presented in this book represent the work developed by Brazilian researchers who investigate ethnomathematics in their contexts by applying their own perspectives, world vision, and cosmologies, which show that here is no linearity in the development of ethnomathematics research. This knowledge is made by developing different processes, common to all sociocultural groups that enable the elaboration and use of mathematical abilities, which include counting, locating, measuring, drawing, representing, playing, understanding, comprehending, and explaining the necessities and interests of diverse groups and individuals.

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[8] Systems are part of reality considered integrally in the mathematical modeling process They are also representations that help the members of distinct cultural groups to understand and comprehend the world around them by using small units of information, called ethnomodels, which link cultural heritage to the development of mathematical practices (Rosa and Orey 2019).

[9] A holistic context consists essentially of a critical analysis of the generation (creativity) of knowledge, and the intellectual process of its production. The focus on history analyzes the social mechanism and institutionalization of knowledge (academics), and its transmission through the educational process (Rosa and Orey 2013).

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Part II
Africanities

It is important to emphasize the recognition of diversity in the cultural contexts marked by *African* (considering the diversity of the ethnicities and cultures on the continent in all its socio-political divisions) and Afro-Brazilian roots. The expression *diversity* has been thought of as a broad, deep, multiple, and complex set of meanings. In this sense, the perspective of *Education for Ethnic-Racial Relations in Brazil* is completed in the ongoing movement for diversity, which includes the thinking related to the eradication of racism and bigotry, which are elements constituted in school contexts and in the greater Brazilian societal context.

Chapter 2
African-Based Cultural Practices and Mathematical Practices: Decolonizing Knowledge

Evanilson Tavares de França and Jackeline Rodrigues Mendes

2.1 One Word First

I know, I know I'm a piece of Africa
hanging on the night of my people.
I carry the mark of my whip
on my body like red steps made of flesh
through which the wagons of progress
sought the fumes of the future
Eduardo de Oliveira

It might be an uncomfortable concern (at least for us) that we should start our writing. Neither we are the first to do, nor give it an end, anyway we are interested to ask, according to Uthui's view (Uthui 2010, p. 18), why "western sciences in general are not also called 'ethno,' given that at some point they emerged from a certain geographical location and then affirmed themselves universally through a lengthy process." Which was probably based on asynchronous power relations.

It is quite true that, in relation to mathematics, D'Ambrosio (2005) someway answers this question. According to him, this field of knowledge represents in fact a set of cultural practices produced in part of Europe that had indispensable contributions from Indian culture and Mediterranean civilization. In another moment and place, this same researcher adds that "of all the cultural manifestations that were tried to impose as a universal character, the only one that dominated was Mathematics" (D'Ambrosio 1994, p. 93).

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However, some clues are given to us, we could add another element to it. Castiano (2010, p. 51, italics in the original, [sic]) reminds us that "the ethno prefix—which refers to ethnicity—wants to underline that it is the kind of knowledge of a certain group of people who share the same culture, which has its own language, in common habits and customs, etc."

This meaning echoes D'Ambrosio's understanding when he uses the same term to refer to the "distinct natural and socioeconomic contexts of reality" (D'Ambrosio 2005, p. 114). The division between culture and nature proposed by D'Ambrosio is meaningless when we think practices/knowledge developed by humans across the relationships that are woven with environmental/natural context—the human being himself is the result of this weaving.

In this sense, to think questions about African and Afro-diasporic's traditional territories, it is necessary an epistemic disobedience, in terms of Mignolo discussion (2008). In other words, it is necessary a frontier thought to move from epistemic and methodological canons imposed by modern/colonial western thinking.

It is possible to say that this notion of epistemic disobedience can dialogue with D'Ambrosio's understanding when he claims, "the program of ethnosciences should not be limited to study of knowledge itself." Furthermore, it should be extended "*to kind of studies that address all cultural dynamics, in which knowledge development appears under internal conceptual frameworks used in each culture*" (D'Ambrosio, quoted by Uthui, 2010, p. 18, italics ours).

In our point of view, we are talking about immersion into peripheral, subalternized, minorized, devalued cultures. Such cultural spaces are allocated, according to Boaventura de Sousa Santos (2009), on the other side of abyssal line where nonexistence is produced, which should be understood as "not existing in any form of being relevant or understandable" (Santos 2009, p. 4). We can say that this is valid for both, practices and people. Moreover, it is necessary to raise a post-abyssal thought, capable of breaking the oblivion of condemnation and the condemned, in other words, able to "recognize the imperative of ethical and epistemic pluriversality of the world" (Maldonado-Torres 2008, p. 99).

In this chapter, we intend to show a theoretical essay that brings studies coming from a decolonial perspective as a contribution for Ethnomathematics field. To choose a decolonial option means to cross other ways of knowledge presentation. The Ethnomathematics understood as a counter-conduct movement inside of mathematics, in a Foucaultian sense, has opened up the possibility of thinking and making mathematics in other ways (Monteiro and Mendes 2019).

From now on, we wish to provoke a reading which is going to demand open-spirits, that is, freedom from any kind of prejudice about science's formalism. In other words, it is necessary that the readers open their bodies (and souls, if they want; however, we are not allowing to be captured by another dichotomy). Even because, as Maldonado-Torres (2019) has taught us, "the open-bodies are questioners, creatives and prepared to act." Also: "critical questions that are founded on our experience with openness [provoke] an emergency of other discourse and another way of thinking" (Maldonado-Torres 2019, p. 47).

It is our purpose to present only one tiny opening for where the sun can illuminate and provoke tensions and possible displacement. To conduct this essay we bring examples from African-based practices. Firstly, the cultural practices are from a *quilombola* community called *Mussuca*, located in Sergipe State. The ethnographic-based study in this community brings some elements of two central practices in life of community: the fishing activity and the *Samba de Pareia* (a traditional dancing in community).

In a second moment, the practices are related to narratives in *Candomblé*, an Afro-Brazilian ritual in Brazil. Then we try to go through (ethno) mathematics studies with African-based practices and finally we raise issues around how a decolonial thinking can cause displacement to think knowledge in these practices.

2.2 Corrupting Dichotomies is a *Sine Qua Non* for Understanding Traditional Community Practices/ Knowledge

Although there are many dichotomies inaugurated or enhanced by modern and colonial Western thinking, in this space and time we are going to focus on only two of them: nature/culture and theory/practice. The discussion/reflection about the nature-culture relationship could not be shallow, since it is embryonic and imperative for understanding the ways in which knowledge and practices are constructed and how they constitute their practices participants. In this regard, Mignolo emphasizes that "the contemplation of *pachamama* ("nature" for Western minds) in the new Bolivia and Ecuador constitution was incorporated by the mere fact that it is inherent in the thinking of communities, leaders and indigenous intellectuals" (Mignolo 2017, p. 6).

Ramose (2011), in despite of using these two terms separately, culture and nature in reference to philosophy, he assures that this separation "would be unduly restrictive and blind if intended to exclude elements of nature, culture, sex, religion or history" (p. 12). Cultural practices are built on very intimate and interdependent relationships with geographic space, which can also (and usually) be changed by these practices. Several examples could be listed here when we look to many African-based practices in Brazil. We will bring up only two examples which were chosen because, besides photographing the intrinsic dialogue between cultural practices and nature, they also record the changes suffered by the body itself (of the subjects and practitioners) for its realization.

Initially, we bring a research with an ethnography focus, which we have been developing in the *Quilombola Mussuca* Community, since 2011. At first, the academic investigation was focused on the mathematical perceptions of *Mussuca* people with the objective of considering both the school space and the socio-cultural practices developed beyond the walls of the community school. In a second moment, the focus has moved to dialogues between curricular practices and cultural practices,

showing how the fishing activity connects "naturally" with the daily life these *quilombolas* people. And it goes through all (or almost) part of life: food, work, leisure, art, etc.

In this practice the fish usually requires a trawl, in the case of guaiamum, aratu[1] and the crab, the fish requires to access the mangroves. In addition, bivalve molluscs such as sururu and sutinga[2] are fished, which, like crabs, are found buried in the mangrove mud under the protection of *Nanã*, an African female deity reigning in the mud places. Fishing crab, guaiamum, aratu as well as sutinga and sururu requires from the fisherman an almost Herculean physical effort. Artisanal fishing has been doing with the aid of the trawl and the net, and also requires an athletic body from those who carry it. Yet, in these cases the demands on the body are even more impetuous, since the fisherman has to put on his leg inside the mangrove mud.

Sometimes this occurs with the mud reaching the belly level and the fisherman has to bend the spine to nearly ninety degrees to introduce his hand and arm into the mud until he finds the fish. This process is repeated several times per minute. In addition, the body is put on a constant risk: there is always a possibility that fragments of shellfish or mangrove twigs tear the skin or even the flesh of fishermen's legs and arms.

In *Quilombola Mussuca* Community, fishing activities are carried out by men and women: 22.22% of them work in the fields and/or fishing; in relation to them, the index corresponds to 9.09%, according to data from Laranjeiras (2006). They are also responsible for the processing of fish, their sale (inside and outside *Quilombo*) and, as a rule, for the preparation of exquisite food based on river products.

Approaching to geography of the *quilombo* a little, *Mussuca* is bordered by two rivers, Sergipe and Cotinguiba, between them are lower regions and hills, where are the residences. Therefore, the Community is divided between up and down, *Mussuca de Cima* and *Mussuca de Baixo*.

Before the implementation of State Government's Project called *Chapéu de Couro* (leather hat), which took place between 1983 and 1987, to promote communities' water supply (Alves Filho 1997), it was quite common to see women—mainly—going up and down the *Quilombo* hills with water cans on their heads or "bundles" of clothes.

These are the same women who play the *Samba de Pareia*, a traditional, oral cultural practice of Bantu origin (Risério 2010). This practice is found only in *Mussuca* community. According to Dona Nadir, singer, songwriter, and also a dancer of this

[1] *Aratu* is the popular name for several semi-terrestrial mangrove crabs, mainly from the Sesarmid family. Of these, the most important is the edible Goniopsis cruentata, which is often called simply "Aratu." It has several popular regional names, "Red Aratu," "Red-black Aratu," "Mangrove *Aratu*," "*Maria-mulata*," etc. Retrieved from http://www.planetainvertebrados.com.br/index.asp?pagina=especies_ver&id_categoria=25&id_subcategoria=24&com=1&id=190&local=2. Accessed 31 May 2019.

[2] It is a seafood very similar to sururu, being, however, smaller than this and having lower market value.

practice, the development of samba occurs as follows: "Samba de Pareia is four people: four there and four here. To change the pair has to be four people: two dance samba here and two dance samba there. Then one pair goes there and the other goes here. Then change the pair again. Then, that is why is *Pareia* [dancing in pairs exchanging], because it changes the pair" (Quoted by Dumas and Brito 2016, p. 48). Teacher Edeise Gomes Cardoso Santos has the following reading of this practice:

> To visualize all the choreographic logic, in my point of view, the dance performing needs at least three pairs: facing each other, they begin to dance in a rhythm that identifies this samba. The songs determine the time of footstep and, consequently, the rhythm is marked by the feet that generate a loud, vibrant and harmonious sound (Santos 2016, p. 93).

The group have 21 participants and, among them, there is only one man, Mr. Acrísio (Mr. Hose, as he is best known), who is responsible for playing the drum. The women play other percussion instruments: Mrs. Sessé plays a kind of bass-sounding *cuica* and Mrs. Nadir plays other specific afro Brazilian instrument called *ganzá*. She is responsible to start all songs. Beyond these instruments all dancers use wooden clogs, which also act as percussive instruments.

When we interviewed the teacher Cleanis Silva, an educator, dancer, dance teacher, and immensely interested in the *Samba de Pareia*, she called our attention to the physical structure of women in this cultural practice, who average ages are 44–72 years:

> Their legs are tough, looking like they did a lot of class. And they are not like people from the cities. I say to girls who are so urban, we have to do class, […] we have to do I don't know what. Not them. They come together and make the circle and start dancing, *because it is their natural, it came from the beginning doing this*. Now, for example, you cannot pick up your mother and make her learn. She may learn something, but it will not be the same. Her body was not conditioned from the beginning to do that (Interview, March 2019).

Cleanis point out that "They [*Samba de Pareia* players] get together and make the circle and start dancing, because it is natural for them, it came from the beginning doing this," seeks to highlight the rigidity of lower limbs muscles, which are the most required in the practice of this kind of *samba*. The practice of fishing and *Mussuca*'s geography also contribute to strengthening of these same members, thus evidencing an intertwining between cultural practices and the environment (nature), which is not carried out with impunity. At time when human bodies live in their territories, they are target of a necessary architecture for survival in those same spaces/times.

These two practices in the life of *Mussuca quilombola* community show how the relations of movements/time/space/ in the cultural practices constitute practitioners as a subject of knowledge; their bodies are affected and constituted by these practices. There is not a division between culture and nature, mind and body. To know how to participate in the practice implies a body integrated in practice.

In order to reinforce the rupture between nature and culture dichotomy, but also reinforcing the breakdown of theory and practice dichotomy, we cite *Candomblé* ritual. From within this African-based religion, several examples could be

highlighted. Iansã (*Yánsàn*[3]) rules the winds, lightning, and storms, *Nanã* (*Náná*) is a singular healer and have the mastery of earth's energy, her territory is the place of mud and slime, which makes her a unique and powerful enchantress (Verger 1981). One way to knowledge these deities is to access their narratives, called *itan* (*ìtán*).

According to Prandi (2001), there is an *itan* about *Nanã*, recognized as one of the oldest female deities (orishas). She was who has found the solution to *Oxalá´s* dilemma. The dilemma of *Oxalá* (*Òsálá*) was how to use several elements in the attempt of man creation: air, stick, stone, fire, water, olive oil, and palm wine did not work as a raw material for shaping. Then this old *Ayabá* (woman) helped *Oxalá* and offered to him the clay taken from the bottom of the pond where she lived, hence the man was created. There is a close relationship of all orishas with nature and, from these narratives, we can assert that human beings are intrinsically linked to these relationships.

There is a profusion of entanglements in orishas' narratives. We chose to highlight only *Ossanha*, also known as *Ossaim* (*Ossaniyn*). This orisha is *Nanã*'s son and he has a brother called *Obaluaiyê* (*Omulu/Omolo*).

> (…) the lord of leaves, sciences and herbs, this orisha is who knows the secret of healing and the mystery of life. All orishas used *Ossaim* to cure any illness, any illness of body. Everyone depends on *Ossaim* in the fight against diseases. All went to *Ossaim*'s house to offer their sacrifices [read: offerings]. In return *Ossaim* gave them magical preparations: baths, teas, infusions, *abô* [a liquid prepared to ritual], drinks (Prandi 2001, p. 153).

All rituals in *candomblé* practice are dependent of leaves that are given by *Ossaim*. Entering the woods in order to search for herbs, with which many preparations are made, such as the *abô* (agbò), highlighted by Reginaldo Prandi in the quote above. *Abô* is a bath prepared by macerated herbs and people need to greet and ask for permission to *Ossaim* when they go to take leaves for this and other kind of baths. One song to salute this deity in this practice is:

Kò si ewé
Kò si Òrìsà
Ewé ò
Ewé Òrìsà

The meaning of this song for participants of *Candomblé* is that without leaf, there is no life (*Kò si ewé*); without leaf, without Orisha (*Kò si Òrìsà*). Then there is a greeting to Orisha: "*Ewé ò! Ewé Òrìsà!*".

We can observe that all knowledge in these practices show an imbrication between culture and nature, the meaning of life (what we could related to culture) is linked and dependent of meanings given to nature. There is no separation. The ways of knowing how to take this leaves and use them in ritual is linked with knowledge present in the narratives.

[3] According to Verger (1981), *Iansã* is the first wife of *Shango*, another *candomblé* deity. Considering the strategies used by the enslaved in face of a strongly racist and dehumanizing society, this deity was portrayed with the figure of Saint Barbara because they could worship their deities.

We can also observe in these examples of Afro-diasporic cultural practices (*Samba de Pareia* and *Candomblé*) an absence of boundaries between theory and practice in the ways of knowing. This perception echoes Mignolo's reading of decolonial thinking, which, in our point of view, is indispensable for transiting through traditional African life forms:

> Decolonial thinking also means decolonial doing, since the modern distinction between theory and practice does not apply when you enter the field of frontier thinking and decolonial projects; when you enter the field of quichua and quechua, aymara and tojolabal, arabic and bengali, etc. categories of thought confronted, of course, with the relentless expansion of the foundations of Western knowledge (i.e. Latin, Greek, etc.), I say epistemology (Mignolo 2008, p. 290–291).

These barriers that separate theory and practice, based on the modern/colonial western world, are also breathless in the universes of *candomblés* as well as *Samba de Pareia*, which were referred lines behind. These practices are grasped/understood while practicing—and it is in the body and soul that they are tattooed, memorized and perpetuated—never statically, of course.

Therefore, when Mignolo refers to decolonial thinking as synonymous of decolonial making, it does not only produce a break with Eurocentric epistemology. It is also his intention to include in this movement (of decoloniality) all ways of doing and being that present other ways of being in the world. Regardless of the ways in which these interactions take place: if by writing alphabetically or orally, whether based on faith or empirically, if based on people's sciences or experiences. The idea is to guarantee pluriversality rather than universality (of totalitarianism); interculturality as opposed to monoculture.

Walter Mignolo (2008) outlines what, in his view, would be a border thinking, which is indispensable when making decolonial: "Frontier thinking or frontier epistemology is one of the consequences and the way out to avoid both western and non-western fundamentalism" (p. 297). It highlights that any decolonial project need to deal with border thinking. In another moment and place he concludes: "Border thinking in action is necessarily critical and decolonial" (Mignolo 2013, p. 25).

Still according to Mignolo, there are three possible ways for those who inhabit the frontier of coloniality, when they "feel" the weight of colonial difference, means that, when it is reached by the always violent ways of hierarchization and valorization/devaluation of cultures (Mignolo 2005, 2013):

> We try to assimilate ourselves, and good luck in assimilating; we adapt as best as we can because we have to live; or third, we adapt and start building projects that point to other life forms. At this moment the consciousness and the frontier being become the frontier thinking into action, we put the experience and thinking into action (Mignolo 2013, p. 24–25).

In other words, putting border thinking into action, we provoke the established, vehemently questiony the hegemonic and monocultural way of defining what is true or false and validating knowledge (we intend ethics), the beautiful and the ugly (we intend aesthetics), we produce fractures with modern/colonial western monoculture.

Another example that seems to us to be quite potent regarding the lack of barriers capable of separating theory and practice can be symbolized by the Ubuntu philosophy that can be found, according to Castiano (2010), in the southern African region, especially in South Africa. The story elaborated by Alexandre Nascimento (2014) greatly illustrates this philosophy. We will repeat it here:

> When I was in South Africa I visited a Zulu ethnic community. There I met people, watched a dance performance and could witness some habits, the main one was do collective things always in the simplest way and in groups, never a single person, as when, at lunch, I used my hands to bring the food to my mouth, sitting on the floor with other people around a beautiful colored fabric where the food was. Everything was very cheerful, supportive, gentle and beautiful. It was almost time to return to the hotel where I was staying and there were many children in that community and they liked soccer, I proposed to them a game as a way for me to say goodbye, a race in which the first child would win a ball as a prize. They immediately bumped into it. So I arranged the starting lines and the finish. All the children positioned themselves at the starting line and it was agreed that when I gave the signal they would start running towards the finish line. With everything ready, I started and the children started the race. Interestingly for me, they ran together and arrived together at the finish line. How did I find that different, I asked them why they did it, I mean, why they left, ran and arrived together. One of them answered me: *It's Ubuntu, sir, we are each and every one of us because we do and do everything together. Didn't you realize that everything we did today we did it together?* My eyes overflowed with emotions. I have never had an experience that affected me so strongly. Ubuntu, tenderness and common constitution of the common. Blackness (p. 1).

To explain how the practical Ubuntu philosophy works, Mogobe Ramose (2009) uses two aphorisms, which, according to the author, clearly shows this way of understanding and acting in and against the world. The first aphorism, *Motho ke motho ka batho*, is supported by two conceptually interrelated principles. The first says that each human being is a subject of intrinsic value in himself, "if it were not so, it would be meaningless base the affirmation of humanity on one person's recognition of the other": "Pointing out that to disparage and disrespect a human being is, first and foremost, depreciate and disrespect oneself, only acquires meaning if one accepts that he or she is someone who deserves dignity and respect" (Ramose 2009, p. 170).

The second principle of aphorism *Motho ke motho ka batho* is connected to the *motho* expression, whereby the humanity of someone is tied to relationships with other human beings, which does not establish hierarchy with other entities of nature, but conditions individual humanity relations with the collectivity, with other human beings.

The second explanatory aphorism of Ubuntu philosophy, *Feta kgomo o tshware motho*, "means that if and when a person faces a decisive choice between wealth and the preservation of another human being's life, he must choose to preserve life" (Ramose 2009, p. 170).

A practical example of life inspired by Ubuntu philosophy is presented to us by Carlos Eduardo Gomes Nascimento. According to this author, both spiritual leader Desmond Tutu and Nelson Mandela resorted to this philosophy as a way of resistance the "fierce racism practiced by the institutions and the search for reconciliation between men in South Africa" (Nascimento 2014, p. 323). And it should be highlighted that were strategies used by Mandela, also inside the prison, where he remained for 27 years, that halted him with the Nobel Peace Prize in 1993.

It is important to focus on our recurrence to Ubuntu philosophy justifying by our interest in showing forms of life in which walls separating theory and practice are not erected. In fact, what we see most of the time in traditional African practices—and, as it was said, in Afro-diasporic territories, in the context of Brazil, *candomblés* symbolize, in our view, an example of this—there is a link between to think and to do. You live what you stand for. Even because "knowledge is triggered from reality. Knowing is to know and to do" (D'Ambrosio 2005, p. 101).

Territories, with all that is in them and always (always floating) conforming to them—and continually reorganizing them—are, in our view, living and dynamic beings. They are sustained in possibilities and limits that are peculiar to them, at least at a given moment—because they are moving. Because of the challenges they provide (and that we provide), they inflate ways of doing and existing that delineate identities. And it is through intersections between subjects and spaces that practices and subjects are born.

Because of this, we believe that using the term ethno to identify knowledge produced colonized territories (formerly and/or now)—ethnomathematics, ethnoscience, ethnophilosophy—instead of symbolizing a healthy concern with the intrinsic relationship between nature and culture (theory and practice), it moves along paths based on modern western thought, which Dussel (2005) had already demonstrated, colonial.

The idea, it seems to us, is forced a unique form of truth, there is a tendency to put Western-modern thinking in center and name others as ethno. The "others," who Mignolo (2013) defines as colonial difference, the rest are housed in the locus of exoticism, in other words, in the space where people and/or practices are inadmissible to Western-modern thinking are enclosed.

2.3 We Need to Talk About (Ethno) Mathematics

We need to talk about (ethno) mathematics, but didn't we? Let's think! According to D'Ambrosio, mathematics should be understood "as a strategy developed by the human species throughout their history to explain, to understand, to manage and to live with the sensible, perceivable reality, and with their imaginary, naturally within a natural and cultural context" (D'Ambrosio 2005, p. 102).

In addressing ethnomathematics, D'Ambrosio points out that the term was created to designate "many ways, techniques, (*ticas*) skills to explain, understand, cope, and live (*matema*) with different contexts natural and socioeconomic conditions (*etnos*)" (D'Ambrosio 2005, p. 114, italics in the original).

> A culture is identified by its systems of explanation, philosophies, theories, and actions and daily behaviors. All of this is based on processes of communication, representation, classification, comparison, quantification, counting, measurement, inferences. These processes take place in different ways in different cultures and change through time. They always reveal the influences of the environment and organize themselves with an internal logic, codify and formalize themselves. Thus is born knowledge (D'Ambrosio 2005, p. 101).

The point is that, as *Dambrosian* thought once again teaches us, "Mathematics is generally conceptualized as the science of numbers" (D'Ambrosio 1994), and if we do not find/visualize them, in the moments that we deal with it, we cannot realize, most of the time, the presence of this language or dialogues with it, in a certain cultural practice. However, it is always relevant to highlight that "the daily life is impregnated with the knowledge and doings of the culture its own" (p. 93).

At all times, individuals are buying, classifying, quantifying, measuring, (…) and somehow evaluating, using the material and intellectual instruments that are their own culture" (D'Ambrosio 2002, p. 22, italics ours). In this way, they are putting into action knowledge that moves other knowledge, among which those in modernity/coloniality were fronted and named Mathematics.

But we could talk about "African mathematics" following Attico Chassot's footsteps and highlight, for example, that "more recent studies show that knowledge attributed to Pythagoras, such as the famous Pythagorean Theorem, is found in other pre-Greek civilizations in Asia and in Africa" (Chassot 2004, p. 42). Professor Cunha Junior (2010, p. 11) echoes Chassot's statement emphasizing that "the Pythagorean theorem, for example, has a geometric demonstration performed in Africa and China at the same time" and before the demonstration of Greek philosopher.

We could highlight that Archimedes, mathematician, engineer, physicist, although he was born in Syracuse (Italy), his studies were done from an early age in Alexandria (Egypt, Africa), as also emphasized by Chassot (2004). Or list the prowess that marked the construction of Egyptian pyramids, which is still unexplained until these days.

From another perspective, Paulus Gerdes's[4] studies, based mainly on African territory, enlighten mathematical knowledge present in traditional African cultural practices. This researcher has listed several strategies used by African communities in which a mathematics (ethnomathematics) is inserted, as in the storytelling techniques, of the Quioco people (ethnic group present in Angola): "famous for your art. They like to decorate the houses' walls with designs. They manufacture decorated mats and baskets. They model ceramics, carve wood and forge iron" (Gerdes 1997, p. 6).

Within this ethnic group are careful storytellers who, while narrate them, make drawings on the floor, using lines and dots:

> (…) the number of lines required to execute varies with the drawing and the dimensions of the dot net. (…) not only in Angola, but also in other African countries, such as Ghana and Congo, many adults and children can tell immediately how many [author's emphasis] lines are needed—just show them the network (Gerdes 1997, p. 23).

Gerdes identifies in these drawings the presence of a mathematical content, in this case the mcd (maximum common divisor), since the elaboration of the drawing

[4] In Mozambique, the research carried out by the team coordinated by Paulus Gerdes is so important that this researcher is, in that country, according to Castiano (2010), considered the true father of ethnomathematics.

requires a correspondence between the number of closed lines and the number of rows and columns of the dot net, and this correspondence is obtained by calculating the greatest common divisor between this one and this one, even though the Quioco do not use this algorithm to produce their drawings.

Another situation presented by Paulus Gerdes (2010) refers to the process of home construction, which begins with the assembly of quads on the ground where the house will be erected: (1) "begins extending two long bamboo sticks on the ground. Both sticks have the length equal to the desired length for the house. These first two sticks are then combined with two other sticks, also of equal length, but usually smaller than the first"; (2) "then move the sticks to form a closed quadrangle" (p. 3). Finally, adjust the figure until the diagonals, measured with a rope—are of equal length. Where the sticks are laid on the floor, lines are then drawn and construction of the house can begin" (Gerdes 2010, p. 21–22).

Gerdes adds that this experience of spontaneous use of geometry represents an alternative axiomatic construction to Euclid's fifth postulate, known as the parallel axiom: "From a point outside a line *m* one can draw a single line parallel to line *m*." He concludes: "mathematical ideas are not alien to African cultures, there is an awareness that not all mathematics comes from Europe" (Gerdes 2010, p. 22–23).

Following this same path, now in the Brazilian context, Costa e Silva (2010), and not the only ones, illuminate mathematical elements (also from other disciplines) in Afro-diasporic practices, such as *capoeira* and the *jogo de buzios* (oracular game). According to these scholars, although research in Brazil, based on ethnomathematics and linked to Africanities are still incipient, they have been transited by three approaches: (1) in the rural environment, revealing practices created or experienced in predominantly black environments, (2) in the urban area, the investigations are moved by religious issues, soccer, carnival, and dances of the periphery, (3), in another direction, researchers have been analyzing pedagogical materials, relations between teachers/students and students/students.

Referring to *capoeira* and the mathematics present in it, Costa e Silva highlight elements of geometry when one observes the action of bodies projecting themselves constantly into a 3D space drawing geometric figures that turn into a rapid sequence as the arms and combatants' legs build dynamic geometry. Regarding the game of shells, among other knowledge, these same authors emphasize that the:

> (…) set of conch shells follows the probabilistic binomial distribution model (for discrete random variables). In the game, it is hypothesized that each conch admits only two results (opening down or up) and that these results are independent of each conch. Combining the various repetitions of the game, we have a range of possibilities for possible alternatives, which gives us, mathematically, many possibilities for reading the results of the game. (Costa and Silva 2010, p. 254–255).

On the other hand, we can quote Professor Henrique Cunha Júnior's research and his Afro Ethnomathematics, which he defines as "the area of research that studies the contributions of Africans and Afro Descendants to mathematics and informatics, as well as develops knowledge about the teaching and learning of mathematics, physics and informatics in the territories majority African descent" (Cunha Júnior 2006, p. 62).

In our understanding, Cunha Júnior refers to the (re)invention of practices and knowledge, which blacks people were impelled to build, thanks to the inhospitable conditions of life—in fact, the situation of death (or almost so)—and the exchanges/negotiations with indigenous peoples in terra brasilis, other African ethnic groups, the colonizer himself and local geography. Obviously, among the knowledge archived in the body, or the body knowledge, there were those who, from a modern/colonial reading, slipped/crept through the area of mathematics.

Also according to Cunha Júnior (2006), in Brazil studies related to Afroetnomathematics are started from strategies of empowerment, rescue, and valorization of black culture promoted by the Black Movement, whose objective was covered from the recovery of mathematical elements present in African communities to the survey of the history of mathematics of that continent. The pedagogical practice, still with Cunha Júnior (2006), was effective in quilombola's communities and in regions where black presence was quite significant.

Taking the motto of Professor Henrique Cunha Júnior, when he emphasizes the leading role of the "Black Movement" regarding the rescue and valorization of black culture (as provided in the immediately preceding paragraph), we consider it essential to highlight that all the ways presented above to show the African presence in the production of mathematics, are powerful and for several reasons. Among them, we could outline the following lists:

1. Compliance with Law 10.639/2003, one of the first instruments signed by President Luiz Inácio Lula da Silva, which "amends Law No. 9.394, of December 20, 1996, which establishes the guidelines and bases of national education, to include in the official curriculum of the Education Network the obligation of the theme 'Afro-Brazilian History and Culture,' and other measures.
2. Compliance with Resolution CNE/CP No. 01/2004, which "establishes National Curriculum Guidelines for the Education of Ethnic-Racial Relations and for the Teaching of Afro-Brazilian and African History and Culture."
3. Compliance with Resolution CNE/CEB No. 08/2012, which "defines National Curriculum Guidelines for Quilombola School Education in Basic Education."
4. Tensions in curriculum movement by educational institutions at all levels and modalities, which, most of the time, are based on a Eurocentric monoculture, continuously putting the colonial difference.
5. Presentation of references (also symbolic) for teachers and students able to contribute to the subjectification processes of these actors, regardless of their ethnic-racial belonging, aiming at the formation of subjects who turn and defend the construction of a plural society and therefore inclusive.

Nevertheless, there are some precautions that deserve extra attention, we think. When we carry a cultural practice to school, although it still has a relationship with the original one, it is no longer the same. For instance, *capoeira* practiced on the Salvador's streets suffers adjustments when it becomes part of the school curriculum. At the very least, logics of time and space are no longer the same that cradled the bodies under the sun of the Bahia's capital. And that requires for those who follows curricular traditional practices understanding and explanation.

But not only that. We have to keep in mind that the game of cowrie shells, a divinatory practice of *candomblés* (sacralized, therefore), has a particular meaning and function very different from the reading of probability in this game, as a possible strategy for teaching and learning. Costa and Silva seem to agree with us by warning that:

> (…) in the game of cowrie shells, there are also present, in a fundamental and inextricable way, a mythical component and another personal and circumstantial—which refer to the interpretation in face of the situation actually lived. These components greatly increase the possibilities of reading the answer given and reveal the richness and complexity of the mythical-religious-corporal ethnomathematics of the Brazilian black (Costa and Silva 2010, p. 255).

We would add yet another aspect regarding to pursuit/perception of (ethno)mathematics in traditional African or Afro-diasporic practices. Let's go back to D'Ambrosio. According to him "Mathematics, with its infallibility, rigor, precision and being an essential and powerful tool in the modern world, had its presence established excluding other forms of thought." And he keeps going: "In fact, being rational is identified with mastering mathematics. Mathematics presents itself as a wiser, more miraculous powerful god than traditional divinities and other cultural traditions" (D'Ambrosio 2002, p. 17). And in this aspect is our concern.

According to Quijano (2005), modernity/coloniality is responsible for the rise of the coloniality of power, which is characterized by the institution of race idea and, from this, determines the place and role to be played in the new global structure of power and control. The coloniality of power is reinforced and become strong by another form of coloniality, that of knowledge. According to Lander (2005), the coloniality of knowledge has historically contributed to development, strengthening, and maintenance of a Eurocentric thinking that has prevented us from solving our problems based on our own knowledge and ways of practicing them.

We think it is a peaceful point to understand that to fix an element component of a particular practice is no longer the practice itself. On the other hand, we think that when we try to highlight the mathematics of certain traditional knowledge (secular and even millenary), we no longer have the practice and the relationship of person with territory where this practice is performed.

Thus, a questioning remains present. Would be our concern to find mathematics in traditional practices, whether African or Afro-diasporic, rather than contributing, strengthening the colonial difference, and strengthening the hierarchy between people and their knowledge?

Trying to contribute with possible answers. When children, youth, and adults (*Mussuca* people or not) learn how to dance *Samba de Pareia*, they do this giving up their whole and integrated body, with no gap between body and mind or theory and practice; nor do they fragment the set of knowledge that makes up this *samba* modality: playing, singing, dancing, the relationship with African ancestry, the maintenance of the circle, the exchange of pairs, etc., everything is connected, everything forms all.

In *Candomblé*, we do not see separations and fragmentations in knowledge that circulates in these practices, nor in teaching and learning processes. To be initiated, it is necessary be included and immerse in ritual daily life and its secrets while

practicing. There is no specific moment and no exclusive place for transmission of knowledge, although religious practices often occur in certain places. It is (co)living, in whose routine the bodies fully engage and intersect, which ensures to know/to do.

2.4 A Last Word. Never the Last.

In a certain moment and place, Ubiratan D'Ambrosio (2005) reminds us that the subject is extremely valuable and that sometimes it is put as the most important thing when it comes to knowledge (from school or not). It is, in fact, ethnomathematics produced in the Mediterranean Europe with some contributions from Indian and Islamic civilizations, even though both groups quoted above are hardly remembered.

The universalization of this ethnomathematics was conceived as the owner of an unquestioning true, it has contributed not only to hide other peoples' cultural practices, including the Africans, it was also responsible for production of nonexistence, in terms of Santos (2009), from its practitioners. The destruction of knowledge and cultures that are not related to European standard and different philosophy which establishes "the difference between beings and those who are under the beings," say Maldonado-Torres (2019, p.37), that is: creating the sub-human or non-human. This understanding is essential and powerful for the construction not only of another world but a world where the standards—walls and covers as well—have as main ingredient love and kindness.

Our concern is related to compartment and hierarchization that produce marginalization, apartheids, and death. We do not believe that ethnomathematics is, naturally, attached to it. However, it will always be present every time when, in name of identification of mathematics knowledge, the original practices are deformed. This will happen every time when European ephystemology is a reference, a standard—which symbolizes a colonial behavior.

A decolonial attitude is tense, it is opening the body, it is a silent scream (synchrony and diachrony). To think in a decolonized way is an act in direction to pluralism and interculturalism within the prevailing universalism.

A decolonial thinking in Ethnomathematics field could even think of mathematics lines in multiples cultural practices. It would never conceive randomly or collect those lines to necessarily glorify this field of knowledge inside cultures. On other way, it would describe knowledge and its chains with a social and ambiental context, essentially composed of individuals that put them on evidence and that are, by them, evidenced.

A decolonial option implies that colonial knowledge (thinking in an eurocentered mathematics) do not give the lenses from which we are able to look for knowledge labeled as other knowledge by colonial difference (Mignolo 2008). It is necessary that knowledge can always be tensioned by its differences, and to question the centrality that conceives an only, possible and true mathematics.

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Chapter 3
The Mandira Quilombola Community and the Production of an Anthropophagic Identity

Diego de Matos Gondim and Roger Miarka

Après tout, le moyen le plus simple d'identifier autrui à soi-même, c'est encore de le manger.

Claude Lévi-Strauss

3.1 Introduction

What is the identity of the community, sir?, a woman asks.
Caiçara-quilombola, replies Mr. Chico.
You have to choose one or the other, says the woman.
So write down quilombola, because it guarantees the territory, concludes Mr. Chico.

In 2019 Ethnomathematics celebrated 35 years since the creation of its political space in Mathematics Education: the pronouncement of Ubiratan D'Ambrosio, in 1984, at the fifth International Congress on Mathematical Education, Adelaide, Australia (D'Ambrosio 1984). Why have we used the adjective *political*? Because, more than a name announced in that congress—Ethnomathematics, which was written ethno-mathematics in that proceeding—, there were already a few studies that operated or showed a concern about the relationship between mathematics and culture (Gay and Cole 1967; Zaslavsky 1973; Fettweiss 1935). However, it is important

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to highlight that, with the nomination of a new research domain in the most important congress of Mathematics Education, a political space was created, in which culture became an important element to be discussed within Mathematics Education. It is political in a way this brought culture to discussion as a kind of commitment among the community of mathematics educators.

At that time, D'Ambrosio (1984) claimed the importance of bringing social and cultural issues to Mathematics Education. His concern was about the articulation between "ethno-mathematics and society, where 'ethnos' comes into the picture as the modern and very global concept of ethno both as race and/or culture, which implies language, codes, symbols, values attitudes and so on, and which naturally implies science and mathematical practices" (p. 5).

With time, *ethno-mathematics* became *ethnomathematics*. This change means that ethnomathematics is not meant to be centered in mathematics. In order to clear that, D'Ambrosio (2016) states that he:

> (…) realized that the prefix ethno- is much broader that ethnic. It means a culturally identified group sharing knowledge and practices, language, and myths. Indeed, what many ethnomathematicians are doing is looking for ethnic-mathematics, which is contradictory. The nature, history, and philosophy of mathematics have shown how inappropriate it is to look for mathematics in different ethnic groups, as well as in different ethnos or cultures (p. 7).

As we can see, the effort is to avoid ethnomathematics to be used as a tool of recognition of Western Mathematics in different cultures, what could restrict its power. The point is the very opposite. How can we think of mathematics in a broader way, taking culture into account? Different researchers have been dealing with this question in diverse ways. Miarka and Bicudo (2012) studied the conceptions of well-known researchers on ethnomathematics of Brazil, New Zealand, and Mozambique. The study came up with three categories nominated as Mathematics in/and/or Ethnomathematics.

Mathematics in Ethnomathematics understands mathematics (or Western Mathematics) as an example of a wider body of knowledge called ethnomathematics, which is enrooted in different cultural groups. *Mathematics or Ethnomathematics* brings up the idea that ethnomathematics is an alternative to talk about a body of knowledge culturally called mathematics. *Mathematics and Ethnomatematics* highlights the importance of culture to talk about mathematics, that would be universal, but with different cultural contributions.

This chapter has not the aim of analyzing these conceptions or of trying to convince our readers about any of them. By bringing light to this study, we just want to point out that there is one point in common among the different conceptions of ethnomathematics: all of them are based on the idea of cultural groups. Generally, ethnomathematical researches work with different cultural identifiers of cultural groups, not always related to ethnicity, in accordance to D'Ambrosio's theory.

In Brazil, for instance, in the *Fifth Brazilian Congress on Ethnomathematics* (CEBEm-5), carried out in 2016, there were researches related to ethnicities, such as indigenous people and quilombolas; to specific professions, such as dressmakers, rural workers, fishermen, craftsmen, merchants, potters, masons, and nurses; to

peoples that share the same space, such as rural or urban communities; and to practitioners of practices that induce the constitution of a group around it, such as music production, dish ornamentation process, construction of houses, players of certain games and school mathematics practitioners (CBEm 2016).

All these studies identify cultural groups by using different criteria through a process of identification, what makes clear that the identity is a crucial issue to be discussed in the midst of ethnomathematics.

The aim of this chapter is to discuss the conception of identity by using anthropological studies and elements of a research in progress with a Brazilian Quilombola community in order to produce and to operate with the concept of *anthropophagic identity*, inspired by the *Anthropophagic Manifest*, written by Oswald de Andrade in 1928.

3.2 Anthropology and Identity: The Construction of the Concept of *Anthropophagic Identity*

Criticism of the theoretical propositions of a static identity—whether based on historical, genealogical, ethnical basis, etc.—constitute a contemporary concern in studies on cultural groups. The very idea of a group already appears to carry some categorization, as it establishes a previous criterion for the constitution of a group. In the case of cultural groups, this criterion is often ethnic—as in the case of indigenous groups—or around practices—as in the case of groups of practitioners, such as stonemasons, seamstresses, etc.

The discussions that stage problematizations around identity operate in the tension between *essentialist* and *non-essentialist* perspectives. As Kathryn Woodward presents in the chapter *Concepts of Difference and Identity*, published in her book *Identity and Difference*, *non-essentialist* perspectives are opposed to *essentialist* perspectives since the latter "identity would suggest that there is one clear, authentic set of characteristics which all (...) share and which do not alter across time" (Woodward 1997, p. 11). On the other hand, the non-essentialists "would focus on differences, as well as common or shared characteristics" (Ibid, p. 11) within certain cultural groups, or between these and others.

This tension between *essential* and *non-essential* is also discussed in the book *Questions of Identity*, edited by Stuart Hall and Paul du Gay. In the introductory chapter, *Who needs 'identity'?*, Hall (1996) intercedes with several contemporary thinkers, especially Michel Foucault and Judith Butler,[1] in order to deal with Anthropologists who defend identity from an essentialist perspective. The author approaches Foucault's theory since, for him, identity is defined as "subject

[1] Hall's passage (1996) to Michel Foucault or even to Judith Butler also considers a passage both to the concepts of Jacques Derrida's philosophy and to the psychoanalysis of Sigmund Freud and Jacques Lacan.

positions," that is, positions that subjects are obliged to assume in a given context, be it economic, political, historical or any other. In his reading of the contemporary philosopher, these subject positions produce a web of power relations that composes the subject as the one that is produced *within/by* the discourse (Hall 1996).

By evoking Michel Foucault's "genealogy of technologies of the self" and the proposal of an "inevitably performativity of the self" presented by Judith Butler in her book *Bodies that Matter*, Hall (1996) defends a *non-essentialist* perspective of identity at stating that he understands it as a *strategic* and *positional* concept. According to his words:

> I use 'identity' to refer to the meeting point, the point of *suture*, between on the one hand the discourses and practices which attempt to 'interpellate,' speak to us or hail us into place as the social subjects of particular discourses, and on the other hand, the processes which produce subjectivities, which construct us as subjects which can be 'spoken.' Identities are thus points of temporary attachment to the subject positions which discursive practices construct for us (Hall 1996, p. 5–6, emphasis added).

The way in which Hall (1996) defends the concept of *identity* is also developed by Woodward (1997) when conceptualizing a non-essentialist perspective as being of the order of *contingency*.[2] For the author, in a non-essentialist perspective: "identity is seen as *contingent*; that is, as the product of an intersection of different components, of political and cultural discourses and particular histories" (Woodward 1997, p. 28, emphasis added).

The more important to us in the discussion made by Woodward (1997) and Hall (1996) are the elements brought by these authors to conceptualize identity. By using them, we can synthetically construct the following hypothesis: *the identity determines a strategic condition of the subject positions that are, inevitably, of the contingency order.* However, we want to add one more element that, in our view, can help us in the construction of our proposal.

While Woodward (1997) and Hall (1996) contrast essentialist and non-essentialist perspectives, Ernesto Laclau—in Chap. 2 of his book *Emancipation(s)*—problematizes identity based on the concepts of *universalism* and *particularism*. To avoid a possible antagonization of these two concepts, the author poses an important question, namely: "Are the relations between universalism and particularism simple relations of mutual exclusion?" (Laclau 2007, p. 22).

By presenting this question, Laclau (2007) develops a problematization from ancient classical philosophy to contemporary movements that does not antagonize both perspectives. In this presentation, the author concludes that the paradox between the definitions of universal and particular cannot be solved given that, for him, "universality is incommensurable with any particularity but cannot, however, exist apart from the particular" (Laclau 2007, p. 34). Still according to the author, the insolvability of this paradox is productive insofar as it is, in his words, the "very precondition of democracy" (Ibid, p. 35).

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[2] It is important to highlight that contingency is also considered by Hall (1996) in the production of identification processes. In his words: "identification is in the end *conditional*, lodged in *contingency*" (Hall 1996, p. 2–3, emphasis added).

It is at the democratic level—where individuals assume their subject positions—that occurs what Ernesto Laclau evidences as *negotiation* (Laclau 2007). It is the negotiation that inaugurates the *differential* and *relational* processes, and it is in the impossibility of solving the paradox between the universal and the particular that democracy emerges. Thus, the process of differentiation in the political perspective of Laclau (2007) occurs through the form of negotiation, because, for the author, "it [the identity] presupposes not only the presence of all the other identities but also the total ground which constitutes the differences as differences" (p. 27).

By assuming these discourses, the hypothesis raised above can be reformulated differently, namely: *the identities determine a strategic condition of the subject positions that are inevitably negotiated in contingency.* We believe that, when considering this hypothesis, we rely on a non-essentialist perspective of identity, because, in this case, individuals assume their subject positions in a process that does not occur in the order of an antagonization between the *self* and the *other*, but in the negotiation and intersection of elements that make it possible to affirm an identity as a life-affirming strategy.

In conceptual terms, by avoiding the antagonization between *self* and *other*, we also avoid the contrasts commonly produced between the concepts of *identity* and *difference*, because, in this case, both are conceived in the process of differentiation. *Identity* is itself the expression of a differentiation without limits, where what matters is the affirmation of a life that perseveres. Otherwise, *identity* and *difference* are mixed in a movement that could be called "cannibal identification," which does not occur in the order of the "similarity" between self/other and identity/difference, but in the order of *devouring*.

It is in the wake of this way of conceptualizing identity that we build our proposal to think of an *anthropophagic identity*. In other words, what we intend with this article is to present the process of producing an identity inspired by the Oswald de Andrade's (1928)[3] *Anthropophagic Manifest*. To this end, we will gradually use data from a survey initiated in 2015 with a *Quilombola* community in the interior of the state of São Paulo, Brazil. In summary, what we seek to gather here are elements that enable us to create *a plan of consistency* for the following hypothesis*: an anthropophagic identity operates in a process of devouring elements to produce an identity that, at the same time, has got characteristics of perseveration and differentiation, without clear borders.*

It is worth mentioning, however, that, although we can locate the subject positions in the presentation of an *anthropophagic identity* in the community in question (their negotiations and strategies), it is the *process* (in its contingency and differentiation) that allows us to think an identity that works in the order of devouring. With this concept, we are not interested in the differences between the *self* and the *other* as the determination of an identity, as put by Woodward (1997) and Hall (1996). As we will see, anthropophagy does not work under a binary logic. On the contrary, it

[3] The *Anthropophagic Manifest* is problematized from several theoretical perspectives and cultures in the books *Where to sit at the dinner table?* and *The forest and the School*. See: (Marques 2014).

claims to be "barbaric" and "savage" in the processes of *feeding of* what is near and far at the same time. In other words, as stated by Ceppas (2017, p. 1390) "[...] this is what Oswald invites us to do, when he defines a thought as being of the order of devouring, of overcoming the opposition self/other ("only what is not mine interests me")."

It is also worth noting that we intend to present that in an anthropophagic identity there is no distinction between *identity* and *difference*, as defended by Silva (2002),[4] because it is founded on (and by) repetition and difference. It is, in its effect, the expression of the possibility of perseveration and differentiation of an identity.

In the next section, we will use the concept of *anthropophagic identity* to operate the data produced by one of the authors of this article, during his stay in a fieldwork with a group of *Quilombolas*, as part of the research of a finished Master's Degree (Gondim 2018a, b) and of a PhD study in course. By doing this, we expect to present how an a*nthropophagic identity* is not predicative or propositive, but always becomes another in its process of devouring.

3.3 The Production of Mandira

According to oral information, the Community of *Quilombola* Remains of Mandira, as it is known today, has been counted since 1868. What is now known as the "*Quilombola* Community" was a rice production farm in the countryside of São Paulo, in the Vale do Ribeira region, Brazil. It is said that:

> The owner of the farm was called Antônio Florenço de Andrade; he had three children, two men and a woman. He had a son with a slave. As for the two men, one was a politician in Cananéia and the other died at the age of 16. His daughter, Celestina Benício de Andrade, and the son he had with the slave, Francisco Vicente, stayed there. Celestina donated the land of Mandira to her brother in 1868, before the Golden Law. *When Princess Isabel signed the Golden Law, the Mandira Negro was already freed, as he had received the land as an inheritance* (Gondim 2018a, b, p. 21, emphasis added).

The expression "the Negro of Mandira was already freed," meaning that he had received the land as an inheritance from his half-sister, reveals the relationship of the *Mandiranos* (Mandiran individuals)—a nomination recognized by them—with the territory. The inheritance of the land, in this case, is not the guarantee of possession in a given geographical place, but, besides that, is the achievement of freedom. In other words, freedom is here combined with the inheritance of a land where they could survive as individuals.

[4] In his text, Silva (2002, p. 66) differentiates identity and difference insofar as he defends impertinences, such as: impertinence 4: "Identity is predicative, propositional: x is this. Difference is experimental: what to do with x"; impertinence 5: "Identity is in the order of representation and recognition: x represents y, x is y. The difference is in the order of proliferation; it repeats, it replies: x and y and z"; impertinence 14: "Identity is. Difference become."

And it is freedom that goes through all the affirmation processes of the Mandiran life and that will, later, assume a subject position. We could say that it is on a "will of freedom" that the Mandira production is sustained. We will see, therefore, as the anthropologist Maria Célia Manzoli Turatti states, that this land "is not just any land, but the land in which they maintained some cultural, social *autonomy* and, consequently, *self-esteem*" (Turatti 2002, p. 11, emphasis added).

The land, in this case, is not a material good extended to the Mandirans, but an existential condition of the community. Thus, if we affirm the production of an *anthropophagic identity*, in terms of devouring the *other*, it is because that *other* is, in some way, connected to the land. Interest in the other is not established in *ontological* terms of an identity assimilation, but in *topological* terms. In other words, what the production of the Mandira presents to us is the constitution of an identity that is, genetically, the production of a *topos* (Gondim 2018a). In our view, sociologist Wanda Maldonado understands this hypothesis by defending the following title: *"Mandira: name of the land, name of people"* (Maldonado 2005, p. 357, emphasis added).[5]

It is important to highlight that, for the Mandirans, identity is not the centrality of their struggles, but, before anything else, the land itself. The evocation of a subject position, if we can call it that, is always evoked in an affirmation of the land. This is expressed several times when Mr. Chico starts introducing the *quilombo* or himself. In the field notebook this is repeated, gradually, as follows:

> *My name is Francisco de Sales Coutinho Mandira. Everyone knows me as Chico Mandira. Mandira is the name of the land* (Field notes 2016).

Mr. Chico's dialogue, presented at the beginning of this article, is an episode narrated by the sociologist Wanda Maldonado in an interview by one of the authors. In this dialogue, it is also the territory that assumes the centrality above the assumed subject position (*quilombola*). Land as an inheritance is the guarantee of freedom and not of an identity embedded in an ethnocentric genealogy. Contrary to this, the Mandira is produced in a genealogy that breaks with the old *human* and *nature* antagonization, being a *geophagic process*, where the *geo* is the composition between people and land. It is in this decentralization of the categories of the *self* and the *other* that we can defend the production of an anthropophagic identity, where *anthropo* does not determine the evolution and the condition of an "absolute self," but of a relational and contingent process, thus expressing what we could call *becoming-land.*[6]

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[5] It is worth mentioning that—although we consider the *topos* (land) as a condition for the creation of the existential territories of the Mandirans and a constituent of their ontological dimension, as can be seen in Gondim (2018a)—, unlike Turatti (2002, p. 37, emphasis added), this ontological dimension, for us, is not "almost an *extension* of the land," but the land itself. In other words, they are the land itself, the very existential *topos*. The naming shared by them, as highlighted by Turatti (2002), constitutes a genealogy, before any other, with the land.

[6] It is in devouring that we find the immanent sense of becoming, that is, when it is no longer possible to defend a separation (or duality) between individual and land, as there is an existential condition that does not determine a "relationship" with the land, but being it as cosmogony. We

3.4 Devouring in the Production of a Right/Freedom

The history of the Mandira community is composed of several lines and interlocutors. With it, it is necessary to surpass the values of an absolute truth and assume a purely relational and emerging condition in its production. As Woodward (1997) and Hall (1996) conceptualize, it is the contingency that ensures its consistency. This emergency is narrated by one of the interlocutors as follows:

> They were not in fact related. They were of several slave ethnicities. Suddenly, they were freed. The boy receives the land and he needs to sign his name, however, who recognized him was his half-sister and not the father. So, he signed the name Mandira, which was the name of the farm. When slavery ended, then, no one had anywhere to go. With the advent of the republic, everyone needed to have a name. Then they all started to sign as Mandira. They were not necessarily related, but of various ethnicities. They came together because they had nowhere to go, adopted the name and did what the boy did, that is, wrote the name Mandira (Gondim 2018b, p. 26–27).

The name, here, does not assume a purely naming character, but it produces guarantees of a right/freedom. What is being produced is not just a designation (in an essentialist sense), but the possibility of saying: "my name is Francisco de Sales Coutinho Mandira. Everyone knows me as Chico Mandira. *Mandira is the name of the land.*" As Andrade (1928) states, "death and life of hypotheses. From the self-equation part of the Kosmos to the Kosmos-axiom part of the self. Subsistence. Knowledge. Anthropophagy" (p. 3, emphasis added). It is in this process of *becoming-land* that we defend a cosmological conception of the Mandiran condition, because, as we will see, their work and subsistence practices happen empirically close to the land (and the sea), concretizing a cosmology.

The right is connected with freedom since the inheritance of the land is, at the same time, the inheritance of freedom. This is expressed in a quote already presented here. We repeat it: "When Princess Isabel signed the Golden Law, *the Mandira Negro was already freed, as he had received the land as an inheritance"* (Gondim 2018b, p. 21, emphasis added). It should be noted, however, that inheriting land or assuming it as a right is not connected with the logic of possession. On the contrary, it is always in the order of sharing. Work is sharing. We quote:

> Mandira, at that time, was a place of brotherhood. The family, everyone was united. There were some people who lived there, but they all considered themselves siblings. They did not own Mandira, but they had the right to work. Many people lived there, grew up, built a house, raised their children there (Gondim 2018b, p. 27, emphasis added).

All residents and former residents constantly talk about freedom as a right. They say: "it was a place where we lived with freedom, we had freedom. We sowed

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will see in the following sections a problematization around this idea. However, it is worth mentioning again the speech of Mr. Chico presented here: "My name is Francisco de Sales Coutinho Mandira. Everyone knows me as Chico Mandira. Mandira is the name of the land." At this moment, Mr. Chico introduces himself not only as a subject, but as the land itself. In other words, as an existential and cosmogonic condition.

anywhere, planted and made a canoe anywhere" (Gondim 2018b, p. 27). Sharing a land is also sharing a freedom. The sharing of work is also the sharing of a right. There were no owners. The other, as a foreigner, was not assumed like the other: *they considered themselves siblings.* What is at stake is not "to be Mandira or not to be Mandira?," But the production of a union: *everyone was united*. If we speak of an *anthropophagic identity*, it is because, given the way Mandira has been produced, it is possible, together with Oswald de Andrade, to affirm: "only anthropophagy unites us. Socially. Economically. Philosophically" (Andrade 1928, p 0.3).

If this union happens, in some way, through "assimilation," it happens through devouring. Francisco Vicente devours the right/freedom to read from his white half-sister, Celestina Benício de Andrade. He devours the land as the production of a right/freedom (Gondim 2019). The other is not conceived as "other": "the people stayed there, then they married one another, a Mandiran with another family, and that was it" (Gondim 2018b, p. 27).

This "anthropophagic reasoning," as conceptualized by Haroldo de Campos (1992), contains a contingent aspect insofar as it produces a "concrete philosophy" (p. 246). Precisely in the community of Mandira, it makes it a reality (and also as the Neighborhood of Residents of Mandira; Residents' Association of the Mandira Neighborhood; Extractive Reserve of Mandira; Community of *Quilombola* Remnants of Mandira, among others). We will see, in the following section, that subject positions are assumed given the community's emergence and struggle. Land and territory (right/freedom) are the constant struggle of the Mandirans. A right that, after the 1988 Federal Constitution, founds a legal dimension, evoking a "*quilombola* identity."[7]

In the narrative told by the Mandirans, this struggle for right/freedom dates to past centuries. The history of the community is presented—whether by the community members or by external interlocutors—from 1860, when the slave son received the land as an inheritance from his half-sister. Francisco Vicente had several children; however, the narrative develops from two of them, namely: João and Antônio. The most prominent figure in the narratives is that of João, often presented as a "super intelligent black man" who, "for going out to defend lands here, became very well known. There are reports that he defended the land from squatters even in Paraná" (Gondim 2018b, p. 22). João's intelligence, according to Mr. Chico, made him known as an "advocate for the poor."

João considered an "advocate for the poor" means he defended a right/freedom. A land. However, as previously mentioned, it is not just any land, but one that allows the statement "Mandira: name of the land, name of people" (Maldonado 2005, p. 357).

[7] This is Article 68 of the Transitional Constitutional Provisions Act, which defines the right to land of what came to be called "Communities of Quilombo Remnants." This article is certainly an important turning point in the struggle of black communities in Brazil, being a constitutional "loophole" of ethnic and racial rights—achieved by the black movements that date back to the 1920s and 1930s, with great intensity between the decades 1950–1980.

> At the time, João Vicente fought a lot because of this land; fought against the famous Cabral, Colonel Cabral. He left Mandira, took the canoe at the outside dock—as we call it—and went to Cananéia. In Cananéia, he went to the beach, went to the rock in Iguape. He took a boat there and went to Santos to fight over this territory. And he succeeded. There he won the case in court and the land remained with the family. It is where we are. The history has been passed from generation to generation and reached us (Maldonado 2005, p. 358).

"We are concretists" (Andrade 1928, p. 7).

As a constituent of the Mandira's existential territories, the land is still today the operator of the community differentiation processes. Its "chewing jaws," as Campos (1992) describes, produces an anthropophagic identity in which "history becomes the product of a construction, of a reconfigured appropriation" (p. 243). Mandira is produced, differentiating itself and, as *intercommunicating monads*, in the sense of Gabriel Tarde, expressing itself in differences, given that they are the causes and objectives (Tarde 2007). In Suely Rolnik's reading of the Oswaldian *Anthropophagic Manifest* and of the Guattari's *Schizoanalysis*, it would be the same to say that what is being produced in the machinic-anthropophagic processes is "a certain malleability to allow itself to be inhabited by a constant variation of universes, as well as a certain freedom to create new masks, territories of existence marked by the hybridization of such universes" (Rolnik 2000, p. 456).

If for Rolnik (1998, 2000) anthropophagy is a "machinic" process, it is because devouring is an act of war, a revolution against the Colonels, against the appropriators of land for value and accumulation of capital. Andrade (1928) calls this revolution the Caraíba Revolution.[8] Thus, it is possible to defend an *emancipatory* hypothesis in the differentiation processes as proposed by Laclau (2007), in the case of the Mandira community; however, it also constitutes the right/freedom fold, thus not ending in the oppressor/oppressed.[9]

We will see, in the following section, how devouring produces identities that—although differences are established in the words of Woodward (1997), Hall (1996) and Laclau (2007)—are contingent and relational, evoked by an emergence that

[8] In view of the text Quilombola Manifest, by Gondim (2019). In this text, the author tries to cannibalize the writing of his data along with the Anthropophagic Manifest and the quilombo of Mandira. We also emphasize that the revolution here is also assumed in Deleuzo-Guattarinian terms, that is, "revolution is the absolute deterritorialization, to the point of appealing to the new land, to the new people" (Deleuze and Guattari 1991, p. 97). In this sense, Mandira produces, as a revolutionary strategy, *new settlements*.

[9] It is not possible to deny the operation of the oppressor/oppressed fold, defended by Laclau (2007), in the processes of emancipation in social movements. Both in the quilombo of the colonial period and in the quilombola communities after the Federal Constitution of 1988, it is the search for emancipation and autonomy (what we called in the beginning "will to freedom") that founded their insurrections. The studies produced by Gomes (1997, 2015) and also Reis and Gomes (1996), for example, are evidence of these processes (the figures of the Colonizer, the Colonels, the farmers, the State, the Police, among others, are always present not only with the slaves of the colonial period, but also with the remaining quilombolas). However, we add here the right/freedom fold in order not to understand the defense of an anthropophagic identity only as a way of affirming emancipation upon the land. It is not just a matter of emancipating oneself, but of being able to exercise the right to the possibility of being the land itself in the constitution of its existential territories.

inaugurates a state of negotiation. The negotiation, therefore, will be assumed in terms of anthropophagy in view of the hypothesis announced here.

3.5 Devouring in the Production of an Emergence/ Negotiation

The dialogue that starts this article will serve us, once again, as a way of giving materiality to the concept of anthropophagic identity. When stating that "then write down *quilombola*, because it guarantees the territory," the emergence of a subject position is presented as a result of a process that, in the dialogue, remains hidden. Assuming yourself as *quilombola* in Mandira is something that expresses, with greater emphasis, the operation of its chewing jaws as constituting an identity in the Brazilian tropics. The emergence/negotiation fold is combined with the right/freedom fold. We will see below how this is produced.

In an interview for the Pessoa Museum, Mr. Chico says:

> After the people sold, they left and we stayed here and there was a priest, he came to the community once to give a mass and he already knew a little of Ivaporunduva's work, I think that here in Ivaporunduva it was the only community that has already a good period of fight over what is *quilombo*, title of the land, until, thanks God, last year they got it, the title of the land, so today the only community that has the definitive title of the land is Ivaporunduva, the others, in some communities they have some extension, some part that is titled, but it is not the territory as a whole like in Ivaporunduva*, and then due to the question of Ivaporunduva and the sisters working with Ivaporunduva, this priest came one day and told us that we were a quilombola community, a black community, an Afro-descendant community and people were really pissed off with him, really angry, like that, because he was calling us black, negro, you know, and then after we, after we started to realize it, what he was trying to say to us, what he was tellingus*, and then the people from Eldorado started talking to the community, still a little timid (Mandira 2013, p. 16, emphasis added).

The subject position assumed by Mr. Chico in the dialogue that begins this article is produced insofar as it enables the exercise of a right/freedom: the land *as a producer of its existential territories*. The opposition is always against those who seek to appropriate this topo-genetic dimension of the *quilombo* (as in the case of Colonel Cabral). This statement, however, expresses the devouring process itself in anthropophagy.

Let us turn to the *Anthropophagic Manifest*: "I asked a man what a Right was. *He replied that it was the guarantee of exercising the possibility*. This man was called Galli Mathias. I ate him" (Andrade 1928, p. 3, emphasis added). If we assume the figure of the priest with that of Galli Mathias, we have him, thus, as the one who affirms the law as the exercise of a possibility: "Mandira: name of the land, name of people" (Maldonado 2005, p. 357). However, at this time, this takes on another dimension, that is, a legal dimension, since Article 68 of the Constitutional Transitional Provisions guaranteed this possibility. "Then write down *quilombola*, because it guarantees the territory."

The emergence/negotiation fold occurs when it is necessary to affirm a subject position that allows the accomplishment of a right/freedom. The previous quote demonstrates that there was a negotiation process, not only between the interlocutors outside the community but also with the members themselves. This negotiation process by which the body is racialized involves a multiplicity of elements that, in this article, will not be problematized, given not only the complexity it presents us but also the text's objective.[10]

It is important to highlight that this emergence/negotiation fold happens as a product of a devouring process in this case, as stated by Santiago (2008),

> Rather than the sign of recognition of a debt that is lost in the vortex of centuries, *anthropophagy is the primacy of a negotiation whose result*—the return or the discount of the legal and official price, as we said—is the lighting of this world and its inhabitants by the absolute extent of full knowledge of the differences in the exercise of their overcoming (p. 21, emphasis added).

In Mandira, emergences/negotiations, produced in the processes of devouring, are expressed not only in the affirmation of this subject position but also in the (cultural) practices themselves. For example, the sustainable production of oysters fattening—today the most important economic activity in the community—occurs in a process that, in terms of anthropophagy, could be understood as inherent to devouring: "only what is not mine interests me. Law of the man. Law of the anthropophagous" (Andrade 1928, p. 3).

The Mandira community previously worked with agriculture. The work with oysters began, more effectively, from the years 1978, 1980. The emergence of this work occurs after the sale, made by some families, of a part of the territory to a farmer. In the data produced in the field (and other researches), it is evident that this sale took place due to several factors, of which three can be highlighted, associated with each other, namely: pressure from the Federal Police; the restrictions of environmental legislation imposed since the 1960s; and the presence of an Environmental Park (Gondim 2018b; Maldonado 2005; Mandira 2013; Sales and Moreira 1996).

The emergence of this work is evidenced in several interviews as the only way to survive in the territory, because in the region where they were placed by the farmer it was not possible to continue with agricultural practices, given the geographical

[10] We emphasize, however, that not assuming to be "black" may, in this case, mean a repression of the racism that they suffered in outside communities. This is shown at different times, both in the interviews conducted in the research in question, and in other interviews. For example, the Mandirans went to Cananéia as a group, because they suffered racism in the city. Going in a group meant, for them, to protect themselves in case of physical violence against the members. See how Mr. Chico narrates: "here we were like a community of blacks, a black community (laughs), it was like that, and we were very tacked on, like this, with very strong discrimination, even for organizational reasons as you mentioned before, for people to leave the community, there was usually a large group. 10, 15 people to go shopping, to go to work, everyone would work together, the leader, because there was always a leader, my grandfather's brother was a very strong leader here in the community and when he said it was time to come back, everyone would come around here and such, and we even had a saying that people said that when someone hit the face of a Mandiran it would hurt the face of all of them, they were very close"(Mandira, 2013, p. 16).

impossibilities that the space offered. Thus, some of the community gave up of trying, believing that "Mandira would be something that would no longer exist" (Gondim 2018b, p. 26). Environmental restrictions, in the words of Mr. Chico, are the cause of this abjuration:

> (…) it was the environmental police, they started to chase everyone, I couldn't walk with a shotgun, I couldn't hunt, I couldn't, if I farmed anywhere here it would burn, because here to grow something you have to burn, the guy, the prick saw the smoke, because smoke goes up, he saw smoke and the guy was already there in the fields looking for the owner and annoying, he would want to fine, he would want to do everything, with heart of palm it was the same thing, the guy would cut down the heart of palm, I had to run, hide, deliver heart of palm at midnight, one o'clock in the morning, hunting was even worse, so that made people give up living in the place, and ended up giving up, how am I going to stay in a place, in the woods, if I can't live out of the woods? (Mandira 2013, p. 9, emphasis added).

We repeat the question raised by Mr. Chico: *how am I going to stay in a place, in the woods, if I can't live out of the woods?* This question resumes the importance of thinking about the production processes of the Mandira, always from the right/freedom fold, since the forest is not, here, just an external nature to the Mandiran individuals (not even an "extension" of its ontological dimension), but a topo-genetic condition of their existential territories.

The emergence of creating a way of working places them in the condition of being clandestine to the right/freedom. To ensure this fold, a new feeding is produced, namely: "after Marcos Campolim came with Renato Sales (from Nupaub/USP) to hold a meeting, so we started talking and he said he wanted to do an experiment with fattening. That is when we made the first attempt" (Maldonado 2005, p. 368).

Another interview offers us more elements that enable us, here, to support our proposal. We quote:

> The foundation at the time hired a technician, an oceanologist, and he had done a job in the South, and he told us and Fernando Cristino that he intended to work with oyster, do a job with the oyster that we worked with, and my uncle called me, said to me, oh uncle let's try it, let's see what this thing is then he said, I did a job in the South, with mussel guard, I took mussel from the stone, foot on the leg, put water on the rope to grow and such, and it was very good, it's a new experience that I learned, and I wanted to do it with the oyster to see if it works, then the people of community said no, we've worked with oysters for a long time, since the seventies and little until today, working with oysters, we already know how to deal with the animal, I said "No, I'm going to do this experiment.," I called a technician from the fishing institute, he saw the area for me here nearby, and we implanted the first oyster fattening nursery (Mandira 2013, p. 12).

In this scenario, both the devouring and the expression of the emergence/negotiation fold are explicit. The production processes of Mandira take place in a devouring that goes beyond any definition of self/other or difference/identity, as we assume the emergence/negotiation fold connected to the right/freedom fold. We believe that these elements already enable us to create a consistency plan that allows us to defend the strength of speaking about Mandira as the production of an anthropophagic identity that does not operate in a logic of self/other, difference/identity, as it

presents itself in a differential and relational way, for the defense of a right/freedom, when realizing that the right of the existential sphere itself is at risk.

In this way, Mandira, then, expresses itself as a contingency, given that its identity operates within the exercise of possibility as a right/freedom. In this way, an anthropophagic identity is not conceived in an essentialist and binary perspective, but dynamic and relational, since its operation is of the order of devouring.

3.6 Final Remarks

What is being built here is a conceptual dispositive, named *anthropophagic identity*, to be operated within processes of production of subjectivities in different groups, where there is a constant process of confrontation between already affirmed identities and what is understood to be outside them, which can also be considered a *state of war*. In the specific case of Mandira, this *state of war* is expressed in the exercise of the possibility as a right/freedom and also as an emergence/negotiation.

The notion of land in the production of this anthropophagic identity, present in the *becoming-land*, calls into question the perspective of territory in the nation-state, considering that, first, the sense of property is not established by the individuality, but by the collectivity. In addition, the appropriation of this land takes place, effectively, from its cultural, agricultural, ecological, economic, etc. practices, thus constituting a cosmogonic perspective that does not figure in the usury of the land, but by its self-making. Thus, the notion of land and territory for the Mandira community brings us a conception of space that is neither Cartesian nor Euclidean, as it is the constitution of life (its absolute immanence), but in a *state of resistance*.

It should be noted that if the notion of space is not based on a Cartesian or Euclidean notion, it is not only because it is not an extension but also because, for the Mandirans, the territory is not something to be measured, attributed as private property, destroyed, etc., that is, mathematicized. In summary, it is because it is the possibility of right/freedom. As Andrade said: "I asked a man what a Right was. He replied that it was the guarantee of exercising the possibility. This man was called Galli Mathias. I ate him" (Andrade 1928, p. 3).

When we think of an anthropophagic identity within the Mandira community, we support the idea that the territory is not an extension of the community, as it is, in its phatic exercise, the affirmation of an existential (cosmogonic) *topos*. The *geo-phagy* practiced by Mandira is what sustains affirming a genealogy that, first of all, is the land. It escapes essentiality by affirming an identity perspective—along with the transformations of the territory, the sun, the wind, the moon etc.—based on spatio-temporal dynamics and not in a state of "untouchable." On the contrary, it is the touch of devouring that affects its dynamisms.

After all this discussion, we ask ourselves *How can the concept of anthropophagic identity contribute to ethnomathematics?*. One first answer is that it may help the researcher in ethnomathematics to assume the dynamicity and multiplicity of the world, peoples, and group of people, when taking culture into account.

Ethnomathematics, in this way, has its political dimension highlighted, assuming itself not as a way of mathematicising or recognizing mathematical practices, but of creating ways to communicate with groups of people by taking into account their processes of producing identity. In this article, for instance, with the concept of anthropophagic identity, it was possible to understand the territory in a cosmogonic "sphere" within the Mandira's identity production process, linked to right/liberty, and not to property organization.

Finally, when we think of such concept of identity—as a *topos* that acts right/liberty—we glimpse the possibility of problematizing the concept of identity in ethnomathematics not only as an "identifier" of a certain cultural group but also as a political, ethical, and aesthetic tool that presents another way of doing genealogy. Moreover, its contingent and contemporary logic allows us to think about the constitution of an identity based not only on a "transfer of knowledge" from generation to generation (genealogical knowledge), but also on an emergency/negotiation that is in the order of the here and now.

In our view, this opens up possibilities for future researches regarding the concepts of identity and identification in Ethnomathematics, making it possible to problematize not only the methodological procedures that the area assumes in its productions with cultural groups, but also thinking of the ways in which the own groups constitute themselves and their identification processes.

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Chapter 4
African and Afro-Brazilian Roots for Mathematics Teaching: Decolonize the Curriculum

Cristiane Coppe de Oliveira and Ana Paula Santos

4.1 Introduction

When talking about the term culture in the academic environment, there are several perspectives in the field of human and social sciences. The discussion of the term culture in a school context, specifically, where this master's research project entitled: *Mathematics teaching, Ethnomathematics and African roots: a proposal for the continued training of teachers who teach mathematics*, is being developed, began in the field of Anthropology. The research appropriated the idea of culture from anthropologist Roque de Barros Laraia. According to Laraia (1999), "men of different cultures use various lenses and, therefore, have divergent views of things" (p. 69).

The definition presented by Laraia (1999) can become an essential element for education. With regard to the school context, culture, in general, is not interpreted as part of the student's daily life and life history. Culture is explored in school always from a minimalist point of view, leading again only to social representations of the artistic expressions of a people. This matter can lead many mathematics teachers with Eurocentric view of curriculum and with traditional teaching practices that focus only on content, for example, to state that there are no evident relations between Mathematics and Culture.

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Considering this reality, it is necessary to think of inserting the discussion of the relation between Mathematics and Culture in the continuing education of the mathematics teacher. Such proposal gains paths for it to become effective, by taking as theoretical framework the Ethnomathematics Program, which, according to D'Ambrosio (2011), is a research program with obvious pedagogical implications, expanding its educational dimension. For that matter, the intention of this chapter is to share experiences of a movement of continuing education of teachers within the school context, favoring the discussion on African and Afro-Brazilian roots with teachers of a public school from a city of São Paulo state in Brazil.

The district where the school is located belongs to the city outskirts, characterized by its surroundings, by terrain invasions (illegal occupations), and where the majority of students (approximately 75%) of this community are Afro-Brazilians. A survey conducted in the school with these students showed that approximately 90% of their parents or guardians have low schooling and work in subordinate jobs with low wages and have minimum per capita income. The cultural diversity of the students in this study became a phenomenon to be considered by school teachers to highlight African and Afro-Brazilian values in educational practices in order to strengthen this group's identity.

However, though Brazil enacted Law 10639/03 that makes mandatory the insertion of African and Afro-Brazilian cultures in curricula, this line was not studied/discussed in the initial education of teachers, and so the proposal of a master's degree research project emerges entitled *Mathematics teaching through African and Afro-Brazilian roots: an Ethnomathematics proposal for a de-colonized curriculum,* developed by the authors in the post-graduation education program of the University of São Paulo (USP), by considering the fact that the second author is a teacher at the school. The research project considers the following statement by Oliveira (2003) when thinking about the discussion of African roots to help in the decolonization of the curriculum.

> It is necessary to rethink Brazilian history from the African legacy. Without this, we would lose in depth and quality the knowledge about ourselves. Brazilianness, is, at large, derived from Africanity. Africanities re-design and redefine the national identity and, with it, the Brazilian political, economic and social project. Even considering that the academic and political discourse has excluded, for centuries, the African experience in Brazil, its influence has not ceased to play a fundamental role in the construction of this country (p. 18) (Our translation).

In the context of the present research and seeking to improve the mathematics teaching practice, it is considered that values of black students can contribute to a historical, social, political, and cultural learning of mathematics, showing to all students the relevance of all identities in the constitution of a society, corroborating Oliveira (2010) assertion that:

> (…) the teacher leaves the objective of teaching contents and offer to students the opportunity to act and reflect their daily life with a mathematic knowledge that is not stable, has history and is not a direct reflex of reality, but rather a knowledge built by man as an action tool and also a philosophy, with internal logics, for humankind to materialize its most human characteristic: to think (p. 5). (Our translation)

The project was designed to collectively and collaboratively build an educational proposal that will contemplate racial and social differences of students seeking to eradicate racism in the school context, expanding the work with non-Eurocentric mathematics, seeking to decolonize the curriculum.

4.2 Invisibility of African and Afro-Brazilian History and Culture

The current Brazilian government disregards the values of Brazilian cultural roots in official documents that present proposals for basic education. This action is shown to be a setback, considering that educators and various social movements (indigenous, black, landless and others) approved in previous governments, laws and documents that gave visibility to the contributions of these movements to the constitution of Brazilian society.

One can mention, for example, the National Curricular Guidelines for the Education on ethnic-racial relations, law 10639/03 that establishes the Black Consciousness Day (optional in each Brazilian state) and a system of racial quotas to enter Brazilian public universities as part of a set of affirmative actions.

It is worth mentioning that art and literature are inexhaustible sources of the creative, reflexive, and transforming capacity of social groups, from where the major part of the Brazilian population comes. These potentialities can transform sociocultural realities if they are highlighted in the reality of black students, valuing their identity, which has no representation in textbooks and narratives about the evolution of ideas.

The diasporic movement that occurred with African peoples in Brazil created "labels" of intellectual, moral, and religious inferiority, causing "historical wounds" that remain in the Brazilian society until today. Though some researchers, institutions, and social movements fight for a historic compensation (because of the existence of structural racism in Brazilian society) for the black population by means of affirmative actions, we can still witness the invisibility of black values, speeches, and contributions of Afro-Brazilian people in the construction of the nation.

The activism in favor of black visibility in Brazil should not be a necessary and constant movement, since, though Afro-Brazilian people can be considered in a minority frame, according to IBGE research publication pointed out by Afonso (2019) when the *Black Consciousness Day* is celebrated in Brazil, 56.10% of the people state to be black in Brazil. According to the author, among the 209.2 million inhabitants in the country, 19.2 million assume themselves as black, while 89.7 million state that they are "grayish-brown." Afro-Brazilian people—which IBGE conceives as the sum of black and "grayish-brown" people—represent, therefore, the majority of the population. The superiority in numbers, however, is not yet reflected in the Brazilian society.

Considering this reality and the actions of disregard to the Afro-Brazilian population by the wrongly considered majority of the Brazilian society (non-black and rich), the idea of retrieving the African and Afro-Brazilian memory, history, and culture in the school context practices is built, as justified by Munanga (2005) statement that:

> The rescue of the collective memory and history of the black community does not interest black ancestry students alone. It also interests students from other ethnic ancestries, chiefly the white one, because, while receiving an education poisoned by prejudices, they also had

> their psychic structures affected. Moreover, this memory does not belong only to Afro-Brazilian people. It belongs to all, considering that the culture from which we feed ourselves is fruit of all ethnic segments that, despite the unequal conditions where they are developed, contributed in their own ways to form the National economic and social wealth and identity (p. 16) (Our translation).

Thus, we can think of school movements with actions that may deconstruct old traditional teaching styles, with strictly Eurocentric orientation (eurocentrism), which reveal psychological and moral damages in black students in the teaching and learning process. As Bortoluci (2009) says:

> (…) One can think of Eurocentrism as a set of categories and images adaptable to changes in the organization of global power, but always expressed from a point of view of the European/Western center of this system. This hegemonic central view ontologizes the differences in relation to other (peripheral) societies, seeing them as incomplete forms of realization of a modern ideal (p. 26).

The invisibility of Afro-Brazilian people is found in studies on the modernity where Europe is considered as the center of the world, which Dussel (1993) calls Eurocentrism, triggering several colonizing movements. It is believed that educators should understand that the choice for an education on ethnic-racial relations may make possible a decolonized educational practice and for that it is necessary the creation of spaces for continuing education on such theme in the school space.

Taking as an example the fights of the Brazilian black movement over many years in favor of racial equality and democratization of power spaces in the society, the present work is intended to deconstruct, in the context and practice of the school involved in the project, the capitalist view, unequal and exclusionary with regard to black students.

Such movements, institutional or not, can assist to make educators (involved in a larger educational projects) help in the construction of the students' identity, excluding hegemonic discourses and actions in the field of culture, science, and religion.

The several forms of organization by and for Afro-Brazilian people in the history of colonization are understood as movements and strategies of survival, overcoming and identity strengthening. One of the spaces that are considered as cultural resistance are the large slums presents in several Brazilian capitals. Comparing them to quilombos, that became spaces of resistance, fight, and shelter of Afro-Brazilian people in the slavery period, favelas were settled as urban quilombos.

The superiorization of some to the detriment of others in the Brazilian society proves the need of educators to adopt practices in favor of an equal education, discussing and offering a dynamic to their students that will favor the understanding of differences and respect, emphasizing that Mathematics is constituted and understood in several ways, bringing a space of construction and deconstructions of the myth that it is inaccessible and for geniuses.

4.3 Circulation of Knowledge: Ethnomathematics, Education on Ethnic-Racial Relations and the Research Context

The presentation of the Ethnomathematics Program in its educational dimension to the teachers of the school where the research was being developed represented a great challenge. The circulation of academic-scientific knowledge in the school space is not yet a natural action, though we have different groups in Brazil working in a collaborative way with mathematic education, involving the university, manager-teachers, mobilizing knowledge in the school space, questioning educational practices and building more significant curricular proposals.

When this type of theme "escapes" the teacher-specific content, such as racism issues, black identity, affirmative action, quotas, and many other terms that circulate in the context of black movements and militancy in universities by means of Afro-Brazilian Studies centers, we can observe another difficulty to establish the bridge between theory and practice. In the first activities of the research project, occurred in the school in 2018, this difficulty to be based on the discourse of teachers involved in the proposed actions was observed.

One can notice that in most mathematics educational practices, there is a distance between cultural themes in classroom, exempting science from any relation to that or the establishment of new discourses for an intercultural education. Themes like diversity, cultural plurality, racism, and discrimination, which are part of the school routine, are not taken into account, creating a distance between teacher-student relation and mathematic learning.

In the context of research in Mathematics Education, according to Domite (2004), the theoretical-methodological option of ethnomathematics research is building a knowledge grounded on the ethnographic experience, the perception of the "other group," from its logics, seeking to understand it in its own rationality and terms. The author also states that

> (…) in general, in the ambit of the ethnomathematics research, the researcher experiences a process of strangeness and tension, since the quantitative/spatial relations perceived in the group investigated—as long as no longer exclusively centered in the explanations of the investigator society—prove to be, many times, for him, disjointed and, in general, the process of resignification and analysis asks for the creation of categories involving articulations between the Mathematics and other fields of knowledge like stories, myths, economy, among others. In fact, such relations don't need articulations in a non-disciplinary dimension of knowledge, but rather transdisciplinary (Domite 2004, p. 420) (Our translation).

This view that mathematics can't be understood by all was well discussed in the trainings and also that this label must be understood and deconstructed, strengthening a real mathematics, understood as part of its context, as humankind production, collective and individual, occurring every time we are handling it, without even noticing that. Our proposal intends to reach the educator and the student, where each one perceives himself as producer of liberating Mathematics, from Freire's (1967) perspective that:

> We could not understand, in a dynamically transitioning society, an education that would lead man to quiet positions rather than the one that led him to seek the common truth, "listening, asking, investigating". We could only understand an education that would make man an increasingly conscious of his transitivity, which should be used as much as possible critically, or with an increasing emphasis on rationality (p. 90) (our translation).

In this context, the research is being developed in a Basic Education School with multifunctional educators[1] that don't live in the school district. Almost 50% live in[1] the city and the others live in nearby cities, which means that the educators must be attentive to the students' reality, because little or almost nothing is known about the locality/reality of this school context.

There is a policy in the municipality of Taboão da Serra, in the state of São Paulo, Brazil, in which the school is inserted, that all school units need to develop an institutional project. In the school where the project is being developed, we opted for the Africa project, at the suggestion of the second author, involving all elementary school teachers I. Thirty-six teachers, considered as teachers in the early years, who are graduated in Pedagogy and Art, participated in the implementation of the institutional project Africa, considered as teachers in the classroom, who are graduated in Pedagogy and Art, and only the second author has a degree in Mathematics, but acts as a teacher of the early years.

The theme involving African and Afro-Brazilian values in 2018 was addressed both in institutional hours and in educational practices at the school. The group with 36 teachers discussed such theme in all subjects in a training movement conducted by the author, who is a mathematics teacher at the school.

The educators organized themselves by teaching series and discussed what would be the subject about Africa that would be developed with their students. The subjects raised presented themselves with great richness, involving African and Afro-Brazilian elements of the arts, musicality, the relevance of black personality, cuisine, and literature. The themes were selected based on the appropriation of specific contents (racism, identity, affirmative actions, and ethnomathematics, among others) by means of investigations involving chiefly academic literature.

The initial discussions were grounded on the Ethnomathematics Program dimensions, because mathematics teachers had limitations in the search for the relations between African and Afro-Brazilian culture and mathematics. This fact showed that the teachers' thought that African and Afro-Brazilian culture comprises only cultural movements of arts (dances, masks, and drawings, among others) and the slavery period of the Afro-Brazilian people.

The main references that discuss the ethnic-racial theme were presented in a collective and collaborative training movement. Studies, debates, planning, and execution of the actions were discussed and reflected by the group in 2018 after classes in the institutional educational meeting called Collective Pedagogic Work Hours (CPWH). The educational practices based on the theoretical framework were

[1] Educator who teaches Mathematics, Portuguese, Science, History, and Geography.

planned, and many times rethought, based on annotations on students associated to age group and social and cultural reality.

After the presentation of objectives and expectations of the project to the school management and pedagogic team, dialogues and debates on the relevance of the ethnic-racial theme occurred in order to think and collaboratively build a decolonizing curriculum that considered the school local reality.

During CPWH meetings, it was said by some teachers that the theme did not favor performing activities in all disciplines and it was difficult to work widely with the theme due to lack of subjects that it brought. Given the difficulty of some educators, a proposal was made for a more in-depth study on the subject and the researcher, bringing some proposals of themes for discussion and reflections of the feasibility or not of the proposal, dialoguing with theoretical contributions that address the subject Africa.

The intention was to demystify the relations that are established in Africa only with poverty, enslavement, religiosity, and other elements that were shown in a pejorative way in the teachers' discourse. Another important fact in the discussion was to always highlight the idea of Africa as a continent, with different values, myths, languages, and customs, depending on the country we will be studying.

After this conversation about the African continent and its values, the researcher interviewed the teachers about how they prepared their pedagogical material of mathematics. About 85% of the teachers said that they always used activities in the textbook used by the school or removed exercises from the Internet, 15% said that, from time to time, they worked activities together with the students in the classroom.

In the following CPWH meetings, the researcher conducted the discussion about the ethnomathematics Program using the theoretical references of her research project, with the objective of dialoguing with her peers about their perceptions about this program. It is worth mentioning an interesting fact in which some teachers, after four meetings, identified their pedagogical practices that valued the reality of the student, as being a dimension of the ethnomathematics Program: everyday practices. They cited proposals in which they considered students' actions as "buying bread for their family," "paying for a bus ticket," and "giving change in a grocery store," among other examples.

After the discussions held in the CPWH meetings about the theoretical assumptions of the Ethnomathematics Program, the teachers, led by the researcher, began the first dialogues about the pedagogical possibilities for practice in mathematics classes. Of the 24 teachers involved, we highlight three blocks of proposals: "the cookbook of African and Afro-Brazilian foods," "African games," and "African masks." The preparation of the proposals followed the same proposal, according to what the teachers decided at CPWH meetings, highlighting the following items:

1. The students, guided by the teachers, investigated the history of the theme.
2. The teacher proposed to the students the manufacture of the material with recyclable materials.
3. Dialogue on mathematical knowledge involved in the preparation of the material.

4. Presentation of the proposal by students in the classroom (Fig. 4.1).
5. Presentation of the material made on display for the internal and external community of the school.

The educators carried out the activities and conducted research to enrich their proposals and the development of activities, culminating in a pedagogical exhibition that was open to the entire school community. The pedagogical exhibition is a space in which all the activities carried out for the implementation, closure, and completion of the institutional project with the date chosen in the CPWH meetings are presented. Figure 4.1 shows the presentation of the proposal by students in the classroom.

It is also worth mentioning that in the first stage of each proposal, which proposed to investigate the subject, several countries were worked, such as Angola, Mali, and Cape Verde, among others. This contributes to strengthening the first dialogues with teachers, of which there are "many Africas." The proposals idealized and applied throughout the year 2018 and were presented in November in an institutional pedagogical exhibition.

This event is organized by the secretary of education of the city of Taboão da Serra in São Paulo. In November in Brazil, the National Day of Black Consciousness is celebrated on November 20, which is an affirmative action that composes Law 10639/03.

Fig. 4.1 Presentation of the proposal by students in the classroom (*Source* Personal file)

4.4 Conclusions

Some educators were not aware of Law 10639/03 and the need for its implementation. Were surprised by the proposal of development of educational practices that contemplated the existence of African and Afro-Brazilian people in the constitution/contribution to the Brazilian society. The movement to direct students' self-identification as black, praising their knowledge and culture, made the possibility of considering the curriculum (and who knows, even the *curriculum trivium*) as a liberating act, untying the chains of racism and social injustice. In the training meeting some possibilities emerged of a new look on the pedagogic discourse that can be worked with Afro-Brazilian students.

In a first analysis of discussions on Ethnomathematics it was observed that Ethnomathematics was considered as a potentiating element in the movement of approximation of mathematics contents and culture of the students and that the ludicity can be an instrument to facilitate and favor learning, involvement, and development of the student. When there is a discourse which one knows in-depth by the knowing/doing and that several actions are already showing effect within an educational space by means of a research project, it is possible to rethink the curriculum and educational practices, promoting decolonization and the dimensions of equality, alterity, and respect.

In this sense, it is believed that a possibility to think of a decolonizing curriculum can be based on the proposal of the of Ethics of Diversity (D'Ambrosio 2011), in which opens transdisciplinary possibilities while considering the ethics of diversity, with basic principles of *respect* to the other and all his differences, *solidarity* with the other in his way of meeting his needs of survival and transcendence and *cooperation* with the other in the preservation of the common natural and cultural legacy. The Ethnomathematics Program, in addition to the ethics of diversity, also proposes the trivium curriculum.

For D'Ambrosio (2014) the proposal of a curriculum should consider three aspects: *literacy* (reading, writing, speaking and listening skills), *matheracy* (assists students in the interpretation and analysis of signs and codes in order to propose models to find solutions to daily problems), and *technoracy* (students' capacity to use and combine different logic instruments that will help them solve problems). The curriculum based on these aspects, called by D'Ambrosio (2014) *curriculum trivium*, can be educationally effective, providing students with indispensable instruments for their transcendence and survival in their schooling path, and also making real the reduction of differences and laceration of human dignity.

Considering mathematics teaching through its social and cultural function may contribute to make students incorporate it with more property and understanding of its use in their reality, making significant associations of academic and personal learning. The *curriculum trivium* (*literacy, matheracy, and technoracy)* seeks to consider the different sociocultural resources, concerned with the being in his individuality and integrality, his function and actions in the context where he is inserted.

This proposal can create several possibilities of thinking mathematics, comprising capacity to construct and not just reproduce, multiple ways of thinking, that is, present the curriculum in a decolonized context, with several aspects of know-how, combined to the respect to the being as a whole in his capacities, understanding and experience of the world. About the proposal of decolonizing the curriculum, Nilma Lino Gomes (2012) states that:

> In a perspective of curriculum de-colonization and in the understanding of epistemological and cultural disruptions brought by the racial issue in the Brazilian education, I agree with the fact that this look is an important alert. The understanding of the ways through which the black culture, gender issues, youth, fights of social movement and popular groups are marginalized, addressed in a way disconnected to the wider social life and even discriminated in the school routine and curricular can be considered an advance and an epistemological disruption in the educational field (p. 104) (our translation).

Associated to the decolonization thought, it is worth highlighting the decolonization of the knowledge which many authors, such as Catherine Walsh (2013), have studied, providing reflections on the possibility of a decolonized pedagogy. The decolonization, in short, makes the real sense and actions of colonialism emerge, which contributed to the dehumanization and classification of human beings that would be considered inferior by the European hegemony, and makes one think in the theoretical criticism of colonialism.

In this perspective and based on practical political action to face, make feasible and transform institutions where the vision is to keep a hegemonic thought of permanence and sustentation of colonialism and its social practices. Starting from the principle of decolonization, it is expected that this project will expand the space for debates in the school context, contributing to establish an anti-racist educational practice, creating an education on ethnic-racial relations.

Such proposal may assist in the understanding of the historical path of education curriculum construction in Brazil where the Eurocentric orientation of knowledge prevails today. These statements that circulate in the historical, academic, cultural, and social construction of our country show the importance of offering a decolonized curriculum, presenting the trajectory of African and Afro-Brazilian persons who, in general, are excluded from the Brazilian history and curriculum.

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Part III
Indigenous Diversities

Brazil has a diversity of indigenous peoples and their practices, mathematical procedures, with a diversity of traditional knowledge articulated by members in these communities. In this regard, the vast majority of ethnic groups in Brazil organize and delimit their educational practices with regard to indigenous school education and indigenous education. In the Brazilian context, indigenous identities and traditions are developing possibilities for their recognition and valorization in both society and the school contexts.

Chapter 5
Ethnomathematics in the Brazilian Indigenous Context

José Roberto Linhares de Mattos and Sandra Maria Nascimento de Mattos

5.1 Introduction

In recent times, indigenous peoples have developed actions to protect culture, their rights, and territorial demarcation of indigenous lands. These aspects were relegated and subordinated by the oppressor when the Brazilian territory was invaded. Understanding himself as the holder of a culture that the indigenous needed to appropriate, the colonizer massacred some indigenous ethnicities and subdued others.

The imposition of a school education begins with the Jesuits in an attempt to enslave them and obtain workers for any kind of services. However, the so-called pacification or humanization of the natives was the basis for the struggles and revolts, the deaths of thousands of indigenous and the erasure of their ancestral origins, but they have resisted and resist to this day.

Although it was imposed, school education became a space for liberation and for the rescue of indigenous culture. Through indigenous school education, established by Brazilian law, indigenous peoples saw the possibility of claiming their rights by restoring traditional knowledge and practices, which contain appropriate knowledge for use in village schools. These laws include the Federal Constitution (Brasil 1988), the Law of Guidelines and Bases of National Education (Brasil 1996), and the National Curriculum Referential for Indigenous Schools (Brasil 2005).

Given the opening of this space in indigenous schools, ethnomathematics has established itself as a way to achieve the visibility desired by the indigenous themselves. The need to contextualize the school contents was remedied with the

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approach of the wise indigenous people, who with their stories and tasks pass the traditional knowledge to younger indigenous.

Accordingly, the research presented here are representations of indigenous culture, facilitating the empowerment of this knowledge and strengthening the identity of each ethnicity. Brazil is a country with immense cultural diversity and, at the same time, with intense biodiversity. Therefore, there is a vast potential of knowledge and practices that indigenous peoples keep for the preservation and protection of the forest, animals, and resources contained in nature.

Combining indigenous traditional knowledge with school content becomes the link between indigenous education and indigenous school education, with the intention of protecting the forest. It allows us to portray peoples who were almost decimated, but who continue to assure the potential existing in Brazilian forests. In short, all existing capacity in the resources of the Brazilian Amazon is safeguarded by indigenous peoples.

The research paths had a qualitative bias with a participatory approach. In this way, conversations and interviews were conducted with audio and video capture, immersion in the place of research and socializing with different ethnicities.

The results support the introduction of ethnomathematics as a way to give visibility to indigenous traditional knowledge, to act interdisciplinary, and to contextualize school contents through their experiences, practices, and customs. They point to the importance of indigenous teachers' formation, showing that only an indigenous teacher of the ethnicity can announce their culture. Finally, the daily activities reaffirm this traditional knowledge and protect indigenous identity.

5.2 Ethnomathematics Program

Ethnomathematics, which Ubiratan D'Ambrosio was one of the creators in the 1970s, in addition to expressing a relationship between mathematics and culture, has a decolonial stance of struggle, resistance, and reaffirmation of identity with regard to marginalized, discriminated, or oppressed groups such as the indigenous peoples of Brazil. Subsequently, ethnomathematics was treated by D'Ambrosio as a research program called Ethnomathematics Program.

This research program seeks to understand the strategies of mathematical nature, in their most varied manifestations, or the nature of knowledge more generally. It tries to understand the dynamics that guide the generation and diffusion of the *knowing* and *doing*. Through this program, we can give place and voice to the indigenous peoples who have been subdued and made invisible for a long time. Consequently, they need to be empowered and to perceive themselves as producers and diffusers of knowledge because their knowledge, which is part of culture, can be an essential factor for meaningful learning.

D'Ambrosio (2011) points out that "ethnomathematics fits into this reflection about decolonization and the search for the real possibilities of access for the subordinate, and the marginalized, and the excluded" (p. 42). In this perspective, we

affirm that the Ethnomathematics Program allows to restore the dignity of the members of the ethnicity and their own culture, relegated by those who dominated them. It is also a way of reestablishing the affective ties with the knowledge and practices of indigenous ethnic groups, by recognizing their roots without neglecting the others "but in a synthesis process reinforcing their own roots" in the exchange, dialogue, and respect by different cultures.

Decoloniality is understood here as a way of problematizing coloniality, reflecting on the socio-historical conditions of domination and oppression. "The decolonial denotes, then, a path of continuous struggle" (Walsh 2017, p. 25), i.e., insurgency ways that contribute to reflect theoretical and political paradigms from other points of view (Mattos 2020). In this perspective, the political dimension of the Ethnomathematics Program deals with this decolonial vision of strengthening the fight against the destruction of indigenous cultures, transcending the coloniality veiled by non-indigenous hegemony.

The various dimensions of the Ethnomathematics Program make it a comprehensive research program with holistic focus on knowledge. In the context of indigenous school education, it implies embracing respect for culture of the ethnicity and conferring dignity on indigenous peoples. The indigenous teachers work with interdisciplinarity on curriculum content in close relationship with indigenous culture. According to Mattos and Mattos (2019):

> In addition, the teacher uses interdisciplinarity to demonstrate that the existing boundaries between the disciplines were created and that they can be crossed naturally, using an indigenous school pedagogy in a dialectical and dialogical manner with the different areas of knowledge (p. 104).

Thus, the subjects are not worked in isolation by indigenous teachers, but with interdisciplinarity and based on elements of indigenous education. In all researched villages, teachers, students, and wise indigenous are participants in the educational process and the Ethnomathematics Program assists with a methodological approach in this dialectical and dialogical relationship.

5.3 Indigenous School Education in Brazil

At the time of Portuguese colonization, school education, practiced with the indigenous peoples of Brazil, had an imposing nature and the reason for this imposition was the mastery of all actions on them. The existence of this education goes back to the origin of colonization, with a domineering education implanted by the church through the Jesuits. The education of the Jesuits followed an oppressive model, characterized by the transmission of information to the indigenous, what mattered to the colonizer (invader). The indigenous culture was excluded and eliminated in this model of education. The function of the school was to make them citizens appropriate to the European model, causing them to abandon their beliefs, their language, and thus cease to be "Indians."

For about four centuries, this education practically remained in this model. Subsequently, it went through several phases until it reached the current model. The first great victory was achieved with the promulgation of the Constitution of the Federative Republic of Brazil in 1988 (Brasil 1988). Article 210 ensures that indigenous communities maintain their culture, language, customs, and ideals and use their own learning process. Although assure of the implementation of an appropriate School Curriculum to its nature, this is not what effectively it happens. The indigenous peoples are still submitted to curriculum and methods of non-indigenous schools.

Backed by the Federal Constitution, in 1996, the Law of Guidelines and Bases of National Education (Brasil 1996) ensured that indigenous school education was intercultural and bilingual, preserving the identities, customs, and traditions of the ethnicities. Currently, the proposal presented in the National Curriculum Referential for the Indigenous Schools (Brasil 2005) is that indigenous school education, besides being bilingual, must be intercultural, respecting and valuing the culture of ethnicity, deconstructing the idea of "inferior culture."

The United Nations Declaration on the Rights of Indigenous peoples (UN 2007), in its article 14, says that "Indigenous peoples have the right to establish and control their educational systems and institutions providing education in their own languages, in a manner appropriate to their cultural methods of teaching and learning" (p. 7). Therefore, it is necessary to provide conditions for indigenous peoples to manage their school education autonomously, with quality. For this, actions need to be implemented in their favor, such as the indigenous teachers' formation and the elaboration of specific didactic-pedagogical materials.

Indigenous school education, consisting of the simple establishment of the school in a village, is nothing more than what we call "school education for the indigenous," but not necessarily a true indigenous school education. Today, indigenous peoples know that a true indigenous school education must be bilingual, specific, differentiated, intercultural, and exercised by indigenous teachers of the ethnicity. Indigenous education, which in our view is one that always was carried out in the village, without the need for school and systematization of educational processes contributes to indigenous school education.

Thus, the Ethnomathematics Program, with its historical-cultural approach, methodologically assists in the educational processes in indigenous school education. To research, through the Ethnomathematics Program, the knowledge of an indigenous ethnicity, not only provides value to their culture and tradition, but also brings cultural knowledge closer to school knowledge, aiming at a resignification of curricular contents. In this perspective, anchoring school mathematics contents through cultural elements in indigenous school education leads students to meaningful learning in the sense of Ausubel (2000).

5.4 Traditional Knowledge: Intellectual Property of Indigenous Peoples

Today, much is said about traditional communities, and understanding of this concept is important. In its constitution, Brazilian society is composed of culturally different groups. At the time of the Portuguese invasion, there were already inhabitants of the land—the indigenous peoples. Later, several other peoples entered Brazil and constituted the Brazilian population, initially a mix of Indigenous, enslaved Africans, and Europeans. With this understanding, traditional communities or peoples are considered to be the origin of this constitution, e.g., indigenous peoples, *quilombolas*[1], and other peoples who, over time, have become vulnerable, such as riverine people, peasants, fishermen, rubber tappers, and *caiçaras*[2].

Brandão (2012) considers traditional community as a local group linked to a collective territory, modified through the work that provides the development of a peculiar wisdom full of knowledge that is transmitted from generation to generation. Its members are autonomous; maintain recognition of their identity, with experiences marked in their memories of struggles and resistances, both past and present, resisting and enabling environmental conservation and biodiversity.

Brandão (2015) brings the idea of the constitution of communities. According to the author, there is the primitive community, native, that was confronted with the colonizer; the typical traditional community that encompasses the poor, dispossessed, resilient, and borderline; the exclusion community, composed by homeless and poor immigrants; and the adherence community, formed by groups of people who are on the fringes and who, by choice, constituted a unit of social action. He notes that in any of these types, the community constitutes the human place. All are represented in the constitution of Brazilian society.

All develop their own know and doing. All of them generate and disseminate traditional knowledge, especially the indigenous peoples. In Brazil, traditional indigenous peoples and communities are most prominent with the 1992 Convention on Biological Diversity—CBD, held in Rio de Janeiro. In Article 8 (j) it is stated that:

> In accordance with their national law, to respect, preserve and maintain the knowledge, innovations and practices of local communities and indigenous peoples with traditional lifestyles relevant to the conservation and sustainable use of biological diversity and encourage their wider application, with approval and participation of holders of this knowledge, innovations and practices; and encourage the equitable sharing of the benefits derived from the use of this knowledge, innovations and practices (Brasil 2000, p. 12).

As a result, it is the search to give visibility to the ancestral knowledge of the Brazilian indigenous peoples. However, it is necessary to safeguard the intellectual property of such knowledge. According to Eloy et al. (2014), traditional knowledge

[1] Black people who are descendants of inhabitants of *quilombos* (places that housed runaway slaves).

[2] Inhabitants of the Brazilian coast that emerged from the sixteenth century, originating from the mix of white men and indigenous.

is the heritage of ancestral peoples. It has peculiarities due to the approximation of these peoples with the nature and local biodiversity. It is empirical, cumulative, and socially constructed, being passed down from generation to generation. Therefore, it is the intellectual property of indigenous peoples.

In 2007 this visibility increases, with the enactment of Decree 6040 establishing the National Policy for Sustainable Development of Traditional Peoples and Communities (Brasil 2007). This, in conjunction with the Commission for the Sustainable Development of Traditional Peoples and Communities, enables to work with these communities to include recent advances in agriculture and healthy eating while preserving indigenous peoples' knowledge.

The concept of traditional knowledge has advanced over time and strengthened indigenous peoples through culture, safeguarding traditional knowledge and practices. In this context, Udry and Eidt (2015) state that traditional knowledge indicates the importance of cultural evolution of human beings in their relationship with the environment. From this perspective, traditional knowledge manifests itself in the daily activities of the village through art; dance; music; instruments for environmental management and biodiversity preservation; of indigenous myths and rituals in which they demonstrate their beliefs and customs.

5.5 The Political Dimension of Ethnomathematics in Indigenous Empowerment and Resistance Through Decolonial Practices

The history of the Brazilian people begins with practices of domination. First, the original peoples were massacred, decimated, and subjugated to lose their identities and accept the domains of the colonizer (invader). With this, its historicity was erased, relegated, and depreciated. Later, the dehumanization process encompassed all those underprivileged, poor, insurgent from various communities who were there. To accept these moments is to be sure that decolonial practices are turning in the pursuit of humanization and the real construction of Brazilian historicity.

This domination process occurred in Brazilian education that, starting from a system of erasure of culture, implemented by the Jesuits, fell into the meshes of education systems coming from Europe or the United States of America (USA). This process of imposition makes the adequacy of practices really focused on the Brazilian people unfeasible. As a result, students are imprisoned with a curriculum that represents nothing, nor does it strengthen the Brazilian identity and, much less, empower existing cultures.

The curriculum, in any education system and in its various presentations, should strive for the knowledge generated and disseminated by humanity throughout its development process and civilization. Therefore, all cultures must be privileged. In Brazil, the colonizing practices inflicted the coloniality of knowledge, based on the imposition of the oppressor's culture in relation to others. Thus, knowledge is seen

as unique, reinforcing the coloniality of knowledge, weakening identity, and causing the loss of traditional knowledge originating from peoples and sociocultural groups.

There is also the coloniality of being, which depreciates, subordinates, and causes the erasure of the existence of these traditional groups and peoples. Thus, the occurrence of the coloniality of *Mother Nature* in which the perspectives of their spiritual and ancestral world are taken away, determining the absence of their myths and rituals. According to Walsh (2008) all this domination requires a transcendence of these colonialities, a moment to promote decolonial practices.

From this perspective, decoloniality is taken as insurgent practices to critically reflect on coloniality and to provide ways of resisting and existing, promoting a break in the established order. Thus, it is understood that it is not possible to undo what has been done, but it is possible to make a new path. This new course involves fighting against what is set in the capitalist world, having as weapons social justice, equity, and ethics.

Teachers and students are responsible for pronouncing the world (Freire 2019). By pronouncing it, they subvert it in acts of creative insubordination (D'Ambrosio and Lopes 2015), unveiling cultures of the invisible peoples. The struggle of some teachers reverts in a loving act towards education and towards students. This struggle, centered in the culture, brings the ways of life of indigenous peoples, the ways of managing the environment and the internal organization of the traditional community, especially regarding the political, symbolic and spiritual aspects of the ethnic identity.

The political dimension of ethnomathematics also demonstrates this domination of the colonizer who depreciated historicity and weakened the roots of indigenous peoples. However, by means of decolonial practices, it aims at the dynamics of the cultural encounter, in which two or more cultures exchange knowledge in a dialogical relationship. Invisible peoples help each other, favoring the strengthening of insurgent practices of resistance. These peoples, through actions and practices political, educational, and collective, reaffirm their existence as an act of struggle against coloniality, be it of power, of knowledge, of being, or of mother nature.

From this perspective, traditional knowledge is used in the classroom to contextualize school knowledge and, at the same time, to develop it with interdisciplinarity, since traditional knowledge does not fragment knowledge. It is built interconnected to different areas of knowledge in a natural way. Consequently, the indigenous teaching practice has a specificity that is to know and to make known the own culture and its ancestral roots to the young of the ethnicity, empowering them.

Through methodological approaches, ethnomathematics makes a connection between indigenous teaching practice and the surrounding world, since it aggregates the culture, considering traditional knowledge and the implicit values in indigenous cosmogony. In this way, it makes it possible to give voice to the indigenous and situate them territorially. Santos (2006) shows that there is a diversity of cultures which produce important knowledge to make known to the other cultures. The author also brings the concept of ecology of knowledge as a useful aspect to give voice to the invisible people. Consequently, the ecology of knowledge is a collective

process, carried out within communities, for the generation and diffusion of knowledge to reinforce the collective struggle for social, political, and cultural emancipation.

5.6 Research with Indigenous Peoples of the Brazilian Amazon in the North and Center-West

In Brazil there are 305 indigenous ethnic groups who speak 274 languages. According to the Socio-Environmental Institute, in Brazil there are 723 indigenous lands of which 486 are homologated. These indigenous lands represent about 13% of the Brazilian territory and most of them are in the North and Center-West of the country.

We will bring here some research carried out in Rondônia, Amapá, Roraima, Amazonas, and Mato Grosso states. We will focus on culture, sustainability, and environmental preservation related to indigenous education in indigenous school education context.

5.6.1 Indigenous Ethno-Architecture Helping in Teaching Practices

The relationship of the indigenous with the territory is directly related to the sociocultural coexistence of each village, where they keep alive their ancestors, their knowledge, their doings, their myths, and their rituals. Territoriality is linked to ethnicity and has spiritual, symbolic, and religious characteristics, at the same time composing the collective identity.

Ethno-architecture is one of the elements of cultural expression regarding the link with the surrounding environment, i.e., with the territory. The constructive structures, which constitute indigenous ethno-architecture, are directly related to the physical space, the social space, and the maintenance of life. Their houses are places where they live and raise children, so they must be safe and well built.

Macuxi and *Wapichana* ethnic groups from Roraima state built their traditional houses with a rectangular base and four-water roof. They are simple constructions, consisting only one room, without internal division and completely covered by *buriti*[3] leaves. Size varies greatly as it depends on how many people make up the family.

The *Paiter Suruí* ethnic group from Rondônia has a more elaborate form for the construction of a traditional house. The columns are made of tree trunks not perpendicular to the ground, i.e., inclined to triangulate. These trunks that are placed

[3] Tree of the Amazon rainforest.

forming triangles are tied to other trunks at the top of the house. The house is completely covered by straw. The area within the house is determined by the number of hammocks that will be placed there. These hammocks are arranged in the width and length of the house. Therefore, they are a way for these indigenous to calculate the area inside the house.

Another indigenous ethnicity that uses triangulation in the construction of the traditional house is *Wajãpi* from Amapá state. The *Wajãpi* house is a bit more elaborate than the previous ones. The tree trunks used are tied with lianas (flexible climbing plants) and the cover is made with straw. The *jura*, as the Wajãpi house is called, is built on two floors. The upper part is the bedroom and the lower part is a living space with multiple uses.

Making original constructions is important to safeguard a traditional knowledge and ensure that young people acquire it. However, a teaching methodology is used for the elaboration of an architectural model of these constructions. Initially, a research is conducted with the elderly wise indigenous about what these houses looked like, and then opting for one to develop the model.

For indigenous people, regardless of ethnicity, older people are considered a source of knowledge, so they are designated as wise. They are those who transmit orally or by doing something allows the youngest to learn their knowledge. Therefore, respect for the elderly is an integral part of indigenous culture.

Based on this knowledge with a focus on indigenous school education, interdisciplinarity and contextualization are aspects that ethnomathematics excels, besides presenting the culture of each ethnicity through its traditional knowledge. In the development of the architectural model, indigenous students learn school mathematical concepts, such as shapes and measurements, using Euclidean geometry content.

They learn biology concepts about the plants used and how they are preserved. In geography they learn spatial localization as a means of finding the materials needed for construction and locating themselves. In history they learn about the origin of ethnicity, its myths and rituals, stories told by the elderly wise indigenous.

5.6.2 Ethnobotany and Use of Medicinal Plants

Brazil is a rich country in its biodiversity, with a great potential of medicinal plants and herbs that can be used for physical and mental health. The interest of indigenous for medicinal plants is not of today, and their use is a resource used by them in their daily lives to heal the body and soul. The use of medicinal plants and herbs is part of the indigenous cosmogony.

From the Portuguese invasion until today there is the exploitation of this knowledge by pharmaceutical, cosmetic, and phytotherapic interests. The exploration of Brazilian nature by Europeans and later by the USA expropriates a traditional knowledge that belongs to indigenous peoples. The indigenous themselves recognize the need to safeguard such knowledge for their own benefit.

Recognition by indigenous peoples as part of nature allows them to withdraw resources and use nature sustainably. As part of nature, they have an obligation to preserve it for their own survival. Taken by this feeling, the use of ethnobotany knowledge is seen as their relationship to nature.

Ethnobotany is a tool that allows the understanding, regarding the culture of indigenous peoples, about the use of medicinal plants and herbs to cure illnesses known to them long before the Portuguese invasion. Indigenous ethnobotany has symbolic, magical, and spiritual character.

The creation of the Medicinal Plant Center, called *Olawatawa*, in *Paiter Suruí* Indigenous Lands in Rondônia is an attempt to preserve medicinal plants and herbs, as well as the knowledge derived from their culture. The center was developed by indigenous *Narayamat Surui*, a nursing technician, with the purpose of rescuing the custom on the use of medicinal plants and herbs for the treatment and cure of some diseases.

The *Olawatawa* Center has resources from the non-governmental organization called Forest Trends, which supports initiatives aimed at local development and conservation of the Amazon rainforest, valuing indigenous traditional knowledge of production. The creation of the *Olawatawa* gives importance to the planting and cataloging in the mother tongue of the native plant species and medicinal herbs of the forest, recognized and extracted by the village's wise elderly. There is also a concern in placing information plates indicating the name of the herb and its use.

The indigenous is not concerned with maintaining rigid spaces in the demarcation of the place for production of medicinal plants and herbs, since it is an open nursery. In this way, it develops on a trail, along the forest, and closer to the village. Using trail is a sustainable development. First, because there is no need to get other plant species out of the way. Second, because it has a natural relationship with the forest.

The *Paiter Surui* understands that all medicinal plants and herbs have spirit. To them the ancestors of plants were people. In this perspective, the plant must be green when removing part of it to make a medicine. When the plant is green it has healing power, but when dried it loses this function. Only bark, leaves, and some roots are used.

The indigenous cosmogony is full of narratives about natural and supernatural beings. They are characters who participated in the construction of indigenous culture, which brings respect to life, the forest with its various natural and supernatural beings and to God. Similarly, to maintain the health of the body, the *Pajés*[4] performed rituals, which summoned the spirits contained in the plants.

In this context, this knowledge is used, in an interdisciplinary way, by *Paiter* indigenous teachers in their teaching practices, in indigenous school education. For example, in an ethnomathematical approach, the indigenous teacher teaches reason and proportion, based on the culture of making teas, ointments, and preparations. The mother tongue teacher appropriates the names of plants and their properties to

[4] Xamãs or Shamans.

reinforce the language. In science, the indigenous teacher teaches types of plants, herbs, and their characteristics. In geography, he works the relationship with space and the location of plants. The use of indigenous cosmogony stories brings reports that explain the creation of the world and, at the same time, of human beings, aspects explored in history classes.

5.6.3 Ethno-Knowledge and Sustainable Use of Forest Resources

According to data from the Brazilian Institute of Geography and Statistics, the Brazilian Amazon is considered the largest biome and has the largest biodiversity in the world. It occupies about half of the Brazilian territory and is mainly concentrated in the North and Center-West of Brazil. In the Amazon rainforest there is a wide variety of plants and trees of different species, sizes, and shapes; it houses small- and medium-sized animals and acts as a reactor for the planet's environmental balance.

Indigenous peoples derive from the forest the resources necessary for their livelihoods. Food collection is a traditional practice and many of them are making it an economic activity. Fruits (or their derivatives) such as babassu, Brazil nut, *açaí*, *tucumã*, and *pupunha* are some commercially exploited examples.

The Brazil nut is considered viable for sustainability, as there is no need to fell the tree. The indigenous catch the fruit that fall to the ground. This practice is performed by many indigenous ethnic groups, mainly by the *Paiter Suruí* from Rondônia and Mato Grosso and by the *Cinta Larga* from Rondônia.

The Rondônia state is the fourth largest producer of Brazil nut and extraction is concentrated in indigenous lands. Traditional extraction is performed, and the collection is made by the whole community, divided by families. The Brazil nut is traditionally consumed, i.e., for thousands of years indigenous communities have been using the nut for food, extracting milk, preparing flour with meat, or grated with honey.

Currently, they reap as much as possible to negotiate. This commercialization started in the middle of the twentieth century and has been increasing significantly due to its nutritional, gastronomic, and cosmetic properties. Proper management of this resource is necessary to ensure quality and sustainability. Thus, enable the exploitation for an indefinite period by indigenous peoples.

Another resource explored by the *Paiter Suruí* from Rondônia is babassu. They use all the babassu. They use babassu straw to cover the houses; babassu mesocarp is used in flour manufacturing; the endocarp, the sturdiest part of babassu, is used in handicrafts; babassu oil is produced with the almonds and the husks are used as firewood to make charcoal.

Babassu is a resource of great interest to indigenous and non-indigenous people, as it is a product that serves culinary, pharmaceutical, and cosmetic interests.

Babassu flour is highly medicinal in nature as it has anti-inflammatory, immunomodulatory, analgesic, and antipyretic properties. The oil is edible but is widely used in soap and cosmetics manufacturing.

Regarding indigenous school education, these activities are used based on the time of collecting each fruit. There is always talk about the fruit production, collection, and marketing. Indigenous teachers work on financial mathematics and statistical information by presenting graphs and data for each product, its production, and trade.

5.6.4 Ethnoecology and Reforestation Practices for Preserving Local Biodiversity

Local ecological knowledge or indigenous ecological knowledge represents indigenous knowledge regarding the environment, which involves the sustainability and preservation of the forest. This knowledge is related to the territory, the collective place of conviviality and the manifestation of the rituals. It allows indigenous peoples to understand where plant species have been taken away and even extinct.

Ethnoecology aims to understand indigenous local ecological knowledge through ways of adapting and modifying the environment as a result of the evolutionary process that encompasses culture and nature. The harmony of this process instigates investigations of these adaptive forms and the type of contact with the forest, the way they establish the welfare of the indigenous collectivity.

Currently, some non-indigenous are interested in indigenous ethnoecology with a focus on conservation and preservation of standing forest and the use of sustainable biodiversity. Indigenous people, for their part, strive for local harmony and focus on the reforestation of degraded areas by those who somehow tried to expropriate the forest from their resources. If the forest does not resist non-indigenous attacks, they will also perish.

The *Zoró* indigenous ethnicity, from Northwestern of Mato Grosso state, is developing a reforestation project with the support of the *Kanindé* Association of Ethno-Environmental Defense in partnership with *Zoró* Indigenous People's Association (APIZ).[5] This project has resources from the Amazon Fund and the National Bank for Economic and Social Development.

The reforestation project has a seedling nursery of native species from the Amazon region. These seedlings are replanted by the *Zoró* in areas devastated by burning or illegal extraction of woody that occurred in the forest belonging to indigenous lands.

The indigenous ethnic group has a school in village called the *Zawã Karej Pangyjej* Indigenous State School, which was planned to be intercultural and in pedagogy of alternation. The school uses on its curriculum the traditional

[5] APIZ is abbreviation in Portuguese for *Associação do Povo Indígena Zoró*.

knowledge of *Zoró* people and therefore the project is included in its actions. From this perspective, teachers, students, and other community members actively participate in the project.

The *Paiter Suruí* ethnic group from Rondônia started, in 2019, a reforestation project. The seedling nursery is located at the *Olawatawa* Medicinal Plant Center. It is a sustainable project aimed at planting fruit trees and hardwood.

The expectation is to ensure the preservation of some almost extinct trees and at the same time provide reforestation of the area around the center, then to continue reforestation in areas degraded by illegal actions carried out within indigenous lands.

In addition, it is a work of raising awareness, for both younger indigenous and non-indigenous people, of the importance of securing the standing forest. In this ethnicity, the project gains strength with the participation of the school, called Izidoro de Souza Meireles Indigenous State School, in which the director *Gamalonô Surui* encourages several projects that involve the culture of ethnicity.

In both projects, teachers and students, as well as other members of the community, get involved and seek to find out which plants in the region are almost extinct, which fruit or not plants should be grown in the nurseries, their importance, and uses. It is an interdisciplinary work, contextualized in practice, with the participation of all.

This type of teaching that utilizes students' traditional knowledge leads to the meaningful learning by David Ausubel (2000). For this author, the new knowledge acquisitions are based on pre-existing knowledge, on the inherent experiences of culture, which are relevant, on the student's cognitive structure. Thus, using a forest preservation project as a pedagogical action in indigenous school education strengthens and empowers the culture.

5.6.5 *The Introduction of New Agricultural Techniques to Produce Clonal Coffee*

The food of indigenous peoples is based on some animal foods, some roots, and fruits. Generally, indigenous peoples feed on birds and other small- and medium-sized animals and some types of fish. Among the foods of plant origin preferred by the indigenous are cassava, *cará*,[6] sweet potatoes, peanuts, and corn.

Not all ethnicities use the same food products. However, many fruits native to the region are included in the diet, such as Brazil nuts, babassu, coconut, *pupunha*, *inga*, and banana, among others. The seasoning used in the preparation of food is basically salt and pepper. The fermented beverage used in festivities is usually prepared with cassava, *cará*, and *pupunha*.

From contact with non-indigenous people, the indigenous ethnicities began to use products from outside the forest. Some technological resources are now being

[6] Kind of yam.

introduced to protect the forest itself, such as Google Earth, which monitors deforestation in the indigenous land of the *Paiter Suruí* ethnicity. In addition to clothing, indigenous peoples added other non-indigenous products, such as appliances, cars, motorcycles, computers, and cell phones.

In the diet there was some adequacy, as they continue hunting, fishing, and planting, but were introduced processed foods such as cookies and other cereals such as rice. For drinks, soft drinks and coffee were introduced. This does not mean that they are losing their culture, but they are adapting to new demands and needs after contact. Culture is dynamic; therefore, it undergoes transformations that the approximation with the urban environment provides.

Indigenous agricultural practice includes *coivara*[7] for cleaning and land preparation. The place (called *roça*) is developed in the middle of the forest, in area demarcated by the leader, who ensures the protection of the forest. The Brazilian Agricultural Research Corporation (Embrapa[8]) is always helping indigenous peoples to develop new plantations. Embrapa expects that indigenous peoples want to develop agricultural projects in their territories.

The indigenous people *Paiter Suruí* from Rondônia have agreed to participate in the clonal coffee project. The seedlings of clonal coffee were developed by Embrapa with its own characteristics to adapt to the region's climate and soil. Clonal coffee is a unique variety, i.e., it is a genetically improved type of coffee, maintaining the genetic characteristics of the matrix.

This coffee ensures the homogeneity of the crop, developing larger beans and providing uniformity in their maturation. Consequently, there is greater production and improved quality of coffee. In the village *Paiter*, Line 09, the production of clonal coffee seedlings has the guidance and supervision of *cacique*[9] Rafael *Mopimop Surui*. The harvested and kiln-dried coffee is of high quality.

In the surroundings of the clonal coffee plantation, traditional plantations such as corn, pineapple, cassava, and the *cará* are maintained. Thus, they maintain the sustainability of local biodiversity, produce subsistence food for families, and the soil is not depleted, due to the diversity of crops produced.

The practice of taking the daily life of the indigenous community into the classroom is also present in this project. Indigenous do not separate what happens outside of school from what happens within it. Almost all daily activities of ethnicity are collective and involve the entire community. Some mathematical concepts are worked out, such as the use of scale to develop a sketch of the clonal coffee area and its surroundings with the placement of different plants.

[7] An indigenous agricultural burning technique to fertilize the land.

[8] Abbreviation in Portuguese for Empresa Brasileira de Pesquisa Agropecuária.

[9] Indigenous chief.

5.6.6 *The Fishing Activity of Beating* Timbó

Indigenous food is considered healthy. Most of the resources used to prepare food is taken directly from the forest or planted by them or fished in the rivers or hunted. Foods of plant origin are free of pesticides, preservatives, and dyes, harmful to human health. It is a diet rich in minerals, vitamins, and other nutrients.

The school lunch is maintained by introducing foods such as rice, beans, spaghetti, and some types of seasonings such as onions and garlic. However, the biggest challenge is to incorporate eating practices of the indigenous ethnicity into the school lunch, by the indigenous, ensuring that traditional food knowledge is safeguarded.

The introduction of processed foods has harmful implications for indigenous health. The government makes certain types of processed foods available for school meals. Some are not part of the food of indigenous, causing some illness such as diabetes, which is an illness reported by some indigenous people. Therefore, it is urgent to resume traditional eating in indigenous schools.

Fish is a food enjoyed by the indigenous, but if not preserved it may become extinct. In *Paiter Suruí* ethnic group from Rondônia, the traditional techniques for fishing are the bow and arrow and beating *timbó*[10] in times of drought. Both men and women perform the fishing with *timbó*. After contact was made with non-indigenous people, fishing with hooks and nets was introduced. The fishing zone, *Morip ey Pãyah*, is described as the protected area along the rivers and streams where the *Paiter Suruí* people fish.

It is not only *Paiter Suruí* ethnicity that uses the *timbó* for fishing. Other indigenous ethnicities also use it, such as *Wajãpi* from Amapá, *Ticunas* from Amazon state, and *Zoró* from Mato Grosso, among others. It is a traditional practice performed by everyone in the village to catch the fish with the hands, as they are stunned. Women make food in a place prepared by men, but they also participate in the fishery. Some of them stay in the nets, others catch the fish.

To carry out the fishing of *timbó* it is necessary to have the knowledge of wise indigenous from choosing the vine to the fishing itself. *Timbó* is a climbing plant from North of Brazil, which contains a substance that can prevent the breathing of fish by choking or stunning them. The *timbó* is cut into pieces which are tied and placed in water, where they will be struck by a piece of wood to release the substance.

It is a technique performed by *Paiter Suruí* in season of low water, when there is greater dilution of the substance and less dispersion in water. This custom is passed from generation to generation as a fishing technique performed only in rivers and streams.

Paiter Suruí Indigenous recognize that the preservation of existing water resources in the region is of paramount importance for the reproduction of fish and, consequently, to ensure food for future generations. This fishery is conducted only

[10] A toxic vine that reduces water oxygenation by causing fish to come to the surface and be caught by hand.

in low water seasons to ensure spawning of fish in the reproductive nests that occur during the flood season.

Some indigenous are positioned in appropriate places to catch the stunned fish trying to escape. One strategy used by indigenous is to place nets at two ends of the river or stream. The choice of where the nets will be placed depends on the amount of water in the place and the concentration of the substance in it.

As the *timbó* fishing is carried out collectively, at the end of each day the fishes caught and kept in sacks are distributed by the oldest to the participating members, separated by family. Some families, after *timbó* fishing, prepare a food made with fish, called *caldeirada*[11]. It is time for socializing and to thank the fishing done.

This type of fishing involves various school subjects, such as history, geography, arts, mathematics, biology, culture of the people, and mother tongue. All school subjects are worked from the preparation of the fishery to the end. It is a collective activity that, according to the indigenous, is important to safeguard this knowledge and show young people the need to be carried out at appropriate season.

5.7 Conclusion

In all research conducted, the focus is on indigenous traditional knowledge used daily and passed on by culture. The indigenous people who inhabit the Brazilian Amazon, in the North and Center-West, have different cultures full of stories and myths that personify animals and plants and rituals that evoke spirits. These symbolic representations encompass the indigenous cosmogony of the ethnicity.

In conversations with us, the indigenous always resort to their stories, be it of creation, personification of some animals, rituals of passage and festivities that surround them. The forest is present in all these manifestations. Forest and biodiversity preservation strategies are developed naturally in practices, from which elderly indigenous transmit orally or when produce some artifacts. These are teaching and learning practices contained in indigenous education.

Belonging to the territory is inherent to indigenous peoples, leading them to constant struggles to ensure and protect it. The history of indigenous peoples is marked by the need to reaffirm their identity and safeguard their culture. Despite the colonization and extinction of some ethnicities, indigenous peoples keep the cultural legacy alive, an aspect of pride for the community.

It is important point out that the colonization was not a natural process and there was an intense massacre of many indigenous ethnicities. Those who have succeeded in escaping oppression have, through wise elderly, been able to restore much of traditional knowledge and practices. Among the identity forms is the valorization of the indigenous traditional knowledge, mainly allied to the indigenous school education.

[11] Fish soup.

Another great indigenous struggle was the validation of their rights to the possession of their lands; the implementation of an indigenous school education that prioritizes each ethnicity and the government's protection against illegal attacks on indigenous territories. Thus, there is a need to create strategies that overcome the invisibility of Brazilian indigenous ethnicities.

In accordance with Brazilian law, among them the federal constitution of 1988 (Brasil 1988), indigenous peoples are guaranteed the protection of the original cultural manifestations, as well as the defense of their rights and interests. Regarding indigenous school education, the constitution ensures the use of the mother tongue and its own learning processes. In line with this guarantee, indigenous peoples use daily activities to contextualize school content.

There is also in the federal constitution (Brasil 1988), in its article 231, the recognition to indigenous peoples of their social organization, customs, languages, beliefs and traditions, and the original rights over the lands traditionally occupied by them. Therefore, it is up to the indigenous to exclusively use the riches of the land, rivers and lakes.

A step back in time reveals that the school education imposed on the indigenous by the Jesuits was a way of turning them into submissive workers. The curriculum implemented in schools in indigenous territories was based on catechesis and the integration of indigenous peoples into the ways of life of the surrounding society, which entered the villages. The school was an instrument of imposition of the colonizer's culture, allowing the negation of the mother tongue and the ethnic culture. Despite this, the indigenous peoples emerged against the oppressor, reaffirming their identity.

At the end of the twentieth century the Law of Guidelines and Bases of National Education was promulgated, imposing a series of modifications with respect to the Brazilian education. In assistance to indigenous communities, article 78 (Brasil 1996) states that indigenous school education must be offered in a bilingual and intercultural manner.

Moreover, in item I (Brasil 1996) of the same article, the recovery of historical memories is assured; the reaffirmation of ethnic identities; the valorization of languages and sciences. From this perspective, the indigenous ethnic groups were able to guarantee the rescue of their traditions, traditional knowledge that is passed from generation to generation.

In conversations with us, the indigenous themselves claim that it was a great advance for the community to have an indigenous school education with its own characteristics for the indigenous. Although it was an imposition, the school currently has the role of ensuring access to non-indigenous knowledge, necessary so that they can continue fighting for their rights.

The school, used with an indigenous pedagogy, assumes the trend of giving possibilities, either due to the empowerment and strengthening of identity, or to give visibility to traditional indigenous knowledge. Indigenous peoples now understand that there is knowledge in all activities in the community. They want to appropriate the non-indigenous linguistic and cultural codes to keep fighting politically.

They are concerned with indigenous teachers' formation so that they do not need to use non-indigenous teachers in their schools. They understand that only an indigenous teacher of ethnicity can present the indigenous culture and make use of it appropriately. Therefore, the importance of bringing daily activities, developed in the village, to the classroom. Based on the National Curriculum Referential for Indigenous Schools (Brasil 2005), indigenous peoples have ensured an intercultural, bilingual, differentiated, community, and specific education.

Each ethnicity guarantees its specifics. Although some ethnicities have suffered losses and changes in their culture, there is a need to rescue the traditional knowledge that is presented by the wise elderly. People who were part of the history of submission and oppression and are respected by the indigenous community, for knowing various stories that narrate the trajectory of struggles of their people. They are voices of the past, with appropriation in the present, to guarantee the future of each indigenous ethnicity in Brazil.

Given these expectations, ethnomathematics enters the villages' schools with the function of presenting and spreading the culture of each indigenous ethnicity. The intention is to strengthen culture and enable indigenous resistance through decolonial and insurgent practices. Some researchers have already given visibility to the existence of the coloniality of power, knowledge, being, and *Mother Nature* imposed on indigenous villages. There are people who think they have the right to expropriate indigenous peoples in their traditional knowledge, others, such as some religious, continue to impose a regime of cultural usurpation, erasing rituals and demonizing the *pajés*.

Ethnomathematics starts from the forms of mathematization developed by indigenous peoples, over time, to reach the traditional knowledge that is presented in a non-fragmented way. In many indigenous artifacts, ancestral knowledge and practices are present. Thus, in all research presented there is the identification of this knowledge and the need to use them in the classroom.

Basic actions, developed naturally in the daily life of the village, are full of knowledge. In the construction of houses is recognized a system of logical ideas, ways of mathematizing space, as well as ways of dealing with reality and overcoming the challenges imposed by it. These aspects allowed indigenous peoples to develop, for example, ways to measure, establish perimeters and areas, and develop geometric figures.

Still, regarding the appropriation of space, they establish the size of the *roça* by number of people that the family contains. This form of measurement is also present in the construction of houses. It is an intuitive measure of their connection to the environment, but it contains mathematical concepts related to space. These ethnoknowledges are markers of identities and belonging to local territoriality. It is a territoriality instrumentalized by the body itself, by the collective living and by artifacts inherent to this collectivity.

The harmonic relationship with space, characterized by local territoriality, translates into the utilization, preservation, and protection of the standing forest. The use of medicinal plants and herbs, the collection of fruits that nature gives them, the ways of fishing and hunting, and the practice of reforestation of degraded areas, are

some aspects that are impregnated in indigenous culture. The use of water, plant, and animal resources gives them identity, as they are part of the forest and the forest is part of their identity.

Thus, they can personify plants and animals in the many stories contained in the evolutionary trajectory of ethnicity. The cure of known diseases assumes that some evil has invaded the body and therefore there is a need to invoke the spirits contained in medicinal plants and herbs. Wise elderly and a nursing technician *Paiter Suruí* have teamed up to develop the nursery for medicinal plants and herbs. It is a way to rescue the culture itself and use natural medicines.

Many of these actions, such as the simple act of fishing, involve numerous ways of contextualizing teaching practice and facilitating meaningful learning for indigenous students. Land preparation, environmental management, living spaces, and teaching and learning spaces are anywhere in the village. The systematization of this knowledge, present in the most varied artifacts produced by the indigenous, is developed in conversation wheels and in daily activities, not needing a school for this teaching to happen.

Forest areas, guaranteed by the demarcation of indigenous lands, are more protected than other areas around them, even though there is still invasion and illegal extraction of wood and ores on indigenous lands. Culturally, nature is more than a means of survival; it is a support for the social, cultural, and historical life of indigenous peoples. They are responsible for maintaining the forest in the Brazilian Amazon. It is important to make this knowledge visible so that non-indigenous learn how to protect and preserve the resources that nature makes available.

So, the various dimensions of ethnomathematics assist teachers in indigenous school education, as a methodological tool, in the protection of culture, the forest and in the awareness of the need to preserve nature. In particular, the pedagogical dimension that promotes reflection on the need to recover their dignity and strengthen their roots. And the political dimension, because it leads the indigenous peoples to reflect on their condition of oppressed and leads them to recognize the importance of a decolonial thought that is translated through practices, very important in the current moment that we pass in the Brazil's government policy.

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Chapter 6
Knowledge Networks in the Training of Indigenous Mathematics Teacher

Maria Aparecida Mendes de Oliveira

6.1 Introduction

> *I find that there is nothing barbarous and savage in this nation, by anything that I can gather, excepting, that every one gives the title of barbarism to everything that is not in use in his own country. As, indeed, we have no other level of truth and reason than the example and idea of the opinions and customs of the place wherein we live. […]. They are savages at the same rate that we say fruits are wild, which nature produces of herself and by her own ordinary progress; whereas, in truth, we ought rather to call those wild whose natures we have changed by our artifice and diverted from the common order […] we have helped to degenerate in these, by accommodating them to the pleasure of our own corrupted palate […]. We have so surcharged her with the additional ornaments and graces we have added to the beauty and riches of her own works by our inventions, that we have almost smothered her* (Montaigne, 1580, Chapter XXXI, Of Cannibals).

Indigenous peoples have been going through an intense process of contact with the surrounding society in Brazil, which has brought and brings to these peoples many transformations in their economic, social, political, and cultural organization. In other words, they experienced and have still been experiencing processes of zcoloniality. According to Quijano (1992), these processes are based on the relationship between European culture, also called Western culture, and the others, which still follow a relationship of colonial domination.

In this context, coloniality consists of a colonization of the imaginary of the dominated and acts within that imaginary and to some extent it is part of it. It is based on a domination process that started with a systematic repression not only of beliefs, ideas, images, symbols or knowledge, but also spread as roots, above all, "regarding the ways of knowing, producing knowledge, producing perspectives, images and image systems, symbols, modes of signification; the resources,

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standards and instruments of formalized and objective expression, intellectual or visual" (Quijano 1992, p. 12).

School education for indigenous peoples was an important instrument of coloniality. A story that goes from an education policy for the Indian to the appropriation of schools by the indigenous people. That can be perceived by different moments and times. It begins with an assimilationist[1] policy, founded on the desire to submit the indigenous to the dominant culture, which constituted itself as a central element of the dominant ideology in the Portuguese colonial world (Meliá 1979).

Followed by an integrationist[2] propose, which becomes no longer exclusive to religious missions, the State begins to control indigenous school education policies. According to Meliá (1979), the education thought for the Indian during this period by "national society" "does not differ structurally, neither in its functioning, nor in its ideological assumptions, from missionary education. It collects similar failures" (p. 48).

Cultural domination has its own characteristics in each of these moments. This form of domination is not only characterized as political, social, and economic domination, but also as being and knowing (Quijano 1992). The school worked for a long time as an instrument of cultural transmission, of ways of thinking and knowing a certain society, delegitimizing and diminishing the knowledge transmitted collectively from generation to generation by societies without written culture. Hence, Tamayo-Osorio (2017) states that these schooling processes expanded across all continents, in different times and spaces as a result of the colonization processes organized institutionally based on a conception of knowledge seen as universal and submitted to the dichotomous category of forms versus content (p. 37).

The domination processes that occurred during the schooling of indigenous peoples do not differ from each other, as many of the objectives that are present in one are also present in the other, as well as many elements of assimilationist and integrationist policies are seen in educational practices in indigenous schools, or even in government speeches. The end of colonialist politics "did not represent the end of the colonialism of mentalities and subjectivities, in culture and epistemology, on the contrary, it continued to reproduce in an endogenous way" (de Santos 2010, p. 8).

On the other hand, it is possible to identify in Brazil, from the 1970s onwards, the constitution of indigenous non-governmental organizations and the formation of indigenous movements throughout the country, characterized by the period of the military dictatorship, which strengthened the need of a specific, differentiated, intercultural, and bilingual indigenous school education, as well as in the struggle that these peoples have been facing in the last decades for the resumption of indigenous territories. In this period, the indigenous people started a movement through processes of autonomy and protagonism. In the midst of processes of struggle for the demarcation of its territories the school is constituted as an important space for the

[1] This period takes place during Colonial Brazil, where the schooling of the indians was in charge of the Jesuit missions.

[2] The integrationist period went from the beginning of the twentieth century to the 1970s and 1980s.

production of internal and external relations. That is, the school is understood as a place that produces connections in the networks of relationships that constitute the non-indigenous world.

The demand for a differentiated indigenous school and the increasing number of indigenous to work in their schools villages have also challenged the institutions, since the demand for the provision of specific courses for the training of indigenous teachers have boosted. Also, the presence of indigenous people in universities resulted in some explicit tensions in the academic space, in the sense of considering knowledge from the difference and of other epistemological logics unlike from those produced by Western culture.

When discussing about the knowledge networks that are being formed in the processes of training indigenous mathematics teachers, it is important to highlight the "order of knowledge" (Walsh 2009b, p. 186) that allows addressing the educational problem that goes beyond the public policies advocated in the current legislation, or the curriculum of indigenous schools or indigenous teacher training courses. We believe in the perspective that considers that educational institutions have contributed and continue to contribute to the "colonization of minds, to the notions of singularity, objectivity and neutrality of science" (Walsh 2009b, p. 186). Also, in particular values constantly present in mathematics as a discipline that, in people's imagination, it is identified by its purity, uniqueness, universality, neutrality and that some people are more inclined than others.

> Mathematics has been since the Ancient Greece, a discipline that focuses on educational systems, and it has been the most stable form of thought in the Mediterranean tradition that has endured until today as a cultural manifestation that imposed itself, undisputed, on other forms (…) mathematics became universal, displacing all other ways of quantifying, measuring, ordering, inferring and serving as a basis, imposing itself, as the rational logical thought mode that started to identify the species itself. (D'Ambrosio 1993, p. 10).

The relationships that are established between indigenous and non-indigenous knowledge in teacher training courses are at the heart of what has been understood an intercultural education. In this sense, more than valuing the knowledge and practices of a given cultural group, there is concern with respect to mathematical knowledge which circulates in indigenous schools, in indigenous teacher training courses and is inserted in processes of an established cultural dynamic through dialogue.

This concern and the need to establish a dialogue between cultures, which are present in the relationships between different knowledge systems—indigenous and non-indigenous—are some of perspectives of the Ethnomathematics Program. It is considered as a reference program and contributes to the understanding of specificity, both in mathematical thinking and in social, political, and cultural contexts, in which it manifests itself and is focused on psychological, social, epistemological, pedagogical, and power issues.

The knowledge networks that are formed in the process of training indigenous teachers to work in a school that proposes to be differentiated, intercultural and bilingual place us in the field of Ethnomathematics. According to Monteiro and Mendes (2015), the networks of relationships that are established in the process of training indigenous mathematics teachers lead us to a resistance process, not to

mathematical knowledge, but in the search for new ways of thinking about this knowledge, new behaviors and norms for constituting this knowledge, as a counter-conduct movement:

> (…) Ethnomathematics appears as a counter-conduct—since there is no rupture with Mathematics nor does it stand against the principles of the field of knowledge, but it claims for another way of thinking and doing mathematics. This other form emerges from doubt and from new questions posed about the uniqueness and universality that this field of mathematical knowledge asserted itself. And, the tracks of this counter-conduct point us especially to the experiences based on cultural and social otherness (Monteiro and Mendes 2015, p. 6).

When considering networks of relationships and the ways in which different knowledge regimes circulate in specific training courses for indigenous teachers, ethnography is configured, according to Cohn (2016), as one of the research bases we have developed with a view to getting to know the educational reality promoted by the indigenous people.

In this study, we take into account the indigenous teacher training courses and indigenous schools where a relationship between indigenous teachers, communities, schools, and universities is established. The analytics present in this text is composed of several elements that go through the understanding of the existing scenario in the formation of Guarani and Kaiowá indigenous teachers, in which interfaces are produced between the social relations that indigenous groups establish with external institutions and with their communities. We identify some common situations to the study context developed in an Indigenous Intercultural Teacher Training course, more specifically in the qualification of indigenous teachers to work with the subject of mathematics, from an ethnographic perspective.

We rely particularly on research carried out by indigenous researchers who also work in school education in their communities as well as in the tensions between different knowledge regimes present in schools and in training spaces, which are evidenced in the dialogue between indigenous education and indigenous school education. We understand that the studies of indigenous researchers can fill the gaps left by "theoretical study on indigenous peoples (their histories, ritual cosmologies, social organization), and those on indigenous schools (their pedagogical practices, use of native languages) as a result of silence of national ethnology regarding educational issues" (Tassinari 2008, p. 219–220).

6.2 Training of Indigenous Guarani and Kaiowá Mathematics Teachers and the Relationship Between Different Knowledge Systems

The whole movement that indigenous population went through in the schooling process, from assimilationism and integrationism to the appropriation of school by indigenous communities, reflects in the training of indigenous teachers in Brazil,

which is today "one of the main challenges and priorities for the construction of an Indigenous School Education guided by the principles of difference, specificity, bilingualism and interculturality" (Grupione 2006, p. 50–51).

This movement is based on the principle that indigenous teachers are guides capable of inventing and building paths that allow the promotion of dialogue between different modes of knowledge production. Teacher education programs have their own pedagogical and curricular proposals that have been constituted in conflicting relationships with national hegemonic education programs in other Brazilian schools. According to Monte (2000), these conflicts are generated because it is an intercultural education:

> (…) experienced in indigenous contexts, confronted for centuries by domination and asymmetry with States and national societies, the affective emphasis on what they call 'own' and 'traditional culture' gives teachers' discourse a strong ideological bias and, to the didactic procedures, an indispensable inductive methodology: the base and the vertebra of the acquisitive processes are based on the culture and the proper language, from where, due to historical nexus and permanent discursive connections, the new knowledge common to other societies, appropriated also by the intercommunication at school (p. 21).

The training of indigenous teachers is located in a *region of borders*, in conflicts of cultures, societies, and identity. Therefore, these courses bring a perspective of intercultural education, given that the curricula of these courses seek to address content from indigenous and other cultures, in addition to teaching the indigenous language and Portuguese. These training courses also stand out for developing, together with indigenous teachers, a reflection on the currently curriculum of indigenous schools.

The offer of specific higher education courses for indigenous population is part of a strong movement that occurs throughout Latin America, aiming the formation of Indian teachers in an Intercultural and Bilingual Education perspective. Many indigenous organizations discuss and carry out actions around bilingual and intercultural education in order to strengthen the various forms of educational thought, with and for indigenous peoples, aiming to meet the growing needs and demands for more and better indigenous school education, being the indigenous political participation one of its strongest aspects. The strengthening of indigenous social movements brings new paradigms to education presented by indigenous organizations, that is, they have endeavored to establish the bases for a policy of difference, as well as for their own intercultural and bilingual education that satisfies the political needs of two societies.

It is in one of these courses, the Indigenous Intercultural Teacher Training course—*Teko Arandu* (Living with Wisdom), which was implemented in 2006 at the Federal University of Grande Dourados (UFGD), that we constitute meanings for the training of indigenous teachers. During its process of creation and implementation, the course takes on a proposal for the training of indigenous teachers at a higher education level from an intercultural perspective in which one of the challenges is to recognize the different ways of producing and communicating knowledge (Oliveira 2009). The course project was taken to the university by indigenous teachers who sought in the institution a public space as a guarantee of ethnic

sustainability and the elaboration of knowledge based on different logics of world understanding.

The reflections made on this study that focuses on the training of indigenous teachers and in the network of relationships that are established in the circulation of different knowledge regimes present in this training process are a result of the research carried out on the training of Guarani and Kaiowá indigenous teachers in Mato Grosso do Sul. The proposed analytics is based on the research developed during different periods of the initial training of these teachers in an Indigenous Intercultural Teacher Training course and from the field research in the villages where we conducted interviews/conversations with graduates of the course with specialization in mathematics and that work in village schools teaching this subject.

The Guarani and Kaiowá indigenous peoples are located in the South of the State of Mato Grosso do Sul. Nowadays, they are the second largest indigenous population in Brazil. They consist of a population composed of approximately 51,800 inhabitants, restricted to a territory that does not exceed in the entire State 30,000 hectares of demarcated land (Cavalcante 2013), in a scenario of land conflict that worsens every year. The land situation in Mato Grosso do Sul and the historical, economic, and social transformations experienced by the Guarani and Kaiowá peoples are quite complex and permeated by the expropriation of their territories, such as the violent disputes between Indians and farmers and countless types of violence undergone by this population throughout its history.[3]

It is in a context of violence/appropriation, as stated by de Santos (2010), that these peoples experience a colonialist relationship imposed on their communities. In this scenario, schools are seen as spaces of resistance as they are located in a territory within the indigenous areas with frontier, as pointed out by Tassinari (2001), also producing as defined by Mignolo (2015) a "frontier thinking."

It is essential to understand the way in which territorialization processes take place throughout the history of these peoples in order to comprehend the network of relationship that they establish with schools in the villages and the multiple ways of being of these peoples (*ava kaiowa reko kuera*). These conditions produce different ways of life given the plurality of environments in which the Kaiowá and Guarani population currently live in Mato Grosso do Sul (Pereira 2004).

The schooling processes of indigenous peoples were produced from the perspective of non-existence. Against this existential invisibility, in the last decades the

[3] We will not go into the conflict and violence against the indigenous peoples of this state. However to have a closer view of this reality the reader can access the "Reports of Violence against the Indigenous Peoples of Brazil," published annually since 2003 by the CIMI-Indian Missionary Council, available at https: //cimi.org.br/observatorio-da-violencia/edicoes-aniores/. These reports are an important tool to denounce the violence suffered by indigenous peoples. It is possible to verify that the violence suffered by the Guarani and Kaiowá increases every year, as highlighted by Don Erwin Karutter in the 2006–2007 edition. He affirms that among the regions of the country "in a particular way in Mato Grosso do Sul, involving the Guarani Kaiowá people and under certain aspects, such violence has increased in the last 2 years. The number and types of violence suffered by indigenous peoples in that state continues to be shocking, notably the countless and persistent suicides, today being mostly practiced by Guarani Kaiowá children and adolescents" (2007, p. 7).

struggles of indigenous social movements have resulted in the expansion of the country's political agenda regarding the recognition of the rights of indigenous peoples. These movements also demanded a differentiated education, intercultural and bilingual. Currently, indigenous communities are going through a process of appropriating school spaces in search of their autonomy, with proposals to build a differentiated, intercultural, and bilingual Indigenous School Education. Among possible experiences and what is available, schools in indigenous[4] areas are seen as a territory of resistance and, consequently, universities are also used as a space of power in which they can place their demands.

The specific training of indigenous teachers has been constituting itself by different ways of understanding about knowledge, impregnated by power relations. This knowledge was produced in a historical context of colonization experienced by the indigenous peoples of the country which started with the homogeneity founded by a single way of thinking and being in the world built under the purposes of modernity (Walsh 2004, 2009a, b, 2012; Mignolo 2000, 2015; Quijano, 1992; Santos 2010, 2019; Maldonado-Torres, 2007). Therefore, we propose a discussion on: How do coloniality processes, which cross and produce ways of being and organizing reflect in the organization and circulation of different knowledge regimes among indigenous peoples? What other ways of knowing indigenous people bring to the university in specific training courses?

6.3 Interculturality, Ethnomathematics, and Decoloniality: Doodles in the Training of Indigenous Teachers

Throughout the colonization process, strongly undertaken by the insertion of the school, the ways in which indigenous peoples produce their knowledge has been silenced. An epistemic, but also ontological, silence of its own way of being (*nhandereko*), as pointed out by the indigenous professor and researcher Claudemiro P. Lescano (2016):

> The integration policy through the institutions to which the indigenous people had access, as well as the school as a teaching and learning environment were instruments that helped to weaken our identity, when, pedagogically, we worked to silence our way of being (p. 16).

de Santos (2010) called this *epistemicide*, given the process of subordinating the knowledge of different peoples caused by European colonialism. The process of colonialism, imposed on the different peoples who were subordinated, produced not only the economic domain, but also the domain of bodies through slavery and

[4] Over the past decade, Guarani and Kaiowá indigenous teachers have been increasily occupying the school's territory, not only as teachers but also as coordinators and directors, assuming the management of these spaces, even experiencing tensions with the municipal departments of education and sometimes with the local community.

physical and cultural genocide. The indigenous researcher Eliel Benites (2014) highlights that:

> The Kaiowá and Guarani alterity, in this sense, for the colonial discourse, is the search for framing the palpable fixed model, the place of the "known" and this known must be reproduced continuously from the perspective of sameness. In this direction, the position of the colonizer is to make the different fit or normalize, adapting to the colonizing context, transforming the other into one and not dialoguing with this other, or imposing the terms of the dialogue (p. 50).

This colonialism is also related to the field of knowledge since it has omitted and still omits the knowledge of the indigenous and other peoples. The processes of knowledge production of different peoples have been subordinated to the colonialist model based on the ideal of modernity. According to Maldonado-Torres[5] (2007) coloniality has to do with:

> (…) long-standing patterns of power that emerged as a result of colonialism, but that define culture, work, intersubjectivity relations and the production of knowledge far beyond the strict limits of colonial administrations. Thus, although colonialism precedes coloniality, the coloniality survives to colonialism. It is kept alive in books, academic performance criteria, cultural standards, common sense, peoples' self-image, subjects' aspirations, and many other aspects of our modern experience. In a sense, we breathe coloniality and modernity daily (p. 131, author's translation).

The colonization process established a power relationship in which knowledge is revealed just like the economy. They are organized from centers of power and subordinate regions, north and south (de Santos 2010). The discourse of modernity created the illusion that knowledge is universal and that it is not related to the place.

> The process of increasing specialization in the modern sciences has been restricting the objects on which the knowledge produced focuses. That is, the increasement of knowledge rigor goes together with the growing arbitrariness and the need to protect/ control the borders between different knowledge. This means that disciplinary knowledge is also disciplined knowledge due to the need to police and repress possible frontier transformations. (Oliveira 2008, p. 29)

According to de Santos (2019), the monoculture of modern Eurocentric knowledge in the form of colonialism of power, knowledge, and being produced the non-existence, invisibility, inferiority, and irrelevance of groups of people and ways of life. They created "the monoculture of valid knowledge, the monoculture of linear time, the monoculture of social classification, the monoculture of the superiority of the universal and the global and the monoculture of productivity" (p. 50). Regarding

[5] "la colonialidad se refiere a un patrón de poder que emergió como resultado del colonialismo moderno, pero que, en vez de estar limitado a una relación formal de poder entre dos pueblos o naciones, más bien se refi ere a la forma como el trabajo, el conocimiento, la autoridad y las relaciones intersubjetivas se articulan entre sí, a través del mercado capitalista mundial y de la idea de raza. Así, pues, aunque el colonialismo precede a la colonialidad, la colonialidad sobrevive al colonialismo. La misma se mantiene viva en manuales de aprendizaje, en el criterio para el buen trabajo académico, en la cultura, el sentido común, en la auto-imagen de los pueblos, en las aspiraciones de los sujetos, y en tantos otros aspectos de nuestra experiencia moderna. En un sentido, respiramos la colonialidad en la modernidad cotidianamente" (Maldonado-Torres 2007, p. 131).

the monoculture of knowledge produced by colonialism and imperialism Linda T. Smith[6] (2018) points out that:

> Imperialism and colonialism are specific ways in which the West came to "see", "name" and "know" indigenous communities. The cultural archive with its classification and representation systems, its codes for deciphering such systems and its fragmented knowledge artifacts allowed adventurers and observers to assign meaning to what they saw and to represent their newly discovered knowledge upon returning to the West, through authorship and authority of their representations" (p. 77).

When addressing geopolitics, epistemologically designed spaces, Mignolo (2000) does not only make a reference to the physical space, located on a map, but also to the historical, social, cultural, discursive spaces. Epistemological spaces which provide a basis for subjectivities (identities).

> This way, the coloniality of power has passed into the field of knowledge, discarding the indigenous people's notion as an intellectual, as someone who can intervene directly in the production of knowledge. As an essential part of this coloniality of knowledge, the hegemony of Eurocentrism is maintained as the only perspective of knowledge (Walsh 2004, p. 2).

According to Walsh (2009b), this geopolitics of knowledge guides education and its conceptions of science of knowledge and the knowledge itself. Bilingual education conceived and proposed by indigenous peoples, as a practice of confronting integrationist, homogenizing, colonizing educational policies and unequal power reforms has a clearly identitarian and demanding political character. For the author, it is necessary to move towards an educational reformulation based on a decolonial orientation, which implies working strategically on different elements, including knowledge.

Discussing an epistemic policy of interculturality, but also political and critical epistemologies in the field of education would help take the debates around interculturality to another level, moving from its roots in ethnic-cultural diversity to the "Science" itself to show how science, as one of the central foundations of the modernity/coloniality project, contributed in a vital way to the establishment and maintenance of the current and historical hierarchical racial order in which White people—European white men—remain at the top (Walsh 2009b, p. 205, author's translation).

The modern world has placed in our minds and in our being the concept of universal (mathematical) knowledge, which we must all know. This is not diffezrent for indigenous peoples. What has been produced by these peoples, and more specifically by the Guarani and Kaiowá, is a movement of resistance to this process. When dealing with the indigenous knowledge present in the training of the indigenous teachers, we verified at different times during the course development, a discourse from the indigenous teachers in training highlighting that in these spaces there is a

[6] Regarding the way in which the monoculture of modern Eurocentric knowledge produces key concepts for research, it is recommended to read: Linda T. Smith's (2018) studies: "the intersections of gender and race," the "conceptualizations of individual and society," the "conceptions of space and time."

revival of knowledge that had been silenced. That is, specific training at the university allows them to bring out knowledge that is there, but that are now modified due to a resistance/existence movement:

> These resistances keep inside of Kaiowá and Guarani peoples the founding elements that are part of their subjectivity and that are directly linked to their constitution in the traditional universe, at a time when traditional logics and values are more present or alive in the specific geography (space and time) of its traditional subjectivity. These aspects come up mainly when claiming their constitutional rights, as differentiated peoples. (Benites 2014, p. 52)

The concept of knowledge geopolitics (Mignolo 2000; Walsh 2009a, b) is important as an element to understand what Benites (2014) points out as the *geography (space/time)* of traditional subjectivity in this geography of coloniality of power. During the training processes, indigenous teachers put another geopolitics of knowledge considering the need to build new cosmologies and epistemologies as a reference to other places of enunciation. It is important to state that there are other geographies of knowledge production, of being production and other places of enunciation. Therefore, producing other networks of relationships from an understanding that human knowledge is not produced in a single region of the globe, as, for example, the history of philosophy is presented (from Greece, Europe to the northern Mediterranean) (Mignolo and Walsh 2002).

It is essential to realize that the relations of knowledge established in the model imposed by structures, power systems, and colonial knowledge are maintained and reproduced by the school. And they still guide the ways of being and perceiving themselves in the world. D'Ambrosio (2018) has used the concept of *epistemological cage* as a metaphor to describe knowledge systems. The knowledge created and disseminated as being located in only one region of the world develops "as birds in a cage that communicate in a language that only them known" (p. 199). This way, interculturality and decoloniality, just like ethnomathematics, are projects that are connected to the struggle for a differentiated indigenous school and in the training of indigenous teachers, permeated by a series of tensions.

When referring to the indigenous teacher training and their performance in an environment which had the potential for colonization, it is necessary to think that overcoming is more than just resistance. It is necessary to take into account the need for incorporation, overcoming, and the radical reconstruction of beings, that is, to create the conditions of existence, knowledge, and power that could contribute to more fair societies (Walsh 2009a).

The entry of indigenous people into universities, which is a place of knowledge produced and legitimized—caged—by processes of coloniality has generated tensions when dealing with specific, differentiated, and intercultural training. These tensions are revealed in indigenous teacher training courses and also in indigenous schools, when there is a need to link these different knowledge systems. Tensions which are related to the recognition, difference, valorization within these institutions and of other ways of knowing and producing knowledge.

The indigenous school and specific training courses for indigenous teachers, in its turn, can be seen as spaces where different cultural practices are addressed and

taken into account. The values that permeate the curricular organization of schools and the university itself are still strongly influenced by Western logic. At the same time these places are also frontier spaces, where processes of translation and mediation of knowledge take place (Tassinari, 2001). There, different knowledge regimes/knowledge meet themselves and are resinified, producing different networks of relationships.

The challenge for universities and for the training of indigenous mathematics teachers in village schools is to establish a permanent dialogue between different meanings of the world and between different knowledge systems. According to D'Ambrosio (2018), this challenge justifies the relevance of the Ethnomathematics Program, as a research program, since "it uses the analysis of the history of ideas and the origin and evolution of the behavior and knowledge of the human species, in different natural environments and socio-cultural" (p. 189).

The training of indigenous teachers at the university faces the need to establish relationships between epistemologies from other places of enunciation. According to Mignolo and Walsh (2002), it is necessary to recognize that knowledge is not produced in a single region of the globe, even Eurocentric knowledge is also local. However, there is a widespread tendency to conceive the thought as constructed from European history and experience, as if it were out of focus.

The dialogue resulting from the action between different systems of knowledge has influenced the teaching of mathematics in indigenous schools and the training of indigenous mathematics teachers. In this regard, analyzing the initial training process developed at the university and indigenous schools, based on the enunciation of indigenous teachers, enables an understanding of the different aspects of mathematics teaching and learning, which presupposes an education for teachers where the dialogue is established from the perspective of intercultural education.

As a starting point we propose ethnomathematics and decoloniality to dialogue with the perspective of an intercultural indigenous school. We take into account the configurations that have been taking place in the field of ethnomathematics which places the need for recognition of different processes of generation, organization, and ways of transmitting knowledge, of different cultural systems in a process of interactions (D'Ambrosio 2001).

The movements that have been produced in the field of ethnomathematics (Tamayo-Osorio 2017) allow us to analyze the crisis of the modernity rationality model and the place occupied by mathematics in this model. Monteiro and Mendes (2019) state that ethnomathematics is as a counter-conduct movement, according to the authors "the discourse that emerges from this place called Ethnomathematics is that it questions some of the main structures of the Mathematical field, especially universality and its power of unique and absolute truth" (p. 5).

The training developed in the Pedagogical Curricular Project of the Indigenous Intercultural Degree course—Teko Arandu, with specialization in mathematics proposes the development of a pedagogical action whose assumption is that, when groups of different cultures meet, there is an exchange of knowledge. Indigenous teachers, in addition to the skills to be developed for the classroom, must develop

other skills such as the management of indigenous schools and the management within their communities.

In the spaces of social organization of indigenous communities, mathematics has been translated into a strategy of dialogue with the outside world. This is so because the need to deal with money and with situations of tension related to the territory has been one of the challenges for indigenous peoples (Benites 2009). Negotiating with non-Indians and dealing with money, for example, are forms of relationship with the outside world, which demands the mobilization of mathematical knowledge. Considering that (mathematical) knowledge has been important in the networks of relationships produced in indigenous communities, mathematics has been seen as an area for teacher education in Indigenous Intercultural Teacher training courses. Understanding, for example, how they deal with (mathematical) knowledge in their practices and in social relations is fundamental to think about the formation of teachers to work in these environment.

The challenge for the institutions is the training of this teacher with a profile that fulfill the demands presented by the indigenous communities. Indigenous teachers are called to assume different tasks in the indigenous school education. This requires them to acquire knowledge that allows the development of the task, that is, being the bridge in a frontier space—the indigenous school. These professionals tend to develop skills and values associated with the individual and the collective. These values are always present in the considerations made by indigenous researchers, regarding the processes of resistance and negotiation in the current social organization configuration.

> The current social organization of the Kaiowá indigenous communities results from a long process of contact, resistance, negotiation and a new social organization. The Kaiowá constitute their knowledge through various internal and external values, that is, they acquire external materials or technologies, but the use of these materials is still strongly based on spirituality and collectivity (Lescano 2016, p. 32).

The necessary knowledge for teacher education of indigenous teachers is built from a framework of values that are defined and recognized in sociocultural contexts (village and the indigenous people) in which they are inserted. This knowledge tends to be collectivized while these individuals are constituted as teachers and to the extent that they relate to others and to the future project of their indigenous communities.

Differentiated teacher education, from an intercultural perspective, has influenced the identity of indigenous teachers. The knowledge and values that permeate these courses and the constant search for dialogue between different knowledge help about the curriculum and teaching practices not only in training courses but also in the indigenous school and in the classroom. There is a need for a deeper understanding of the challenges that exist in the development of inclusive education for the training institutions. This challenge is to enable access to knowledge that demands universality in a globalized society and that values its own practices and knowledge.

The need for the development of a mathematical culture has now also become a concern for the indigenous peoples of Latin America and more specifically for the

Guarani and Kaiowá. These options are often the result of a struggle for Independence directed especially against the colonial school that imposed programs, language of teaching, and values (Cauty 2006). Thus, the mathematical activity that developed throughout the world, following the spread of industrial cultures closely linked to the development of sciences and techniques, has now reached even the most remote of the territories. Indigenous teachers seem to notice this form of organization of mathematics.

In the initial training of indigenous (mathematics) teachers, it is necessary to take into account that in indigenous cultural contexts the teaching of mathematics cannot be separated from the cultural practices of the group. Indigenous (mathematics) teachers often assume the role of mediator, not only of the knowledge to be taught in schools, but also of the relationships that are established outside and within their villages. The values to be considered in the network of relationships between different knowledge systems, which are perceived in the participation of indigenous teachers in different training courses, coincide with the values presented by Vergani (2007) and D'Ambrosio (1993). They propose a curricular structure that possess principles of an ethnomathematical action: formative, utilitarian, sociological, cultural, aesthetic, and ethical.

This curricular structure is articulated to what the Guarani and Kaiowá peoples defined as a system of axes formed by three fundamental elements: *teko* (culture), *tekoha* (territory), and *ñe'ë* (language) that express place, time, and sociocultural diversity. This structure presents a methodological vision that comprehends the joint organization to the theoretical dimensions, to the communicative and evaluative practices present in the training course of indigenous teachers who seek a movement towards a curricular proposal that considers the tensions presented here, which can be guided by an ethnomathematics perspective in which according to D'Ambrosio (2005):

> (…) is to make mathematics something alive dealing with real situations, in real time [now] and space [here]. And, through criticismo to question the here and now. In doing so, we dive into cultural roots and practice cultural dynamics. In effect, we are recognizing in education the importance of various cultures and traditions in the formation of a new civilization, transcultural and transdisciplinary (p. 43).

In this sense, we understand that the training of teachers in order to work in the field of Mathematics in indigenous schools can be focused on the Ethnomathematics perspective. According to D'Ambrosio (2012)[7]:

> What justifies the central role of mathematical ideas in all civilizations [ethnomathematics] is the fact that it provides the intellectual instruments to deal with new situations and define strategies for action. Therefore, the indigenous ethnomathematics is useful, efficient and adequate for the things of that cultural context in that society. There is no reason to replace it. The ethnomathematics of white people serves to other equally importante things proposed by modern society and there is no way to ignore it. To pretend that one is more

[7] D'Ambrosio, Ubiratan. The Mathematicians' Responsibility in the Search for Peace. Retrieved from: http://vello.sites.uol.com.br/ubi.htm. Accessed on: 17th May, 2005.

> efficient, more rigorous, ultimately better than the other if taken from the contexto is a false and falsifying question (para. 63).

The mastery of the two ethnomathematics offers greater possibilities for explanations, understandings, management of situations that are placed in the life of the community, problem solving and definition of strategies for organizing and systematizing traditional knowledge. For example, the mathematics that is taught and that is present in the curricula of indigenous schools has still been treated as neutral knowledge with its universality as a science and it is taught from a rationality that does not meet different cultural practices.

The teaching of mathematics in indigenous schools has little to do with local culture. However, when in the training of indigenous teachers, teachers are willing to dialogue with other forms of knowledge production, placing the indigenous student to the production of this knowledge. The logic that is placed for teaching academic mathematics[8] is subverted and other ways of knowing and producing knowledge are also used. There is resistance to an imposed model, reflected in the different forms of subversion produced in the indigenous school, in the classroom where it can be found teachers and students indigenous. Movements that are present in the practices themselves of teaching mathematics in indigenous schools.

The indigenous mathematics teacher Lidío C. Ramires (2016) states that Kaiowá and Guarani education based on the principles of own teaching-learning knowledge "strengthens the importance of territory/territoriality—*tekoha* in the construction, reconstruction and resignification of knowledge" (p. 43). Eliel Benites (2014) affirms that the "traditional Kaiowá and Guarani knowledge as a spiritual vision is acquired from social and environmental experience in traditional territory (*ñane retã*), from a physical and spiritual relationship." According to the author, it is through this territory experience that can be acquired "the ability to contemplate time and space, with its own sensitivity, and that constitutes the Kaiowá and Guarani cosmology" (p. 66).

This way, even if schools follow a course determined or established by a policy of a colonialist state, such as the resumption of territory, these teachers and the school produce ways to scape when they rehabilitate their own ways of teaching and learning, transgressing or adapting into the Guarani style the school contents, routines, and divisions of time/space.

[8] It is interesting to observe the experiences presented in different academic researches in the field of Ethnomathematics which address the training of indigenous teachers, to name a few Correa (2001), Mendes (2001), Mendonça (2007), Oliveira (2009), Leite (2014), Monteiro (2016), Melo (2016), among other researches who present different possibilities of working with mathematics in indigenous schools and in the training of indigenous math teachers.

6.4 Further Considerations

Thinking about mathematical education which overcame the rationality of eurocentered scientific knowledge is moving towards an action project that can establish a relationship and negotiation among conditions of respect, legitimacy, symmetry, equity, and equality (Walsh 2009a, b) in order to overcome the vision of objective knowledge in the way mathematics is conceived.

Both in teacher training and in indigenous schools, what is presented as a challenge for trainers and indigenous teachers in an intercultural education perspective is to interchange knowledge—mathematics in dialogue with other mathematics—which as Walsh (2012) points out can open space for "new perspectives for the geopolitical order of knowledge production." However, according to the author:

> More than interchange knowledge, what these reformulations and perspectives suggest is a different epistemological relationship, a relationship that intentionally allows to situate this knowledge (and its logics, ways of thinking and its thinkers) from a social, political and ethics posture and the goal of a transformation project (p. 83).

In this regard, Ethnomathematics, when considering the complex relationship between the different ways of knowing has been configured as a potential tool to think about a pedagogical action for indigenous schools and for the formation of indigenous teachers in the perspective of an intercultural education, founded on a dialogue between indigenous knowledge and non-indigenous knowledge.

In teacher training, the dialogue with the difference is essential to think about the curriculum and teaching practices in the classroom, at the university, and at the indigenous school. Training institutions need to deeper their understanding on the challenges in order to build an inclusive education that allows access to knowledge, demands universality in a globalized society and values practices and its own knowledge. The production and circulation of different knowledge systems in indigenous teacher training courses, at the university and in indigenous schools take place in a field of many conflicts, of connection and of opposition.

Although the school in the indigenous areas still has a colonialist role, it has been resinified by the indigenous people. They produce other ways of dealing with reality, resisting to different forms of coloniality of the indigenous being, producing decolonial movements, even through schools. In this regard, we can highlight the ambiguity in the Guarani and Kaiowá indigenous schools as well as in the training that is developed in specific courses. These movements are produced in the extent that in their pedagogical practices they produce their own way of teaching, subvert and confront the model of knowledge and school imposed on them.

These movements escape from the school model conceived from a universal paradigm of knowledge, in a Eurocentric perspective (Walsh 2009a). Bringing to university other ways of knowing and relating to knowledge by questioning the individualist perspective present in the European paradigm of rationality founded on the total absence of the "other" can also be seen as a way of escaping (Quijano 1992). These escapes, even if subtle, are important to reflect about the effects that coloniality produces in the indigenous context by the school and by the training

institutions and how indigenous teachers after their training contribute with the decoloniality in the indigenous mathematics teachers network and with pedagogical practices in the indigenous school.

According to D'Ambrosio (2008), the objective of Ethnomathematics is to give meaning to different ways of knowing and doing, from different cultures, to recognize how and the reason groups of individuals organized as families, communities, professions, tribes, nations perform their mathematical practices. We can consider the ethnomathematics program as an element that contributes to the construction of an intercultural and decolonizing education. Ethnomathematics indicates decolonization processes of school dynamics through socialization, reconstruction, and claim of knowledge produced from other enunciation places.

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Chapter 7
Ethnomathematics Research on Indigenous Peoples' Knowledge and Practices

Maria Cecilia Fantinato and Kécio Gonçalves Leite

7.1 Introduction

Brazil has an indigenous population of 896 thousand, who belong to 305 peoples, and speak 274 languages in different sociolinguistic situations (IBGE 2012). These peoples are distributed across 723 indigenous lands at different stages of the demarcation process; among them only 486 are ratified and reserved (ISA 2020). This entire ethnic, linguistic, and cultural diversity also constitutes a diversity of knowledge forms and practices, including those of a mathematical nature. Such diversity has been the subject of studies and research in ethnomathematics on indigenous issues carried out by some national institutions. This has taken place through research groups, graduate programs, and, more recently, specific courses for the training of indigenous teachers.

Brazilian ethnomathematics research on indigenous themes began in Brazil during the 1980s. Eduardo Sebastiani Ferreira pioneered fieldwork in the area, carrying out and advising investigations whose empirical research was developed in indigenous communities of the upper Xingu and Amazonas (Knijnik 1996). Over the past few decades, new research groups in ethnomathematics emerged from different national institutions, bringing together researchers and developing research in indigenous contexts on themes related to indigenous knowledge and practices or, more specifically, to Indigenous School Education (ISE). Ethnomathematics is apparently undergoing a movement of capillarization: although the subject was

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initially limited to graduate-program research, nowadays it permeates undergraduate conclusion papers and scientific initiation studies (Leite 2017).

This expansion of ethnomathematics research in Brazil—particularly in respect to studies investigating indigenous themes—is directly related to the achievements of the indigenous movement. Through the articulation of its organizations and beginning with the Federal Constitution of 1988, this movement was able to change the legal framework that underlays the rights of indigenous peoples. The transformations in the relations between the Brazilian State and indigenous peoples—brought forward by the new and up-to-date constitutional text—induced changes in the organization of indigenous territories' school education. This created the necessity and opportunity for researches on the different indigenous peoples, so as to support the construction of curricula, the development of new intercultural pedagogical practices, the elaboration of specific bilingual or multilingual teaching materials, and the design of differentiated courses for the training of indigenous teachers (currently, there are 25 intercultural indigenous teaching-degree courses distributed across 16 states of the federation) (Nascimento 2017).

This study presents an overview of Brazilian ethnomathematical research on indigenous themes, based on the academic production carried out between 2002 and 2019. We focus specifically on ISE-related works. Therefore, for a better understanding of the historical context in which Brazilian researchers are inserted—and which partially explains the motivations behind their research—it is necessary to present a brief history of school education among indigenous peoples in Brazil.

7.2 Indigenous School Education in Brazil

The recognition that indigenous peoples are entitled to a specific and differentiated school education is relatively recent in Brazil, just as the recognition of the existence and value of indigenous forms of knowledge—including mathematical knowledge. Thus, in order to understand the contributions of ethnomathematical research to the construction, implementation, and development of public policies in the country's indigenous educational contexts, it is important to analyze, according to a historical perspective, the different phases of indigenous peoples' self-organization towards the preservation of their identities, knowledge, languages, cultures, and territories (which are currently divided into the ethno-educational territories instituted by Presidential Decree no. 6.861) (Brasil 2009).

The history of school education in Brazil's indigenous territories is marked by four phases. The first began in the second half of the sixteenth century, with the introduction of Jesuit schools in villages in Rio de Janeiro and Bahia. These functioned as "schools for [learning how to] read, write and count" (Freire 2004, our translation). Teaching was carried out exclusively by missionaries, and speaking mother tongues was forbidden. Indigenous peoples were subjected to catechesis and taught agricultural techniques and handicrafts, while "the most skilled [among them] learned to read and write" (Freire 2002, our translation). According to Ferreira

and Silva (2007), the education practiced in these first Jesuit schools was at the behest of a Eurocentric policy whose main goal was to assimilate indigenous peoples to Christian civilization, by means of evangelization.

Thus, regarding this first phase, one could say that Jesuit schools provided education *to* indigenous people, but were not schools *of* indigenous people. Catechesis was their main objective, and they completely ignored these peoples' originary educational processes, languages and cultures. Colonizers' resistance to recognize indigenous peoples' own teaching and learning processes was constant throughout the colonial period. According to Brandão (2007), even anthropologists of the early twentieth century—specialists in the "primitive cultures" of the tribal societies of the Americas, Asia, Africa, and Oceania—avoided using the term *education* to designate their subjects' everyday relationships, ceremonies, and rituals.

Throughout the colonial period, school education in the villages was under the responsibility of Catholic missionaries. This scenario did not change with the transition to the imperial period. On the contrary, the Empire went so far as to turn catechesis into an official practice, with Law No. 16 of August 12, 1834 establishing that the Provincial Legislative Assemblies would be responsible for "promoting, cumulatively alongside the Assembly and the General Government, the organization of the province's statistics, catechesis, the civilization of indigenous peoples, and the establishment of colonies" (Brasil 1834, our translation). Aiming to reinforce catechesis as a way of "civilizing" indigenous peoples, Salesian missionaries arrived in Brazil in 1883. Their missions were installed in Mato Grosso starting in 1884 (Tacca 2001). By the Republic, the only novelty in the indigenous village school-education scenario was the participation of other religious groups, including Protestants.

The second phase in the educational policy for indigenous peoples began in 1910, with the creation of the Service for the Protection of Indians and Placement of National Workers (*Serviço de Proteção aos Índios e Localização dos Trabalhadores Nacionais*—SPILTN), which from 1918 onwards became known simply as the Service for the Protection of Indians (*Serviço de Proteção aos Índios*—SPI). According to Oliveira and Nascimento (2012), the SPI marked the institutionalization of Brazilian indigenism, replacing the religion-oriented policies that characterized the colonial and imperial regimes. SPI proposed a more humanist indigenous policy, linked to an alleged concern for the linguistic and cultural diversity of indigenous peoples (Ferreira 2001).

However, when assuming the responsibility of providing secular assistance, seeking to distance the Catholic Church from the practice of indigenous catechesis, SPI—with its positivist orientation—ultimately sought to induce indigenous peoples into gradually adopting "civilized" habits, transforming them into a workforce to be integrated into the national production system (Paladino and Almeida 2012). Thus, indigenous education surpasses the paradigm of *assimilation*, which characterized the previous phase, and is now governed by a paradigm of *integration*.

The apparent advances in public educational policies for indigenous peoples promoted by SPI did not effectively translate into the recognition of these people's right to autonomous territorial existence, or into a societal appreciation of their forms of

knowledge. On the contrary, the educational policy of the country's first indigenous organ was guided by the positivist ideology of progress, which considered it necessary to modernize indigenous societies through their incorporation into national society, with a view to the progress and national modernization (Oliveira and Nascimento 2012).

To implement its educational policy, the SPI installed a network of new schools in villages across different parts of the national territory. According to Cunha (1990), by 1953 the indigenous organ maintained 66 schools. Their curricula were similar to those of rural schools, and they employed methods and teaching materials that were alien to the specific sociocultural aspects of indigenous peoples. Thus, its secular character notwithstanding, school education provided *to* indigenous peoples as implemented by SPI was hardly different from the educational policies of the previous phase—after all, it upheld the goal of "civilizing" these ethnicities, assuming their inferiority on the linear scale of human cultural evolution. This paradigm survived the Brazilian constitutions of 1934, 1946, and 1967.

The third phase of the history of school education in indigenous contexts in Brazil began in 1967, with the extinction of the SPI. The country's military dictatorship replaced it with the National Indian Foundation (*Fundação Nacional do Índio*—FUNAI). The new indigenist agency continued the paradigm of integration. However, as part of its educational proposal for indigenous peoples, FUNAI developed a bilingual education program, guided by the experience of the *Instituto Indigenista de América Latina* and by resolutions from indigenous congresses held in Mexico (Paladino and Almeida 2012). Moreover, it signed agreements with religious organizations, such as the American Summer Institute of Linguistics (SIL), reintroducing missionaries into indigenous village schools. Under the influence of the SIL, a transitional bilingualism was introduced in the educational system. Thus, literacy in the mother tongue was used as a means for the religious conversion of indigenous people, who were to become readers of the translated Gospel (Ferreira and Silva 2007).

From the 1970s onwards, the Brazilian indigenous movement grew stronger, giving rise to and articulating different indigenous organizations throughout the national territory, with the support of universities and non-governmental entities. From then on, there is a marked challenge to the educational practices developed by FUNAI and by religious institutions, with the building of alternative proposals for differentiated educational models in the indigenous villages. These proposals were intended to overcome official educational policies and their integrationist nature. Unofficial initiatives for the training of indigenous teachers emerged, together with the first ethnomathematical field researches in the country's indigenous contexts.

With an ever-increasing mobilization, the indigenous movement started to demand changes in the country's legislation and indigenous policy, finally obtaining the recognition of its own forms of social organization, languages, and cultures—as well as its originary right over traditional indigenous territories—in the new Federal Constitution of 1988. Thus, the new Constitution surpassed the integrationist

principles of the constitutional texts that preceded it, detaching itself from the positivist conception of indigenous peoples' assimilation or integration into national society. In lieu of this conception, the principle of all peoples' right to existence (no matter their cultural differences) prevailed—also ensuring the rights of all peoples to a school education respectful to their knowledge and forms of teaching and learning.

This was the beginning of the fourth phase of school education in Brazil's indigenous contexts. One could say it was also the birth of *Indigenous School Education* proper, since it was no longer a question of school education *to* indigenous peoples, but rather school education *of* indigenous peoples, *with* indigenous peoples, and *by* indigenous peoples. Thus, in 1991 the responsibility for indigenous school education was transferred from FUNAI to the Ministry of Education. Its execution, meanwhile, became the responsibility of states and municipalities (Grupioni 2004). However, as reported by Paladino and Almeida (2012), in the years following this transfer of responsibility, there was still a dispute between FUNAI and MEC for the prerogative of conducting school education in different indigenous contexts.

The current phase of indigenous school education in Brazil is no longer characterized by the religious bias of catechization—be it civilizatory or colonial—that marked the previous phases. It is based on the recognition of the right to cultural difference and to the affirmation of identity. Cultural difference is also recognized from an epistemological point of view, and that is why research in ethnomathematics developed with indigenous peoples has the potential to contribute with subsidies for the reformulation of curricula, the elaboration of specific didactic materials, the formulation of differentiated public policies, and the training of indigenous teachers.

Regarding the training of indigenous teachers at the higher-education level in particular, the beginning of the 2000s marked a new phase for Brazilian public policies. The year 2002 saw the creation of the Diversity at the University Program (*Programa Diversidade na Universidade*—PDU), resulting from the demands of social movements and from the international agenda assumed by Brazil after the Durban Conference. The PDU was followed by the Support Program for Higher Education and Indigenous Licentiate Programs (PROLIND), created in 2005 on the basis of the work by the Special Commission for Indigenous Higher Education Policies (Nascimento 2017).

In this favorable context, the first specific courses offering exclusively indigenous licentiate (teaching habilitation) degrees emerged. The first was created by the Mato Grosso State University in 2001, followed by the Federal University of Roraima in 2003. Currently, there are 25 courses across institutions in 16 Brazilian states. The spaces created by these courses have been configured as instances for the production of new knowledge, including ethnomathematical research. These researches have been prompted by the need for training the non-indigenous educators who are part of educational institutions' teaching staff, or even by the interests of indigenous teachers themselves, by way of their course completion works, scientific initiation projects, and the production of specific teaching materials.

7.3 Methodology

This is a bibliographic research (Moreira and Caleffe 2006) regarding the state-of-the-art Brazilian ethnomathematical research on indigenous themes, with a focus on indigenous school education. Research of this nature seeks to map and discuss academic works in a given field of knowledge, trying to identify which aspects and dimensions have been highlighted at different times and places, as well as the conditions under which these researches have been produced for different types of publications. They make use of "a methodology [that seeks to carry out] an inventory and description of academic and scientific production on the investigated subject, in light of categories and facets that are characterized as such in each work and in the entire body of works" (Ferreira 2002, our translation). The phenomenon is then analyzed using these categories and facets.

Other works on the Brazilian academic production in the field of ethnomathematics have already been carried out. A few highlights are Conrado's master's research (2005) on theses and dissertations in ethnomathematics produced from 1985 to 2003, the study by Costa (2012) on articles in the area published in national journals, as well as the works by Fantinato (2013), Martins and Gonçalves (2015), and Rosa and Orey (2018) analyzing the proceedings of Brazilian congresses. With regard to indigenous education, there is the work by Oliveira (2018), who analyzed the approximations between interculturality and ethnomathematics in academic productions on the indigenous theme, as well as our own previous works, which represent the starting point for the present article: our study of Brazilian research in mathematics education located at the interface between ethnomathematics and indigenous school education (Leite 2017), and our research on trends of studies on indigenous education at Brazilian ethnomathematics congresses (Fantinato and Silva 2019).

Since it would be impossible to discuss the entire Brazilian ethnomathematical production on indigenous themes, we opted to establish a few specific focus points, so as to build our database—this is the first step in our chosen methodology.

The first focus point was established by the choice of a temporal filter encompassing the period between 2002 and 2019. The year 2002 was chosen as an initial milestone, since it marked the beginning of the implantation of intercultural indigenous licentiate degrees across different regions of Brazil. From the standpoint of curricular proposals, these courses—whose students are indigenous people in search of an academic training that meets the demands of their communities—brought significant challenges to universities. Many teachers with indigenous licentiate degrees began to take an interest in indigenous issues and to seek training at the master's and doctoral level. On the other hand, indigenous licentiate undergraduates began to develop research projects in partnership with university professors. This entire movement increased the academic production on indigenous themes.

As for data sources, we worked with three types of production: theses or dissertations from the Catalog of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (CAPES), texts published in the

proceedings of Brazilian Ethnomathematics Congresses, and articles published in two mathematical education journals: Bolema and Zetetiké.

CAPES is a foundation linked to the Ministry of Education. It operates in the expansion, consolidation, and evaluation of *stricto* sensu graduate programs in Brazil. It also invests in researcher training and in international scientific cooperation. Master's and doctoral courses must regularly update the CAPES database with information on dissertations and theses defended by their students. For our research, we carried out a survey using the descriptor *ethnomathematics* and applying the aforementioned temporal filter. This resulted in 436 theses or dissertations. We then read the abstracts to separate works that fit within the indigenous theme. This resulted in 64 papers.

The Brazilian Congress of Ethnomathematics is the only national event specific to this research area. The First Brazilian Congress of Ethnomathematics (CBEm1) took place at the University of São Paulo, in 2000. Since then, these events have occurred every 4 years. The Second Brazilian Congress of Ethnomathematics (CBEm2) took place in 2004 at the Federal University of Rio Grande do Norte; the Third (CBEm3) was held at the Federal Fluminense University in 2008; the Fourth (CBEm4) took place at Federal University of Pará, in 2012, and, finally, the Fifth (CBEm5) was held at the Federal University of Goiás. In view of its 4-year periodicity and the organizers' intention of holding the event in different Brazilian regions, the next instalment, the Sixth Brazilian Congress of Ethnomathematics (CBEm6), should take place at the Federal University of Tocantins, in 2020.

Since 2000, CBEms have represented the main scientific space for the dissemination of ethnomathematical academic production. For this reason, we chose to work with the texts published in CBEms' proceedings, as long as they remained within our temporal filter. This restricted our data source to texts published in the proceedings of the CBEm2, CBEm3, CBEm4, and CBEm5. Specifically, we selected works presented as scientific communications or lectures, leaving posters and experience reports out of the sample. From this group, we selected 40 additional works.

There are many Brazilian journals (in the area of education in general and in the field of mathematical education) where we might be able to find publications related to our theme. Due to the limits of this work, however, we chose to focus on the production of two journals that "have become broad, diversified and representative references of Brazilian academic production in Mathematical Education" (Costa 2012, our translation): the Mathematics Education Bulletin (*Boletim de Educação Matemática*—Bolema) and the *Zetetiké* Journal.

Although it originated and is based at the State University of São Paulo (UNESP), Rio Claro, Bolema has become a national journal with an editorial body and consultants from throughout Brazil and the world. "It received a high score in an evaluation done by CAPES, the national body that guides the policies of the Ministry of Education regarding graduate study and research,[1]" and has published uninterruptedly since 1985.

[1] http://www.scielo.br/revistas/bolema/iaboutj.htm, Access on February 18, 2020.

Zetetiké is an institutional publication of the Faculty of Education of the University of Campinas (UNICAMP), currently brought forward in partnership with the Federal Fluminense University (UFF). Since its first publication in 1993, it has contributed to the development and dissemination of research in Mathematical Education, as well as to "training researchers in this field."[2] Among works published in these two journals between 2002 and 2019, we found 22 addressing the indigenous theme. Thus, the total sample of our research consisted of 126 works. Table 7.1 shows the sample's totals according to the type of publication and in general.

Among works published in these two journals between 2002 and 2019, we found 22 addressing the indigenous theme. Thus, the total sample of our research consisted of 126 works. Table 7.1 shows the sample's totals according to the type of publication and in general.

After this selection, we prepared a spreadsheet listing all 126 works, placed in chronological order and grouped according to the three types of sources consulted, as presented in this chapter's Annex. We then moved on to the second methodological stage: reading the abstracts and/or full texts, identifying the following characteristics: work's title; year of production; authors' names; first author's institution; Brazilian region where said institution was located.

Simultaneously, we sought information on indigenous or non-indigenous authorship and the ethnicities represented in the works. To this end, we had to consult the works' full texts. During this reading, we tried to identify texts that fit within the theme of Indigenous School Education (ISE), grouping them according to the following categories, considering what we regarded as the work's central theme: *Teacher education*; *Curriculum*; *Culture and cultural identity*, *Pedagogical practices*; *Teaching material*. This selection refinement resulted in 61 texts for analysis. Some studies were left out of this thematic categorization because they could not be considered ISE works. Examples were articles describing indigenous knowledge, or papers that did not specifically deal with ethnomathematics.

The third and final methodological stage consisted of re-reading the 61 previously selected ISE texts, in order to qualitatively analyze them. Aspects such as the works' main themes, the area's current theoretical and methodological challenges, and the growing indigenous author participation were emphasized.

Table 7.1 Brazilian production in ethnomathematics on indigenous themes from 2002 to 2019

Type	Quantity
Scientific presentation	40
Article	22
Thesis or dissertation	64
Total	126

[2] https://periodicos.sbu.unicamp.br/ojs/index.php/zetetike/about, Access on February 18, 2020.

7.4 Research Overview

According to the resulting database, the Brazilian ethnomathematical production on indigenous themes has grown over the years.

7.4.1 Temporal Distribution

We separately analyzed the temporal distribution of each type of production, so as to avoid disregarding their specificities.

Theses or dissertations are registered annually in the CAPES Catalog by the respective Graduate Programs. Figure 7.1 shows the temporal distribution of these productions within the survey period:

As shown in Fig. 7.1, the indigenous theme acquired a greater representativeness in ethnomathematical research starting from 2005, while 2011 was the year with the largest number of works in the entire period (9). The period between 2006 and 2016 had a significant production, with 57 theses or dissertations throughout 11 years. This may have been the result of the expansion of intercultural indigenous licentiate degrees, promoted by the Support Program for Higher Education and Indigenous Licentiate Degrees (*Programa de Apoio à Formação Superior e Licenciaturas Indígenas*—Prolind), created in 2005 under the supervision of the Ministry of Education.

Other factors may have been the creation of new federal universities, since their number jumped from 45 in 2003 to 59 in 2010, accompanied by the expansion of vacancies in undergraduate courses—there were 504 thousand in 2001, and 938 thousand in 2010 (INEP 2012)—with the consequent emergence of new graduate programs.

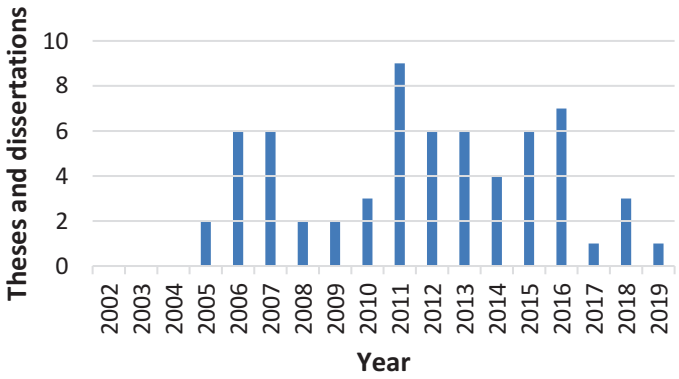

Fig. 7.1 Temporal distribution of theses and dissertations in ethnomathematics on indigenous themes—2002 to 2019

There is an apparent decline in production starting from 2017. This can be explained, perhaps, by the *dismantling* of public policies aimed at indigenous populations during the Temer and Bolsonaro governments,[3] possibly discouraging the development of studies with these peoples.

As for the works sampled from Brazilian Ethnomathematics Congresses, one must be reminded that the period in question saw the organization of four events (CBEm2, CBEm3, CBEm4, and CBEm5), separated from each other by 4-year intervals. Table 7.2 indicates the number of communications on indigenous issues per congress:

The table points to a trend of growth within the time period of the first three congresses, a higher incidence of works at the 2012 congress (14 communications) and a slight decrease in CBEm5, with 11 scientific communications on the topic. It should be noted, however, that our sample did not include posters or experience reports. This may have biased this partial result.[4]

The temporal distribution of articles published in the Bolema and Zetetiké periodicals proved more irregular than the one observed in other data sources. Figure 7.2 illustrates the annual number of publications on the subject in question during the period established by our temporal filter:

The figure shows years with no occurrence whatsoever, followed by years with a significant number of publications. In fact, 2008 saw no published articles, while in 2009 there were 5. The year 2018 featured the largest number of articles (11), but was preceded and followed by 2 years (2017 and 2019) in which the number of published articles on indigenous communities was zero. We believe that this quantitative result could be better interpreted had we included a wider range of journals as data sources.

Table 7.2 Scientific communications on indigenous themes in CBEms held between 2002 and 2019

Event	Quantity
CBEm2	5
CBEm3	10
CBEm4	14
CBEm5	11
Total	**40**

[3] Since the impeachment of President Dilma Roussef in 2016, sucessive governments have made clear that policies aimed at the indigenous population have a low priority. The Secretariat for Continuing Education, Literacy, Diversity and Inclusion (SECADI/MEC), created in 2012, worked on the development of education systems, so as to valorize difference and socio-cultural diversity. It also developed actions in the field of Indigenous Education and alongside *quilombola* peoples, among other groups. On January 2, 2019, by means of Decree n° 9.465, the newly installed president Bolsonaro extinguished SECADI, a clear picture of the *dismantling* of protection policies targeting Brazil's originary peoples.

[4] Fantinato and Silva (2019) observed a proportional growth trend among works on indigenous themes from CBEm4 to CBEm5, as compared to the total works of each event.

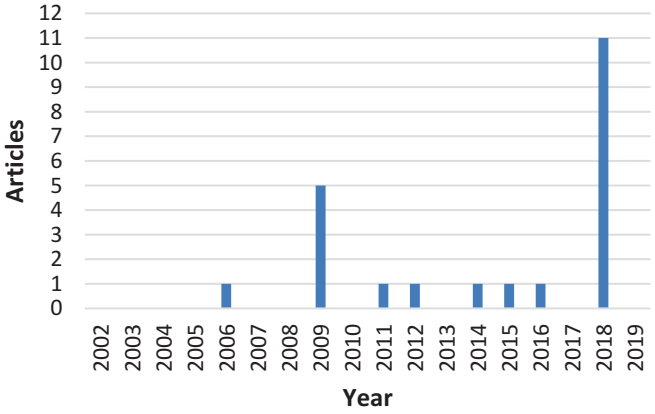

Fig. 7.2 Temporal distribution of articles on indigenous themes published in Bolema or Zetetiké

However, it should be noted that journals operate not only in a continuous flow mode, but also in thematic dossier mode, which seems to have an important role in stimulating publications on a given topic. In our research, the high incidence of works in 2018 can be explained by the thematic dossier organized by Zetetiké during that year, entitled *Saberes e práticas matemáticas na educação (escolar) indígena* (Knowledge and mathematical practices in indigenous [school] education). The total number of articles for this year corresponds to the number of articles of the dossier.

7.4.2 Regional Distribution

Data on the higher education institutions (HEI) of the first authors of the 126 works allowed us to evaluate the geographical distribution of authorship. Although all Brazilian regions were represented—North (N), Northeast (NE), Midwest (MW), Southeast (SE), and South (S)—there were greater concentrations in certain regions than others, as can be observed in Table 7.3 and Fig. 7.3:

The predominance of the Southeast region (with 40% of works) is not surprising, since most Brazilian universities are concentrated in states of this region. However, the high percentage (27%) of works produced in the Midwest region is noteworthy. By consulting the data on the region's HEIs, we see a high incidence of works from the Federal University of Goiás—UFG (8 works), from the State University of Mato Grosso—UNEMAT (7 works), from the Federal University of Mato Grosso—UFMT (6 works), and from the Federal University of Grande Dourados—UFGD (4 works). We believe that this expressive concentration can be explained by the existence of intercultural indigenous licentiate degrees in the region, and also by the existence of research groups coordinated by ethnomathematics researchers who are professors at these licentiate-degree courses.

Table 7.3 Ethnomathematics production on indigenous themes according to HEI region—2002 to 2019

Region	Quantity
SE	51
MW	34
S	18
N	17
NE	6
Total	126

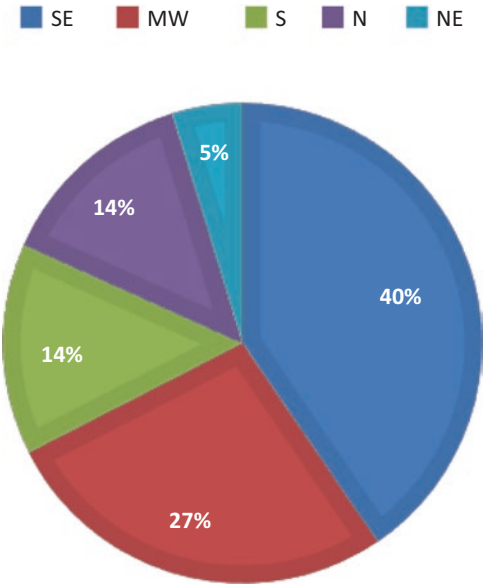

Fig. 7.3 Regional distribution of authors' HEIs—2002 to 2019

7.4.3 Indigenous Ethnic Groups

Sixty-seven (67) indigenous peoples were represented in our database. We recorded all referenced peoples and the number of works in which they appear (shown below in descending order): Guarani (24), Tapirapé (9), Xerente (9), Kaingang (8), Tupinikim (8), Xavante (8), Karajá (7), Javaé (6), Paiter (6), Apinayé (5), Ticuna (5), Pataxó (4), Krahô (3), Makuxi (3), Rikbaktsa (3), Terena (3), Bororo (2), Gavião (2), Ikpeng (2), Kadiwéu (2), Kaiabi (2), Karipuna (2), Mehinako (2), Paresi (2), Tukano (2), Wapixana (2), Xacriabá (2), Aparai (1), Apiaká (1), Ashaninka (1), Assurini (1), Aweti (1), Bakairi (1), Chiquitano (1), Cinta Larga (1), Enawene Nawê (1), Galibi (1), Guató (1), Irantxe (1), Kalapalo (1), Kamaiurá (1), Kaxinawá (1), Kaxuyana (1), Kuikuro (1), Matipu (1), Mebêngokrê (1), Nafukua (1), Noke Koi (1), Oro Win (1), Palikur (1), Panará (1), Pankará (1), Pàrkatêjê (1), Saterê-Mawé (1), Taliáseri (1), Taurepang (1), Tiriyó (1), Tuxaua (1), Umutina (1),

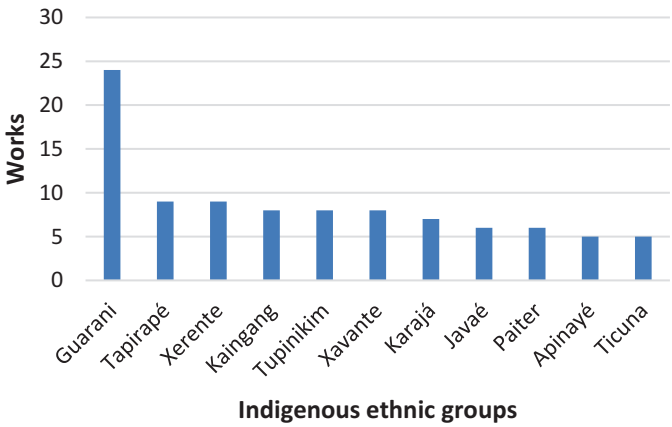

Fig. 7.4 Indigenous ethnicities with the highest representation—2002 to 2019

Waimiri-Atroari (1), Wajãpi (1), Wari (1), Waurá (1), Wayana (1), Xambioá (1), Yudja (1), and Zoró (1).

By considering only the most-often represented ethnicities, we obtained the graph in Fig. 7.4.

With no distinction between their subgroups, the Guarani people stand out from the rest in terms of number of citations (24). This is not surprising, considering that the Guarani are the largest indigenous population in Brazil (85,255 people, according to the 2016 Guarani Continental Map[5]).

Moreover, their territories are spread across the South, Southeast, and Central-West regions of Brazil, which have the largest concentrations of research groups. This direct relationship—which occurs more frequently with research groups of the SE, MW, and S regions—is also observed in the case of the Kaingang, Tupiniquim, and Xavante peoples. In the North region—the one with the largest indigenous population in the country, according to the 2010 Census[6]—the Xerente, Paiter, Apinayé, Ticuna ethnic groups stand out in terms of mentions in academic works. References to the Tapirapé, Karajá, and Javaé, on the other hand, are distributed between works produced in the MW and N regions. The low representativeness of indigenous peoples in works produced in the Northeast—they are entirely absent from Fig. 4—is worthy of notice.

References to indigenous peoples were made in different ways. Often, researches approached a specific people, their knowledge, practices, and conceptions about school. In other cases, works discussed several ethnic groups simultaneously, as in reports on formative experiences carried out by universities. We should also comment on the presence of works (13, about 10% of our sample) that did not specify

[5] https://pib.socioambiental.org/pt/Povo:Guarani. Access on February 21, 2020.

[6] https://pib.socioambiental.org/pt/O_Censo_2010_e_os_Povos_Ind%C3%ADgenas. Access on February 21, 2020.

which ethnic group participated in the research. These employed generic terms, such as *peoples of the Xingu*, *peoples of São Paulo*, or simply *indigenous people*. We believe that naming the indigenous people involved in the study is an important strategy. This confers visibility to the ethnicity in question and to indigenous movements in general. It is also our understanding that identifying ethnicity should be a primary requirement for works claiming to develop research in the field of ethnomathematics.

Developing work *with* indigenous people, preferably involving *indigenous authors*, should be another requirement. Very few publications had this characteristic: only 6 out of 126 (4.7%) were co-authored with indigenous people; among these, there were no theses or dissertations. Had we included posters[7] in our sample, this number would have been slightly higher.

We believe that congresses are more accessible spaces for the dissemination of works than academic journals such as Bolema and Zetetiké. In order to write dissertations or theses, indigenous people must attend graduate programs. Although this is already happening in some universities, until 2019 this phenomenon was yet to generate ethnomathematical works on ISE that had been included in the Capes thesis catalog.

7.4.4 Thematic Areas of ISE Research

Among the 120 works in our sample, only 61 dealt with the theme of Indigenous School Education proper. The other works either described knowledge and mathematical practices unrelated to school education, or were proposals for teaching school mathematics with the use of materials produced by local cultures, an attempt to contextualize the curriculum of a non-indigenous school education installed in indigenous communities.

For this characterization, we took into account not only the presence of the category "Indigenous School Education" in the texts, but also evaluated whether its definition and the way it was conceived by the authors were in accordance with the current conception of ISE—which regards it as a basis for differentiated organizational proposals of school education in Brazil's indigenous territories. Such a conception, originated in the fourth phase of ISE's history (as presented at the beginning of this chapter), encompasses the presuppositions of interculturality, autonomy of peoples, confrontation with colonialism, as well as the promotion and valorization of these peoples' original forms of knowledge, together with their languages, cultures, and identities.

In this sense, it is worth mentioning that, although some of the mapped productions mentioned ISE, a qualitative analysis made it possible to ascertain that they sought to prescriptively propose teaching methodologies or pedagogical practices.

[7] For example, in CBEm5, there were three posters with indigenous and non-indigenous authorships.

Table 7.4 Main themes of Brazilian research in ethnomathematics and indigenous school education—2002 to 2019

Topic	Quantity
Teacher education	32
Culture and cultural identity	10
Curriculum	10
Pedagogical practices	8
Teaching material	1
Total	**61**

While starting from local knowledge, these prescriptions ultimately aimed at promoting the kind of school mathematics that has been institutionalized in the curricula and teaching materials of indigenous schools since the early stages of the history of indigenous school education in Brazil. Thus, through a qualitative analysis of the 61 mapped productions that properly contemplate ISE, we identified five main research themes, as shown in Table 7.4.

Table 7.4 shows that the research theme "Teacher education" is covered by most of the identified works. This may be related to the implantation of intercultural indigenous licentiate-degree courses in Brazil from the 2000s onwards, as a result of the demands of the indigenous peoples for the formation of higher-education indigenous teachers. Such demands were converted into public policies for indigenous school education during the Lula (2003–2010) and Dilma (2011–2016) governments, through government programs (PROLIND, PIBID-Diversidade, *Saberes Indígenas na Escola*), specific legislation, and the creation of courses in public universities.

With the entry of students from different indigenous peoples into universities, especially those with intercultural licentiate degrees, collectives of professors and researchers were created in the departments, faculties, institutes, and academic centers. These collectives required training regarding the cultures, languages, and knowledge of indigenous peoples. In this scenario, the training of formative professors in graduate programs started to include, in several cases, the development of ethnomathematical research on indigenous themes. Hence it is possible to verify that, to a large extent, the authors of research in ethnomathematics and ISE are also responsible for training indigenous teachers in intercultural licentiate courses.

Other topics that are no less important to ISE were also contemplated by the evaluated works, namely: culture and cultural identity (10), curriculum (10), pedagogical practices (8), and teaching material (1). This significant quantitative gap between research in ethnomathematics regarding the training of indigenous teachers and research on other topics will likely be reduced as the training of formative professors is consolidated and training spaces are occupied by indigenous people, so that other important aspects of ISE organization can become predominant themes of researches and academic works in the country.

7.5 Ethnomathematics Research on Indigenous Knowledge and Practices: Current Challenges

The partial results of our bibliographic study on Brazilian ethnomathematics research regarding indigenous themes point to some recurring aspects and indicate some topical challenges for the area.

Firstly, the academic production on this subject has grown over the past 18 years. In our sample, this growth appears mainly in congresses' proceedings, as compared to theses/dissertations and the analyzed journals. With the increasing access of indigenous people to the academic environment, as in the case of graduate courses, we assume that this growth will also be verified in other types of publications. This development, however, may be affected by the recent policies of the Brazilian government, which have made a point of ignoring the demands of indigenous peoples in Brazil—particularly those of indigenous movements—for intercultural and bilingual education.

Regional distribution shows that, although works are still predominantly from the Southeast, there is a tendency of expansion in the Brazilian regions where there are large concentrations of indigenous populations. Intercultural indigenous licentiate degrees at public universities have boosted production, bringing forward new themes and actors. The *Teacher education* theme predominated among ISE works, as a result of the many researches reflecting on the experiences of teacher formation courses. Ethnomathematics researchers who were coordinators of research groups at their universities have played an important role in the expansion of this production in their respective regions, advising dissertations and theses on the theme.

The represented ethnicities are distributed across all Brazilian regions, but make up only 22% of the total of indigenous peoples living in Brazil. Due to its political dimension (D'Ambrosio 2001), it is our belief that the field of ethnomathematics should contribute to the visibility of these indigenous nations, emphasizing their knowledge, practices, and education. The reference to the ethnicity alongside which the work was done is fundamental for this visibility.

Indigenous authorship is still not very expressive in the context of academic production, but it is gradually gaining space. In our sample, it started appearing in some scientific communications and articles from 2008 onwards, but always in co-authorship with non-indigenous researchers. Although timid, the participation of these indigenous authors already points to certain changes in the profile of academics in the field of ethnomathematics. This change may lead to the deepening of the epistemological, methodological, and pedagogical aspects of Brazilian research on indigenous themes. Stimulating the development of indigenous authorship in academic works that relate to their sociocultural contexts means moving towards an "ecology of knowledge[8]" (Santos 2019, our translation), and is consistent with the perspective of decolonialism (Lander 2000).

[8] The epistemological diversity of the world, which Boaventura de Souza Santos termed an "ecology of knowledges," is the "recognition of the co-presence of different knowledges of [something]

Due to the volume of data generated in this work and its length restriction, it was not possible to deepen the qualitative analysis of the identified works, especially those that specifically approached ISE. Thus, in this study's follow-up, we will attempt to identify the methodological trends of these works and the theoretical references supporting their development.

Appendix: Ethnomathematics productions on indigenous themes—2002 to 2019

Type	Authors	Title	Year
Scientific communication	Roseli de Alvarenga Correa	A Etnomatemática na Escola Indígena	2004
Scientific communication	Giani R. da Silva	Breve relato de experiências em Etnomatemática na Escola Indígena Kadiwéu, Mato Grosso do Sul	2004
Scientific communication	Jackeline Rodrigues Mendes, Claudido Lopes de Jesus	Relações entre saberes tradicionais e saberes provenientes do contato com o mundo não indígena: um exemplo na formação de auxiliares de enfermagem indígenas no Xingu	2004
Scientific communication	Wanderleya Gonçalves Costa	A simetria na cultura Bororo	2004
Scientific communication	Roseli de Alvarenga Correa, Caroline Mendes dos Passos, Dirceu Cenem dos Santos	Uma proposta pedagógica de Matemática para a Escola Indígena	2004
Dissertation	Rodrigo Alexandro Rodrigues	As "ticas" de "matema" dos índios Kalapalo: Uma interpretação de estudos etnográficos'	2005
Dissertation	Ozirlei Teresa Marcilino	Uma abordagem étnomatemática no ensino e aprendizagem de Matemática nas aldeias Tupinikim e Guarani do Espírito Santo	2005
Dissertation	Claudio Lopes de Jesus	A etnomatemática das práticas cotidianas no contexto de formação de profissionais indígenas no Xingu	2006
Thesis	José Pedro Machado Ribeiro	Etnomatemática e formação de professores indígenas: um encontro necessário em meio ao diálogo intercultural	2006

(continued)

and the need to study the affinities, divergences, complementarities and contradictions that exist between them, so as to maximize the effectiveness of resistance struggles against oppression" (Santos 2019, p.28, our translation).

Type	Authors	Title	Year
Thesis	Rogério Ferreira	Educação escolar indígena e etnomatemática: a pluralidade de um encontro na tragédia pós-moderna	2006
Dissertation	Katia Cristina de Menezes Domingues	Interpretações do papel, valor e significado da formação do professor indígena do Estado de São Paulo	2006
Dissertation	Vanilda Alves da Silva	Noções de contagens e medidas utilizadas pelos Guarani na reserva indígena de Dourados - Um estudo etnomatemático	2006
Dissertation	Adailton Alves da Silva	A Organização Espacial A ÙWe – Xavante: um olhar qualitativo sobre o espaço	2006
Article	Wanderleya Nara Gonçalves Costa; de Menezes Domingues, Kátia Cristina	Educação Matemática, Multiculturalismo e Preconceitos: que homem é tomado como medida de todos os outros?	2006
Dissertation	Adão Oliveira	Etnomatemática dos Taliáseri: medidores de tempo e sistema de numeração	2007
Dissertation	Augusta Aparecida Neves de Mendonça	Práticas pedagógicas nas aulas de matemática: um estudo exploratório nas escolas Xacriabá	2007
Dissertation	Dóris Reis de Magalhães	Concepções, crenças e atitudes dos educadores Tupinikim frente à matemática	2007
Dissertation	Elisângela Aparecida Pereira de Melo	Investigação etnomatemática em contextos indígenas: caminhos para a reorientação da prática pedagógica	2007
Thesis	Walmir Thomazi Cardoso	O Céu dos Tukano na escola Yupuri: construindo um calendário dinâmico	2007
Thesis	Wanderleya Nara Gonçalves Costa	A etnomatemática da alma A'uwe-xavante em suas relações com os mitos	2007
Thesis	Aparecida Augusta da Silva	Em busca do diálogo entre duas formas distintas de conhecimento matemático	2008
Dissertation	Leila Andrade	Etnomatemática: a matemática na cultura indígena	2008
Scientific communication	Elisângela Aparecida P. de Melo, Hélio Simplício R. Monteiro, José Ricardo Mafra	Cultura, história e educação etnomatemática no contexto dos indígenas karajá do estado do Tocantins	2008
Scientific communication	Elisângela Aparecida Pereira de Melo	Alguns aspectos da tradição da cultura xerente saberes e fazeres: perspectiva para o ensino de matemática na escola indígena Srêmtôwe	2008
Scientific communication	Ozirlei Teresa Marcilino, Circe M. Dynnikov	Ensino e aprendizagem na educação indígena do Espírito Santo: A busca de um diálogo com a etnomatemática	2008

(continued)

Type	Authors	Title	Year
Scientific communication	Giani R. da Silva, Giovani J. da Silva	Educação matemática em contextos culturais indígenas: apontamentos preliminares sobre Guató e Kadiwéu	2008
Scientific communication	Wanderleya Nara Gonçalves Costa	Educação matemática em escolas indígenas- A câmara de espelhos	2008
Scientific communication	Dóris Reis de Magalhães	"Matemática: que bicho é esse?" a matemática para os educadores Tupinikim e Guarani no Espírito Santo	2008
Scientific communication	Jackeline Rodrigues Mendes	Educação matemática no contexto indígena: processos identitários que emergem das relações entre conhecimento e escrita	2008
Scientific communication	Dóris Reis de Magalhães, Ozirlei Teresa Marcelino	Uma visão tupinikim a partir de elaboração de recursos didáticos em matemática	2008
Scientific communication	Eduardo Sebastiani Ferreira, Maria Betriz Rocha Ferreira,Glauco Gomes de Matos, Marcelo Waimiri, Joanico Atroari	Desafios - atividades corporais e etno-matemática entre os Waimiri-Atroari	2008
Scientific communication	Adailton Alves da Silva	Etnomatemática no contexto cultural/ escolar do povo Tapirapé	2008
Dissertation	Giovana Maciel	A Didática da Matemática na formação do professor indígena: possibilidades de relação com a Etnomatemática	2009
Dissertation	Maria Aparecida Mendes de Oliveira	Práticas vivenciadas na constituição de um curso de licenciatura indígena em matemática para as comunidades indígenas Guarani e Kaiowá de Mato Grosso do Sul	2009
Article	Maximino Rodrigues, Rogério Ferreira, Maria do Carmo Domite	A formação de professores e suas relações com cultura e sociedade: a educação escolar indígena no centro das atenções	2009
Article	Wanderleya Nara Gonçalves Costa, Kátia Cristina Domingues, Silvanio de Andrade	Uma análise de práticas discursivas e não discursivas sobre o ensino de matemática em contextos indígenas	2009
Article	Circe Mary da Silva, Lygia Arantes Sad	Avaliação em Matemática no contexto da Educação Indígena:	2009
Article	Wanderleya Nara Gonçalves Costa	Do Labor aos Mitos: uma nova linha no mapa das pesquisas em Etnomatemática	2009
Article	Ole Skovsmose, Helle Alro, Paola Valero, Pedro Paulo Scandiuzzi, Ana Pauloa Silvério	"Antes de dividir temos que somar": 'entre-vistando' foregrounds de estudantes indígenas	2009

(continued)

Type	Authors	Title	Year
Thesis	Helena Alessandra Scavazza Leme	Formação superior de professores indígenas de matemática em Mato Grosso do Sul: acesso, permanência e desistência	2010
Dissertation	João Severino Filho	Marcadores de tempo indígenas: educação ambiental e etnomatemática	2010
Thesis	Cláudia Alessandra Costa de Araújo Lorenzoni	Cestaria guarani do espírito santo numa perspectiva etnomatemática	2010
Dissertation	Claudia Angela da Silva	Práticas de um professor de matemática em contexto multicultural	2011
Dissertation	Hélio Simplício Rodrigues Monteiro	Magistério indígena: contribuições da etnomatemática para a formação dos professores indígenas do Estado do Tocantins	2011
Dissertation	Iozodara Telma Branco De George	Conhecimentos (Etno)Matemáticos de Professores Guarani do Paraná	2011
Thesis	Lucí Teresinha Marchiori dos Santos Bernardi	Formação continuada em matemática do professor indígena Kaingang: enfrentamentos na busca de um projeto educativo	2011
Dissertation	Luzia Voltolini	Conhecimentos matemáticos: um contexto em transição na comunidade indígena Serra da Moça	2011
Dissertation	Rejane Maria Caldas Freitas	Sistemas numéricos e ensino de ciências: construção do conhecimento matemático de povos indígenas do Alto Rio Negro	2011
Dissertation	Sérgio Florentino da Silva	Sistema de numeração dos Guarani: caminhos para a prática pedagógica	2011
Dissertation	Timóteo Gama da Silva	Construção da canoa indígena: contribuições para o ensino da matemática nos anos iniciais da escola da comunidade indígena de Terra Preta no Baixo Rio Negro	2011
Dissertation	Carlos Eduardo de Souza	VËNHNÏKRÉN: um estudo sobre a matemática kaingang	2011
Article	João Severino Filho, Elias Januário	Os marcadores de tempos indígenas e a etnomatemática: a pluralidade epistemológica da ciência	2011
Dissertation	Gleide de Almeida Ribeiro	Etnomatemática: situações, problemas e práticas pedagógicas na realidade do sistema educacional Macuxi em Roraima	2012
Dissertation	Lozodara Telma Branco de George	Conhecimentos (etno) matemáticos de professores guarani do Paraná	2012
Dissertation	Lucélida de Fátima Maia da Costa	A etnomatemática na educação do campo, em contextos indígena e ribeirinho, seus processos cognitivos e implicações à formação de professores	2012

(continued)

Type	Authors	Title	Year
Dissertation	Patrícia Magalhães Pinheiro	A construção da escola Apyãwa/Tapirapé a partir da práxis dos professores em formação na Licenciatura Intercultural Indígena da Universidade Federal de Goiás	2012
Dissertation	Rafaella Rodrigues Santos	Análise crítica das ações pedagógicas dos professores Apyãwa/Tapirapé graduandos do curso de Licenciatura Intercultural da Universidade Federal de Goiás	2012
Dissertation	Ruana Priscila da Silva Brito	Apropriação de práticas de numeramento em um contexto de formação de educadores indígenas	2012
Article	Luci dos Santos Bernardi, Ademir Donizeti Caldeira	Educação Matemática na Escola Indígena sob uma Abordagem Crítica	2012
Scientific communication	Rafaella Rodrigues Santos, Rogério Ferreira	Oralidade, escrita e escola: visões de um pretérito inacabado	2012
Scientific communication	Adailton Alves da Silva	Alguns aspectos do sistema de contagem A'uwe/Xavante: outros saberes, outros conhecimentos, outras soluções	2012
Scientific communication	José Roberto Linhares de Mattos, Geraldo Aparecido Polegatti	Educação escolar indígena através de uma currículo etnomatemático	2012
Scientific communication	Eulina Coutinho Nascimento, Marcos Paulo Souza da Silva	Da aldeia para a cidade: a matemática da etnia Paresi	2012
Scientific communication	Leilane Pereira da CostaSilivan Oliveira ApinayeFrancisco Edviges Albuquerque	As potencialidades matemáticas das escolas indígenas das aldeias Apinaye São José e Mariazinha	2012
Scientific communication	Kécio Gonçalves Leite	Notas sobre a etnomatemática Wari	2012
Scientific communication	Erika Mesquista	Alguns conhecimentos da etnomatemática Ashaninka	2012
Scientific communication	Tanabi Sufiatti, Lucí dos Santos Bernardi, Cláudia Glavam Duarte	Geometria em práticas e cestaria dos Kaingang da Terra Indígerna Xapecó	2012
Scientific communication	João Severino Filho	Os marcadores de tempos indígenas e a solidariedade entre o ambiente e os povos que o habitam: um olhar etnomatemático	2012
Scientific communication	Hélio Simplício Rodrigues Monteiro, Erasmo Borges de Souza Filho	Professores indígenas do Estado de Tocantins em formação inicial: a etnomatemática no contexto do Magistério Indígena	2012
Scientific communication	Sérgio Florentino da Silva, Ademir Donizeti Caldeira	Sistema de numeração guarani: aspectos básicos	2012

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Type	Authors	Title	Year
Scientific communication	Lucélida de Fátima Maia da Costa, Evandro Ghedin, Isabel Rodrigues de Lucena	Um olhar etnomatemático sobre a formação de professores indígenas	2012
Scientific communication	Luci dos Santos Bernardi, Elemcristiane Borges	Oralidade, cuidado e aprendizagem da criança Kaingang: um projeto colaborativo	2012
Scientific communication	Ruana Priscila da Silva Brito, Maria da Conceição Fonseca	Apropriação de práticas de numeramento em um contexto de formação para educadores indígenas: um estudo etnomatemático	2012
Thesis	Adailton Alves da Silva	Os artefatos e mentefatos nos ritos e cerimônias do danhono: por dentro do octógono sociocultural A'uwẽ/Xavante	2013
Dissertation	Antônio Ferreira Neto	A Etnomatemática no cotidiano do ensino indígena em aldeias Paiter Suruí	2013
Dissertation	Geraldo Aparecido Polegatti	A Matemática Rikbaktsa para o povo Rikbaktsa: Um olhar da etnomatemática na Educação Escolar Indígena	2013
Dissertation	Marcos Paulo Souza da Silva	Da aldeia para a cidade: a matemática da etnia Paresi e a inserção escolar indígena	2013
Thesis	Sinval Oliveira	O Saber/fazer/ser e conviver dos Educadores Indígenas Apinayé: algumas reflexões no campo da Teoria da Complexidade e da Etnomatemática	2013
Dissertation	Everton Melo de Melo	Katsitĩ: um estudo sobre a matemática Noke Koĩ	2013
Thesis	Augusta Aparecida Neves de Mendonça	Fechando pra conta bater: a indigenização dos projetos sociais Xacriabá	2014
Thesis	Kécio Gonçalves Leite	Nós Mesmos e os Outros: etnomatemática e interculturalidade na escola indígena paiter	2014
Thesis	Ozirlei Tereza Marcilino	Educação Escolar Tupinikim e Guarani: experiências de interculturalidade em aldeias de Aracruz, no estado do Espírito Santo	2014
Dissertation	Tanabi Sufiatti	O currículo de matemática como dispositivo na constituição do sujeito indígena kaingang contemporâneo da Terra Indígena Xapecó	2014
Article	Bruno José Ferreira da Costa, Thaís Tenorio, André Tenorio	A Educação Matemática no Contexto da Etnomatemática Indígena Xavante: um jogo de probabilidade condicional	2014

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Type	Authors	Title	Year
Dissertation	Abner Márcio Oliveira Teixeira Cicarini	GEOMETRIA PLANA E O GRAFISMO INDÍGENA: O estudo de suas relações no contexto histórico do grupo Tukano de alunos da Licenciatura Intercultural dos Povos Indígenas do Alto Rio Negro	2015
Dissertation	Aldenora Perrone Amador	A geometria das pinturas corporais e o ensino da geometria: um estudo da escola indígena Warara-Awa Assuriní, Tucuruí, PA	2015
Dissertation	Jonatha Daniel dos Santos	Saberes etnomatemáticos na formação de professores indígenas do curso de licenciatura intercultural na Amazônia	2015
Thesis	Michael Lopes da Silva Rolim	Estudantes indígenas nos cursos de bacharelado e licenciatura em matemática da Universidade Federal de Roraima	2015
Thesis	João Severino Filho	Marcadores de tempo Apyãwa: a solidariedade entre os povos e o ambiente que habitam	2015
Dissertation	Suellen de Kássia Lemos dos Reis	Análise da prática docente na Escola Estadual Indígena Maluá da Aldeia Santa Isabel do Morro/Hawalò (TO): diálogo com a formação de professores no curso de educação intercultural da UFG	2015
Article	João Severino Filho	A experiência etnográfica: sobre habitar e ser habitado pelo mundo Apyãwa	2015
Thesis	Aldrin Cleyde da Cunha	A contribuição da Etnomatemática para a manutenção e dinamização da cultura Guarani e Kaiowá na formação inicial de professores indígenas	2016
Dissertation	Darlane Cristina Maciel Saraiva	O ensino e a aprendizagem da matemática na educação escolar indígena da etnia Saterê-Mawé	2016
Thesis	Elisângela Aparecida Pereira de Melo	Sistema Xerente de educação matemática: negociações entre práticas socioculturais e comunidades de prática	2016
Dissertation	Gabriela Camargo Ramos	Sistema de numeração e pinturas corporais javaé: a etnomatemática por uma relação dialógica entre cultura e educação escolar	2016
Thesis	Hélio Simplício Rodrigues Monteiro	O ensino de matemática na educação escolar indígena: (im)possibilidades de tradução	2016
Dissertation	Ronaldo Cardoso da Silva	A arte indígena como instrumento para o ensino da geometria	2016

(continued)

Type	Authors	Title	Year
Dissertation	Wiliam Gonçalves Silva	Sentidos que os estudantes Pataxó da EJA conferem aos conhecimentos matemáticos para as suas vidas	2016
Article	Sérgio Florentino da Silva, Ademir Donizeti Caldeira	Etnomatemática do Sistema de Contagem Guarani das Aldeias Itaty, do Morro dos Cavalos, e M'Biguaçu	2016
Scientific communication	Gerson Franco, Helena Scavazza Leme	Dificuldades em ensinar matemática pelos professores indígenas na aldeia Amambai (Guapoy) município de Amambai - MS	2016
Scientific communication	José Sávio Bicho-Oliveira, José Linhares de Mattos	Saberes Etnomatemáticos de Professores Indígenas em Formação Inicial: Diálogos Interculturais	2016
Scientific communication	Gabriela Camargo Ramos, José Pedro Ribeiro	Reflexões sobre os conhecimentos etnomatemáticos Javaé	2016
Scientific communication	Kécio Gonçalves Leite, Erasmo Borges de Souza Filho	Etnomatemática e etnicidade em contexto indígena: reflexões a partir de discursos de professores paiter	2016
Scientific communication	Ezequiel Oliveira, Aldrin Cunha, Heiracles Dias Batista	As fontes de renda da Aldeia Indígena Te' Yikue e sua contribuição para o desenvolvimento do município de Caarapó – MS	2016
Scientific communication	João Severino Filho	Os marcadores de tempo indígenas e a relação entre o Ethos e a Visão de Mundo de um povo	2016
Scientific communication	Luzinete Benites	O ensino de cálculo de área na escola municipal indígena na aldeia Amambai-MS	2016
Scientific communication	Josinalva Estacio Menezes	Etnomatemática na transposição de algoritmos matemáticos usados pelos estudants indígenas de graduação para os algoritmos acadêmicos e suas inserções no ensino formal e informal	2016
Scientific communication	Noemi dos Reis Corrêa, Geraldo Polegatti, Lucy Gutierrez de Alcântara, Gleka Debacker	Do Mito Dokoi à Engenharia de pesca. A etnomatemática Enawene Nawê nas suas barragens de pesca do Rio Juruena	2016
Scientific communication	Aldrin Cleyde da Cunha, Janielle Melo da Cunha	Tecnologias educacionais: representações sociais de professores indígenas em formação	2016
Scientific communication	Ozirlei Teresa Marcilino	Formação continuada em serviço com os educadores indígenas Tupinikim de Aracruz, Espírito Santo	2016
Dissertation	Aline da Silva Lima	Licenciatura intercultural indigena da UEPA: saberes matemáticos e prática pedagógica	2017
Thesis	José Sávio Bicho de Oliveira	Etnomatemática e práticas pedagógicas: saberes matemáticos escolares e tradicionais na educação escolar indígena Karipuna	2018

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Type	Authors	Title	Year
Thesis	Antônio Ferreira Neto	Ensino e aprendizagem de matemática na educação escolar indígena paiter suruí	2018
Dissertation	Vanessa Nascimento Silva	Projetos Extraescolares do curso de Educação Intercultural e a educação escolar indígena: um olhar etnomatemático sobre os saberes e fazeres Javaé'	2018
Article	Adriano Pawah Suruí, Kécio Gonçalves Leite	Etnomatemática e Educação Escolar Indígena no contexto do povo Paiter	2018
Article	Magda Natália Marin Pires, Wagner Roberto do Amaral, Jean Carlos da Silva Kuboyama	O ciclo intercultural de iniciação acadêmica em uma universidade estadual do Paraná: o fortalecimento dos indígenas na Educação Superior em uma experiência com a Educação Matemática	2018
Article	Mara Rykelma da Costa Silva, Edcarlos Miranda de Souza, Itamar Miranda da Silva	Percurso formativo de professores que ensinam Matemática em escolas indígenas do Acre	2018
Article	Gabriela dos Santos Barbosa	Formação continuada de professores Guarani: um estudo de conceditos sobre números racionais	2018
Article	Luzia Voltolini, Carmen Teresa Kaiber	Saber cultural e a matemática escolar: encontro necessário na educação escolar indígena	2018
Article	José Roberto da Silva, Evilásio Clécio de Souza, Maria Aparecida Rufino	O ritual do Toré como organizador prévio para o conceito de círculo	2018
Article	Hélio Simplício Rodrigues Monteiro	Contribuições da Etnomatemática para formação dos Professores Indígenas do Estado do Tocantins	2018
Article	Maria Aparecida Mendes de Oliveira, Jackeline Rodrigues Mendes	Formação de professores Guarani e Kaiowá: interculturalidade e decolonialidade no ensino de matemática	2018
Article	Ruana Priscila da Silva Brito, Maria da Conceição Fonseca	Aldeia, Matemática e Escola Indígena: apropriação de práticas discursivas por estudantes Pataxó	2018
Article	Lucí Teresinha Marchiori dos Santos Bernardi, Jorge Alejandro Santos	Etnomatemática y Pedagogía Freireana: una experiencia intercultural con la comunidad Kaingang	2018
Article	Elisângela Aparecida Pereira de Melo, Tadeu Oliver Gonçalves	A Comunidade de Prática Xerente e suas matemáticas	2018
Dissertation	Barbara de Medeiros Marinho	A educação escolar indígena do povo Guarani M'bya: uma visão etnomatemática	2019

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Part IV
Urban Diversities

It has been long known that urban contexts offer rich opportunities for ethnomathematics research. Such diversity occurs in the Brazilian context of the schools and distinct cultural specificities that bring out knowledge developed in everyday life. Thus, urban diversity is a relevant issue discussed in relation to inclusive dimensions of education. This is important in the contexts of race, social class, sexual orientation, and deaf culture; all the contexts are important when highlighting non-school knowledge developed in educational practices. It is necessary to seek ways for the development of educational practices that articulate specific/local knowledges.

Chapter 8
Intertwining the Ethnomathematics and the Deaf Culture to Promote Financial Education for Deaf Students

Rodrigo Carlos Pinheiro and Milton Rosa

8.1 Initial Considerations

Deaf[1] students' history shows that, for centuries, they were considered as beings incapable of thinking, rationalizing, and also learning. Aristotle, for example, alleged that *deaf* people were not capable of rationalizing because they did not have a language (Honora and Frizanco 2009). Historically, the Deaf were banished from the listening sociocultural context, since they were identified as individuals who needed charity and, therefore, they could not work, study, and attend social spheres.

Nevertheless, currently, Deaf Education in Brazil has been a heavily debated theme and it has been studied by numerous researchers, resulting in many works, mainly: linguistic issues related to Brazilian Sign Language (Libras), processes of translation and interpretation of Libras to Portuguese and vice versa, teaching Portuguese as a second language to Deaf people, the acquisition of language for Deaf children and the alphabetization and literacy issues in Deaf Education. However, researches about mathematics education for Deaf people in an ethnomathematical perspective can be considered recent and still scarce in Brazil.

[1] In this chapter, the term *Deaf*, with a capital letter, is used to differ it from the terms *hearing-impaired* and *deaf*, which refer to individuals who have a hearing loss and are not characterized by their deficiency, but belonging to their own culture, language, behavior, and knowledge. The use of the term d*eaf*, with lowercase letters, refers to the pathological and audiological condition of individuals who cannot hear (Moura 2000).

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The current context encourages the recognition of the relevance of education networking that makes Deaf students adopt an attitude that can contribute, effectively, to make them more independent, critical, and reflective, and also makes them feel included in their society. Hence, there are some necessary curricular adaptations to be made, so schools become a way of cultural, scientific, and social development to Deaf learners.

In this regard, the National Curricular Parameters (PCN) of mathematics (Brasil 1998) pointed to the inclusion perspective alerting that, in this way, it is necessary to make adaptations related to the creation of physical, environmental, and material conditions; here the authors emphasize Deaf students should be given better levels of communication and interaction with individuals whom they live with in school and community.

Thus, the fourth competency of the Curricular Common National Base—BNCC (Brasil 2017) quotes the Brazilian Sign Language (Libras) when it mentions the importance of using knowledge of verbal (oral and written) and/or visual-verbal (like Libras), physical, multimodal, artistic, mathematical, scientific, technological, and digital languages to express themselves and share information, experiences, ideas, and feelings in different contexts and, with them, produce senses that lead to mutual understanding.

For example, Healy and Ferreira dos Santos (2014) affirmed that attending more closely to the mathematical expressions of those who communicate in sign language may not only contribute to including deaf learners in mathematical practices, it may also open windows on the role of gestures and visual expressions in mathematics learning more generally.

However, it is important to emphasize that, according to this assertion, BNCC (Brasil 2017) considers Libras as a way of communication and not a language. Libras compromises the Deaf educational process from the perspective of providing their independency because it disrespects the subjectivity and cultural identity of a given school population. In this regard, this document has a logic which prioritizes the Portuguese Language (oral and written) and the phonological methods of the teaching and learning process.

Similarly, the BNCC (Brasil 2017) is structured into five fields of experience where learning goals are defined. For example, the fourth field of experience is related to *orality and writing*, which symbolizes a setback in Deaf improvement, since it can involve students who must learn the hearing language through oralization, and thereby excludes this population from active participation in the mathematics teaching and learning process.

In this context, besides the necessary curricular adaptations, it is essential to know and understand Deaf students' specificities in order to trace teaching strategies and methodologies appropriate to this audience. It is also convenient to say that Deaf people belong to a distinct cultural group, specially, when they adopt their own sign language, which is Libras in Brazil, and also develop deaf identities and different world perspective from that one developed by hearing people. In this language, the jargons, the customs, and the ways of thinking and acting are specific from the *Deaf Culture* (Santana and Bergamo 2005).

Considering that Deaf Culture conveys values and beliefs that, many times originated and transmitted by Deaf individuals from past generations or successful Deaf leaders through deaf associations (Strobel 2009), an educational proposal or a research in Education that focus on Deaf individuals cannot be developed without a reflection on the cultural aspects involved in the educational processes. Deaf Education, however, is considered a challenge to many teachers and researchers precisely due to the cultural and linguistic conditions of these people.

Therefore, in proposing mathematics teaching and learning strategies and techniques for this audience, it is necessary to know and understand Libras and Deaf Culture specificities (Pinheiro 2017). It is also important to consider aspects stipulated in the Law N° 10.436, April 24, 2002, which provided regulations regarding Libras, as well as the Decree N° 5.626, December 22, 2005, which regulated this specific law.

These laws are considered indicators in the development of Deaf Education in Brazil because they provide the right of education in their own language by assuring accessibility in many areas of Brazilian society as well as the valorization of their own culture. For example, the first article of the law N° 10.436/2002 acknowledges the Brazilian Sign Language (Libras) and other expression resources associated with this language as a legal means of communication and expression. Libras originated from a motor-visual nature that has its own grammar structure, which enables it to constitute a Deaf linguistic system.

Hence, Deaf Culture can be understood as the way in which Deaf people understand the world and modify it to make it accessible and habitable by adjusting to it with their own perspective, which contributes to the definition of deaf identity and the soul of the Deaf Communities. This means that it covers the language, the ideas, the beliefs, the customs, and the habits of Deaf people (Strobel 2009). It is convenient to explain that the specificity of Deaf cultural groups is associated with the existence of Sign Language in which the jargon, slang, habits, and ways of thinking and acting are common to Deaf Culture and emphasizes the notion of these members' identity (Pinheiro and Rosa 2016).

Deaf students also have multiple identities that were developed in specific sociocultural environments, which were socially, culturally, and politically developed (Morais and Lunardi-Lazzarin 2009). Seen in these terms, Deaf people are considered plural and multifaceted individuals whose identities are built by meeting other Deaf people who use the sign language. This identity is related to a way of constituting themselves as a specific cultural group whose differences politically strengthen sign language, culture, and the Deaf community (Müller 2012).

In relation to Libras, it is possible to state that it is an independent language from the oral language, which has the same linguistic levels of analysis and it is as complex as verbalized languages. In this context, Libras' main feature is its *visual-spatial* modality (Pinheiro 2017). It is important to emphasize that Pereira and Vieira (2009) argue that if bilingualism is defined as the use of two or more languages, then Deaf people who use the sign language and the majority language are bilingual. In this regard, Deaf people who communicate in Libras and use Portuguese in written modality are bilingual as well.

Being bilingual means that people need to know words and sentence structures, understand how to use two grammars, as well comprehend the social and cultural meanings of linguistic communities in which they belong. Therefore, bilingualism, in the context of Deaf people, is only possible if it is associated to biculturalism, which means the identification of specific linguistic groups who use sign language as well the majority language mainly in written modality (Pereira and Vieira 2009).

It is important to emphasize that, from the 1970s, there has been awareness that sign language should be utilized regardless of oral language abilities. That way, the teaching proposal of Bilingual education emerges, and has been disseminated around the world. In this proposal, Deaf people are not considered disabled, but members of a unique and diverse culture that has its own language, which possesses a peculiar way of thinking and acting that must be respected (Goldfeld 2002).

Bilingualism is an educational proposal, which recommends that the Deaf people should be trained in two languages to make access to school and social contexts possible. This proposal considers the Sign Language as the first language of the Deaf people and their native language, Portuguese in Brazil, as the second one. The main assumption of bilingualism is that Deaf people are bilingual because it is important for them to acquire Libras as well as their native language as the second language (Perlin and Strobel 2009).

Because it provides the communication development and performs a significant role of thinking support and stimulator of social and cognitive development in all knowledge fields, sign language is important to the development of all Deaf people (Brito 1993). As previously stated, in Brazil, bilingualism obtained legal recognition through the Decree No.5.626/2005, which establishes:

(a) The insertion of Libras as a curricular subject.
(b) The training of Libras teachers, instructors, and translators/interpreters.
(c) Libras proficiency certification.
(d) Teaching Portuguese as a second language to Deaf students.
(e) The inclusion of Deaf students with the organization of bilingual education in Brazilian education systems.

Because it made the official regulation of Deaf educational rights in Brazilian law possible, this decree initiates the deconstruction of hegemonic views of deafness, and it supports the need to deepen existing cultural dynamics in the classroom so that the value of Deaf Culture and its implications for the teaching and learning process in mathematics are recognized in the educational process.

Seen in these terms, it is crucial to think of teaching practices which acknowledge Deaf living and experiences as well as their history, language, and culture to create diverse and flexible teaching and learning processes in mathematics. Considering these factors, it is possible to state that there is a relation between Deaf Culture and the assumptions related to an ethnomathematics program, which values the mathematics practices developed in other rationalities as it is necessary to offer other possibilities to the teaching and learning process of school mathematics (Rosa 2010). Included, in this context, is financial education.

An important ethnomathematics program component enables a critical view of reality using instruments of mathematical nature which facilitate a comparative analysis of prices, accounts, and budget because it provides an excellent teaching material (D'Ambrosio 2005) that can help in the teaching and learning process of contents related to financial education.

In this context, it is important to state that Deaf students who communicate in Libras and use Portuguese in written and spoken forms are considered bilingual since they are inserted in a specific culture named Deaf Culture. Although there is a need to understand that the development of financial education is important to Deaf students, yet there are also challenges so that the main objective of this study was to comprehend how ethnomathematics program can contribute to the development of financial education for this student population.

Thus, in the next sections, the authors discuss how this investigation is theoretically grounded to the connection between Deaf Culture, ethnomathematics, and financial education. Then, the authors outline methodological procedures based on Grounded Theory that were used to conduct this study. Subsequently, the authors present the analysis and interpretation of some results. As well as the appreciation for the daily experiences of Deaf students, the respect for Deaf Culture, and the valorization of the cultural aspects of mathematics that help the development of pedagogical actions in the classrooms in order to promote the development of Deaf students' financial education. Finally, the authors develop some final considerations about this study.

8.2 Interweaving Deaf Culture, Ethnomathematics, and Financial Education

In Brazil, Deaf people have many traditions and stories created through participation in organizations, associations, federations, and churches, which arose in the face of a need for Deaf people to have a place to meet and learn to combat listening practices that do not respect the needs of the Deaf people (Strobel 2009). However, although Brazilian educational legislation does not yet cover the country's various language communities, political pressures in defense of linguistic rights of different cultural groups have achieved in the last two decades two important accomplishments: recognition of indigenous peoples' rights and the rights of the Deaf Community to access bilingual education, in their first language and in Portuguese.

According to Felipe (2008), this context enables Deaf people to develop a unique way of understanding the world that generates values, shared common behavior and socio-interactive traditions. Consequently, this *modus vivendi* can be called *Deaf Culture*. For example, the term *culture*, in the area of deafness, is generally related to sign language, social strategies, and compensatory mechanisms that the Deaf people develop and use so they can participate in Brazilian society.

It is important to emphasize that, historically, the disability is a label that does not characterize Deaf people because it does not mean sociocultural self-representations, which are not familiar to them. Thus, when Deaf people debate their deafness, they use terms deeply related to their language, origins, traditions, and community (Padden and Humphries 1988).

Sign language is related to Deaf Culture, and refers to the identity of individuals who live with both communities: deaf and hearing. The difference between these communities is one because it is one of the most important aspects that is responsible for the development of the Deaf culture. The recognition and valorization of this difference depends on the ability to apprehend the educational potential of Deaf people regarding to their acquisition of a language whose communication channel is the visual-gestural as well to their linguistic ability, which is manifested in the creation, use, and development of that language (Dorziat 2004).

In addition, Strobel (2009) argues that, for Deaf people to have access to information, knowledge, and the construction of their identity, it is very important that they create bonds with the Deaf Community. It is important to note that this community uses a common sign language, which is one of the main features and peculiarities of the Deaf Culture, as it is a form of communication that captures their visual experiences in order to provide the acquisition and diffusion of their knowledge.

Hence, Strobel (2008) presents eight cultural *artifacts*[2] that illustrate the Deaf Culture and are related to behaviors and attitudes of being Deaf in order to perceive, interact, and change the world. These cultural artifacts are related to:

(a) The *visual experience*, which means the use of sight in total replacement to hearing as a means of communication.
(b) The *linguistic aspects of Libras*, which is the priority of sign language for the Brazilian Deaf people.
(c) The *familiar aspect,* which are the behaviors of families that have Deaf people in their coexistence.
(d) The *Deaf literature*, which discloses the memory of deaf experiences of several generations that translates into different genres.
(e) The *social and sport life*, which is the sociocultural relations, such as parties, leisure, and activities in associations, in marriages between Deaf people, in baptisms performed in sign language and at sporting events.
(f) The *visual arts*, which are visual artistic productions that demonstrate the emotions, stories, subjectivities, and culture of the Deaf people.
(g) The *politics*, which consists of countless movements and struggles of Deaf people for their rights.
(h) The *materials*, which are instruments to assist in the accessibility of Deaf people's daily life.

[2]The concept of *artifacts* does not only refer to cultural materialisms, but also to their cultural aspects that constitute by their members' productions who have their own way of being, perceiving, understanding, and changing the world (Strobel 2008).

These artifacts show the specific characteristics of Deaf Culture, which are related to jargon, scientific and mathematical knowledge, ideas, notions, techniques, and procedures, beliefs, language, and their own diverse customs and habits. According to this assertion, ethnomathematics embraces Deaf Culture because it deals with social issues and school practices that have as a starting point the student daily lives (Rosa 2010). Thus, D'Ambrosio (1993) argues that ethnomathematics restores mathematics as a natural and spontaneous practice practiced by members of distinct cultural groups.

From an ethnomathematical perspective, there is a need to value Deaf sociocultural experiences so that Deaf students are able to link their own mathematical and financial knowledge to those presented by educational institutions. Mathematics teaching in this conception allows students to link concepts worked in classrooms to their everyday experiences in accordance to their own cultural, social, and natural environments. It is important to state that this is not related to the rejection of academic mathematics, but to incorporate values that are experienced in group experiences by considering their historical, social, and cultural ties (Carneiro 2012).

Then, it is possible to establish a connection between Deaf Culture and ethnomathematics to highlight the importance of a school curriculum that centralizes the importance of sign language for the members of this distinct cultural group. However, it is important to emphasize that the literature does not address investigations related to Deaf people regarding the connection between mathematics and financial education in an ethnomathematical perspective.

Nevertheless, research shows that there are challenges and difficulties that Deaf students face as they learn and come to understand mathematical content in classrooms. Studies conducted by Allen (1995) and Marschark and Everhart (1999) show that Deaf students have difficulties in problem solving involving logical thinking and reasoning. However, it is relevant to emphasize that the study results conducted by Zarfaty et al. (2004) reveal that Deaf students' abilities concerned to work with numerical representations to similar to their hearing classmates.

Investigations conducted in the first decade of the twenty-first century also showed that Deaf students present some difficulties in understanding basic mathematical concepts (Kritzer 2008), such as numerical sequences (Leybaert and Van Cutsem 2002), mathematical representation and relations (Blatto-Vallee et al. 2007), and mathematical calculations and problem solving (Traxler 2000).

Similarly, Nunes and Moreno (2002) argued that Deaf students also struggle to understand the concepts of ratios, proportion, and mental calculation. In this direction, almost two decades before, the results of the study conducted by Post et al. (1984) showed that the importance of understanding numerical concepts is related to the development of proportional reasoning for deaf students.

Consequently, the understanding of these contents is basic and central for understanding the concept of percentage and proportional reasoning, as well mathematical content related to financial education (Bailey et al. 2014). Thus, Markey (2003) stated that the evolution of proportional reasoning, the understanding of situations and phenomena that involve problem solving, and the understanding of the applications of financial education concepts are basic mathematical skills that are necessary for the development of critical, reflective, and transformative citizens.

Then, Bertoli (2012) stated that for the development of the process of teaching and learning in mathematics to occur satisfactorily in the classrooms with Deaf students, teachers must support the development of three factors: (a) sign language, (b) mathematical knowledge, and (c) appropriate methodologies. These factors can be considered the basis of an *educational tripod* that helps Deaf students to master Libras, and enables the expansion of their communication process.

However, Bull (2008) argues that it is necessary to recognize that Deaf students frequently need more time to improve mathematical abilities related to problem solving, numerical representation, estimation and measurement concepts, fractions, percentage, and ratio and proportionality. In relation to the development of financial education for Deaf students, it is necessary to link mathematical knowledge to their daily lives, such as economics, sustainability, professional and personal lives, and family financial situation.

These contexts can contribute to the professional and personal growth of Deaf students, as well to their insertion as active citizens in society (Pinheiro and Rosa 2016). In this direction, Campos (2013) affirmed that financial education can be considered as social practices that are concerned about contributing to the acquisition of information regarding to the functioning of financial and economical means, such as interest rate, installments, credit cards, and loans, but also about the consumption decision-making process as consumers.

In this regard, financial education aims to contribute to the development of students' criticality, autonomy, and citizenship through the elaboration of activities that enable the expansion of their critical reflection on the influence that mathematical content related to this field of knowledge can trigger in schools. Consequently, financial education is important for personal well-being and socially oriented actions, as the consequences of financially wrong decisions can disrupt citizens' personal lives, as well compromise their professional careers. In this regard, Lelis (2006) argued that financial education can be used as a tool that enables citizens to manage their own money and their finances.

According to this point of view, financial education can be considered as the ability of making clever judgments and effective decisions in relation to money management (Gallery et al. 2011). In this context, Lelis (2006) highlighted the importance of Financial Education for the development of citizenship, which aims to provide useful economic and financial information so that citizens can increase their income, reduce their expenses, and manage their investment funds by applying mathematical content. However, the results of the study conducted by Pagliaro and Kritzer (2005) indicated that, in general, Deaf students' teachers have difficulties to structure the teaching and learning process in Mathematics to provide them with educational opportunities so that they can succeed.

According to Johnson (1991), a factor that also contributes to complicate Deaf students' learning is that the lack of knowledge in most educators in relation to their culture, even, if unconsciously, they make use of stereotyped views regarding this school population. This fact has ramifications for teacher education and professional development.

A constrained mathematical curriculum continues to be used in classrooms with Deaf students, despite the recognition of their educational demands for a pedagogical reform in the teaching and learning process in mathematics (Ramnarain 2003), mainly with the specific content of financial education, such as ratio, proportion, fractions, and percentage.

According to Bull (2008), mathematical content is considered a challenge for the teaching and learning processes for Deaf students. Thus, it is necessary that teachers contribute to the development of their mathematical abilities and competencies in order to enable the improvement of their autonomous, critical, and reflective reasoning so that they can actively live in society as transformative citizens. For example, Scott-Wilson (2009) argues that school communities need to develop different curriculum in relation to mathematical content.

8.3 Methodological Procedures

The investigation that originated this discussion was conducted with two groups totaling 20 Deaf students, who communicate in Libras, with ages ranging from 15 to 56 years old. These students we enrolled in a public school, in a teaching modality named *Youth and Adult Education*, who were attending the second and third grades of final years of Middle School, in the morning and evening shifts, respectively. This school is located in Belo Horizonte, Minas Gerais, Brazil.

In this context, the first author under the guidance of the second one proposed three blocks of mathematics activities related to financial education, which were developed according to sociocultural realities of the students. The specific blocks of activities in this study were planned applying the perspective of an ethnomathematics program that values Deaf culture by using sign language as the main way of communication as well as the respect for their limitations in relation to the Portuguese language. The first block of activities was in relation to the *History of Currency and the Brazilian Monetary System*, the second block of activities was in relation to *Percentages,* while the third block of activities was related to *Profit and Discount* content.

For data collection, two questionnaires were applied, a semi-structured interview, the notes of the teacher-researcher's field diary, and three blocks of activities. The data were analyzed and interpreted in the course of the research according to the theoretical framework based on the ethnomathematics program, Deaf Culture, and financial education, and also by using the methodological assumptions of *Grounded Theory*. Analysis was developed by using the data collected through activities in Libras that were translated and transcribed by the first author.

The main objective of the first questionnaire was to outline a general profile of the participants in order to obtain information about their sex, age, and economic level, as well to determine how participants understood issues involving money and the importance of mathematics classes to solve the problems they face in their daily lives. The second questionnaire aimed to identify students' perception of the presence of

mathematics in their daily lives, as well to verify if the activities developed from the ethnomathematical perspective contributed to the process of teaching and learning financial education content, as well to identify the existence of signs for mathematical symbols through the elaboration of activities contextualized in their daily experiences.

The main objective of the interviews was related to the verification of whether participants were positively interacting with the proposed activities that were developed according to the ethnomathematics perspective, as well another objective was to verify the proposed activities contributed to the teaching and learning process of financial education content.

It is important to state here that grounded theory was chosen as a research method that it enabled the authors to develop a holistic understanding of the pedagogical action of ethnomathematics regarding to financial education for Deaf students. It also propitiated a comprehension that mathematical knowledge is not static, but is always emerging and transforming, and is interpreted by both authors and participants. Meaning was conveyed through dialogue and action that was embedded in understanding, experience, emotion, and interaction, which enabled meanings to be unlocked and conveyed to the authors.

From this perspective, grounded theory provided a method that enabled the authors adduce true meaning and comprehension to the theme of this study. It allowed the authors to get into the ethnomathematics and financial education fields, and quickly acquire an empirically grounded understanding of sociocultural understanding of Deaf students' culture.

Grounded theory also assumes a set of raw data named *theoretical sampling* that was collected in Sign Language during this study, and was transcribed to facilitate its analysis, as well to interpret the information obtained during the interpretative phase of this study. Subsequently, the data were encoded by using *open and axial coding* to elaborate conceptual categories that made it possible to interpret the results obtained in this investigation.

In *open coding*, after the data collection (theoretical sampling) the researchers carefully analyzed each sentence, line, and paragraph obtained in the collection instruments to identify the preliminary codes. Figure 8.1 shows some examples of preliminary codes identified in the open coding process.

After analysis of raw data, the development of 23 preliminary codes were used to classify conceptual categories. In the axial coding, there was an improvement of the preliminary codes resulting from the open coding. Thus, the authors selected the preliminary codes to group them by relevant information contained in the data. During this analytical coding process, the authors alternated between open and axial coding as conceptual categories were constantly verified by the data that composed them in order to regroup and reorganize them to obtain dense and complete information on the studied problem (Baggio and Erdmann 2011). The application of this methodological method yielded to the elaboration of a set of categories coupled with a subcategory, which provided the authors with an opportunity of having the data inform the development of this research and consequently discovering the theoretical principles that were relevant to the situation under investigation.

Collected Raw Data	Open Coding (Preliminary codes)
"So, you have to do a subtraction to know the change value!" **(2).** "He took a while, so you answered first!" **(8).** "This text is big **(6)**. What does *will be left* mean?" **(6).** "Yes, you have to add them! **(4)**. You have to multiply them!" **(4).** "This activity is too hard! **(5)** I'll help you!" **(9).** "Wow! That's a lot of coins! It's better to draw it on the board" **(7).** "It's really important to study a lot of Mathematics to be able to learn how to do the operations **(1)**, to know the right nomenclature, do addition, multiplication, division, well, geometry itself, so we need to know how to do Math" **(3).**	**(1)** Importance of Mathematics **(2)** Mathematical content application **(3)** Using mathematical knowledge **(4)** Mathematical content **(5)** Difficulties and/or easiness of mathematical content **(6)** Difficulty with written Portuguese **(7)** Strategies to overcome learning obstacles **(8)** Teacher as a mediator **(9)** Learning motivation

Fig. 8.1 Examples of preliminary codes identified in the open coding process (*Source* Personal file)

Therefore, from the preliminary codes obtained in the open coding, the researchers elaborated 03 (three) conceptual categories and a subcategory during the axial coding process. The data were regrouped in new ways, as the authors sought to relate the subcategories and categories, which promoted development of other conceptual codes. Figure 8.2 shows conceptual categories identified in the axial coding process.

After the axial coding development, the authors performed the interpretation of the results obtained during the analytical phase of this study by writing the conceptual categories determined during the coding process.

8.4 Analyzing Some Results: Valuing Deaf Students' Experiences and Respecting Deaf Culture

The results of this study supports the assumption that an important contribution to the ethnomathematics program for the development of financial education for Deaf students is to respect and value their own culture. Also, it is necessary to apply their daily experiences in the process of teaching and learning mathematics, which is relevant to the promotion of a significant relation between everyday knowledge and the one systematized by schools (Pinheiro 2017). This methodological approach was related to the contextualization of everyday facts which was possible to negotiate mathematical meanings in the classroom, thus favoring the construction of financial concepts.

The interpretation of the results of his study showed that it is important to use contextualized activities in the classroom, as they challenge students to relate the knowledge built during the development of their experiences with those developed in the school practice. For example, participant *B25* commented that:

Conceptual Categories and Subcategories
***Category*: Pedagogical Action for Financial Education**
(1) Importance of Mathematics
(2) Mathematical content application
(4) Mathematical content
(5) Difficulties and/or easiness with mathematical content
(7) Strategies to overcome learning obstacles
(8) Teacher as mediator
(9) Learning motivation
(18) Resolution of mathematical operations involving money
(20) Preference for Libras
***Subcategory:* Difficulties and differences in communication for the development of mathematical knowledge**
(6) Difficulty with written Portuguese
(19) Communication problems
***Category*: Mathematics Contextualization**
(3) Using mathematical knowledge
(10) Contexts or problem situations contributing to the connection with Mathematics
(11) Connections between Mathematics and everyday life
(12) Mathematical knowledge diffused from generation to generation
(16) World knowledge
***Category*: Citizenship development**
(13) Financial thinking (reasoning)
(14) Critical and reflective positioning
(15) Financial illusion
(17) Financial and monetary knowledge
(21) Autonomy
(22) Financial dependency
(23) Dependence on a hearer to perform tasks

Fig. 8.2 Conceptual categories identified in the axial coding process (*Source* Personal file)

> If you are going to go shopping, for example, you can use the debit card or pay in cash and receive a discount, you need to do a subtraction and subtract a portion of that amount, so when we take it out we are getting a discount. So, the discount is related to the subtraction operation.

Similarly, participant *B19* argued that the activities carried out in the classroom helped him to calculate the "ticket change and buy things with discounts" while participant *A11* stated that these activities brought "knowledge to solve daily problems." For example, the activity called *Mini Market*[3] promoted the effective participation of all students by directing them toward to the use of their mathematical knowledge so that they were able to think about what they could buy during that simulation.

[3] This activity was developed in group and it aimed to promote knowledge developments related to buying and selling products, to recognize and identify Brazilian money, to make purchases and payments and to check if the change was correct, as well to verify if the money available was enough to buy a specific product. In addition, it was intended to discuss how to save money on purchases and the importance of the discounts on the product prices.

For example, participant *B10* reported that "I will buy a cell phone for my daughter." Then, when she checked if the money she had was enough to purchase this product, this participant used a calculator to calculate the 30% discount determined on a poster located next to that product. It is important to emphasize that this process incorporates the cultural aspects of mathematical knowledge in curricular activities as it aims to enable the understanding of the qualitative character of the quantitative mathematical content proposed in classrooms (Rosa and Orey 2007).

Hence, Miranda and Miranda (2011) argue that mathematics teachers must provide teaching and learning situations in which students can build mathematical concepts such as the use of mathematical games. Besides, it is important that teachers create problem situations that can stimulate Deaf students to elaborate daily connections with mathematics.

The interpretation of data analysis showed that some participants use their own global knowledge to contribute to the development of mathematical activities proposed in this study. For example, participant *A9* affirmed that "when buying a food or good, taxes are already included in the product price." On the other hand, it is important that teachers value the *tacit*[4] knowledge students bring to the classroom because when their previous experiences are appreciated and used as a way of promoting the teaching and learning process, the mathematical knowledge will naturally emerge (Rosa 2010).

In this perspective, the *National Mathematics Curricular Parameters*—PCN (Brasil 1998) revealed the appreciation of this mathematical knowledge as it is brought closer to the school knowledge. The insertion of the students in the center of the educational process is essential to the development of teaching, learning, and pedagogical action.

The proposed activities enabled participants to think and discuss financial education content through dialogues developed in the classroom by applying Brazilian Sign Language—Libras. Figure 8.3 shows when some participants of *A* and *B classes* were simulating purchases in the *Mini Market* in the classroom.

According to this result, it is possible to infer that these participants used their knowledge in a critical and reflective way in order to understand daily financial situations. For example, participant *B27* reported that the "debit card is good to do some shopping and to get a discount" while participant *B19* commented that "I have already made a profit when I sold my food supplement to my co-worker."

Thus, it is important to provide reflective moments in the classroom to help Deaf students to develop their autonomy, critical, and reflective thinking and reasoning. For example, during classroom discussions, participant *B19* affirmed that "sometimes

[4] Tacit knowledge is embedded in personal experience; it is subjective, contextualized and analogous. For example, an individual does not learn to ride a bicycle by reading a manual, as he needs personal experimentation and practice to acquire the necessary skills to learn this action. Therefore, this knowledge is acquired and accumulated through individual experience, as it involves intangible factors such as beliefs, perspectives, perceptions, value systems, ideas, emotions, norms, presentiments and intuitions (Rosa and Orey 2012).

Fig. 8.3 Participants of *A* and *B* classes simulating purchases in the *Mini Market* (*Source* Personal file)

I will make a purchase, but the product is very expensive, so I ask for a discount in order to pay less for it."

8.4.1 Ethnomathematics as a Pedagogical Action: Cultural Features of Mathematical Problems

In order to solve mathematical problems, participants used strategies, such as the use of their fingers to represent money bills or coins. For example, participant *A2* used his fingers to add and to multiply as he had not memorized the multiplication table yet. In this context, we selected an episode from the interpretation of the results obtained in this study that was related to a class about the monetary system in which a participant used a specific strategy from the members of the Deaf Culture to solve a mathematical problem proposed in the classroom.

This episode occurred due to the curiosity showed by some participants in the study about addition of money bills and, then, the first author proposed the following problem-situation: "If I have four coins equivalent to 25 cents, then how, much do I have in total?". The analysis of the given answers to this question showed that 3 (three) participants of *Group A* answered, immediately, that the answer was one real, while 4 (four) participants did not answer this question. However, participant *A7* asked the first author if he could explain to the other participants the procedure that he had used to solve this question.

The images represented in Figs. 8.4, 8.5, 8.6, and 8.7 show the reproduction of the movements used by participant *A7* to explain his own way (ethnomathematical jargon) to solve this specific question. The authors emphasize that this sequence of figures are being shown this way because of the difficulty in transcribing participant *A7* movements, which were performed in Libras.

Image A, in Fig. 8.4, represents numbers 2 and 5, respectively. This image expresses the number 25, in Libras, which is formed next to the little finger, indicating the first 25 cents coin. In image B, in Fig. 8.4, the photographs represent the numbers 2 and 5, respectively. This image expresses the number 25, in Libras,

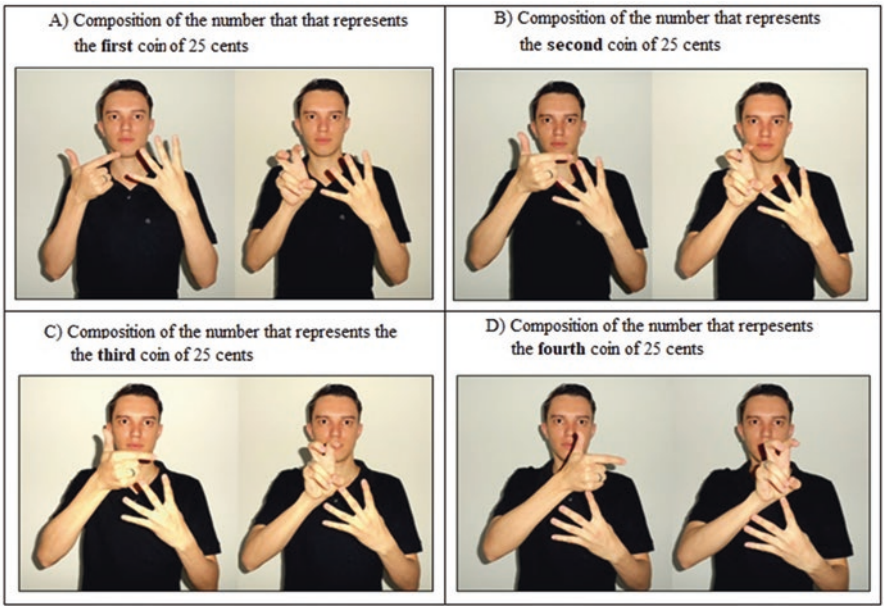

Fig. 8.4 Representation of the four coins of 25 cents in Libras (*Source* Personal file)

which is formed next to the ring finger, indicating the first 25 cents coin. Similarly, image C represents the third 25 cents coin, which was signalized next to the middle finger, while image D represents the fourth 25 cents coin and was signalized next to the index finger.

Image A, in Fig. 8.5, indicates the first and second coins of 25 cents together, which were represented by the little and ring fingers. The photographs in image B, in Fig. 8.5, represent numbers 5 and 0, respectively, which correspond to number 50 in Libras so; the join of the two first coins of 25 cents equals 50 cents.

Image C, in Fig. 8.5, indicates the third and fourth coins of 25 cents together, which were represented by the middle and index fingers. Image D, in Fig. 8.5, represents numbers 5 and 0, respectively, which correspond to number 50 in Libras; in other words, the join of the two last coins of 25 cents equals 50 cents.

The two images represented in Fig. 8.6 show the join of all fingers, in other words, the sum of the four coins.

The representation of the two images in Fig. 8.7 shows that the joining of all fingers, in other words, the sum of the four coins, equals one real, considering that the first image indicates the number 1 (one) and the second image represents the word real. Therefore, it is important to emphasize that the explanation of participant *A7*, by using his own jargon, enabling other participants the opportunity to understand the given problem.

It is also necessary to emphasize that the explanation developed by the first author was not enough for the majority of the participants to understand the solu-

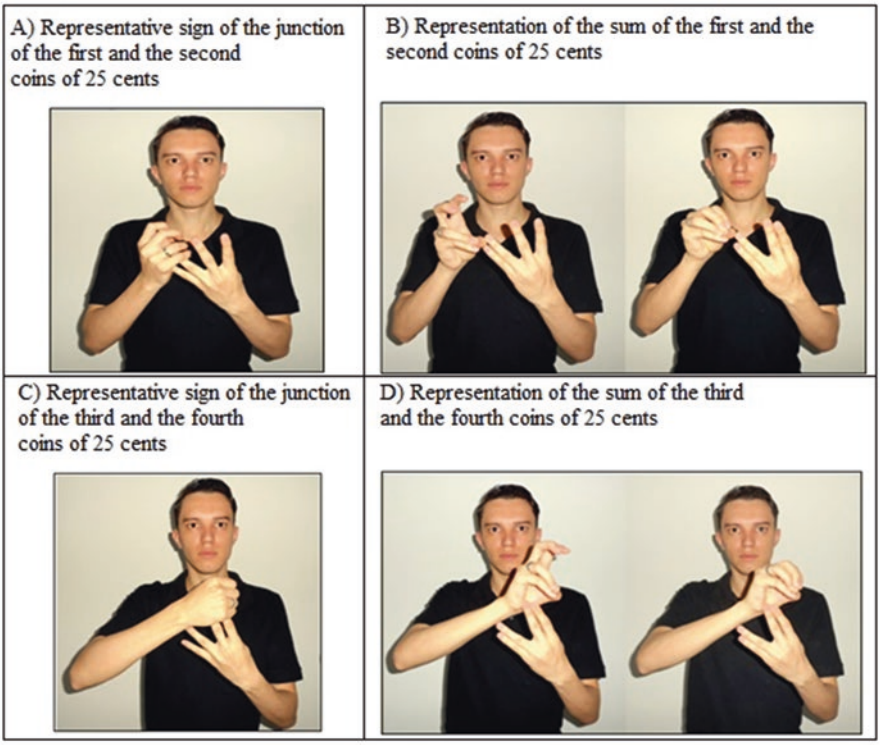

Fig. 8.5 Representation of the four coins of 25 cents altogether in Libras (*Source* Personal file)

tion of the proposed problem because this professional did not utilize strategies of resolution that are particularly used by the members of the Deaf Culture. Thus, the data support the supposition that ethnomathematics is considered a pedagogical action that enables mathematical content (financial education) as developed in the proposed activities. The fieldwork supports the development of autonomy for Deaf students who participated in this study. For example, participant *B21* commented that "I learned to count money and I will help my mother in her shopping time."

It was also inferred that it is important to use sign language and specific *jargons*[5] developed by the Deaf student participants in this investigation, who are members of this cultural group, as well to understand their behavior codes that are an important part of the Deaf Culture, as they contribute to the teaching and learning process of financial education contents.

In this context, it is necessary to value Deaf students' voices, regardless of whether they are expressed as spoken words or visual-gestural signs is a way of supporting their engagement with school mathematics. It is also a way of demon-

[5] These specific forms of jargon can be seen in the Figs. 8.4, 8.5, 8.6, and 8.7.

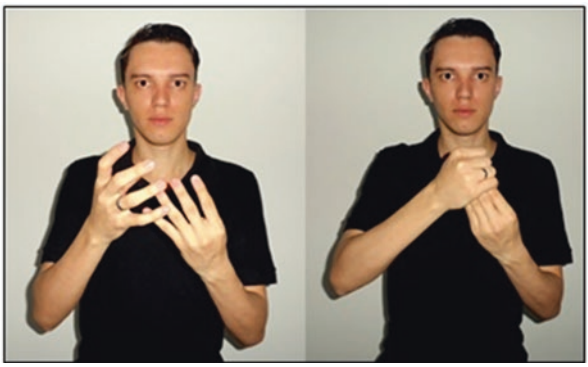

Fig. 8.6 Representation of the sum of the four coins of 25 cents in Libras (*Source* Personal file)

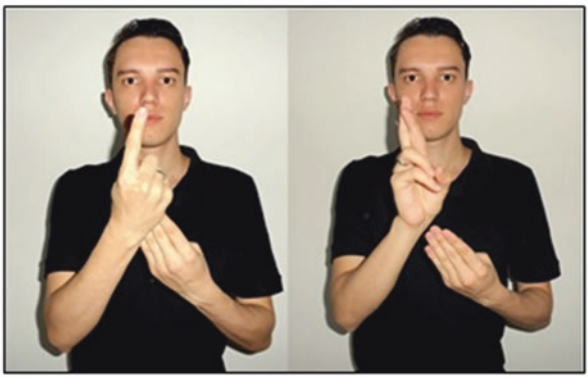

Fig. 8.7 Representation of 1 (one) real in Libras (*Source* Personal file)

strating how they are valued as members of the learning community, of publically acknowledging how different perspectives and different ways of communicating mathematically enrich school mathematics (Healy and Ferreira dos Santos 2014).

Therefore, it is inferred that the activities applied in the classroom using ethnomathematical perspectives provided an environment that encouraged the development of financial content to Deaf students and enabled the understanding of problem solving that involved financial concepts related to activities practiced in daily basis.

8.5 Final Considerations

Mathematical content related to Financial Education that were addressed in this study contributed to the participants better understanding the world in which they live so that they are able to comprehend and discuss information conveyed by the media. Then, there is the need of thinking about mathematics education for Deaf students, who are different sociocultural individuals who have their own language, culture.

The results obtained in this study show that ethnomathematics promotes reflection on the teaching and learning process in mathematics in relation to the contents and methods, as well on the role of this knowledge area in the construction of citizenship. The purpose of an ethnomathematics program is to highlight and analyze a contextualizing proposal based on a pedagogical action for the educational development of Deaf students in society by strengthening their own culture. In this context, Rosa (2010) argues that ethnomathematics seeks to relate school mathematics to everyday life because for students to learn mathematical content there is a need for them to analyze phenomena and problems faced in their own communities.

Consequently, one of the contributions of ethnomathematics to the development of the participants in this study was related to the identification of the existence of a bond between learned knowledge in school and that one used in everyday life to solve problems related to commercial and financial activities practiced on daily basis.

This proposal for the pedagogical action of ethnomathematics for the development of financial education for Deaf students who communicate in Libras demands from the agents involved in the educational process and, especially from teachers, an educational involvement that allows them to analyze the teaching and learning process in mathematics based on a critical discourse reasoned on the connection between practice and theory.

Finally, this approach aims to enable the development of democratic spaces for critical reflection on the phenomena faced in daily life. Consequently, through ethnomathematics, students can appropriate financial education in a critical and reflective way so that they can actively participate in society, fully exercising their rights and duties, in short, and their citizenship.

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Chapter 9
Practices in Ethnomathematics, University, and Basic Education Schools: Possible Interconnections

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9.1 Introducing to the Theme

This chapter aims to examine questions related to a set of pedagogical practices used with Basic School students, theoretically and methodologically supported by the field of Ethnomathematics. Such practices were designed by Basic School teachers, researchers, and undergraduate and graduate students of a university located in the state of Rio Grande do Sul, Brazil. This idea of gathering diverse players of the educational setting in a research/intervention emerged from the conclusion of the involved researchers. According to them, continuing education training of Basic Education teachers consisted of courses or workshops offered by university faculty, and were generally connected to *stricto* sensu graduate programs.

In brief, Basic School teachers should, within this context, apply in their classrooms the teaching ways designed by researchers, by including teaching strategies considered relevant by the latter. Consequently, school teachers would be left with little space—or none at all—to build pedagogical and reflecting practices about their work. Nor would they be invited to research their classrooms, thus being restricted to the job of "teaching" and solely providing researchers—those with the consistent theoretical frameworks—with the empirical material to write about practices carried out in venues they did not occupy.

This scenario disturbed a group of Mathematics teaching researchers who had been working with school teacher training for over a decade in southern Brazil. Although the themes were chosen by the teachers and there used to be a constant exchange between them, the prevailing idea was that math classes should be taught as designed by the researchers. Then teachers would teach accordingly, and in subsequent meetings with the researchers, they would share the progress or limitations

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observed. Such method lacked consistency, once those designing the practices were not the ones familiar with the school setting where they would be carried out. Furthermore, teachers were limited to the role of narrators.

The feeling of disturbance was amplified when a teacher—who had been teaching multi-level classes for more than 30 years in a school located in the countryside of a small town encompassed by the university region—was interviewed. Because they were in a small number, students of different educational levels were all placed in the same classroom, and taught by the same and only teacher; the interviewee taught classes simultaneously from the first to the fourth years.

When asked whether he would accept to give the interview and have one of his classes taped, he immediately asked how the investigation data would be treated, how the results would be published, and how he and the school would benefit from participating in the research. He clarified that his inquiry derived from recent visits of researchers whose sole concern was "observing my classes, describing what I do, assigning a theoretical framework to it and criticizing my teaching methods."

This event led the researchers to reflect on the continuing education processes of Mathematics teachers in Basic School. In the core of this issue, not knowing the work context of these teachers implied in being impossible to operate with pedagogical practices that only minimally took into consideration the students' culture. Therefore, it became vital to include the teachers of these schools, and undergraduate and graduate students in the investigation team.

Due to this, the team submitted a research/intervention project to a Brazilian governmental entity supporting education; it was called Observatory of Education, developed between 2014 and 2018. The broader purpose of this program was integrating undergraduate and graduate students, researchers, and Basic Education teachers in investigative research projects and pedagogical practices. This would assure that teaching processes would take into consideration the school students' culture.

This was the beginning of a partnership between the Higher Education Institution and the administrators of five public Basic Education schools in the university surroundings. The students who attended them at that time were mostly from urban working families, whose parents had jobs in the industries and commerce in the area. In addition to that, it started to receive people from other states, due to the job openings available at that time in these towns. Thus, the youth belonging to these families started to attend the schools associated with the project.

Each school chose a Mathematics teacher to integrate the investigation team, which also included undergraduate and graduate students. With the exception of the researchers, the other participants earned a scholarship from the governmental entity supporting education.

While the project was being carried out, the team met weekly at the university to prepare pedagogical practices whose theoretical and methodological framework was the field of Ethnomathematics. Therefore, undergraduate and graduate students visited the Basic Education schools and there conducted their research. The school teachers were in charge of receiving the other team members when these visited the

schools. Separately, they actively participated in the continuous training activities that addressed their colleagues.

In the next session, the theoretical and methodological framework supporting the pedagogical practices based on the field of Ethnomathematics as described by Knijnik (2016) will be explained.

9.2 Theoretical Framework: The Field of Ethnomathematics and Its Intertwining with Investigative Pedagogical Practices

Three remarks have to be made about the theoretical-methodological frameworks that supported the investigations within this project. The first one relates to the need to choose one theoretical perspective of the field of Ethnomathematics, among so many. As stated by Knijnik (2016), functioning with this idea may seem obvious if "many times there was not in the academic realm a dispute to utter which is 'the best theory to be used, who are the best authors to be mentioned" (p. 22).

Moreover, this author asserts that she opposes this notion, once it seeks to find an all-encompassing theory, which would state "all the truth about the objects we research" (Ibidem, p. 22). Thus, the selection was based on two philosophical angles: the studies by Wittgenstein and some ideas by Michel Foucault. Thus, the ethnomathematical perspective chosen by the team may be described as follows "A theoretical toolbox that enables analyzing the mathematical language games of diverse forms of life and their similarities within the family; also examining school math discourses and their real effects" (Knijnik 2016, p. 23).

As pointed by Knijnik (2016), the ideas of the two philosophers put together may become strong so that we may answer "how to philosophically justify the existence of other mathematics, other than the academic and the school ones?" (p. 23) What particularly interested the investigation team in the perspective chosen was the first part of Knijnik's (2016) definition, i.e., examining the mathematical language games generated in distinct forms of life. When mentioning the existence of distinct language games, Ludwig Wittgenstein's ideas on maturity start to echo.

In fact, Wittgenstein "sees language not anymore as the imprint for perfection and orderliness, as if it existed prior to human actions" (Knijnik et al. 2012, p. 29). In the light of this, the authors would dispute "the existence of a universal language" (Ibidem, p. 29), by problematizing "the notion of a total and a priori rationality, by expecting the constitution of distinct rationalities (Ibidem, p. 29). Furthermore, to Wittgenstein, as the meaning is assigned by the use, each of those that we make of words, these meanings may change.

Thus, "we remove words from their metaphysical employment and place them back in their daily employment" (Wittgenstein 2004), in touch with the "rough soil." He adds that, "the meaning of a word is its use in the language" (Wittgenstein 2004, p. 38). According to Condé (2004), Wittgenstein signifies "grammar and

language games as rationality shaped from social practices in a form of life that is no longer established on ultimate foundations" (p. 29). When we abandon the idea of a single and natural structure, producing sense, we begin to understand rationality as a "fabrication," a "construction" (Ibidem, p. 29).

Furthermore, the author expresses that the notion of the form of life is central to Wittgenstein's work on maturity because it is "the ultimate haven to language" (Condé 1998, p. 104). In other words, "the meaning of words, gestures, and one may say, of mathematical languages and of the criteria of rationality in them are produced within the context of a certain form of life" (Knijnik et al. 2012, p. 30). Wittgenstein's theoretical production and that of some of his scholars such as Condé (1998, 2004) and Glock (2006) enable one to infer that the language games and the rules constituting them are strongly intertwined through the use we make of the first, that is, they are part of a specific form of life. This means that the language games should be understood as being immersed in a form of life, vigorously blended with the "non-linguistic activities" (Glock 2006, p. 174). Glock states that "a form of life is a cultural or social construct, the entirety of community activities that are included in our language games" (Ibidem, p. 174).

Indeed, with the meaning being assigned by its use, with every use we make of the words, these meanings may be modified. Such ideas are in consonance with the field of Ethnomathematics. Wittgenstein's theoretical production and that of some of his scholars such as Condé (1998, 2004) and Glock (2006) enable one to infer that the language games and the rules constituting them are strongly intertwined through the use we make of the first, that is, they are part of a specific form of life. This means that the language games should be understood as being immersed in a form of life, vigorously blended with the "non-linguistic activities" (Glock 2006, p. 174). Glock states that "a form of life is a cultural or social construct, the entirety of community activities that are included in our language games" (Ibidem, p. 174).

Indeed, with the meaning being assigned by its use, with every use we make of the words, these meanings may be modified. Such ideas are in consonance with the field of Ethnomathematics. In short,

> Academic Math, School Math, Peasant Math, Indigenous Math, that is, the Mathematics generated by specific groups may be understood as sets of language games intertwined with distinct forms of life, having specific rationality criteria added to them. Nevertheless, these distinct games do not possess an invariable core that makes them totally impenetrable to each other, nor a property common to them all, but some analogies or kinships—what Wittgenstein (2004) calls family similarities.

According to this theoretical framework, when it is stated that two language games present family similarities, one is not expecting a supposed similarity between them, but rather pointing to similar aspects randomly distributed, without a supposed uniform repetition. Thus, one may understand that a language game presents similarities and differences regarding others, and there might be connections and contraposition among them.

The language games mentioned above were designed within the investigative pedagogical practices. As mentioned at the beginning of this chapter, the project in question was created with a focus on design, development, and problematizing of

investigative pedagogical practices addressing Primary School students. These required that the teachers involved changed the school venues by turning them into environments of potency and of creation of knowledge through research (Buhring 2017). Within this context, it is worth pointing that:

> The issue is not trying to propose a mere merge of the words teacher and researcher, resulting in the teachers-researchers of Basic Education, once these are "(in)secure", "afraid of novelty" (…). What I propose is just the opposite! It is a teacher-researcher constituted within and by research, by guiding education in (re)creative, (re)invented ways and of creating teaching. These teachers-researchers stimulate questions, awe, curiosity, and creativity (Buhring 2017, p. 143).

Such a viewpoint had been discussed in other investigations by the team. Grasseli (2012) carried out an investigative pedagogical practice with a group of students from public Secondary School in Rio Grande do Sul. Its aim was to examine what mathematical rules emerged while students analyzed issues connected to the culture of viticulture, and what meanings they assigned to such rules, as well as to those usually present in the School Mathematics.

In this situation, the teacher concluded that he and his students became researchers along the process, a notion that emerged from some information supplied by a grower about the alcohol content of the wine produced, that, according to the latter, to be found, required a multiplication by 0.6. Puzzled by this rule, considered as indisputable by the grower, the students and the teacher himself chose to research it. Thus, the

> (…) students became effective researchers, no longer interviewers. They consulted agricultural technicians, enologists, their own parents, who made wine, with no convincing answer. They did so very enthusiastically, for not even the teacher knew it, and also needed to become a researcher, thus enabling his students to become "his equals (Grasseli 2012, p. 73–74).

As in Grasseli (2012), most investigations—including investigative pedagogical practices—make use of the so-called methodologies with ethnographic inspiration. Studies such as those of Wanderer and Scheffer (2016) point that the ethnographic research, connected with the qualitative type, is widely used by researchers in the field of Ethnomathematics. To the authors, a significant part of the investigations following this trend utilize, in their empiric natures, this kind of methodology, by including mainly observations and interviews.

Moreover, such techniques "have been widely used in the ethnographic research emerging in the twentieth century, inspired by the works of anthropologists Boas and Malinowski" (Grasseli 2012, p. 39). The said researchers state that the educational ethnography appeared in 1954, in the State of California, where it influenced the academia, even in Brazil; the discussions about this methodology held from then on "opened paths to other studies and to the dissemination of research with anthropological bases in the area of Education" (Ibidem, p. 41).

At this point, it is worth presenting some notions expressed by Scheffer in her dissertation (2015). Initially this author supports them on André (1995) when she asserts that "if the ethnographists' focus is the description of the culture of a social

group, the education scholars' main concern is the educational process," enabling "certain requirements of ethnography not to be followed, or not to be needed to be followed by those investigating educational issues" (Ibidem, p. 28).

This proposition relies on working with this methodology "focusing on teaching," which does not require "coming to a field with *a priori* definitions whether regarding the object and the informers, the length of stay and collection tools)." Therefore, there is an attempt "to verify values, customs, faiths, rites, objects (artifacts) **so as to understand a group's practices**" (Ibidem, p. 43). [bold type by the proponent]. The authors still express that, in this methodological perspective

> (…) even when considering the plurality of concepts that guide the research projects of the "ethnographic type", one may say that they make use of techniques such as participating observation, collection of documents, records of talks and events, semi-structured and open interviews, as well as the use of photographic images and videotapings (Wanderer and Scheffer 2016, p. 45).

Thus, the research material generated by the investigative pedagogical practices was constituted of field journals of the participating teachers, materials written and produced by the students, and videotaping of the classes taught.

9.2.1 What Some Productions Emphasized

This section focuses on a set of investigations whose locus was the six partner schools. Such investigations occurred after communicating with the school administrators and teachers, who became aware of the ethical procedures to be followed. Thus, the researchers involved agreed on presenting them with the outcomes of the investigations. Another relevant fact concerns the way these were designed, sometimes involving undergraduate students, graduate students, Basic School teachers, and researchers.

It is worth pointing out that the following analysis is based on Michel Foucault's ideas. In one of his works, Foucault (2005) mentions some historians' tendency to examine "how economic conditions for existence may find their reflection and expression in men's awareness" (Ibidem, p. 8). To him, the agent of knowledge and the forms of knowledge are, somehow, "previously and definitively" provided, and "the economic, social and political conditions for existence are only deposited or imprinted in this definitively provided agent" (Ibidem, p. 8).

In one of the five lectures he gave at the *Pontificia Universidade Católica of Rio de Janeiro*, in May 1973, when connecting systems of truth with social practices, the philosopher stated that if "the linguistic nature of the language facts used to be a relevant discovery in a specific time" (Foucault 2005, p. 9), the time has come to consider such discourses not only through their essentially linguistic aspects but "as games, *strategic games of action and reaction, of question and answer, of domination and dodging, as well as of fight*" (Ibidem, p. 9) [italicized by the author].

Thus, upon analyzing the research material with theoretical tools originated in Michel Foucault's considerations, the philosopher's notion of discourse becomes central. In his work *The Archeology of Knowledge,* he points out that discourses are constituted by

> (…) practices that systematically form the objects they refer to (…) are made of signs; but what they do is more than using these signs to name things. And this *more* makes them irreducible to language and to the act of speech. It is this "more" that one has to make stand out and describe (Foucault 1995, p. 56). [italicized by the author]

Bearing these references in mind, some productions of the project will be analyzed. Peransoni (2015) carried out a research in two partner schools, aiming to investigate the pedagogical implications originated from discussions in two study groups, one in each school, with teachers of Initial Years of Primary School. For 1 year, meetings were held to study theses and papers based on ethnomathematical investigations, where teachers were constantly stimulated to contrast these texts with their pedagogical practices. Therefore, some care was taken in order to avoid procedures that took into consideration the theory-practice duality.

This research material consisted of recorded discussions, which were subsequently transcribed; its analysis yielded three units. In the first one, the researchers observed that teachers of both groups were attached to mathematical formality, as clearly put by one of the latter:

> Sometimes they say, Miss, I did the calculation in "my head", but I don't know how to transfer it to paper. I say: but try to do it, do it somehow, the way you engendered the answer, tell me where you got it from, I need the calculation… He is disobedient too, then sometimes I keep an eye on him. But on one occasion, I was watching, he wasn't cheating, he did it in his head, or wrote it on the desk, and crossed it all out. I said: Where is the calculation? It is there somewhere, I'm gonna show it to you, just wait, but he didn't show anything, he just does it in his head (Peransoni 2015, p. 99).

Peransoni (2015) also noticed that these teachers attempted to establish relations with the students' daily lives; however, calculations needed to be developed according to the algorithms of School Mathematics.

One of them reported an activity in the classroom, which depicted this exact situation. According to her, the mathematical problem given to the students consisted in calculating the difference when purchasing laptop computers in installments by two distinct purchasers. The teacher said, when explaining she was strict when it came to using the rules of School Mathematics, "I'll kill them with these two questions" [when referring to the requirement of presenting the calculations] (Peransoni 2015, p. 100).

Nevertheless, with the progress obtained with the discussions and the study of papers and theses recently published, both study groups began to acknowledge the existence of math language games originated in non-school venues. In this regard, one of the participating teachers reported that, after interviewing a painter, she understood the need to think of aspects sometimes not associated with Mathematics, such as if there was the need to sand the wood before painting, or for a bigger amount of paint. In this case, a mere mathematical calculation would not solve the

question. Moreover, she stated, "when we give them calculations in the classroom, we did not consider other language games; then we give them a calculation."

In addition to that, the research material has shown how significant it is for teachers to take into consideration the influence of the form of life in the emergence of math language games. Indeed, one of them stated that she started to notice that some students calculated the four operations distinctly, and she added "only now I started to realize, I guess, it seems that I have found where it all comes from, and I see in these ways a possibility to better teach Mathematics" (Peransoni 2015, p. 105). Due to this, the researcher affirms that:

> Therefore, acknowledging the influence of culture in applying and determining the math language games, in distinct forms of life where they are applied, may represent a major step in valuing and validating such games, originated in diverse cultures, in school and academic venues (Peransoni 2015, p. 112).

Analyzing the culture present in the classroom was also determining to Azevedo and Giongo (2014) research. The authors developed a pedagogical practice in the ninth year of Primary School focusing in the culture of construction workers, bearing in mind that a considerable number of the students' family members worked in this area. The researchers asked an architect, an engineer, and a construction worker several questions. More specifically, the interviewees shared how to proceed to find if a house was at right angles, or the number of bricks needed to build, of the materials used in the mass to lay them, and that of tiles to cover part of the floor; and finally, how much was spent with the purchasing of the construction material.

Moreover, the students and the teacher followed some work practices of these three professionals. The transcriptions of interviews and individual notes were problematized in the classroom. Then, the questions related to aspects directly connected with math contents were discussed, by emphasizing in what context they had been generated, what led them to understand that every culture produces math knowledge, and that it makes sense in these individuals' work activities. Consequently,

> When the students had access to the transcribed interviews, they started to establish relations between the math studied in the classroom and the one used by the professionals mentioned previously; such relations had not been noticed by them prior to that in the classes of Mathematics (Azevedo and Giongo 2014, p. 83).

One student stated that "Er, when he speaks about [while reading the transcribed interview with the professional] chalking 60 and 80 cm on the two sides of intersecting walls, and then joining these two points to find a measurement equal to 100 cm, the construction workers obtain a right angle" (Azevedo and Giongo 2014, p. 84). Subsequently, he asked his classmates if "that was a practical application of Pythagoras Theorem. Is this what in construction workers' language means to have it at right angles"? (Ibidem, p. 84).

The pedagogical practices done with the teachers enabled the emergence of Bernstein's research (2017). When observing that the students of the fourth year of two partner schools showed special interest in digital games—inside and outside the school—the general goal of her research was investigating what math language

games emerged in the digital form of life of the said students, and their family similarities with those usually expressed in School Math. Briefly put, the outcomes pointed that the math language games generated by these students in their digital form of life showed greater family similarity to those usually present in School Math when the formality of algorithms and that of the Decimal Numbering System was used.

Those with smaller similarities were related to the ways they calculated by using the upper limbs of the human body, producing numeric sequences, and converting decimal numbers into natural ones. Thus, a great part of the games used by them made recurring use of the four fundamental operations. In order to score some points, the students should, for instance, get right the amounts paid in supposed purchases.

In one of these occasions, when trying to do a calculation of how much she should pay for three goods which cost R$15,00, R$15,00, and R$13,00, respectively, a student answered that she proceeded as follows: "five plus five equals ten, I still have three, and this three is left, it becomes a unit, and five plus five equals ten. It became a set of ten, one plus one equals 20, plus one equals 30. And there is that set of ten left, then it equals 43" (Bernstein 2017, p. 86).

In another circumstance, a student should add 0.20 + 0.18 + 0.09 + 0.15. His explanation—without using pencil and paper—was abridged to stating that the calculation should start with "nine plus five, because in nine, one to go to ten. And there is the five, and you subtract one from five, equals four, then you get 14." (Bernstein 2017, p. 88). The student added "14 plus eight equals 22. Twenty-two, two up there, plus one, plus two, equals four, five, six. Sixty-two cents. (Ibidem, p. 88)."

In her final remarks, the author mentions that those moments considered to be of free expression, that students could enjoy, enabled the emergence of math knowledge that would possibly pass unnoticed in the school culture, mainly for not being structured according to the rules of School Math. Gorgen and Peransoni (2014) also carried out an investigation with fifth-year students in the Primary School, in a school located in the city outskirts.

Some of the students of the said class were struggling, mainly with multiplication and division operations; but on the other hand, their teacher observed that these same students did calculations orally instead of in writing. In one situation, she asked them to divide 126 by 3. One of them, immediately, without pencil and paper, reported that "I thought three plus three equals six. Plus six equals twelve. Answer: forty-two [after dividing six by three]" (Gorgen and Peransoni 2014, p. 190). The authors showed that, upon using his fingers to work out the answer, this student

> (…) divided a hundred and twenty-six by three by using addition and duplication of the results. He began by adding three plus three, obtaining six as the result. Afterward, he added six to the first addition, thus obtaining twelve. Such procedures led them to conclude that when the number three is multiplied by four, the result was twelve. Subsequently, he divided six by three, obtaining two. In conclusion, he reported the final answer was forty-two. (Gorgen and Peransoni 2014, p. 191).

At this point, when students finished the activities involving oral calculation, the teacher asked students to assess them. Unanimously they showed they perceive the

existence of other ways to calculate, to which she questioned them why people think distinctly. One of them answered that "we are not the same person," and if everyone were alike, "what would be the fun of it."

Gerstberger et al. (2016) also highlighted distinct ways to operate with math concepts by carrying out a pedagogical practice with students of two fifth-year classes of another school partner in the project. Most of these struggled with math contents, mainly with fractions. This activity focused on observing the writing of fractions in cake recipes, and what utensils were used to measure them, thus observing the similarities between the culinary mathematics and the school one. To do so, students took home a sheet to write a recipe used by their grandparents or parents; through several means, such as symbols and drawings, they presented to their classmates the recipes and the utensils used for measurement.

One of the students' grandmothers visited the school to show how she handled the cake recipes; she said she rounded and approached when she needed to measure, for instance, one and a half cup of an ingredient. Thus, she explained that she used her fingers to determine in the cup the amount she needed to pour into the container, especially when it came to halves and quarters. Afterward, the students questioned her how she proceeded with the measurements when preparing a recipe with a cup bigger than the one usually used. She said that she needed to pour a smaller amount of ingredients or to use eggs to increase the recipe dough; she added that fraction symbols are not usually employed in recipes, they are simply written in full "a cup and a half." Eventually, the researchers concluded that

> Generally speaking, the grandmother's and the students' remarks are related to rationalities that are present in school math—adding, dividing, fractioning, measuring—that are usually used inside and outside the school scope. Somehow, there was an exchange of knowledge between the students and their family members while they prepared the cooking recipes (Gerstberger et al. 2016, p. 91).

In this context, Gerstberger (2017) revealed that ninth-year students of a partner school regularly used their smartphones in the school, although this was not permitted. Thus, his investigation occurred in two scopes: in the first one, aspects related to the culture of using smartphones emerged and were identified, what made the researcher use the term "ethnomobile" referring to the fact that all students owned a smartphone and that, regardless of their social class, this technological device was ubiquitous. In the second one, possibilities and limitations of applications and features of the said gadget were explored with the students, more specifically to analyze the contents of school mathematics.

Therefore, regarding the first scope, questions such as bullying, excessively high expenses with constantly exchanging the device, as well as a shift in relationships due to the emergence of social networks, were discussed. The abrupt changes in communication were also problematized, through the talks with three students' relatives who discussed them in the classroom; their opinions follow in this excerpt:

Guest 1: There were only landlines in the old days, and few of them; if you wanted to call someone, you had to walk for kilometers, right, because there wasn't, there was no other way.

Teacher: Why did you have to walk for kilometers?
Guest 1: Because we lived in the countryside, then we needed to go to the city, to a business, in order to use the telephone.
Guest 3: I remember that, I know what it was like, because my father ran a business, and we had one of those telephones. Then everybody in the neighborhood came to make his or her calls. It was a paid phone within the business.
Student G: But who had to pay for it?
Guest 3: Yes, yes, we used to charge a (…) a fee.
Guest 1: Back then was not as it is today, we waited for hours. Not today.
Teacher: How did people know they were going to get a call?
Guest 1: The person called the business and let them know he/she was going to call. They arranged the callback (Gerstberger 2017, p. 90).

In the second scope, math contents were analyzed through the use of the smartphones; among them, rule of three, with the use of the cameras of the devices, and Financial Math, as well as three-dimensional graphs through the applications. Furthermore, contents as mean, average, and fashion were problematized, although they were not included in the syllabus, similarly to the three-dimensional graphs. The researcher affirms that, at the end of his experiment, the Internet and the smartphones have also affected school venues, in an ever-changing society, "and classrooms cannot ignore such facts, by keeping their activities and practices unchanged" (Gerstberger 2017, p. 142). In light of this, to him, there are "other propositions and possibilities to teach and approach math contents (…) as well as the distinct ways to teach and to restructure the teachers' practices in the classroom" (Ibidem, p. 142).

Other ways to teach and to learn mathematics were also emphasized by Giongo et al. (2018), with a pedagogical practice carried out with a fourth-year class in primary school. This school is located in a small town whose economy is based on family farming. Due to this, the tasks assigned to students were about buying and selling farming goods. A student found expensive paying R$0.40 for an egg at a local grocery shop after she took into consideration how much her family had received when selling eggs by the dozen. The teacher suggested that they continued the calculations to check the prices to be paid in each scenario.

Teacher-researcher: Now I want to know how much four eggs cost? Can anybody answer that?
Student: One real and sixty cents.
Teacher-researcher: One real and sixty cents. How did you get this result?
Student: I used plus.
Teacher-researcher: Why plus?
Student: Forty plus forty plus forty and plus forty.
Teacher-researcher: And if I want to know the price of nine eggs?
Student: It's three and sixty.
Teacher-researcher: How did you get to the price of three and sixty?
Student: I did [a calculation] with plus, three and twenty, that referred to eight [eggs], and here nine [eggs], equals three and twenty plus forty, equals three and sixty (Giongo et al. 2018, p. 5).

These investigations, included in the studies carried out by the research team, produced some theoretical-methodological lessons for future research. Such lessons are described in the last section of this article.

9.3 Lessons Learned

Carrying out the project and, therefore, the investigative practices provided the investigation team with some lessons. The first one regards the pressing need to have Basic School teachers participating in research projects connected with Graduate Programs in Teaching, at Higher Education institutions. Thus, university and the Basic School also become partners, and abstain from activities that solely aim to make the knowledge produced in the first be consumed by the latter.

Concerning developing research in the classroom, Moreira and Candau's (2007) ideas are worth highlighting. They state that, although distinct from the ones carried out at universities, research can take place in school syllabuses. Moreover, they consider that participating in investigations may contribute to teachers' work, overall, higher valuation.

> In short, we support that syllabuses, in every school, become a site of research. We reaffirm that research, broadly put, is not limited to universities. As teachers/scholars who work in schools, we need to face this challenge, by becoming researchers of knowledge, values, and practices that we teach and/or develop, thus focusing our teaching on research. Along this path, we might be able to better our professional performance, locate ourselves in the world, we might even engage in the fight to make it a better place. Along this path, we may stimulate in students the spirit of researching, seeking, taking pleasure in learning, in knowing new thing. (Moreira and Candau 2007, p. 43).

The second lesson refers to large scale continuing teacher education. This has typically used theories and propositions of pedagogical practices that are not in consonance with the teachers' circumstances. The role of teachers in Basic School would be restrained to receiving knowledge produced at universities, and problematizing particular cultural questions would not be considered. Moreira and Candau (2007, p. 42) also draw attention to how important teachers' intellectual activities are, by pointing out that this "implies in questioning what seems to be written in stone, what is presented to us as being natural; basically a questioning that seeks to show us that things are not inevitable." They add that the "intellectual activity is thus focused on critiquing the culture that surrounds us" (Ibidem, p. 42).

The third lesson refers to the relevance of teachers registering their practices and presenting them in events. In fact, the development of this project showed some paths waiting to be treaded, related to carrying out research in classrooms with teachers' participation. At the beginning, they showed resistance, but throughout 4 years, four books with theoretical-methodological texts were written, in many cases by distinct authors. They also prepared, developed, problematized, registered, and presented outcomes of pedagogical practices in congresses, together with researchers and undergraduate and graduate students.

Finally, one may conclude that there has been significant progress regarding teachers' participation in research projects. However, some hindrances come to mind, mainly concerning the time allocated to them; quite often teachers' compulsory hours in the school restricted their participation in special activities. Another hindrance refers to the significant decrease in funding by Brazilian governmental entities to promote teacher education. An alternative to it, executed by the research team, consists of the institutional partnership signed between university and cities' departments of education. Indeed, in operation now for 2 years, it has enabled the interested teachers to use part of the time originally allocated to pedagogical meetings with participation in the research teams.

Therefore, the pedagogical practices thus designed were problematized within the theoretical-methodological frameworks, more specifically those of the field of Ethnomathematics. Some of the questions currently discussed focus on evaluation and teaching/learning processes in classes with immigrant students.

Originated in the former one, the fourth lesson refers to the need for some ethnomathematical studies to focus on contemporary issues, such as problematizing the use of social networks in school venues, of technological resources in the teaching/learning processes in Basic School, and in classes with immigrant students. Briefly put, one has to reflect on other pedagogical practices for multicultural classes in times of the internet.

At this point, a remark has to be made. Both School and Academic Mathematics, as we know, are also outlined as an Ethnomathematics practiced by particular groups. Especially concerning Basic Schools, the relevance of teaching language games associated with them is not disputed. As pointed out by Knijnik et al. (2012, p. 82), there is a need for "making accessible [to students] the set of language games that has been called Mathematics."

The authors add that such games are socially legitimized as scientific knowledge that, in its turn, "has supported (…) and been fed by new technologies typical of our times" (Ibidem, p. 82). They additionally state that technologies have been crucial for improvements in people's life quality, by extending life expectancy and promoting early diagnosis of diseases, among other benefits.

> But these same technologies have placed those who have access to scientific breakthroughs further from those who do not. In light of this that we consider relevant that the new generations may master, in their complexity and magnitude, the grammar that constitutes the academic mathematical knowledge (Knijnik et al. 2012, p. 82–83).

Such ideas are in consonance with those of Zygmunt Bauman. In his work *Liquid Modernity,* the sociologist (2001) uses terms such as "fluidity" and "liquidity"—associated with the idea of "lightness"—to describe "the current phase, a *new* one in many aspects, in the history of mankind" (Ibidem, p. 9) [highlights by the author]. Such expressions point to an opposition to "solidity"—related to "fixity"—once liquids are not stable and change according to the container in which they are placed.

He adds that, if Solid Modernity advocated seeking for "solids with a *lasting* solidity, a solidity one could trust, and that would make the world predictable, and therefore, manageable" (Ibidem, p. 10) [highlights by the author], in Liquid

Modernity the ideal of a fair society to all gives way to uncertainties, contingencies, and instability: “the relationship between space and time should, from now on [in the liquid society] be procedural, changeable and dynamic, not pre-determined or stagnant” (Ibidem, p. 131).

Precisely these are the ideas that may be productive in rethinking teaching, learning, and evaluation processes in liquid times, supported by the framework of Ethnomathematics. The challenges for doing this are multifold, and for their minimal attainment, partnerships among universities, Basic Education schools and Graduate Programs point to a possibility.

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Part V
Diverse Cultural Practices

Many dimensions of the Ethnomathematics Program can be revealed in the category of research that prioritizes cultural practices, such as the political and educational dimension, and the challenges of daily life, which are themes concerned with research in this field in Brazil. A diversity of symbols, languages, and cultures circulate in the continuous movement of diversity of thought and reasoning that permeate a rich diversity of cultural practices, evidencing reflections on the *others* and/or on ourselves, as well in the ways we organize the world.

Chapter 10
Research in Ethnomathematics in the Context of the Professional Master's Degree: An Analysis of Five Theses Developed at UFRN

Paulo Gonçalo Farias Gonçalves and Francisco de Assis Bandeira

10.1 Introduction

In order to meet demands for professional qualification as of the production of academic knowledge focused on the labor market, the Coordination for the Improvement of Higher Education Personnel (CAPES) regulated in Brazil, through Ordinance Nº 080, of December 16, 1998, in the modality of professional master's degree.

According to CAPES assessment area and its latest document (Capes 2016), the area of Teaching brings together 76 stricto-sensu professional postgraduate courses, which have in common the development of translational research, seeking to establish "bridges between academic knowledge generated in education and teaching, for its application in educational products and processes in society" (p. 3).

Among these courses, the Professional Master in Teaching Natural Sciences and Mathematics began in 2002 at the Federal University of Rio Grande do Norte, bringing together research in the teaching and learning lines of Sciences, Chemistry, Physics, Astronomy, Biology, and Mathematics, as well as the study of its historical, philosophical, and sociological aspects.

We highlight among the studies developed in the referred Program the research in Ethnomathematics, considering the relevance of this field of Mathematical Education in the investigation, understanding, and appraisal of the different knowledge and practices of social groups in order to integrate them into the school context.

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Considering the singularities of the research developed in professional master's degree courses and the greater need for investigations that addresses the educational dimension of ethnomathematics (Martins and Gonçalves 2015), this article aims to analyze five researches on ethnomathematics produced within the Master's degree in Professional Teaching of Natural Sciences and Mathematics from UFRN.

10.2 Ethnomathematics: Some Approaches

Ethnomathematics, an Ambrosian conception that emerged in the 80s of the twentieth century, is one of the trends in Mathematical Education that appeared by questioning the universality of Mathematics taught in the educational system of formal education unrelated to the social, cultural, and political context by giving visibility to the Mathematics of different sociocultural groups, especially those subordinated in a socioeconomic point of view (Bandeira 2016).

Before the denomination of Ethnomathematics, this area of knowledge received other nomenclatures, acknowledge: Sociomathematics—Claudia Zaslavsky; Spontaneous Mathematics—Ubiratan D'Ambrosio; Oppressed, Hidden or Frozen Mathematics—Paulus Gerdes; Popular Mathematics—Mellin-Olsen (Gerdes 1991).

Because there are different ways of mathematical know-how in the seeking to deal with the immediate and remote environment, some privilege comparing, classifying, quantifying, measuring, explaining, generalizing, and inferring, according to D'Ambrosio (2015). This author understands Ethnomathematics as "… mathematics practiced by cultural groups such as urban and rural communities, working groups, professional classes, children of an age group, indigenous societies and so many other groups identified by goals and traditions common to those groups" (D'Ambrosio 2015, p. 9).

Etymologically, it is constituted by three terms: ethno which means the natural, social, cultural, and imaginary environment, mathematics that comes from explaining, learning, knowing, dealing with, and ethics understood as modes, styles, art, techniques. In short, it is conceptualized as the "art or technique of explaining, of knowing, of understanding in different cultural contexts" (D'Ambrosio 1998, p. 5).

Because the perspective of Ethnomathematics is quite broad, that is, it is not limited to identifying mathematics created and practiced by a specific cultural group, D'Ambrosio (2001) establishes the concept as part of a research program that consists of a holistic investigation generation, intellectual and social organization of mathematical knowledge, with broad pedagogical implications.

As a pedagogical proposal, this field of knowledge has several goals, among which, we will mention only two: the first, to demystify an unquestionable, absolute, and neutral form of mathematical knowledge; and the second, to illustrate intellectual achievements of civilizations, cultures, peoples, professions, and genres, that is, to understand that real people in all parts of this world and in all times of history also developed mathematical knowledge because they needed to solve everyday survival problems (D'Ambrosio 2002).

According to the conceptions of Frankenstein and Powell (2002), one of the objectives of Ethnomathematics, in the educational field, is to enable students to discover that they already think mathematically and, therefore, they can learn school mathematics. Thus, "we defend the connection of their mathematical understandings with a history of deconstructed mathematics and with the academic mathematics they are studying" (p. 1).

According to Knijnik et al. (2012), although Ethnomathematics seeks to break the demarcated boundaries of the school environment, its interest is still focused on school mathematics, as D'Ambrosio (2015) states that, due to historical circumstances; Ethnomathematics does not reject the academic mathematics symbolized by Pythagoras:

> Whether we like it or not, the peoples who, from the 16th century, conquered and colonized the entire planet, were successful thanks to the knowledge and behavior that were based on Pythagoras and his fellow Mediterranean basin. Today, it is this knowledge and behavior, embodied in modernity that drives our daily lives. It is not a matter of ignoring or rejecting modern knowledge and behavior. But, to improve them, incorporating to it values of humanity, synthesized in an ethic of respect, solidarity and cooperation (D'Ambrosio 2015, p. 42–43).

This mathematics should not be understood as a mere set of contents and methods to be transmitted to the student in order to develop their logical reasoning; however, it should be a subject directly implicated in the production of subjectivity, giving meaning to our lives and things of this world.

In line with these conceptions, Vergani (2007) adds that ethnomathematics decentralizes from the usual references of a uniform curriculum to which the school population is obliged to conform. It is aware of the need to train young people to be capable of integrating themselves in a globalizing world, nonetheless, still recognizing the specific sociocultural values of the environment in which they operate.

One of the difficulties of inserting Ethnomathematics in the Brazilian educational context is the resistance among some mathematical educators who seem to be indifferent to the influence of culture in the understanding of mathematical ideas (Domite et al. 2004). However, this author explains that Ethnomathematics, when linked to the educational field, must start from the interpretation of the role of culture of each identified sociocultural group.

10.3 Research in Ethnomathematics at UFRN

At the Federal University of Rio Grande do Norte (UFRN), Brazil, there is no specific research group in Ethnomathematics. The research group Mathematics and Culture in which the second author of this article participates, currently researches on Islamic mathematics, under the coordination of Professor Dr. Bernadete Morey. This teacher is retired from this University, however, she still in activity, developing studies on the Theory of Objectification, together with Luis Radford, its main theorist.

Under a supervision concluded, in 2002, a master's research based on the conceptions of the D'Ambrosio's notions of ethnomathematics, together with the Programa de Postgraduate Program in Education of the mentioned University. It aimed to investigate how mathematical concepts present in the daily activities of a community of horticulturists, located 30 km from the center of the city of Natal, in the state of Rio Grande do Norte, Brazil, on the North Coast.

People in this community live primarily of informal work, growing and selling vegetables at street markets, grocery stores, supermarkets, and in surrounding cities. In the development of this research in this community, some informal mathematical knowledge, developed by the people who lives there, often in different codes of academic mathematics, was unveiled. This closer elaboration of the daily life of this community is rooted in its cultural uses and has proven effective in solving its everyday problems (Bandeira 2002).

In this doctoral research, the second author under the guidance of Professor Dr. Bernadete Morey. Whereas the dissertative studies related Mathematics and Culture, one of the strands of Ethnomathematics, because the interest at that time was the nature of the thought and the mathematical activity of a certain sociocultural group, the gardeners of the Gramorezinho community. In this thesis, now transformed into a book, there was the integration between Culture and Mathematical Education, another aspect of Ethnomathematics, or, more precisely, it was shown that Mathematical Education can be more effective if it becomes an example of culturally specific contexts.

One of the results of this research showed students of the municipal school belonging to this community that mathematics does not exist only in the classroom, in books, but everywhere, and that it is different because cultures are also different, as one of the students said, one who understood both maths: the one from the school and the one from the garden, however the last one was better because the subject already worked with it (Bandeira 2009, 2016).

Upon completion of the doctorate, which occurred in 2009, the second author became a teacher of *Postgraduate Program in Science and Mathematics Teaching*, *Center for Exact and Earth Sciences*, UFRN. Among the works in the area of Ethnomathematics, the object of analysis in this chapter: Araújo Júnior (2013); Gonçalves (2013); Morais (2016); Pereira (2017) and Lima (2018), as which will be analyzed in section five, entitled, *Results and Discussions*. Let us see, then, a demonstration of the dissertation works already concluded.

The work of Araújo Júnior (2013) in qualitative character in an ethnographic approach, aimed to investigate the mathematical knowledge of potters[1] of the Currais Novos settlement, who uses it as tools in the production and marketing of colonial type ceramic tiles.

[1] In the Houaiss electronic dictionary of the Portuguese language (2009), potter means an individual who makes and /or sells ceramic objects; potter; one who works in pottery (factory). Pottery means the place where ceramic pieces are made; the technique of making clay objects; the potter's art.

The Peruvian Ceramics researcher's field of research is located in this village, a district of the Municipality of Jardim do Seridó, in the state of Rio Grande do Norte, with an estimated population of 500 families, whose main economic base is the manufacture and marketing of red ceramic tiles for Rio Grande do Norte and surrounding states.

In this worked Ceramics, without the period in which the research was carried out, 35 potters, all male. Most of these students did not finish high school, others just completed elementary school and some of them attended up to the fifth year of basic education in basic education. Many of them work in ceramics due to the lack of work options in this village, or even due to the influence of family members.

In visits to Cerâmica Peruana, in addition to analysis of photographs, interviews, logbook, field notes, interviews, among other documents, this researcher identified mathematical knowledge of tile making and fabrication, several times different from formal Mathematics, handled in the following activities: no method of counting tiles when loading trucks; the purchase of raw materials and the sale of tiles; no shape of the tiles, in the cooking ovens as tiles and in the tile storage sheds; no calculation of the volume of water used by potter workers when mixing it with clay; in cubing the clay at the time of purchase; in the cubing of composite wood for food; and other knowledge.

This researcher, when investigating the mathematical knowledge of these potters to help in the elaboration of a pedagogical proposal to dialogue with the formal mathematical knowledge of the school of that village, emphasizes that, by working pedagogically the mathematical knowledge of this community in tune with academic mathematics will make the students understand the meaning of this knowledge, and value it.

Gonçalves' research (2013) was conducted in the city of Russas, in the state of Ceará. Being one of the main economic activities of this city, the industries of red ceramic or simply *ceramics*, as they are popularly known in this locality, place the city as an important producer of tiles and bricks in the Brazilian Northeast. In fact, because it houses part of the ceramics of this city, this researcher developed his research in the Sítio Ingá community, located seven kilometers from the urban area of Russas, specifically with students from the José Ricardo de Matos Municipal School of Elementary Education.

This research aimed to develop an educational intervention in a sixth grade elementary school class coming from a community of workers of red ceramic industries, located in the rural area of this municipality, as we already mentioned, and from this intervention, elaborate a set of pedagogical recommendations aimed at teachers of Basic Education.

These students were chosen because they concentrated the largest number of students/workers in *ceramics* among all other classes of elementary school in that school. Formed by 24 students aged 12–17 years old, nine of these students, even at an inappropriate age, worked in the *ceramics* during the day they attended school. In addition, they all had family members who worked in these industries.

The initial stage of the pedagogical process, using field research while, as a pedagogical resource, the students, mediated by the teacher/researcher, managed to characterize and identify four ethnomathematical practices related to the work of ceramic workers, namely: counting the production of bricks and tiles and loading the production of bricks and tiles into trucks.

These ethnomathematics practices were systematized by the students, under the guidance of the teacher/researcher, showing similarities and singularities with the school knowledge contained in the sixth grade curriculum of that school, which served as a foundation for the elaboration of activities related to the workers' ethnomathematics knowledge. Ceramics and school content: multiplication, introduction to proportionality, and division.

The process of development and application of the educational intervention carried out by Gonçalves (2013) was undertaken based on the following steps: (1) preparation for research; (2) field research and data analysis; (3) activity planning; and (4) application of activities. As a result, this researcher found that the use of field research as part of the teaching and learning process favored the placement of students as critical subjects of their own reality.

Morais' research (2016) aimed to analyze evidence of the mathematical knowledge implicit in the commercial operations of three fair marketers, located in the New Natal Housing Estate, located 30 km from the city center of Natal, in the state of Rio Grande do Norte. To achieve this goal, this researcher was based on Ubiratan D'Ambrosio's conceptions of Ethnomathematics, as well as other researchers aligned with this field of knowledge, as well as qualitative research in an ethnographic approach and its techniques.

This author emphasizes that before a range of items displayed on the stalls of three marketers, such as handcrafted products, metal tools, applications for a general shape, medicinal herbs, temperatures and a number of other objects, such as rope, lamps, punches and a variety of bottles included. Opt for goods in which users use improvised instruments to quantify, but which are not measured using standardized measurement units, currently in force in Brazil.

In this perspective, then, investigate the mathematical knowledge used in the purchase and sale of black pepper, garlic, color, and bilberry. As the first two wares were chosen because they are widely sold in the open market, a third ware, or color, was chosen because it is a condition widely used in the kitchen and, or boldo, the fourth ware, or less sold among the selected wares. These goods are sold in glasses, a retail outlet, and in plastic bags, measured in an approximate way. For example, a black pepper is sold in two types of glass. The Garlic is sold per *head* or in a bag with garlic *cloves*, paprika in the glass or in the 1 kg package, and the bilberry in the glass or in the plastic bag.

From the data analysis, this researcher elaborated an *Activity Book* with problem situations arising from the interpretations, in the light of Ethnomathematics, of the purchases and sales made by the fair marketers. Such problem situations try to unveil the true meanings that mathematical contents have or should have for students by applying them to solve problems inherent in their experiences. In

this perspective, according to this author, the notebook intends to subsidize the pedagogical work of teachers, with problem situations and their respective orientations, containing teaching-learning actions in mathematics teaching directly focused in the classroom of Basic Education.

The research of Pereira (2017) aimed to investigate, through the professions of students of Youth and Adult Education, in one of the schools of Natal, in the state of Rio Grande do Norte what are the mathematical knowledge used and/or produced in their professional activities to then build pedagogical activities based on the ethnomathematics of these students' professions and problem solving as intervention tools in the teaching and learning of mathematics in basic education.

To achieve this goal, this researcher made use of ethnographic procedures such as a field diary, semi-structured interview, questionnaires, and participant observation to identify the empirical mathematical knowledge of these students and their professional experiences.

From the data analysis, a didactic material, called Educational Product, was produced, an indispensable technical production for the conclusion of the Professional Master's Degree in Teaching Natural Sciences and Mathematics from UFRN, a tool from educational nature, which should be disseminated, analyzed, and used by other teachers.

The educational product, produced from this research, was made of problem situations related to the professional experience of an automotive painter, one of the students of the school from which the research was conducted, since it was possible to perceive mathematical ideas present in several moments in the execution of his professional activity.

Let us see, then, two problem situations that are part of this educational product. The first problem situation is as follows: defining what price to paint a shock from a certain car model is proportional to its width and height, so if the shock is caused by the wear and tear used as the following dimensions: 1.80 m wide 0.70 m high, then answer the following questions: (a) Knowing that 100 mL of paint corresponds to a painting of an area of approximately 0.250 m^2, how much would the ink cost in ml to paint this shock? (b) How much would this painting cost, considering only the amount spent on paint?

You are now in the second problem situation: Notice, in Fig. 10.1, the advertisement for the sale of an automotive paint. A painter needs to choose one of the most economical paint packages to buy. To define your purchase, select compare or yield. In your opinion, which can of paint or paint should you stop using?

Lima's research (2018) was carried out with a group of prospectors who develop their work activities in prospectors located in the municipality of Parelhas, in the state of Rio Grande do Norte. This city, 245 km from the state capital, Natal, is made up of approximately 20,354 inhabitants. In the Seridó region of the state of Rio Grande do Norte, more precisely in this city, from the year 1937, the prospectors were digging in the mountains of the region in search of minerals, especially beryl, columbite, and tantalite.

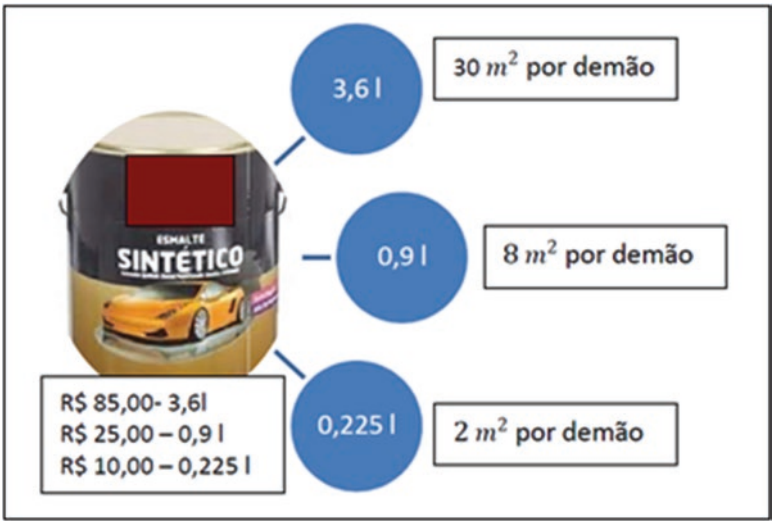

Fig. 10.1 Sales announcement (Source: Pereira (2017, p. 98))

This research aimed to investigate the mathematical knowledge that is used by prospectors in the process of extraction and commercialization of these minerals in two mines located in the rural area of Parelhas, in the state of Rio Grando do Norte, and how they can contribute to the elaboration of a pedagogical action proposal for the teaching and learning of mathematics in basic education.

To achieve these goals, this researcher was based on the assumptions of Ethnomathematics, Ubiratan D'Ambrosio's conception, as well as other researchers aligned with this theme and the teaching and learning methodology through Problem Solving. In the methodological field, some elements of ethnography were used, such as the field diary, the semi-structured interview, and the participant observation.

From the data analysis, an Educational Product or Activity Book was elaborated and pedagogically elaborated with third grade students from Monsenhor Amâncio Ramalho State School, mostly participated in the local communities in the municipality of Parelhas, in the state of Rio Grande do Norte, Brazil, from the perspective of teaching and learning methodology through Problem Solving.

This activity book had as one of the objectives, to provide students with a new look at Mathematics, in the promotion of an education that values cultural diversity, respecting the singularities and particularities of this student in their social reality, offering activities and actions of pedagogical actions that rescue a historical and cultural heritage without escaping the reality of academic mathematics.

Let us see, then, how these problem situations were structured and the link to access them.

As problem situations that make up this Activity Book were divided into five stages, four of these characterize a sequence of tasks that are performed without mining for the production of minerals and a fifth that concerns the use of these ores.

In the first stage, they are considered as problematic situations related to the mineral extraction process, followed as situations in the export stage of these minerals, later, a third stage considered as problematic situations in the mineral transport and storage process. A fourth stage, in turn, concerns the commercialization and, finally, as problem situations involving the use of ores.

Each stage of problem situations is structured in the following way, a saber: only an introductory text will be presented, including a brief explanation of the questions about practices used, as well as the contents that are used. Subsequently, the reference texts will be displayed, which were extracted from participating participants in the researched mines and in the interviews with the garimpeiros. Then, turn as situations related to the reference text previously.

As a result of the intervention, the students obtained an active and effective participation in the construction of mathematical concepts during the process of solving problem situations inherent to the sociocultural context of mining, understanding and valuing in the same way the different ways of understanding and dealing with mathematics that manifest themselves in extra environments.

Given the results, the actions of this author, it is possible to insert in the scope of Mathematics Education, the pedagogical actions that link the ethnomathematical knowledge of different sociocultural groups with problematic situations to be worked through the teaching and learning method through the Problems.

10.4 Methodology

In the view of the proposed mapping and discussion of the convergences and divergences between researches that deal with a given theme, obtained according to a previously defined cutout, this study can be understood as the State of Question type. According to Nóbrega-Therrien and Therrien (2004) the research State of Question aims to "lead the researcher to record, from a rigorous bibliographic survey, how the theme or object of research is in the current state of Science at your reach" (p. 7).

Thus, within the scope of this article, we defined as the theme to be investigated *Ethnomathematics*, focusing on the works produced at the Federal University of Rio Grande do Norte (UFRN) from 2013. The option for this specific cut was given so there is a better discussion and dedication about the paths taken by the list of research developed in this Higher Education Institution.

From the theme and the defined cutout, the research was searched through the electronic repository of the UFRN[2] Digital Library of Theses and Dissertations, from the descriptor "ethnomathematics" and the filter for the dissertations from students of the Professional Master in Teaching Natural Science and Mathematics.

[2] The webpage ofthe cited digital repository is: https://repositorio.ufrn.br.

Table 10.1 List of selected theses

Title	Authors's surname	Year
Ethnomathematics in a pottery of the Seridó Region/RN	Araújo Júnior	2013
The Ethnomathematics of Ceramics Workers of Russas-CE and the school context: Outlining pedagogical recommendations from an educational experience	Gonçalves	2013
Ethnomathematics of Feira Livre: Contributions to a didactic-pedagogical proposal of mathematics teaching-learning in Basic Education	Morais	2016
Dialogues between Ethnomathematics and EJA Problem Solving	Pereira	2017
Mining ethnomathematics: A pedagogical action proposal for mathematics teaching and learning from the perspective of Problem Solving	Lima	2018

Source: Prepared by the authors

The search results were refined by reading the abstracts of all works and selecting those that were in line with the subject matter of this State of the Question. After the refinement step, five works were selected, which are better detailed according to the title, author's last name, and year of publication in Table 10.1.

Based on the objective and the specific questions defined in the previous section, the results will be analyzed and discussed according to the following categories: research participants, ethnomathematics, and school mathematical knowledge, methodological path, relationship between Ethnomathematics and other trends in Mathematical Education, and the educational product.

10.5 Results and Discussions

Regarding the participants, all works focused on knowledge and doings related to specific professions. In particular, 60% of the researches also had the participation of students from different levels of Basic Education, either directly (Gonçalves 2013; Lima 2018) or indirectly (Pereira 2017).

The research subjects of Araújo Júnior (2013) were Peruvian Ceramics potters responsible for the manufacture of colonial type red ceramic tiles, located in the Currais Novos village, rural area of Jardim do Seridó, in the state of Rio Grande do Norte, Brazil.

In Morais' research (2016), the ethnomathematic knowledge investigated was related to the practices of fair traders (who are also popularly known as mangeiros for selling homemade products or food from small farms) of a free fair at Conjunto Nova Natal, in Natal, in the state of Rio Grande do Norte.

Among the investigations that also had the participation of students of school education, Gonçalves (2013) investigated together with a class of Early Childhood Education and Elementary School's sixth grade at José Ricardo de Matos Municipal School, in rural Russas, in the state of Ceará, the ethnomathematics knowledge from the practice of workers of the red ceramic industry of that municipality.

In Lima (2018), the researcher addressed the ethnomathematic practices of prospectors in the city of Parelhas, in the state of Rio Grande do Norte, during his work and, subsequently, undertook an educational experience with students from the third grade of Monsenhor Amâncio Ramalho State School, structured based on the ethnomathematic knowledge unveiled.

From a survey of the students' professions of a Youth and Adult Education group (EJA) at the Professor Ulisses de Góis Municipal School, in Natal, in the state of Rio Grande do Norte, Brazil, Pereira (2017) selects as object of his research the ethnomathematic knowledge and doing of an automotive painter, student of the class accompanied during the investigation.

In view of the knowledge and practices investigated in each of the researches, we began to analyze and discuss the ethnomathematic practices and knowledge and their relationship with knowledge derived from school mathematics. In Table 10.2 below we list what were the ethnomathematic knowledge unveiled by the research and what correspondence of school mathematics were made.

Regarding the ethnomathematic practices and knowledge of each research group, although immersed in different contexts, we noticed common skills, such as calculating, for example. The ability to calculate was present in production counting processes (Gonçalves 2013; Lima 2018), purchase and sale of products (Araújo Júnior 2013; Morais 2016), and services budget (Pereira 2017). Although common, the way calculations are performed by the professionals studied varies according to the needs and instruments available to put the aforementioned skills into action.

Regarding knowledge of school mathematics, according to the Common National Curriculum Base (BNCC) of Early Childhood Education and Elementary School (Brazil 2019), the area of Mathematical Knowledge lists important elements for the formation of mathematical thinking (named fundamental ideas), which articulate and permeate various contents of the curriculum. The document cites: proportionality, equivalence, representation, order, interdependence, approximation, and variation (Brazil 2019). In addition to the fundamental ideas, the BNCC subdivides mathematics into thematic units, namely: Numbers, Algebra, Geometry, Quantities and Measures, and Probability and Statistics.

From this classification, we found that proportionality was a fundamental idea addressed in all works. In addition, the presence of contents such as numerical operations and percentage (Numbers), functions and rule of three (Algebra), flat and spatial figures (Geometry), means and medians (Probability and Statistics), and units of measurement and volume calculation (Quantities and Measurements) shows that the list of investigations analyzed discusses mathematical knowledge that make up all five thematic units of the BNCC.

Regarding the methodological path taken by the investigations, while Araújo Júnior (2013), Morais (2016), Pereira (2017), and Lima (2018) have defined qualitative researches of ethnographic approach, Gonçalves (2013) developed a qualitative research of the action research type. In all cases, the data collection instruments used were participant observation, interviews, and/or questionnaires.

Taking as reference the typification of the perspectives of research in Ethnomathematics elaborated from the proposed State of Art, Conrado (2005), Araújo

Table 10.2 Ethnomathematical content and its relation to school mathematics in each research

Author	Ethnomathematical practices and knowledge	Knowledge of school mathematics
Araújo Júnior (2013)	– Purchase and extraction of clay – Dough preparation – Automatic tile cutter – Transport of shingles to sheds and patios – Cutting and cubing the firewood – Unmolds the redneck pottery ovens – Marketing of tiles	– Numerical operations – Units of measure – Flat figures – Polyhedra – Reason and proportion – Percentage – Three simple rule – Affine function – Tables and graphs
Gonçalves (2013)	– Brick production count – Tile production count – Loading bricks in trucks – Loading tiles on trucks	– Multiplication – Reason and proportion – Division
Morais (2016)	– Purchases of goods and calculation of expenses – Change in price of goods according to the time of year – Calculation of profit margin – Development of merchandise sales strategies – Estimates	– Numerical operations – Reason and proportion – Percentage – Estimated calculation – Units of measure – Breaks – Tables – Arithmetic mean – Weighted average – Median – Rol – Classes – Frequencies – Sample space – Event – Geometric solids – Cylinder volume – Cone volume – Affine function
Pereira (2017)	– Estimated amount of ink and other materials – Budgeting of painting – Car surface preparation – Ink applications	– Numerical sets – Proportional quantities – Spatial geometry
Lima (2018)	– Separation and classification of material containing ores – Approximate calculations of stocked ore volume and mass – Transportation of ores	– Numerical operations – Number sequences – Percentage – Reason and proportion – Rule of three – Units of measure – Graphics – Arithmetic mean – Geometric figures – Cylinder volume – Regular rectangular prism volume

Source: Prepared by the authors

Júnior (2013), Morais (2016), and Pereira (2017) would be *anthropological*studies. According to the author, this type of research "seeks to investigate, describe, unveil, understand the knowledge, expertises, practices, mathematical practices—or ethnomathematics—of a social, ethnic or cultural group" (Conrado 2005, p. 95). On the other hand, Gonçalves (2013) developed a work with pedagogical focus, which according to Conrado (2005) is characterized by:

> (…) in addition to addressing the knowledge produced in the groups, it develops a pedagogical intervention proposal for the school of the investigated community. In this case, the ethnographic research with the group takes place in parallel with the researcher's performance at school or in some unofficial educational nucleus, as an educator, teacher or participating observer, in which the proposal is built within the school context (p. 96).

Thus, the research cited sought to evaluate possibilities and potentialities of ethnomathematics, such as pedagogical action, reflecting on the perspectives present in this action from the context in which it develops and its implications.

In the case of Lima (2018), there is an intermediate position between the two perspectives. At first, the research focused on unraveling the ethnomathematical knowledge inherent in the practice of mining. Next, the author elaborated a pedagogical action proposal that brought ethnomathematic knowledge as elements for the teaching and learning of mathematics for a high school class.

In the context of research in Ethnomathematics that bring contributions to the school context, Martins and Gonçalves (2015) found that one of the strategies used by the investigated works consisted in the interrelation of this field with other trends in Mathematical Education. Communicating with this perspective, the list of investigations, object of this work, adopted the integration of Ethnomathematics with Problem Solving.

In general, all the research analyzed addressed problem solving from the elaboration of a set of situations with convergences between the knowledge of school mathematics and the ethnomathematic practices and knowledge unveiled. In this sense, research has adopted a perspective, in the light of ethnomathematics, of teaching to solve problems (Onuchic 1999).

In addition, some work has also combined problem solving with a problem-solving teaching perspective using the Teaching-Learning-Problem Solving Assessment Methodology (Onuchic 1999; Onuchic and Allevato 2004).

In the works of Morais (2016) and Pereira (2017), teaching through problem solving is proposed as a guide for the use of educational products developed within each of the researches. On the other hand, Gonçalves (2013) and Lima (2018), besides presenting the Teaching-Learning-Assessment Methodology through Problem Solving, also employed this strategy in the application of the educational experiences developed in their investigations.

Considering that all the works analyzed in this research were developed within the scope of a Professional Master in Teaching Natural Sciences and Mathematics, we focus on an important element that makes up the dissertations that come from this type of course, the educational product.

The educational product can be understood as a didactic material developed from a research, detachable from the rest of the dissertation and that makes possible, as

directly as possible, contributions to the educational context. Among the dissertations analyzed, we classify educational products in two types.

Although they adopted different nomenclatures, such as Pedagogical Sequence (Araújo Júnior 2013), Problem Situation Booklet (Pereira 2017), or Activity Booklet (Morais 2016; Lima 2018), the first type of educational product was constituted as a set of problems, elaborated from the ethnomathematics practices and knowledge of the investigated sociocultural groups. In addition to the problems, the aforementioned products also provide complementary guidance for teachers who wish to apply the activities, adapt them or elaborate new ones through integration with other subjects.

Based on the premise that the activities elaborated and applied within the scope of the pedagogical experience would be more relevant only in the sociocultural context for which they were elaborated, the educational product developed by Gonçalves (2013) gathered a list of methodological orientations to support the planning, application, and evaluation of new classroom interventions directed to other environments and participants, according to the premises of Ethnomathematics and Problem Solving adopted by the investigation.

Although all educational products elaborate elements that make it possible to contribute to the spread of ethnomathematics in the school context, some of them (Araújo Júnior 2013; Morais 2016; Pereira 2017) did this without performing interventions in educational contexts. Another particularity of educational products was the range of these materials for use by teachers in other educational contexts.

The first type constitutes more restricted references with transferability, bringing a set of problems that would be more relevant only if used by teachers in the surroundings where the research was conducted. However, the complementary guidance provided in the educational products and the diversity of elaborated problems make it possible to alleviate this limitation by providing teachers with access to materials on how they could adapt the proposals in new pedagogical interventions.

By providing detailed methodological guidance, according to each stage in which the educational experiment was developed, the second type of educational product has a greater transfer potential. Nevertheless, fewer problems (compared to the first type) could be an obstacle for teachers to be inspired and organize their proposals for the classroom.

10.6 Final Considerations

Given the relevance of the professional master's proposal in improving the qualification of human resources for the labor market and in order to investigate the particularities of research in ethnomathematics developed under this type of course, this article analyzed a clipping of research in ethnomathematics produced in

Professional Master in Teaching of Natural Sciences and Mathematics from UFRN, highlighting the following aspects: research participants; ethnomathematic knowledge and school mathematics; methodological route; relationship between ethnomathematics and other trends in mathematics education; and educational product.

Concerning the research participants, most of the dissertations studied professional groups in line with students of Basic Education, suggesting an advance in the integration of ethnographic and pedagogical bias of research in Ethnomathematics.

The consonance between the knowledge unveiled and the contents of all thematic units proposed in the Common National Curriculum Base (BNCC) of Elementary Education portrays diversity between the ethnomathematic knowledge of the social groups studied, allowing different forms of integration in the school mathematics curriculum.

Regarding the methodological approach adopted by the works, although the studies (80%) that define themselves as "anthropological" prevail, we found at least three approaches: research on the ethnomathematic knowledge of social groups without intervention in the school context, investigation of the knowledge of the students' groups with later insertion in the classroom, and instigation of the groups through educational experiences. This plurality indicates that there are different methodological possibilities for ethnomathematics studies focused on contributing to the school.

Regarding the relationship between ethnomathematics and other trends in mathematics education, all researches opted for the use of problem solving. From the perspective of teaching problem solving as well as teaching through problem solving, the unanimity of the work indicates a convergence among researchers in exploring Problem Solving as one of the ways of integrating ethnomathematics in the classroom.

In relation to the educational product produced by the dissertations, the prevalence of didactic systems containing complementary tasks and orientations, elaborated from the systematized ethnomathematic knowledge in the investigations, allows us to affirm that there is a concern with the diffusion of ethnomathematics through the elaboration of ready-made materials used by teachers acting in Basic Education. However, given the limited transferability of these materials to different school contexts, it is essential that teacher training for the construction of their own materials focusing on ethnomathematics be better explored in research on the subject.

In light of what has been presented so far, it is of utmost importance that research in ethnomathematics with educational focus continue to be undertaken on different fronts, particularly within the framework of stricto sensu postgraduate programs in education and teaching, aimed at both a closer approach to teaching practice and exploring the potential of this course format in the construction of educational products focused on the diffusion of this investigative field in the school context.

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Chapter 11
Unity in Differences: Reflections on the *Doing* of Ethnomodelling and Dialogue Through Connecting Ethnomathematics and Modelling

Daniel Clark Orey and Diego Pereira de Oliveira Cortes

In a time of destructsion, create something: a poem, a parade, a community, a school, a vow, a moral principle; one peaceful moment.

Maxine Hong Kingston

Developing an unbiased attitude does not mean we have to think of all paths as equal, as this is simply not true. Each has its own flavor and strengths, and therefore what we are trying to do is develop greater awareness of what diversity has to offer. Our aim is to clearly distinguish between their differences, respecting each as a skillful means to guide different sentient beings towards greater happiness.

Khentral Rinpoche

11.1 Introduction, Background, and Context

Prof. Orey, in November 2019, returned from a fifth trip to Nepal, where he worked with the ethnomathematics research group, colleagues in ABI-Nepal,[1] and gave two talks and a workshop for teachers. The workshop happened to be near one of the planet's most sacred places: *Lumbini*, the birthplace of *Lord Buddha*. Conversation

[1]Activity-Based Instruction Project (ABI) has been working with teachers in Nepal to upgrade curriculum and instructional practices for mostly rural public schools.

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with colleagues in Nepal, especially at the *Buddhist University of Lumbini*, encouraged him to think more about dialogue and communication in relation to how we create further ethnomodelling experiences.

In our work in ethnomodelling we are concerned and mindful of the harm that colonization has created and are deeply interested in the ideas of emic-etic perspectives[2] and the creation of dialogue in modelling phenomena at a local level; this is especially true in the context of educators, learners, and schools in Brazil. The ideas related to the outsider and insider and how their interaction combine to create unique and healthy dialogue is what is of deep interest here. We see it manifested in what we are doing at a deeply personal level, as Orey for example, for almost 9 years now, is both an insider and outsider (immigrant) in his community in Brazil.

So, it was with great anticipation that work began with an investigation conducted by Diego Pereira de Oliveira Cortes, in 2017. Cortes's work represents the first masters' thesis that we know of, using ethnomodelling. His very creative and innovative study looked at how ethnomodelling might be applied in a practical way in a school setting. His work brought together a group of high school mathematics students and a neighborhood farmer seller. Besides forming the data for his masters' work, his work has been presented in various congresses and published in books and journals in English and Portuguese. He has shown that the idea of ethnomodelling can be successfully examined and applied at both a deep and practical level. The *ethnomodels*[3] his students developed are of the utmost quality and profundity, and we are proud here to share a few of them below.

That being said, the work of ethnomodelling and ethnomathematics is ongoing, many mathematical and scientific traditions are being revived in many places across the world, chiefly in both Brazil and Nepal in the context of both higher and teacher education. There is a great need to couple powerful western mathematical and scientific ideas to peaceful and spiritual or moral aspects of its implementation.

One movement of note is related to *nonkilling mathematics*, a mathematics-science paradigm built on peace and justice, something Ubiratan D'Ambrosio (D'Ambrosio 2011) links to the ethnomathematics program. Before we talk about nonkilling mathematics, we would like to share our perspective on ethnomathematics and ethnomodelling.

[2] Etic perspectives or approaches refer to the interpretations of aspects of another culture from the view or perception of those who observe, that is, outsider viewpoints (How we think they do it). Emic perspectives or approaches seek to understand specific culture based on the insider views (this is how we do it) or on their own worldviews or cosmologies (Rosa and Orey 2012).

[3] Ethnomodels are rooted in sociocultural contexts and represent small units of information used in the interpretation and understanding of the reality of the members of distinct cultural groups. The translation between emic and etic approaches can be carried out with the elaboration of dialogical ethnomodels that consider sociocultural influences in the ethnomodelling process (Rosa and Orey 2019).

11.2 Ethnomodelling/Ethnomathematics

Much of the current work in ethnomodelling and ethnomathematics looks at how diverse members of distinct cultural groups use mathematical thinking. What is common across all cultures is the basic human endowment that allows all people to count, to order, to pattern, to measure, and to model.

The emic/etic dialogue that forms the basis to ethnomodelling came about because of current concerns in ethnomathematics towards colonization, and the imposition of powerful western academic mathematics and sciences while traditions and science that served diverse communities for millennia are being lost at an alarming rate. Often before these traditions have been shared, catalogued, or saved for future generations.

Many of the diverse ways of counting, reasoning, classifying, ordering, and thinking seem exotic or strange for those of us nurtured in the western-academic tradition of science and mathematics, yet, they are part of daily life for those not blinded by this paradigm. What colonization gave was also taken from many cultures. For instance, while giving access to very powerful mathematics and science, it has also ignored, discounted, and often erased traditions, mathematics, economics, science, and ways that diverse people ordered, counted, patterned, measured, and modelled their environments.

Orey has been working with ideas of ethnomathematics since the late 1980s; his work looks at forms of technology (Orey 1988) and ethnomathematics taking him to Guatemala, México, Brazil (where he lives now), and Nepal. Professors Rosa and Orey have been keen at looking how they might take the "mystical way and walk it with practical feet" (Orey and Rosa 2016, p. 7), that is creating practical applications of ethnomathematics, or the actual doing of ethnomathematics. So, it is that we have been exploring aspects that we call ethnomodelling (Rosa and Orey 2013).

Currently, Professors Rosa and Orey have had a number of graduate students looking at ethnomodelling, modelling, and ethnomathematics in one form or another, including Cortes who is the first researcher in ethnomodelling, who developed a study with a farmer seller (Cortes 2017). Ethnomodelling and ethnomathematical related ideas previously explored include modelling a Sioux tipi (Orey 2000), financial education for deaf students (Pinheiro 2017), a gym (Bastos 2019), ethnomodelling in a bilingual Palestinian-Israeli school (Naftaliev and Gara 2019), and the theorization of teaching practices in mathematical modelling contexts through the examination of teacher scaffolding in the ethnomodelling perspective (Lewis 2018).

A recent collaboration between the University of Oslo and the Universidade Federal de Ouro Preto have created a project beginning in March 2020 looking at aspects of *math trails* in both locations and ethnomodelling of cultural artifacts, monuments, and aspects of out of school mathematics. Future and ongoing ethnomodelling work is related to students and exploring coffee production and very powerful study in a *favela* (slum) in Belo Horizonte that aims to empower middle school students to successfully compete in a state and national mathematical/scientific competition.

As well, from an aspect of pure mathematical modelling, aspects of transportation and river pollution in courses taught online at Universidade Federal de Ouro Preto (UFOP) have been undertaken. For example, Orey and Rosa's ongoing work documents the emic/etic (insider/outsider) perspectives, using the necessary dialogue between them. This ongoing research, as well as the work of numerous students, Cortes included, has brought us to a place that considers what and how dialogue might be structured as it naturally arises.

At the same time, when we reflect on the world around us, what is becoming more and more evident is the real difficulty in communication and respect for diverse opinions and new ideas. By refocusing on the moral aspects of mathematics teaching and learning, and how it relates to the *how and what*, our teaching has been the cause for us to refocus on how to truly create dialogue in our ethnomodelling work.

While consulting in Nepal and working with the ethnomathematics research group in Kathmandu, Prof. Orey was deeply moved by his colleagues work and how they are implementing and reflecting South Asian Hindu-Buddhist perspectives of peace through their research in creating dialogue in ethnomathematics, including the peaceful uses of mathematics as described by D'Ambrosio (2011).

What many of our students and teachers are noticing is a lack of curiosity and interest in the world around them; this is especially true in regard to mathematics; this is coupled with a passive disregard for the dangerous aspects of the mathematics and science we teach and use. Meanwhile communities across the world are suffering from the effects of financial disasters, injustices of rich versus poor, environmental degradation, and violence of all types.

Questions related to indeed mindfulness of what is mathematics itself and its nature and how to teach it empower learners and communities to resolve some of these pressing issues. We have seen this is the abovementioned projects. It might be appropriate here to look briefly at what a *nonkilling education* movement adds to this discussion.

11.3 Nonkilling Mathematics

Essentially, a *nonkilling mathematics*, as outlined by D'Ambrosio (2011), is the mathematics taught in a way that is both rigorous and critical, yet in a form of criticality that is both positive and inclusive. In this context, Rosa and Orey (2019) demonstrate that by helping participants to look at data, often emotional or angry moments can be transformed into productive opportunities that encourage us to communicate, to share ideas, and to solve basic problems in communities, what Kingston (2020) refers to as *creating one peaceful moment*.

Mathematics can and should be used as an instrument for peace, a discipline where learners using ethnomodelling techniques come to see the power and beauty of mathematics and its humanizing features by finding how it directly affects their lives and that of their own community. In this regard, D'Ambrosio (2011) affirms that:

> Particularly in Mathematics, there is a general acceptance that if we do Mathematics well, thus instilling attitudes of rigor, precision and correctness in the student's behavior, we are fulfilling our broad responsibilities. Undeniably true. But this is not enough. This must be subordinated to a much broader attitude toward life and toward how mathematics can be used for good or bad (p. 133).

This aspect comes about when awareness or a true sense of ethics in the mathematics develops when we teach, learn, and use mathematics, and includes:

1. A respect for the other with all the differences (which are inevitable, since the individual and the other are different).
2. Solidarity with the other.
3. Cooperation with the other (D'Ambrosio 2011, p. 132).

How diverse groups create dialogue between researchers and practitioners using ethnomodelling is becoming more and more relevant (Rosa and Orey 2019). This dialogue takes place when an outsider and insider encounter each other and create dialogue and new perspectives (Rosa and Orey 2013). This dialogue allows for all parties to develop a certain sense of mindfulness related to the forms and kinds of mathematics being used to create powerful decisions in community life or to solve problems, and perhaps in this dialogue to create peaceful moments in which people can come together to resolve conflict.

What is missing in current research are specific ways in which ethnomodelers might construct meaningful dialogue and begin to understand how to create it between two diverse groups observing a given object or mathematical context. What is especially important in creating this dialogue is how it occurs in a respectful climate of tolerance, receptivity, curiosity, and flexibility. That is, a sense of mindfulness that includes respect for diverse perspectives. This includes how a diversity of point of view can come together to look at data and create even more new perspectives.

A powerful discussion by Rinpoche (2019), not necessarily related to mathematics, but instead to the creation of peaceful dialogue, offers a valuable perspective. In so doing, he offers us a tool for *dialoguing* as it were, or at very best, he offers four aspects that we can be mindful of as we develop strong dialogically based ethnomodels. In his practice related to the importance of diversity and unity, we see how this offers a guideline to structuring dialogue in resolving conflict or solving problems in mathematical contexts in our work.

In creating dialogue, he speaks of four areas: tolerance, receptivity, curiosity, and flexibility in thought. Let us take a moment to briefly outline these four areas, and then connect them to aspects of dialogue and ethnomodelling through what we observed in Cortes' work (2017).

11.3.1 Tolerance

Tolerance is built on a base of mutual respect, being able to connect to a person in such a way that even if we do not agree with their view we still value their right to hold those views. Tolerance makes it possible to establish a basis for connection with another person or set of ideas. What we experience when we apply an ethnomathematics perspective is a certain sense of *intolerance* towards local or ancient forms of mathematics and science.

In many places of the world research coming from North America or Europe is valued much higher than that of Latin America or Asia. This still happens despite 500 years of colonization where thousands and thousands of scholars have been trained and returned to their homelands. Could it be that new forms of mathematics and science are awakening from ancient non-western traditions or from unique ecological or socio-historical contexts?

In the context of his project, Cortes (2017) was interested in using ethnomodelling as a pedagogical action towards learning how to respect and listen to alternative views and perspectives; we can all agree that tolerating new ideas might not be enough, but a more tolerant perspective leads us to being receptive to new or innovative possibilities. In a world where young people often come to see the food they eat, wrapped in plastic and sold in supermarkets, getting to know the farmer himself was a valuable opportunity. Seeing the value in fresh food and being able to converse with the producer himself is important. Thus, seeing the food they eat from the perspective of the producer offered an important opportunity to connect mathematics and real-life contexts.

11.3.2 Receptivity

All forms of communication involve the transmission of ideas and the reception of new ideas. Our main focus is on acquiring new information, so that we need to cultivate a greater *quality* of receptivity. In our increasingly confusing world, being receptive to alternative possibilities being created by an accelerated *glocalized society*[4] and coming to see the difference between fake and real news (or data), science, research, and its consequences are important now more than ever.

Being receptive to moral and spirituality-based perspectives on data might cause us to rethink what we are doing is important; this is especially true in the context of huge disparities between the rich versus poor, or the current debate on

[4] According to Rosa and Orey (2017), in a glocalized society there is an acceleration and intensification of interaction and integration among members of distinct cultural groups. In this context, glocalization reinforces positive aspects of worldwide interaction in textual translations, localized marketing communication, socio-political considerations, and in the development of scientific and mathematical knowledges.

the environment and climate. An ethnomathematical perspective addresses cultural issues in order to guarantee the effectiveness and unprejudiced methods that distinguish achievements between individuals from different cultural groups.

In the context of this project, becoming receptive to the perspective of the farmer, Corte's (2017) students realized how the farmer sold the products he produced, how he calculated, and created wealth for his family. When he came to speak to the students in the school, they saw how moved and honored he was to be there, to have the students be receptive to his experiences, knowledge, and way of life. The students were able to see, directly, just how mathematics is used in alternative contexts.

11.3.3 Curiosity

When we are around preschool children, we see that they are intensely interested in the how and why things work. When young people encounter new ideas, there is a hunger for more answers. When these new ideas are introduced into the mind, a process where the mind tries to reconcile what is new information means in relation to previous or existing ideas an inquisitive mind desires to understand, it is a reaction to uncertainty.

What many teachers in numerous locations in the world are lamenting is a lack of curiosity or engagement in our students and colleagues. How can we assist learners to appreciate this uncertainty or create opportunities where we can engage creativity and new understanding right when communities seem stuck in the old and incapable of resolving conflict, problems, or day-to-day aspects of life? We have shown that engaging students in problems that affect them and are real increases curiosity and a need to know more.

Educators worldwide are regretting the lack of interest in indeed lack of curiosity in learning mathematics. Cortes's students demonstrated a high level of motivation and curiosity in creating powerful ethnomodels that described the context of the farmer, and how he sold his products. Feedback from the students demonstrated this curiosity and excitement in seeing mathematics as something alive, humanized, and useful in their lives.

11.3.4 Flexibility

Tolerance, receptivity, and curiosity all combine together to form a powerful engine for acquisition of information. A person cultivating these qualities is very much like a sponge, pulling in as much as they can whenever they can, because they actively engage in clarifying their understanding, the quality of their view will be very strong and very broad (Rinpoche 2019).

The more people learn about diverse approaches, to similar problems, the more flexible of mind they are able to exhibit. Exposing the mind to a wide variety of experiences creates a discriminating awareness. In this regard, Rosa (2010) argues that teaching mathematics that is needed solely for elite students to pass exams needed to enter universities misses the point and is not enough. Instead we must create sponges where we are eager to learn and exchange new ideas! Maybe even create new forms of mathematics!

11.4 How Cortes's Students Created Dialogue

During the functioning of a daily farmer's market, Cortes (2017) found that it was possible to recognize the development of local mathematical practices and then model them by making use of ethnomodels. According to Cortes (2017), Cortes et al. (2018), and Rosa and Orey (2019), ethnomodels are the representations composed of small units of information that allow us to interpret and understand the reality of the mathematical thinking produced by the members of distinct cultural groups.

In this context, Cortes's (2017) pioneering work looked at the mathematical practices developed by a farmer seller, and he has provided insight into powerful pedagogical resources that involve mental calculations used to quickly calculate discounts, profits, and losses. He also found that this included the notion of proportional thinking related to mathematical functions.

In his study, the farmer's street market was the multicultural setting in which the farmer and the students shared and used their mathematical knowledge to shape a form of *transdisciplinarity* of the out-of-school environment and reality. The results demonstrated that the daily life of the market is impregnated with very rich mathematical procedures and practices that are rooted in the culture of farmer seller. Cortes (2017) showed us how they are related to concepts of *quantification, measurement, classification, comparison, and modelling*, and it can be studied conducting an ethnomodelling process.

Probably the most important contribution by Cortes (2017) and his students to ethnomodelling is that they developed a process of re-signification of mathematical concepts of functions and did this by sharing an analysis of local strategies applied by the farmer seller. They did this with applying global academic techniques and employed the students in their own cultural context that allowed them to constitute effective exchange of mathematical knowledge through the elaboration of the ethnomodels created by Cortes' students.

The farmer seller developed an *emic* (local) *ethnomodel* by mathematizing the calculation of the price of his products:

> Let's assume that you buy a 10 kg box of tomatoes for 40 *reais,*[5] and the kilogram is sold at 4 reais, thus each 100 g cost 40 cents, then you cannot sell it at that price because we have expenses like gas, transportation, employees, packaging, etc. Thus, I sell each kilogram of tomatoes by 5 or 6 *reais* because it should be more expensive since you do not go to the market to buy the products and sell them at the same price. In this case, I increased the price by 25 or 50%. Sometimes, I need to sell my products, for example, at 100 or 60% more, depending on the price I buy them and the expenses I have. This system is used to determine the price of any of the products I sell. For example, if I buy a product for 80 or 100 reais each box, then the price of the kilogram should be 16 and 12.80 reais [60% Mark-up] or 20 or 16 reais [100% Mark-up] (Cortes 2017, p. 135).

The particular ethnomodel stemming from this observation included a number of perceptions and understandings valued and deemed appropriate by the farmer-vendor himself. Cortes (2017) showed us that the primary goal of *local approaches* is related to descriptive idiographic orientations of mathematical phenomena. This is because it places an emphasis on the uniqueness of each mathematical idea, procedure, or practice developed by the members of diverse cultural groups (Rosa and Orey 2013).

In this regard, Cortes's (2017) work found that *glocal* approaches provide cross-cultural contrasts and/or comparisons by using aspects of academic mathematics that translate, in this case, the farmer seller's practices in order to create connections and new understandings related to how individuals from a non-academic cultural background use mathematical thinking. This approach is unique as looks at how we can come to explain this particular mathematical practice as a whole, as well as from the point of view of the students as outsiders.

For example, students developed a*n etic* (global) *ethnomodel*, which is an approximation of the *local ethnomodel* used by the farmer-vendor:

> A product, whose cost price is 40 reais, has a sale price between R$ 5.00 and R$ 6.00. Another product, whose cost price is 80 reais, has a sale price between R$ 12.00 and R$ 16.00. And a third product whose cost price is 100 reais, has a sale price between R$ 16.00 and R$ 20.00. However, it is important to note that these sales prices may be increased by other costs related to the market's expenses (Cortes 2017, p. 166).

The interpretation of these results discussed how the farmer determined his prices, besides being related to the quantity of products purchased, is also bounded to emic constructs developed by the unique labor experiences of the farmer seller. Figure 11.1 shows the dialogical ethnomodel developed by the students. This ethnomodel represents the sale process developed by the farmer-vendor.

If CP(m) = 40, then SP(m) = V. M, where $5 \leq V \leq 6$ If CP(m) = 80, then SP(m) = V. M, where $12 \leq V \leq 16$ If CP(m) = 100, then SP(m) = V. M, where $16 \leq V \leq 20$	CP = Cost Price SP = Sale Price M = Mass (kg) of the product V = Variation of price including expenses and charges

Fig. 11.1 A glocal ethnomodel developed by the students. (*Source*: Cortes (2017))

[5] The Brazilian *real* or *reais* (R$) is the official currency of Brazil, which is subdivided into 100 cents.

Cortes (2017) and his students developed one more emic (local) ethnomodel with rhetorical characteristics observed by the students in their visit to the farmer's market. This model is important in relation and respect to the mental calculations developed by the farmer-vendor when he solved the following problem: *Based on the price of 8 reais a kilogram, how would you calculate the price of a product that weighed 1 kg and 450 g?* (p. 167).

The farmer seller explained to the students that "First, I put 8 reais aside because I am not very good at doing some calculations. Then, I know that 200 g will be 1.60 reais, and twice 1.60 will be 3.20 reais, then I find the price of 50 g, which is 40 cents, and the value will be 3.60 reais. Therefore, the price is 8 plus 3.60 that is 11.60 reais. That is it." (Cortes 2017, p. 168).

Although the farmer seller did not develop what we as academics might refer to as *formal knowledge* in regard to the study of functions and related mathematical characteristics, his experience and observation of the world enabled him to use this concept implicitly in his work in the market. This allowed Cortes's students to elaborate a dialogical ethnomodel through the mathematization of this labor practice developed by the farmer seller, such as: "$SP\,(v) = V.\,m$ or $SP\,(V) = 8 + v.\,m$, in which $0.1 \leq V \leq 0.45$".

As part of this process, Cortes's students mathematized the price of some of the products sold by the farmer by using a functional representation including: "(u) = 1.50(u)" to determine the price of chives, coriander, and parsley and "(u) = 2.50(u) and that 2(u) = 2.50 (2u) − 1.00 = 4.00" to determine the price of lettuce. What was heartening there was Cortes's students in the mathematization process, they defined that the variable *u* represents the lettuce unit and that *2u* represents two units of this product. This approach demonstrates how the students translated the promotion offered by the farmer to his customers in which 2 lettuce units are sold at a discount of 1 *real*, for example, one lettuce unit costs 2.50 *reais* and two lettuce units costs 4 *reais*.

In our work in ethnomodelling we use the term *translation* to describe the process of modelling *local* (emic) cultural systems and how it may enable *global* (etic) forms of academic representations (Rosa and Orey 2013). A very effective application of ethnomathematics uses modelling in order to move away from exotic interpretations of this program and enables us to establish relations between local conceptual frameworks and mathematical concepts embedded in global practices. This act of translation as applied in this process is best referred to as ethnomodelling.

The data (Cortes 2017) shows that this mathematization process, as used by Cortes's students, demonstrates the existence of glocal (dialogical)[6] connections between the knowledge developed in the school environment (global) and those perceived in the non-school environment of the market (local). This supports the work of Rosa and Orey (2015) where they argue that connections between these two

[6] Glocal (dialogical) approach mixes the local and global forms of knowledge, experience, outlook, and worldviews.

approaches should be prioritized in school curricula. Cortes's (2017) work reduced the gaps between theoretical and practical mathematical knowledge.

Ethnomodelling has evolved to allow educators and researchers to assist, reconceptualize, and apply their mathematical concepts through the elaboration of activities that originate in their own unique sociocultural contexts and communities. By learning to elaborate their own ethnomodels in the mathematics curriculum, this approach enables the development of a glocal (dialogical) approach between the ideas, procedures, and mathematical practices intrinsic to the labor practices of farmer seller (local/emic approach) and school mathematical contents (global/etic approach) with the use of problem situations that emerged from the context of a fair (glocal/dialogical approach).

11.5 In Closing…

Being ever mindful of how we create a climate of tolerance, receptivity, curiosity, and flexibility in emic-etic based dialogue is essential for cres. Teaching both learners and educators to focus on data that they themselves may have created is a powerful opportunity to new foster forms of nonkilling mathematics. When we create, focus on, and mindfully discuss findings, and respectfully listen to and critique alternative perspectives is nonkilling mathematics at its best.

More often than not, millions of learners experience mathematics as something mind numbing and dull, they are stuck in a purgatory of mathematics, which makes little to no sense, and is free of context. To many, it is if that had been learning English and never ever written an essay or a poem. This is exactly how most learners see mathematics. The ethnomodels that Cortes's students and many other researchers are developing are mathematical prose and they are example of nonkilling mathematics.

In this context, we found that the sociocultural perspectives in the ethnomodels contributed to the appreciation and know-how of the farmer seller who performs his own mathematical practices, such as counting, measuring, comparing, classifying, and modelling. These activities performed in the market unveiled an environment full of ideas, notions, procedures, and mathematical practices that are inherent to the marketing process, which became obvious, indeed implicit in this context that are distinct from those practiced in the school environment.

Ethnomodelling has allowed for an insertion and reconceptualization of function concepts in the mathematical curriculum by means of the elaboration of mathematical activities originating in the sociocultural context of the school and its community. This approach made possible a dialogical development between ideas, notions, procedures, and mathematical practices intrinsic to the farmer seller mathematical processes (emic) and the school mathematical contents (etic) with the use of problem situations that emerged from the context of a market free in an encounter of distinct cultures (dialogical).

The sociocultural perspective of ethnomodelling provided an integrative approach to school curriculum, which in addition to considering it as an *etic* approach to mathematical knowledge recognizes what is as necessary as the *emic* features of this knowledge. Through this approach, teachers and students might come to understand a more respectful, connected, real, and holistic (dialogical) way for the cultural information of the members of distinct cultural groups that make up the school community.

Experiencing and learning tools that allow us to move away from emotional solutions that many of us are manipulated with towards looking at the data, helps us become empowered, and to create moments of peace, and move us towards creating something in times of confusion and emotion. Therefore, Cortes and his students offer us excellent examples of how to create a tiny space and steps towards a more peaceful world.

> Whatever merits there might be in this endeavor, may they help alleviate the suffering of all beings; may they help us humans to create a more peaceful world.
>
> Thumpten Jinpa.

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Chapter 12
An Analysis of the Subjection of (Ethno) Mathematical Knowledge in the Study Plan of Brazil's Landless Workers' Movement

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12.1 Introduction

Brazil is a country marked by inequality—whether it is economic, social, educational, income inequality, in the access to land, etc. North and South, downtown and working-class suburbs, coast and inland, city and countryside, white and black people, men and women, many are the possibilities of dichotomizing such inequality.

It is in this context that several social movements have been organized throughout the twentieth century. We highlight here the Landless Workers' Movement (MST), officially founded in 1984.

Over the years, MST has led much of the effort for a more equal distribution of land in the country, with occupations and other forms of action (negotiations, marches, public acts, petitions, and open debates), demanding that the State expropriate and distribute these lands. As stated by Caldart (2012, p. 32), this way of acting "stirs up the very social structure of a country marked by large country estates, closely related to slavery."

This has been a fight for popular land reform, which will substantially alter the distribution of land in the country, questioning the legitimacy of private property in the face of social inequalities. Martins (1999) points out "that the fight for land, from which the fight for land reform derives, is also a fight for inclusion, for active, productive, participatory and creative social integration into society, it is a fight for dignity and respect" (p. 100).

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The fight for land reform is associated with the fight for changes in living conditions in rural areas, and among them is the fight for education. Illiteracy rates, for example, reflect the precarious situation in this respect: in 2010, while in cities the percentage of illiterate people was approximately 8.6%, in rural areas this rate reached 23.7% (Instituto Brasileiro de Geografia e Estatística 2012).

The organization of MST in sectors[1] allows its actions and proposals to be directed at related areas and government bodies, in order to achieve results in public policies or spaces for action—such as encampments and settlements.[2] One of these sectors is Education, which has obtained important results in the last 20 years in the country.

The sectors, in each state of the federation—considering that MST is present in all of them—have a certain autonomy to carry out their propositions, as long as they maintain the coherence with the national collective. In this text, we analyze a document prepared by the Education Sector of the State of Paraná (located in the Southern region of Brazil), called Study Plan (Movimento dos Trabalhadores Rurais Sem Terra 2013).

In general terms (it will be better presented below), it is a curricular proposal for Middle School (children from 11 to 14 years old)[3], organized by complex method, in which the pedagogical work of the semester is detailed. The educational objectives are explained, as well as the group of complexes proposed each semester, and some essential elements are indicated: the portion of reality; the subjects involved; the contents; the justifications; the teaching objectives; the prerequisites; and the expected achievements (Movimento dos Trabalhadores Rurais Sem Terra 2013).

As in other subjects, with regard to Mathematics in particular, the contents there are very close to what is indicated by other curriculum documents, such as the State Curriculum Guidelines for Basic Education in Paraná (Paraná 2008). Our objective is to discuss the subjection of the (ethno)mathematical knowledge present in the Study Plan, resulting from the power relations present in the differentiation between

[1] Some sectors of MST are: mass front or mass mobilization; production; education; political formation or activism; health; gender; communication; and culture.

[2] "The encampment is a place of fight and resistance. It is the materialization of a collective action that publicly proclaims the intention to claim the right to the land for production and housing. The encampment is a permanent manifestation to pressure governments in the realization of Land Reform" (Caldart et al. 2012, p. 23). Settlement is when the consolidation of Land Reform occurs, when the land is conquered by the campers. Thus, an "expressive number of workers who participated in land occupation processes cease to be encamped to become, in a following stage, settlers" (Caldart et al. 2012, p. 23).

[3] Due to the difficulty in the translation and equivalence of school grades, we chose to name the four school grades (sixth, seventh, eighth and ninth grades) as "Middle School", which refer to what in Brazil are the final grades of *Ensino Fundamental*, when students are between 11 and 14 years old. Similarly, we use "Elementary School" for what in Brazil are the initial five grades of *Ensino Fundamental* (first, second, third, fourth and fifth grades), aimed at students from 6 to 10 years old.

scientific mathematics[4] and what is called ethnomathematics. What was left out and what is in the curriculum?

In this chapter, using Foucaultian tactics, inspired by genealogy (Foucault 2015), we do an exercise to raise doubts about the naturalness, coherence, and certainty about what constitutes (and, consequently, what does not) the curriculum, with regard to mathematics, in the Study Plan.

12.2 Study Plan

Since the first years of operation of MST, there has been a concern to ensure access to education for the children and young people participating in the Movement. Although this is a national social movement, with common agendas in the different regions of the country, there are some local and regional specificities. In the state of Paraná, the itinerant schools were created in 2003, inspired by a previous experience in the state of Rio Grande do Sul. These schools are temporary in nature and are built in rural encampments, to guarantee school education in and from the rural areas, even before the land reform is carried out in the area. In this way, the schools migrate together with the encampments and with the students—that is why they are "itinerant".

At first, the development of the pedagogical proposals of these schools was based on Paulo Freire and the generative themes.[5] Until the middle of 2010, according to Sapelli (2013), they were very present in the itinerant schools of Paraná, but one problem pointed out by the author was that they did not ensure that all the contents of the various subjects were addressed.

Between 2010 and 2012, the Education sector of MST in Paraná organized meetings with specialists and educators to evaluate the development of a new pedagogical proposal, with the incorporation of the complex method—based on the experience of the Commune Schools of the former Union of Soviet Socialist

[4] In this chapter, we use the expression "scientific mathematics" to designate the knowledge produced by mathematicians or mathematics professors, present in textbooks or academic and scientific research, differentiating it from what we call ethnomathematics. As will be clear below, we consider scientific mathematics to be qualified knowledge, according to Foucault (1980).

[5] The generative theme is the starting point for the process of creating the discovery. Because they emerge from popular knowledge, the generative themes are extracted from the practical life of the learners, replace the traditional contents and are searched for by 'searching their vocabulary universe'. It is important to emphasize that the political character of the Freirean pedagogy is radically present in the generative themes; that is, generative themes only generate action-reflection-action if they are loaded with social and political contents with concrete meaning in the lives of the students (Tozoni-Reis 2006, p. 103).

Republics (Sapelli 2017). The collective work culminated in the publication of the Study Plan for Middle School (Movimento dos Trabalhadores Rurais Sem Terra 2013). In that sense, the document states:

> It is well known that throughout its development, the school has increasingly strayed its relationship with the lives of students, becoming artificialized and dependent on increasingly limited motivational situations, thus making it difficult for students to perceive the meanings of what they learn. On the other hand, it is known that the experiences aimed at developing a closer relationship between the school and life have trivialized the teaching content and promoted a low mastery of the knowledge necessary for the students' understanding and insertion into the contemporary world. This study plan is an exercise which aims to overcome these limitations (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 9).

More broadly, complex method is a curricular proposal that brings together the dimensions of nature, of society, in connection with work, so that together they deal with the complexity of a part of reality, called a portion of reality (Freitas 2009). Pistrak (1934 as cited in Freitas 2009), one of the Russian idealizers of the proposal, lists some characteristics of the complex method, based on the experience of the Commune Schools.

An important point is that this is a way of overcoming knowledge isolation in the old or traditional school, which is also quite strong in the pedagogical proposal of MST in Paraná, thus justifying the interdisciplinary work. In the words of Pistrak (1934, p. 120–121 as cited in Freitas 2009), "when the goal becomes not studying the subject, but studying the living reality, it is natural that the boundaries between the disciplines become more mobile; that the link between the disciplines becomes stronger," and thus "the complex method requires collective, joint work of all teachers" (p. 45).

In the preparation of the curriculum proposal with the complex method, by MST in Paraná, the importance of building an inventory of the reality is highlighted, which "consists of [an] accurate and detailed ethnographic diagnosis of the reality in which the schools are located and its construction" (Hammel et al. 2015, p. 74). Thus, complex method can be constituted from themes, the portions of reality, which articulate with the local reality. Table 12.1 shows the distribution of the portions of reality presented by the material. Highlighted (bold) are those in which the discipline of Mathematics appears—the focus of this research.

Thus, the idea is that mathematical content—like other areas of knowledge—is connected with portions of reality. Hammel et al. (2015) detail the process of building the material, even clarifying that the discipline of Mathematics was included after the other disciplines, "because at first there were no specialists in this area to establish the connections" (p. 86).

Next, we present the theoretical framework adopted to discuss the subjection of (ethno)mathematical knowledge present in the Study Plan.

Table 12.1 Distribution of portions of reality

Grade	Semester	Portions of reality
Sixth (11-year-old children)	First	• **The fight for land reform** • **Food production** • The forms of collective organization within and outside the school • Peasant culture
	Second	• **The fight for land reform** • **Ecosystem management** • Self-service • The forms of collective organization within and outside the school
Seventh (12-year-old children)	First	• The fight for land reform • **Animal husbandry** • Agribusiness • Organization of the encampment/settlement and at school
	Second	• **The fight for land reform** • **Food production** • **Organization at the encampment and settlement and at school**
Eighth (13-year-old children)	First	• The fight for land reform • **Agroecosystem management** • Forms of organization of the encampment and the school
	Second	• **The fight for land reform** • **Agribusiness** • Forms of organization of the encampment and the school
Ninth (14-year-old children)	First	• The fight for land reform • **Production milling and processing** • Agribusiness (monoculture and cooperative or other enterprises) • Collective organization within and outside the school (encampment or settlement)
	Second	• **The fight for land reform** • Sales/marketing of products • Collective organization within and outside the school (encampment or settlement)

Source: Adapted from Movimento dos Trabalhadores Rurais Sem Terra (2013)

12.3 Subjugated Knowledge

In order to carry out the discussion on subjugated knowledge[6], as we intend in this text, it is necessary to clarify what we understand by these terms and in which theoretical grounds they are established. Michel Foucault develops the tactic of genealogy, from the writings of Friedrich Nietzsche, as:

[6] In French, *savoirs assujettis*, translated as subjugated knowledges (Foucault 1980) or dominated knowledges (in Portuguese, *saberes dominados*) (Foucault 2015).

> What it really does is to entertain the claims to attention of local, discontinuous, disqualified, illegitimate knowledges against the claims of a unitary body of theory which would filter, hierarchise and order them in the name of some true knowledge and some arbitrary idea of what constitutes a true science and its objects (Foucault 1980, p. 83).

As Carvalho (2012) states, it is characteristic of the philosophy of Nietzsche to "dismiss deceptions about the causes and effects around what history has established as the true trajectory of facts, events and perspectives made visible to men (p. 221)." It is in this sense that Foucault (1980) presents the genealogy as a procedure to "bring out the different perspectives from those that have been falsified by the very historical culture, that is, by the means that consolidated the origin (*Ursprung*) of the phenomena taken as true" (Carvalho 2012, p. 222).

For Foucault (2015), therefore, genealogy places on history a "persistent questioning about the obscure memory that accompanies the appearance of things" (Carvalho 2012, p. 228). As stated by him, "what is found in the historical beginning of things is not the inviolable identity of their origin; it is the dissension of other thing. It is a disparity" (Foucault 2015, p. 59).

Subjugated knowledge is, according to Foucault (1980), "a whole set of knowledges that have been disqualified as inadequate to their task or insufficiently elaborated: naïve knowledges, located low down on the hierarchy, beneath the required level of cognition or scientificity" (p. 82). Thus,

> By comparison, then, and in contrast to the various projects that aim to inscribe knowledges in the hierarchical order of power associated with science, a genealogy should be seen as a kind of attempt to emancipate historical knowledges from that subjection, to render them, that is, capable of opposition and of struggle against the coercion of a theoretical, unitary, formal and scientific discourse. It is based on a reactivation of local knowledges—of minor knowledges, as Deleuze might call them—in opposition to the scientific hierarchisation of knowledges and the effects intrinsic to their power; this, then, is the project of these disordered and fragmentary genealogies (Foucault 1980, p. 85).

Regarding mathematical knowledges, Damázio Júnior (2014, emphasis added by the author) reiterates this subjection through what he calls scientific mathematics. For him:

> By considering itself as a form of superior knowledge, scientific mathematical knowledge subjugates other types of knowledge, because this conception based on a supposed *universal* mathematics denies the legitimacy of all other forms of knowledge that are not in accordance with its epistemological principles and its methodological rules (p. 1164).

Thus, genealogy allows, and that is what it proposes, the insurrection of subjugated knowledges. More specifically with regard to mathematics, Damázio Júnior (2014) proposes that ethnomathematics should move closer to genealogy and allow this insurrection of mathematical knowledges, subjugated by the totalizing discourse of scientific mathematics:

> (…) genealogy is not about reflecting on a certain universal truth, but on the true discourses, so that each truth has its history: the truth of madness; the truth of sexuality; the truth of imprisonment; the truth of the education system and the truth of mathematics (Damázio Júnior 2014, p. 1162).

Sachs (2019), for example, presents subjugated knowledges in the curriculum and pedagogical practice in rural schools. The thesis defended by the author is that there is no uniformity of mathematical knowledges, but a multiplicity of them. For this, an analysis is made of a collection of textbooks intended for rural schools and of assignments proposed by teachers in Elementary School (children from 6 to 10 years old) in a rural school, showing that there are totalizing discourses in the curriculum, covering up a large part of the knowledges, treating what is known to be multiple as singular, pretending to encompass what has been left out.

In this text, we raise doubts about the naturalness, coherence, and certainty about what constitutes and does not constitute the mathematics curriculum in the Study Plan. The idea here, as Foucault (2015) states, is to shake up what seems well established: "the search of descent is not the erecting of foundations: on the contrary, it disturbs what was previously considered immobile; it fragments what was thought unified; it shows the heterogeneity of what was imagined in consistent with itself" (p. 63–64).

We assume here a post-structuralist understanding of curriculum, questioning transcendental knowledges and meanings (Silva 2010), which are no longer understood as pre-existing and, even less, as "truths", since the idea of truth is abandoned and replaced by the idea of veridiction (Foucault 2010)—that is, the process for something to be considered true. We assume, therefore, that the knowledges, which are part of the curriculum or not, are culturally produced.

12.4 Analysis

To discuss the subjection of (ethno)mathematical knowledges present in the Study Plan, we have undertaken a movement to highlight the subjugated knowledges. What has been swept under the carpet? What was left aside? On the other hand, what is present, visible to anyone who wants to see it?

There are at least two methodological possibilities for that. One is to search for the conflicts. After all, curriculum is conflict; it is the result of power relations (Silva 2010). What conflicts have been present (historically) so that something is or is not part of it? Another possibility is to question the naturalness, coherence, and certainty of what constitutes and does not constitute a curriculum. There is always justification for a knowledge to be on a curriculum; but is it enough? What does this justification hide? Would there be other knowledges more justifiable or consistent with the justifications presented that are not there? And why are they not there? What in these other knowledges, which Foucault (1980) calls subjugated knowledges, prevents them from being part of the curriculum? Is there room, in a curriculum, for discontinuity, for multiplicity, for difference, for what is uncertain, for what is disqualified?

While the first possibility turns to the process of building the curriculum, the second looks at the outcome of that process. The choice made in this research is for the second possibility, precisely because of the difficulty of accessing information

related to the process, while the outcome is already materialized in the Study Plan (Movimento dos Trabalhadores Rurais Sem Terra 2013).

We then selected some excerpts from the document, with the details of three complexes (portion of the reality, school year, justification, contents, teaching objectives, prerequisites and expected achievements), regarding the subject of Mathematics, for scrutiny and to present other possibilities of approaches. This selection of complexes took place in order to contemplate different portions of reality, without exhausting the analyses to be undertaken.

The contents are delimited within the major themes of the Study Plan disciplines. In the case of Mathematics, the major themes are: Numbers and Algebra; Quantities and Measures; Geometries; and Information Processing. They are the same as those present in the State Curriculum Guidelines for Basic Education in Paraná (Paraná 2008), called structuring contents.

In a comparative analysis between the two documents, we observed that there is a great deal of similarity between the themes addressed and the ways in which they are presented. With a few exceptions[7], the contents are the same, with equal distribution among the grades.

In the Study Plan, the contents are coded with a letter followed by a number, the letter indicates the discipline ("M" for Mathematics) and the number indicates a formative objective (among 28 listed, common to all disciplines and to all grades).

The three complexes chosen for our analysis are:

- Complex 1 of the first semester of the sixth grade of Middle School, whose portion of reality is "Fight for Land Reform"
- Complex 2 of the second semester of the sixth grade Middle School, whose portion of reality is "Ecosystem Management"
- Complex 2 of the second semester of the seventh grade of Middle School, whose portion of reality is "Food Production"

Let us begin with complex 1 for the first semester of the sixth grade of Middle School, present, in part, in Table 12.2.

The *numeral system* content, in the Study Plan, refers to the history of mathematics (in the justification, "analogies between the history of Mathematics itself as a field of dispute and the Fight of the Movement" (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 43)) and ethnomathematics (in the expected achievements, "associating appropriate knowledge to the fight for land reform by working on the history of numbers and different systems, the appropriation by Europeans, the colony process and its supremacy over the other ethnomathematical knowledges

[7] The differences between the documents are some of the contents present in the State Curriculum Guidelines, but not in the Study Plan, namely: in the sixth grade (11-year-old children), volume and time measurements; in the seventh grade (12-year-old children), first-degree equation and inequity, statistical research, arithmetic mean, mode and median and simple interest (this appears in the ninth grade); in the eighth grade (13-year-old children), graph and information, population and sample; and in the ninth grade (14-year-old children), irrational equations, notions of probability and statistics.

Table 12.2 Part of complex 1 of the first semester of the sixth grade of Middle School

Sixth grade of Middle School (Mathematics) Complex 1—first semester Fight for Land Reform				
Discipline justification	Contents	Teaching objectives	Prerequisites	Expected achievements
From the point of view of the subject of Mathematics, by relating specific contents to the fight for land reform, it is possible to make analogies between the history of Mathematics itself as a field of dispute and the Fight of the Movement, as well as to research on the origin of families, numbers of campers/settlers, processing this information, quantifying it and organizing it to critically analyze it.	**Numbers and algebra** **M1—Numeral system** Numerical language. Mayan; Babylonian; Egyptian; Roman; Decimal (Indo-Arabic) numeral system	**M1—**Knowing the different numeral systems and establishing relationships with the decimal system	**M1—**Knowing the history of numbers	**M1—**Associating appropriate knowledge to the fight for land reform by working on the history of numbers and different systems, the appropriation by Europeans, the colony process and its supremacy over the other ethnomathematical knowledges of different peoples.

Source: Adapted from Movimento dos Trabalhadores Rurais Sem Terra (2013)

of different peoples" (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 43)).

The Mayan, Babylonian, Egyptian, Roman, and Indo-Arabic numeral systems are present there—as indicated in the structuring content Numbers and Algebra of the State Curriculum Guidelines (Paraná 2008). Several others are not present and we can exemplify, in reference to the ethnomathematical knowledges referred to by the document, with indigenous counting and numbering methods or those used by cultural groups from different regions of the country.

The research developed by Scandiuzzi (1997) explains the counting processes of indigenous people in the Kuikuro village, in the Xingu National Park, in the state of Mato Grosso, and its numeral systems, and suggests significant changes in the History of Mathematics books, based on the results presented.

Groups of horticulturists, as explained by Bandeira (2016), in turn, use a numeral system called "pair of five," as an auxiliary base for the base ten numeral system. The author states that this could be used to help understanding the decimal system in teaching processes.

In this respect, the Study Plan indicates a "supremacy over the other ethnomathematical knowledges of the different peoples" (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 43). It is not clear whether the supremacy of one

knowledge over another (ethnomathematical) is defended by the document or whether it wishes to indicate a process of colonization that implies the subjugation of one culture by another, resulting in a supposed supremacy. In any case, we may question what actions take place in order for one knowledge to be subjugated to another, so that one overrides and erases the other.

Furthermore, it is important to be clear that, in the proposal, the content relates to the portion of reality, "The Fight for Land Reform". In this case, the justification presented indicates a possibility of using the history of Mathematics to explain the field of dispute in any historical narrative, as in relation to the theme of land reform in the country. If, on the one hand, the analogy can be made, on the other hand, doubts remain as to why the choice of this content, numeral system (and not any other), for such analogy. Table 12.3 shows part of complex 2 for the second semester of sixth grade of Middle School:

The *notions of point, line, plane, ray, and line segment* are indicated as specific Geometry content in this complex. Likewise, they are present in the State Curriculum Guidelines (Paraná 2008). These are concepts of mathematics developed by the

Table 12.3 Part of complex 2 for the second semester of sixth grade of Middle School

Sixth grade of Middle School (Mathematics) Complex 2—second semester Ecosystem management				
Discipline justification	Contents	Teaching objectives	Prerequisites	Expected achievements
From the point of view of mathematics, it is possible to identify the application of decimal numbers, in the use of the spaces of the encampment/settlement, relate them to the income of the land and also showing through the basic elements of geometry, relating to scales, the location of houses, schools, churches, among others and mapping the encampment/settlement. We can also identify the geometric shapes, polygons and angles by observing the environment. We can organize data from the observation of activities carried out in the management of ecosystems.	**Geometry**[a] **M10—Concept of point, line and plane, ray and line segment**	**M10—** Recognizing and representing the three basic elements of geometry (point, line and plane)	**M10**—No prerequisites	**M10—**Showing through the basic elements of geometry and relating to scales, the location of houses, schools, churches, among others and map the encampment/settlement.

Source: Adapted from Movimento dos Trabalhadores Rurais Sem Terra (2013)

[a]The document contains the term "Geometry" (in the singular form) at sometimes and "Geometries" (in the plural form) at others. We preserved the writing the way it is presented in the document

Greeks, present in the classic work "Elements," by Euclid. These knowledges carry with them "a theoretical, unitary, formal and scientific discourse" (Foucault 1980, p. 85), still today. Indeed, as Roque (2012) states, in Ancient times there was not even a unified body of knowledge called "Greek mathematics" (p. 94) as it is called today.

The presence of this content is justified for the location in space ("show through the basic elements of geometry, relating to scales, the location of houses, schools, churches, among others and mapping the encampment/settlement" (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 80–81)). Other knowledges, besides these notions, could also be approached in this sense.

The research by Lima and Monteiro (2009), for example, addresses the construction of maps by community health agents of a rural settlement in Mato Grosso. The researchers requested that these agents represented, through drawings, the spatial organization of their work and the plots they served. It is noted that the complexity of these representations is quite different from what is usually approached and objectified in Mathematics (or Geography) classes, as it considers elements of social life and local nature.

Thus, the construction of maps would consider cultural, religious, and affective elements, the difficulties of access due to distance, climate, physical-geographical accidents, and the transportation used. In this sense, it would not be possible to separate these elements from the activity of mentally tracing or drawing a path taken on a daily basis.

This ethnomathematical knowledge is substantially different from that conveyed in didactic materials in the construction of maps. The concept of scale, which appears in the expected achievements of this complex, no longer makes sense in the traditional way. Each map would bring with it a singularity and a subjectivity, modifying not only what is represented, but how it is done—with other possible scales.

It is important to emphasize that the notions of point, line, plane, ray, and line segment have a strong abstraction characteristic. A cartographic representation—as indicated in the complex or in the example presented here—modifies this characteristic, making these notions concrete. On a map, the point would be the representation of a place, such as a house, a school, or a church in the encampment or settlement (as presented in the justification); however, these elements have a certain dimension and can be represented in this way—differently from what is commonly thought of as a point.

The document does not present how to proceed from practical knowledge to abstract knowledge, as in this case.

Table 12.4 shows part of the complex 2 of the second semester of the seventh grade of Middle School:

Non-Euclidean geometries are addressed in this complex. It is worth mentioning that the State Curriculum Guidelines for Basic Education in Paraná (Paraná 2008) innovated by inserting this theme, which is not present in the current national curriculum documents and in several other states. The research by Caldatto (2011) presents a historical narrative of the process of preparation of this document and the inclusion of non-Euclidean geometries as a content of Middle School and High School (children from 151 to 17 years old) in Paraná. Like the curriculum

Table 12.4 Part of complex 2 of the second semester of the seventh grade of Middle School

Seventh grade of Middle School (Mathematics) Complex 2—second semester Food production				
Discipline justification	Contents	Teaching objectives	Prerequisites	Expected achievements
About food production, mathematics contributes to the measurement of data and calculations to evaluate the planned production using scales, measures of angles and percentages.	**Geometries** **M12—Non-Euclidean geometries** Differentiating from Euclidean geometry Presenting concrete situations of non-Euclidean geometry	**M12—** Understanding the topological notions of non-Euclidean geometry		**M12**— Realizing that not everything is Euclidean or Cartesian. The world is not given as points, lines and planes, but it is three-dimensional, real and not everything may be quantified. Often to achieve the expected results, we have to rely on information that is not always pragmatic, but that involves the perception of material and immaterial reality.

Source: Adapted from Movimento dos Trabalhadores Rurais Sem Terra (2013)

guidelines issued by the state, the Study Plan presents non-Euclidean geometries as a content for Middle School, in several complexes.

The absence of non-Euclidean geometries in several curriculum documents is not due to the subjection of knowledge, as we have argued in relation to some knowledges; it is not because they are considered disqualified knowledges that they are not part of the curricula, but because they are overqualified. As well as a considerable part of the mathematical knowledge developed in more recent times, non-Euclidean geometries, developed since the nineteenth century, present a high level of abstraction and are not usually addressed in Basic Education—and even in higher level courses.

Despite this characteristic of abstraction, present at the origin of non-Euclidean geometries, the Study Plan suggests "presenting concrete situations of

non-Euclidean geometry" (Movimento dos Trabalhadores Rurais Sem Terra 2013, p. 158), without detailing how this can be done.[8] Greenberg (2001), who presents the beginning of the historical and mathematical development of non-Euclidean geometries, especially by researchers Nikolai Ivanovich Lobachevsky, János Bolyai, and Carl Friedrich Gauss, indicates the importance of the advance of symbolic logic to facilitate a knowledge that is so disconnected from reality. The hyperbolic geometry, developed by Lobachevsky, for example, replaced the fifth postulate of Euclid with a postulate that states that, for a point outside a line, infinite parallels go alongside a given line—which questions even the understanding of what a line is.

This does not mean that non-Euclidean geometries have not been developed since then that are close to reality—for example, taxicab geometry or spherical geometry itself—but it is important to observe that they have not emerged as an antidote to the lack of connection of practical and concrete situations from Euclidean geometry, but exactly the opposite.

On the other hand, there is ethnomathematical research with concrete situations of knowledges that could be called geometric knowledges. A classic example of this is the research developed by Knijnik (2006), which discusses quite different ways of using area calculations (land cubage) and volume calculations (wood cubage) in the rural context.

Regarding land cubage, for example, Knijnik (2006) presents two different methods of area calculation (the method suggested by Adão and the method suggested by Jorge), used in real situations. In this research, there was an interesting situation where the teacher asks the students (who are teachers in training) to show the techniques they use in their land plots and spaces where they live but using simple data to facilitate the calculations. One of them replied: "We need to think of an actual piece of land, which is not all plain" (p. 68). The researcher comments on this:

> At the time I accepted the arguments, even though I felt that the work could be jeopardized. Later, I interpreted them from a different perspective: the "simplification" suggested by me made the problem situation artificial, less "concrete"; therefore, the possible benefit it would provide—in this case, the simplification of the calculations—would be hindered by the possible introduction of greater pedagogical obstacles (Knijnik 2006, p. 68–69).

[8] In this specific case, there seems to be a misalignment between the items in the complex (justification, contents, objectives and expected achievements). The justification, which refers to various contents within the complex, is not related to the theme; in the contents, the focus is on the differentiation between Euclidean and non-Euclidean geometry and the presentation of concrete situations; the objective deals with topological notions (although it does not make much sense to say "topological notions of non-Euclidean geometry", since topology itself can be understood as non-Euclidean geometry); and, finally, the expected achievements address other items ("not everything may be quantified", "to achieve the expected results, we have to rely on information that is not always pragmatic, but that involves the perception of material and immaterial reality"). It should also be noted that the column for prerequisites is blank, as in other complexes not presented here. It is not clear whether there are no prerequisites or whether they have not been determined for some reason.

Perhaps the "Food Production" portion of reality was closer to ethnomathematical knowledges such as the one mentioned, than to non-Euclidean geometries themselves. However, knowledges originally proposed by the Landless Workers, such as those pertaining to land cubage, are not present in the curriculum, even though they are known by several encamped and settler teachers, who composed the team that prepared the Study Plan. They are illegitimate knowledges, they do not have a unitary theoretical instance, such as that of non-Euclidean geometries, that qualifies them; therefore, they are left out.

Power relations, which differentiate between what is scientific knowledge and what is not, subjugate illegitimate knowledges in the curricula. With the Study Plan, this also happens, as we have seen. The ways of counting and numbering of certain cultural groups, the construction of maps that take into account cultural, religious, and affective aspects, and methods for calculating area, such as land cubage, are examples of subjugated knowledge, absent from the curriculum, giving space to others that are considered legitimate, scientific, exact. Thus, a curriculum proposal that aims to get closer to reality ends up distancing itself from it, in some moments when this could be different.

12.5 Conclusions

We highlight here, despite any criticism that may be made, the importance of building a curriculum document, such as the Study Plan, which innovates by using portions of reality as articulators between the reality lived by those in the encampments and settlements and the various areas of knowledge. In many aspects, the proposal advances in relation to other curriculum documents, elaborated by governmental institutions—with all the incentive that may result from this. Based on the experiences of MST in Paraná in rural schools, the Study Plan was proposed, with a view to overcoming problems and achieving well-established educational objectives.

It should also be noted that this curriculum proposal, adopted by rural state schools (whether itinerant or not), cannot be separated from what the official curriculum documents of the Education Departments propose—or impose, depending on the case. Although they are considered "educational experiences," itinerant schools respond to state requirements and are evaluated by the state—the same goes for rural settlement schools, linked to the state or municipalities. Thus, the similarity observed in the Study Plan with the State Curriculum Guidelines for Basic Education in Paraná is justified.

After the analysis undertaken, we indicate here some situations in which there is a subjection of (ethno)mathematical knowledges in the Study Plan—specifically, in three complexes, with counting and numbering methods of certain cultural groups, with the construction of maps and with methods for area calculation. We add to this the possibility of inserting these knowledges into the curriculum, without this leading to a denial of access to the foundations of science, as defended by the creators of the complex method (Pistrak 2009) and MST.

Why not have a dialogue between different knowledges, breaking the hierarchy between knowledges and explicitly showing the power relations present in the constitution of curricula, both by selecting what should and should not be present, and in the reasoning that is or is not allowed by them? In this text, we present some possibilities for this, showing ethnomathematical knowledges that could be present in the curriculum proposal of the Study Plan.

However, we can debate the extent to which Mathematics, placed as universal and unquestionable, allows this dialogue. Perhaps it is no wonder that dialogue does not take place in class; it may undermine mathematics. And then we enter into a political dispute, a dispute about territory, power, and access—and there is no doubt that this single undisputed mathematics has won all the disputes imposed to it. To realize so, one must only look at the school curriculum, the space and time dedicated to it, the external evaluations, and the teacher training courses.

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Part VI
Conclusion

The main objective of this conclusion is to revise the 11 contributions in this book that were organized into four categories of: *Africanities*, *Indigenous Diversities*, *Urban Diversities*, and *Diverse Cultural Practices*. This volume provides examples of new trajectories for investigations related to the Brazilian cultural context and offers a glimpse of new directions of ethnomathematics as a program.

Chapter 13
Concluding Remarks About Diverse Mathematical Practices in the Brazilian Context in an Ethnomathematical Perspective: Past, Present, and Future

Cristiane Coppe de Oliveira

13.1 Introduction

Over the past four decades, the amount of research, investigations, thesis, and dissertations that dealt with both the theoretical and practical aspects of ethnomathematics has expanded exponentially. At the same time, numerous articles, book chapters, and books have been written in many countries about the relation between culture, mathematics, and mathematics education. Over the same time, the early founders of ethnomathematics have developed countless new voices that have gone on to make even more new discoveries, and offered new insights.

As well as in study groups around the world, numerous studies involving ethnomathematics have been discussed and debated during a succession of local, regional, national, and international meetings, seminars, conferences, and congresses. Additionally, some journals in Brazil have provided their readership with dossiers and special editions on ethnomathematics; these have been created in both English and Portuguese languages.

From the emergence of Ethnomathematics in the early 1980s to the expansion of its perspectives, this program proposed by professor, educator, and philosopher Ubiratan D'Ambrosio, currently lists political, educational, and everyday dimensions. This research field also has constituted an interface between other knowledge areas, such as philosophy, anthropology, sociology, cognition sciences, and history. According to Knijinik et al. (2012):

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> Almost four decades after D'Ambrosio first dared to present his ideas to the international community at the 6th International Congress on Mathematics Education (ICME-6) in Adelaide, [Australia], Ethnomathematics is now recognized as a field of research, developed in major research centers and universities around the world (p. 27).

In this regard, the movements and moments of Ethnomathematics in terms of academic and scientific production find paths that distance themselves from the first studies in this research area that prioritized ethnography as the main methodological approach that described culturally distinct communities, especially in the various Brazilian regions in which indigenous people population predominate. In this context, Knijinik et al. (2012) consider that:

> (…) if D'Ambrosio is positioned as the one who instituted Ethnomathematics as a Mathematics Education perspective, Eduardo Sebastiani Ferreira (1991; 1993; 1994) was the pioneer in Brazil, in the fieldwork in this area, when he conducted and oriented investigations whose empirical research were developed in the urban periphery of Campinas and in indigenous communities of Xingu and Amazon regions (p. 19–20).

Currently, most of the scientific production in Ethnomathematics is concentrated in a theoretical-conceptual universe, favoring the interfaces between other knowledge areas. In any case, the first investigations with culturally distinct Brazilian communities showed relevant elements for the continuity of research in accordance to the dimensions of Ethnomathematics Program. As Marchon (2016) points out, Ethnomathematics has been considered (or identified) since the beginning of the 1980s by some researchers as a *trend*, *sub-area,* or *aspect* of Mathematics Education.

This book seeks to bring contributions of research conducted in culturally distinct communities by favoring traditions, educational experiences, and the *knowing* and *doing* developed by the members of diverse cultural groups. This context brings to our minds the theme most present in the first academic and scientific productions in Ethnomathematics, which highlights the values of current research with traditional communities in Brazil.

It is important to emphasize that these contributions are organized into four ethnomathematical research areas in the Brazil context.

13.2 Contextualizing Four Ethnomathematical Research Areas in Brazil

This book is authored by 21 authors from 11 states in Brazil. These authors were invited to discuss their ideas, perspectives, and share their research on ethnomathematics. The 11 chapters presented here represent the Southeast, North, Center-West, South, and Northeast regions of Brazil.

The content of this book is composed of a preface, an introduction section, four sections entitled: *Africanities*, *Indigenous Diversities*, *Urban Diversities*, and *Diverse Cultural Practices*, and a concluding section. Therefore, the four sections of this book deal with diverse ethnomathematical research areas currently developed in Brazil.

In this context, Ubiratan D'Ambrosio was invited to open this book by writing its preface in which he argues that it is important to develop research that deal with the

mathematical knowledge and beliefs of Brazilian communities, with special attention to Africa, Afro-Brazilians, and Indigenous Peoples, yet by considering Brazilians of European and Asiatic ancestry and mixed populations. In his point of view, current Brazilian population is a dynamic encounter of several ancestries.

The introduction of this book, which is its first chapter entitled: *An Overview of Diverse Mathematical Practices in Brazil: AnEthnomathematical Perspectivein Action,* written by Milton Rosa, discusses ethnomathematics as a research program that has an agenda that offers a broader view of mathematics, which embraces the ideas, processes, methods, and practices that are related to the diverse mathematical practices developed in Brazil. In his opinion, this aspect leads to an increased evidence of cognitive processes, learning capabilities, and attitudes that may direct a learning process occurring in investigations and in the classrooms in the Brazilian context.

13.2.1 Africanities

The three chapters of the first section of this book deal with discussions regarding the epistemology and features of Africanities and Afro-Brazilian population.

The second chapter entitled: *African-Based Cultural Practices and Mathematics Practices: Decolonizing Knowledge*, written by Evanilson Tavares França and Jackeline Rodrigues Mendes, proposed a decolonized look at the curriculum by considering African ancestral memories and presenting interfaces between knowledge and traditional knowledge, which circulates and articulates in a quilombola community in the state of Sergipe in Brazil. Cultural practices originating in Africa pulse in the context of the quilombola community Mussuca, which provides an environment to reflect on how popular knowledge can become an element that belongs to the curriculum.

The third chapter entitled: *The Mandira Quilombola Community and the Production of an Anthropophagic Identity*, written by Diego de Matos Gondim and Roger Miarka, present findings from the Quilombola Mandira community in the Ribeira Valley region. The concept of anthropological identity was discussed in the text as the participants of this community interact with/in their own dynamic investigative process, revealing Afro-Brazilian identities. The authors created a concept of *Anthropophagic Identity* in which the identity of a community is never static, but always dynamic, having to digest different elements and circumstances that it goes through, what includes the presence of the researcher in the group.

The fourth chapter entitled: *African and Afro-Brazilian Roots for Mathematics Teaching: Decolonize the Curriculum*, written by Cristiane Coppe de Oliveira and Ana Paula dos Santos, took into account the fact that there is still a Eurocentered hegemony in educational relations and curricula presented to students by invisibilizing *African* and *Afro-Brazilian*history and culture in the school context. Their proposal is to collaboratively think about a decolonized curriculum for a public school in the state of São Paulo, in Brazil, in which the majority of its students are Afro-Brazilian. In addition to fostering debate among teachers in an institutionalized

training space in the school context, their proposal seeks to implement the Federal Law 10639/03 that makes African and Afro-Brazilian history and culture mandatory in the school curriculum in the Brazilian context.

The first section of this book emphasizes the recognition of diversity in three cultural contexts marked by African (considering the diversity of the African continent in its socio-political divisions) and Afro-Brazilian roots. According to Conrado (2019), the expression diversity has been thought of as a broad, deep, multiple, and complex set of meanings. In this sense, the perspective of Education for Ethnic-Racial Relations in Brazil is completed in this movement for diversity, thinking about the eradication of racism, which is an element that is constituted in school contexts and in Brazilian society.

In this context, Coppe (2012) comments that it is necessary to think about pedagogical aspects that open up possibilities of breaking up racism by seeking the decolonization of the curriculum, which was highlighted in the first and third chapters of this section. This author also points out that:

> Particularly, in the teaching of Mathematics, there are recurrent discourses that the teaching of Mathematics must be directed to the understanding of reality, of social phenomena, of the development of citizenship, contributing to socio-historical transformations. However, many mathematics teachers consider that, in the teaching of the subject, it is not their job to explore issues of fundamental importance, such as racial and/or cultural prejudices. Others claim that their (traditional) education has not contributed to them by making the necessary associations between mathematical content and such problems. In fact, it is not rare for those who express a desire, but also the difficulties of resizing their actions in order to harbor reflections regarding cultural and racial diversity (p. 11).

A look at the Brazilian quilombola communities, their treasures and cultural values presented in this section, can favor the approximation, association, and articulation of traditional knowledge and school knowledge. Such proposal may highlight invisible identities in society and, especially, in the school environment.

13.2.2 Indigenous Diversities

The three chapters of this second section deal with the discussion regarding the education practices in relation to Indigenous peoples, which shows that the results of these investigations come from the voice of indigenous peoples in Brazil.

The fifth chapter entitled: *Ethnomathematics in the Brazilian Indigenous Contexts*, written by José Roberto Linhares de Mattos and Sandra Maria Nascimento de Mattos, points out under the perspective of the Ethnomathematics Program, important elements for a decolonial posture by considering pedagogical practices in indigenous school contexts, in the North and Center-West regions of Brazil, which respect the identities and cultural values of the people living in these regions. According to these authors, Brazilian research that contemplate indigenous diversity should value the traditional knowledge of each Brazilian ethnicity in a way that favors the circulation of their knowledge in the school context.

The sixth chapter entitled: *Knowledge Networks in the Training of Indigenous Mathematics Teacher*, written by Maria Aparecida Mendes de Oliveira, considers interculturality to be a common thread in the Indigenous teacher education program in the context of the state of Mato Grosso do Sul, in the Midwest region of Brazil. The experiences with indigenous teachers highlighted the values of Ethnomathematics as a pedagogical tool that considers different forms of knowledge/doing/thinking in Mathematics. This study analyzes the process of indigenous teacher program in the course of specific habilitation in mathematics.

The seventh chapter entitled: *Ethnomathematics Research on Indigenous Peoples' Knowledge and Practices*, written by Maria Cecília Fantinato and Kécio Gonçalves Leite, emphasizes the field of ethnomathematics research, which presents a mapping of Brazilian research on Ethnomathematics developed along with the knowledge, practices, and education of indigenous peoples. Indigenous peoples in Brazil have always been in the spotlight of political interests and the domination by wealthy agribusiness entrepreneurs in demarcated lands. The investigation presented the diversity of Brazilian indigenous peoples with different ethnic groups and different languages.

The second section of this book deals with Ethnomathematics in the Brazilian indigenous context. Brazil has a diversity of indigenous peoples and their practices, mathematical procedures, and traditional knowledge that are articulated in these communities. In this regard, the vast majority of ethnic groups in Brazil organizes and delimits their educational practices with regard to indigenous school education and indigenous education. According to Barbosa and Magina (2014), the state of Rio de Janeiro has assured that the members of indigenous communities, in regular elementary school, need to use their mother tongues and their own learning processes. It also guaranteed bilingual education (mother tongue and Portuguese language) and the state's protection of indigenous cultural manifestations.

The identity of indigenous peoples, its (ethno) knowledge, and its mathematical practices were put into focus in this section. It is not a question of explaining what may be evident as a certain *indigenous* or *non-indigenous* cultural distance, formal or informal, and tradition or modernity. It is about recognizing and highlighting cultural values and different ways of thinking mathematically and other knowledge in indigenous contexts. They are other logics, other senses, and other worldviews from the cultural traditions of each community or group.

13.2.3 Urban Diversities

The two chapters of this third section discuss issues related to urban diversities in different contexts in Brazil, specifically that which is related to Deaf Culture and the relations between public schools and universities.

The eighth chapter entitled: *Intertwining the Ethnomathematics and the Deaf Culture to Promote Financial Education for Deaf Students,* written by Rodrigo Carlos Pinheiro and Milton Rosa, presents deafness as a specific cultural context for

inclusive mathematics education. The authors state that an ethnomathematical perspective can contribute to the teaching and learning process of mathematics by considering the needs and specificities of Deaf students. The cultural group of Deaf students should be favored with a bilingual education with moments in which the integration of the Brazilian sign language (LIBRAS) and mathematical language can contribute to the inclusion of this student population in relation to decision-making process in society based on an ethnomathematical perspective.

The ninth chapter entitled: *Practices in Ethnomathematics, University, and Basic Education Schools: Possible Interconnections*, written by Ieda Maria Giongo, Márcia Jussara Hepp Rehfeldt, and Marli Teresinha Quartieri, presents basic education and its relations that can be established between public schools and universities. The authors discuss limits and possibilities of the development of work between school and non-school knowledge in educational practices, with an investigative gaze that is supported by the perspective of the Ethnomathematics Program. The authors also show the emergence of teaching practices using school and non-school knowledge.

It is known that urban diversity is also a concern of ethnomathematics researchers. Such diversity occurs in the context of the schools and in cultural specificities that bring out knowledge developed in everyday life. Thus, the third section of this book presents urban diversity in the inclusive dimension of education, both when thinking about deaf culture and when highlighting non-school knowledge in educational practice. In speaking of diversity and difference we agree with Conrado (2019) who states that:

> Diversity as difference can be interpreted from a cultural matrix that dialogues with what social movements, of an identity nature, articulate in terms of what is affirmed as the "right to be different", which requires the recognition, in the public and private spheres, of groups defined as "minority" or "subaltern" (p. 72).

This dimension brings out the posture of the teachers who must be sensitive to these diversities and differences by seeking formation and awareness. The chapters of this section have alerted researchers and educators to know and recognize differences between students in order to seek ways for educational practices that articulates specific/local knowledge. About teacher's attitude towards differences we agree with Gerdes (2010) who argues that teachers should be aware of the students' cultural reality by preparing pedagogical proposals centered on Ethnomathematics.

In this context, Rosa and Orey (2007) affirm that mathematical knowledge is perceived as a version of ethnomathematics because *ethno* is defined as culturally identifiable groups with their own jargons, codes, symbols, myths, and specific ways of reasoning and inferring; *mathema* is defined as categories of analysis; and *tics* is defined as methods or techniques for solving problems faced daily. In this context, in mathematics classrooms, teachers build from students' tacit knowledge (*ethno*) and direct lessons toward their culture and experiences (*mathema*) while developing their critical thinking strategies and techniques (*tics*).

This environment enabled Gerdes (2010) to state that each people, every culture and every subculture, including each social group for example, artisans in basketry

and pottery in the examples given – and each individual, builds and develops its mathematics, in a certain way, particular. When a teacher is not aware of how cultural differences can generate different developments in mathematics this could lead to significant problems for mathematics learners.

13.2.4 Diverse Cultural Practices

The three chapters in the fourth section deals with diverse cultural practices developed in the distinct Brazilian contexts.

The tenth chapter entitled: *Research in Ethnomathematics in the Context of the Professional Master Degree: An Analysis of Five Theses Developed at UFRN*, written by Paulo Gonçalo Farias Gonçalves and Francisco de Assis Bandeira, presents cultural practices and discuss research investigations developed in the Professional Master's Degree Program in Science and Mathematics at the Universidade Federal do Rio Grande do Norte. The authors consider that research of ethnomathematical knowledge of the members of professional groups (cultural practices) when working in classrooms with students allows for the possibility of dialogue between the official national curriculum (currently the *Common National Curriculum Base—BNCC*) and various teaching methodologies in mathematics, such as problem solving.

The eleventh chapter entitled: *Unity in Differences: Reflections on the Doing of Ethnomodelling and Dialogue through Connecting Ethnomathematics and Modelling*, written by Diego Pereira de Oliveira Cortes and Daniel Clark Orey, addresses labor practices of farmer vendors in the state of Minas Gerais, in the Southeast region of Brazil. In addition to presenting the importance of dialogue in relation to the distinct labor practices used to understand cultural conceptions in the elaboration of ethnomodels, the authors presented the possibility of this dialogue reaching the classroom context as a pedagogical action from the perspective of ethnomathematics, modelling, and ethnomodelling. This proposal was based on the formal school mathematics content of functions in the mathematics curriculum. As well, its main point was to understand the importance of cultural conceptions and the elaboration of ethnomodels extracted from the mathematical practices developed by this farmer vendor for use as a pedagogical action in a high school mathematics classroom. According to the authors, ethnomodelling can be considered a tool that can bring school knowledge and work practices closer together by valuing local mathematical knowledge and context.

The twelfth chapter entitled: *An Analysis of the Subjection of (Ethno)mathematical Knowledge in the Study Plan of Brazil's Landless Workers' Movement,* written by Línlya Sachs and Amanda Araújo Coelho Nogueira, is related to an area considered to be in agrarian reform in the state of Paraná, in the southern region of Brazil. The (politically constituted) environment comprises the actions of the *Landless Workers' Movement* in Brazil (MST) related to a curriculum proposal for secondary education students from that locality. A look at the Foucaultian perspective helps us

to understand the (ethno) knowledge present in such a document by revealing possible cultural practices of the rural communities in southern Brazil.

The fourth section of this book deals with diverse cultural practices related to the presentation of some elements concerned to research that stand out in this field in Brazil. In this regard, Tamayo et al. (2018) state that:

> When studying cultural practices, research does not compare them with a supposed universal mathematics, in order to identify how this mathematics happens in these practices. It is evident that the purpose is to think about the mobilization of knowledge, not disciplinarily, in cultural practices through the centrality of language, daring to explore its uses (p. 601).

Many dimensions of the Ethnomathematics Program can be revealed in the category of research that prioritizes cultural practices, such as its political and educational dimensions, and the challenges of daily life (D'Ambrosio 2019). The pulsions of survival and transcendence permeate realities and contexts presented in the three chapters of this section that deal with: (a) labor practices of professionals from the Northeastern region of Brazil, (b) farmer vendors and their practices to determine the price of products, in the state of Minas Gerais, in the Brazilian Southeast region, and (c) MST militants and their problemizing practices in a rural school context in the Southern region of Brazil.

Chapter in this section can be thought of when considering Lourenço's (2018) idea of naming ethnomathematics as a technography of diverse cultural practices that are mathematically interpretable in the studied societies. Symbols, languages, and cultures circulate in the continuous movement of diversity of thought and reasoning that permeate cultural practices, evidencing reflections on the *others* and/or on ourselves, as well in the ways we organize the world.

Finally, the thirteenth chapter entitled: *Concluding Remarks about Diverse Mathematical Practices in the Brazilian Context in anEthnomathematical Perspective: Past, Present, and Future*, written by Cristiane Coppe de Oliveira, provides examples of new trajectories for research regarding ethnomathematics and offers a glimpse of new directions of this program in the Brazilian context.

13.3 Final Considerations

In order to elucidate, clarify, and perhaps to facilitate new discussions about ethnomathematics, we truly hope that readers will be able to capture the authors' thoughts and concepts regarding this program. From the authors own particular vantage points they each have done a great deal to add to the growing body of scientific discourse of ethnomathematics.

As well, we would like to state at the outset here that there are still other perspectives and innovative views on ethnomathematics emerging from other researchers and in other knowledge fields. We hope that future special editions, dossiers, and books will consider them as well, and that particular collection provokes new ideas, perspectives, and research and give voice to those who currently do not have one. In

this context, we would also like to emphasize that the questions, answers, comments, conceptions, and discussions made in the chapters in this book are the authors' personal views on ethnomathematics.

We are certain that not all educators, mathematicians, and philosophers will agree upon these views and conceptions on ethnomathematics, yet we are also confident that, in some cases, the approaches and perspectives presented here may be in discordance with views of other ethnomathematicians. Yet, we are pleased that this book illustrates ethnomathematics new voices and investigations that continues to evolve and has spread itself worldwide, and that includes a diversity of schools, colleges, universities, local communities, and informal environments in which education takes place, in a relatively short period of time.

We have no doubt that ethnomathematics is alive, it is evolving, and as more and more research is uncovered worldwide, we also understand that it will continue this growth process. It is a research field that has not yet crystallized, and that to us this is very, very exciting! As it stands currently, ethnomathematics seeks to document and understand widely diverse mathematical ideas, procedures, and practices developed by the members of distinct cultural groups in order to empower them and value their tacit knowledge. As this diversity of voices begins to speak, they have remarkably similar, yet different points of view and cosmologies.

From the discussions provided in the chapters presented in this book, we conclude that mathematical knowledge as we currently experience it is constructed by developing different processes, common to the members of all sociocultural groups that enable them to elaborate and use abilities, and which include universal processes of *counting, locating, measuring, drawing, representing, playing, understanding, comprehending, explaining*, and *modelling.* Currently, ethnomathematics developed in the Brazilian context investigates the roots of mathematical ideas and practices, starting from the way members of distinct cultural groups develop, use, and apply their mathematical knowledge in accordance to their own environments and surroundings.

It is important to state that in the diverse ethnomathematical studies, as presented in this book, many identify mathematical ideas, procedures, and practices that begin with the knowledge of the *others* in their own terms and rationality. In order to know and understand the value of the plurality of the nature of the Brazilian diverse social, cultural, economic, and political realities there is a deep necessity to take a firm stand against prejudices based on cultural differences, social classes, beliefs, gender, sexual orientation, ethnics, or other social, cultural, political, and individual characteristics. We must encourage these voices to produce and present their research in future publications.

The authors in this collection have shared the necessity for issues regarding mathematics education, classroom practices, and the knowledge of the members of distinct cultural groups. Ethnomathematics clearly has a role in helping us to clarify the nature of mathematical knowledge and of knowledge in general.

With the exchange of knowledge discussed in the chapters in the categories: *Africanities*, *Indigenous Diversities, Urban Diversities*, and *Diverse Cultural Practices*, presented in this book, it is hoped to propitiate the constitution and

diffusion of new knowledge about research in the field of ethnomathematics by searching, like birds, for "new *out of the cage* perspectives" (D'Ambrosio 2016, p. 224). Therefore, it is important to acknowledge that:

> The birds only see and feel what the cage allows, they only feed on what they find in their confinement, they only fly inside of their limits, they only communicate in a language known to them, and they breed and reproduce in the cage. But they do not know what color the cage is painted on the outside (p. 224).

According to this assertion, D'Ambrosio (2015) uses the concept of *epistemological cages*as a metaphor to describe knowledge systems. In his point of view, "traditional knowledge is like a birdcage (p. 23) and a:

> (…) similar situation may happen with specialized scholars. The scholars in the cage develop their own jargon and adhere to rigorous methodological and ontological standards. To overcome academic sameness is a big challenge. It is common to see researchers subordinating their students to themes proposed by the advisors, restricting their space for creativity (p. 23).

Accordingly, D'Ambrosio (2015) points out that, as researchers, it is our mission to encourage academic discussions and debates circulating about Ethnomathematics Program so that "Mathematics Educators must be aware of the dramatic prospect facing humankind and should be careful when their students learn well how to play mathematical games implicit in curricula, but do not reveal any ethical thinking" (p. 23).

Finally, this book discussed the contributions of investigations conducted in the perspective of ethnomathematics by researching diverse mathematical ideas, procedures, and practices in the Brazilian context, which demonstrates the vibrance of innovative and creative development of this program.

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Milton Rosa and Cristiane Coppe de Oliveira

Correction to: Milton Rosa, *Ethnomathematics in Action*, https://doi.org/10.1007/978-3-030-49172-7

This book was inadvertently published with incorrect affiliation of Dr. Cristiane Coppe de Oliveira in the front matter (page iv and xiii), chapter 4 (page 57) and chapter 13 (page 229).

The affiliation has now been updated as follows:

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