Chapter 1 The Measurement of Time



I. As long as we don't leave the domain of consciousness, the concept of time is relatively clear. Not only do we easily distinguish the present sensation from the memory of past sensations or the anticipation of future sensations, but we know perfectly well what we mean to say when we affirm that, of two conscious phenomena of which we have kept a memory, one came before the other or else that, of two expected conscious phenomena, one will come before the other.

When we state the two conscious facts are simultaneous, we mean to say that they enter so deeply into each other that analysis cannot separate them without damaging them.

The order in which we line up conscious phenomena has no arbitrariness. It is imposed on us and there is nothing about it that we can change.

I have only one observation to add. For a set of sensations to become a memory which could be ranked in time, it must have stopped being actual, that we might have lost the sense of its infinite complexity, without which it would remain real. It must have, to say it that way, crystallized around a center of associations of ideas which will be like a kind of label. It is only when they will have thus lost all life that we will be able to order our memories in time, like a botanist ordering dried flowers in their herbarium.

But, there can only be a finite number of these labels. On account of this, psychological time would be discontinuous. What is the origin of this idea that between two arbitrary moments there are other moments? How would we know that there were empty slots, if these slots were only revealed to us by their content?

II. But that isn't all; in this form we want not only to bring back the phenomena from our consciousness, but those for which other consciousnesses are the theater. Even more, we want to bring back to it physical facts, these little things with which we populate space and which no consciousness sees directly. It has to be done, because without that science could not exist. In a word, psychological time is given

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to us and we want to create scientific and physical time. That is where the difficulty starts, or rather difficulties, because there are two of them.

Here are two consciousnesses which are like two mutually impenetrable worlds. By what right do we want to bring them into a single mold, to measure them with the same gauge? Isn't it as if one wanted to measure length with a gram or to weigh with a meter?

And furthermore, why do we speak of measurement? We may know that one fact is before another, but not *how much* earlier it is.

Therefore the two difficulties are:

- 1) Can we transform psychological time, which is qualitative into a quantitative time?
- 2) Can we reduce facts which happen in different worlds to a single measurement?

III. The first difficulty was commented on long ago; it was the subject of long discussions and we may say that the question is resolved.

We do not have the direct intuition of the equality of two time intervals. People who believe they have this intuition are tricked by an illusion.

When I say that from 12:00 to 1:00 the same time has passed as from 2:00 to 3:00, what does that statement mean?

The least thought shows that on its own it has none at all. It will only have what I want to give it via a definition which will have to comprise some degree of arbitrariness.

Psychologists could have gotten by with this definition; physicists and astronomers cannot; let's see how they got out of it.

To measure time, they make use of a pendulum and accept by definition that all beats of this pendulum are of equal length. But, this is only a first approximation; the running of the pendulum is changed by temperature, air resistance and barometric pressure. If we escape from these sources of error, a much better approximation will result. New sources, neglected until now, whether electric, magnetic or other, could manage to contribute small disturbances.

In fact, the best clocks must be corrected from time to time, and the corrections are done using astronomical observations; it is set up so that the sidereal clock marks the same time when the same star passes the meridian. In other words, it is the sidereal day; meaning the time for rotation of the Earth which is the constant unit of time. We accept, via a new definition substituted for the one which was drawn from the swinging of the pendulum, that two complete rotations of the Earth around its axis have the same length.

However, astronomers are still not content with this definition. Many think the tides act as a brake on our globe and that the rotation of the Earth becomes slower and slower. In this way the apparent acceleration of motion of the moon would be explained (it would seem to go faster than theory allows it to) because our clock, which is the Earth, would be slowing down.

IV. One could say that none of that matters much; most likely, our measuring instruments are imperfect, but it is sufficient that we could conceive of a perfect instrument. This ideal could never be reached, but it would be enough to have conceived it and have in that way given rigor to the definition of the unit of time.

Unfortunately that rigor has not yet been found. When we use a pendulum for measuring time, what is the postulate that we implicitly accept?

It is that *the length of two identical phenomena is the same*; or, if one prefers, that the same causes take the same time to produce the same effects.

And it is, at first glance, a good definition of the equality of two intervals.

However, let's be careful about it. Is it possible that one day experience will falsify our postulate?

Let me explain; I assume that at some point in the world the phenomenon α happens leading as a consequence at the end of some time to the effect α' . At another point in the world very far away from the first, the phenomenon β happens, which leads as a consequence to the effect β' . The phenomenon α and β are simultaneous as are the effects α' and β' .

At some later time, the phenomenon α occurs again under almost identical circumstances and simultaneously the phenomenon β occurs again also at a very far away point of the world and under nearly the same circumstances.

The effects α' and β' are also going to occur again. I assume that the effect α' takes place substantially before the effect β' .

If the experiment made us witnesses of such an event, our postulate would be falsified.

The experiment taught us that the first length $\alpha \alpha'$ is equal to the first length $\beta \beta'$ and that the second length $\alpha \alpha'$ is smaller than the second length $\beta \beta'$. In contrast, our postulate would require that the two lengths $\alpha \alpha'$ be equal to each other and also the two lengths $\beta \beta'$. The equality and inequality deduced from the experiment would be incompatible with the two equalities drawn from the postulate.

Now, can we affirm that the hypotheses that I just made are absurd? They hold nothing contrary to the principle of contradiction. Most likely they would not be able to occur without the principle of sufficient reason seeming to be violated. But, to justify such a fundamental definition, I would prefer another guarantee.

V. But that isn't all.

In physical reality, one cause does not produce one effect, but a multitude of distinct causes contribute to producing it, without us being able to discern the contribution of each of them.

Physicists seek to make this distinction; but they only do it approximately and whatever progress they may make, they will only ever do it approximately. It is approximately true that the motion of the pendulum is due solely to the attraction of the Earth; but in all rigor, even the attraction of Sirius could act on the pendulum.

Under these conditions, it is clear that the causes which one day produced a certain effect will only ever be approximately reproduced.

And then we will have to change our postulate and our definition. Instead of saying:

"The same causes take the same time to produce the same effects."We have to say:

"Nearly identical causes take about the same time to produce about the same effects."But our definition is therefore no more than close.

Further, as Calinon very rightly remarked in his recent monograph (*Études sur les diverses grandeurs*; Paris, Gauthier-Villars, 1897), "One of the circumstances of an arbitrary phenomenon is the rotational speed of the Earth; if this rotational speed varies, it constitutes, in the reproduction of these phenomena, a circumstance which no longer remains identical to itself. But assuming this rotational speed is constant is assuming that one knows how to measure time."

Our definition is therefore still not satisfactory; it is certainly not the definition that the astronomers, whom I talked about above, implicitly adopt when they state that the Earth's rotation is slowing down.

Coming from their mouth, what is the meaning of this statement? We can only understand it by analyzing the evidence that they give for their proposition.

They first sate that friction from the tides produces heat which reduces the energy. They therefore invoke the principle of conservation of energy.

They next state that the secular acceleration of the moon, calculated from Newton's law, would be smaller than the acceleration which is deduced from observations if the correction relating to the slowing of the Earth's rotation is not made.

They therefore invoke Newton's law.

In other words, they define duration in the following way: time must be defined such that Newton's law and the equations of motion are respected.

Newton's law is an experimental truth; as such, it is only approximate, which shows that we still only have a definition by approximation.

If we now suppose that another way to measure time is adopted, the experiments on which Newton's law is based would still retain their same meaning. Just the statement of the law would be different, because it would be translated into another language; it would obviously be much less simple.

In that way, the definition implicitly adopted by astronomers could be summarized as:

Time must be defined such that the equations of mechanics are as simple as possible.

In other words, there is no way to measure time which is truer than another; the one which is generally used is only more *convenient*.

We are not allowed to say about two clocks that one works well and the other does not; we can only say that it is better to refer to the indications of the first clock.

The difficulty with which we just concerned ourselves was, as already stated, frequently reported; among the most recent works where it was dealt with, I will cite, beyond the short work of Calinon, the treatise on mechanics from Andrade.

VI. The second difficulty has until now attracted much less attention; it is however entirely analogous to the preceding and so logically I should have talked about it earlier.

Two different psychological phenomena occur in two different consciousnesses; when I say that they are simultaneous, what do I mean?

When I say that a physical phenomenon, which occurs outside of any consciousness, is before or after a psychological phenomenon, what do I mean?

In 1572 Tycho Brahe noticed a new star in the heavens. An immense conflagration had occurred in some very distant star; but it had occurred much earlier—it took at least.

200 years before the light leaving that star had reached our Earth. This conflagration was therefore prior to the discovery of America.

So, when I say this, when I consider this gigantic phenomenon which perhaps had no witness, because the satellites of this star perhaps did not have inhabitants, when I state that this phenomenon is prior to the formation of the visual image of Hispaniola in the conscience of Christopher Columbus, what do I mean?

A little thought is sufficient for understanding that all these affirmations have no meaning on their own.

They can only be meaningful after an agreement.

VII. We must first ask ourselves how one can have the idea of bringing into a single framework all the mutually impenetrable worlds.

We wish to represent the external universe for ourselves and it is only at this price that we could believe we know it.

We will never have this representation, we know it: our infirmity is too great.

We want to know at least that it is possible to conceive of an infinite intelligence for which this representation would be possible, a sort of large consciousness which would see all and which would classify all *in its time*, as we classify, *in our time*, the little that we see.

This hypothesis is coarse and incomplete; because this supreme intelligence would only be a demigod; infinite in one sense, this intelligence would be limited in another because it would only have an imperfect memory of the past; and it couldn't have any other, because without that all memories would be equally present to it and there would be no time for it.

And however when we talk of time, for everything which happens outside of us, don't we unconsciously adopt this hypothesis; don't we place ourselves in the place of this imperfect god; and the atheists themselves, don't they put themselves in the place where God would be, if he existed?

What I just said perhaps shows us why we have sought to bring all physical phenomena back to a single framework. But that cannot substitute for a definition of simultaneity, because this hypothetical intelligence, if it even existed, would be incomprehensible for us.

We have to look for something else.

VIII. Ordinary definitions which are sufficient for psychological time, can no longer suffice for us. Two simultaneous psychological facts are linked so closely that analysis cannot separate them without damaging them. Is it the same for two physical facts? Isn't my present closer to my past from yesterday than the present of Sirius?

We also said that two facts must be regarded as simultaneous when the order of their succession can be freely inverted. It is obvious that this definition could not be suitable for two physical facts that occur very far apart, and that, as it involves them, we no longer even understand what this reversibility could be; further, it is first the succession itself that needs to be defined.

IX. Let us therefore seek to consider what is understood by simultaneity or anteriority and to do that let us analyze some examples.

I write a letter; it is next read by the friend to whom I sent it. Here are two facts which played out in two different consciousnesses. While writing this letter I had the visual image of it, and my friend in their turn had this same image while reading the letter.

Although these two facts happen in impenetrable worlds, I do not hesitate to regard the first as earlier than the second, because I believe that one is the cause of the other.

I hear thunder and I conclude that there had been an electric discharge; I do not hesitate to consider the physical phenomenon as earlier than the audible impression experienced by my consciousness, because I believe that one is the cause of the other.

Here is the rule that we follow, and the only one that we could follow; when one phenomenon appears to us as the cause of another, we regard it as earlier.

We therefore define time by the cause; but most often, when two facts appear connected to us by a constant relation, how do we recognize which is the cause and which is the effect? We accept that the earlier fact, the antecedent, is the cause of the other, the consequent. It is then by time that we define the cause. How do we extricate ourselves from this circular reasoning?

Sometimes we say *post hoc, ergo propter hoc*; sometimes *propter hoc, ergo post hoc*; will we ever get out of this vicious circle?

X. Let us therefore look, not at how one manages to get out of it, because one can't completely do that, but how one tries to get out of it.

I perform a voluntary act A and I next experience a sensation D, which I regard as a consequence of act A; additionally, for an arbitrary reason, I infer that this consequence is not immediate, but that outside my consciousness two facts B and C, of which I was not a witness, occur and in such a way that B is the effect of A, that C is that of B and D is that of C.

But why that? If I think that I'm right to regard the four facts A, B, C and D as related to each other by a causality chain, why arrange them in causal order A, B, C and D and also in chronological order A, B, C and D instead of in some other order?

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I see clearly that in the action A I have the feeling of having been active, whereas in experiencing the sensation D, I have that of having been passive. That is why I regard A as the initial cause and D as the ultimate effect; it is why I order A at the beginning of the chain and D at the end; but why put B before C instead of C before B?

If one is asked this question, one ordinarily responds: it is well known that it is B which is the cause of C because one *always* sees B occur before C. These two phenomena, when they are witnessed, occur in a certain order; when the analogous phenomena occur without a witness, there is no reason for this order to be inverted.

That seems likely, but be careful about it; we never directly know the physical phenomena B and C; what we know are sensations B' and C' produced respectively by B and C. Our consciousness immediately teaches us that B' proceeds C' and we allow that B and C are in succession in the same order.

This rule in fact seems wholly natural and just the same one is often led to exceptions from it. We hear the sound of thunder only a few seconds after the electric discharge from the cloud. With two lightning strikes, one far away, the other nearby, couldn't the first be earlier than the second, even though the noise from the second reaches us before that from the first?

XI. Another difficulty: do we have the right to speak of the cause of a phenomenon? If all the parts of the universe are connected to some extent, one arbitrary phenomenon will not be the effect arising from a single cause, but the result of infinitely many causes; it is, we often say, the consequence of the state of the universe at an earlier moment.

How can the rules applicable to such complex circumstances be stated? And however it is only at this price that the rules can be generalized and made rigorous.

In order to not lose ourselves in this infinite complexity, let us make a simpler hypothesis; consider three bodies, for example the Sun, Jupiter and Saturn; but for more simplicity look at them as reduced to material points and isolated from the rest of the world.

The positions and speeds of the three bodies at a given moment suffice for determining their positions and their speeds at the following moment and consequently at an arbitrary moment. Their positions at the moment t determine their positions at the moment t + h, as well as their positions at the moment t - h.

There's even more; the position of Jupiter at a moment t, combined with Saturn at a moment t + a, determines the position of Jupiter at an arbitrary moment and that of Saturn at an arbitrary moment.

The set of positions the Jupiter occupies at the moment $t + \varepsilon$ and Saturn occupies at the moment $t + a + \varepsilon$ is related to the set of positions which Jupiter occupies at the moment *t* and Saturn occupies that the moment t + a by laws fully as precise as those of Newton, however much more complicated.

Henceforth why not regard one of these sets as the cause of the other, which would lead to considering as simultaneous the moment *t* for Jupiter and the moment t + a for Saturn?

For that, there can only be reasons of convenience and simplicity; which are very powerful, it is true.

XII. Let us move to less artificial examples; so we can understand the definition implicitly accepted by scholars, let us look at their work and try to find according to what rules they seek simultaneity.

I will take two simple examples; the measurement of the speed of light and the determination of longitudes.

When an astronomer tells me that a stellar phenomenon, which his telescope revealed to him at that moment, had however occurred 50 years ago, I try to understand what he means and for that, I will first ask him how he knows it, meaning how he had measured the speed of light.

He started by *allowing* that light has a constant speed, and in particular that its speed is the same in all directions. It is a postulate without which no measurement of this speed could be tried. This postulate can never however be directly verified by experiment; it could be contradicted by it, if the results of various measurements were not consistent. We will have to think that we are fortunate that this contradiction has not happened and that the small discrepancies which can happen can be easily explained.

The postulate, in any case satisfying the principle of sufficient reason, was accepted by everyone; what I want to hold on to is that it provides us a new rule in the search for simultaneity, completely different from what we had stated earlier.

Having accepted this postulate, let's see how the speed of light is measured. It is known that Roemer made use of the eclipses of the satellites of Jupiter and sought how much the event was delayed from prediction.

But, how is this predicted? It is predicted by using astronomical laws, for example Newton's law of gravitation.

Couldn't the observed facts be explained just as well if a value slightly different from the adopted value were given to the speed of light, and if it were accepted that Newton's law is an approximation? Only, one would be led to replace Newton's law by another more complicated.

Thus, a value for the speed of light is adopted such that astronomical laws compatible with this value are as simple as possible.

When sailors or geographers determine a longitude, they have to solve exactly the same problem that concerns us: without being in Paris, they must calculate the time in Paris.

How do they go about it?

Either, they carry a chronometer with them set to Paris. The qualitative problem of simultaneity is referred back to the quantitative problem of the measurement of time. I'm not going back to the relative difficulties of this latter problem because I described them at length above.

Or else, they observe an astronomical phenomenon such as a lunar eclipse and accept that this phenomenon is observed simultaneously at all places on the Earth.

This is not entirely true, because the propagation of light is not instantaneous; if absolute accuracy is desired, it would be necessary to make a correction according to a complicated rule.

Or else, finally, they can make use of the telegraph. To start with, it is clear that receiving the signal at Berlin, for example, comes after the transmission of this same signal from Paris. It is the rule of cause-and-effect analyzed above.

But later, how much later? In general, one neglects the length of the transmission and considers both events as simultaneous. But, to be rigorous, one would have again to make a small correction by a complicated calculation; it isn't done in practice, because it would be much smaller than the observational errors; its theoretical need remains undiminished from our point of view, which is that of a rigorous definition.

I want to take away two things from this discussion:

1) The rules applied vary widely.

2) It is difficult to separate the qualitative problem of simultaneity from the quantitative problem of the measurement of time; either one makes use of a chronometer, or one has to include the transmission speed, like that of light, because one could not measure such a speed without *measuring* a time.

XIII. It is appropriate to conclude.

We do not have direct intuition of simultaneity any more than that of the equality of two durations.

If we believe that we have this intuition, it is an illusion.

We compensate for this by using some rules that we apply nearly always without our being aware of it.

But what is the nature of these rules?

No general rule, no rigorous rule, but a multitude of small rules applicable to each specific case.

These rules are not imposed on us and one could take pleasure in inventing others; however, one would not be able to set them aside without greatly complicating the statement of the laws of physics, mechanics and astronomy.

We therefore choose these rules, not because they are true, but because they are the most convenient and we could summarize them by stating:

"The simultaneity of two events, or the order of their succession, the equality of two durations, must be defined such that the statement of the natural laws is as simple as possible. In other words, all these rules, all these definitions are solely the fruit of an unconscious opportunism."

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