Crack Growth and Plastic, Fracture Process and Damage Zones Behavior Under Mixed Mode Loading

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Abstract. Fatigue crack paths for inclined cracks are studied through experiments and computations under different mixed-mode loading. The experimental study of fatigue crack growth in the aluminum and titanium alloys and two types of the steels is performed on compact tension shear specimens. The cyclic fracture process and damage zone concepts are introduced and used for numerical analysis of the crack tip fields. The elastic K_1, K_2 and plastic K_P stress intensity factors are calculated by finite element method to characterize the fracture resistance along the curvilinear trajectories in compact tension shear specimens. The influence of mode-mixity and elastic-plastic material properties on the behavior of three crack tip regions as a function of cyclic loading is demonstrated.

Keywords: Crack path · Fracture process and damage zones · Mixed mode

1 Introduction

To date, several of models have been presented in the literature for the purpose of analyzing the process of fatigue crack growth from the viewpoints of micromechanics, fracture and damage mechanics. Many fracture mechanics models are based on a critical distance local to the crack tip. It is considered a fundamental characteristic parameter that distinguishes damage at the microscopic and macroscopic scale levels. This characteristic distance is often identified with a fracture process zone or fracture damage zone. A general assumption regarding the distance criterion under elastic–plastic cyclic loading conditions is that a crack increment occurs when the fracture resistance parameter (stress, strain, or energy) reaches a critical value at a characteristic distance from the crack tip. It is well known that mixed-mode conditions appear when the direction of the applied loading does not coincide with the orthogonal K_I-K_{II}-K_{III} space. The main feature of the mixed-mode fracture is that the crack growth no longer takes place in a self-similar manner and does not follow a universal trajectory. Various fracture criteria and phenomenological approaches can predict the behavior of cracks in

brittle and ductile materials loaded in combined modes. Even though many studies have been carried out in the literature to determine fracture characteristics of ductile materials under cyclic loading, it is important to investigate the local crack tip zones behavior due to low cycle fatigue for quasi-brittle and moderate ductile materials under mixed mode conditions.

2   Material Properties and Loading Conditions

Compact tension shear specimens (CTS, Fig. 1a) are made from P2M and 34X steels as well as aluminum Al7050 and titanium Ti-6Al-4 V alloys and used for cyclic mixed-mode fracture test with the loading direction variation from 90° (pure Mode I) to 0° (pure Mode II) with respect to the initial crack plane. Main static and low-cyclic mechanical properties of the tested materials are listed in Tables 1 and 2. The mixed-mode parameters expressed through the elastic and plastic stress intensity factors (SIFs) were varied by changing the load direction. Both Mode I and Mode II elastic SIFs K_1 and K_2 as well as general plastic SIF K_P are evaluated along experimental crack path of each tested material based on the series of numerical calculations. On the CTS was realized the full range of mixed-mode cyclic fracture from tensile to shear loading. Figure 1b represents experimental fatigue crack growth trajectories with initial pure Mode II loading for tested steels, aluminum and titanium alloys.

Table 1. The static properties of materials.

Material	E (GPa)	σ_0 (MPa)	σ_b (MPa)	σ_u (MPa)	N	α
P2M steel	226.9	362.4	636.0	1190.0	4.13	4.141
34X steel	216.21	714.4	1040.0	1260.4	7.89	0.529
Al-7050	70.57	471.6	524.4	701.0	10.85	1.570
Ti-6Al-4 V	118.0	118.0	963.8	1289.6	12.59	1.225

Table 2. The cyclic properties of materials.

Material	σ'_f	B	ε'_f	c	K'	n'
P2M steel	1785.0	−0.087	0.424	−0.58	1963.5	0.15
34X steel	1890.6	−0.087	0.381	−0.58	2079.6	0.15
Al-7050	1170.7	−0.095	0.35	−0.69	1128.6	0.11
Ti-6Al-4V	2153.6	−0.095	0.35	−0.69	2076.2	0.11

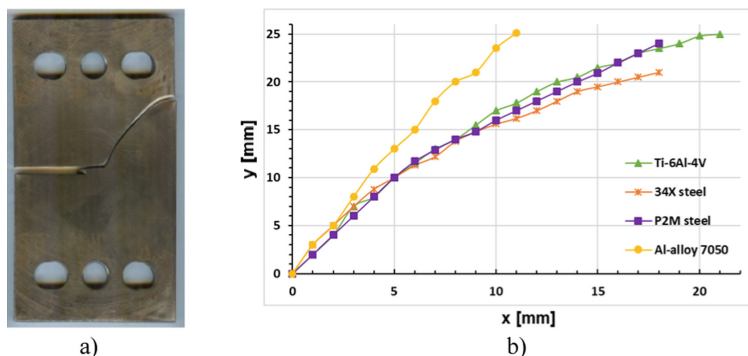**Fig. 1.** Experimental crack paths in CTS specimens at pure mode II.

Crack growth in the CTS specimen with the applied load angle β = 0° has its own characteristics. Initially, the deformation mode corresponds to the pure shear with $M_E = M_P = 0$. With an increase in the number of loading cycles N, the contribution of mixed modes of deformation and fracture gradually increases. As follows from Fig. 1, the crack path in an aluminum alloy Al7050 differs significantly from the experimental trajectories for other materials under the same loading conditions.

3 Plastic, Cyclic Fracture Process and Damage Zones Behavior

In front of a stable growing crack, we may distinguish three regions. The first outer contour represents the reversed plastic zone where the current values of stresses and deformations exceed the yield stress or strain of the material. The second region is fatigue fracture process zone. The third small area exists immediately ahead of the crack tip which is called the damage zone. For a strain-hardening material the elastic-plastic boundary around the crack tip can be obtained using the equivalent von Mises stress as a function of principal stress ratio $\lambda = \sigma_2/\sigma_1$

$$\sigma_e = \sigma_1 \eta_i \tag{1}$$

where for plane stress $\eta_i = \sqrt{1 - \lambda + \lambda^2}$ and plane strain $\eta_i = \sqrt{1 - \lambda + \lambda^2 + v(v-1)(1+\lambda)^2}$.

Full-field 3D FE-analyses are carried out to determine stress-strain fields, elastic and plastic SIFs along the curvilinear crack path in the CTS subjected to different mixed mode loadings. Figure 2 shows that, for pure mode I (Fig. 2a) and pure mode II (Fig. 2b) the plastic zones have a symmetrical shape while for mixed mode (Fig. 2c) they is non-symmetrical with respect to the initial crack plane. The dimensions of the plastic zones in Fig. 2 for titanium alloy are satisfied to the conditions of small-scale yielding. Similar shapes of plasticity zones are observed for other tested materials.
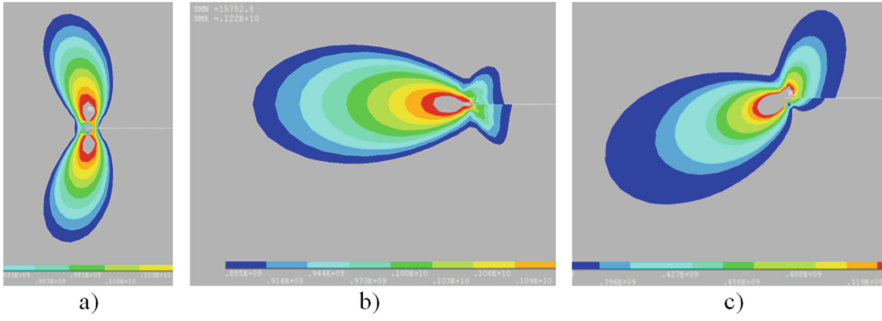
a)   b)   c)

Fig. 2. Plastic zone under different mode mixity in CTS specimen for Ti6Al4 V.

The variable critical distance is one of important parameter in modern fracture mechanics, denoted to as the fracture process zone (FPZ) size. A general assumption regarding the distance criterion under elastic–plastic cyclic loading is that a crack increment occurs when the fracture resistance parameter (stress, strain, or energy) reaches a critical value at a characteristic distance from the crack tip. The following expression was proposed by Shlyannikov [1] for normalized fracture process zone size based on the nonlinear crack-tip stress field and the main low cycle fatigue material properties

$$\bar{r}_f = \left(\frac{\sigma_n}{\sigma_0}\right)^2 (\bar{S}_1 + S_P) / \left[4\frac{\sigma_f' \varepsilon_f' E}{\sigma_0^2}\left(2N_f\right)^{(b+c)}\right] \tag{2}$$

$$\bar{S}_1 = \frac{(1+v)}{2}\left(a_{11}Y_1^2 + a_{12}Y_1Y_2 + a_{22}Y_2^2\right); \ \bar{S}_P = \frac{\alpha n'}{(n'+1)}\frac{\pi \tilde{\sigma}_e^{n'+1}}{I_n}\left(\frac{K_{eqv}}{\sigma_n\sqrt{\pi a}}\right)^2 \tag{3}$$

where n' is low cycle fatigue hardening exponent, ε_f' and σ_f' are the fatigue ductility and strength coefficients, respectively, a is the crack length, σ_u is the ultimate true tensile strength, σ_0 is the yield stress, σ_n is the nominal stress, $\tilde{\sigma}_e(\theta)$ is dimensionless equivalent stress angular function, I_n is the governing parameter of the nonlinear crack tip field [1], N_f is the number of cycles to failure, Y_1 and Y_2 are geometry dependent for SIF correction factors. The equivalent elastic stress intensity factor (SIF) for mixed mode fracture can be determined by using energy release rate criterion [2]

$$K_{eqv} = \cos\frac{\theta^*}{2}\sqrt{K_1^2(1+\cos\theta^*) - 4K_1K_2\sin\theta^* + K_2^2(5 - 3\cos\theta^*)} \tag{4}$$

$$K_1 = \sigma_n\sqrt{\pi a}\cdot Y_1(a/w); \ K_2 = \sigma_n\sqrt{\pi a}\cdot Y_2(a/w) \tag{5}$$

where, θ^* is the crack deviation angle, w is the cracked body width. The most general criterion of mixed mode crack growth direction is obtained [3] on the basis of the limiting state theory under multiaxial loading.

During low-cycle fatigue, the accumulation and growth of damage occurs in the local fracture process zone that covers the crack tip. The size of the fracture process zone is typically regarded as a distance-dependent parameter that physically represents the increase in crack size during the basic cyclic fracture process. Figure 3 shows the dependence of the both size and shape of the fracture process zone, according to Eqs. (2–5), on the accumulated number of loading cycles for the pure mode (Fig. 3a), pure mode II (Fig. 3b) and mixed mode (Fig. 3c, d) loading. It can be clearly observed that the FPZ contour behavior is not constant-size and were dependent on the accumulated loading cycles, mode mixity and both the static and low-cycle fatigue material properties. Figure 3c presents the process of changing the FPZ in the 7050 aluminum alloy according to the stages of cyclic loading, which are accompanied by corresponding changes in the mixed modes of fracture. Figure 3d give a comparison of the dimensions of the mixed-mode fatigue fracture process zone in the CTS specimen for all tested materials having different crack paths at final the number of cycles of loading, $N/N_f = 1$, as shown in Fig. 1b. Initially, owing to the same pure mode II plastic deformation in the crack tip region in several first cycles, the size of the low-cycle fatigue fracture process zone due to mode mixities is changed in different degree as a function of the tested material main mechanical properties.
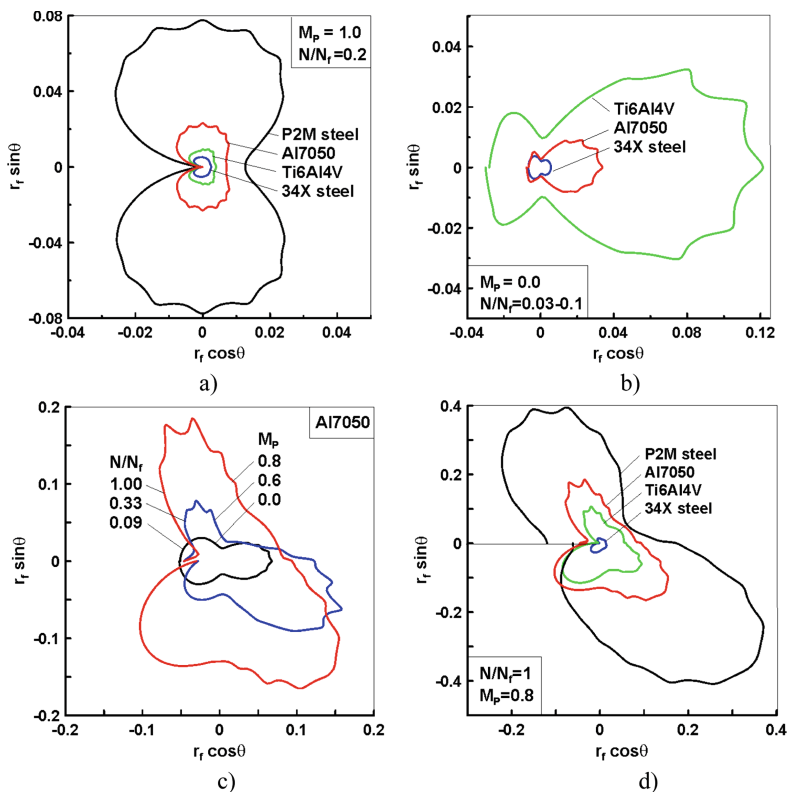

Fig. 3. Fracture process zone contour under (a) pure mode I, (b) pure mode II and (c, d) mixed modes cyclic fracture.

A fatigue damage evolution equation was proposed by the authors [4] through connecting the damage variable with the static toughness exhausting during cyclic loading. This equation has the following form

$$\omega_N = \frac{\omega_{N-1}}{\ln N_f} \ln\left(1 - \frac{N}{N_f}\right); \tag{6}$$

where ω_{N-1} denotes the critical value of the damage variable, that is expressed by

$$\omega_{N-1} = 1 - \frac{\Delta\sigma^2}{2E\sigma_u} \tag{7}$$

An amplitude of cyclic stress at the crack tip can be determined by applying the singular HRR-type stress field

$$\Delta\sigma = \bar{K}_P \bar{r}_c^{\frac{-1}{n'+1}} \tilde{\sigma}(\theta) \tag{8}$$

where K_P is the plastic stress intensity factor and r_c is the critical distance. The damage variable ω_N indicate in Eq. (6) the measure of fatigue damage with $\omega_N = 0$ denoting the undamaged state and $\omega_n = 1$ the fully damaged state.
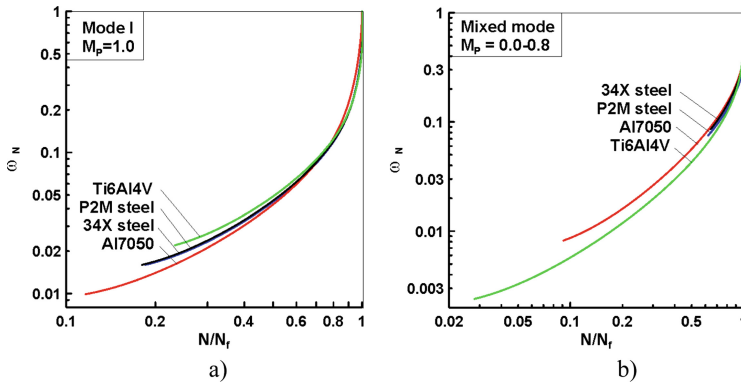

Fig. 4. Damage distributions as a function of accumulated number of loading cycles (a-pure mode I, b-mixed mode).

Figure 4 presents a comparison of the predicted values of the fatigue fracture damage evolution for pure mode I (Fig. 4a) and mixed modes (Fig. 4b) according to Eqs. (6–8) for all considered structural metallic materials. It may be seen that the mode mixity changed significantly the processes of fatigue damage accumulation only in the range $N/N_f < 0.5$.

In this work, an expression for the low cycle fatigue fracture process is derived; this includes static and cyclic material properties, as well as mixed mode fracture parameters. The elastic–plastic crack tip singularity fields are employed. The main finding,

based on the numerical and experimental results, is that the effect of mixed mode fracture during low-cycle fatigue crack growth in considered steels, aluminum and titanium alloys leads to the conclusion that under pure mode I, pure mode II and mixed mode loading conditions and shape and size of the plastic, fracture process and damage zones do not coincide with each other and have a hierarchical structure.

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