



Chapter 38

Shaft Bending to Zero Nodal Diameter Disc Coupling Effects in Rotating Structures Due to Asymmetric Bearing Supports

G. Tuzzi, C. W. Schwingshackl, and J. S. Green

Abstract In a flexible shaft-disc assembly, coupled shaft-disc vibration modes are likely to occur, provided that the natural frequencies of the two components are close. It is well known that the shaft axial and bending modes can couple with the zero and one Nodal Diameter (ND) modes of the disc, respectively. In a previous work, it has been shown that in presence of asymmetric axial-radial bearing supports, combined axial-bending shaft modes can occur, which are further impacted by gyroscopic forces when the system is rotating.

Extending the previous findings, the impact of disc flexibility on this new coupling family has been investigated in more detail. The obtained results show the emergence of shaft whirling modes with an axial component, that can couple with 0ND or 1ND disc modes. As a result, a 0ND disc mode can possibly be excited by an out of balance mass on the shaft, leading to a previously unobserved vibration behaviour.

Keywords Rotor dynamics · Shaft-disc assembly · Modes coupling · Whirl · Axial vibration

38.1 Introduction

Rotor systems combining shafts and flexible bladed discs are extensively used in industry, with applications ranging from aircraft propulsion and power gas turbines to vacuum cleaners. Due to new requirements such as higher rotating speeds and lower weight, these components are subjected to higher dynamic loads, challenging today's vibration analysis. The dynamic analysis of shaft and discs has historically been carried out independently, first analysing the shaft with rigid discs, and the discs dynamics was then studied separately. This uncoupled analysis is considered valid as long as the disc resonances are well above the shaft ones. However, modern design trends, particularly in the aero engine sector, led to more flexible discs and blades [1], and consequently the dynamic interaction between the two components cannot be neglected. Detailed studies of the shaft and disc coupled vibration [2–5] have shown that axial and bending modes of the shaft can couple with the zero and one Nodal Diameter modes of the disc respectively. These investigations suggest more generally that shaft-disc mode coupling is the result of force/moment transmission at the interface between components. The effect of rotation on these coupled modes has also been investigated, showing that shaft whirling modes involving 1ND disc modes can appear. More recently Ma, Li et al. [6] studied the full coupled dynamics of shaft and bladed discs, showing that disc transverse and blade bending can interact with shaft bending and torsional vibration. It is important to point out that, throughout all these works, shaft lateral, torsional and axial modes have been considered as independent from each other. In [7], Berger showed that the nonlinear behaviour of a thrust hydrodynamic bearing can lead to a cross coupling between shaft axial and lateral displacements but its impact on the overall shaft dynamics has not been investigated further.

In a previous work [8], the authors followed up the observations from [7] and investigated the coupling behaviour of a shaft carrying a rigid disc in the presence of general asymmetric bearing supporting structures, which leads to a coupling of axial and lateral shaft displacements. It was found that under those circumstances shaft bending and axial modes can combine

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together, leading to the emergence of a new class of axial-bending mixed modes. When the system is rotating, the combined effect of axial-bending coupling and gyroscopic forces leads to shaft whirling modes involving axial motion as well.

To understand the impact of these new mode class on a general rotor-dynamic system, this work will extend these findings to the case of a shaft carrying a flexible disc, showing that those whirling-axial modes can potentially couple with disc 0ND and 1ND modes and that an out-of-balance load on the shaft is able to excite a disc 0ND mode.

38.2 Model Description

In order to capture the coupling effects described above, a simple shaft-disc assembly has been chosen (Fig. 38.1a). The shaft is solid and circular, with a length of 0.8 m and a diameter of 0.02 m. The disc has a 0.2 m diameter and a 3 mm thickness and is rigidly connected to the shaft at 0.3 m from the left end. The whole system is made of Aluminium, which is considered elastic and homogeneous. The shaft is supported by identical plain bearings at each end.

Each bearing is then supported by four flexible rods connected to ground. In the Y-Z plane, the two rods are perpendicular to the shaft, whilst in the X-Y plane they are inclined towards the axial direction (see Fig. 38.1b) to introduce a coupling with the axial displacement. The combined stiffness of the bearing and the supporting rod is described by a lumped spring of stiffness k_i , oriented by an angle α_i with respect to the shaft midline. This applies to the rods in both planes, even if in the Y-Z plane the angle α_i is fixed to 90° . The stiffness of the two springs in the X-Y plane varies with the following law $k_{1,2} = k_0(1 \pm \delta)$.

This choice has been made to introduce an asymmetry in the supports without changing the total axial and lateral stiffness of the system, which is proportional to the sum $k_1 + k_2$. In order to connect the mono-dimensional springs to the three dimensional shaft cross section, a Multi Point Beam constraint has been adopted.

In order to study in detail the system dynamics, 40,128 3D second order brick finite elements have been selected. The reason behind this choice can be explained with the three-dimensional nature of mode coupling; since it is the result of modal forces between the subsystems, displacements and stresses have to be computed and then integrated across the three dimensional interface surface to capture it correctly. All the analyses (modal analysis and forced response) were carried out in ABAQUS.

38.3 Numerical Results

In previous work [8] it was shown that, when the shaft supports are asymmetric ($\delta = 0.5$), the shaft second bending mode (occurring at 218.9 Hz) and the axial rigid mode (occurring at 279.67 Hz) combine themselves into two Mixed Modes at 205 Hz and 255 Hz, which exhibit both axial and bending vibration (Fig. 38.2a, b). When the system is rotating, the Mixed Modes combine with the corresponding orthogonal bending mode, giving rise to traveling waves (or whirling modes) which also involve axial vibration (Fig. 38.2c). It must be pointed out that these analyses were carried out with a rigid disc.

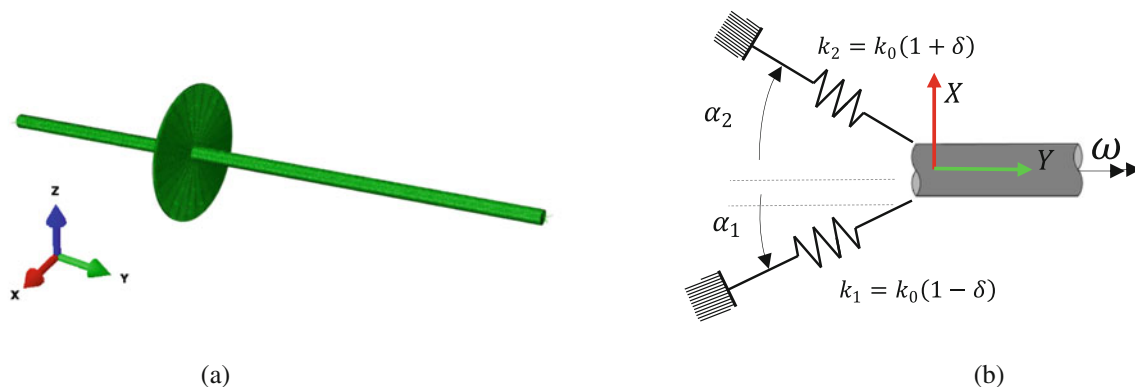


Fig. 38.1 Assembly under study – (a) solid geometry and mesh and (b) lumped parameters approximation of the bearing supporting structure

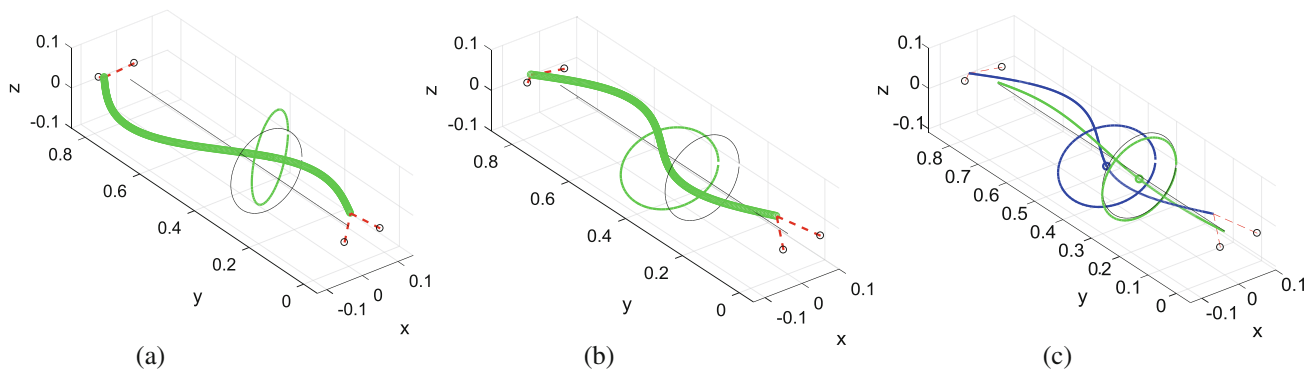


Fig. 38.2 Rigid disc model results (a) shaft axial-bending mode at 205.62 Hz, (b) shaft axial-bending mode at 255.02 Hz and (c) shaft forward travelling wave with axial component (real part in blue and imaginary part in green)

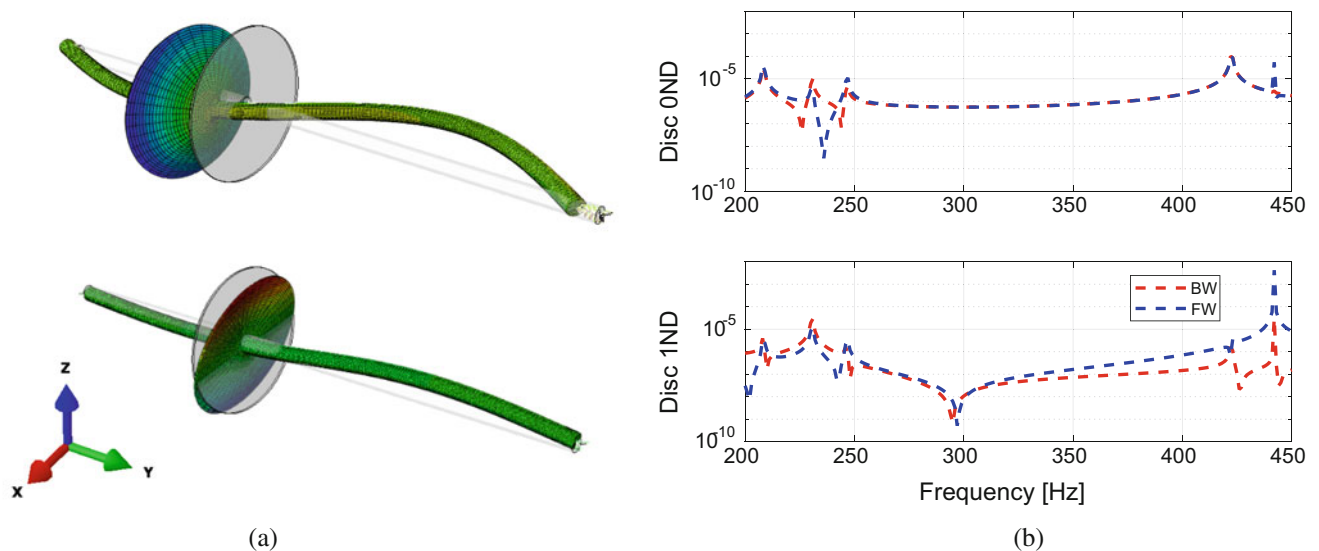


Fig. 38.3 (a) Real part (top) and imaginary part (below) of a XW whirling mode occurring at 246 Hz involving shaft axial and disc 0ND and 1ND vibration patterns and (b) Disc forced response with forward and backward rotating unitary force at left end bearing

When the disc flexibility is introduced in a rotordynamic system, shaft and disc modes couple. It is well known from the literature that shaft bending modes are prone to couple with disc 1ND modes, due to the transmission of an inertia moment between the two subsystems. On the other hand, shaft axial modes can couple with disc 0ND modes, as an axial force is transmitted. When both axial and bending shaft vibration are present at the same time, it can be seen that the disc vibrates on a superposed 0ND-1ND pattern.

A modal analysis of the rotating system is then performed considering the disc flexibility, with $\delta = 0.5$ and $\Omega = 1000 \text{ rpm}$. Due to gyroscopic forces, eigenvectors are complex and whirling modes with axial components appear. Due to the disc flexibility, those shaft modes couple with disc 0ND and 1ND simultaneously, leading to travelling waves involving disc 0ND and 1ND components. An example of such a mode, occurring at 246 Hz, is shown in Fig. 38.3a. The real part (top) is dominated by a shaft first mixed mode and involves disc 0ND mode. The imaginary part (below) has a strong shaft bending component in the YZ plane and consequently disc 1ND is excited. Since the imaginary part is ahead of the real one ($j e^{j\omega t} = e^{j(\omega t + \frac{\pi}{2})}$), this mode leads to a backward travelling wave.

In order to better understand these features, a forced response analysis has also been performed. The assembly is excited with a unitary forward and backward rotating force applied to the left end bearing (asynchronous excitation). Disc 0ND and 1ND components are then extracted at each frequency and shown in Fig. 38.3b. The most significant finding is that a rotating force in the XZ plane (which is the asynchronous equivalent of an out-of-balance load) is able to drive a 0ND disc mode, via the combined effect of bearing supports asymmetry, gyroscopic effects and shaft-disc coupling. This coupling effect has not been described previously but is suspected to be the source of some recently observed behaviour in an industrial application.

It is also worth noticing that both FW and BW excitation can drive all modes in the frequency range, regardless of their excitation direction, due to the presence of an axial component in all these modes.

38.4 Conclusions

This work investigates the effect of an axial-radial bearing supporting structure on the modes of a rotating flexible shaft-disc assembly. It was shown that an asymmetric axial-radial support leads to the appearance of axial-bending coupled shaft modes, which then can simultaneously couple with 0ND and 1ND disc modes. When the system is rotating, gyroscopic forces lead to a coupling of the orthogonal shaft bending modes, which interacts with the previously explained coupling, resulting in travelling waves with axial components showing 0ND and 1ND disc vibration patterns.

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