Chapter 1
Benfield-Hruda Damping and Residual Vectors

Paul Blelloch, Scott Sharp, and William Zornik

Abstract The Benfield-Hruda component mode synthesis (CMS) method is commonly used in the coupled loads analysis (CLA) community because it provides a very convenient means of applying component mode damping to selected components. However, since it is a CMS method, it results in approximate, rather than exact, system modes. In this paper, we will show that the approximation in the Benfield-Hruda modes can result in unacceptable errors in the calculation of residual vectors, resulting in incorrect quasi-static responses. A solution is proposed and demonstrated. This uses the Benfield-Hruda method for damping synthesis but not to calculate the system modes. It results in a much more robust convergence of results and eliminates the issue with residual vectors.

Keywords Damping · Coupled loads analysis · Simulation · Component mode synthesis

Acronyms

CMS component mode synthesis
CLA coupled loads analysis
DOF degree of freedom

1.1 Introduction

One of the most difficult-to-define things to model in any structural dynamic analysis is the damping. The simplest approach is to assume a fixed modal damping applied to each system mode, sometimes called diagonal or system mode damping. This approach is straightforward but does not easily allow for the incorporation of either localized damping, such as that arising from isolators, or for knowledge of damping in the modes of one or more components. The most commonly used method for addressing the second issue is the Benfield-Hruda component mode synthesis (CMS) method [1, 2]. This first calculates a set of "intermediate" modes by removing the component modes from the system. It then reassembles the system using the intermediate modes and the component modes that had been removed and calculates the system modes. These system modes are approximate, but the advantage of the process is that damping can be applied to both the "intermediate" modes and the component modes separately, which is then transformed into a fully coupled damping matrix in the space of the system modes. This provides a very natural method for incorporating knowledge of component mode damping into a system-level damping matrix.

The biggest disadvantage of the Benfield-Hruda method is that the system modes are approximations, and while they do converge on the "exact" system modes as the number of intermediate modes are increased, this convergence can be slow and it is difficult to know a priori how many intermediate modes to calculate. In the absence of residual vectors, this

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D. S. Epp (ed.), *Special Topics in Structural Dynamics & Experimental Techniques, Volume 5*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-3-030-47709-7_1

slow convergence is not a large problem because it is not necessarily critical that system modes be exact. However, in this paper, we will show that the method that commercial versions of the popular Nastran finite element solver package use to calculate residual modes [3] are only correct for exact normal modes, and do not necessarily work with Benfield-Hruda modes. This can result in very significant errors in quasi-static results, even when the system modes might otherwise appear to be sufficiently accurate. This makes the convergence properties of the Benfield-Hruda method much more critical, and an impractical number of intermediate modes may be required to converge on an acceptably accurate solution.

A very simple correction is proposed. This is to calculate the intermediate modes and system modes separately, and directly transform the intermediate mode damping to the system modes. This approach avoids calculating approximate Benfield-Hruda modes, and therefore completely avoids the issue with residual vectors. The convergence is now only with respect to the damping matrix, but we will show that this is very robust and that a relatively small number of intermediate modes will typically result in very small errors. This approach can be numerically more expensive than the traditional Benfield-Hruda approach, since it requires two eigensolutions on the full assembled matrices, but in practice, it is typically less expensive since the number of intermediate modes required is so much fewer. The approach has been implemented in DMAP and applied to a small number of small and large example problems.

This paper is organized as follows: First, we show the equations for the traditional Benfield-Hruda method and how these are used to synthesize a system damping matrix. This is followed by a development of the equations used to calculate residual vectors in commercial versions of Nastran, with an explanation of why these only work for the case of exact normal modes. Finally, we present the equations for the proposed correction. The main body of the paper then consists of two examples that demonstrate the issue with the traditional Benfield-Hruda method along with the accuracy of the proposed solution.

1.2 Review of Benfield-Hruda Component Mode Synthesis

The Benfield-Hruda modal synthesis method is predicated on the system model having some number of degrees of freedom (DOFs) that are identified as component modes from upstream components. These are not necessarily (and, in fact, rarely are) all the component modes, but they are just those component modes for which modal damping will be applied. However, for the purposes of this discussion, we refer to these DOFs as the component mode DOFs. The first step in the Benfield-Hruda method, therefore, is to identify the component mode DOFs and partition the left-hand side of the system equations of motion as follows[1] :

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^{\mathrm{T}} & K_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \tag{1.1}$$

where (x_2) are the component mode DOFs, and (x_1) are the remaining DOFs. An eigensolution on the remaining DOFs (x_1) is then performed to calculate the intermediate modes[2] $\left[\tilde{\Phi}_1\right]$ so that

$$\left[\tilde{\Phi}_1\right]^T [K_{11}] \left[\tilde{\Phi}_1\right] = \left[\Omega_1^2\right], \left[\tilde{\Phi}_1\right]^T [M_{11}] \left[\tilde{\Phi}_1\right] = [I] \tag{1.2}$$

The assembled equations are then expressed in terms of the intermediate modes and the component modes as

$$\begin{bmatrix} I & M_{12}\tilde{\Phi}_1 \\ \tilde{\Phi}_1^T M_{12}^T & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \Omega_2^2 & K_{12}\tilde{\Phi}_1 \\ \tilde{\Phi}_1^T K_{12}^T & K_{22} \end{bmatrix} \begin{Bmatrix} q_1 \\ x_2 \end{Bmatrix} \tag{1.3}$$

Now, a second eigensolution is performed on the assembled equations so that

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} I & K_{12}\Phi_1 \\ \Phi_1^T K_{12}^T & K_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \left[\Omega^2\right] \tag{1.4}$$

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[1]For the case where the component modes are associated with a Hurty/Craig-Bampton component, the K_{12} is zero, M_{22} is the identity, and $K_{22} = \Omega_2^2$, where Ω_2 is the diagonal matrix of component modal frequencies.

[2]Note that K_{11} and M_{11} include the physical stiffness and mass matrices from all components, so these are sometimes referred to as mass-loaded modes.

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} I & M_{12}\Phi_1 \\ \Phi_1^T M_{12}^T & M_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = [I] \tag{1.5}$$

The matrix $[\Omega^2]$ is the diagonal matrix of system frequencies squared, and the system modes are $\begin{bmatrix} \tilde{\Phi}_1 \Phi_1 \\ \Phi_2 \end{bmatrix}$. In the limit where all modes are retained at the intermediate eigensolution, the Benfield-Hruda modes will be identical to the system modes that would have been calculated with a single eigensolution.[3] If the intermediate modes are truncated, the secondary eigensolution will produce system modes that are an approximation to the single-eigensolution modes. There is not much advantage to the two-step synthesis process, except that it allows damping to be assigned to both the intermediate and component modes as follows[4] :

$$[B] = \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \tag{1.6}$$

This damping can then be transformed into the system modal coordinates as follows:

$$[B_{hh}] = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} \tag{1.7}$$

Note that while the damping matrix in the space of intermediate and component modes is diagonal (uncoupled), the damping matrix in the space of the final system modes ($[B_{hh}]$) is fully coupled.

The advantage of the Benfield-Hruda damping synthesis is that it allows the user to choose a subset of component modes for component modal damping while applying damping to the remaining DOFs in a way that is as close to system damping as possible.

1.3 Benfield-Hruda and Residual Vectors

Residual vectors are used in Nastran to approximate the contribution of neglected modes [3, 4]. They provide a capability that is equivalent in many ways to mode acceleration data recovery [5]. The basic equations are fairly simple. The first step is to orthogonalize the applied loads with regards to the calculated modes as follows:

$$\hat{P} = \left(I - M\Phi\hat{M}^{-1}\Phi^T \right) P \tag{1.8}$$

where

M - Physical mass matrix
Φ - Retained eigenvectors
$\hat{M}$ - Diagonal generalized mass matrix (usually identity)
P - Applied load matrix (includes inertial load vectors)

Note that $\hat{P}$ is orthogonal to the normal modes Φ in the sense that $\Phi^T \hat{P} = \left(\Phi^T - \Phi^T \right) P = 0$. This is true for any shape Φ, as long as $\hat{M} = \Phi^T M \Phi$.

The next step calculates the shapes associated with the orthogonalized loads:

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[3]Because the Benfield-Hruda modes are only identical to the system modes when all intermediate modes are included; the user needs to be careful to include enough intermediate modes to get an accurate solution.

[4]The Benfield-Hruda method is not limited to diagonal component mode damping ($2\xi_1\Omega_1$). The component can have a fully populated damping matrix if available. It is also possible to have discrete damping elements distributed anywhere in the structure that result in additional coupled damping terms. However, the most common application is with diagonal damping for both the component and the intermediate modes, which still results in a fully coupled damping matrix at the system mode level.

$$\hat{R} = K^{-1}\hat{P} \tag{1.9}$$

The deflection due to any set of shapes Φ is $X = \Phi\hat{K}^{-1}\Phi^T P$, where $\hat{K} = \Phi^T K \Phi$. The residual solution is then the exact solution $K^{-1}P$, minus the contribution of the shapes $\Phi\hat{K}^{-1}\Phi^T P$, or.

$$\hat{R} = \left(K^{-1} - \Phi\hat{K}^{-1}\Phi^T\right) P \tag{1.10}$$

Equation (1.9) is only correct if the right-hand sides of (1.9) and (1.10) are equal, or if $K^{-1}\hat{P} = \left(K^{-1} - \Phi\hat{K}^{-1}\Phi^T\right) P$. Substituting in for $\hat{P}$ from Eq. (1.8) leads to the following identity:

$$\left(I - K\Phi\hat{K}^{-1}\Phi^T\right) P = \left(I - M\Phi\hat{M}^{-1}\Phi^T\right) P \tag{1.11}$$

This is true if $K\Phi\hat{K}^{-1} = M\Phi\hat{M}^{-1}$. If the columns of Φ are eigenvectors, and $\hat{K}$ is a diagonal matrix of eigenvalues λ_i, then $K\Phi\hat{K}^{-1} = M\Phi\hat{M}^{-1}$ can be rewritten as $[K]\{\phi_i\} - \lambda_i[M]\{\phi_i\} = 0$, which is the generalized eigenvalue problem. So, $K\Phi\hat{K}^{-1} = M\Phi\hat{M}^{-1}$ is true if and only if the columns of Φ are eigenvectors. This shows that although residual vectors can be developed relative to any set of basis functions, the residual vectors calculated in Nastran are only correct if the basis functions are, in fact, the exact modes of the system. Residual vectors will work for basic vectors that are not solutions to the eigenvalue problem if they are calculated from Eq. (1.10), but not if they are calculated as the solution of Eq. (1.9).

It has proven to be the case in practice that the small error in Benfield-Hruda synthesized mode shapes results in sufficient errors in the residual vectors that quasi-static results using a combination of Benfield-Hruda modes and residual vectors can have very significant errors. While the errors are very much case-specific, attempts to correct them by increasing the frequency cutoff of the intermediate modes are difficult to implement. In the next section we propose a solution, and in the following two sections demonstrate the solution by example.

1.4 Proposed Solution

There are a few potential solutions to the fact that residual vectors calculated in Nastran are not correct for Benfield-Hruda synthesized mode shapes. These include the following:

1. Keep increasing the frequency cutoff of the intermediate modes until the errors are sufficiently small.
2. Start with the exact system modes and use these to approximate the intermediate modes, rather than starting with intermediate modes to synthesize system modes. This might use a method similar to the one documented in previous work [6].
3. Calculate system and intermediate modes separately and just use the intermediate modes to map damping.

Both methods 2 and 3 solve the issue with residual vectors, since both use the "exact" system modes, which satisfy Eq. (1.11). This results in residual vectors that are complete with respect to the system modes and therefore generate correct quasi-static results.

As mentioned previously, the first solution has proven to be very unsatisfactory. The convergence of the Benfield-Hruda modes using quasi-static responses as a metric can be extremely slow. In practice, the only feasible way to ensure convergence is to calculate "all" intermediate modes. This is only practical in a model that is constructed of superelements, such that the size of the system matrices are moderate, but even in that case the number of intermediate modes required can be very large and numerically inefficient. This is illustrated in the second example.

The second method is feasible and would only require solving the full eigensolution once, since the intermediate modes could be approximated based on a relatively small number of system modes. However, the method outlined in [6], is fairly complex, can be numerically sensitive, and would require significant development.

The third method is effectively a brute force method. It neither uses the intermediate modes to approximate the system modes, nor does it use the system modes to approximate the intermediate modes. It does require two full eigensolutions, but for most practical problems this is not an issue. In practice, it has proven to be much faster than the approach of increasing intermediate modal frequency cutoff and it is robust in the sense that no studies are required to demonstrate convergence.

To implement this method, the user starts by partitioning the matrices into component modes DOF $\{x_2\}$ and all other DOF $\{x_1\}$ as in Eq. (1.1). The user solves for the intermediate modes $[\Phi_1]$ as is Eq. (1.2). The method now diverges by not assembling the system consisting of intermediate and component modes as in Eq. (1.3). Instead, the system modes Φ are calculated directly from Eq. (1.1) such that

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} \Omega^2 \end{bmatrix} \tag{1.12}$$

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = [I] \tag{1.13}$$

Note that these are exact system modes and not approximations. The goal is now to map damping from the intermediate modes to the system modes. Noting from Eq. (1.2) that $\left[\tilde{\Phi}_1\right]^T [M_{11}] \left[\tilde{\Phi}_1\right] = [I]$, so the pseudo-inverse of $\left[\tilde{\Phi}_1\right]$ is $\left[\tilde{\Phi}_1\right]^T [M_{11}]$. The intermediate modes $\left[\tilde{\Phi}_1\right]$ can therefore be mapped to the system modes $[\Phi_1]$ by pre and post-multiplying by $\left[\tilde{\Phi}_1\right]^T [M_{11}] [\Phi_1]$ which is the cross-orthogonality between intermediate and system modes relative to the $[M_{11}]$ partition of the mass matrix. The Benfield-Hruda damping can then be expressed as follows:

$$[B_{hh}] = \begin{bmatrix} \left[\tilde{\Phi}_1\right]^T [M_{11}] [\Phi_1] \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \left[\tilde{\Phi}_1\right]^T [M_{11}] [\Phi_1] \\ \Phi_2 \end{bmatrix} \tag{1.14}$$

This approach requires two eigensolutions to calculate both the intermediate modes $\left[\tilde{\Phi}_1\right]$ and the system modes $\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$; however, there is no need to retain a large number of intermediate modes, and since the system modes are exact there is no issue with residual vectors. In practice, the method has been found to be very robust and efficient relative to worrying about modal convergence using a traditional Benfield-Hruda method.

1.5 First Example

As a first example, consider the Northrop Grumman OmegA launch illustrated in exploded view in Fig. 1.1. For this example, the cryogenic third stage is simulated, with bending moments recovered in an element connecting the two liquid tanks.

A simulation was performed for an empty cryogenic third stage, with axial thrust ramped up over 1 s and held steady over 1 s on both engines. The third-stage model contains a Hurty/Craig-Bampton of the payload, and a Hurty/Craig-Bampton model of each of the two engines. The remainder of the model is represented by standard finite elements.

The transient bending moment in plane 1 on end A of a beam connecting the two liquid tanks was recovered. All solutions used system modes to 60 Hz and a residual vector associated with the engine thrust. The baseline was run without Benfield-Hruda CMS. Traditional Benfield-Hruda solutions were then run based on intermediate frequency cutoffs of 60 Hz, 100 Hz, 200 Hz, 500 Hz and 1000 Hz. The alternate Benfield-Hruda solution was run with intermediate frequency cutoff of 60 Hz. The run times are summarized in Table 1.1. The traditional Benfield-Hruda solution with intermediate modes to 60 Hz takes only slightly longer than the baseline solution, and the alternate Benfield-Hruda solution with intermediate modes to 60 Hz only takes a little longer than that. Increasing the frequency cutoff of the intermediate modes, however, significantly increases the run time.

Now consider the transient bending moment solution plotted in Fig. 1.2. The alternate Benfield-Hruda solution is effectively exact. The error in the traditional Benfield-Hruda solution is about 13% for frequency cutoffs of either 60 or 100 Hz, but drops to less than 1% for frequency cutoffs of 200 Hz or higher.

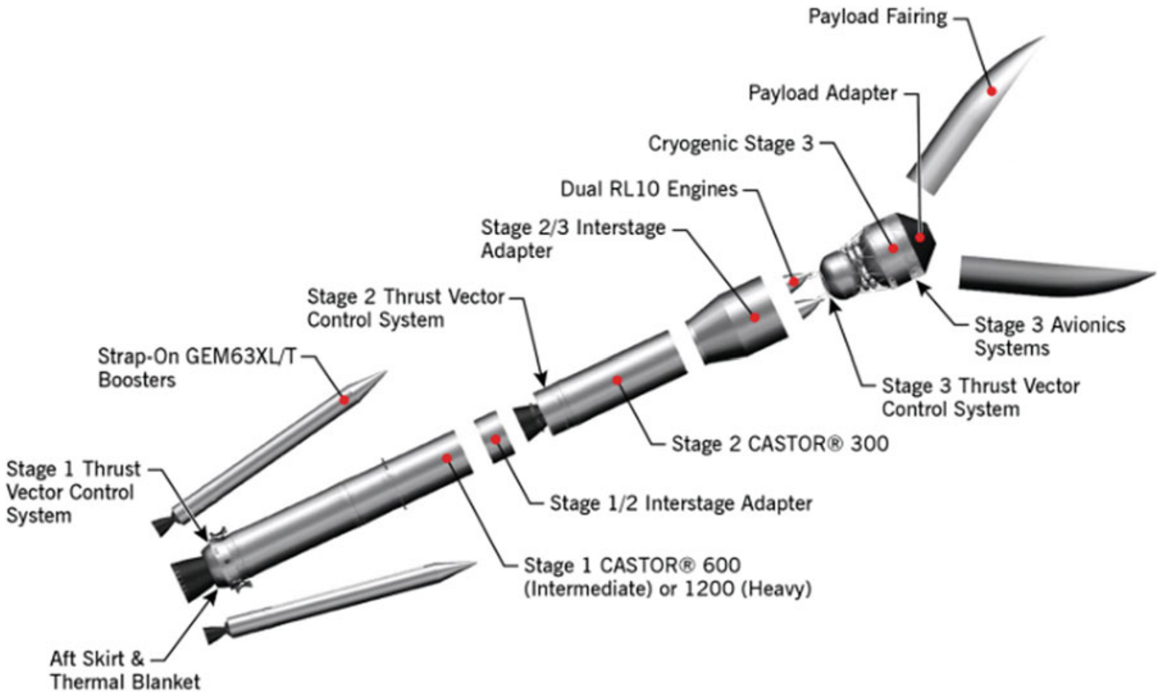

Fig. 1.1 Expanded view of Northrop Grumman OmegA Launch Vehicle (https://www.rollcall.com/sponsored-content/omega-rocket-air-force-pursuit/)

Table 1.1 Comparison of run times for various Benfield-Hruda solutions

Case	Run Time
No Benfield-Hruda	0:16
Traditional Benfield-Hruda (60 Hz)	0:18
Traditional Benfield-Hruda (100 Hz)	0:30
Traditional Benfield-Hruda (200 Hz)	1:10
Traditional Benfield-Hruda (500 Hz)	5:07
Traditional Benfield-Hruda (1.000 Hz)	9:34
Alternate Benfield-Hruda (60 Hz)	0:25

For this particular output, an intermediate frequency cutoff of 200 Hz ($4\times$ the system frequency cutoff of 60 Hz) gives an acceptable answer. However, in practice it is rarely practical to perform this type of convergence study, and every output will converge at a different rate. The alternate Benfield-Hruda, which uses “exact” system modes has effectively no error.

The error in the Benfield-Hruda system frequencies as function of intermediate mode frequency cutoff is illustrated in Fig. 1.3. The error does decrease, although it is difficult to map frequency error to error in transient results.

For this example, the intermediate mode damping was set to 10%, the damping on the two engine component modes to 5%, and the damping on the payload modes to 1%. Since the traditional Benfield-Hruda solution with intermediate modes to 1000 Hz is converged, it was treated as the “truth” solution for damping. The error in diagonal damping ratios for each of the other Benfield-Hruda solutions are plotted in Fig. 1.4. The error in damping ratios also decreases as the intermediate modal frequency cutoff increases. All solutions except for the two with a 60 Hz intermediate mode cutoff frequency have negligible error. The alternate solution with a 60 Hz cutoff has a lower error than the traditional solution with a 60 Hz cutoff (1% versus 3%). These errors are probably negligible and well within the uncertainty in damping.

1.6 Second Example

The second example consists of a substructured model of a large launch vehicle. The assembled matrices have approximately 40,000 DOFs, meaning that it is feasible, although painful, to calculate all the intermediate modes. In this case, the exact solution was the traditional Benfield-Hruda solution retaining all the intermediate modes. A total of 2165 system modes were

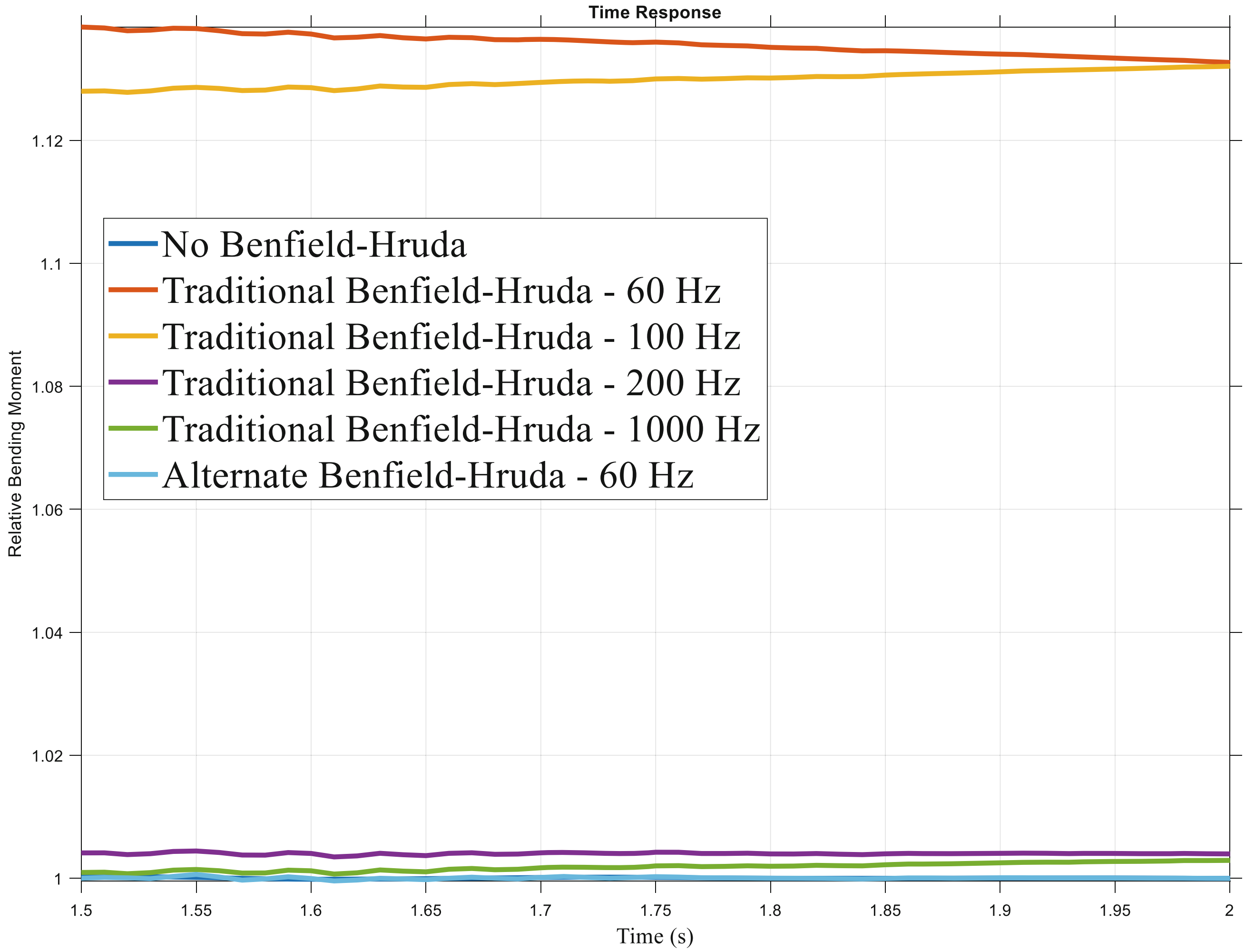

Fig. 1.2 "Steady-state" bending moment for various Benfield-Hruda solutions

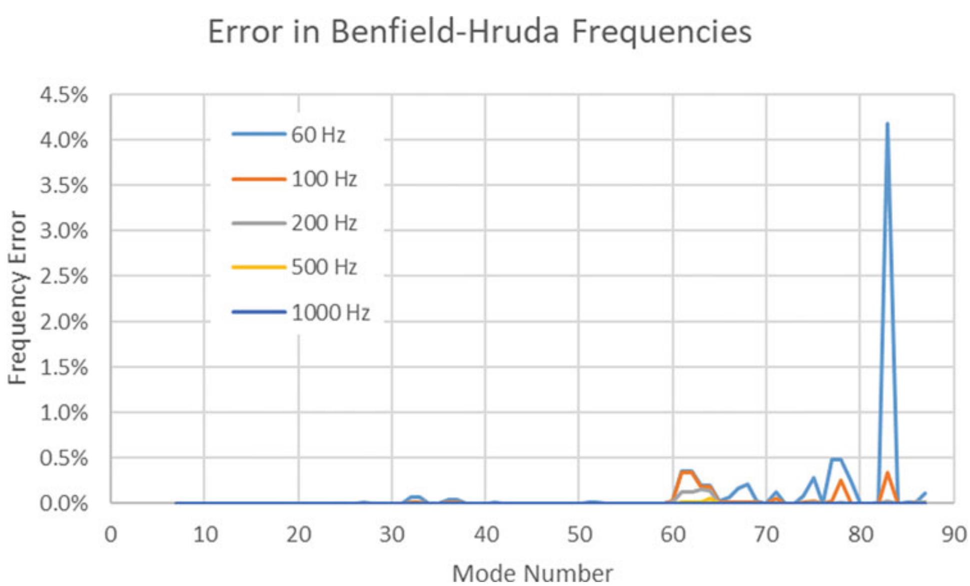

Fig. 1.3 Error in Benfield-Hruda system mode frequencies for different intermediate mode frequency cutoffs

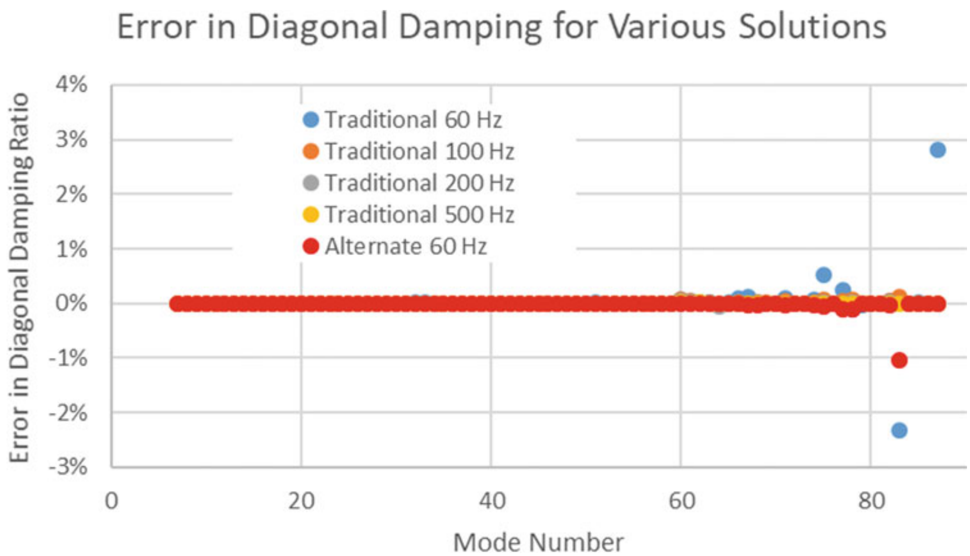

Fig. 1.4 Error in Benfield-Hruda diagonal damping ratios for different solutions

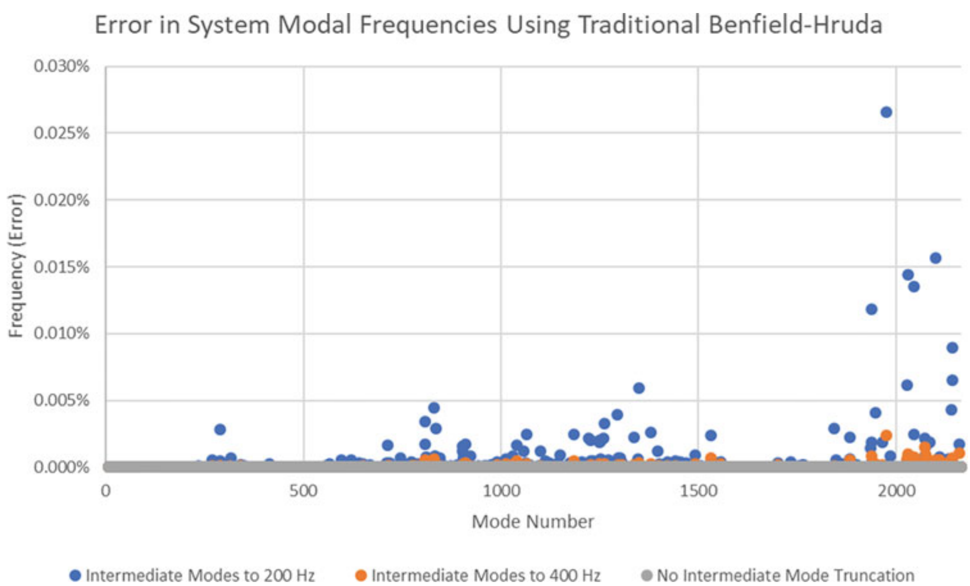

Fig. 1.5 Error in Benfield-Hruda system mode frequencies for different intermediate frequency cutoffs

calculated up to 60 Hz. The Benfield-Hruda solution was then repeated with intermediate frequency cutoffs of 200 Hz and 400 Hz. The error in system mode frequencies are Fig. 1.5. The frequencies are all fairly close.

Again treating the traditional Benfield-Hruda with no intermediate frequency cutoff as the truth, the error in the diagonal damping ratios for traditional Benfield-Hruda with either 200 or 400 Hz cutoff and the alternate Benfield-Hruda with either 65 Hz or 120 Hz cutoff were calculated and are plotted Fig. 1.6. In this case, all diagonal damping ratio errors are fairly small, but the alternate Benfield-Hruda results are much closer, even with an intermediate frequency cutoff of 65 Hz.

This demonstrates that the alternate method is feasible on a large model, and that the diagonal damping ratios are very accurate.

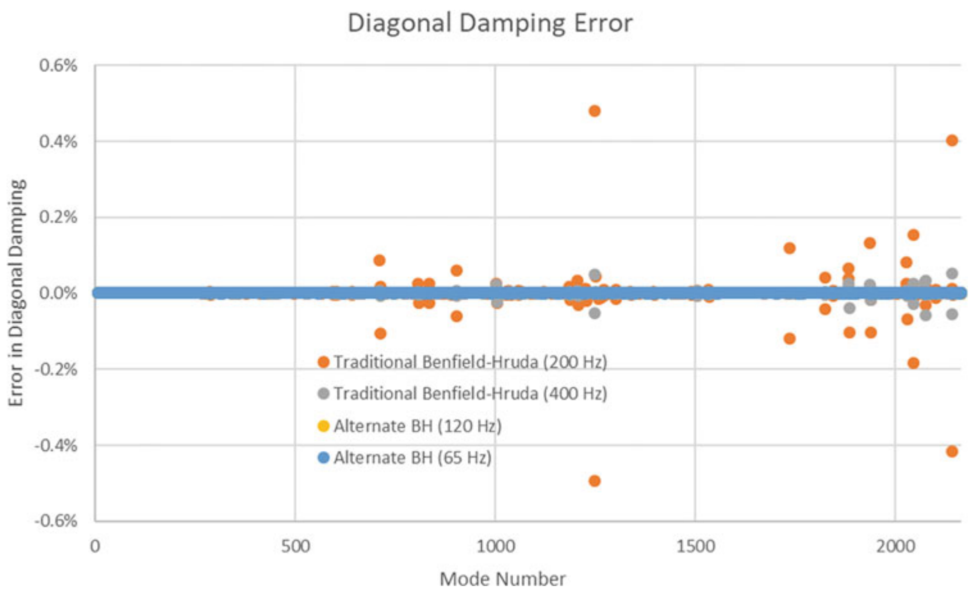

Fig. 1.6 Error in Benfield-Hruda diagonal damping ratios for different models

1.7 Summary

It has been demonstrated that errors introduced in the Benfield-Hruda component mode synthesis method can result in residual vectors that do not correctly capture steady-state response. An alternate procedure, which calculates the system and intermediate modes separately has been suggested. This method eliminates the problem with the residual vectors by using "exact" system modes while still allowing for the Benfield-Hruda damping synthesis method to be applied. Two examples were used to demonstrate the shortcomings in the traditional method and the performance of the alternate method. The first example showed how the use of a traditional Benfield-Hruda method can result in significant errors in quasi-static internal loads when using residual vectors, and how this error is eliminated using the alternate method. The second example illustrates the accuracy of the damping achieved using the alternate method for a large launch vehicle model.

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