The Einstein, Podolsky, and Rosen Paper

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A. Einstein, B. Podolsky, and N. Rosen, Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?,
Physical Review, **47**, 777–780 (1935).

2.1 Reprint of the Paper

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? A. Einstein, B. Podolsky, and N. Rosen, Physical Review, Volume 47, Page 777–780, published in 1935 by the American Physical Society. Reprinted with permission https://doi.org/10.1103/PhysRev.47.777

2.2 Critical Summary

The paper by Einstein, Rosen and Podolsky, that we shall call the EPR paper in the following, is not long. It is a little less than four pages long and does not contain any references to other publications. It was submitted to *Physical Review* on March 25, 1935, and published on May 15, 1935.

Besides an introduction, the EPR paper consists of two chapters without heading. In the first part, the authors claim that there is a difference between an assumed objective reality and its description through a physical theory:

> Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates.

C. Kiefer (ed.), *Albert Einstein, Boris Podolsky, Nathan Rosen*, Classic Texts in the Sciences, https://doi.org/10.1007/978-3-030-47037-1_2

They continue to stress that the success of a theory should be determined by asking the following questions. First: Is the theory correct? Second: Is the description given by the theory complete? To answer the first question, the authors refer to the agreement with measurement outcomes; this question is not going to be the subject of their paper. The EPR paper focusses on the question of completeness, as implied in the paper's title.

What is completeness supposed to mean? Quantum mechanical states are described through wave functions (more generally: vectors in Hilbert space). If this description is complete, i.e., if no further ('hidden') variables exist that would allow for a simultaneous determination of, e.g., position and momentum, then, the theory shall be called complete.

EPR propose the following, necessary criterion for completeness, which according to them seems unavoidable:

> [...] every element of the physical reality must have a counterpart in the physical theory.

In a letter to Schrödinger on June 19, 1935, Einstein wrote more specifically:

> In quantum mechanics, one describes a real state of affairs of a system by means of a normed function Ψ of the coordinates (of configuration space). The temporal evolution is uniquely determined by the Schrödinger equation. One would now very much like to say the following: Ψ stands in a one-to-one correspondence with the real state of the real system. The statistical character of measurement outcomes is exclusively due to the measurement apparatus, or the process of measurement. If this works, I talk about a complete description of reality by the theory. However, if such an interpretation doesn't work out, then I call the theoretical description 'incomplete'.[1]

In order to apply this criterion one needs to know what elements of physical reality are. Then, according to the authors, the question of completeness can be easily answered. On these grounds they present a criterion that they consider sufficient for their purpose. It is this criterion of reality that would cause much concern and many misunderstandings later on. It goes like this (italics by EPR):

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

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[1] "Man beschreibt in der Quantentheorie einen wirklichen Zustand eines Systems durch eine normierte Funktion ψ der Koordinaten (des Konfigurationsraumes). Die zeitliche Änderung ist durch die Schrödinger-Gleichung eindeutig gegeben. Man möchte nun gerne folgendes sagen: Ψ ist dem wirklichen Zustand des wirklichen Systems ein-eindeutig zugeordnet. Der statistische Charakter der Meßergebnisse fällt ausschließlich auf das Konto der Meßapparate bzw. des Prozesses der Messung. Wenn dies geht rede ich von einer vollständigen Beschreibung der Wirklichkeit durch die Theorie. Wenn aber eine solche Interpretation nicht durchführbar ist, nenne ich die theoretische Beschreibung 'unvollständig'." (von Meyenn [158], English translation taken from Fine, A. [73]. *The Shaky Game*. Second edition. University of Chicago Press, Chicago, p. 71.)

The key passage here is “without in any way disturbing a system”. We will get back to this.

The first part of the EPR paper focuses on the quantum mechanical description of a particle in a single space dimension. The authors highlight the fundamental importance of the notion of a state, “which is supposed to be completely characterised by the wave function Ψ.” According to quantum mechanics, Ψ renders a complete description. Further, the authors assume the validity of the probability interpretation. It states that the probabilities of measuring certain values of classical quantities can be obtained from the square of Ψ’s absolute value.

Now, if Ψ is the eigenfunction of an operator A with eigenvalue a, the probability interpretation holds that the physical quantity given by the operator A (such a physical quantity is called an observable in quantum theory) will in this state have the value a with certainty. EPR apply their reality criterion to this situation and infer from it that in this eigenstate an element of physical reality exists which corresponds to the physical quantity A. They take a momentum eigenstate with eigenvalue p_0 as an example[2] and conclude that it is thus reasonable to say that momentum of the particle in this state is real.

If the state is not an eigenstate of the operator A, one cannot deduce this anymore: There is no longer a certain value ascribed to the physical quantity described by A. Take the case where you ask for coordinate values of a particle (corresponding to its position) that currently is in a momentum eigenstate. As a matter of fact, in this case all coordinate values have the same probability! According to EPR, the only way to obtain a specific value for the coordinate is through a direct measurement, which, however, disturbs the particle and its state; the particle then no longer is in the momentum eigenstate. Interestingly, EPR assume a collapse (or reduction) of the wave function, i.e., a violation of Schrödinger’s equation, cf. Sect. 1.4; as was common back then, they do not describe this collapse dynamically but insert it by hand. EPR go on to generalise their conclusions and emphasise that, according to quantum mechanics (where the description through a wave function is considered complete), when the operators corresponding to two physical quantities do not commute, the two quantities cannot have simultaneous reality.

The conclusion of the first part of the EPR paper can thus be summarised as follows. Consider the two propositions:

P_1 The description of reality with wave functions is complete.

P_2 When the operators corresponding to two physical quantities do not commute, the two physical quantities have simultaneous reality.

EPR’s conclusion then is that either P_1 or P_2 is false. That is, either the description of reality with wave functions is incomplete, or non-commuting quantities cannot be

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[2] Such a state is described by a wave function that in every measurement yields the value p_0 for the momentum.

simultaneously real. It is important to note that the phrasing is “not simultaneously real” instead of “not simultaneously measured”: the authors do not doubt that non-commuting quantities cannot be *measured* simultaneously. Up to this point, the paper contains a mostly non-controversial application of the quantum formalism as would be expected in 1935. The second part ist the controversial one. Herein, EPR find that the result of the first part leads to a contradiction.

They demonstrate this with the help of a thought experiment (remember Einstein’s fondness for thought experiments). In it, they consider two systems I and II that interact for a certain time, but are disconnected ever after. Think, e.g., of a decay of a particle resulting in two new particles that fly away in opposite directions and that, in principle, can be separated by arbitrarily large distances. EPR assume that at the beginning both systems are described by states on their own (each by its own wave function), and that these states are known. Following the interaction, there is only one (today we call it entangled) wave function Ψ for the combined system I plus II. The two subsystems do not have a state (a wave function) on their own anymore.

EPR now assume that a measurement is taken on one of the two systems (system I), leading to a collapse (or reduction) of the wave function. As they put it in the first paragraph of part 2.:

> We cannot, however, calculate the state in which either one of the two systems is left after the interaction. This, according to quantum mechanics, can be done only with the help of further measurements, by a process known as the reduction of the wave packet. Let us consider the essentials of this process.

EPR now consider (as part of the thought experiment) measurements that are conducted on system I alone. To this end, they consider two different physical quantities (observables) A and B in I. The combined wave function[3] $\Psi(x_1, x_2)$ can then be expanded either in terms of the eigenfunctions of the operator A (EPR’s equation (7)) or in terms of the eigenfunctions of the operator B (EPR’s equation (8)). If one now measures in system I the quantity A, one finds a certain value a_k and the corresponding eigenfunction $u_k(x_1)$. According to the postulate of the reduction of a wave function, the wave function in EPR’s equation (7) is thus reduced to a product $\Psi(x_1, x_2) = \psi_k(x_2)u_k(x_1)$. But that means that the system II, on which no measurements can be taken (and that by consequence was not ‘disturbed’), now is in a concrete state, namely the one given by $\psi_k(x_2)$. The entanglement of the two systems is broken; the new state of the system is a product and therefore no longer describes a correlation between systems I and II.

But instead of A one can also measure quantity B in I, yielding a value b_r and an eigenfunction $v_r(x_1)$. The combined wave function thus reduces to a different product, namely $\Psi(x_1, x_2) = \varphi_s(x_2)v_s(x_1)$. EPR conclude from this:

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[3] Let x_1 denote the coordinate of the first, x_2 the coordinate of the second particle.

> We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions.

According to EPR, one can therefore assign two different wave functions to the same reality. They consider especially important the case where the alternative wave functions ψ_k and φ_s of system II are eigenfunctions of non-commuting operators P and Q. EPR's example for this case is so important that we shall discuss it explicitly. This means that our discussion will become somewhat formal now. However, the conclusions drawn from it can be understood without a detailed knowledge of the formalism.

In their example, systems I and II are two particles I and II that have a common wave function $\Psi(x_1, x_2)$. EPR choose the following form of the wave function:

$$\Psi(x_1, x_2) = h\delta(x_1 - x_2 + x_0) \equiv \int_{-\infty}^{\infty} \mathrm{d}p \; \mathrm{e}^{2\pi \mathrm{i}(x_1 - x_2 + x_0)p/h}, \tag{2.1}$$

where x_0 is a constant and h is Planck's constant. Due to the delta function the difference of x_2 and x_1 is virtually fixed to x_0.

Now A is the momentum operator[4] of particle I, whose eigenfunctions read:

$$u_p(x_1) = \mathrm{e}^{2\pi \mathrm{i} p x_1/h}, \tag{2.2}$$

where the usual normalisation constant is missing and where the number p is the eigenvalue. The combined state (2.1) can now be expanded in terms of these momentum eigenfunctions:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \mathrm{d}p \; \psi_p(x_2) u_p(x_1), \tag{2.3}$$

where

$$\psi_p(x_2) := \mathrm{e}^{-2\pi \mathrm{i}(x_2 - x_0)p/h} \tag{2.4}$$

is the eigenfunction of the momentum operator of particle II, itself given by

$$P := \frac{h}{2\pi \mathrm{i}} \frac{\partial}{\partial x_2}.$$

P's eigenvalue is $-p$, which is of course a direct consequence of momentum conservation: The total momentum of the initial state is zero and retains that value (as long as no measurement of position is taken on any of the particles).

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[4] Given explicitly by $(\hbar/\mathrm{i})\, \partial/\partial x_1$.

Now EPR consider an alternative where B is the position operator of particle I, whose (improper) eigenfunction v_x is the delta function:

$$v_x(x_1) = \delta(x_1 - x). \tag{2.5}$$

Then one can expand the combined state (2.1) in terms of these position eigenfunctions instead of the momentum eigenfunctions:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \mathrm{d}x \; \varphi_x(x_2) v_x(x_1), \tag{2.6}$$

where

$$\varphi_x(x_2) := \int_{-\infty}^{\infty} \mathrm{d}p \; \mathrm{e}^{2\pi \mathrm{i}(x - x_2 + x_0)p/h} = h\delta(x - x_2 + x_0) \tag{2.7}$$

is the eigenfunction of the position operator $Q = x_2$ of particle II. Its eigenvalue is $x + x_0$, this being the coordinate value of particle II.[5] Because

$$[P, Q] \equiv PQ - QP = \frac{h}{2\pi \mathrm{i}},$$

$\Psi_p(x_2)$ and $\varphi_x(x_2)$ are indeed eigenfunctions of non-commuting operators, namely of position and momentum of particle II. Non-commuting operators correspond to physical quantities that cannot be measured simultaneously and are therefore subject to an uncertainty relation.

This is EPR's example, a special case of the general situation they had presented in the beginning of their second part. EPR now get back to their general discussion and reach a conclusion. They assume that the wave functions Ψ_k and φ_r of system II are eigenfunctions of non-commuting operators P and Q with eigenvalues p_k and q_r, respectively. The (thought of) experimenter now has a free choice to take on system I a measurement of A *or* B. If he chooses to measure A, system II will be described by the wave function $\Psi_k(x_2)$ (in the above example this is $\Psi_p(x_2)$ in (2.4)) with eigenvalue p_k (above: p). If he chooses to measure B in I, the wave function of II will be $\varphi_r(x_2)$ (above: $\varphi_x(x_2)$) with eigenvalue q_r. But in neither case system II is being disturbed! EPR now bring forward their reality criterion a second time:

> In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality. But, as we have seen, both wave functions Ψ_k and φ_r belong to the same reality.

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[5] Note that the difference $x_1 - x_0$ and the sum $p_1 + p_2$ correspond to commuting operators and are thus simultaneously 'measurable'.

The EPR paper now concludes as follows. In the first part, EPR found that of the two propositions P_1 and P_2 either P_1 or P_2 is false, i.e., either the description with wave functions is incomplete or non-commuting quantities do not have simultaneous reality. In the second part, they found $P_1 \Rightarrow P_2$, i.e., that the simultaneous reality of physical quantities corresponding to non-commuting operators follows from the assumption of completeness. According to the rules of elementary logic the findings of both parts of the paper are both true only if either P_1 and P_2 are both false or if P_1 is false and P_2 is true. Since EPR's thought experiment showed P_2 to be true (under the assumption of separability), P_1 must be false; i.e., the description of reality with wave functions is incomplete. This is the essential conclusion of the EPR paper.

As a matter of fact, the trueness of proposition P_2 follows directly from the locality criterion or from separability (although these notions[6] are not mentioned directly in the paper): events in system I cannot affect the spatially separated system II (it is too far out). In their second part, EPR only show P_2 to be true if one assumes locality: P_1 is not really involved. One can deduce directly from the result of part I (either P_1 or P_2 false) that P_1 must be false, i.e., that i.e., that the description with wave functions is incomplete. These are EPR's closing remarks:

> While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

However, they did not conclude their paper without having presented a possible loophole through which one could avoid the conclusion of quantum theory's incompleteness (second last paragraph):

> Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted.*

Since the simultaneous measurement of non-commuting quantities A and B in system I is not possible, P and Q could not have simultaneous reality in system II, although system II is arbitrarily far out. Here is what EPR have to say about this:

> This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way.

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[6] These notions are sometimes used synonymously, and sometimes as meaning something different. We apply the meaning that d'Espagnat later called *Einstein-separability* [47, p. 132] and that was described by Einstein in 1949 in the following words[59, p. 84 (85)]: "Now, however, the real situation of S_2 must be independent of what happens to S_1." ("Der reale Sachverhalt (Zustand) des Systems S_2 ist unabhängig davon, was mit dem von ihm räumlich getrennten System S_1 vorgenommen wird.")

EPR do not expect this to be a reasonable definition of reality. Bohr, however, clearly thought differently (cf. further below).

2.3 Bohm's Version of the Thought Experiment

The combined state (2.1) that EPR used in their thought experiment shows some disadvantageous traits. For example, it is not only mathematically quite intricate to include the delta function in the formalism of quantum mechanics (it is not a state in Hilbert space that can be normalised), it is also dynamically unstable. By this we mean that a state described by this function widens with time – a strongly localised wave packet delocalises rapidly. An easier version of EPR's thought experiment, mathematically and conceptually, was introduced by David Bohm (1917–1992) in his textbook on quantum mechanics [21, p. 610–623], written under strong influence of the Copenhagen interpretation. It uses the particle's spin and has since its appearance replaced EPR's original thought experiment in most discussions of their arguments. Interestingly, it is also the version that is more accessible to an experimental realisation. However, the experiments could only be realised decades after Bohm's introduction of the thought experiment. Yet, also EPR's original state (2.1) can be realised experimentally as a two-mode squeezed state[7] (see, e.g., Leonhardt [111, p. 74]). The realisation was reported in a paper by Ou et al. [121]. In fact, this state plays an important role in cosmology and the physics of black holes (see discussion in chap. 6). It exhibits, in a well-defined way, maximal correlation [8].

Now, Bohm in his thought experiment considers a molecule consisting of two atoms, each of spin $\hbar/2$. The total spin of the combined system is assumed to be zero. So there is an anticorrelation for the orientation of the atoms' spins with respect to a given but arbitrary direction.

Now the molecule is thought to be fragmented into the two atoms by some dissociation process, and the atoms then separated to an arbitrarily large distance (in principle, their distance can be of astronomical magnitude). 'Arbitrarily large distance' means that an interaction between the two atoms is made impossible, in complete analogy to the two particles in EPR's original thought experiment. Note that the value of the total spin of the combined system is conserved.

The argument then continues as in the EPR paper. If one takes a measurement on the z-component of the first atom's spin, then one can deduce from it the z-component of the second atom's spin due to their anticorrelation; if one finds $+\hbar/2$ for atom 1, atom 2 must have $-\hbar/2$. Now the point is that one can of course take a measurement on atom 1's spin with respect to any other direction, for example the x-direction. If one finds $+\hbar/2$ for

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[7] A squeezed state is a state with a very small uncertainty in either the position or the momentum coordinate, resulting in a very large uncertainty of the conjugated quantity (i.e., momentum and position, respectively) due to the uncertainty relation.

atom 1, atom 2's spin component in x-direction must be $-\hbar/2$. So just like EPR one can conclude that all spin orientations of atom 2 have simultaneous reality. The atom's spin orientations play the role of the conjugated quantities position and momentum. Just like those, the spins in different orientations do not commute and can therefore not be measured simultaneously, according to the laws of quantum mechanics. Thus, one can conclude the incompleteness of quantum mechanics from this version, too.

What does the mathematical description look like (for the formalism, also cf. appendix A)? Following the rules of quantum mechanics, one has the following four basic states at one's disposal when constructing the combined state of a system consisting of two spin-½-systems:

$$\begin{aligned} |\Psi_a\rangle &= |\uparrow\rangle_1 |\uparrow\rangle_2, \qquad |\Psi_b\rangle = |\downarrow\rangle_1 |\downarrow\rangle_2, \\ |\Psi_c\rangle &= |\uparrow\rangle_1 |\downarrow\rangle_2, \qquad |\Psi_d\rangle = |\downarrow\rangle_1 |\uparrow\rangle_2. \end{aligned} \tag{2.8}$$

The last two of these basic states,$|\Psi_c\rangle$ and $|\Psi_d\rangle$, describe situations in which each of the two atoms has a certain spin value in z-direction and the two spins are antiparallel. But neither of those two states corresponds to a well defined total spin value. In other words: Those states are no eigenfunctions of the total spin operator. The state with zero total spin is only obtained through an interference of $|\Psi_c\rangle$ and $|\Psi_d\rangle$ with a very specific phase relationship; this is the 'singlet-state':

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\Psi_c\rangle - |\Psi_d\rangle\right). \tag{2.9}$$

This is the state that takes over the role of EPR's state (2.1) in Bohm's version of the thought experiment. If one chose a minus sign instead of a plus sign in (2.9), one would obtain a state with total spin 1; so the phase relationship[8] between the two components is essential.

Whereas the state (2.9) corresponds to a definite total spin (of zero value), the individual spins of the atoms are undetermined. In a measurement of atom 1's spin, this state with definite total spin but undetermined individual spins turns into a state with undetermined total spin but definite individual spins. Namely, it turns into the state $|\Psi_c\rangle$ or $|\Psi_d\rangle$ (this is the wave function collapse as discussed above).

The form of the state (2.9) can be generalised to all spin orientations. For two antiparallel spins in x-direction and total spin zero, *the same* state (2.9) can be decomposed into spin eigenfunctions with respect to the x-direction:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\rightarrow\rangle_1 |\leftarrow\rangle_2 - |\leftarrow\rangle_1 |\rightarrow\rangle_2\right). \tag{2.10}$$

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[8] Because the wave functions are complex quantities, they are characterised by an amplitude and a phase (an angle). The phase relationship then is the *relative* angle between wave functions; in (2.9), this is 180 degrees due to the minus sign. With a plus sign one would have 0 degrees.

An equal expression holds for any direction; the quantum state is independent of direction.

If a measurement shows that atom 1's spin points to the right, so atom 2's spin must point to the left after following that measurement, and vice versa. The argument continues as shown above. Since you can measure atom 1's spin *either* in z- *or* in x-direction without disturbing atom 2, both spin directions must have a physical reality. Because the state is independent of direction, this argument holds for all spin directions. The description with a wave function that doesn't allow that must thus be incomplete.

Remember that in the measurement it is not possible to send signals from atom 1 to atom 2.[9] For atom 2, both state decompositions (2.9) and (2.10) yield the same reduced density matrix (cf. appendix). Whatever the direction, this matrix has always the form

$$\rho_{\text{red}} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.11}$$

For the z-direction, this reduced density matrix corresponds to an ensemble of 50% atoms with spin in the positive and 50% atoms with spin in the negative direction. The same is true for all directions. So a measurement on atom 2 cannot help to decide whether a measurement on atom 1 has taken place or not.

How did Bohm interpret EPR's thought experiment? He questions EPR's local reality criterion and states that only on a classical level there is a unique correspondence between a mathematical theory and "elements of reality". In contrast to this, all there is in quantum theory is a statistical relation between the wave function and the system (Bohm uses the term *potentiality*). Because of this purely statistical trait of nature, one just cannot talk about a precisely defined element of reality for, e.g., the position of an electron. And because EPR's assumption does not apply to quantum theory, one cannot deduce from it this theory's incompleteness. In Bohm's own words [21, p. 622]:

> [...] the present form of quantum theory implies that the world cannot be put into a one-to-one correspondence with any conceivable kind of precisely defined mathematical quantities, and that a complete theory will always require concepts that are more general than that of analysis into precisely defined elements.

It should, however, be stressed once again that for EPR it is precisely this statistical character of quantum theory that embodies its incompleteness.

Bohm's discomfort with the quantum theory led him to his own interpretation in the following year, that is known today as de Broglie-Bohm or Bohm-interpretation. In it, the election is described by a wave function and an *additional* position variable; we are going to get back to this further below.

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[9] A general proof of this can be found in d'Espagnat [47, p. 117*ff.*], where this fact is referred to as 'parameter independence'. In particular, there can be no communication with superluminal velocity.

Bohm concludes this chapter of his textbook with a short argument to prove that the concept of local reality (represented by hidden local variables) is incompatible with quantum theory. A convincing mathematical expression of this incompatibility are the Bell inequalities that shall be discussed further below (Sect. 5.2). With them, it can be decided *experimentally* if the concept of local reality is correct or not.

2.4 The Contributions of Einstein's Co-Authors

We have already briefly discussed the life journey of Einstein's co-authors up to their meeting in Princeton in the first chapter. How did their journey continue?

There are indications that Podolsky was the one who did the actual writing of the paper. Einstein was not really happy with it on a linguistic level.[10]

For example, he wrote to Schrödinger on June 19, 1935:

> For reasons of language it was written by Podolsky, after several discussions. It did not become clear what I actually intended, though; rather, the essential thing was, so to speak, smothered in learnedness. The true difficulty lies in the fact that physics is a kind of metaphysics; physics describes 'reality'. But we do not know what 'reality' is; we only know it through the physical description![11]

That Podolsky was the one responsible for language[12] can also be guessed from the missing definite article in the title of the article. In "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" there is a *the* missing after *can*. An omission that would not be atypical for a scientist of Russian origin. While Bohr in his answer chose to quote the title as it is, the American physicist Arthur E. Ruark inserted the definite article in his commentary [139]. In his biography of Einstein, Abraham Pais

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[10] Whitaker [166, p. 78] noted: "And it was Podolsky who put the argument together and wrote the account of the ideas that was published. But unfortunately in doing so, he irritated Einstein very much, because Podolsky was an expert in logic, and wrote the paper rather as an exercise in formal logic, instead of the comparatively straightforward argument that Einstein thought was possible." This is why Einstein later sought to bring his argument forward in his own terms, his contribution to the journal *Dialectica* that is reprinted in this book being an example for these attempts.

[11] "Diese ist aus Sprachgründen von Podolsky geschrieben nach vielen Diskussionen. Es ist aber nicht so gut herausgekommen, was ich eigentlich wollte; sondern die Hauptsache ist sozusagen durch Gelehrsamkeit verschüttet. Die eigentliche die Physik eine Art Metaphysik ist; Physik beschreibt 'Wirklichkeit'. Aber wir wissen nicht, was ist; wir kennen sie nur durch die physikalische Beschreibung!" (von Meyenn [158], parts of the English translation taken from Fine, A. [73]. *The Shaky Game*. Second edition. University of Chicago Press, Chicago, p. 35).

[12] It might be suspected that the only American in the trio, Rosen, was not chosen to write the article because he was judged too young and unexperienced or, what seems to be more probable, because Podolsky was too dominant a personality.

furthermore noted that Einstein would have used the term Ψ*-function* instead of *wave function* [122, p. 499].

After the EPR paper was finished, it seems Einstein had no further contact with Podolsky. This however was not so much due to the fact that the "essential thing was [. . .] smothered in learnedness", but was rather provoked by a capricious act of Podolsky's that left Einstein in anger. The *New York Times* in its Saturday issue on May 4, 1935 (ten days before the EPR paper was published in *Physical Review*!) published an article entitled "Einstein Attacks Quantum Theory", reporting to quite some extent on EPR's work.[13] The article was supplemented by a résumé of the paper, written by Podolsky. Podolsky was the one to initiate this article, without having talked about it to Einstein or Rosen. Einstein's unease with this can be read off his following statement, published in the *New York Times* on May 7, 1935 [quote taken from Jammer [98, p. 190]]:

> Any information upon which the article 'Einstein Attacks Quantum Theory' in your issue of May 4 is based was given to you without authority. It is my invariable practice to discuss scientific matters only in the appropriate forum and I deprecate advance publication of any announcement in regard to such matters in the secular press.

Boris Podolsky was appointed professor at the University of Cincinnati, United States, in 1935 and went on to the Xavier University, also in Cincinnati, in 1961. He died in 1966. His scientific work focused on generalisations of electrodynamics. His later statements on EPR's work emphasised its main point, namely the incompleteness of quantum theory, cf. especially his contributions to a conference at the Xavier University from October 1–5 in 1962 that he had helped organise and that was devoted to the foundations of quantum theory. Among the conference participants were Rosen, Dirac and Wigner. The talks and discussions contributions are available online and offer plenty of interesting thoughts [172].[14]

Nathan Rosen's contribution to the EPR paper seems to have been to propose the entangled state (2.1) and to have worked out the details of the actual calculation. In view of Rosen's preliminary experiences this should not come as a surprise. In 1931, he had published a paper on the hydrogen molecule [135], related to his dissertation at MIT, wherein he had used the wave function

$$\Psi = \Psi(a1)\Psi(b2) + \Psi(b1)\Psi(a2), \tag{2.12}$$

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[13] It says in the article: "Professor Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is 'correct' it is not 'complete'." Quote taken from Jammer [98, p. 189]

[14] Photo (Fig. 2.1) taken from: http://www.titanians.org/about-bob-podolsky/
The page also includes a photograph of Boris Podolsky.

Fig. 2.1 Group photograph taken at the conference at Xavier University in October 1962. Bottom row (from left to right): Eugene Wigner, Nathan Rosen, Paul Dirac, Boris Podolsky, Yakir Aharonov and Wendell Furry. (Photograph by Babst Photographic Services found in University Archives and Special Collections, Xavier University Library, Cincinnati, Ohio)

where a and b refer to the two nuclei and 1 and 2 refer to the two electrons (this is equation (10) of Rosen's paper).

At first sight, (2.12) looks like an entangled state similar to the state (2.1) that was later used in the EPR paper. But in fact, it is only the result of a formal symmetrisation that becomes necessary if one is working with the classical construct of ascribing numbers to particles (cf., i.e., Zeh [180, p. 10]). A true entanglement involves the entanglement of relative coordinates, in the general case (where the spin-orbit coupling cannot be neglected) also the spin. In 1931, Rosen probably had not yet recognised this difference. An entangled state for the hydrogen molecule was used by Hylleraas [94], who had already in 1929 formulated an entangled state for the helium atom, see equation (11) in Hylleraas [93].[15] Only when taking into account this entanglement, one obtains the correct energy levels for the ground state. This important remark was already made by Heisenberg in 1935:

> Furthermore, one can point to the fact that the natural character of quantum mechanics is very tightly bound to the formal circumstance that its mathematical frame-work of wave functions operates in multi-dimensional configuration space, not in ordinary space, and that

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[15] A discussion of Hylleraas' method can, e.g., be found in Sommerfeld [152, p. 677*ff.*] or in Bethe and Salpeter [20, p. 232 *ff.*].

> precisely this feature of quantum mechanics has been exactly confirmed through the correct reproduction of the more complicated atomic spectra. [16]

In contrast to Podolsky, Einstein did not cut off contact to Rosen. As a matter of fact, from 1935 on, a fruitful collaboration developed between the two on the subject of problems related to general relativity. While working on the EPR paper, they had worked on the perhaps most important publication on this subject. This article, entitled "The Particle Problem in the General Theory of Relativity" was submitted to *Physical Review* on May 8, 1935 (one week before the publication of the EPR paper) and was published on July 1 of the same year [64]. In it, the authors present what would later be called the 'Einstein-Rosen bridge' or the 'Einstein wormhole'.

What is this about? According to general relativity, the exterior geometry of a spherical mass distribution is described by a solution of the field equations that had been found by the astronomer Karl Schwarzschild in 1916 and is known as the Schwarzschild solution. In its originally adopted form this solution features an irritating trait: at a certain distance from the centre, it becomes singular and therefore meaningless.[17] While trying to eliminate this singularity, Einstein and Rosen found a solution that connects two copies of the exterior geometry by a small bridge (a 'wormhole') (see Fig. 2.2).

Because such a solution also exists in the presence of an electromagnetic field, Einstein and Rosen interpreted it as a possible model for describing elementary particles (such as protons and electrons). This would offer a way to infer the existence of matter directly from the field equations, i.e., without having to insert it into the equations by hand. Towards the end of their paper, Einstein and Rosen write:

> ...one does not see *a priori* whether the theory contains the quantum phenomena. Nevertheless one should not exclude *a priori* the possibility that the theory may contain them.

And in his letter to Schrödinger on June 7, 1935, Einstein wrote [158, vol. 2, p. 536]:

> I have found that in general relativity neutral and charged particles can be described as singularity-free fields, with no need of additional terms. From the point of view of principles, I absolutely do not believe in a statistical basis for physics in the sense of quantum mechanics,

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[16] "Ferner kann man darauf hinweisen, daß der natürliche Charakter der Quantenmechanik aufs engste mit dem formalen Umstand verknüpft ist, daß ihr mathematisches Schema von Wellenfunktionen im mehrdimensionalen Konfigurationsraum, nicht im gewöhnlichen Raum handelt, und daß eben dieser Zug der Quantenmechanik durch die korrekte Wiedergabe der komplizierteren Atomspektren eine genaue Bestätigung erfahren hat." [87, p. 418]. English in: W. Heisenberg, Elise Crull, Guido Bacciagaluppi (2011), Translation of: W. Heisenberg, 'Ist eine deterministische Ergänzung der Quantenmechanik möglich?' Available online: halshs-00996315.

[17] We are talking about the coordinate singularity occurring at the Schwarzschild radius.

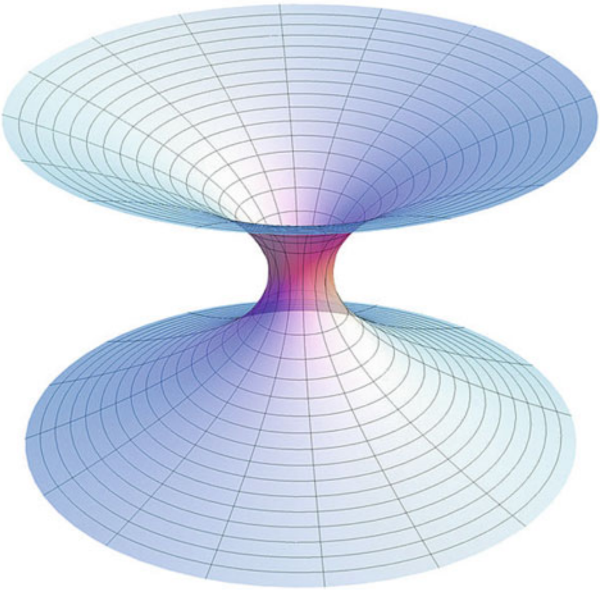

Fig. 2.2 Einstein-Rosen bridge. (Figure drawn by Allen McC.; redistribution permitted under a *Creative Commons Attribution-Share Alike 3.0 Unported* license)

> despite the singular success of the formalism of which I am well aware. I do not believe such a theory can be made general relativistic.[18]

And that is where their work is related to the EPR paper! With the help of concepts like the Einstein-Rosen bridge Einstein sought to complete the quantum theory. We will get back to this. It was found later that Einstein's and Rosen's solution from 1935 is not suited for this purpose, because it is unstable and therefore not able to describe (stable) elementary particles. Also, being a classical concept it could not explain the EPR situation.[19]

Einstein and Rosen cooperated on two further papers. One dealt with the two-body problem in general relativity [65], the other one with cylindric gravitational waves [66].

Following the year 1936, Rosen was initially appointed professor at the University of Kiev, Ukraine (back then USSR), and from 1941 on at the University of North Carolina in

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[18] "Ich habe gefunden, daß allgemein relativistisch neutrales Massenteilchen und elektrisches Teilchen sich ohne Zusatzglieder als singularitätsfreie Felder darstellen lassen. Es besteht aber eine ernst zu nehmende Möglichkeit, die Atomistik relativistisch-feldtheoretisch darzustellen, wenn es auch mathematisch überaus schwierig erscheint, zu den Mehrkörper-Problemen vorzudringen. Ich glaube vom prinzipiellen Standpunkt absolut nicht an eine statistische Basis der Physik im Sinne der Quantenmechanik, so fruchtbar sich dieser Formalismus im Einzelnen auch erweist. Ich glaube nicht, daß man eine derartige Theorie allgemein relativistisch durchführen kann." ([158, vol. 2, p. 536, parts of the English translation are taken from Fine, A. [73]. *The Shaky Game*. Second edition. University of Chicago Press, Chicago, p. 68])

[19] Still, Maldacena and Susskind [115] did speculate about a possible connection between the two.

Chapel Hill, United States. From 1953 until his death in 1995, he was a professor at the Israel Institute of Technology (Technion) in Haifa, Israel. Just like Einstein and Podolsky, throughout the rest of his career, Rosen kept emphasising the main point of the EPR paper: the incompleteness of quantum mechanics (see, e.g., Rosen [137]). One of his students was Asher Peres (1934–2005), who contributed to the foundations of quantum theory, especially to the comparatively young field of quantum information.

2.5 Critical Evaluation

In their paper, EPR concluded the incompleteness of quantum theory. We have discussed the details of this conclusion in the preceding section. Now, we shall discuss the implicit and explicit assumptions on which EPR based their conclusion and will, to this end, deviate from the historic course of events in favour of a more systematic approach. We will rejoin the chronological path of events in the following chapter on the reception and impact of the EPR paper.

First, there is the reality criterion, according to which there is an element of reality corresponding to a physical quantity, if we can predict with certainty the value of this quantity without disturbing the system, cf. the beginning of Sect. 2.2. As Beller and Fine [18] have pointed out, EPR use this criterion merely once in their paper, and only indirectly. Following their equation (1), EPR mention the undisputed quantum mechanical fact that a physical quantity A has with certainty the value a, whenever the wave function of the system is the eigenfunction with associated eigenvalue a of the operator A that corresponds to the physical quantity ('eigenfunction-eigenvalue link'). Then, EPR continue, "in accordance with our criterion of reality […] there is an element of physical reality corresponding to the physical quantity A." When EPR mention the reality criterion in part 2 of their paper, they really only talk about this eigenfunction-eigenvalue link. They do *not* show that P and Q have *simultaneous* reality in system II; in order to show this through the reality criterion, A and B would need to be *simultaneously* measurable in system I, but quantum mechanics prohibits that; and EPR do not doubt the validity of quantum mechanics. They only *do* show that system II has a reality that can be described by a momentum eigenfunction as well as by a position eigenfunction. This fact will be essential when evaluating Bohr's response in Sect. 4.2.

According to Fine [73, p. 5], Einstein never again mentioned EPR's reality criterion. It appears that – at least to Einstein – the criterion did not much matter for EPR's line of argument. As he put it in the letter to Schrödinger quoted above, for him "the essential thing was […] smothered in learnedness." However, what happens if the learnedness were pushed aside and the core of EPR's argument laid bare? What was Einstein's key assumption of their work? He repeatedly commented on this, initially in his letters to Schrödinger, later in his essay on "Physik und Realität" ("Physics and reality") [57], and in the article published in the journal *Dialectica* [58] included in this book, as well as in his contributions to the anthology edited by Paul Arthur Schilpp [59, 60] and the Born

Festschrift [61]. All these texts make it quite clear that Einstein's point was based on the locality or separability of physical systems; in EPR's case, of systems I and II. It is worth taking another, detailed look at Einstein's letter to Schrödinger on June 19, 1935 – i.e., one month after the publication of the EPR paper–, in which Einstein highlights the important points giving a simple example. Einstein wrote:

> I have in front of me two boxes, with lids that can be opened. I can look into the boxes, when the lids are open; this is called 'to make an observation'. There is also a ball that is always either in one or the other box when making an observation. I will describe the situation as follows: 'The probability to find the ball in the first box is ½.' Is this a complete description?[20]

Einstein then goes on to discuss the two possible answers. If one considers the statement that the ball is either in the first box or it is not, a complete description, then according to Einstein the answer must be *no*. But if one assumes (as most of the quantum theorists did at the time) that the ball is in neither of the boxes before the lid is opened, then according to Einstein the answer must be *yes*. The state then is completely described by the probability ½, and there is no reality beyond the statistical character of the world of experience. Einstein then establishes the analogy with quantum theory:

> We face similar alternatives when we want to explain the relation of quantum mechanics to reality. With regard to the ball-system, naturally, the second, 'spiritualist' or Schrödinger interpretation is hackneyed, and the man on the street would only take the first, 'Bornian' interpretation seriously. But the Talmudic philosopher dismisses 'reality' as a bogy of naiveté and declares the two conceptions differing only in their terminology.[21]

Now, Einstein lets the cat out of the bag and presents his separation principle, from which in the EPR paper followed – though smothered in learnedness – the incompleteness of quantum theory:

> Now *my* mode of thought is thus: One cannot get by the Talmudic philosopher without the help of an additional principle: the 'separation principle'. Namely: 'The second box including

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[20] "Vor mir stehen zwei Schachteln mit aufklappbarem Deckel, in die ich hineinsehen kann, wenn sie aufgeklappt werden; letzteres heißt "eine Beobachtung machen". Es ist außerdem eine Kugel da, die immer in der einen oder anderen Schachtel vorgefunden wird, wenn man eine Beobachtung macht. Nun beschreibe ich einen Zustand so: *Die Wahrscheinlichkeit dafür, daß die Kugel in der ersten Schachtel ist, ist 1/2.* Ist dies eine vollständige Beschreibung?" ([158], volume 2, p. 537, English translation by S. Linden and A. K. Hudert)

[21] "Vor der analogen Alternative stehen wir, wenn wir die Beziehung der Wirklichkeit deuten wollen. Bei dem Kugel-System ist natürlich die zweite, 'piritistische' oder Schrödingersche Interpretation sozusagen abgeschmackt und nur die erste 'Bornsche' würde der Bürger ernst nehmen. Der talmudische Philosoph aber pfeift auf die 'Wirklichkeit' als auf einen Popanz der Naivität und erklärt beide nur der Ausdrucksweise nach verschieden." (ibid., English translation following Fine, A. [73]. *The Shaky Game*. Second edition. University of Chicago Press, Chicago, p. 69)

> everything concerning its content is independent of what is happening with the second box' (separated subsystems). If we hold fast to this separation principle, then the second (the 'Schrödinger') interpretation is excluded and only the Bornian one remains according to which, however, the above given description of the state is an *incomplete* description of *reality* or of the real states, respectively.[22]

Einstein then, by using the mathematical formalism and explicitly applying his separation principle, once again summarises the essential line of thought of the EPR paper.

So the locality or separability criterion is of pivotal importance to Einstein; when you dispose of it, says Einstein, you lose the basis for a reasonable description of nature. In his later essays on the topic, he elevated the thoughts he had formulated in the letter to Schrödinger on a philosophical level of general significance. For example, in his *Dialectica* contribution [58, p. 321] he writes:

> Without assuming such an independence of the existence (of the 'being-thus') of spatially distant objects, which first originates from everyday thinking, physical reasoning would not be possible in the manner familiar to us.[23]

From this, Einstein concludes the incompleteness of quantum theory because different wave functions can be attributed to the same local reality. Assuming the completeness would correspond to implying "the hypothesis of action-at-a-distance, an hypothesis which is hardly acceptable"[24], which for Einstein is incompatible with the theory of relativity. Only later, in connection with the formulation of the Bell inequalities some years after Einstein's death, it became completely clear that the assumption of local reality not only contradicts the completeness of quantum theory, but more than that also contradicts its consistency and feasible experiments. It seems pointless to retrospectively try to imagine Einstein's reaction to this finding, had he still lived.

It also follows from Einstein's general remarks that it is indeed sufficient to consider only one variable (e.g., the position coordinate). To consider two non-commuting and therefore not simultaneously measurable variables sharpens the argument, but is rooted mostly in the historical tide of events, namely that it was preceded by the discussion about

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[22] "*Meine* Denkweise ist nun so: An sich kann man dem Talmudiker nicht beikommen, wenn man kein zusätzliches Prinzip zu Hilfe nimmt: Nämlich: 'die zweite Schachtel nebst allem, was ihren Inhalt betrifft, ist unabhängig davon, was bezüglich der ersten Schachtel passiert' (getrennte Teilsysteme). Hält man an dem Trennungsprinzip fest, so schließt man dadurch die zweite ('Schrödingersche') Auffassung aus und es bleibt nur die Bornsche, nach welcher aber die obige Beschreibung des Zustands eine *unvollständige* Beschreibung der *Wirklichkeit*, bzw. der wirklichen Zustände ist. " (ibid., English translation by S. Linden and A. K. Hudert)

[23] "Ohne die Annahme einer solchen Unabhängigkeit der Existenz (des 'So-Seins') der räumlich distanten Dinge voneinander, die zunächst dem Alltags-Denken entstammt, wäre physikalisches Denken in dem uns geläufigen Sinne nicht möglich."

[24] "[. . .] die Hypothese einer schwer annehmbaren Fernwirkung" [58, p. 323].

the uncertainty relations. When we consider an entangled wave function $\Psi(x_1, x_2)$, we can conclude as follows. Because of the entanglement, x_1 and x_2 do not have a reality on their own. Only when the state is reduced to a product by taking a measurement on, e.g., x_1, and when one also applies Einstein's separation principle, a reality can be ascribed to x_2. Edward Teller (1908–2003) was one to very early highlight this fact, as we can see in a letter he wrote to Schrödinger in June 1935 [158, p. 530]. For this reason, Teller did not even want to speak of a reality.

Einstein expressed his 'epistemological credo' in his contributions to Schilpp's Festschrift. Therein, he wrote [59, p. 31]:

> Physics is an attempt conceptually to grasp reality as it is thought independently of its being observed. In this sense one speaks of 'physical reality'.[25]

In his remarks to the essays appearing in this collective volume, Einstein pointed out [60, p. 236]:

> 'Being' is always something which is mentally constructed by us, that is, something which we freely posit (in the logical sense).[26]

Here, Einstein essentially turned against classical positivism and its motto *esse est percipi* ('to be is to be perceived'), according to which being results from being perceived. The thinking conveyed in these quotes differs remarkably from the first part of the EPR paper. Whereas in that paper, reality is an objective quantity and the terms and definitions one applies serve the only purpose of corresponding to the elements of this reality, Einstein now promotes the freedom of choice when selecting the terms and definition in an attempt to approach the reality that is somewhere beyond its being observed. He wrote [59, p. 12]:

> Although the conceptual systems are logically entirely arbitrary, they are bound by the aim to permit the most nearly possible certain (intuitive) and complete co-ordination with the totality of sense-experiences; [. . .][27]

A whole lot of these remarks resemble the *Philosophical Investigations* ('Philosophische Untersuchungen'), published years later, in 1953, by the philosopher Ludwig Wittgenstein (1889–1951, cf. Wittgenstein [171]) who to some extent was influenced by

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[25] "Die Physik ist eine Bemühung, das Seiende als etwas begrifflich zu erfassen, was unabhängig vom Wahrgenommen-Werden gedacht wird. In diesem Sinne spricht man vom 'Physikalisch-Realen'." (English translation by Schilpp [141, p. 81])

[26] "Das 'Sein' ist immer etwas von uns gedanklich Konstruiertes, also von uns (im logischen Sinne) frei Gesetztes." (English translation by Schilpp [141, p. 669])

[27] "Die Begriffssysteme sind zwar an sich logisch gänzlich willkürlich, aber gebunden durch das Ziel, eine möglichst sichere (intuitive) und vollständige Zuordnung zu der Gesamtheit der Sinneserlebnisse zuzulassen; [. . .]" (English translation by Schilpp [141, p. 13])

the Vienna Circle chaired by Moritz Schlick. In some ways, Einstein's notion of a free posit of terms and definitions corresponds to Wittgenstein's notion of language-games. The main difference between the two is of course that the terms and definitions one chooses in physics are subject to empirical testing.

Since Einstein was convinced to have discovered the incompleteness of the quantum theory, he tried to complete it. But he did not attempt a completion from within: "I believe, however, that this theory offers no useful point of departure for future development."[28]

Instead, Einstein saw remedy in his search for a unified field theory. Since the 1920s, Einstein had tried to unify gravitation and electromagnetism following the example of his general relativity. He expected particles – that is, the behaviour of atoms and electrons, usually the realm of quantum theory – to turn out to be singularity-free solutions of his fundamental field equations. The above mentioned paper he wrote with Rosen was supposed to be the first brick of this beautiful building: "Look at the small paper on a possible relativistic interpretation of matter that I recently published with Mr. Rosen in the Physical Review. If the mathematical difficulties can be overcome, this might lead to something."[29]

In his contribution to the de Broglie Festschrift, Einstein once again made the point:

> We partly owe my efforts to complete the theory of general relativity by generalising the gravitational equations to the surmise that a reasonable general relativistic field theory might offer the key to complete the quantum theory. This is but a modest hope, not a conviction.[30]

We will discuss further below why ultimately Einstein's hope was in vain.

Quite early, Einstein felt appalled by the statistical character of quantum theory. There is a passage in a letter he wrote to Born on December 4, 1926 (i.e., ten years before the EPR paper), that has gained fame:

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[28] "Ich glaube aber, daß diese Theorie keinen brauchbaren Ausgangspunkt für die künftige Entwicklung bietet." ([59, English translation by S. Linden and A. K. Hudert])

[29] "Schau Dir die kleine Arbeit an, die ich mit Herrn Rosen in der Physikalischen Review jüngst über eine denkbare relativistische Deutung der Materie publiziert habe. Dies könnte zu etwas führen, wenn sich die mathematischen Schwierigkeiten überwinden lassen." Einstein in a letter to Schrödinger on August 8, 1935, see von Meyenn [158, vol. 2, p. 562].

[30] "Meine Bemühungen, die allgemeine Relativitätstheorie durch Verallgemeinerung der Gravitationsgleichungen zu vervollständigen, verdanken ihre Entstehung zum Teil der Vermutung, daß eine vernünftige allgemein relativistische Feldtheorie vielleicht den Schlüssel zu einer vollkommeneren Quantentheorie liefern könne. Dies ist eine bescheidene Hoffnung, aber durchaus keine Überzeugung." ([62, p. 17]), English translation by S. Linden and A. K. Hudert

> Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one'. I, at any rate, am convinced that *He* is not playing at dice.[31]

As early as in May 1927, i.e., one month after having studied Heisenberg's paper on the uncertainty relations, Einstein gave a talk at the Prussian Academy of Sciences in Berlin where he discussed the question whether Schrödinger's wave mechanics describes the dynamics of a system completely or only in a statistical sense. Einstein withdrew the submitted manuscript before publication, but today it is available online [56].[32] In it, Einstein introduces an interpretation that shows some similarities to de Broglie's theory of pilot waves. But what is important for our purpose here is the fact that, already in 1927, Einstein was bothered by the question of the completeness of the quantum theory; also cf. Brown [32]. The EPR paper and his later writings clearly show that Einstein had accepted the statistical nature of quantum theory. But to him, it was only an expression of the theory's incompleteness. Determinism is not a presupposition of EPR's analysis (see the quote by John Bell towards the end of Sect. 5.2).

In his contribution to the Born Festschrift, Einstein later highlighted the importance of the classical limit [61]. He takes as an example a wave function that in the macroscopic realm corresponds to a superposition of macroscopically different momenta. So it does not describe a classical behaviour. (Schrödinger got to the heart of this situation in his famous cat argument, see further below.) As a consequence, according to Einstein, quantum mechanics can only tell the probability of finding a certain macroscopic state when measuring. This is what he wrote:

> The result of our consideration is this: The only acceptable interpretation of the Schrödinger equation is the statistical interpretation given by Born. However, this does not provide a real description of an individual system, but only statistical statements about ensembles of systems.[33]

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[31] "Die Quantenmechanik ist sehr achtung-gebietend. Aber eine innere Stimme sagt mir, daß das doch nicht der wahre Jakob ist. Die Theorie liefert viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, daß *der* nicht würfelt." ([69, English translation taken from Born, M. (ed.), Born, I. (trans.) (1971) *The Born-Einstein Letters.* New York: Walker and Company, p. 90])

[32] This is what Schrödinger referred to when he wrote to Einstein on July 13, 1935, i.e., a few days after the publication of the EPR paper: "True, we discussed those things much and heatedly in the seminars after you brought them up in Berlin years ago." ("Wir haben ja die Dinge, nachdem Du schon vor Jahren in Berlin darauf hingewiesen hattest, in den Seminaren viel und mit heißen Köpfen diskutiert.", [158], p. 551, English translation by S. Linden and A. K. Hudert)

[33] "Das Ergebnis unserer Betrachtung ist dieses. Die einzige bisherige annehmbare Interpretation der Schrödinger-Gleichung ist die von Born gegebene statistische Interpretation. Diese liefert jedoch keine Realbeschreibung für das Einzelsystem, sondern nur statistische Aussagen über System-Gesamtheiten." ([61, p. 40]), English translation by S. Linden and A. K. Hudert

It still took a long time to truly understand how to obtain the classical limit from quantum theory (see Sect. 5.4). Macroscopic wave functions necessarily become entangled with their environment's degrees of freedom, yielding a combined state that emulates classical behaviour for the variables of the macroscopic system. So Einstein's argument from 1953 is ineffective, because it assumes a strictly isolated state.

For the rest of his life, Einstein considered the quantum theory to be a statistical description of nature, similar to the statistical mechanics of the nineteenth century, that was to be replaced by a microscopic theory without such a fundamentally probabilistic character. Amongst other things, it was this belief that motivated his search for a unified field theory based on classical principles – an endeavour that was ultimately in vain.

of lanthanum is 7/2, hence the nuclear magnetic moment as determined by this analysis is 2.5 nuclear magnetons. This is in fair agreement with the value 2.8 nuclear magnetons determined from La III hyperfine structures by the writer and N. S. Grace.[9]

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[9] M. F. Crawford and N. S. Grace, Phys. Rev. 47, 536 (1935).

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

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In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one such way, whenever the conditions set down in it occur. Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.

To illustrate the ideas involved let us consider the quantum-mechanical description of the behavior of a particle having a single degree of freedom. The fundamental concept of the theory is the concept of *state*, which is supposed to be completely characterized by the wave function ψ, which is a function of the variables chosen to describe the particle's behavior. Corresponding to each physically observable quantity A there is an operator, which may be designated by the same letter.

If ψ is an eigenfunction of the operator A, that is, if

$$\psi' \equiv A\psi = a\psi, \tag{1}$$

where a is a number, then the physical quantity A has with certainty the value a whenever the particle is in the state given by ψ. In accordance with our criterion of reality, for a particle in the state given by ψ for which Eq. (1) holds, there is an element of physical reality corresponding to the physical quantity A. Let, for example,

$$\psi = e^{(2\pi i/h)p_0 x}, \tag{2}$$

where h is Planck's constant, p_0 is some constant number, and x the independent variable. Since the operator corresponding to the momentum of the particle is

$$p = (h/2\pi i)\partial/\partial x, \tag{3}$$

we obtain

$$\psi' = p\psi = (h/2\pi i)\partial\psi/\partial x = p_0\psi. \tag{4}$$

Thus, in the state given by Eq. (2), the momentum has certainly the value p_0. It thus has meaning to say that the momentum of the particle in the state given by Eq. (2) is real.

On the other hand if Eq. (1) does not hold, we can no longer speak of the physical quantity A having a particular value. This is the case, for example, with the coordinate of the particle. The operator corresponding to it, say q, is the operator of multiplication by the independent variable. Thus,

$$q\psi = x\psi \neq a\psi. \tag{5}$$

In accordance with quantum mechanics we can only say that the relative probability that a measurement of the coordinate will give a result lying between a and b is

$$P(a, b) = \int_a^b \bar{\psi}\psi dx = \int_a^b dx = b - a. \tag{6}$$

Since this probability is independent of a, but depends only upon the difference $b-a$, we see that all values of the coordinate are equally probable.

A definite value of the coordinate, for a particle in the state given by Eq. (2), is thus not predictable, but may be obtained only by a direct measurement. Such a measurement however disturbs the particle and thus alters its state. After the coordinate is determined, the particle will no longer be in the state given by Eq. (2). The usual conclusion from this in quantum mechanics is that *when the momentum of a particle is known, its coordinate has no physical reality.*

More generally, it is shown in quantum mechanics that, if the operators corresponding to two physical quantities, say A and B, do not commute, that is, if $AB \neq BA$, then the precise knowledge of one of them precludes such a knowledge of the other. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first.

From this follows that either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.* For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness. If then the wave function provided such a complete description of reality, it would contain these values; these would then be predictable. This not being the case, we are left with the alternatives stated.

In quantum mechanics it is usually assumed that the wave function *does* contain a complete description of the physical reality of the system in the state to which it corresponds. At first sight this assumption is entirely reasonable, for the information obtainable from a wave function seems to correspond exactly to what can be measured without altering the state of the system. We shall show, however, that this assumption, together with the criterion of reality given above, leads to a contradiction.

2.

For this purpose let us suppose that we have two systems, I and II, which we permit to interact from the time $t=0$ to $t=T$, after which time we suppose that there is no longer any interaction between the two parts. We suppose further that the states of the two systems before $t=0$ were known. We can then calculate with the help of Schrödinger's equation the state of the combined system I+II at any subsequent time; in particular, for any $t>T$. Let us designate the corresponding wave function by Ψ. We cannot, however, calculate the state in which either one of the two systems is left after the interaction. This, according to quantum mechanics, can be done only with the help of further measurements, by a process known as the *reduction of the wave packet.* Let us consider the essentials of this process.

Let $a_1, a_2, a_3, \cdots$ be the eigenvalues of some physical quantity A pertaining to system I and $u_1(x_1), u_2(x_1), u_3(x_1), \cdots$ the corresponding eigenfunctions, where x_1 stands for the variables used to describe the first system. Then Ψ, considered as a function of x_1, can be expressed as

$$\Psi(x_1, x_2)=\sum_{n=1}^{\infty} \psi_n(x_2)u_n(x_1), \qquad (7)$$

where x_2 stands for the variables used to describe the second system. Here $\psi_n(x_2)$ are to be regarded merely as the coefficients of the expansion of Ψ into a series of orthogonal functions $u_n(x_1)$. Suppose now that the quantity A is measured and it is found that it has the value a_k. It is then concluded that after the measurement the first system is left in the state given by the wave function $u_k(x_1)$, and that the second system is left in the state given by the wave function $\psi_k(x_2)$. This is the process of reduction of the wave packet; the wave packet given by the infinite series (7) is reduced to a single term $\psi_k(x_2)u_k(x_1)$.

The set of functions $u_n(x_1)$ is determined by the choice of the physical quantity A. If, instead of this, we had chosen another quantity, say B, having the eigenvalues $b_1, b_2, b_3, \cdots$ and eigenfunctions $v_1(x_1), v_2(x_1), v_3(x_1), \cdots$ we should have obtained, instead of Eq. (7), the expansion

$$\Psi(x_1, x_2)=\sum_{s=1}^{\infty} \varphi_s(x_2)v_s(x_1), \qquad (8)$$

where φ_s's are the new coefficients. If now the quantity B is measured and is found to have the value b_r, we conclude that after the measurement the first system is left in the state given by $v_r(x_1)$ and the second system is left in the state given by $\varphi_r(x_2)$.

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, *it is possible to assign two different wave functions* (in our example ψ_k and φ_r) *to the same reality* (the second system after the interaction with the first).

Now, it may happen that the two wave functions, ψ_k and φ_r, are eigenfunctions of two noncommuting operators corresponding to some physical quantities P and Q, respectively. That this may actually be the case can best be shown by an example. Let us suppose that the two systems are two particles, and that

$$\Psi(x_1, x_2)=\int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1-x_2+x_0)p}dp, \qquad (9)$$

where x_0 is some constant. Let A be the momentum of the first particle; then, as we have seen in Eq. (4), its eigenfunctions will be

$$u_p(x_1)=e^{(2\pi i/h)px_1} \qquad (10)$$

corresponding to the eigenvalue p. Since we have here the case of a continuous spectrum, Eq. (7) will now be written

$$\Psi(x_1, x_2)=\int_{-\infty}^{\infty}\psi_p(x_2)u_p(x_1)dp, \tag{11}$$

where

$$\psi_p(x_2)=e^{-(2\pi i/h)(x_2-x_0)p}. \tag{12}$$

This ψ_p however is the eigenfunction of the operator

$$P=(h/2\pi i)\partial/\partial x_2, \tag{13}$$

corresponding to the eigenvalue $-p$ of the momentum of the second particle. On the other hand, if B is the coordinate of the first particle, it has for eigenfunctions

$$v_x(x_1)=\delta(x_1-x), \tag{14}$$

corresponding to the eigenvalue x, where $\delta(x_1-x)$ is the well-known Dirac delta-function. Eq. (8) in this case becomes

$$\Psi(x_1, x_2)=\int_{-\infty}^{\infty}\varphi_x(x_2)v_x(x_1)dx, \tag{15}$$

where

$$\varphi_x(x_2)=\int_{-\infty}^{\infty}e^{(2\pi i/h)(x-x_2+x_0)p}dp$$

$$=h\delta(x-x_2+x_0). \tag{16}$$

This φ_x, however, is the eigenfunction of the operator

$$Q=x_2 \tag{17}$$

corresponding to the eigenvalue $x+x_0$ of the coordinate of the second particle. Since

$$PQ-QP=h/2\pi i, \tag{18}$$

we have shown that it is in general possible for ψ_k and φ_r to be eigenfunctions of two noncommuting operators, corresponding to physical quantities.

Returning now to the general case contemplated in Eqs. (7) and (8), we assume that ψ_k and φ_r are indeed eigenfunctions of some noncommuting operators P and Q, corresponding to the eigenvalues p_k and q_r, respectively. Thus, by measuring either A or B we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P (that is p_k) or the value of the quantity Q (that is q_r). In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality. But, as we have seen, both wave functions ψ_k and φ_r belong to the same reality.

Previously we proved that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted.* On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.