Chapter 4
Optimal Forecast Models for Clean Energy Stock Returns

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Abstract This chapter searches for optimal models for forecasting the Wilder Hill Clean Energy Index (ECO), the Standard and Poor's Global Clean Energy Index (SPCLE), the MAC Global Solar Energy Index (SUN), and the European Renewable Energy Index (EURIX). These indices measure the stock market performance of renewable energy companies. We employ fat-tailed distributed models, and we analyze their in-sample and out-of-sample performance for the returns and the 1%-Value-at-Risk (VaR) of renewable energy indices. Heavy-tailed distributed GARCH and GAS are optimal for all renewable energy returns. They also have the lowest out-of-sample mean-squared error and the best coverage for 1%-VaR of renewable energy returns. These findings highlight the relevance of modeling the kurtosis for renewable energy returns, and they are relevant for policymakers and investors who invest in the renewable energy sector.

Keywords Clean energy · GARCH · GAS · Heavy-tailed distributions

JEL Classifications C22 · C52 · G11 · Q42

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4.1 Introduction

Renewable energy assets have gained consideration among investors in recent years. Several studies document a growing demand for renewable energies and an increment in clean energy investments in emerging and developed economies (Kaldellis and Zafirakis 2011; Teske et al. 2011; IRENA 2017; REN21 2017). The increasing interest in clean energy equities by investors may be explained by the growth prospects of the sector, which in turn are based on three reasons. First, there is an increasing concern about the natural environment and decarbonizing the energy system, reflected in the Kyoto protocol and the Paris agreement (Schellnhuber et al. 2016; Klein et al. 2017; Grubb et al. 2018). Moreover, there is a need for mitigating energy security issues such as political instability in oil supplying countries or unexpected increments in the oil demand (Lieb-Dóczy et al. 2003; Ang et al. 2015; Bondia et al. 2016). According to the International Energy Agency, energy security is defined as "the uninterrupted availability of energy sources at an affordable price." Finally, it is important to invest in technological innovation and developing new energy storage technologies to attain a clean energy system (Sagar and van der Zwaan 2006; Wilson and Grubler 2011; Kittner et al. 2017).

Clean energy indices, such as wind and solar energy, are sold in financial markets that share the same dynamics of highly volatile assets. Besides, clean energy returns may exhibit heavy-tailed distributions since financial returns follow fat-tailed distributions (Gabaix 2009). It is important to model the volatility of renewable energy returns for investors since it affects the performance of their portfolios on renewable energy. Many research papers employed generalized auto-regressive conditional heteroskedasticity (GARCH) models of Bollerslev (1986) to model the volatility of renewable energy data (Henriques and Sadorsky 2008; Kumar et al. 2012; Sadorsky 2012; Wang and Wu 2012; Managi and Okimoto 2013; Ahmad et al. 2018; Kocaarslan and Soytas 2019). Nevertheless, to the best of our knowledge, no study has applied generalized auto-regressive score (GAS) models to model renewable energy returns. GAS models are flexible models that are robust to misspecifications of the conditional density (Creal et al. 2013; Harvey 2013).

This chapter contributes to the literature on clean energy returns as follows. First, it searches for optimal models for forecasting the Wilder Hill Clean Energy Index (ECO), the Standard and Poor's Global Clean Energy Index (SPCLE), the MAC Global Solar Energy Index (SUN), and the European Renewable Energy Index (EURIX). These indices measure the stock market performance of clean energy companies in the world and Europe. This chapter employs 37 flexible and fat-tailed GAS and GARCH models for modeling clean energy returns. No previous study has used GAS models to forecast clean energy returns and risk. Besides, it compares the out-of-sample performance of all models to find the optimal forecast model for clean energy returns. Finally, this chapter performs several backtesting approaches for daily 1%-Value-at-Risk (VaR) forecasts of clean energy returns. It is important to measure correctly VaR to fulfill market risk capital reserves of the Basel Agreements.

Our findings suggest that heavy-tailed distributed GARCH and GAS models are optimal for all renewable energy returns considered. They also have the best out-of-sample forecast performance and the best coverage for 1%-VaR of renewable energy returns. Therefore, fat-tailed distributed models enhance both in-sample and out-of-sample performance of renewable energy returns and risk. These findings illustrate the relevance of modeling the kurtosis for renewable energy returns.

The rest of the chapter proceeds as follows. Section 4.2 outlines the data and the methodology. Section 4.3 presents the empirical results and discussion. Finally, Sect. 4.4 concludes.

4.2 Data and Econometric Methodology

We employ data on 1458 daily observations of the Wilder Hill Clean Energy Index (ECO), the Standard and Poor's Global Clean Energy Index (SPCLE), the MAC Global Solar Energy Index (SUN), and the European Renewable Energy Index (EURIX). The ECO index is based on US stocks that operate in the promotion and preservation of clean energy. The SPCLE index comprises 30 companies from around the world operating in clean energy-related businesses. The MAC index is made up of US companies involved in the solar energy industry, and the EURIX is built on the largest European renewable energy companies. Our sample period spans from November 14, 2013, to September 18, 2019. We selected this period because of the data availability. We obtained all series from DataStream. Given the closing price P_t of a renewable index at time t, we calculate its logarithm returns as $r_t = 100 \times \ln(P_t/P_{t-1})$.

Table 4.1 reports descriptive statistics (in percentages) for the daily returns on the renewable energy indices. The clean energy indices are nonstationary at the 5% level, whereas the log-returns are stationary. Besides, all returns are non-normally distributed and volatile, with a standard deviation greater than the mean. All returns are negatively skewed, illustrating the usefulness of heavy-tailed distributed models for the conditional volatility of clean energy returns.

We estimate AR(1)-GARCH(1,1) models of Bollerslev (1986) for the daily returns on renewable energy indices as:

$$r_t = c + \varphi_1 r_{t-1} + u_t, \;\; u_t = \sigma_t \varepsilon_t, \tag{4.1}$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{4.2}$$

where $|\varphi_1| < 1$ and ε_t follow a white noise process. Let $z_{t-1} = u_{t-1}\sigma_{t-1}^{-1}$ and $1(\cdot)$ be an indicator function. We model different GARCH specifications for the conditional volatility of u_t:

Table 4.1 Summary statistics: Clean energy returns (in %)

	ECO	SPCLE	SUN	ERIX
Mean	0.01	0.01	-0.03	0.04
Median	0.10	0.03	0.02	0.08
Minimum	−6.46	−5.10	−8.78	−8.05
Maximum	6.34	4.94	8.19	5.72
Standard dev.	1.46	1.10	1.74	1.41
Skewness	−0.24	−0.21	−0.30	−0.38
Kurtosis	1.06	1.80	1.95	2.63
Jarque–Bera	0.00	0.00	0.00	0.00
ADF level	−1.35	−1.69	−1.18	−3.08
ADF first diff.	**−25.21**	**−24.86**	**−23.77**	**−26.26**

Notes: We report summary statistics (in %) for the daily returns on the closing price of the Wilder Hill Clean Energy Index (ECO), the Standard and Poor's Global Clean Energy Index (SPCLE), the MAC Global Solar Energy Index (SUN), and the European Renewable Energy Index (EURIX). Our sample period spans from November 14, 2013, to September 18, 2019. Jarque–Bera is the p-value of the normality test of Jarque and Bera (1980), where the returns are normally distributed under H_0. ADF level and first diff. are the augmented unit root test of Dickey and Fuller (1979) on the level and on the log-returns of the renewable energy indices, respectively. Boldface values of the ADF test statistic indicate the rejection of the null hypothesis of a unit root at the 5% significance level

$$\text{ALL-GARCH}(1,1): \sigma_t^\delta = \omega + \alpha\delta\sigma_{t-1}^\delta\left[|z_{t-1} - b| - \gamma(z_{t-1} - b)\right]^\delta + \beta\sigma_{t-1}^\delta, \tag{4.3}$$

$$\text{APARCH}(1,1): \ \sigma_t^\delta = \omega + \alpha(|u_{t-1}| - \gamma u_{t-1})^\delta + \beta\sigma_{t-1}^\delta, \tag{4.4.}$$

$$\begin{aligned}\text{CGARCH}(1,1): \sigma_t^2 &= \xi_t + \alpha\left(u_{t-1}^2 - \xi_{t-1}\right) + \beta\left(\sigma_{t-1}^2 - \xi_{t-1}\right),\\ \xi_t &= \omega + \rho\xi_{t-1} + \eta\left(u_{t-1}^2 - \sigma_{t-1}^2\right),\end{aligned} \tag{4.5}$$

$$\text{E-GARCH}(1,1): \ \log\left(\sigma_t^2\right) = \omega + \left[\alpha z_{t-1} + \gamma(|z_{t-1}| - E|z_{t-1}|)\right] + \beta\log\left(\sigma_{t-1}^2\right), \tag{4.6}$$

$$\text{GJRGARCH}(1,1): \ \sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 1(u_{t-1} < 0) + \beta\sigma_{t-1}^2, \tag{4.7}$$

$$\text{I-GARCH}(1,1): \ \sigma_t^2 = \omega + \sigma_{t-1}^2 + \alpha\left(u_{t-1}^2 - \sigma_{t-1}^2\right), 0 < \alpha \leq 1, \tag{4.8}$$

$$\text{NGARCH}(1,1): \ \sigma_t^\delta = \omega + \alpha|u_{t-1}|^\delta + \beta\sigma_{t-1}^\delta, \tag{4.9}$$

$$\text{T-GARCH}(1,1): \ \sigma_t = \omega + \alpha|u_{t-1}| + \gamma|u_{t-1}|1(u_{t-1} < 0) + \beta\sigma_{t-1}, \tag{4.10}$$

where ξ_t is a permanent component of σ_t^2. Equation (4.3) displays the ALL-GARCH model proposed by Hentschel (1995) that encompasses the most important GARCH models. Equation (4.4) describes the Asymmetric Power ARCH (APARCH) model of Ding et al. (1993) that considers long memory for absolute returns, and it estimates the power of heteroscedasticity (δ) from the data. Equation (4.5) displays the Component GARCH (CGARCH) model of Engle and Lee (1999) that decomposes the conditional variance of the returns into a permanent and a short-run component. Equation (4.6) shows the Exponential GARCH (E-GARCH) model of Nelson (1991) that specifies an asymmetric impact of negative shocks to σ_t^2. Equation (4.7) presents the Glosten–Jagannathan–Runkle (GJRGARCH) model of Glosten et al. (1993) that represents asymmetric shocks to σ_t^2 by applying an indicator function to negative shocks.

Equation (4.8) illustrates the integrated GARCH (I-GARCH) model of Engle and Bollerslev (1986) that assumes persistency in GARCH models. Equation (4.9) shows the nonlinear GARCH (NGARCH) model of Higgins and Bera (1992) that estimates the power of heteroscedasticity (δ) from the data. Finally, Eq. (4.10) displays the threshold GARCH (T-GARCH) model of Zakoian (1994), in which σ_t (instead of σ_t^2) reacts differently to negative and positive shocks. For each model in Eqs. (4.2)–(4.10), the innovations ε_t follow Gaussian (N), t-Student (t), or skewed t-Student (St) distributions.

We employ GAS models based on time-varying parameters, which are flexible and avoid the problem of incorrect specification. We define the conditional distribution of the returns at time t as $P(r_t; \boldsymbol{\theta}_t)$, given a time-varying vector of parameters $\theta_t \in \Theta \subseteq \mathbb{R}^N$ that fully characterizes $P(\cdot;\cdot)$ as follows:

$$\boldsymbol{\theta}_{t+1} = \mathrm{A}_0 + \mathrm{A}_1 \boldsymbol{S}_t(\boldsymbol{\theta}_t) \frac{\partial \log P(r_t; \boldsymbol{\theta}_t)}{\partial \boldsymbol{\theta}_t} + \mathrm{A}_2 \boldsymbol{\theta}_t, \tag{4.11}$$

where A_0, A_1,and A_2 are matrices of coefficients, and $\boldsymbol{S}_t(\boldsymbol{\theta}_t)$ is a positive-definite scaling matrix. Following Creal et al. (2013), we specify $\boldsymbol{S}_t(\boldsymbol{\theta}_t)$ as

$$\boldsymbol{S}_t(\boldsymbol{\theta}_t) = \boldsymbol{I}, \tag{4.12}$$

$$\boldsymbol{S}_t(\boldsymbol{\theta}_t) = E_{t-1}\left[\frac{\partial \log P(r_t; \boldsymbol{\theta}_t)}{\partial \boldsymbol{\theta}_t} \frac{\partial \log P(r_t; \boldsymbol{\theta}_t)}{\partial \boldsymbol{\theta}_t}'\right]^{-\frac{1}{2}}, \tag{4.13}$$

where $\boldsymbol{I}$ is the identity matrix. We denote the specifications for $\boldsymbol{S}_t(\boldsymbol{\theta}_t)$ of Eqs. (4.12) and (4.13) as Identity (Id) and Inverse Squared (InvSq), respectively. We employ the following conditional distributions for calculating the score function: asymmetric t-Student with a left-tail (AST1) or two decay parameters (AST), Gaussian, t-Student, and skewed-t-Student.

We estimate all GARCH and GAS models for all renewable energy returns, and we evaluate their Akaike information criterion (AIC), Bayesian information criterion (BIC), and the maximum value of the log-likelihood (LogLik) function. We test for serial correlation on the GARCH residuals by applying the Ljung–Box test on the standardized squared residuals. To test for correct specification of GAS models, we employ the probability integral transform (PIT) test proposed by Diebold et al. (1998) on the estimated conditional distribution of GAS models. We also run 500 one-step-ahead rolling out-of-sample forecasts to analyze the out-of-sample performance of all models. The out-of-sample period spans from September 22, 2017, to September 18, 2019. We compare the root-mean-squared error (RMSE) of the out-of-sample forecasts of all models.

Further, we apply backtests on 1%-Value-at-Risk (VaR) forecasts for each renewable energy index return. We employ the conditional coverage (CC) test of Christoffersen (1998) on the conditional density of the returns $f(r_t|r_{t-1}, r_{t-2}, \ldots, r_1)$ and the dynamic quantile (DQ) test of Engle and Manganelli (2004). We apply the quantile loss measure developed by González-Rivera et al. (2004) to evaluate 1%-VaR forecasts as follows:

$$QL_{t+1}(1\%) = (1\% \ - e_{t+1})(r_{t+1} - \text{VaR}_{t+1}(1\%)),$$

where $e_{t+1} = 1(r_{t+1} < \text{VaR}_{t+1}(1\%))$ is a VaR exceedance for a $\text{VaR}_{t+1}(1\%)$ forecast at $t + 1$. We also calculate the ratio between VaR exceedances and the expected values a priori, the Actual over Expected ratio (AE), $AE = \sum_j^{500} e_{t+j}/(1\% \times 500)$. VaR forecasts with an AE ratio equal to one are optimal. In addition, we compare the mean and maximum Absolute Deviation (ADmean and ADmax) of the 1%-VaR forecasts among all models, which deliver the expected loss given a VaR exceedance (McAleer and Da Veiga 2008). VaR forecasts with lower ADmean and ADmax are preferred.

4.3 Empirical Analysis

Tables 4.2 and 4.3 report the estimation results of the GARCH models of Eqs. (4.2)–(4.10) and the GAS models of Eqs. (4.11)–(4.13) for ECO returns. The Ljung–Box test shows that all GARCH residuals are serially uncorrelated at the 5% level. On the other hand, the PIT test rejects the correct specification of the GAS-N-Id, GAS-N-InvSq, and GAS-t-InvSq at the 5% level (Table 4.3). The AR(1)-ALLGARCH(1,1), AR(1)-E-GARCH(1,1), and AR(1)-T-GARCH(1,1) with a skewed *t*-Student distribution have the lowest AIC and BIC for the ECO returns (Table 4.2). The GAS model with a skewed *t*-Student together with an inverted square score displays the lowest AIC and BIC among all GAS models. Therefore, fat-tailed distributed models display a better in-sample fit for ECO returns. Further, the GAS-t-Id has the lowest out-of-sample RMSE, followed by the AR(1)-CGARCH(1,1)-*t* and the AR(1)-I-GARCH(1,1)-*t*.

Table 4.4 presents backtesting measures for daily 1%-VaR forecasts of ECO returns. None of the GARCH models with the lowest AIC and BIC are optimal for 1%-VaR forecasts. For instance, the DQ test rejects that the AR(1)-ALLGARCH(1,1)-St model has a correct specification for 1%-VaR forecasts at the 1% level, although this model has the lowest AIC among all models. Nevertheless,

Table 4.2 GARCH models for ECO returns

GARCH model	AIC	BIC	LogLik	$Q^2(10)$	RMSE
ALLGARCH(1,1)-N	3.512	3.541	−2552.56	0.79	1.3418
ALLGARCH(1,1)-St	**3.487**	3.523	−2531.99	0.88	1.3419
ALLGARCH(1,1)-t	3.500	3.533	−2542.83	0.87	1.3401
APARCH(1,1)-N	3.517	3.542	−2556.81	0.93	1.3421
APARCH(1,1)-St	3.488	3.521	−2533.66	0.94	1.3419
APARCH(1,1)-t	3.503	3.532	−2545.44	0.94	1.3401
CGARCH(1,1)-N	3.527	3.552	−2564.20	0.71	1.3400
CGARCH(1,1)-St	3.503	3.535	−2544.48	0.71	1.3395
CGARCH(1,1)-t	3.515	3.544	−2554.31	0.71	**1.3390**
E-GARCH(1,1)-N	3.515	3.537	−2556.75	0.90	1.3420
E-GARCH(1,1)-St	**3.487**	**3.516**	−2533.91	0.91	1.3411
E-GARCH(1,1)-t	3.501	3.527	−2545.56	0.91	1.3401
GJRGARCH(1,1)-N	3.516	3.538	−2557.48	0.92	1.3417
GJRGARCH(1,1)-St	3.488	3.517	−2534.39	0.93	1.3416
GJRGARCH(1,1)-t	3.502	3.527	−2545.98	0.94	1.3399
I-GARCH(1,1)-N	3.531	3.545	−2570.02	0.26	1.3404
I-GARCH(1,1)-St	3.504	3.526	−2548.32	0.35	1.3399
I-GARCH(1,1)-t	3.516	3.534	−2558.31	0.34	**1.3390**
NGARCH(1,1)-N	3.527	3.549	−2565.31	0.50	1.3404
NGARCH(1,1)-St	3.502	3.531	−2545.24	0.57	1.3397
NGARCH(1,1)-t	3.514	3.540	−2555.06	0.56	1.3391
GARCH(1,1)-N	3.527	3.545	−2566.17	0.52	1.3404
GARCH(1,1)-St	3.501	3.527	−2545.46	0.60	1.3397
GARCH(1,1)-t	3.514	3.536	−2555.61	0.59	1.3391
T-GARCH(1,1)-N	3.516	3.538	−2557.50	0.92	1.3421
T-GARCH(1,1)-St	**3.487**	**3.516**	−2534.38	0.92	1.3419
T-GARCH(1,1)-t	3.502	3.528	−2546.13	0.92	1.3401

Notes: $Q^2(10)$ is the p-value of the Ljung–Box test on the standardized squared residuals. RMSE is the root-mean-squared error for 500 one-day-ahead rolling forecasts. The out-of-sample period spans from September 22, 2017, to September 18, 2019. The best model for each criterion is in boldface

the AR(1)-NGARCH(1,1)-St is the optimal model for 1%-VaR forecasts of ECO returns since it has the best AE and AD mean ratios together with the highest conditional coverage (CC) of 1%-VaR forecasts. The GAS-St-Id and GAS-St-InvSq also display an AE close to the unity and the highest CC of 1%-VaR forecasts. Overall, both the AE and AD mean ratios enhance when we employ fat-tailed distributed modelsfor 1%-VaR forecasts of ECO returns.

Table 4.3 GAS models for ECO returns

GAS model	AIC	BIC	LogLik	NP	PIT	RMSE
GAS-AST-Id	5119.75	5188.45	−2546.87	13	0.44	1.3710
GAS-AST1-Id	5121.13	5173.97	−2550.56	10	0.26	1.3611
GAS-N-Id	5147.43	5179.14	−2567.71	6	0.01	1.3398
GAS-St-Id	5120.38	5162.66	−2552.19	8	0.10	1.3396
GAS-t-Id	5131.87	5168.87	−2558.94	7	0.07	**1.3384**
GAS-AST-InvSq	5165.67	5234.37	−2569.83	13	0.30	1.3666
GAS-AST1-InvSq	**5107.94**	**5160.79**	−2543.97	10	0.48	1.3571
GAS-N-InvSq	5150.19	5181.89	−2569.09	6	0.01	1.3396
GAS-St-InvSq	5120.38	5162.66	−2552.19	8	0.10	1.3396
GAS-t-InvSq	5133.81	5170.80	−2559.90	7	0.02	1.3393

Notes: NP is the number of parameters. PIT is the *p*-value of the PIT test of Diebold et al. (1998). We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

Table 4.4 Backtesting measures for daily 1%-VaR forecasts: ECO returns

Model	AE	AD mean	AD max	DQ	CC
ALLGARCH(1,1)-N	0.8	0.52	1.29	0.997	0.869
ALLGARCH(1,1)-St	1.0	0.57	1.40	0.003	0.951
ALLGARCH(1,1)-t	0.8	0.54	1.31	0.973	0.869
APARCH(1,1)-N	1.8	0.58	1.63	0.062	0.230
APARCH(1,1)-St	1.8	0.52	1.67	0.064	0.230
APARCH(1,1)-t	1.6	0.61	1.61	0.038	0.407
CGARCH(1,1)-N	1.0	0.59	1.45	0.002	0.951
CGARCH(1,1)-St	1.0	0.55	1.41	0.002	0.951
CGARCH(1,1)-t	1.0	0.63	1.43	0.002	0.951
E-GARCH(1,1)-N	1.8	0.66	1.76	0.064	0.230
E-GARCH(1,1)-St	1.6	0.57	1.69	0.053	0.407
E-GARCH(1,1)-t	1.8	0.63	1.77	0.066	0.230
GARCH(1,1)-N	1.8	0.64	1.77	0.063	0.230
GARCH(1,1)-St	2.0	0.67	1.74	0.059	0.115
GARCH(1,1)-t	1.8	0.65	1.73	0.072	0.230
GAS-AST1-Id	1.2	0.49	1.50	0.013	0.845
GAS-AST1-InvSq	1.0	0.66	1.47	0.003	0.951
GAS-AST-Id	1.4	0.54	1.56	0.020	0.632
GAS-AST-InvSq	0.4	0.51	**0.77**	0.930	0.306
GAS-N-Id	1.8	0.65	1.82	0.059	0.230

(continued)

Table 4.4 (continued)

Model	AE	AD mean	AD max	DQ	CC
GAS-N-InvSq	2.0	0.65	1.68	0.002	0.115
GAS-St-Id	**1.0**	0.57	1.59	0.967	**0.951**
GAS-St-InvSq	**1.0**	0.57	1.59	0.967	**0.951**
GAS-t-Id	1.6	0.65	1.79	0.036	0.407
GAS-t-InvSq	1.6	0.71	1.71	0.045	0.407
GJRGARCH(1,1)-N	1.0	0.60	1.46	0.002	0.951
GJRGARCH(1,1)-St	**1.0**	0.53	1.34	0.988	**0.951**
GJRGARCH(1,1)-t	1.2	0.59	1.33	**1.000**	0.845
I-GARCH(1,1)-N	**1.0**	0.49	1.30	0.986	**0.951**
I-GARCH(1,1)-St	1.0	0.54	1.41	0.002	0.951
I-GARCH(1,1)-t	**1.0**	0.49	1.29	0.985	**0.951**
NGARCH(1,1)-N	1.0	0.87	1.65	0.003	0.951
NGARCH(1,1)-St	**1.0**	**0.47**	1.35	0.981	**0.951**
NGARCH(1,1)-t	1.4	0.58	1.58	0.981	0.632
T-GARCH(1,1)-N	1.8	0.62	1.76	0.070	0.230
T-GARCH(1,1)-St	1.8	0.62	1.68	0.090	0.230
T-GARCH(1,1)-t	2.0	0.56	1.74	0.070	0.115

Notes: We report the Actual over Expected ratio (AE), the mean Absolute Deviation (AD mean), and the maximum Absolute Deviation (AD max) of 1%-VaR forecasts of ECO returns. CC and DQ are the p-values of the tests of Christoffersen (1998) and Engle and Manganelli (2004), respectively, where the model is correctly specified for 1%-VaR forecasts under H_0. We perform 500 one-day-ahead rolling forecasts. The forecasting period spans from September 22, 2017, to September 18, 2019. We denote the best models (for each criterion) in boldface

Tables 4.5 and 4.6 show the estimation results of the GARCH and GAS models for SPCLE returns. The Ljung–Box test results indicate the residuals are serially correlated for the AR(1)-ALLGARCH(1,1)-St, AR(1)-ALLGARCH(1,1)-t, AR(1)-T-GARCH(1,1)-St, and AR(1)-T-GARCH(1,1)-t at the 5% level. Conversely, all GAS models are correctly specified at the 5% level. The AR(1)-GJRGARCH(1,1) with a skewed t-Student distribution and with a t-Student distribution have the best in-sample fit for the SPCLE returns (Table 4.5). The GAS-t-Id and the GAS-AST1-InvSq present the lowest AIC and BIC among all GAS models. Consistent with the results for ECO returns, fat-tailed distributed models provide a better in-sample fit for SPCLE returns. Further, the AR(1)-CGARCH(1,1)-N, AR(1)-CGARCH(1,1)-t, AR(1)-NGARCH(1,1)-t, and AR(1)-GARCH(1,1)-t have the lowest out-of-sample RMSE.

Table 4.7 displays backtesting results for one-day-ahead 1%-VaR forecasts of SPCLE returns. Consistent with the backtesting results for ECO returns in Table 4.4, none of the GARCH models with the best in-sample fit is optimal for 1%-VaR forecasts of SPCLE returns. The AR(1)-APARCH(1,1)-St is the optimal model for

Table 4.5 GARCH models for SPCLE returns

GARCH model	AIC	BIC	LogLik	$Q^2(10)$	RMSE
ALLGARCH(1,1)-N	2.925	2.954	−2124.47	0.13	0.9255
ALLGARCH(1,1)-St	2.903	2.939	−2106.25	0.03	0.9247
ALLGARCH(1,1)-t	2.902	2.935	−2106.72	0.03	0.9239
APARCH(1,1)-N	2.935	2.960	−2132.65	0.54	0.9246
APARCH(1,1)-St	2.909	2.941	−2111.49	0.42	0.9249
APARCH(1,1)-t	2.909	2.938	−2112.31	0.40	0.9242
CGARCH(1,1)-N	2.936	2.961	−2133.30	0.89	**0.9236**
CGARCH(1,1)-St	2.912	2.945	−2113.84	0.88	0.9240
CGARCH(1,1)-t	2.911	2.940	−2114.33	0.88	**0.9236**
E-GARCH(1,1)-N	2.938	2.960	−2136.16	0.24	0.9245
E-GARCH(1,1)-St	2.911	2.940	−2114.31	0.07	0.9249
E-GARCH(1,1)-t	2.911	2.936	−2115.00	0.06	0.9242
GJRGARCH(1,1)-N	2.934	2.956	−2133.07	0.62	0.9245
GJRGARCH(1,1)-St	**2.908**	2.937	−2111.74	0.54	0.9248
GJRGARCH(1,1)-t	**2.908**	**2.933**	−2112.58	0.54	0.9241
I-GARCH(1,1)-N	2.950	2.965	−2146.84	0.43	0.9239
I-GARCH(1,1)-St	2.917	2.939	−2120.49	0.45	0.9242
I-GARCH(1,1)-t	2.916	2.935	−2121.10	0.44	0.9237
NGARCH(1,1)-N	2.941	2.963	−2138.01	0.81	0.9237
NGARCH(1,1)-St	2.914	2.943	−2116.04	0.77	0.9241
NGARCH(1,1)-t	2.913	2.939	−2116.73	0.76	**0.9236**
GARCH(1,1)-N	2.940	2.958	−2138.03	0.81	0.9237
GARCH(1,1)-St	2.912	2.938	−2116.15	0.77	0.9241
GARCH(1,1)-t	2.912	2.934	−2116.83	0.76	**0.9236**
T-GARCH(1,1)-N	2.936	2.957	−2134.12	0.19	0.9246
T-GARCH(1,1)-St	2.909	2.938	−2112.71	0.05	0.9249
T-GARCH(1,1)-t	2.909	2.934	−2113.42	0.04	0.9243

Notes: $Q^2(10)$ is the p-value of the Ljung–Box test on the standardized squared residuals. We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

1%-VaR forecasts of SPCLE returns since it has the best AE and the highest p-values of the DQ and CC tests. The AR(1)-ALLGARCH(1,1)-N also displays an AE close to the unity and the lowest AD mean ratio. Moreover, the AR(1)-GARCH(1,1)-St, AR(1)-GARCH(1,1)-t, GAS-AST1-Id, GAS-N-Id, and the AR(1)-T-GARCH(1,1)-N also exhibit the optimal AE ratio and the highest conditional coverage for risk forecasts of SPCLE returns.

Table 4.6 GAS models for SPCLE returns

GAS model	AIC	BIC	LogLik	NP	PIT	RMSE
GAS-AST-Id	4259.56	4328.26	−2116.78	13	0.95	0.9285
GAS-AST1-Id	4254.58	4307.43	−2117.29	10	0.92	0.9287
GAS-N-Id	4293.59	4325.30	−2140.79	6	0.11	0.9289
GAS-St-Id	4265.89	4308.17	−2124.94	8	0.95	0.9281
GAS-t-Id	4259.17	**4296.16**	−2122.58	7	0.93	0.9260
GAS-AST-InvSq	4383.11	4451.81	−2178.55	13	0.16	0.9282
GAS-AST1-InvSq	**4254.35**	4307.20	−2117.17	10	0.89	0.9287
GAS-N-InvSq	4292.26	4323.97	−2140.13	6	0.07	**0.9253**
GAS-St-InvSq	4265.89	4308.17	−2124.94	8	0.95	0.9281
GAS-t-InvSq	4262.19	4299.18	−2124.10	7	0.97	0.9277

Notes: NP is the number of parameters. PIT is the p-value of the PIT test of Diebold et al. (1998). We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

Table 4.7 Backtesting measures for daily 1%-VaR forecasts: SPCLE returns

Model	AE	AD mean	AD max	DQ test	CC test
ALLGARCH(1,1)-N	**1.0**	**0.19**	0.55	0.750	**0.951**
ALLGARCH(1,1)-St	0.6	0.34	0.45	0.923	0.613
ALLGARCH(1,1)-t	0.8	0.27	0.60	0.629	0.869
APARCH(1,1)-N	0.8	0.35	0.63	0.667	0.869
APARCH(1,1)-St	**1.0**	0.32	0.50	**0.995**	**0.951**
APARCH(1,1)-t	0.8	0.32	0.59	0.711	0.869
CGARCH(1,1)-N	0.8	0.34	0.44	0.964	0.869
CGARCH(1,1)-St	0.6	0.34	0.45	0.924	0.613
CGARCH(1,1)-t	0.8	0.34	0.59	0.368	0.869
E-GARCH(1,1)-N	1.6	0.34	0.72	0.002	0.128
E-GARCH(1,1)-St	0.8	0.33	0.55	0.564	0.869
E-GARCH(1,1)-t	1.4	0.41	0.90	0.135	0.632
GARCH(1,1)-N	1.2	0.33	0.68	0.251	0.845
GARCH(1,1)-St	**1.0**	0.41	0.73	0.306	**0.951**
GARCH(1,1)-t	**1.0**	0.45	0.84	0.224	**0.951**
GAS-AST1-Id	**1.0**	0.28	0.52	0.194	**0.951**
GAS-AST1-InvSq	0.6	0.34	0.54	0.937	0.613
GAS-AST-Id	0.8	0.45	0.64	0.258	0.869
GAS-AST-InvSq	0.4	0.33	0.42	0.933	0.306
GAS-N-Id	**1.0**	0.44	0.73	0.127	**0.951**

(continued)

Table 4.7 (continued)

Model	AE	AD mean	AD max	DQ test	CC test
GAS-N-InvSq	1.2	0.41	0.92	0.057	0.845
GAS-St-Id	0.6	0.54	0.69	0.787	0.613
GAS-St-InvSq	0.6	0.54	0.69	0.787	0.613
GAS-t-Id	0.6	0.57	0.75	0.826	0.613
GAS-t-InvSq	0.6	0.57	0.81	0.819	0.613
GJRGARCH(1,1)-N	0.6	0.37	0.49	0.924	0.613
GJRGARCH(1,1)-St	0.8	0.28	0.56	0.629	0.869
GJRGARCH(1,1)-t	0.8	0.33	0.68	0.557	0.869
I-GARCH(1,1)-N	0.8	0.25	0.52	0.625	0.869
I-GARCH(1,1)-St	0.8	0.31	**0.41**	0.962	0.869
I-GARCH(1,1)-t	0.6	0.30	0.49	0.964	0.613
NGARCH(1,1)-N	0.8	0.33	0.54	0.567	0.869
NGARCH(1,1)-St	0.8	0.30	0.63	0.639	0.869
NGARCH(1,1)-t	1.2	**0.19**	0.61	0.814	0.845
T-GARCH(1,1)-N	**1.0**	0.42	0.78	0.289	**0.951**
T-GARCH(1,1)-St	1.4	0.38	1.00	0.318	0.632
T-GARCH(1,1)-t	1.2	0.33	0.68	0.239	0.845

Notes: We report the Actual over Expected ratio (AE), the mean Absolute Deviation (AD mean), and the maximum Absolute Deviation (AD max) of 1%-VaR forecasts of SPCLE returns. CC and DQ are the p-values of the tests of Christoffersen (1998) and Engle and Manganelli (2004), respectively, where the model is correctly specified for 1%-VaR forecasts under H_0. The best models for each criterion are in boldface. We perform 500 one-day-ahead rolling forecasts as in Table 4.4

Tables 4.8 and 4.9 present the estimation results for SUN returns. The Ljung–Box test results indicate the residuals are serially correlated for the AR(1)-E-GARCH(1,1)-N and AR(1)-T-GARCH(1,1)-N models at the 5% level. The PIT test rejects the correct specification of the GAS-t-Id and GAS-AST-InvSq at the 5% level. The AR(1)-CGARCH(1,1)-St and AR(1)-NGARCH(1,1)-t have the lowest AIC and BIC, respectively, for the SUN returns (Table 4.8). In addition, the GAS-AST-Id displays the best AIC and BIC among the GAS models. In line with the results for ECO and SPCLE returns, fat-tailed distributed models for the residuals have an optimal in-sample fit for SPCLE returns. Further, the AR(1)-E-GARCH(1,1)-St displays the lowest out-of-sample RMSE among all models.

Table 4.10 shows the results of backtests for daily 1%-VaR forecasts of SUN returns. Consistent with the backtesting analysis of ECO and SPCLE returns in Tables 4.4 and 4.7, none of the GARCH models with the lowest AIC and BIC is optimal for 1%-VaR forecasts of SUN returns. Nevertheless, the AR(1)-E-GARCH(1,1)-St is one of the optimal models for both out-of-sample forecasts of SUN returns and for 1%-VaR forecasts; it has an AE ratio statistically equal to one and the highest

Table 4.8 GARCH models for SUN returns

GARCH model	AIC	BIC	LogLik	$Q^2(10)$	RMSE
ALLGARCH(1,1)-N	3.828	3.857	−2782.76	0.11	1.4848
ALLGARCH(1,1)-St	3.799	3.835	−2759.38	0.19	1.4836
ALLGARCH(1,1)-t	3.800	3.833	−2761.55	0.19	1.4831
APARCH(1,1)-N	3.829	3.854	−2784.34	0.11	1.4847
APARCH(1,1)-St	3.800	3.833	−2761.37	0.12	1.4837
APARCH(1,1)-t	3.802	3.831	−2763.47	0.13	1.4832
CGARCH(1,1)-N	3.825	3.851	−2781.56	0.32	1.4843
CGARCH(1,1)-St	**3.798**	3.831	−2759.96	0.32	1.4829
CGARCH(1,1)-t	3.799	3.828	−2761.34	0.33	1.4828
E-GARCH(1,1)-N	3.830	3.852	−2786.14	0.04	1.4859
E-GARCH(1,1)-St	3.799	3.828	−2761.31	0.07	**1.4822**
E-GARCH(1,1)-t	3.800	3.826	−2763.31	0.08	1.4827
GJRGARCH(1,1)-N	3.828	3.846	−2785.38	0.11	1.4844
GJRGARCH(1,1)-St	3.800	3.826	−2763.44	0.17	1.4829
GJRGARCH(1,1)-t	3.801	3.823	−2765.21	0.18	1.4828
I-GARCH(1,1)-N	3.828	3.849	−2784.37	0.11	1.4845
I-GARCH(1,1)-St	3.800	3.829	−2762.23	0.15	1.4832
I-GARCH(1,1)-t	3.801	3.826	−2764.02	0.16	1.4829
NGARCH(1,1)-N	3.828	3.842	−2786.29	0.08	1.4843
NGARCH(1,1)-St	3.801	3.823	−2765.29	0.11	1.4830
NGARCH(1,1)-t	3.802	**3.820**	−2766.72	0.12	1.4828
GARCH(1,1)-N	3.828	3.850	−2784.88	0.11	1.4843
GARCH(1,1)-St	3.802	3.831	−2763.41	0.17	1.4829
GARCH(1,1)-t	3.803	3.828	−2765.21	0.18	1.4828
T-GARCH(1,1)-N	3.833	3.854	−2788.07	0.05	1.4863
T-GARCH(1,1)-St	3.799	3.828	−2761.77	0.10	1.4839
T-GARCH(1,1)-t	3.801	3.827	−2764.08	0.10	1.4833

Notes: $Q^2(10)$ is the p-value of the Ljung–Box test on the standardized squared residuals. We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

conditional coverage rate. In addition, the AR(1)-APARCH(1,1) and the GAS-t-InvSq have the lowest AD mean and AD max ratios for 1%-VaR forecasts, respectively, among the models with an optimal AE ratio. The GAS-t-Id and the AR(1)-NGARCH(1,1)-N models also have an optimal AE together with good AD mean and CC ratios for 1%-VaR forecasts of SUN returns. In consonance with the findings for ECO and SPCLE returns, all backtesting measures enhance when we employ fat-tailed distributed models for 1%-VaR forecasts of SUN returns.

Table 4.9 GAS models for SUN returns

GAS model	AIC	BIC	LogLik	NP	PIT	RMSE
GAS-AST-Id	**5545.20**	**5598.04**	−2762.60	10	0.23	1.4920
GAS-AST1-Id	5546.99	5599.83	−2763.49	10	0.23	1.4926
GAS-N-Id	5550.41	5619.11	−2762.20	13	0.15	1.4875
GAS-St-Id	5663.12	5731.82	−2818.56	13	0.22	1.4901
GAS-t-Id	5595.26	5626.96	−2791.63	6	0.01	1.4825
GAS-AST-InvSq	5592.90	5624.61	−2790.45	6	0.00	1.4863
GAS-AST1-InvSq	5571.23	5613.51	−2777.61	8	0.22	1.4875
GAS-N-InvSq	5571.23	5613.51	−2777.61	8	0.22	1.4875
GAS-St-InvSq	5573.90	5610.89	−2779.95	7	0.38	**1.4866**
GAS-t-InvSq	5578.68	5615.67	−2782.34	7	0.23	1.4867

Notes: NP is the number of parameters. PIT is the *p*-value of the PIT test of Diebold et al. (1998). We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

Table 4.10 Backtesting measures for daily 1%-VaR forecasts: SUN returns

Model	AE	AD mean	AD max	DQ test	CC test
ALLGARCH(1,1)-N	0.8	0.79	1.44	0.807	0.869
ALLGARCH(1,1)-St	0.8	0.82	1.29	0.785	0.869
ALLGARCH(1,1)-t	0.8	0.71	1.59	0.719	0.869
APARCH(1,1)-N	0.8	1.10	1.77	0.685	0.869
APARCH(1,1)-St	**1.0**	0.75	1.33	0.837	**0.951**
APARCH(1,1)-t	0.8	1.05	1.61	0.665	0.869
CGARCH(1,1)-N	0.8	0.72	1.12	0.876	0.869
CGARCH(1,1)-St	0.8	0.77	1.27	0.762	0.869
CGARCH(1,1)-t	0.8	0.81	1.26	0.745	0.869
E-GARCH(1,1)-N	1.4	0.79	1.71	0.305	0.632
E-GARCH(1,1)-St	**1.0**	0.82	1.47	0.702	**0.951**
E-GARCH(1,1)-t	1.4	0.81	1.74	0.209	0.632
GARCH(1,1)-N	1.4	0.80	1.70	0.214	0.632
GARCH(1,1)-St	1.6	0.75	1.76	0.055	0.407
GARCH(1,1)-t	1.2	0.88	1.86	0.230	0.845
GAS-AST1-Id	0.8	0.85	1.79	0.792	0.869
GAS-AST1-InvSq	0.8	0.80	1.79	0.837	0.869
GAS-AST-Id	0.8	0.80	1.74	0.801	0.869
GAS-AST-InvSq	0.4	0.75	1.40	**0.937**	0.306
GAS-N-Id	1.2	0.88	1.77	0.424	0.845

(continued)

Table 4.10 (continued)

Model	AE	AD mean	AD max	DQ test	CC test
GAS-N-InvSq	1.6	0.67	1.66	0.005	0.407
GAS-St-Id	0.6	0.98	1.37	0.920	0.613
GAS-St-InvSq	0.6	0.98	1.37	0.920	0.613
GAS-t-Id	**1.0**	0.73	1.58	0.639	**0.951**
GAS-t-InvSq	**1.0**	0.69	1.52	0.682	**0.951**
GJRGARCH(1,1)-N	0.8	0.81	1.31	0.762	0.869
GJRGARCH(1,1)-St	0.8	0.86	1.61	0.658	0.869
GJRGARCH(1,1)-t	0.8	1.56	5.92	0.000	0.000
I-GARCH(1,1)-N	0.8	0.82	1.56	0.660	0.869
I-GARCH(1,1)-St	0.8	**0.68**	**1.08**	0.872	0.869
I-GARCH(1,1)-t	0.8	0.82	1.40	0.626	0.869
NGARCH(1,1)-N	**1.0**	0.84	1.49	0.725	**0.951**
NGARCH(1,1)-St	0.8	0.75	1.64	0.724	0.869
NGARCH(1,1)-t	0.8	1.03	1.64	0.799	0.869
T-GARCH(1,1)-N	1.2	0.87	1.91	0.245	0.845
T-GARCH(1,1)-St	1.4	0.85	1.84	0.211	0.632
T-GARCH(1,1)-t	1.6	0.74	1.76	0.068	0.407

Notes: We report the Actual over Expected ratio (AE), the mean Absolute Deviation (AD mean), and the maximum Absolute Deviation (AD max) of 1%-VaR forecasts of SUN returns. CC and DQ are the p-values of the tests of Christoffersen (1998) and Engle and Manganelli (2004), respectively, where the model is correctly specified for 1%-VaR forecasts under H_0. The best models for each criterion are in boldface. We perform 500 one-day-ahead rolling forecasts as in Table 4.4

Tables 4.11 and 4.12 show the estimation results for ERIX returns. Table 4.11 indicates that all GARCH residuals are serially uncorrelated at the 5% level. Conversely, the PIT test rejects the correct specification of the GAS-t-Id and GAS-AST-InvSq at the 5% level. The AR(1)-T-GARCH(1,1)-St and AR(1)-T-GARCH(1,1)-t models present the best AIC and BIC, respectively, for the ERIX returns (Table 4.11). Besides, the GAS-AST1-Id and GAS-St-InvSq attain the minimum AIC and BIC among the GAS models for the ERIX returns. Consistent with the results for the other renewable energy returns, fat-tailed distributed models obtain a better in-sample fit for ERIX returns. Yet, the AR(1)-I-GARCH(1,1)-N attains the lowest out-of-sample RMSE for ERIX returns among all models.

Table 4.13 shows the results of backtests for daily 1%-VaR forecasts of ERIX returns. The DQ test rejects the correct specification of almost all models at the 1% significance level. In line with the backtesting results for the other renewable energy returns, none of the GARCH models with the lowest AIC and BIC is optimal for 1%-VaR forecasts of ERIX returns. The GAS-N-Id is the optimal model for 1%-VaR forecasts of ERIX returns since it attains the lowest AD mean and AD max

Table 4.11 GARCH models for ERIX returns

GARCH model	AIC	BIC	LogLik	$Q^2(10)$	RMSE
ALLGARCH(1,1)-N	3.428	3.457	−2491.11	0.90	1.3031
ALLGARCH(1,1)-St	3.375	3.411	−2450.04	0.85	1.3033
ALLGARCH(1,1)-t	3.379	3.411	−2454.14	0.84	1.3036
APARCH(1,1)-N	3.427	3.453	−2491.48	0.90	1.3034
APARCH(1,1)-St	3.375	3.408	−2451.35	0.85	1.3032
APARCH(1,1)-t	3.378	3.407	−2454.88	0.85	1.3035
CGARCH(1,1)-N	3.452	3.477	−2509.31	0.96	1.3037
CGARCH(1,1)-St	3.396	3.429	−2466.78	0.97	1.3041
CGARCH(1,1)-t	3.398	3.427	−2469.26	0.96	1.3048
E-GARCH(1,1)-N	3.428	3.450	−2493.21	0.91	1.3033
E-GARCH(1,1)-St	3.375	3.404	−2452.50	0.87	1.3221
E-GARCH(1,1)-t	3.378	3.404	−2455.77	0.87	1.3663
GJRGARCH(1,1)-N	3.433	3.455	−2496.95	0.91	1.3035
GJRGARCH(1,1)-St	3.381	3.410	−2456.94	0.90	1.3039
GJRGARCH(1,1)-t	3.384	3.409	−2459.71	0.90	1.3046
I-GARCH(1,1)-N	3.468	3.483	−2524.21	0.68	**1.3030**
I-GARCH(1,1)-St	3.401	3.423	−2473.25	0.79	1.3031
I-GARCH(1,1)-t	3.403	3.421	−2475.57	0.76	1.3036
NGARCH(1,1)-N	3.447	3.468	−2506.51	0.97	1.3035
NGARCH(1,1)-St	3.394	3.423	−2466.43	0.93	1.3038
NGARCH(1,1)-t	3.396	3.422	−2468.95	0.93	1.3045
GARCH(1,1)-N	3.448	3.466	−2508.40	0.96	1.3039
GARCH(1,1)-St	3.395	3.420	−2467.66	0.91	1.3041
GARCH(1,1)-t	3.396	3.418	−2470.00	0.91	1.3047
T-GARCH(1,1)-N	3.426	3.448	−2491.48	0.90	1.3034
T-GARCH(1,1)-St	**3.374**	3.403	−2451.44	0.86	1.3032
T-GARCH(1,1)-t	3.377	**3.402**	−2454.92	0.86	1.3035

Notes: $Q^2(10)$ is the *p*-value of the Ljung–Box test on the standardized squared residuals. We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

ratios together with the highest *p*-values of the CC and DQ tests. The AR(1)-E-GARCH(1,1)-N and AR(1)-GARCH(1,1)-N also have a similar out-of-sample performance for risk forecasts. However, these models exhibit an AE ratio far from the unity, indicating an excessive number of actual 1%-VaR exceedances over expected ones. Therefore, normally distributed GAS and GARCH models have good performance for 1%-VaR forecasts of ERIX returns, in contrast to our findings for the other renewable energy returns.

Table 4.12 GAS models for ERIX returns

GAS model	AIC	BIC	LogLik	NP	PIT	RMSE
GAS-AST-Id	4951.64	5004.49	−2465.82	10	0.88	1.3174
GAS-AST1-Id	**4949.51**	5002.36	−2464.76	10	0.74	1.3199
GAS-N-Id	4956.62	5025.32	−2465.31	13	0.78	1.3186
GAS-St-Id	4952.40	5021.10	−2463.20	13	0.90	1.3096
GAS-t-Id	5040.02	5071.72	−2514.01	6	0.01	1.3034
GAS-AST-InvSq	5028.71	5060.41	−2508.35	6	0.01	1.3035
GAS-AST1-InvSq	4950.36	4992.64	−2467.18	8	0.72	1.3043
GAS-N-InvSq	4950.36	4992.64	−2467.18	8	0.72	1.3043
GAS-St-InvSq	4949.82	**4986.81**	−2467.91	7	0.72	1.3047
GAS-t-InvSq	4950.90	4987.90	−2468.45	7	0.66	**1.3042**

Notes: NP is the number of parameters. PIT is the p-value of the PIT test of Diebold et al. (1998). We calculate the RMSE as in Table 4.2. The best model for each criterion is in boldface

Table 4.13 Backtesting measures for daily 1%-VaR forecasts: ERIX returns

Model	AE	AD mean	AD max	DQ test	CC test
ALLGARCH(1,1)-N	1.0	1.32	2.75	0.001	0.951
ALLGARCH(1,1)-St	1.0	1.17	2.97	0.001	0.951
ALLGARCH(1,1)-t	1.0	1.22	2.73	0.001	0.951
APARCH(1,1)-N	1.2	1.23	2.95	0.001	0.845
APARCH(1,1)-St	1.4	0.97	3.01	0.009	0.632
APARCH(1,1)-t	1.0	1.27	2.86	0.003	0.951
CGARCH(1,1)-N	1.0	1.27	2.94	0.001	0.951
CGARCH(1,1)-St	0.8	1.41	2.89	0.000	0.869
CGARCH(1,1)-t	1.0	1.16	2.97	0.001	0.951
E-GARCH(1,1)-N	**1.8**	0.96	3.28	0.014	**0.230**
E-GARCH(1,1)-St	1.0	1.33	3.05	0.001	0.951
E-GARCH(1,1)-t	1.4	1.28	3.52	0.001	0.632
GARCH(1,1)-N	**1.8**	0.95	3.28	0.016	**0.230**
GARCH(1,1)-St	1.2	1.47	3.22	0.001	0.845
GARCH(1,1)-t	1.2	1.41	3.21	0.001	0.845
GAS-AST1-Id	1.0	1.38	3.11	0.001	0.951
GAS-AST1-InvSq	1.0	1.37	3.04	0.002	0.951
GAS-AST-Id	1.0	1.37	3.10	0.001	0.951
GAS-AST-InvSq	0.8	1.55	3.04	0.000	0.869
GAS-N-Id	**1.8**	**0.90**	**3.20**	**0.017**	**0.230**

(continued)

Table 4.13 (continued)

Model	AE	AD mean	AD max	DQ test	CC test
GAS-N-InvSq	2.0	0.87	3.39	0.007	0.115
GAS-St-Id	1.0	1.39	3.13	0.001	0.951
GAS-St-InvSq	1.0	1.39	3.13	0.001	0.951
GAS-t-Id	1.2	1.22	3.20	0.002	0.845
GAS-t-InvSq	1.2	1.39	3.59	0.002	0.845
GJRGARCH(1,1)-N	0.8	1.47	2.93	0.000	0.869
GJRGARCH(1,1)-St	1.0	1.30	2.79	0.001	0.951
GJRGARCH(1,1)-t	2.0	1.39	7.58	0.000	0.000
I-GARCH(1,1)-N	1.0	1.24	2.75	0.001	0.951
I-GARCH(1,1)-St	0.8	1.51	2.83	0.000	0.869
I-GARCH(1,1)-t	0.8	1.31	2.67	0.000	0.869
NGARCH(1,1)-N	1.2	1.16	3.13	0.001	0.845
NGARCH(1,1)-St	1.0	1.28	2.77	0.001	0.951
NGARCH(1,1)-t	1.2	1.24	2.91	0.001	0.845
T-GARCH(1,1)-N	1.2	1.43	3.21	0.001	0.845
T-GARCH(1,1)-St	1.4	1.27	3.23	0.000	0.632
T-GARCH(1,1)-t	1.4	1.26	3.37	0.003	0.632

Notes: We report the Actual over Expected ratio (AE), the mean Absolute Deviation (AD mean), and the maximum Absolute Deviation (AD max) of 1%-VaR forecasts of ERIX returns. CC and DQ are the p-values of the tests of Christoffersen (1998) and Engle and Manganelli (2004), respectively, where the model is correctly specified for 1%-VaR forecasts under H_0. The best models for each criterion are in boldface. We perform 500 one-day-ahead rolling forecasts as in Table 4.4

In sum, heavy-tailed distributed GARCH and GAS models have the best in-sample fit for all renewable energy returns. They also exhibit the best out-of-sample forecast performance and the best coverage for 1%-VaR of renewable energy returns. These findings highlight the relevance of modeling the kurtosis for renewable energy returns. For instance, the GAS-t-Id, AR(1)-CGARCH(1,1)-t, and AR(1)-E-GARCH(1,1)-St have the lowest out-of-sample RMSE for ECO, SPCLE, and both SUN and ERIX returns, respectively. In addition, the AR(1)-NGARCH(1,1)-St, AR(1)-APARCH(1,1)-St, AR(1)-E-GARCH(1,1)-St, and GAS-N-Id models are optimal models for 1%-VaR of renewable energy returns. Therefore, fat-tailed GARCH and GAS enhance both in-sample and out-of-sample performance of renewable energy returns and risk. These findings are important for policymakers and investors who invest in the renewable energy sector.

4.4 Conclusions

Clean energy indices, such as wind and solar energy, are sold in financial markets that share the same dynamics of highly volatile assets. Clean energy returns may also exhibit heavy-tailed distributions since financial returns follow fat-tailed distributions (Gabaix 2009). It is important to model the volatility of renewable energy returns for investors since it affects the performance of their portfolios on renewable energy. In this chapter, we search for optimal models for clean energy returns using 37 flexible and fat-tailed GAS and GARCH models. Besides, we compare the out-of-sample performance of all models to find the optimal forecast model for clean energy returns. We also conduct several backtesting approaches for daily 1%-Value-at-Risk (VaR) forecasts of clean energy returns.

Fat-tailed distributed GARCH and GAS models have the best in-sample fit for all renewable energy returns. They also exhibit the best out-of-sample forecast performance and the best coverage for 1%-VaR of renewable energy returns. For instance, the GAS-t-Id, AR(1)-CGARCH(1,1)-t, AR(1)-E-GARCH(1,1)-St have the lowest out-of-sample RMSE for ECO, SPCLE, and both SUN and ERIX returns, respectively. In addition, the AR(1)-NGARCH(1,1)-St, AR(1)-APARCH(1,1)-St, AR(1)-E-GARCH(1,1)-St, and GAS-N-Id models are optimal models for 1%-VaR of renewable energy returns. These findings illustrate the relevance of modeling the kurtosis for renewable energy returns, which are relevant for policymakers and investors who invest in the renewable energy sector.

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