Spontaneous Collapse Theories and Cosmology

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Abstract The account for the emergence of the primordial seeds of structure in the universe as a result of "quantum fluctuations" during the inflationary epoch, is, on one hand, remarkably successful at the empirical level, and, on the other hand, it faces severe conceptual shortcomings tied to the conceptual difficulties that afflict quantum theory in general. In the cosmological context, such problems become exacerbated by the relative simplicity of the form that the questions takes. This, at the same time, makes their investigation rather direct, and the case for novel physics, such as that represented by spontaneous collapse theories, extremely compelling. We will discuss those aspects and argue that the most natural framework for the consideration of the relevant issues in this context is that provided by semi-classical gravity. We will see that such line of research offers a path to deal with the conceptual difficulties alluded. Moreover, in this particular case, it also offers a natural resolution of one of the few instances where predictions of the standard approaches to the subject are in tension with the empirical results, namely that referring to the primordial gravity waves.

[1] We will use "spontaneous collapse" to refer to the kind of state reduction considered in theories that attempt to address the "measurement problem" via a modification of Schrödinger's evolution, and that does not explicitly tie such "reduction" to "measurement situations" or "interactions" with "observers". We will make use of the generic term "collapse" to include both the situations above, as well as possible considerations where the reduction of the quantum state is meant to be triggered by "measurements", "interaction with measuring apparatuses" or observers as in the Copenhagen or Von Neumann's approaches. That distinction will not be made explicitly when the context prevents any possibility of confusion and the wording would become too cumbersome, including discussions involving "collapse operators" and "the collapse rate constant".

[2] That dilution is often taken as characterized by a factor of e^N, with N "the number of e-folds" of inflation (the logarithm of the factor by which the scale factor of the Universe grows during the inflationary regime) usually taken to be at least 60.

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1 Introduction

As clearly anticipated by Bell [1], cosmology provides a particularly clear context where the difficulties of quantum theory, viewed as a fundamental description of nature, are exposed. The issue was also recognized in [2], driving the authors to look for an alternative formulation of quantum theory that was appropriate to deal with cosmology. The focus was on an approach [3] which was soon shown to suffer from serious deficiencies [4, 5]. It is, thus, perhaps not surprising that spontaneous collapse [1] theories are increasingly being considered as playing an important role in such context. Indeed, current approaches to cosmology which rely on the hypothesis of an early inflationary epoch to address various "naturalness" difficulties [6] in the traditional Big Bang model, all have quantum theory playing a central role. In particular, inflation is supposed to smooth out all inhomogeneities and dilute all sorts of matter content present in the pre-inflationary stage.[2] The result is a universe that is devoid of matter and is homogeneous and isotropic to an exceedingly high degree. Therefore, any successful scenario must account for both the repopulation of the universe with matter,[3] and for the emergence of the primordial inhomogeneities that eventually grow to form all cosmic structure we see around us. The latter is where the present cosmological models strongly rely on quantum aspects, more specifically, on the so called "quantum fluctuations" of the vacuum. The usual treatment is, however, plagued with unjustifiable steps based on serious misunderstandings of quantum theory, as we will note shortly. This issue was first discussed in [7], where we argued that something like a spontaneous collapse theory could help in addressing the problems behind the unjustified steps. That took place before we learned that, by that time, the research program on spontaneous collapse theories was well underway with concrete and viable proposals represented by the GRW and CSL theories [8–14], where G.C. Ghirardi left his most unerasable marks, and had already made extraordinary advances [15]. The event illustrates, not only our level of ignorance regarding the field at that time, a condition that still afflicts the great majority of the cosmology community,[4] but also, the compelling force of the spontaneous collapse idea in the cosmological context.[5]

In this paper, we want to make the case that, not only is inflationary cosmology an ideal ground to contemplate the role of spontaneous collapse theories, and, in particular, their interface with gravitation, but also that the actual predictions of inflation can be substantially modified. Actually, the modified predictions appear to be, at the time of the writing of this article, more empirically adequate than the standard ones. This is so, at least, within what we believe to be the most appropriate implementation of the marriage between the description of space-time and quantum theory.

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[3] A process known as "reheating". For a discussion see for instance [16].

[4] Although, there are some well known texts that do acknowledge the problem. See for instance [17, 18].

[5] Besides the embarrassment, something positive can be found in our lack of knowledge at the time.

This manuscript is organized as follows: In Sect. 2, we offer a brief review of the basic aspects of inflationary cosmology, including the usual account of the "prediction" regarding the primordial inhomogeneities (for simplicity, many technical details will be just mentioned in passing or completely ignored). In Sect. 3, we present a discussion of the problematic aspects of such accounts, as well as a discussion of the approach we have taken to deal with these and related problems. Section 4 is devoted to a practical implementation of the ideas developed in the previous section to the inflationary cosmological context. This includes the analysis of the emergence of seeds of cosmic structure, and the calculation leading to our prediction of the primordial spectrum of scalar perturbations, as well as a very brief discussion of that corresponding to the primordial gravity waves. As we will see it is in regards to the latter that we find the most dramatic modifications in the theoretical predictions. We finish in Sect. 5 with a short review of what has been accomplished so far, and the aspects of the approach that require further elucidation and development.

We will use the $(- + ++)$ signature for the space-time metric, and Wald's conventions for the Riemann tensor [19]. Greek indices will be used to denote space-time coordinates, and latin indices to denote spatial coordinates on suitable identified spatial sections.

2 Cosmological Inflationary Model

The basic idea of inflation is that the standard radiation-dominated Big Bang regime is preceded by a period of accelerated expansion, controlled by something that behaves as a large cosmological constant, but which is later "turned off", as a result of its own dynamics. The specific realization of this idea is based on the introduction of a new scalar field ϕ with a potential V, which acts as the cosmological constant when the field is away from its minimum, usually taken to correspond to $\phi = 0$ (with $V(0) = 0$), and the value the scalar field evolves towards, as the expansion progresses. The theory is specified by the action:

$$S = \int d^4x\sqrt{-g}\{\frac{R}{16\pi G} - (1/2)\nabla^{\mu}\phi\nabla_{\mu}\phi - V(\phi) + \mathcal{L}_{Matt}\} \tag{2.1}$$

where G is Newton's constant, R stands for the Ricci scalar of the space-time metric $g_{\mu\nu}$, indices are raised using the inverse metric $g^{\mu\nu}$, and ∇_{μ} are metric compatible derivative operators (which when acting on scalar fields coincide with the usual coordinate partial derivative operators). $\mathcal{L}_{Matt}$ stands for the Lagrangian of the of matter fields other than ϕ (i.e. say the standard model of particle physics).

Standard variational principle leads to Einstein's equation for the space-time metric,

$$R_{\mu\nu} - (1/2)g_{\mu\nu}R = 8\pi G(T^{(Matt)}_{\mu\nu} + T^{(\phi)}_{\mu\nu}) \tag{2.2}$$

where $R_{\mu\nu}$ stands for the Ricci tensor of the space-time metric, the first term in the RHS represents the energy momentum of ordinary matter, which will be absent during inflation,[6] and the second that of the scalar field which is given by,

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - (1/2)g_{\mu\nu}(\nabla^\rho\phi\nabla_\rho\phi + 2V), \tag{2.3}$$

as well as the Klein -Gordon equation for the scalar field:

$$\nabla^\mu\phi\nabla_\mu\phi = -\frac{\partial V}{\partial\phi}. \tag{2.4}$$

The space-time metric one considers is, up to small perturbations (restricted to the relevant degrees of freedom for the problem at hand, namely the scalar perturbation known as the Newtonian potential ψ and the tensor perturbation h_{ij}), and using a specific gauge,[7] that of a spatially flat Robertson Walker cosmology[8]:

$$ds^2 = a^2(\eta)\{-[1 + 2\psi(\vec{x}, \eta)]d\eta^2 + [(1 - 2\psi(\vec{x}, \eta)\delta_{ij} + h_{ij}(x, \eta)]dx^i dx^j\} \tag{2.5}$$

where a stands for the cosmological scale factor, while the scalar field is expressed as $\phi = \phi_0(\eta) + \delta\phi(\vec{x}, \eta)$. Therefore, here, one is separating the treatment of the homogeneous (or zero) mode, from the modes that exhibit nontrivial spatial dependence.

The background (on top of which the perturbations of interest will be considered) is taken to represent a spatially flat homogeneous and isotropic space-time, and corresponds to setting $\psi = 0$, $h_{ij} = 0$, $\delta\phi = 0$. For such situation, Einstein's equations yield:

$$3\mathcal{H}^2 = 4\pi G(\dot{\phi}_0^2 + 2a^2 V_0), \tag{2.6}$$

$\mathcal{H} \equiv \dot{a}/a$ where "$\cdot$" $= \frac{\partial}{\partial\eta}$, while scalar field equation is:

$$\ddot{\phi}_0(\eta) + 2\dot{\phi}_0(\eta)\mathcal{H} + a^2\frac{\partial V}{\partial\phi} = 0 \tag{2.7}$$

Note that the relation between the standard co-moving time t and the conformal time η we are using is given by $dt/d\eta = a$. A further assumption regarding this

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[6] In accordance to the view that the dilution caused by even the very early stages of inflation is sufficient to essentially erase all contributions to the energy momentum coming from other sectors.

[7] In working with perturbation theory in a general relativistic context, one invariably encounters ambiguities known as "the gauge freedom", and the cosmological setting is no exception. Fortunately, in this situation there are various approaches to deal with it in a satisfactory manner. One approach works with gauge "invariant variables" [20], and another just fixes the "gauge". We will not discuss this issue further, and will work in the context of a fixed gauge.

[8] The spatially flat Robertson Walker space-time metric corresponds to that in Eq. (2.5) only when setting $\psi(\vec{x}, \eta) \equiv 0$ and $h_{ij}(\vec{x}, \eta) \equiv 0$.

background is that the solution corresponds to a "slow roll" situation characterized by the smallness of the so "called slow parameters", in particular $\epsilon \equiv 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}$.

Under those conditions, the cosmological expansion is almost exponential, corresponding, in conformal time, to $a(\eta) \approx Ce^{H_I t} = -\frac{1}{\eta H_I}$. For definiteness, we set $a = 1$ at the "present cosmological time", the starting time for inflation $\eta = -\mathcal{T}$ and its end point at $\eta = \eta_0 < 0$ so that the inflationary regime correspond $\eta \in (-\mathcal{T}, \eta_0)$. The inflationary epoch is supposed to be followed by a standard hot Big Bang cosmological development, with radiation and matter dominated epochs.[9]

On top of this background, one considers the perturbations or fluctuations characterized by nontrivial $\psi(\vec{x}, \eta)$, $h_{ij}(\vec{x}, \eta)$, and $\delta\phi(\vec{x}, \eta)$. Until this point, our treatment coincides with the standard one. So, let us start by reviewing and examining the "established lore".

Before proceeding, however, let us clarify that the goal of the kind of study one wants to undertake is to obtain an expression of the "power spectrum" of the various kinds of perturbations occurring in the universe. In particular, we will be considering the scalar density perturbations (and later the so called tensor or primordial wave perturbations). In any event, it is worthwhile clarifying for the reader the usage of these concepts.

The question is conveniently discussed representing the quantity of interest $\chi(\eta, \vec{x})$, which are functions of $(\vec{x}, \eta)$, in terms of their spatial Fourier transform coefficients $\chi_{\vec{k}}(\eta)$:

$$\chi(\vec{x}, \eta) = \frac{1}{(2\pi)^{3/2}} \int d^3k \chi_{\vec{k}}(\eta) e^{i\vec{k}.\vec{x}}. \tag{2.8}$$

For our universe, these variables, and thus the corresponding Fourier coefficients, take some specific values. However, cosmologists often proceed by considering an hypothetical ensemble of possible universes of which ours is a "fair" or "typical" representative. The idea is then to describe the statistical distributions of the relevant coefficients over such imaginary ensemble of universes.

In this way, one defines $\mathcal{P}_\chi(\eta, k)$, the power spectrum of χ, through the expression:

$$\overline{\chi_{\vec{k}}(\eta)\chi^*_{\vec{k}'}(\eta)} = \delta^3(\vec{k} - \vec{k}')\mathcal{P}_\chi(\eta, k), \tag{2.9}$$

where the overline represents the average of the quantity in question over such ensemble of universes. It follows from the above definitions that one might characterize the spectrum by

$$\overline{\chi(\vec{x}, \eta)\chi(\vec{y}, \eta)} = \frac{1}{(2\pi)^3} \int d^3k \mathcal{P}_\chi(\eta, k) e^{i\vec{k}.(\vec{x}-\vec{y})}. \tag{2.10}$$

The empirical analysis then proceeds by considering that our universe should be a typical random element of that ensemble.

[9]This transition is thought to be the result of the "reheating" process [16], where the "energy" stored in the inflaton field is transferred to ordinary components of the hot Big Bang universe.

2.1 The Standard Treatment

As it is customary, we will first focus on the so called scalar perturbations. The first step in this treatment requires dealing with the fact that the perturbed evolution equations link $\psi(\eta,\vec{x})$ and $\delta\phi(\eta,\vec{x})$. This is resolved in this context by passing to a characterization of the situation in terms of the combined variable introduced in [36, 37]:

$$v \equiv a\left(\delta\phi + \frac{\dot{\phi}_0}{\mathcal{H}}\psi\right), \tag{2.11}$$

This new field variable is now subjected to a quantum treatment, which is carried out in the standard manner appropriate for quantum fields in a curved space time background. That is, one constructs a Fock- Hilbert space and writes the field variable in terms of suitable creation and annihilation operators.[10]

$$\hat{v}(\eta,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left(\hat{a}_{\vec{k}} v_{\vec{k}}(\eta)e^{i\vec{k}.\vec{x}} + \hat{a}^{\dagger}_{\vec{k}} v^{*}_{\vec{k}}(\eta)e^{-i\vec{k}.\vec{x}}\right), \tag{2.12}$$

The specific choice of the mode functions corresponds to the selection of a particular vacuum state $|0\rangle$ (characterized by the requirement that $\hat{a}_{\vec{k}}|0\rangle = 0$). In the situation at hand, the natural choice for the mode functions $v_{\vec{k}}(\eta)$ corresponds to that which mimics the usual choice in Minkowski space-time, in the limit in which $\eta \to -\infty$.[11] This leads to the so called "Bunch Davies" state, but other possibilities are available.

As noted before, and as a result of the exponential expansion, after a short time into the inflationary regime the Universe is taken to be homogeneous and isotropic (H&I), both in the part that could be described at the "classical level", as well as that which is characterized at the quantum level.

A fundamental observation is that this vacuum state (as well as most of the alternatives considered) is a fully homogeneous and isotropic state. This can be readily seen by considering a spatial displacement of the state by $\vec{D}$. This is just given by $e^{i\hat{\vec{P}}.\vec{D}}|0\rangle = |0\rangle$ (where $\hat{\vec{P}}$ is the operator representing generator of spatial displacements in the Hilbert space, namely the "total momentum operator"), as follows immediately from the fact the $\hat{\vec{P}}|0\rangle = 0$, indicating that the state is completely homogeneous. Similar considerations hold regarding isotropy, namely the invariance of the state under rotations.

[10] Applying such procedure to gravitational perturbations themselves is rather worrisome. First, because the resulting interacting theory is non-renormalizable (a problem that is now a days deemed as non essential, as follows from the effective field theory point of view). On the other hand, and more importantly, the causal structure of the quantum field theory might deviate from that "true causal structure" dictated by the combination of the metric background together with the perturbations [34, 35].

[11] That is, one sets initial conditions for the mode functions that would correspond to the "positive energy solutions" if used as initial data in Minkowski space-time.

At this point, following the standard approach, one is directed to consider that the relevant quantity to investigate the behavior of these "perturbations", is the so called two point correlation function evaluated in the vacuum state $\langle 0|\hat{v}(\vec{x},\eta)\hat{v}(\vec{y},\eta)|0\rangle$. The argument is that such object represents the "quantum fluctuations".[12] From there, one extracts the so called "power spectrum":

$$\langle 0|\hat{v}(\vec{x},\eta)\hat{v}(\vec{y},\eta)|0\rangle = \frac{1}{(2\pi)^3}\int d^3k e^{i\vec{k}(\vec{x}-\vec{y})}\mathcal{P}_v(k). \tag{2.13}$$

The result is (in the limit of infinitely slow roll conditions) $\mathcal{P}_v(k) \sim k^{-3}$.

Now, the point is that the characterization of the quantity $\delta(\eta,\vec{x}) \equiv \frac{\delta\rho(\eta,\vec{x})}{\bar{\rho}(\eta)}$, where $\bar{\rho}(\eta)$ is the spatial average of the universe's density $\rho(\eta,\vec{x})$, with $\delta\rho(\eta,\vec{x}) \equiv \rho(\eta,\vec{x}) - \bar{\rho}(\eta)$, in terms of the "power spectrum" as

$$\overline{\delta(\vec{x},\eta)\delta(\vec{y},\eta)} = \frac{1}{(2\pi)^3}\int d^3k\mathcal{P}_\delta(\eta,k)e^{i\vec{k}.(\vec{x}-\vec{y})} \tag{2.14}$$

where now the average is over pairs of equally separated points in our universe seems to correspond to a Harrison-Zeldovich (HZ), or scale invariant[13] spectrum namely $\mathcal{P}_\delta(k) \propto k^{-3}$. Indeed, after the inclusion of well understood late time physical effects, such as plasma oscillations, etc. (as well as a slight "tilt" or small deviation from the exact exponent of -3, which depends on the small roll parameter ϵ) that spectrum is in excellent agreement with the observations [21].

The issue is, however, that the "power spectrum" we obtained from quantum considerations is now being taken to characterize the primordial inhomogeneities. These include the seeds of all cosmic structure, and thus are what eventually lead to the generation of galaxies, stars, planets, eventually life, and then creatures like ourselves, capable of wondering about the origin of it all.

This all sounds like a really astonishing account, which is, moreover, so full of profound poetic undertones that is hard to resist. Furthermore, as noted above, the theoretical account seems to fit observations to a remarkable extent, so it is not surprising that the prevailing attitude among cosmologists is "what else can one ask for ?".

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[12] In my view, a substantial amount of the confusion among practitioners is due, in part, to the unfortunate usage of the word "*fluctuations*" to refer to various rather different notions: *(i)Statistical variations in an otherwise symmetric ensemble, (ii) Spatial variations in a single extended object, which is homogeneous at large scales, and (iii) quantum indeterminacies.* In our case, these are uncertainties or indeterminacies **in** the quantum state, **for** the field and conjugate momentum operators. I.e. an instance of (iii), which is often taken by cosmologists to represent either an instance of (i) or an instance of (ii).

[13] Note that as the LHS of (2.14) is dimensionless, so $\mathcal{P}_\delta(k)$ has to have dimensions of $length^3$ (as k has dimensions of $length^{-1}$. Thus, any kind of power law would require a length scale L_0 so that $\mathcal{P}_\delta = L_0^{3+n}k^n$, unless $n = -3$. This spectrum was favored in early phenomenological cosmology considerations [23, 24].

The reader should already find some discomfort in the fact that all 3 notions of fluctuations seem to be subject to a rather liberal interchange through the usage of the same wording "*power spectrum*" for the characterization of quantities appearing in Eqs. 2.10, 2.13 and 2.14. We will ignore for the most part in this work the distinction between ensemble averages and spatial averages over a single inhomogeneous universe, and focus on the more problematic relationship between the quantum characterization and the former two (for further discussion on these issues see [22]).

One often hears in the cosmology community this issue characterized as the problem of the "quantum to classical transition". This, in my view, is a misnomer. There is presumably nothing that is truly classical at a fundamental level, and, therefore, a classical description can, at best, be one corresponding to some kind of approximated characterization of the situation. Thus, the framing of the issue in that manner is really missing the point. The real question is the transition from a situation of full homogeneity and isotropy to one that is not. Moreover, as the process is taken to be described in quantum mechanical terms, what we need to account for is a transition from a quantum state that is homogeneous and isotropic to one that is not.[14] The degree to which the latter might be suitably described in classical terms is a question of the accuracy of the approximation, and although important, it is, in a sense, secondary to the main issue that concerns us here, and which we will be expanding on below.

3 The Case for a Modified Approach and a Specific Proposal

We have so far hinted at, and will shortly discuss in more detail, a problematic aspect of the presently accepted accounts of the emergence of primordial perturbations during inflation as a result of quantum fluctuations. It is easy to see that this is, of course, a particular instance of a broader problem afflicting quantum theory, namely the so called measurement problem. In this regard, it is convenient to remind ourselves of the result of [25] establishing the logical inconsistency of accepting the following three postulates regarding quantum theory (viewed as a fundamental theory):

(A) The description of an isolated physical system by its quantum state is complete (and thinking about our case, we ought to note that the universe is the epitome of an isolated system). (B) The evolution of such system is always dictated by Schrödinger's equation.[15] (C) Individual concrete experiments lead to definite results.

[14]This is actually only a problem for those accounts of quantum theory, that posit that the characterization of a system by its quantum state is complete, and could be easily addressed by approaches involving theories with (non-local) hidden variables, such as de-Broglie-Bohm type approaches.

[15]We ought to emphasize that for instance, the Copenhagen, interpretation explicitly forsakes this postulate as it includes the clause that the state of a system instantaneously turns into one of the eigen-states of the quantity that is being measured (thereby failing to evolve according to the Schrödinger's equation at that time) in a stochastic manner with probabilities dictated by Born's

The need to forsake (at least) one of the above forces one towards a specific conceptual path depending on the choice one makes. Concretely speaking, forsaking (A) seems to lead naturally to hidden variable theories, such as de-Broglie Bohm or "pilot wave" theory. Forsaking (B), one is naturally led to collapse theories, which for the cosmological case seem to leave no option but those of the spontaneous kind, (as there is clearly no role for conscious observers or measuring devices that might be meaningfully brought to bear to the situation at hand). Finally, forsaking (C) seems to be the starting point of approaches such as the Everettian type of interpretations. These, again, seem quite difficult to be suitably implemented in the context at hand, simply because observers, minds, and such, notions that play an important role in most attempts to characterize the world branching structure in those approaches, can only be accounted for within a universe in which structure has already developed, well before the emergence of the said entities.

We will focus in the present manuscript on the consideration of path (B), although it seems clear that at least path (A) seems to offer quite a reasonable alternative. However, before entering that discussion, we want to make the case that the account described in Sect. 2.1 is not satisfactory at all. That is, despite its phenomenological success, the picture it offers about the historical development of our universe is, frankly speaking, conceptually inadequate.

3.1 Conceptual Difficulties in the Standard Approach

The first problem, as already noted, is that, according to the above framework, the universe was H&I, both at the level of the part that is described in classical terms, the background metric and background scalar field, as well as in those aspects that are treated in quantum mechanical terms, the metric and scalar field perturbations. The latter can be seen in the fact that the quantum state, the so called Bunch Davies vacuum, is invariant under rotations and translations. Actually, this symmetry is itself the expected result from the early stages of inflation.[16]

rule. Of course the point is that the theory is rather unclear about what kind of interaction does qualify to be considered a measurement.

[16]The argument is that, even if the situation was not exactly homogeneous and isotropic at the classical level, and the state of the quantum field was not exactly the vacuum, the inflationary process itself would drive relatively a broad set of initial conditions towards precisely such homogeneous and isotropic stage for the space-time metric and the quantum state of all fields towards the vacuum. One might then expect small deviations of order e^{-N} to survive after N e- folds. However such minuscule relics from the initial stages of inflation are deemed just too small to be of any relevance. Note that, if those were relevant at all, they would destroy the predictability of the model. The point is that they would be completely unpredictable in the absence of a fundamental quantum gravity theory including a precise prescription of "initial conditions", or whatever replaces that, if such theory is a timeless theory, such as a canonical version of quantum gravity. We will thus ignore any such possible remnants from the pre-inflationary regime and talk as if the situation is exactly homogeneous and isotropic, where the caveat "*up to possible corrections of order* e^{-70}" is implicitly understood.

The issue is the following conundrum: According to the inflationary characterization of the very early universe, the starting point of the analysis corresponds to a situation that is completely homogeneous and isotropic, while the dynamics controlling the evolution explicitly preserves such symmetries. How is it then that we end up with a situation that does not share those symmetries? Indeed, how is it that we are able to make predictions about those inhomogeneities and anisotropies at all? Multiple attempts have been made [26, 27] to try addressing these questions within orthodox and traditional physical practice, but all those have come up short. For a detailed discussion see [28].

The second conceptual difficulty lies, as we discuss below, in the fact that it is rather unclear what is the theoretical framework one is relying on when proceeding according to the standard treatment. Regarding the matter fields, one might consider that one is working with quantum field theory on curved spacetime (QFT in CS), a subject with a rather well developed formalism (see [59]). This is so even though the scalar field is being separated into the "classical background" and the perturbations. The point is that one might regard the separation of the zero mode of the quantum field $\hat{\phi}_0(\eta)$ from the other $\delta\hat{\phi}(\eta, \vec{x})$ (space-dependent modes) and consider that the quantity $\phi_0(\eta)$ appearing in say Eq. (2.7) actually represents the expectation value of the zero mode $\langle\phi_0(\eta)\rangle$ in some highly excited coherent state, while the vacuum state refers only to the state of the spatially nontrivial modes. Regarding gravitation, however the issue is much more delicate. The fact is that one is certainly not working with a quantum gravity theory, or any approximation thereof, simply because we do not have a developed and workable version of such theory.[17] One is not working with classical gravity either, as there are at least some parts of the space-time metric that are being treated in a quantum language (ψ and h_{ij}). What one is doing is separating the metric into background and perturbations, treating the first part in a classical language, and the second part as a quantum field theory on the background space-time provided by the first. This has several problematic features. First, the separation of the metric into background and perturbations can only be viewed as a matter of convenience, and not as something of a fundamental nature. Actually, by suitable changes of coordinates one can pass from one such particular separation into another. Thus it is rather unclear in what sense such a distinct treatment for both is justified. One might think that what one is doing here is similar to what was done regarding the inflaton field, and that was just described above. However, that is not what is going on, simply because, as we noted, we do not have the full quantum

[17]There are multiple approaches in the attempt to construct a fully satisfactory theory of quantum gravity. These include the most popular such as String Theory and Loop Quantum gravity, both still confronting severe obstacles. There are others which are less well known but not for that less deserving, such as causal dynamical triangulations, causal sets, non-commutative geometry, etc. The point is, nevertheless, that none of those approaches is able, at this time, to both recover in a rigorous way general relativity as an approximation, and truly contend with the full quantum nature of what one can expect to be a fundamental theory of space-time, namely one that can deal, not only with causal structures subject to quantum indeterminism, but can also incorporate notions of space-time that accommodate states of matter in superposition of substantially different energy momentum distributions.

gravity theory that would justify that (i.e. we do not have, for quantum gravity, any theory that might be said to be playing the analogous role as quantum field theory on curved space-time). Moreover, we must note that when considering the treatment of the metric perturbations as a QFT in CS, one is, actually, doing severe violence to that framework from the start. The point is that the construction of QFT on CS has as **a basic postulate** that the quantum fields so constructed must have causal commutation relations (i.e. fields at space-like separated points must commute). In the present context one would be imposing the commutation relations for the fields according to the causal structure of the background space-time, rather than that of the "actual" physical space-time, which would be in part characterized by those quantum objects themselves. These and related issues have been discussed by other authors (see for instance [34, 35]). One might dismiss all those concerns and argue somehow that what one is doing is justified as an approximation, and that would probably be something one would tend to agree with. However it seems clear that a conceptually satisfactory picture would only be at hand if one had a clear idea of "an approximation to what" one is supposed to be considering.

We will be motivated by the quest for a clear explanation, framed within a general theoretical setting, that offers at least plausible answers to reasonable questions naturally arising in the situation at hand.

3.2 Spontaneous Collapse of the Quantum State and Einstein's Semi-classical Equations

First, we note that what seems to be required to address the issue at hand is to consider a physical process occurring in time, explaining the emergence of the seeds of structure. After all, emergence means (in this context): *something that was not there at an early time, is there at a later time*.

We need to explain the breakdown of the symmetry of the initial state. We do not want to have to put the inhomogeneities by hand at the start (as that would remove all the hope of predictability of the inflationary model). The theory we are dealing with does not lead through its standard Hamiltonian evolution to a breakdown of the symmetries we are considering. Therefore, something else is required. Spontaneous collapse theories do naturally contain the elements to achieve what is needed: departure from unitary evolution and stochasticity.

Thus, we will *add* to the standard inflationary accounts of very early cosmology, the spontaneous collapse of the wave function. On the other hand, that is not something that can be done straightforwardly. We need to discuss the obstacles such program faces, and the paths we have taken in the quest to overcome those.

We should start by noting that as the spontaneous collapse theories are described by a modification of the dynamics, concretely the time evolution of the state of quantum systems (in our case quantum fields), we seem to be forced to rely on a classical description of the space-time geometry. However, as we will see, for the sake of conceptual clarity,[18] there are even stronger reasons to proceed in this way.

It is worthwhile emphasizing at this point that the interface between quantum theory and gravitation need not involve the *Planck regime*: Consider, for instance, the issues that would have to be confronted in attempting to describe the space-time associated with a macroscopic body in quantum superposition of states localized in two distant regions. A rather influential work [29] considers such an experiment and claims to show semi-classical GR is simply not viable. The core of the argument is the following dichotomy: (1) If there are no quantum collapses, then semi-classical GR conflicts with their experiment. (2) If there are quantum collapses, then semi-classical GR equations are internally inconsistent. In this last regard, the issue is that a quantum collapse would generally be associated with failure of one side of Einsteins's equation, namely that containing the expectation value of the energy momentum tensor, from being divergence free, while the other side is automatically divergence free as a result of Bianchi's identities (see for instance Eq. (3.1)). This and related issues have been considered by various authors, but there is no clear consensus in the conclusions (see for instance [30–33]).

Thus, if the conclusions of [29] were correct, how could one possibly make sense of our approach? The point is that we might regard semi-classical GR, not as a fundamental theory but just as an *approximated description with limited domain of applicability*. We then consider the present line of research as an attempt to push that domain beyond what is usually viewed as a natural boundary. In the present context, we want to consider spontaneous collapses as the missing element that provides plausible resolution to the basic questions discussed in the previous subsection. Indeed, as it is clear that during the spontaneous collapse the equations can not be valid, we can not hope to consider the approach as fundamental. The proposal is, then, to follow an hydro-dynamical analogy: The Navier-Stokes equations for a fluid can not hold in some situations, for instance when a wave is breaking in the ocean. But they can be taken to hold to a very high approximation before and after that. Thus, we take semi-classical GR equations to hold before and after a spontaneous collapse, but not at the "time" it is occurring. In order for such approach to be fully specified, it must be supplemented by a well defined formalism that includes a recipe of how to join the descriptions "just before" and "just after" the spontaneous collapse.

In order to make a concrete proposal for considering the ideas described above, we need to specify a manner which might sensibly incorporate spontaneous collapse into the context of semiclassical GR. At the formal level we take as starting point a slight modification of what is normally described as semi-classical gravity, the theory of classical gravitation, together with the theory of quantum fields on a curved space-time. We will proceed by relying on the notion of *Semi-classical Self-consistent Configuration* (SSC) introduced in [61].

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[18]In the pursuit of the goal of conceptual clarity, we will try to avoid perturbation theory from playing the dual role it is often relied upon: That of making the calculations manageable, and at the same time helping hide from explicit view some of the most serious foundational difficulties. We will not be able to avoid the former, but we will make all efforts to prevent the latter.

Definition: The set $\{g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \in \mathcal{H}\}$ represents a SSC iff $\hat{\varphi}(x), \hat{\pi}(x)$ $\mathcal{H}$ corresponds to QFT (that is, $\mathcal{H}$ is a Hilbert space and $\hat{\varphi}(x)$, $\hat{\pi}(x)$ represent the quantum field and canonical conjugate momentum operators, as distributional valued operators acting on it and realizing the canonical commutation relations, and satisfying the corresponding evolution equations) constructed over the space-time with metric $g_{\mu\nu}(x)$, and the state $|\xi\rangle \in \mathcal{H}$ is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G\langle\xi|\hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)]|\xi\rangle \tag{3.1}$$

The scheme is simply the standard QFT construction on a given space-time, except for the requirement that there be a special quantum state taken to be the one corresponding to the physical situation at hand, and such that the above equation holds. That requirement gives the whole scheme the kind of self referential features which occur in the Schrödinger-Newton system [41–44]. One might regard the SSC formalism as the General Relativistic version of the latter. We note that most other states in $\mathcal{H}$ will fail to satisfy Eq. (3.1)[19] and thus would have to be considered as un-physical.[20]

Next, let us consider a spontaneous collapse *a la* GRW, i.e. a sudden transition from a given quantum state $|\xi\rangle$ to another $|\tilde{\xi}\rangle$, as dictated by the theory and the "stochastic choice". That would leave us with something that is no longer a SSC as the new state will fail to satisfy Eq. (3.1).[21] In order to remain as close as possible to such formalism, we will contemplate, instead, a spontaneous jump from one complete SSC to another one. That is, the usual "GRW jumps" must be considered now as generalized jumps of the form one full SSC to another, i.e. the spontaneous transition must now be regarded as $SSC1 \to SSC2$. It is, however, clear that generically the Eq. (3.1) will not hold during the jump itself.

In order for the scheme to be well defined, we must supply matching conditions: for both space-time and for the states in the Hilbert space. This has been studied in some detail in [61] and it involves various delicate issues.[22] All those aspects will

[19]To see this, simply consider a given SSC, and an arbitrary smooth function of compact support f, take $\hat{\phi}(f) \equiv \int \sqrt{-g}d^4xf(x)\hat{\phi}(x)$, and define the new state $|\chi\rangle = \hat{\phi}(f)|\xi\rangle$. It should be quite clear that the expectation value of the energy momentum for the state $|\chi\rangle$ will differ from the corresponding one for $|\xi\rangle$. So if the latter satisfies Eq. (3.1), then the former will not.

[20]In this sense, we are advocating a point of view where there is already, at this stage, a breakdown of the superposition principle, because, even if there are two states $|\xi\rangle$ and $|\tilde{\xi}\rangle$ which happen to satisfy Eq. (3.1), generic superpositions of these states would not. The principle would have to be considered valid within the present formalism only as a certain type of approximation.

[21]Indeed, one of the most important characteristics of, say GRW or CSL theories, is their tendency to increase the localization of states in the sense of suppressing superpositions of states corresponding to rather different mass densities distributions in physical space. As such, it seems clear that a spontaneous collapse would generically imply an important change in the spatio-temporal form of the expectation value of the energy momentum tensor.

[22]One of the requirements is that a reasonable spontaneous collapse be such that, when starting with a state with a reasonably defined renormalized energy momentum tensor(i.e. a so called Hadamard state), the spontaneous collapse dynamics leads to a state with the same characteristics (i.e. another Hadamard state). That issue has been explored in [60]. A second problem is that the Hilbert space of

certainly become even more complex and vexing when considered in the context of a theory involving continuous spontaneous collapse such as CSL.

Next, let us consider how could all this possibly fit with our current views regarding an ultimate theory including quantum gravity. Let us start by recalling that the whole quantum gravity program is still confronted with various outstanding issues and conceptual difficulties. Among those we should mention (i) *The Problem of Time* [45] i.e. the fact that canonical approaches to quantum gravity lead to timeless theories, and (ii) the difficulties concerning the identification of suitable observables [46]. More generally, there is the issue (iii) of how to recover space-time and something resembling general relativity from various of the existing approaches, and specifically those of the canonical type.

Solutions to (i) often rely on the use of a dynamical variable as a physical clock and consider relative probabilities (and wave functions). Following that line seems to lead to something like an approximated version of the Schrödinger equation, but with corrections that violate unitarity (see [47]). Although tantalizing, it is not clear, however, that the specific form that such issue takes in the analysis of that work is of the kind that could lead to a resolution of the questions at hand here.

Regarding (iii) there are many suggestions indicating space-time might be an emergent phenomena (see for instance [48–50]). In that case, it is not clear that g_{ab}, as such, should be "quantized" any more than the heat equation should. Under such circumstances, the classical level of description might be the only setting in which notions of space-time may be talked about meaningfully.[23]

The point is that any talk about space-time concepts, in anything close to the standard sense, implies that one is already working within some classical description, as we simply have no idea of how to think of a quantum space-time. Therefore, even when considering that one might have started from some hypothetical satisfactory and fully workable theory of quantum gravity, by the time we reach the level of discussion where we can talk of space-time in the usual terms, we would have proceeded through a long chain of approximations and simplifications. Under those conditions, it does not seem unnatural to expect that some traces of the full quantum gravity regime might survive and remain relevant at the stage we are dealing with. Moreover, these might, from the perspective of the standard space-time language

the second SSC might not be unitarily equivalent to the first one. Even if they are, the unitary mapping might not be unique, making it difficult to identify the state in the new Hilbert space resulting from the spontaneous collapse theory applied to the state of the SSC previous to the collapse. Initial proposals to deal with this issue were discussed and implemented in simple situations in [61, 62], and further studies are under development [63].

[23]In thinking about the heat equation, it seems clear that, while at macroscopic effective level, heat flow is a concept that can be truly made sense of, when considering the situation at the more fundamental level, of say, many particle quantum mechanics, the notions involved would become, at best, secondary. Raising, for instance, a question regarding the "quantum operator" characterizing heat is unlikely to lead to any meaningful answer. Analogously, it might well be, although we have, of course, no proof one way of the other, that no sensible definition of a quantized space-time metric is truly compatible with whatever the fundamental quantum description of gravity is.

"look like spontaneous collapses".[24] Let us think again of the hydrodynamic analogy. Here, we might consider the situation where, say, foam forms after the breaking of an ocean wave, a situation that presumably poses no great problems (at least in principle) if we go all the way down to the description of what we call the fluid, in terms of molecular dynamics. It clearly would look rather strange if we attempt to incorporate some kind of phenomenological terms describing such effects within the Navier-Stokes formalism.

One might characterize the present approach to the consideration of gravity / quantum interface as an essentially bottom up approach, in contrast with the usual "top-down" approach, where one starts with what is presumably a well defined proposal for the full theory of quantum gravity, and then work towards establishing a connection between the formalism and the empirical world. The strategy adopted here starts by considering theories that are rather well understood, supported by substantial experimental evidence, specifically general relativity and quantum field theory treatment of matter fields (in this curved space-time version). Then we push their range of applicability towards the domain where presumably new physics might be required, seeking, in the process, to obtain clues about the features of the ultimate theory.

4 Practical Treatment Adapted to the Cosmological Setting

Trying to apply the above formalism in any specific concrete situation of interest, having no extraordinarily simplifying features, and many relevant degrees of freedom, is evidently an almost impossible task at the practical level.[25] We will, therefore, work by making several suitable simplifications, including using the simplest inflationary model where the potential[26] is just $V = (1/2)m^2\phi^2$; focusing our attention on the usage of the formalism in the inflationary cosmological setting, while trying to follow its basic rules. Furthermore, instead of constructing a new complete quantum field theory in a curved space-time that results from previous spontaneous collapses, we will keep using a single QFT theory construction, that corresponding to the background space-time. That is, we will consider jumps in the quantum state, and the corresponding changes in the space-time metric. However we will neglect the

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[24] At this point, and taking a completely agnostic posture regarding what the fundamental theory of quantum gravity might look like, it is hard to offer anything beyond a simple analogy. Thus, one might want to consider a scientist trying to come to terms with, say, the formation of foam when a ocean wave breaks on the shore, while having no clue about the molecular nature of what he normally describes as a fluid using standard tools of hydrodynamics.

[25] In [61], the treatment was applied to the excitation of a single anisotropic mode in the space-time metric. The formalism was later applied [62] to the case of the consecutive excitation of a second mode, allowing the study of essential aspects of the generation of tensor modes.

[26] This specific potential is usually taken as disfavoured by observations [53]. More recent analysis claim quite generally that convex potentials are excluded at the 95% confidence levels [54]. As noted below, that conclusion does not extend to our approach.

requirement of simultaneously changing the Fock-Hilbert space, which will be kept fixed (i.e. we will keep using the Hilbert space constructed for the background SSC, namely the one corresponding to the situation before the breakdown of homogeneity and isotropy).

This simplified treatment seems justified by the smallness of the so called perturbations of the metric (characterized by the 10^{-5} deviations from isotropy observed in the cosmic microwave radiation), and the corresponding smallness of the modification of the Hilbert space. The second simplification is that, although the zero mode of the scalar field (i.e. the mode corresponding to $\vec{k} = 0$ and thus involving no spatial dependence) must, according to our formalism, be treated quantum-mechanically (as should all matter fields), we will describe it as a classical background in the practical calculations. The point is that the zero mode of the field will be taken, as described in Sect. 3.1, to be in a highly excited (and sharply peaked) state, and take $\phi_0(\eta)$ (of Sect. 2) as corresponding to the expectation value of the zero mode $\langle\hat{\phi}_0(\eta)\rangle$. The quantum treatment of the zero mode was included in the works [61, 62] mentioned above. That analysis indicates that, to the level of approximation one is working with, the final results are the same as those we will describe now. The space dependent modes will, just as in the standard approach, be treated quantum mechanically, and taken to start in the "vacuum state". However, the field in question is now the scalar field $\delta\phi$, rather than the composed field v (involving both, matter and metric perturbations) of Sect. 2.1.

It turns out to be convenient to make a change of variables in field space and work with the re-scaled field $y \equiv a(\eta)\delta\phi(\eta, \vec{x})$. The momentum canonical conjugate to that field in conformal time is $\pi = a\delta\phi'$. These objects are now treated according to the standard methods of quantum field theory on curved space-time [59]. In our case, that space-time background is provided by the metric (2.5) with the perturbations set to 0. Thus, the field operator can be represented by the operator as:

$$\hat{y}(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_{\vec{k}} y_{\vec{k}}(\eta) e^{i\vec{k}.\vec{x}} + \hat{a}^{\dagger}_{\vec{k}} y^{*}_{\vec{k}}(\eta) e^{-i\vec{k}.\vec{x}} \right), \tag{4.1}$$

with the modes $y_{\vec{k}}(\eta)$ chosen again according to the Bunch Davies prescription, so that the state satisfying $\hat{a}_{\vec{k}}|0\rangle = 0$ is the Bunch Davies vacuum.

The basic scheme of the analysis is now the following: During the early stages of inflation taken to correspond to $\eta = -\mathcal{T}$, the starting point of inflation (see Sect. 2), the state of the field $\hat{y}$ is the Bunch-Davies vacuum, and the space-time is homogeneous and isotropic. In that state, the operators corresponding to the Fourier components of the field and momentum conjugate ($\pi = a\delta\phi'$), $\hat{y}_k$, $\hat{\pi}_k$ are characterized by gaussian wave functions centered at 0 with uncertainties Δy_k and $\Delta\pi_k$. To the extent that the spontaneous collapse dynamics is ignored, and as we are working in the Heisenberg picture, the state of the field does not change with time.

The spontaneous collapse dynamics, treated using the interaction picture, modifies the evolution of the quantum state during the inflationary epoch, resulting in a change of the expectation values of $\hat{y}_k(\eta)$ and $\hat{\pi}_k(\eta)$. We will assume the spontaneous collapse occurs mode by mode,[27] and is described by some version of a spontaneous collapse theory, suitably adapted to the situation at hand (we will be more explicit in this point later on).

It is worth noting that, in this approach, it is clear that our universe would correspond to one specific realization of the stochastic objects or functions occurring in the spontaneous collapse dynamics. Moreover, we will consider that to each field mode $\vec{k}$ corresponds an individual and independent stochastic object. We will shortly see schematically how this is realized within the context of a specific theory.

Let us now consider the scalar metric perturbations $\psi(\eta, \vec{x})$, as well as the quantities of direct observational interest. We will do so in a schematic way ignoring at this point the late time physics, which we take as well understood, and which is usually incorporated by the introduction of so called "*transfer functions*" [51], which modify the results coming directly from the inflationary regime.

The Fourier decomposition of the relevant semi classical Einstein's Equations takes the form (see Eq. (64) of [7] in the limit of exponential expansion):

$$- k^2\psi(\eta)_{\vec{k}} = \frac{4\pi G\phi_0{}'(\eta)}{a}\langle\hat{\pi}(\vec{k}, \eta)\rangle = c\langle\hat{\pi}(\vec{k}, \eta)\rangle. \tag{4.2}$$

At $(\eta = -\mathcal{T})$ the state is the vacuum, an homogeneous and isotropic state, and as noted, in the absence of the spontaneous collapse part of the dynamics, that will remain the case forever. In particular we would have $\langle\hat{\pi}(\vec{k}, \eta)\rangle = 0$, and, therefore, the space-time would also be completely homogeneous and isotropic. The spontaneous collapse will change that, so that by the end of inflation those expectation values will generically differ from zero. Note that from Eq. (4.2) we can reconstruct the space-time Newtonian potential simply by taking $\psi(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k\psi_{\vec{k}}(\eta)e^{i\vec{k}.\vec{x}}$. Of course we have to keep in mind that, if we are interested in the regimes that are more directly empirically accessible, we must consider the evolution of the physical situation from the end of inflation at $\eta = \eta_0$ through the reheating epoch and standard radiation and matter dominated eras of the hot Big Bang. We will be rather schematic in considering those aspects.

Let us focus on the quantity of main observational interest $\frac{\Delta T(\theta,\varphi)}{\bar{T}}$. It is the relative deviation from the sky mean of the temperature of the CMB, coming from a certain direction in the sky, specified by the angles θ, φ, and corresponding to the point on the intersection of our past light cone with the last scattering surface (i.e. the hypersurface corresponding to the moment the hot cosmic plasma cools sufficiently for hydrogen atoms to form, and photons to effectively decouple) at $(\eta = \eta_D)$. This is related to the fact that photons emanating from that region undergo, besides the

[27] There are two arguments that seem to justify such assumption: on the one hand, GRW and CSL dynamics seem to function in that manner in situations involving multiple degrees of freedom. More importantly, as we are dealing with changes in the state that could be regarded as very small perturbations, it seems clear that the linear level treatments should be accurate enough, and that the kind of correlation generating collapses would only occur in higher order treatments. Indeed, we have seen an indication of that kind of correlation-generation occuring in the second order treatment carried out in [62].

cosmological red shift associated with the universe's expansion, an additional red-shift as they overcome a local gravitational barrier characterized by the value, at the emission event, of Newtonian potential ψ. Including effects of plasma physics, that quantity is then given by:

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = (1/3)\psi(\eta_D, R_D, \theta, \varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta,\varphi) \tag{4.3}$$

where the last expression provides the decomposition of the sky map into spherical harmonics and defines the coefficients α_{lm}.

In our approach, we can therefore directly obtain the expression:

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = (c/3)\int d^3k e^{i\vec{k}\cdot\vec{x}} \frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta_D)\rangle, \tag{4.4}$$

where the Newtonian potential is evaluated on the η_D, the conformal time corresponding to the decoupling surface (also known as the surface of last scattering), and at the co-moving radius R_D of that surface intersection with our past light cone at the corresponding direction in the sky. The specific version and implementation of the spontaneous collapse theory, as well as the specific realizations of the stochastic processes involved, characterize the quantity $\langle\hat{\pi}(\vec{k},\eta_D)\rangle$. This, of course, depends also on the part of the evolution that does not directly tied to the spontaneous collapse dynamics (i.e. that tied to the scalar field free Hamiltonian in the given background space-time).

Thus we find,

$$\alpha_{lm} = c\int d^2\Omega Y^*_{lm}(\theta,\varphi)\int d^3k e^{i\vec{k}\cdot\vec{x}} \frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta)\rangle. \tag{4.5}$$

We note that we can not really extract a direct prediction from this expression, simply because the complex quantities $\langle\hat{\pi}(\vec{k},\eta)\rangle$ are determined by stochastic processes (one for each $\vec{k}$). However, we would obtain an explicit prediction for each angle, if we knew the result of all such stochastic processes. It is worth noting that no analogous to this expression exists in the standard approaches. Those simply do not offer an expression, not even in principle, for this quantity, which is actually that of direct observational interest. The above quantity, in the usual approach, would actually be simply zero.

Following the present approach, we are in a better situation, at least in principle, but how can this approach produce actual predictions? The point is that the Eq. (4.5) shows that the quantity of interest is the sum over a large number (actually an integral) of stochastically determined complex quantities (one for each $\vec{k}$). So the quantity of interest can be thought of as a result of a "random walk" on the complex plane.

As it is usually the case in such situations, one cannot predict the end point of such "random walk", but one can focus on the equivalent to the magnitude of the "total displacement", $|\alpha_{lm}|^2$, and estimate its most likely value, which we denote $|\alpha^{(ML)}_{lm}|^2$.

We will evaluate the latter by identifying it with that corresponding to the ensemble average over the possible realizations of the set of stochastic processes.

Thus, we proceed to compute the ensemble average (represented by an overline) at "late times". The relevant quantity is then:

$$\overline{(\langle\hat{\pi}(\mathbf{k},\eta)\rangle\langle\hat{\pi}(\mathbf{k}',\eta)\rangle^{*})} = f(k)\delta(\mathbf{k}-\mathbf{k}'). \tag{4.6}$$

where we have used the fact that the different modes are taken as statistically independent,[28] and that, at the ensemble level, we are dealing with an isotropic system, even though each individual element of the ensemble (and, in particular, the realization that actually corresponds to our universe) is **not** isotropic.

Therefore, we are led to the following estimate,

$$|\alpha_{lm}^{(ML)}|^2 = \overline{|\alpha_{lm}|^2} = (4\pi c/3)^2 \int_0^\infty dk j_l(kR_D)^2 \frac{1}{k^2} f(k). \tag{4.7}$$

Agreement with observations requires $f(k) \sim k$ (which would correspond to a Harrison-Zeldovich scale free power spectrum $\mathcal{P}(k) \sim k^{-3}$). Note that, if this was actually the exact form of the spectrum, we would have that $|\alpha_{lm}^{(ML)}|^2$ would end up being independent of R_D (changing variables to $z \equiv R_D k$, the LHS of (4.7) would take the form $(4\pi c)^2 \int_0^\infty dz j_l(z)^2 \frac{1}{z} = (4\pi c)^2 \frac{\pi}{l(l+1)}$), reflecting the "scale invariance" of this particular spectral shape. This would result in a statistically featureless CMB, which is not what is really observed. The very interesting and famous oscillations that have been the focus of the recent CMB studies, are thought to be the result of features generated by late time physical process acting on top of the primordial flat spectrum emerging from the inflationary regime.[29] The point, however, is that, when on top of the flat spectrum one places the effects of the late time physics, including acoustic plasma oscillations, all of which is encoded in the so called "transfer functions" [51], that we are ignoring (for simplicity), one obtains, after fixing a few free parameters a spectrum that represents a remarkable match to the observations.[30]

A detailed analysis of the problem based on one of the simplest inflationary models (a single scalar field with a simple quadratic potential term), together with a version of CSL adapted to the situation at hand (i.e. one involving quantum fields in a cosmological setting) has been performed in [38].

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[28] As we noted before, at higher order in perturbation theory, within the approach developed here and based on the SSC formalism, one naturally encounters deviations from statistically independence [62]. An early exploration of the possible consequences of the kind of correlations that naturally emerge include possible modifications in the part of the spectrum that refers to the very large angular scales [72]. See also [71].

[29] There is an additional feature coming from the inflationary regime itself, known as the tilt in the spectrum, which we will be ignoring in the present simplified treatment.

[30] This success after the best fit matching of few parameters, is achieved both, when following the standard accounts [54] for the primordial spectrum, and when following ours [55].

The starting point is the version of the theory described by the following two equations: The modified Schrödinger equation, whose solution is:

$$|\psi, t\rangle = \hat{\tau} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2 \right]} |\psi, 0\rangle \tag{4.8}$$

where $\hat{\tau}$ is the time-ordering operator, λ is a parameter of the theory, $\hat{A}$ is a self adjoint operator on the system's Hilbert space (usually referred to as the "collapse-driving-operator" or "collapse-operator" for short), and $w(t)$ is a random classical function of time, of white noise type, with a probability rule given by the equation,

$$PDw(t) \equiv \langle\psi, t|\psi, t\rangle \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \tag{4.9}$$

The state vector norm evolves dynamically (does *not* equal 1), so expectation values, such as those needed in expressions such as (4.5), must be computed with suitably re-normalized states.

The version of the theory adapted to the cosmological case at hand was based on an equation of the form:

$$|\psi, t\rangle = \hat{\tau} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}} \int_{-\mathcal{T}}^{\eta} d\eta' \int d\mathbf{x}[w(\mathbf{x},\eta') - 2\tilde{\lambda}\tilde{y}(\mathbf{x})]^2} |\psi, -\mathcal{T}\rangle. \tag{4.10}$$

where the operator playing the roll of the collapse operator[31] $\hat{A}$ of Eq. (4.8) was taken to be linear in the field $\hat{y}(\eta, \vec{x})$ (the quantum field operator corresponding to the re-scaled field $y = a(\eta)\delta\phi$). The result of the analysis indicated that the choice for a collapse operator that leads to results compatible with the scale free HZ spectrum was

$$\tilde{y}(\mathbf{x}) \equiv (-\nabla^2)^{1/4} \hat{y}(\mathbf{x}) \tag{4.11}$$

The study also considered the alternative where the operator playing the roll of the collapse operator $\hat{A}$ of Eq. (4.8) was taken to be linear in the field $\hat{\pi}(\eta, \vec{x})$ (the momentum conjugate to $\hat{y}$ and given by $\pi(x) = a(\eta)\frac{\partial\delta\phi}{\partial\eta}$) so that the evolution was given by the equation:

$$|\psi, \eta\rangle = \hat{\tau} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}} \int_{-\mathcal{T}}^{\eta} d\eta' \int d\mathbf{x}'[w(\mathbf{x}',\eta') - 2\tilde{\lambda}\tilde{\pi}(\mathbf{x}')]^2} |\psi, -\mathcal{T}\rangle. \tag{4.12}$$

In this case, the collapse operator leading to adequate results turned out to be $\tilde{\pi}(\mathbf{x}) \equiv (-\nabla^2)^{-1/4}\hat{\pi}(\mathbf{x})$.

We do not know, at this point why are those particular choices of collapse operators the ones that work in this situation. Actually, it is clear that we need a general recipe for extending the spontaneous collapse theories that have been developed for the context of non-relativistic many particle quantum mechanics to more general settings

[31] The operator that drives the spontaneous collapse dynamics of the theory.

involving quantum fields, as well as special and general relativity. The general form of the collapse operator should be framed in such terms, and should, of course, reduce in the former context to the smeared position operators that have proven to successfully deal with the measurement problem, in the corresponding settings. This is clearly, at this stage of the research program, an open task. However, we might note that these seem to be rather natural choices, dimensionally speaking, in the sense that the constant $\tilde{\lambda}$, appearing in the above equations is of the correct dimensionality (i.e. s^{-1}). For further discussion on this point see [38].

Moreover, as shown in [38], the specific resulting prediction for the power spectrum is:

$$\mathcal{P}_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \tag{4.13}$$

where $\mathcal{T}$ is the duration of the inflationary stage in conformal time taken for standard inflationary parameters as 10^8 MpC, V is the starting value of the inflationary potential, and ϵ is the slow roll parameter which is known to lead to a slight amplification of the spectrum even, in the context of spontaneous collapse theories [52]. When using standard estimates for the inflationary model, including the GUT scale for the inflation potential, and standard values for the slow-roll, the result of the detailed calculation leads to agreement with observation if one sets $\tilde{\lambda} \sim 10^{-5} MpC^{-1} \approx 10^{-19}$ s^{-1}. This is not very different from the GRW suggestion (a standard characteristic GRW value for this quantity in many particle non-relativistic applications of GRW and the corresponding one in CSL is often taken to be 10^{-17} s^{-1}). We find that result rather encouraging, in the sense that it provides hope for the existence of a general CSL like theory capable of simultaneously dealing with the present problem, and reducing to the standard versions of spontaneous collapse theories in the regimes appropriate to non relativistic many particle quantum mechanics, and thus adequate for the laboratory situations described elsewhere in this volume.

We should point out that treatments based on CSL adapted to the inflationary cosmology problem, but based on rather different specific implementations, have been carried out by other groups [39, 40].

4.1 Primordial Gravity Waves

We have seen that, while spontaneous collapse theories have, in principle, the features that allow them to resolve the very serious shortcoming of traditional inflationary accounts for the emergence of the seeds of cosmic structure out of quantum uncertainties in the early universe, a detailed qualitative and quantitatively successful treatment which is empirically adequate, requires rather specific features. In the preceding subsection we adjusted those so that the emergent "predictions" matched observations. This might be regarded as a search for clues of how a theory that was developed with the non-relativistic many particle quantum mechanics settings in mind could be extended to work in contexts involving quantum field theory in curved space-time, and, in particular, the cosmological contexts. It is clear we do not, at this time, have a general proposal for the universal form of such theory. However, once this is done, one can go beyond such adjustments and consider truly novel predictions. In particular, we will next concentrate on the generation, by the same mechanism, of the so called primordial gravity waves, also known as primordial tensor modes. Such waves are, indeed, also a generic prediction of the standard approaches. Extensive efforts are currently under way to detect the traces such gravity waves would leave in the CMB. The effects in question are expected to be observable in a certain type of anisotropy in the polarization patterns of the CMB radiation, known as the polarization B modes[32] [51]. The results of the search for such primordial B modes have, so far, failed to find any clear evidence of their existence at the levels that are expected from the simplest, and otherwise more compelling specific inflationary models. Empirical bounds on their amplitude are currently being employed to rule out many specific proposals [66]. We will show here that, regarding this specific issue, the results from the approach outlined in this review are rather different from those of the standard accounts. The former predict a much smaller amplitude for these primordial gravity waves than the usual approach [68], thus dramatically altering the conclusions regarding the viability of most inflationary models.

Our starting point for this calculation is again the corresponding component of the semi-classical version of Einstein's Eq. (3.1). This is the equation of motion for the tensor perturbations h_{ij}, which under the conditions we have previously set, and retaining only dominant terms, takes the form:

$$(\partial_0^2 - \nabla^2)h_{ij} + 2(\dot{a}/a)\dot{h}_{ij} = 16\pi G \langle(\partial_i \delta\phi)(\partial_j \delta\phi)\rangle_{Ren}^{tr-tr} \tag{4.14}$$

$tr - tr$ stands for the transverse trace-less part of the expression. The fact that the quantity has been subjected to a standard renormalization is indicated in the suffix (Ren) (for in-depth discussions of this see [59]).

We have retained in the right hand side just the largest non-vanishing term in the perturbation expansion of $\langle T_{\mu\nu}\rangle$, which, in this case, is quadratic in the collapsing quantities. The point is that the energy momentum tensor, for a field with simple quadratic potential, is simply quadratic in the field. As we have seen, the field might be written as $\phi(\eta, \vec{x}) = \phi_0(\eta) + \delta\phi(\eta, \vec{x})$. The part containing the relevant spatial dependences (leading to actual gravity waves) involves two kinds of terms (**a**) the term linear in ϕ_0 and linear in $\delta\phi$ and (**b**) the term quadratic in $\delta\phi$. When focusing on the component $\langle T_{00}\rangle$, the dominant contribution comes from terms of type (**a**), but those are absent when considering the component $\langle T_{ij}\rangle$ ($i \neq j$) simply because ϕ_0, does not depend on $\vec{x}$ and thus $\frac{\partial}{\partial x^i}\phi_0 = 0$.

Passing to a Fourier decomposition, we need to solve the following equation,

$$\ddot{\tilde{h}}_{ij}(\vec{k}, \eta) + 2(\dot{a}/a)\dot{\tilde{h}}_{ij}(\vec{k}, \eta) + k^2\tilde{h}_{ij}(\vec{k}, \eta) = S_{ij}(\vec{k}, \eta), \tag{4.15}$$

[32] The other polarization modes, the so called E modes in the anisotropy in the polarization patterns, are expected to arise from well understood late time plasma physics effects, and have, in fact, been observed in complete accordance with expectations.

with vanishing initial data (i.e. $h_{ij} = \dot{h}_{ij} = 0$ on the initial hypersurface $\eta = -\mathcal{T}$), and source term given by:

$$S_{ij}(\vec{k}, \eta) = 16\pi G \int \frac{d^3x}{\sqrt{(2\pi)^3}} e^{i\vec{k}\vec{x}} \langle (\partial_i \delta\phi)(\partial_j \delta\phi) \rangle_{Ren}^{tr-tr}(\eta, \vec{x}). \tag{4.16}$$

As it is well known, general relativity has a well posed initial value formulation, thus once the gauge is fixed, the solution to the evolution equation is completely determined by the initial data (which are vanishing in this case), as well as the source terms.

Again, the quantity of interest is the following average over the ensemble of realizations of the stochastic processes, expressed as:

$$\overline{h_{ij}(\vec{k})h_{kl}(\vec{k}')} = \delta_{ikjl}^{tr-tr}(\vec{k})(2\pi)^3\delta^3(\vec{k} - \vec{k}')\mathcal{P}_h(k) \tag{4.17}$$

where the symbol $\delta_{ikjl}^{tr-tr}(\vec{k})$ is 1 for the index structure compatible with the transverse traceless nature of the gravity waves, and the fact that distinct modes are uncorrelated and 0 otherwise. This expression can be taken as the adapted definition to that in Eq. (2.9) when considering the indexed quantity $h_{ij}(\vec{k})$, and thus defining the tensor mode power spectrum.

The calculation is rather involved, and the details can be found in [68]. The result turns out to be formally divergent, involving an integral over pairs of modes $\vec{q}$, $\vec{p}$ such that $\vec{p} + \vec{q} = \vec{k}$ arising from the term $\langle \partial_i \delta\phi(\eta, \vec{x}) \rangle \langle \partial_j \delta\phi(\eta, \vec{x}) \rangle$ (evaluated on the state resulting from the spontaneous collapse), which is the leading contribution left after the renormalization.[33] However, there are various clear physical reasons indicating that we must introduce an ultra-violet (or short wavelength) cut-off p_{UV}^C on the integral. In particular, we must consider the diffusion effects at late times, which would affect the gradients of density perturbations on very short scales, to the point of effectively damping their contribution to the generation of the gravity waves in question. The fact that other physical sources for a cut-off (for instance that which could be expected to result from GUT scale physics effects during the radiation domination) correspond to higher values of k suggests that we should take the cut-off at the scale of diffusion (Silk) dumping with $p_{UV}^C \approx 0.078$ MpC^{-1}.

After a long calculation, the resulting prediction for the power spectrum of the tensor perturbations is:

$$\mathcal{P}_h(k) \sim (1/k^3)(V/M_{Pl}^4)^2(\tilde{\lambda}^2\mathcal{T}^4 p_{UV}^5/k^3) \tag{4.18}$$

We note that the relation between the above power spectrum and that for the scalar perturbations given in Eq. (4.13) indicates that the latter is substantially smaller than

[33]This divergence is not the same as the standard divergences occurring in the evaluation of the expectation value of the energy momentum tensor. That is dealt with via a well established renormalization procedure, which in this setting corresponds to the so called "minimal subtraction" based on the Bunch Davies vacuum.

the former. Indeed, by comparing Eqs. (4.18) and (4.13), in terms of dimensionless quantities, we have that $\mathcal{P}_h(k)k^3 = \epsilon^2(\mathcal{P}_S(k)k^3)^2(\mathcal{T}^2 p_{UV}^5/k^3)$. Given that the scalar power spectrum is extremely small for the relevant values of k, it is clear that the tensor power spectrum is suppressed by a huge factor.

That is very different from the standardly obtained relation between them, which indicates that $\mathcal{P}_h(k) = r\mathcal{P}_S(k)$ with $r = 16\epsilon$ [70], (i.e. it is dictated by the slow roll parameter ϵ, which is, in turn, related to other observables, such as the scalar spectral tilt). Thus, in contrast with the expectations of the standard approach, we do not expect to see primordial tensor modes (and the corresponding polarization B modes) at the level they are currently being looked for. In this regard, our approach can be said to be empirically more adequate than the standard one, which, with the exception of few specific inflationary models, indicates that a detection should already have been made [67].

In [68], we have also considered a simpler spontaneous collapse model (the naive one designed in [7], with just this specific cosmological application in mind), and again obtained a **substantially reduced tensor mode amplitude**, but with a slightly, different shape. We take this, together with the general discussion before Eq. (4.14), as an indication of the robustness of the generic prediction of a substantial reduction for amplitude of the primordial gravity waves spectrum, in comparison with that of standard approaches. Further studies considering multiple specific inflationary models have been used to determine their viability within the present context [69].

5 Discussion

We have seen that despite its phenomenological success, the usual inflationary account for the emergence of the seeds of cosmic structure out of primordial quantum fluctuations of the vacuum suffers from a serious conceptual flaw: it is unable to account for the transition from a completely homogeneous and isotropic situation, as described both by the classical background and the quantum mechanical state of matter fields, to one that is not.

We have argued that spontaneous collapse theories contain, in principle, the elements needed to deal with that conceptual shortcoming. We have provided a theoretical framework whereby such spontaneous collapse theories might be incorporated within a semi-classical treatment of gravitation, and argued how that view might be reconciled, as a suitable effective treatment, with more traditional expectations regarding the nature of quantum gravity.

We have then proceeded to employ such approach in a simplified fashion within the context of inflationary cosmology, and obtained predictions for the primordial spectrum of both the scalar and tensor perturbations. We saw that by a suitable choice of collapse operators, using a simple inflationary model, with typical values of the inflationary parameters, the spontaneous collapse constant $\tilde{\lambda}$ of the theory can be adjusted so as to fit the scalar perturbation observations with values that are of a similar order of magnitude as those considered in the context of spontaneous collapse theories as applied to laboratory situations.

Furthermore, we have seen that, once such adjustments have been made, the approach leads to specific predictions for the tensor mode spectrum that differ substantially from the traditional expectations in their regard. Actually, according to this approach, the tensor perturbations occur only at a higher order in perturbation theory, and thus imply a substantially reduced amplitude, which naturally accounts for the lack of their experimental detection so far. Moreover, with the present approach, the predicted tensor spectrum has a mush steeper shape, indicating the the possibility of detection increases as one looks at longer wave-lengths. If such expectations were confirmed, this might represent the first case where spontaneous collapse theories lead to different predictions than those of the usual practice in quantum theory, which are such that the former are actually empirically preferred.

The above listed findings illustrate, not only the accuracy of J. Bell's observations concerning the need for cosmologists to become concerned about the conceptual problems surrounding quantum theory, but also the fact that present theoretical frameworks dealing with the early stages of our cosmological models, namely the quantum aspects of inflation, are actually inadequate without such a solid quantum theoretical foundation. We have seen that the incorporation of spontaneous collapse theories into the setting provides both a conceptual and an effective path for addressing such issues. We have also shown that some of the actual predictions naturally emerging from the approach differ from the standard ones in a manner that is favored by current observations.

A related development along this line of research is the analysis, within the context of quantum field theory of scalar fields, of the kind of collapse operators that have the property of maintaining the renormalizability of the expectation value of the energy momentum tensor. In other words, a characterization of spontaneous collapse theories for which the dynamics preserves the Hadamard properties of the quantum state [75]. Other works include an analysis showing how correlations arising from the spontaneous collapse dynamics could naturally account for an anomalous low power in the scalar CMB spectrum [71] at large angles [72]; the proposal of a speculative scenario based on spontaneous collapse dynamics that could dynamically account for the apparently spacial conditions characterized in Penrose's hypothesis[73] regarding the Weyl curvature of the initial state of the Universe [80]; the deveopment of a scheme whereby spontanous collapse theories could restore the viability of Higgs inflation [74]. Furthermore, we should mention a recent proposal that ties the violation of energy conservation characteristic of spontaneous collapse theories (as well as of other approches to deal with the "measurement problem" [81]) to the small current value of the cosmological constant [82], and a more recent refinement of that idea connecting the violation of energy momentum conservation to a space-time discreteness associated with quantum gravity which does a superb job in predicting the correct magnitude [83]. It must be said however that the latter is, at this time, not explicitly connected to spontaneous collapse dynamics. This last feature might decrease one's enthusiasm for some of the ideas discussed in this manuscript. However, this need not be so. The point is that it is not unreasonable to expect that the two might be connected, because, as argued in [41, 44], spontaneous collapse might be ultimately tied to quantum aspects of gravitation. Furthermore, the kind of connection that would be required in this specific context (i.e. one between some sort of space-time granularity and energy momentum non-conservation with spontaneous collapse theories), is, in my view, made plausible by the simultaneous consideration of a *flash type ontology*, the approach employed in [84] for the construction of a relativistic spontaneous collapse theory, and the proposal of [87] to incorporate intrinsic diffusion into the spontaneous collapse theories. Finally, it should be mentioned that we have applied spontaneous collapse theories to deal with a different problem involving the interface of quantum theory and gravitation, namely the so called "black hole information puzzle" [57, 58, 76–79] resulting in what we see as an overall self consistent and reasonable picture of the situation.

Needless is to say that there remain many issues requiring a deeper study and substantial development. Among those is the construction of a universal version of the spontaneous collapse theory that is applicable in general situations, including those pertaining to the laboratory conditions on which spontaneous collapse theories have been traditionally considered, as well as regimes such as cosmology, where they need to coexist with general relativity and quantum field theory. As far as the inflationary context is concerned, we made some adjustments in the theory particularly regarding the choice of the field operators that play the role of the collapse operators (or equivalently, the dependence of parameter on the mode's comoving wave number k) which we found to be dimensionally appropriate, but which we, at this point, could not otherwise justify in a clear manner. In this regard, the continuous search for versions of spontaneous collapse theories that are fully compatible, not just with special relativity (where there is already noteworthy progress [84–86]), but also with general relativity seems as an obligated path for future investigations.

More broadly speaking, the discussion presented in this manuscript offers support for my strong conviction that ignoring "the measurement problem", when discussing issues at the Gravity/Quantum interface, can be a serious source of confusion. The incorporation of proposals to seriously address it, such as spontaneous collapse theories, the subject to which G. C. Ghirardi contributed so dramatically to create, develop, and establish, will not just help in clarifying the overall physical picture, but, has the potential to contribute to the resolution of seemingly unconnected problems.

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