Developing Teachers' Didactic Analysis Competence by Means of a Problem-Posing Strategy and the Quality of Posed Mathematical Problems

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Abstract. The study was designed to improve teachers' didactic analysis competence by means of problem-posing tasks and evaluate the quality of mathematical problems posed by them. For this purpose, a problem-posing strategy has been implemented which sample consisted of in-service mathematics teachers. This strategy involves a reflection stage that is very close to mathematical practices and it encourages to develop didactic analysis competence. The quality of the mathematical problems was evaluated through qualitative criteria. Some findings of the research are related to didactic analysis competence and it means that the posers could formulate better problems with educational purposes.

Keywords: Problem posing · Didactic analysis competence · Quadratic function · In-service mathematics teachers · Quality of mathematical problems

1 Introduction

Problem posing has long been recognized as a critically important intellectual activity in scientific investigation [1]. This importance has been reflected in the development of empirical investigations, where those whose focus of study is the didactic analysis competence in the teaching of mathematics based on problem posing (PP) tasks are more significant [2–4].

Problem-posing tasks demand a person to expose his mathematical knowledge. However, if the posed problem is aimed at contributing to the student's knowledge – or more specifically, to understanding and solving other more complex problems – then the teachers' didactic-mathematical knowledge must also intervene. This aspect is closely related to the teachers' didactic analysis competence, which has been broadly studied within the onto-semiotic approach of cognition and mathematics instruction (OSA) [5].

In the literature on mathematics education research, several studies analyze the relationship between the mathematics teachers' competence and the mathematical tasks for the learning of mathematics. For instance, [6] presented a review of the empirical research done on mathematics teachers, and it concluded that these researches show teachers have difficulties to analyze the mathematical tasks (and their educational potential) that their

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students propose. In order to overcome these difficulties, it is fundamental for teachers to have the ability to analyze their own mathematical tasks and we consider that our research provides specific means to do so, through problem-posing strategy with a phase of didactic reflection.

On the other hand, the quality of a mathematical problem is a subjective concept that cannot be measured in an objective way. However, it is mandatory to deep into this concept by means of quality criteria and suitability to have a picture of what are the indicators that a problem should have to be considered as a *good* problem from a didactic perspective. In this study, some theoretical tools from OSA framework are considered to approach to this concept through the notion of didactical suitability.

2 Theoretical Framework and Methodology

In this study, we consider two theoretical frameworks, which gave us some tools to analyze the data.

2.1 Onto-Semiotic Approach of Cognition and Mathematics Instruction (OSA)

We adopt the OSA as framework because we are interested in teachers' competences when analyzing the mathematical activities that they develop. Likewise, we believe it is relevant to use an approach that provides us with categories to analyze both teachers' mathematical knowledge and didactic knowledge. In this framework, didactic-mathematical knowledge is understood as the deepest knowledge of mathematics and its teaching, which a mathematics teacher must have to design, implement and assess the complex processes of mathematics teaching. In addition, important OSA theoretical constructs for the analysis of mathematical objects, such as concepts, procedures, propositions and arguments, are the epistemic and cognitive configurations, which we will explain next.

According to [7], when a person carries out a mathematical practice and assesses it, he or she has to activate a mixture composed by some or all of the mathematical objects, that is to say: problem situations, languages, propositions, definitions, procedures and arguments. These objects will be interrelated, making configurations defined as webs of objects that intervene and emerge from the systems of practice (Fig. 1); such configurations are *epistemic configurations* (EC) when they are webs of objects considered from an institutional perspective, and they are *cognitive configurations* (CC) when they are webs of objects considered from a personal perspective. Analyzing these configurations allows us to obtain relevant information about a problem and its solution. We call it *anatomy of the problem* because it can give us the different objects, which is involved in a mathematical problem, so the teachers could modify them according their educational purposes.

The Quality of a Mathematical Problem (A Didactic Perspective)

The quality of a mathematical problem can be understood from several perspectives. For instance, the analysis of the problem can be carried out from a qualitative perspective that focuses on its cognitive demand [8]. Or even more, this aspect can be deepened by restricting our analysis in the treatment of the mathematical objects that are immersed into the solution to the problem [4, 9]. This kind of research allows us to clarify the

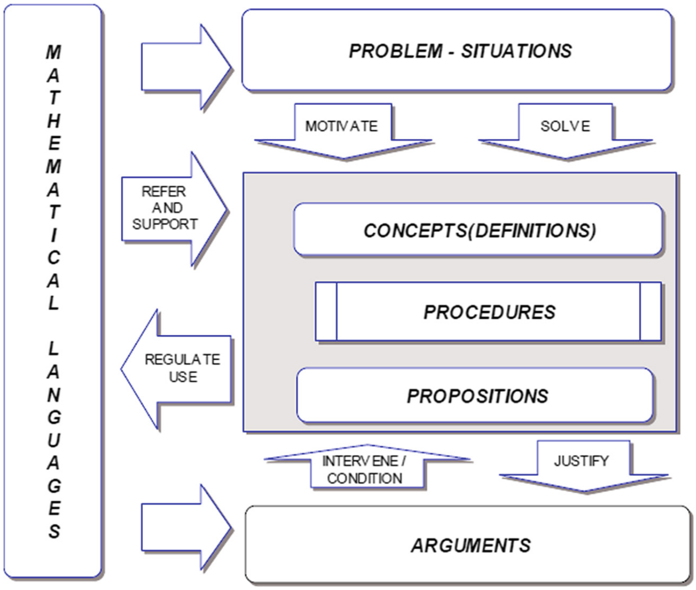


Fig. 1. Configuration of primary mathematical objects

quality of the problem based on its intentionality. Another approach implies a cognitive framework which could face the quality of the problem taking into consideration the cognitive load that requires its resolution. The cognitive load theory [10] explains it in greater depth. The notion of complexity could be another approach to analyze the quality of a mathematical problem. Thus, the complexity of the problem is determined by the conceptual density into the problem, which comprises the number of concepts and properties essential for its solution [11]. All of these perspectives involve some aspects related to the meaning of quality. However, we must focus our analysis in a general view to study this quality. It means to adopt an eclectic posture which should be closed to educational perspective.

In our study, we want to focus this quality on didactic perspective for the teaching and the learning of mathematics. According to [12], the characterization of the quality of a mathematical problem requires tools of description and explanation. This scholar presents a proposal, which takes into consideration the notion of didactical suitability of the OSA framework. This suitability can be subdivided into six specific categories [13]:

- Epistemic Suitability refers to the teaching of "good mathematics". In order to achieve this, in addition to considering the approved curriculum, the intention is to refer to institutional mathematics that have been incorporated into the curriculum.
- Cognitive Suitability refers to the extent to which applied/desired learning is within the parameters of the students' potential development, as well as the correlation between what the students indeed learn and the applied/desired learning.

- Interactional Suitability is the extent to which the means of interaction allow conflicts of meaning to be identified and resolved and how interaction methods favor autonomous learning.
- Mediational Suitability is the degree of availability and aptness of time and material resources necessary for the development of teaching-learning processes.
- Affective Suitability refers to the degree of the students' involvement (interest, motivation) in the study processes.
- Ecological Suitability is the extent to which the process of study is adapted to the center's educational project, the curricular norms and the social environment etc.

As we mentioned before, [12] uses these six specific categories to formulate some property indicators and describe the quality of a mathematical problem. Here we present these indicators related to a "good" mathematical problem from a didactic perspective:

- The difficulty is not too great and it is perceived by the student that the solution is achievable. (Cognitive Suitability)
- It favors to an intuitive way to obtain the solution or to conjecture a solution. (Interactional Suitability, Affective Suitability and Cognitive Suitability)
- It favors making some verifiers, eventually with the help of a calculator or computers for maintaining or rejecting the conjectures. (Interactional Suitability and Mediational Suitability)
- It is perceived by the student that it is interesting or useful to solve the problem. (Affective Suitability and Ecological Suitability)

Taking in mind those six categories that contain the didactic suitability, we highlight the epistemic, cognitive and ecological suitability. This choice is justified in the conception taken from [14], because the importance of the cognitive aspect of problem posing activities for a curricular design that considers these activities as a means to improve the learning and the teaching of mathematics. Moreover, it implies stronger cognitive demand. Likewise, [14] state that the problem posing tasks allow developing more elaborate and advanced problem solving strategies. This last aspect, we think, corresponds very well with the epistemic and ecological suitability.

2.2 Problem Posing and Mathematics Teachers' Didactic Analysis Competence

It is worth mentioning that there are different positions in terms of what researchers understand by engaging in problem posing activities [1]. In our study, we adopt the proposal from [15], according to which problem posing is a process through which a new problem is formulated. Moreover, in this proposal, if the new problem is obtained by modifying a given problem, it is said that the new problem was obtained by *variation*. At the same time, if the new problem is obtained from a given situation or from a specific requirement, whether mathematical or didactic, it is said that the new problem was obtained by *elaboration*. Taking into consideration our research goals, we focus in the first one, it means getting new problems by variation. Likewise, these scholars consider that problems have four fundamental elements: information, requirement, context and mathematical environment; in that sense, problem posing by variation entails quantitative or qualitative modifications of one or more of these elements in a given problem. We analyzed these elements in problem posed by in-service teachers.

Additionally, [3, 15] implemented a strategy in workshops with in-service teachers in order to stimulate their ability to pose problems by variation. This is the EPP strategy since it stands for Episode, Pre-problem and Post-problem, by considering the problem posing by variation, where there were evidences that mathematics teachers lack didactic analysis competence to pose problems with didactical purposes. In this sense, given the importance that mathematics teachers must develop this competence, especially when they pose mathematics problems with emphasis on teaching, in our study we implement the EPP strategy for problem posing by considering a phase (R) of metacognitive and didactic reflection; therefore, the strategy name would be ERPP. In the new strategy ERPP, there is a phase where the teachers must elaborate a CC of their solutions to the problem presented in the episode (episode problem (EP)) and – based on it – reflect on their practices. In the next lines, we propose some phases for implementing this new strategy.

3 Method

In this research, we used a multiple case study with 16 in-service high school mathematics teachers who participated in a problem-posing workshop. Our study is exploratory, descriptive and analytical, taking as unit of analysis the problems posed by the teachers participating in the workshop. We analyse these problems using OSA tools, it means EC and CC for solving and posing practices. The use of EC and CC is a methodology previously used in some researches done in the OSA framework [2, 16], with the aim of examining the mathematical solutions of pupils. In addition, we evaluate the quality of the mathematical problem using the expert triangulation, which qualifies it by means of the four indicators.

3.1 Problem-Posing Workshop on Quadratic Function

In our study, we implemented the ERPP strategy in the *Problem-Posing Workshop on Quadratic Functions* that purpose goes on to stimulate the development of the ability to pose problems by varying a given problem. We focused our attention on pre-problem posing, since it requires didactic criteria from the person proposing the problem, so it should have the characteristic to facilitate the comprehension and resolution of a previously given problem.

In the next lines, we summarize the dynamics of the problem-posing workshop.

- *First session*: a test on quadratic functions was applied which purpose was to go deep into participants' mathematical competence. In addition, a class episode on affine function that we designed in a previous research [3] was presented. Indeed, this episode includes an EP on affine function. Moreover, in this session an EC associated to EP of this class episode was discussed which objective was to initiate the participants on their CCs elaboration.

- *Second session*: Based on their solution for the EP on affine function, the participants elaborated their CC associated to it, later they reflected on their mathematical practices of solving through a previously elaborated questionnaire. At the end of this session, some CC were socialized.
- *Third session*: Another class episode was presented. The participants solved the EP that is involved in this episode and elaborated their CC based on their solution to episode problem (CCPe). Then, the EP solution and some CCPe were analyzed in detail between all of the participants. Subsequently, they were asked to pose and solve a pre-problem (P1) considering the student's reactions to the EP, as well they were asked to elaborate the CC of the solution to P1 (CCPp1). Afterward, each participant reflected individually about his or her mathematical practices related to problem posing. To reinforce the didactical reflection, the participants formed pairs to discuss the pre-problems posed and their CCPp1. For digging into this reflection, it was conducted by the researchers taking into account a comparison between CCPe and the CCPp1 for each participant. Next, some P1 and the results of their reflection process were socialized with the intention of broadening the problems analysis with didactical emphasis among the assistants.
- *Fourth session*: The participants were asked to pose another pre-problem (P2) associated to the reflection made on P1 and its CCPp1. After a specified time, the participants, working individually and then in pairs, reflected on the P2. As the culmination of the workshop, some P2 were socialized.

Considering teachers' didactic experiences in teaching functions in high school, the research team selected the following episode, in order to present it to the teachers participating in the workshop. This episode includes some comments from students whom aged between 14 and 15 years old and they were exposed to the episode problem:

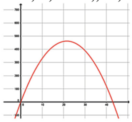
> Mr. Pérez proposed the following problem to eighth-grade students in a mathematics class on functions:
>
> *Find a pair of numbers whose sum is 43 and their product is the maximum possible. Solve the problem and explain your procedure in detail.*
>
> After a few minutes, some students commented:
>
> **Pedro**: The numbers are 21 and 22.
>
> **Isabel**: You cannot know the maximum product.
>
> **Santiago**: What good does it do for me to solve this problem?

An expert solution was adopted for the problem, and the EC of such solution was made in order to have it as a reference to analyse and compare it to the CC of the participants' solutions (CCPp1).

3.2 Expert Solution and EC of the Episode Problem (ECPe)

The expert solution to this problem implies defining a function that allows us to obtain a pair of numbers which sum is known and which product must be the maximum. In this way, the function $f(x) = x(43 - x)$ is defined, where *"x" and "43 − x"* are numbers

Table 1. Epistemic configuration of the solution to the episode problem (ECPe).

Languages
• ***Expressions:*** First number, second number, maximum, product function, completing squares, concave, function, product, first component, vertex, abscissa, parabola, represents, sum. • ***Verbal representations:*** Number, function, maximum, product, parabola, vertex. • ***Symbolic representations:*** *43, x, 43-x,, =, 2, /, 1849, 4, f (x), 43/2, (;), 0* • ***Graphic representations*** 
Problem-situation
• **Information:** The sum of two numbers is 43. • **Requirement:** Find two numbers that meet the given information, whose product is maximum. • **Context:** Intramathematical • **Mathematical environment:** quadratic functions, linear equation
Concepts
Function, vertex, graph of a quadratic function, linear equation
Propositions
• Since the product of two numbers must be the maximum, the function is defined $f(x) = x(43 - x)$ • Since *f* is a concave quadratic function it will have maximum for $x = \frac{43}{2}$ • The abscissa of the parabola's vertex is the maximum value that *f* can take. • The maximum is at the top of the inverted parabola and corresponds to the point $\left(\frac{43}{2}; \frac{1849}{4}\right)$
Procedures
• The information behind of the problem is identified. • The variable "*x*" and function *f* are defined: $f(x) = x(43 - x).$ • Complete the square is applied to find the maximum of the function. • The result is interpreted considering the concavity of the parabola that represents the function under the conditions of the problem. • The linear equation $x - \frac{43}{2} = 0$ is posed to obtain the abscissa that maximizes the function. • The graph of the product function is outlined and interpreted to respond to the problem.
Arguments
Thesis The product function $f(x) = x(S - x)$ would have maximum if $x = \frac{S}{2}$ **Argument** Using complete square technique, we can represent product function in another algebraic expression: $f(x) = -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4}$ Now, since the function is a quadratic function, the graph for this will be a concave parabola. So the maximum is when $x = \frac{S}{2}$.

which product must be maximum. Therefore, both numbers are equal to 43/2 and their product is 1849/4. On the other hand, this answer can also be found by associating the number to the value of the abscissa that maximizes the function *f*. Another strategy for solving this problem, entails relating the vertex of the parabola that represents *f,* so the vertex would be the coordinates of the point (43/2, 1849/4). This last strategy makes use of the graphic representation of the function.

While elaborating the EC of the solution to the episode problem (see Table 1), we could recognize different mathematical objects whose area involved in the mathematical practices. They are the languages used (verbal, symbolic and graphic representations); the information, requirement, context and mathematical environment; the concepts involved (quadratic function, linear equation, the maximum of a quadratic function, vertex, parabola, graphs of functions). Also, the emerging proposition (the function given by $f(x) = x(S - x)$ will have a maximum for $x = S/2$, where S is the sum of the two numbers), the procedure which follows to the solution and the arguments explained to tell the truth about the given proposition, which derives in the conclusion. All of them are explicitly stated.

4 Analysis of Data

In relation to the solutions of EP, it was observed that most of the teachers consider the quadratic function as an object associated with the problem and this is closer to what was posed in the expert solution. From the sample, only 12 of the participants solved the problem correctly. In addition, it is significant that, even though 10 participants define a variable for solving the problem, 6 of them make explicit the function to be maximized. On the other hand, in order to find the maximum value of the function, 5 use the completing square strategy, while 5 do it by using the algorithm to find the vertex of the quadratic function and, as a result, to study its corresponding maximum. Three participants solved the problem with the support of a table of values. However, a teacher used the table partially. It gave us an idea to state that he or she recognized the numeric sequence related to the problem. Precisely, the use of the tables led some teachers (4) to make their analysis in the set of natural numbers and to fail the correct answer.

By analyzing in a qualitative way the CCPe made by the teachers, it was observed that most of them recognized the mathematical objects, at least partially. However, only some of the participants were able to elaborate with greater certainty the objects so-called propositions and arguments. In the same way the lack of robustness in their propositions and/or arguments, correspond to a lack of practice in the analysis of their mathematical chore, and by extension we can say that they lack or they do not show the competence of the didactic analysis.

On the other hand, the P1 were categorized by using the analysis of content and the methodology of expert triangulation. From this categorization, based on the ECPp1 and CCPp1, we can say that most of the participants have an idea of function typified by an epistemic configuration focused in formalist approach instead of empiricist approach [17]. This tendency is showed in posed problems as well, where intra-mathematical environment prevails.

4.1 Case of Study: Teacher T11

Because of space limitations, in this paper we only present the case of a teacher that hereinafter we will call T11 and in this section, we analyze ECPe and ECPp1.

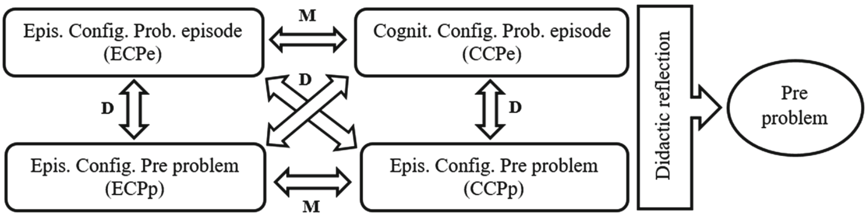


Fig. 2. Scheme to compare configurations

For the analysis of the configurations and the reflection on the mathematical practices of solving and posing for this case, we used a scheme (Fig. 2) according to the strategy ERPP. In this scheme it is shown the interaction among different configurations, whether epistemic or cognitive, in order to study the mathematical competence (M) or didactic analysis competence (D) of the teacher. In this context, looking forward to our interests, we focus on studying the interactions involving the competence in didactic analysis (D).

As an example of the pre-problem posing task, we present the first pre-problem posed by T11 (associated to CCPp1):

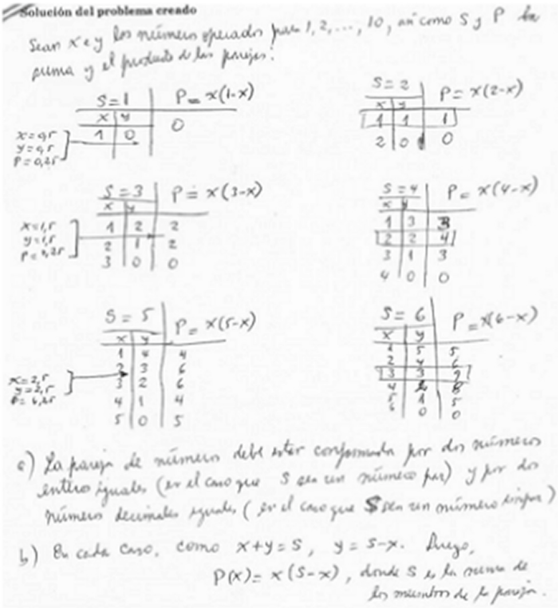
> Determine the pair of numbers whose sum will be, respectively, 1, 2, 3, …, 10; but in such a way that the product of that pair's components will be the maximum possible. (a) On the basis of what was observed, could you indicate which are the features that, in each case, the pair of numbers must meet? (b) If you must formulate each product as a mathematical function, express it.

Along with the problem posed, T11 showed a possible solution to his problem and it allowed us to move into not very explicit aspects of the problem (Fig. 3). Likewise, the teacher elaborated the CCPp1. Finally, he answered the questionnaire about the mathematical practice of posing.

4.2 Analysis: ECPe and ECPp1

From the analysis of ECPe and ECPp1, it was observed that the information is similar and that it was only modified quantitatively. Even though it is true that the amount of pairs of numbers is greater, these are more manageable numbers for a high school student. In the same way, the requirement suffered a change, since in the P1 there are two questions that invite the student to reflect on his/her procedures and solution.

In addition, the problems keep up the intra-mathematical context. In order to answer the requirement of the EP it becomes necessary to use algebraic expressions, however, in P1 posed by T11, it is not necessary to use this resource, except in part "b" in which the notion of generalization using functions is a request. This would be an advantage from the didactic point of view in order to solve the EP, without using the quadratic function.



Translation:

Let us call "x" and "y" the possible numbers which sum and product are S and P respectively, so…

a) The pair of numbers should be two equal integers numbers (if S is an even number) and equal decimal numbers (if S is an odd number).

b) In each case, x+y=S and y=S-x. Then, P(x)=x(S-x), where S is the sum of the numbers.

Fig. 3. Solution to PP1 proposed by T11

Based on an expert solution to P1 and the solution proposed by T11, we state the following: for the language, in general terms, the use of graphical representations is highlighted: the parabola for EP and the table of values for P1. Likewise, in EP and P1 the requirement of maximum is evident. Thus it becomes explicit the use of the quadratic function in P1 regarding EP. Unlike P1, in EP it is necessary to use linear equations to formalize and give rigor to the problem solution. For the considered concepts in both configurations, it is easy to see the coincidence in many of them, like the case of the concepts of function, product of function, the idea of maximum of a function, completing squares, among others.

The propositions and arguments in the EP are more formal and rigorous. In P1, the fundamental feature of the propositions and arguments has a lower level of formality, in such way that allows using the inductive and deductive reasoning easily. As a sample to highlight this reasoning, T1 suggests a table of values in order to recognize a pattern that will allow solving the EP easily.

Talking about the procedures, the use of similar strategies is emphasized, in the sense that both problems require the identification of the main information and requirements. However, in P1, the situation becomes more intuitive, since it is not required to formulate a correspondence rule to solve it, except for the explicit requirement. Certainly, in EP there is no need of this, but due to reasons of effectiveness.

The arguments of the EP are more rigorous and formal, since they use concepts that are closer to the quadratic function, for example: concavity, vertex of a quadratic function. Moreover, in P1 the arguments are closer to an inductive reasoning, since the plan is to elaborate a strategy that makes the solution to the EP. For this last case, the use of a value table will permit to guess the practical rule that emerges as consequence of the analysis of the given values and that is one of the purposes of the problem.

4.3 Approaching to a Didactical Problem Posed by Quantitative Variation

The teacher T11 was asked to reformulate his pre-problem (P1), considering a didactic analysis of the problem posing process through the comparison between CCPe and CCPp1. Thus, T11 posed a second pre-problem (P2) that was analyzed by expert triangulation taking into consideration the four indicators explained in the Sect. 2.1. Next comes the problem P2:

> Determine the pair of numbers whose sum will be, respectively, 5 y 6, in such way that the product of the pair's components will be the maximum as possible. Elaborate a table for the next cases: (a) For the case whose sum is 6, which are the components of the pair of numbers whose product is maximum? (b) For the case whose sum is 5, is there just one pair of numbers whose product is maximum? Explain your answer. (c) If we consider that there is just one pair of non-natural numbers whose sum is 5 and whose product is maximum and it is not a natural number, which are those numbers? (d) Elaborate a strategy in order to obtain the maximum product, in case of the sum of the pairs' components will be an even number. Is this strategy different in case of the sum will be an odd number? Explain your answer. (d) Formulate the product of the numbers whose sum is 5 as a mathematical function.

From the expert triangulation, there is evidence to say that the P2 posed by T11 has the features of a pre-problem, since it makes easier to conjecture a pattern that allows to solve the EP. It is also observed that it was well conceived and detailed in order to achieve subsequently a solution of the EP easily. Thus part "c" fosters an intuitive solution, against the part "d" that invites to the generalization. The section (e) would be a simple exercise that helps a lot to think about how to solve the EP. In addition, we claim that the problem posed has a closer approach to cognitive, interactional, affective and epistemic suitability. This statement is made based on the experts' opinion under the indicators proposed for evaluating the quality of a mathematical problem from a didactic perspective. Moreover, following the answers given by T11 in the post-workshop questionnaire, it is observed that he makes an explanation of the benefits of his problem, using the elements of his configuration and considering its didactic emphasis. Precisely, these aspects are highlighted by the experts when they analyzed the pre-problem posed by T11.

5 Final Consideration

At this time, in the mathematical education field, there are several theories and theoretical approaches for researching. A reason for the existence of different theories and theoretical approaches is the complexity of the topic of research itself [18]. We belief that with our study, we contribute in a way to use different theories for analyzing the mathematical practices of solving and posing problems. Indeed, our proposal to use theoretical notions from OSA and the conception of problem posing tasks gave us evidence for promoting the didactic analysis competence. This competence is crucial and its core represents an advance in teacher education.

Our study proposes a new problem-posing strategy that includes EC and CC tools taken from the OSA framework to analyze the teacher's mathematical practices. Because of this implementation, we have evidence to state that in-service teachers' didactic analysis competence shows to be incipient and urge to develop it. Certainly, our position about the conception of the didactic analysis complements that proposal of [19, 20], it implies *the subject–didactical competence* and the *competence of reflection*, since we consider a strategy for problem posing which includes individual and group reflections taking into account the posed problem using a phase of didactic reflection. Therefore, there is a need of going deeper in our study.

The use of indicators to evaluate the mathematical problems from a didactic perspective is important and relevant, in such a way that it collaborates with the decision-making process for didactic reflection before or after the problem posing process. We can state, based on the data analysis, that the reflection stage is more enriching and profound for education of prospective teachers.

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