An Open Educational Game for Learning Fractions in the Brazilian Context

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Abstract. The areas of Science, Technology, Engineering and Mathematics (STEM) careers, are important areas of knowledge for contemporary society. Among the basics subjects of these careers is Mathematics, but many young students present difficulty to understand basic mathematical concepts. This learning process is complex and demands a lot of motivation by part of the students. In this sense, educational games can act as a stimulating helping tool for them. In Brazil, the levels of proficiency on rational numbers in their fractional representation presented by students aged between 9 and 12 years, in general, are low. We present such a game, called Fracpotion, developed as an Open Educational Resource to teach about fractions to children guided by Didactic Situation theory. This game has been tested by a group of students in an elementary school at São Paulo city, Brazil, and the preliminary results were positive.

Keywords: Open educational resources · Open educational games · Fractions · Didactic situations

1 Introduction

Professions are very important for society so people study and specialize in specific areas to play roles they like, and think it's important, in the job market. In this context, the STEM (Science, Technology, Engineering and Mathematics) careers are become each time more important for contemporary society [1].

Some of basic skills of STEM careers are on Math, but many young students present difficult to understand very fundamental Mathematical concepts [2]. If the student present difficulties in fundamentals of Mathematics still in his childhood or youth and choose to avoid of Mathematics and similar areas, probably they will be increasingly unmotivated through the years and will also avoid STEM careers in the future.

If we want to prepare our children and teenagers to have possibilities of work in future STEM careers, we should offer more didactical and motivational tools and challenges to them. According to [3] we should not to force young students to learn with same methods their grandparents have been exposed to, because traditional school learning based on predefined sequential curricula cannot completely meet requirements for the 21st century children, so modern tools (like digital games, for example) should be applied

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to complement and demand a big amount of motivation by students. Thus, naturally motivating tools like educational games can be interesting as vectors of stimuli for students [1].

Mathematics has many sub-topics that makes impossible to develop a digital game to cover all subjects once, so it is appropriate to select a specific topic to guide the design and application of educational game.

In Brazil the mean level of proficiency in Mathematics, in general, presented by students aged between 9 and 12 years, is low; this situation is more serious when they comes to learn some topics that are not common in their daily activities, like rational numbers in their fractional representation. Different from its neighbors in Latin America, in Brazilian culture the fractional representation is not a commonplace – even ½ is often represented in its decimal form (0,5 – with comma).

The learning of fractions is a complex process for students; difficulties can arise when students try to apply the properties of natural numbers to fractions, without understanding the differences between the two sets [4].

[5] claims that fractional numbers are avoided by students, because they do not like it or do not feel familiar with it. [6] clarify that when students are encouraged to solve problems using acquired knowledge and able in their daily activities and with symbolic representations, they can gradually evolve and use the daily knowledge to solve more complex problems and learn new skills, but it is not the case to fractional representations in Brazil's culture.

On the other hand, digital games are each more common in the daily activities of the young people, including students, so the development of a game about fractions would be of great value to complement the learning of this topic, especially if the game offers some facility of adaptation by teachers. Therefore, in this paper we present an Open Educational Game [7] called FracPotion to help students about fractions. The game was applied to 18 students of an elementary school at São Paulo, Brazil, and the results were positive.

The paper is organized as follows: Sect. 2 describes the panorama and difficulties in teaching fractions in Brazil; Sect. 3 explains Guy Brousseau's theory of Didactic Situations; Sect. 4 demonstrates an overview of educational games for mathematics teaching; the fifth section presents the proposal of the paper; Sect. 6 presents the experiment and results, and Sect. 7 completes the paper.

2 The Brazilian Reality on Learning of Fractions

This section will describe the panorama about learning fractions at elementary schools since 1980s to 2015, according some important national and international educational assessments; and will show some difficulties in teaching fractions.

2.1 Panorama According Important Educational Assessments

According to [8], among 1980s and 1990s, official assessment reports, such as the National Basic Education Assessment System (SAEB), the Programme for International

Student Assessment (PISA) and the National Student Performance Exam (ENADE), confirm the consensus pointed out in Brazilian and international studies on the difficulties of students and teachers in dealing with the concept of rational number.

The SAEB of 2001 [9] reveals that only 35% of Brazilians students were able to solve simple problems involving fractions; many of those that failed had problems with the part-whole relation, which is considered the most basic concept in this topic. The São Paulo State Performance Assessment System (SARESP) in 2005 [10] revealed that only 37% of students answered simple questions regarding such a topic. The results of the PISA (2012) [11] showed that two out of three students do not know how to work with simple operations involving fractions.

A survey recently conducted by the International Student Assessment Program in (2015) [12] showed a drop in scores in the three areas assessed: Science, Reading, and Math. The drop in scores also reflected Brazil's drop in the world ranking: the country ranked 63rd in science, 59th in reading and 66th in Mathematics. The Fig. 1 shows the results of proficiency in Sciences, Reading and Mathematics of the Brazilian students in elementary schools, according to OCDE and PISA 2015 [12].

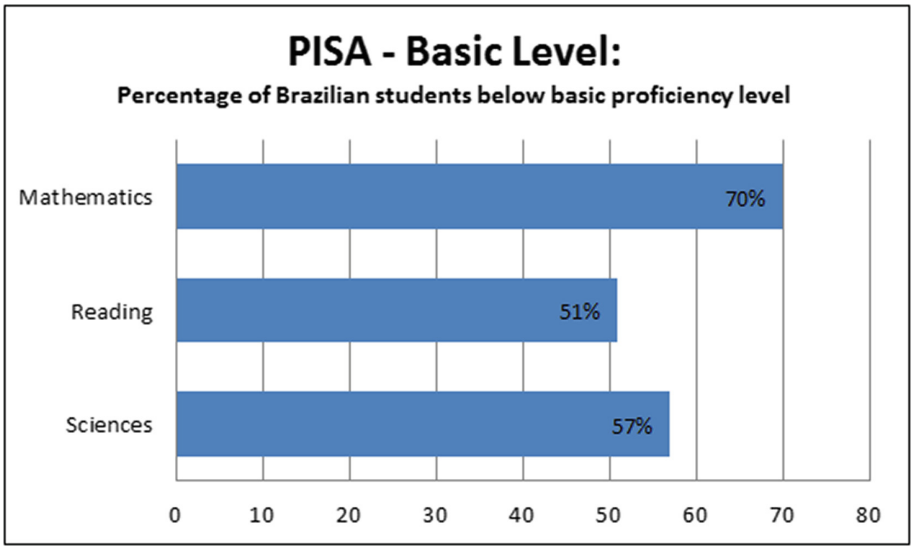

Fig. 1. Results of proficiency in Sciences, Reading and Mathematics of the Brazilian students in elementary schools, evaluated by OCDE and PISA 2015 (Source: adapted of [1]).

As we can see in Fig. 1, of the three areas evaluated in the assessment the Mathematic is the area with the worst performance of the Brazilian students.

2.2 Difficulties in Learning and Teaching Fractions

The teaching and learn of fractions is one of the most difficult areas in elementary school mathematics; is believed that failure to learn fraction arithmetic hinders children to learn more advanced math and difficult their success in careers that use math [13].

Children who have not yet understood fractions generally believe that the properties of whole numbers are the same for all type numbers [14].

Among children misconceptions of fraction are: fraction equivalence, common denominators and fractions arithmetic operations.

In the set of natural numbers, each number has a single representation, but when working with fractions, the same number can be represented by infinite fractions; And when multiplying a natural number by another natural number (except for the values 0 and 1) the result will be greater than the numbers used in the multiplication, this does not always happen with the multiplication between fractions [4].

There is a high rate of difficulties presented by students in understanding the concept of rational number and thinking about multiplication; a survey of 4th to 8th graders of elementary school showed that even students with good performance in mathematics generally achieved poor performance on issues with the whole part, quotient and multiplier operator in work with fractions; These same students performed well in multiplying natural numbers [15].

Based on [16] and [8] we found four important aspects to the practice and learning of fractions [10]:

- First (I), the practical aspect, in which fractions in their different representations appear, often in several situations related to the expression of measures and quantities – this fact highlights the need for extension of the set of natural numbers.
- Second (II), the psychological aspect, since working with fractions appears as a privileged opportunity to leverage and expand mental structures necessary for intellectual development.
- Third (III), the mathematical perspective aspect, since it will be precisely the first studies with the fractions that will ground more complex ideas such as operations.
- Finally (IV), order of numbers in their fractional representations.

We believe that an Educational Game that presents fractions and give player the task of reorganize and perform arithmetic operations with fractions them can offer experiences that address the four aspects mentioned by [16] and [8].

As we can see in (I) different representations of number in fractional form should be presented to the students in a more natural as possible way; in (II) the Brousseau's theory of didactic situation can guide the educational game application (because helps to expand the knowledge and achieve at target knowledge); in (III) and (IV), about the third and fourth aspect explained by [10], many authors show different approaches to the mathematical perspective in the teaching of fractions, as examples we can cite, according to [12] the focus of mathematical programs should be on fractions as quantities, to allow students to make a correlation with their previous knowledge of natural numbers as quantities, and [13] suggests that teachers need to introduce a variety of fractional interpretations for students, as students whose fraction learning was previously focused on regular fractions of tend to have an impoverished understanding of the rational numbers.

According to [8] young students often process numerator and denominator as two separated whole numbers, because it is uncommon to them, and it can cause errors like:

- $1/4 + 1/2 = 2/6$, what is incorrect and refers to III; And
- $1/5 > 1/3$, refers to ordering the fractional numbers as showed in IV.

Still according to [8], to overcome these errors young students have to interact with rational numbers, experiment, modify and compare results to get familiar with fractions and understand the rational number set as a distinct set of whole number set and accept that this new set has his own rules and functioning; here we make a relatives with de Didactic Situation theory by Guy Brousseau and the application of educational games to teach fractions, because the main idea of Didactic Situation and usage of educational games is also allows the student experiment, modify and compare results to learn in a less intimidating and more independent way.

3 Didactic Situation Theory by Guy Brousseau

Guy Brousseau proposed the Didactic Situation theory that consists, basically, of three elements: knowledge, divided in the knowledge that the student already owns and the knowledge that student have to learn (the target), teacher and student. These three elements relate to affect the pedagogical relationship that takes into account professor, student, knowledge (of the student and of the professor), rules, informations, environment and the resources (objects of study or tools) [17]. The Fig. 2 illustrates the Didactic Situation.

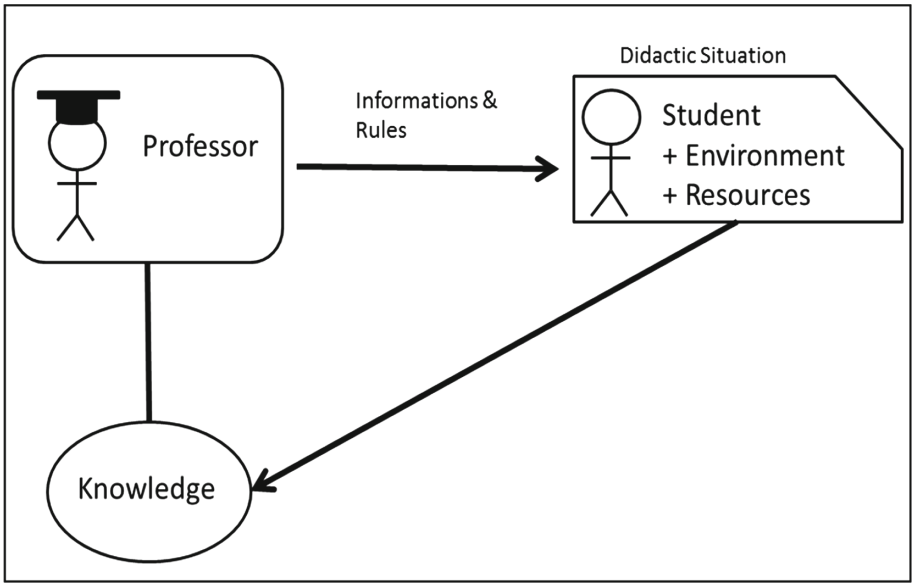

Fig. 2. Brousseau's Didactic Situation illustration: pedagogical relationship (Source: authors).

In the Didactic Situation theory, the professor must provide favorable situations to encourage the students, so that students can transform information and data into knowledge. The role of the student is to experiment, observe concepts, construct hypotheses

and strategies, follow theories, follow rules and build models. The role of knowledge in the Didactic Situation is to anticipate theory and provide information to guide the student on their experiments [18].

The Didactic Situation theory defines that the teacher, in order that the student reaches the knowledge to which he intends to reach, must create didactic situations to motivate the student to learn the content independently, without intimidation and understanding the process of building that knowledge.

According Silva, Ferreira and Tozetti [19], the teacher must be an educator-researcher and the student must be a student-researcher; The difference is in the degree and difficulty of the content that the teacher can research and the problems that the teacher can solve, being that the teacher is a totally independent researcher (in the subject/discipline in question) and able to formulate the problems to be solved.

The didactic situation is an activity that must be dosed so that the teacher does not create challenges for the student, which are too difficult, to the point of discouraging him, or too easy to the point of compromising the student's dedication in building his knowledge, independently.

In the context of the Didactic Situation, an Educational Game fits well, because a game provides independence to the student, the possibility of experiment actions, observing and learning from errors, build strategies to meet the challenges of the game and the game's own rules can provide theory and information to the Student.

4 Digital Educational Games to Teach and Learn Mathematics

This section will be describes the educational games overview in practical perspective in the teaching of mathematics; and will show some related works.

4.1 Educational Games Overview and Applications to Teach Mathematics

Today we live in a world that offers many technology tools for children and young people; usually these young people's first contact with electronic equipment happens through Digital Games [3]. These can be defined as engaging, interactive environments that capture player attention by offering challenges that require increasing levels of dexterity and skills [20, 21].

Many teachers believe that regardless of whether a digital game has or not some educational purpose, it could contribute to psychomotor skills, development of analytical skills and computational skills of the player. This is mainly due to the difficulties faced during the game, the need to create new strategies (when strategies used before are no longer good), the pressure to develop strategic thinking, among other common aspects.

Educational games could be effective learning resources, mainly when applied to courses such as Mathematics or Science, often considered as difficult, abstract courses. They also have the potential of influencing students' social and daily life, affecting their behavior with colleagues [22]. However, many authors – like [23] and [24] suggest that educational games need more empirical evidence of effectiveness, requiring more evidences in this field. According to [25] the research field of educational games still has a limited quantity of empirical evidence about the effectiveness of games, especially

in the domain of Mathematics. Some other works, like [26], bring stimulating results regarding to the use of digital games in the specific topic of fractions.

4.2 Related Works

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In [27], an educational game called "Animo Math" was developed to help 5–7 years old children to learn Mathematics. The game has five levels with different difficulties, beginning with calculus that uses only single digit numbers (0–9) to teach/reinforce addition and subtraction operations; the other levels present more digits and different difficulties in calculus. In this work, there were expected the following four benefits in his game:

1. Make children more interested in positive learning in Mathematics.
2. Gain more of children's attention by using cartoon animations and fun sound effects.
3. Enable children to see the fun of Mathematics.
4. Parents perceive how to use modern digital technology and computers for children in mathematical learning.

In [25] an educational game was developed to teach about decimal numbers. In this game the students are introduced to a group of some fantastic characters that act as guides to Decimal Point Game and encourage students to play, congratulating them when they correctly solve problems. The game is composed by several mini-games inside a kind of Amusement Park. However, the game does not present numbers in factionary format. After playing a mini-game and correctly solving the problem, the student was prompted to explain his or her solution, by choosing possible pre-listed explanations from multiple-choice options available.

5 The Proposal: FracPotion Educational Game

FracPotion is a 2D educational game that was created for elementary school students. It is expected to evaluate whether students find the game funny and whether they can better identify, practice and understand educational content about fractions.

For the game to be better accepted by students, a narrative was created that justified the need to work with fractions in the game. In this sense, a wizard is proposed as a character since he employs fractions to combine potion ingredients that allow player to

have progress in the game, when potions are properly created with the correct fractional quantity calculations. The background history to support the narrative follows:

"The game takes place in a kingdom called Camelot, ruled by Arthur. This kingdom is being threatened by a wicked witch named Morgana who, along with an army of wizards is heading toward Camelot to defeat Arthur and rule over humanity, but with the help of Merlin and his apprentice Arthur intends to defeat her with Magic potions.

For the creation of potions, it is necessary to define the right dose that is in the form of fraction and thus create the potion to defeat Morgana and save the kingdom from destruction."

5.1 The Open Digital Educational Game Overview

The game FracPotion is a simple adaptation of the classic history of King Arthur and his knights, where to defeat the villain Morgana, the player (as an apprentice) will have to hit the right amount of ingredients to complete the potion represented by fractions. This kernel of the game is explained (in Portuguese) on screen of game illustrated in Fig. 3.

Fig. 3. Game title screen with options; Play, Instructions, About and Exit (Source: the authors)

Players are not punished for their mistakes; they only earn stars according to the number of attempts they used to reach the required amount of ingredient that was thrown into the cauldron. Figure 4 is an exhibition of the first level of the game. Contains a question the player must answer to complete the potion. In Fig. 4 we can see the score markers, error markers and the elapsed time. The character is the wizard in the lower right corner of the screen and the alternatives A, B, C and D are the possibilities to complete the potion in the cauldron.

Fig. 4. Screenshot of gameplay at first level of the FracPotion game (Source: [1])

It must be noted that, even though the game was conceived in Portuguese language, it can be easily translated to any other language, as well as it is possible to modify most of its aspects, given that it was designed as an Open Educational Resource Source code is available (https://github.com/josivanSilvaCodes/FracProc) and is made available under open source license.

5.2 Design and Development of the Game

The Game uses a 2D view and was developed in JavaScript and WebGL to facilitate the visual work that forms the look-and-feel of the game. Game's target audience is composed by students who want to discover or review contents about basic fractions.

Cocos2D-x is a multi-platform framework for developing games and graphic applications [22] – the "x" letter in the name of the framework means that it can be changed by one between several languages supported; in our case, the chosen language was JavaScript (JS). This framework is a branch of another framework with a similar name called Cocos2D, which is focused on development for Apple devices running the IOS operating system. The big advantage of Cocos2D-x was to bring improvements to the then Cocos2D:

- It has a simple but also very powerful phase creator.
- Accessible through all major operating systems.
- Make it possible to publish games, for various platforms such as Windows, Mac OS, Linux, Android, iOS and also to the Web, in any HTML5-supported browser.

- Possibility to create your applications through various programming languages, such as C++, Lua, JavaScript, Objective-C, Swift, C#. Figure 5 exposes de Cocos2D-x framework used in this paper.

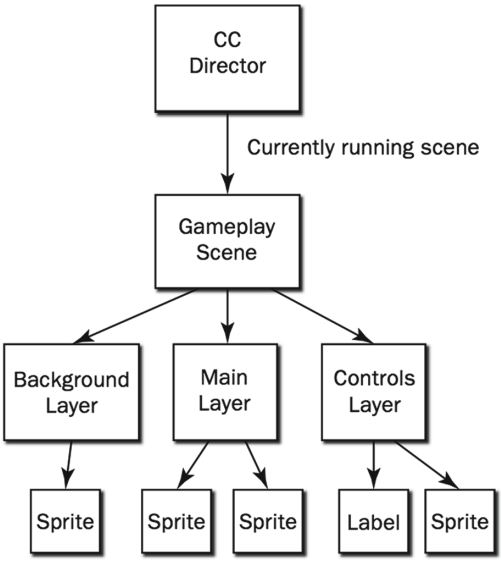

Fig. 5. Cocos2D framework hierarchy (Source: [1])

The Fig. 5 illustrates the Cocos2D framework hierarchy, the concept evolve a hole of Director that runs a Scene that contains several elements, as: background layer, controles layer and etc.

The way the framework works is by using the concept of a director and scenes, being each object on the screen a node in a component tree. Everything that will be drawn is controlled by a class called CCDirector, where it has the ability to modify the order of how components will be drawn and also change the game scenes, being responsible for defining the initialization of the components that are requested by the framework calls. The Cocos Creator tool tends to hide most of this complexity.

Cocos Creator is an editor that works on top of the Cocos2D-JS framework, assisting in the development of the graphical application whatever it is, showing the properties of the components, also providing a preview of what the game scene will be at the moment that it is loaded and a way to graphically manage the files and scripts that will be within the game. The main structure of Cocos Creator can be seen in Fig. 6.

The scenes within Cocos2D-x are the most basic component of each game window, within each scene, will be all the components, effects, texts and images of the game being then managed by the Director. The Cocos Creator offers a Scene View and a Game Preview to facilitate the development the game and provides fast feedback to the developers; it can be seen in the illustration of Fig. 6. The Fig. 7 shows the UML Class Diagram of the game.

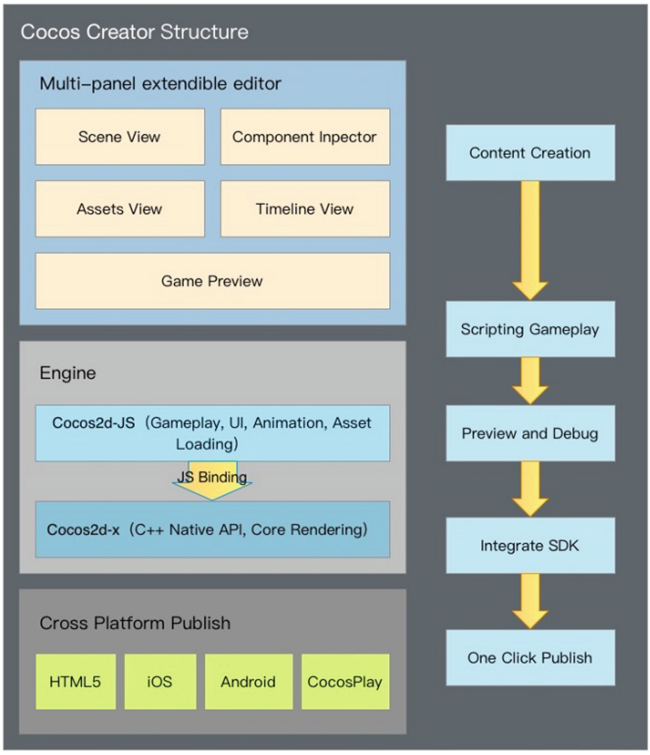

Fig. 6. Cocos Creator main structure: about cocos creator framework structure (Source: [28])

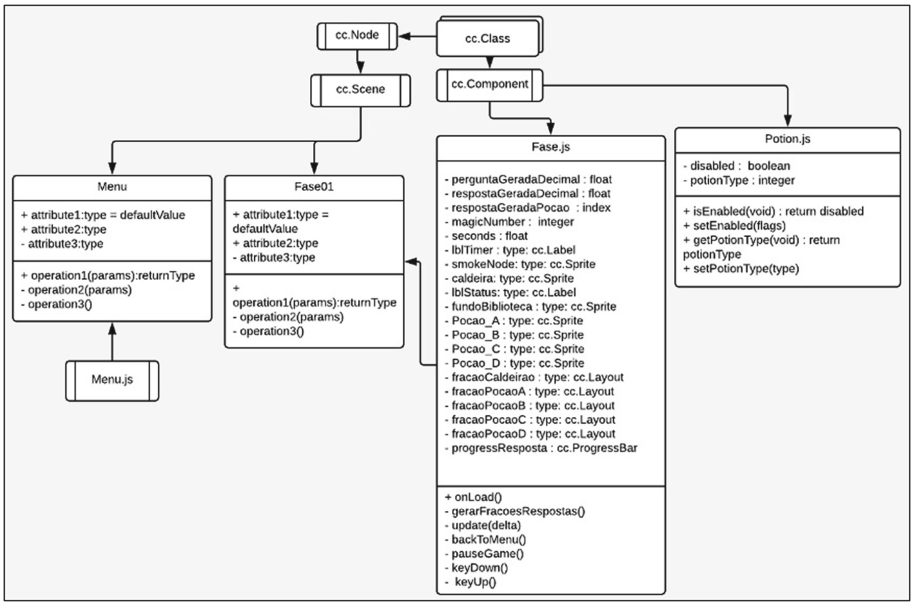

Fig. 7. UML Class Diagram of the FracPotion game (Source: the authors)

In the Class Diagram of the Fig. 7, we can see the composition of the game with the relationships of the classes, mainly between the Fases and Potions to be completed.

6 Evaluation of the Game and Educational Content

For the evaluation of the game FracPotion, an experiment was carried out with elementary school students. As we wanted to evaluate the experience of the children under a qualitative approach, we studied two different frameworks in the evaluation process. The first one was the generic ITU BT500 assessment (https://www.itu.int/rec/R-REC-BT.500) –and the other one was the MEEGA+ model [29]. After we have analyzed these two assessment models, we have chosen the second one, so the MEEGA+ model was used to evaluate the quality of the game and educational content, since it is directly focuses in educational game evaluation instead the generically approach of the ITU BT 500 for all type of software.

6.1 The Experiment

The experiment was carried out with elementary school students. The participants of the research were 18 students and the criterion of choice was that they already knew the basic concepts of fractions learned in the classroom. The profile of the students of the experiment presented a mean age of 12 years, there was a balance with respect to gender, but the girls slightly predominated on boys; 92% of volunteers make regular use of digital games - 47.6% on a daily basis and 9.5% weekly. Players first had an experience of interacting with the game, without the help of any instructor. Soon after, they answered a survey about the aspects of the game, involving the visual characteristics of the game, the interface, the easiness that the game possibly presents and other aspects of usability of the game. In addition, the survey had extra questions about the educational content presented and its relationship with the game, as a way to verify if the game met the educational purpose regarding fractions. The students answered the questions using five possible responses in a Likert-like scale (Too little, Little, Neutral, Very, and Very much).

The questions to evaluate the educational content of the game were:

(1) How motivated did you feel to learn about fraction with this game?
(2) Did you learn about fractions with this game?
(3) Would you recommend this game to a friend?
(4) Would you rather learn fractions using this Game?
(5) Did you feel challenged by this Game?

6.2 Results of the Work

The combination of texts, colors and sources were pointed out as good combination and consistency by 61% (11 students) of the interviewees and the other almost 17% (3 students) answered neutrally (neither agree nor disagree). As for the game being

intuitive and easy to adapt approximately 28% (5 students) agreed with this aspect and approximately 44% (8 students) responded with neutrality. As for the easiness to understand rules 50% (9 students) judged them easily understandable and the other almost 28% (5 students) responded with neutrality.

About the questions regarding the educational content, the students have answered to the question (1) positively or neutral 61% (11 students); question (2) have 56% of positive answers (10 students); third question (3) had 94% of positive or neutral answers (17 students); question (4) had 78% of positive answers (14 students); and question (5) had 67% of positive or neutral responses (12 students). The Table 1 was created to simplify this visualization.

Table 1. Evaluation of the game by students in 1 to 5-scale based on MEEGA+.

Characteristics of the game and questions to evaluate	(1) Too little	(2) Little	(3) Neutral	(4) Very	(5) Very much
(1) Motivation	3 (16.66%)	4 (22.22%)	3 (16.66%)	4 (22.22%)	4 (22.22%)
(2) Learn fractions	1 (5.55%)	2 (11.11%)	5 (27.77%)	6 (33.33%)	4 (22.22%)
(3) Recommend to others	1 (5.55%)	0 (0%)	7 (38.88%)	7 (38.88%)	3 (16.66%)
(4) Prefer this game against traditional methods	1 (5.55%)	1 (5.55%)	2 (11.11%)	9 (50.00%)	5 (27.77%)
(5) Feel challenged	3 (16.66%)	3 (16.66%)	5 (27.77%)	2 (11.11%)	5 (27.77%)

7 Conclusions

The teaching and learning processes in the field of Mathematics require an important effort on building knowledge over a very abstract body of knowledge, but with many practical applications in real-world situations. Fractions, for instance, tend to represent an extremely abstract concept, mainly when local culture do not make regular use of this kind of representation in common situations – which is the case for Brazilian culture, which have adopted the floating point decimal representation in detriment of the fractional one. Tools that help to stimulate students to keep motivational aspect when learning such subjects are potentially useful to these process.

This paper presented a game designed to support students to learn or revise some basic fraction concepts. Regarding the group to which this game was applied, the players/students showed interest in game's subject and they reported to have enjoyed the learning process supported by the game. However, the results point out that more research is needed in order to improve the game itself and to apply it in other study groups. Further work includes the development of other games for different subjects in Mathematics and other STEM areas. The current game is in its beta version; as improvements, new phases are to be developed with other types of fractions and operations, as well as the translation and adaptation capabilities of the game as an Open Educational Resource are to be tested.

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