Dynamic Optimization Model for Planning of Multi-echelon Logistic System Activity

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Abstract. In our paper, we consider functioning in dynamics (with discrete time) of logistics network including set of suppliers, set of manufacturers and set of points of destination of finished product. The optimization problem is formulated for determination of supply, production and transportation joint plans. It is assumed that total demand at destinations either is given over the planning horizon or is random with given probability densities. The objective function is total logistic cost along the whole network.

Keywords: Logistics network · Suppliers · Manufacturers · Finished product · Transportation problem · Joint planning · Total logistic cost · Dynamic optimization

1 Introduction

It is well-known that most part of real logistic systems may be described as a network with inventory of material or product at each node. Therefore, it is natural to use for modeling, optimization, and analysis of logistic systems (or supply chains) the results of inventory control and optimization theories. The book (Zipkin 2000), for example, covers many recent developments related to or impacting inventory such as ERP systems, supply chain management, JIT, etc. It covers also a wide spectrum of stochastic inventory models. Now a number of different models are developed in logistics describing many different real situations in industry and business (Bramel and Simchi-Levi 1997; Shapiro 2001; Postan 2006; Brandimarte and Zotteri 2007; Smith and Tan 2013). At the same time the inventories in logistic systems have been controlled by some specific for logistical management rules and methods. The most important among them is the "demand driven principle", that is, management taking into account the feedback between real volumes of finished product sale and its manufacturing. Note also that main goal of any logistical system functioning is material flows movement along the whole supply chain from one supply chain to another one or

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to end consumer with the minimum total cost (Christopher 2011). These and some other peculiarities of logistical management do not allow to apply immediately the "ready" models from inventory control theory in logistics practices.

The most part of existing applications of inventory control theory for logistic systems modeling and optimization takes into account one type of inventory only, e.g. raw material (Bramel and Simchi-Levi 1997; Shapiro 2001; Postan 2006). In the articles (Morozova et al. 2013; Postan et al. 2014) the multi-echelon logistic systems were under consideration with two types of inventory: raw materials and finished product. The corresponding models were based on generalization of the Wagner-Whitin dynamic model from inventory control theory. Besides, in the cited works the transportation problem was included in model for the purpose of joint optimization of supply, production and finished product transportation plans of integrated supply chain.

Our paper is devoted to further development of the approach to modeling and optimization of logistical networks initiated in the works (Morozova et al. 2013; Postan et al. 2014).

2 Problem Statement

Let us consider logistic network including the S enterprises-suppliers manufacturing different complete set for further manufacturing the finished products by M enterprises-manufacturers, transportation network, and set of points of finished product delivery.

Each enterprise-supplier purchases the raw materials and/or complete set from the vendors. We make the following assumptions and simplifications:

* The market of raw materials is unlimited.
* All ordering of materials, complete set, and delivering the finished products occurs at the start of each period.
* The lead time is zero: that is, an order arrives as soon as it is placed.
* The production equipment of all enterprises is absolutely reliable.
* The capacities of production lines of all enterprises are limited only by capacities of warehouses' for storage of raw materials and finished products.

The sth enterprise-supplier manufactures the L_s kinds of complete set from the R_s kinds of raw materials and other industrial resources. For manufacturing by the sth enterprise-supplier the lth kind of complete set's unit it is needed to use the rth kind of raw materials in the amount of $a_{srl}^{(1)}, s = 1, 2, \ldots, S; l = 1, 2, \ldots, L_s; r = 1, 2, \ldots, R_s$.

The initial inventory level of the rth kind of raw material at the sth supplier's warehouse is $q_{sr}^{(1)}$. It is assumed that $\sum_{r=1}^{R_s} q_{sr}^{(1)} \leq W_{1s}$, where W_{1s} is the warehouse's capacity, s = 1,2,…, S. The capacity of warehouse for storage of produced complete set by the sth supplier is W_{2s} and initial inventory level of complete set is $q_{sl}^{(2)}, \sum_{l=1}^{L_s} q_{sl}^{(2)} \leq W_{2s}$.

The complete set manufactured by enterprises-suppliers are purchased by the mth enterprise-manufacturer for manufacturing the K_m types of finished products.

The capacity of warehouse for storage of the complete set at the mth enterprise-manufacturer is denoted by W_{3m} and initial inventory level of each kind of complete set is $q_{slm}^{(3)}$. It is naturally to assume that following constraint is valid

$$\sum_{s=1}^{S}\sum_{l=1}^{L_s} q_{slm}^{(3)} \leq W_{3m}, m = 1,2,\ldots,M.$$

Let $a_{slmk}^{(2)}$ be the amount of the lth of complete set manufactured by the sth supplier needed for manufacturing the kth type of finished product's unit in the mth enterprise-manufacturer, $s = 1,2,\ldots,S;\ l = 1,2,\ldots,L_s;\ k = 1,2,\ldots,K_m;\ m = 1,2,\ldots,M$. The finished products of the mth enterprise-manufacturer come to the warehouse with the capacity W_{4m} from which they must be delivered at the N points of destination (or finite consumption). The initial inventory level of the kth type of finished product at warehouse of the mth enterprise-manufacturer is $q_{mk}^{(4)}, \sum_{k=1}^{K_m} q_{mk}^{(4)} \leq W_{4m}, m = 1,2,\ldots,M$.

Let d_{mkn} be the total demand for the kth type of finished product produced by the mth manufacturer at the nth destination over the planning horizon T. To avoid the trivial situation, we will assume that following conditions hold true

$$q_{mk}^{(4)} < \sum_{n \in B_{mk}} d_{mkn},$$

where $B_{mk} = \{n | d_{mkn} > 0, n = 1,2,\ldots,N\}, k = 1,2,\ldots,K_m$. Note that the values d_{mkn} may be determined as a result of market research and demand forecasting at points of destination of finished product.

Let us introduce the control variables and corresponding variables describing the inventory levels fluctuation over the planning horizon T:

* Let $x_{srt}^{(1)}$ be the amount of the rth kind of material ordered and purchased by the sth enterprise-supplier in period t, for t = 1,2,…,T.
* Let $x_{slt}^{(2)}$ be the lth kind of complete set which the sth enterprise plans for output at the end of period t, for t = 1,2,…, T.
* Let $x_{slmt}^{(3)}$ be the lth kind of complete set produced by the sth supplier purchased by the mth enterprise-manufacturer at the end of period t, for t = 1,2,…,T.
* Let y_{mkt} be the amount of the kth type of finished product planned for output by the mth enterprise-manufacturer at the end of period t, for t = 1,2,…,T.
* Let z_{mknt} be the amount of the kth type of finished product produced by the mth enterprise-manufacturer which is planned for delivery from warehouse to the nth destination at the end of period t, for t = 1,2,…,T.
* Let $I_{srt}^{(1)}$ be the inventory level of the rth kind of material at the warehouse of the sth supplier at the end of period t, for t = 1,2,…,T.
* Let $I_{slt}^{(2)}$ be the inventory level of the lth kind of complete set manufactured by the sth supplier at the end of period t, for t = 1,2,…,T.

* Let $I^{(3)}_{slmt}$ be the inventory level of the lth kind of complete set produced by the sth supplier at the warehouse of the mth manufacturer at the end of period t, for $t = 1,2,\ldots,T$.
* Let $I^{(4)}_{mkt}$ be the inventory level of the kth type of finished product manufactured by the mth manufacturer at the end of period t, for $t = 1,2,\ldots,T$.

To describe the economic effectiveness of supply, production and delivery plans we need the additional initial parameters, namely:

* Let $c^{(1)}_{srt}$ be the per unit order cost and $K^{(1)}_{srt}$ be the fixed order cost for the rth kind of material ordered by the sth enterprise-supplier in period t, for $t = 1,2,\ldots,T$.
* Let $e^{(1)}_{slt}$ be the per unit production cost of the lth type of complete set produced by the sth supplier in period t, for $t = 1,2,\ldots,T$.
* Let $c^{(2)}_{slmt}$ be the per unit order cost and $K^{(2)}_{slmt}$ be the fixed order cost for the lth kind of complete set produced by the sth supplier ordered by the mth enterprise-manufacturer in period t, for $t = 1,2,\ldots,T$.
* Let $h^{(1)}_{srt}$, $h^{(2)}_{slt}$ be the holding cost per unit of the rth kind of material and the lth kind of produced complete set of the sth enterprise-supplier correspondingly in period t, for $t = 1,2,\ldots,T$.
* Let $h^{(3)}_{slmt}, h^{(4)}_{mkt}$ be the holding cost per unit of the lth kind of complete set produced by the sth supplier at warehouse of the mth manufacturer and the kth finished product of the mth manufacturer correspondingly in period t, for $t = 1,2,\ldots,T$.
* Let $c^{(2)}_{slt}$ be the per unit production cost of the lth kind of complete set manufactured by the sth supplier in period t, for $t = 1,2,\ldots,T$.
* Let $c^{(3)}_{slmt}$ be the per unit cost for purchasing the lth kind of complete set by the mth manufacturer in period t, for $t = 1, 2,\ldots,T$.
* Let $e^{(2)}_{mkt}$ be the per unit production cost of the kth type of finished product produced by the mth manufacturer in period t, for $t = 1,2,\ldots,T$.
* Let $c^{(5)}_{mknt}$ be the cost of transportation of the unit of the kth type of finished product from the mth manufacturer to the nth destination in period t, for $t = 1,2,\ldots,T$.

It is obvious that the following inventory-balanced equations are valid:

$$I^{(1)}_{srt} = I^{(1)}_{sr,t-1} + x^{(1)}_{srt} - \sum_{l=1}^{L_s} a^{(1)}_{slr} x^{(2)}_{slt}, \quad s = 1,2,\ldots,S;\ r = 1,2,\ldots,R_s, \tag{1}$$

$$I^{(2)}_{slt} = I^{(2)}_{sl,t-1} + x^{(2)}_{slt} - \sum_{m=1}^{M} x^{(3)}_{slmt}, \quad s = 1,2,\ldots,S;\ l = 1,2,\ldots,L_s, \tag{2}$$

$$I^{(3)}_{slmt} = I^{(3)}_{slm,t-1} + x^{(3)}_{slmt} - \sum_{k=1}^{K_m} a^{(2)}_{slmk} y_{mkt}, \\ 1,2,\ldots,S;\ l = 1,2,\ldots,L_s;\ m = 1,2,\ldots,M, \tag{3}$$

$$I_{mkt}^{(4)} = I_{mk,t-1}^{(4)} + y_{mkt} - \sum_{n \in B_{mk}} z_{mknt}, \qquad k = 1,2,\ldots,K_m;\; m = 1,2,\ldots,M;\; t = 1,2,\ldots,T, \tag{4}$$

where $I_{sr0}^{(1)} = q_{sr}^{(1)}, I_{sl0}^{(2)} = q_{sl}^{(2)}, I_{slm0}^{(3)} = q_{slm}^{(3)}, I_{km0}^{(4)} = q_{km}^{(4)}$.

The Eqs. (1)–(4) describe the dynamics of inventory levels in warehouses in each period of time.

After solving the difference Eqs. (1)–(4), we obtain

$$I_{srt}^{(1)} = q_{sr}^{(1)} + \sum_{j=1}^{t} x_{srj}^{(1)} - \sum_{j=1}^{t} \sum_{l=1}^{L_s} a_{slr}^{(1)} x_{slj}^{(2)}, \qquad s = 1,2,\ldots,S;\;\; r = 1,2,\ldots,R_s, \tag{5}$$

$$I_{slt}^{(2)} = q_{sl}^{(2)} + \sum_{j=1}^{t} x_{slj}^{(2)} - \sum_{j=1}^{t} \sum_{m=1}^{M} x_{slmj}^{(3)}, \qquad s = 1,2,\ldots,S;\; l = 1,2,\ldots,L_s, \tag{6}$$

$$I_{slmt}^{(3)} = q_{slm}^{(3)} + \sum_{j=1}^{t} x_{slmj}^{(3)} - \sum_{k=1}^{K_m} \sum_{j=1}^{t} a_{slmk}^{(2)} y_{mkj}, \qquad m = 1,2,\ldots,M;\;\; s = 1,2,\ldots,S;\;\; l = 1,2,\ldots,L_s, \tag{7}$$

$$I_{mkt}^{(4)} = q_{mk}^{(4)} + \sum_{j=1}^{t} y_{mkj} - \sum_{j=1}^{t} \sum_{n \in B_{mk}} z_{mknj}, \qquad k = 1,2,..,K_m;\;\; t = 1,2,\ldots,T. \tag{8}$$

Since the total inventory levels

$$\sum_{r=1}^{R_s} I_{srt}^{(1)}, \quad \sum_{l=1}^{L_s} I_{slt}^{(2)}, \quad \sum_{s=1}^{S} \sum_{l=1}^{L_s} I_{slmt}^{(3)}, \quad \sum_{k=1}^{K_m} I_{mkt}^{(4)}$$

for any t can't exceed the values W_{1s}, W_{2s}, W_{3m}, W_{4m} correspondingly, from (5)–(8), it follows

$$\sum_{r=1}^{R_s} q_{sr}^{(1)} + \sum_{j=1}^{t} \sum_{r=1}^{R_s} x_{srj}^{(1)} - \sum_{j=1}^{t} \sum_{l=1}^{L_s} \sum_{r=1}^{R_s} a_{slr}^{(1)} x_{slj}^{(2)} \le W_{1s}, \;\; s = 1,2,\ldots,S;\; t = 1,2,\ldots,T, \tag{9}$$

$$\sum_{l=1}^{L_s} q_{sl}^{(2)} + \sum_{j=1}^{t}\sum_{l=1}^{L_s} x_{slj}^{(2)} - \sum_{j=1}^{t}\sum_{m=1}^{M} x_{slmj}^{(3)} \le W_{2s},\ s = 1,2,\ldots,S;\ t = 1,2,\ldots,T, \tag{10}$$

$$\sum_{s=1}^{S}\sum_{l=1}^{L_s} q_{slm}^{(3)} + \sum_{j=1}^{t}\sum_{s=1}^{S}\sum_{l=1}^{L_s} x_{slmj}^{(3)} - \sum_{j=1}^{t}\sum_{s=1}^{S}\sum_{l=1}^{L_s}\sum_{k=1}^{K_m} a_{slmk}^{(2)} y_{mkj} \le W_{3m}, m = 1,2,\ldots,M; t = 1,2,\ldots,T, \tag{11}$$

$$\sum_{k=1}^{K} q_{mk}^{(4)} + \sum_{j=1}^{t}\sum_{k=1}^{K_m} y_{mkj} - \sum_{j=1}^{t}\sum_{n\in B_{mk}}\sum_{k=1}^{K_m} z_{mknj} \le W_{4m}, \quad m = 1,2,\ldots,M; t = 1,2,\ldots,T. \tag{12}$$

$$\sum_{r=1}^{R_s} q_{sr}^{(1)} + \sum_{j=1}^{t}\sum_{r=1}^{R_s} x_{srj}^{(1)} - \sum_{j=1}^{t}\sum_{l=1}^{L_s}\sum_{r=1}^{R_s} a_{slr}^{(1)} x_{slj}^{(2)} \le W_{1s},\ s = 1,2,\ldots,S;\ t = 1,2,\ldots,T,$$

On the other hand, the enterprises-suppliers for complete set manufacturing in period t can use only inventories of materials which are in warehouses in the end of period $t - 1$, that is

$$\sum_{l=1}^{L_s} a_{slr}^{(1)} x_{slt}^{(2)} \le I_{sr,t-1}^{(1)}, \quad s = 1,2,\ldots,S;\ r = 1,2,\ldots,R_s; t = 1,2,\ldots,T. \tag{13}$$

Further, the all manufacturers in period t can use only inventories of complete set of the sth supplier, which are at warehouse in the end of period $t - 1$, therefore

$$\sum_{m=1}^{M} x_{slmt}^{(3)} \le I_{sl,t-1}^{(2)}, s = 1,2\ldots,S; l = 1,2,\ldots,L_s; t = 1,2,\ldots,T. \tag{14}$$

Similarly, for the mth enterprise-manufacturer the following restrictions must be fulfilled

$$\sum_{k=1}^{K_m} a_{slmk}^{(2)} y_{mkt} \le I_{slm,t-1}^{(3)},\ s = 1,2,\ldots,S; l = 1,2,\ldots,L_s; t = 1,2,\ldots,T; \quad m = 1,2,\ldots,M. \tag{15}$$

In period t it can't be delivered the kth type of finished product at the all destinations in amount more than inventory level in period $t-1$, that is $I^{(4)}_{mk,t-1}$. Therefore

$$\sum_{n=1}^{N} z_{mknt} \le I^{(4)}_{mk,t-1}, \; k = 1,2,\ldots,K_m; m = 1,2,\ldots,M; t = 1,2,\ldots,T. \tag{16}$$

At last, the kth finished product must be delivered at the nth destination in amount d_{mkn} over the planning horizon, i.e.

$$\sum_{t=1}^{T} z_{mknt} = d_{mkn}, \quad k = 1,2,\ldots,K_m; n = 1,2,\ldots,N; m = 1,2,\ldots,M. \tag{17}$$

From (13)–(16), taking into account the relations (5)–(8), we obtain the following constraints

$$\sum_{l=1}^{L_s}\sum_{j=1}^{t} a^{(1)}_{slr} x^{(2)}_{slj} \le q^{(1)}_{sr} + \sum_{j=1}^{t-1} x^{(1)}_{srj}, \; s = 1,2,\ldots,S; r = 1,2,\ldots,R_s; \tag{18}$$

$$\sum_{j=1}^{t}\sum_{m=1}^{M} x^{(3)}_{slmj} \le q^{(2)}_{sl} + \sum_{j=1}^{t-1} x^{(2)}_{slj}, \; s = 1,2,\ldots,S; \; l = 1,2,\ldots,L_s, \tag{19}$$

$$\sum_{j=1}^{t}\sum_{k=1}^{K_m} a^{(2)}_{slmk} y_{mkj} \le q^{(3)}_{slm} + \sum_{j=1}^{t-1} x^{(3)}_{slmj}, \; s = 1,2,\ldots,S; \; l = 1,2,\ldots,L_s; \tag{20}$$

$$\sum_{j=1}^{t}\sum_{n\in B_{mk}} z_{mknj} \le q^{(4)}_{mk} + \sum_{j=1}^{t-1} y_{mkj}, m = 1,2,\ldots,M; k = 1,2,\ldots,K_m; \quad t = 1,2,\ldots,T. \tag{21}$$

The following conditions of non-negativity of control parameters must be added to the constraints (9)–(12), (16)–(19)

$$x^{(1)}_{srt}, x^{(2)}_{slt}, x^{(3)}_{slmt}, y_{mkt}, z_{mknt} \ge 0, \forall s, r, l, m, k, n, t. \tag{22}$$

As an objective function, we choose the total logistic cost for the whole supply chain under consideration over the planning horizon. Taking into account the designations introduced previously, the expression for this total cost takes the form

$$
\begin{aligned}
C = \sum_{t=1}^{T} \{ & \sum_{s=1}^{S} \sum_{r=1}^{R_s} [c_{srt}^{(1)} x_{srt}^{(1)} + K_{srt}^{(1)} \delta(x_{srt}^{(1)}) + h_{srt}^{(1)} (q_{sr}^{(1)} + \sum_{j=1}^{t} x_{srt}^{(1)} \\
& - \sum_{j=1}^{t} \sum_{l=1}^{L_s} a_{slr}^{(1)} x_{slt}^{(2)})] + \sum_{s=1}^{S} \sum_{l=1}^{L_s} [e_{sl}^{(1)} x_{slt}^{(2)} + h_{slt}^{(2)} (q_{sl}^{(2)} + \sum_{j=1}^{t} x_{slj}^{(2)} - \sum_{j=1}^{t} \sum_{m=1}^{M} x_{slmj}^{(3)})] \\
& + \sum_{m=1}^{M} \sum_{s=1}^{S} \sum_{l=1}^{L_s} [c_{slmt}^{(2)} x_{slmt}^{(3)} + K_{slt}^{(2)} \delta(x_{slmt}^{(3)}) \\
& + h_{slmt}^{(3)} (q_{slm}^{(3)} + \sum_{j=1}^{t} x_{slmj}^{(3)} - \sum_{j=1}^{t} \sum_{k=1}^{K_m} a_{slmk}^{(2)} y_{mkj})] \\
& + \sum_{m=1}^{M} \sum_{k=1}^{K_m} [e_{mkt}^{(2)} y_{mkt} + h_{mkt}^{(4)} (q_{mk}^{(4)} + \sum_{j=1}^{t} y_{mkj} - \sum_{j=1}^{t} \sum_{n \in B_{mk}} z_{mknj})] \\
& + \sum_{m=1}^{M} \sum_{k=1}^{K_m} \sum_{n \in B_{mk}} c_{mknt}^{(3)} z_{mknt} \},
\end{aligned}
\tag{23}
$$

Where $\delta(x) = 1$ if $x > 0, \delta(0) = 0$.

Thus, we can formulate the following optimization problem: it is needed to find out the variables $x_{srt}^{(1)}, x_{slt}^{(2)}, x_{slmt}^{(3)}, y_{mkt}, z_{mknt}$ satisfying the constraints (9)–(12), (17)–(23) and minimizing the function (23). This optimization problem may be solved, for example, by dynamic programming algorithm or by the method based on reduction of our optimization model to partly integer linear programming problem.

3 Optimization Model for Random Demand Over Planning Horizon

Now we will assume that values $d_{mkn}(\omega)$ are the continuous mutually independent random variables with the given probability densities $\varphi_{mkn}(d)$. Here we will apply the approach proposed in the work (Williams, 1963). Put

$$
u_{mkn} = \sum_{t=1}^{T} z_{mknt},
$$

where u_{mkn} is total amount of the kth finished product planned for delivery to the nth destination before realization of random demand $d_{mkn}(\omega)$. In result of its realization, one of two risks may occur:

1. $u_{mkn} < d_{mkn}(\omega)$, i.e. demand will not be met;
2. $u_{mkn} > d_{mkn}(\omega)$, i.e. there is necessity of storage of the kth finished product's surplus at the nth destination.

We assume that both risks belong to the destinations, i.e. the all manufactured finished products will be sold. Let π_{mkn} be the penalty for the kth finished product's deficit at the nth destination, and h_{mkn} be the holding cost for storage per unit of the kth product at the nth destination. Then average total logistic cost over the planning horizon is

$$\bar{C} = C + \sum_{m=1}^{M} \sum_{k=1}^{K_m} \sum_{n=1}^{N} \{\pi_{mkn} \int_{0}^{u_{mkn}} (u_{mkn} - w)\varphi_{mkn}(w)dw + h_{mkn} \int_{u_{mkn}}^{\infty} (w - u_{mkn})\varphi_{mkn}(w)dw\}, \tag{24}$$

where C is defined by (23). The function $\bar{C}$ is concave in respect to the variables u_{kn}. Taking second derivative of $\bar{C}$ with respect to u_{kn} and applying the Leibnitz rule, we obtain

$$\frac{\partial^2}{\partial u_{mkn}^2}\bar{C} = -(\pi_{ik} + h_{kn})\varphi_{kn}(u_{mkn}).$$

Since $\pi_{kn} + h_{kn} > 0$ by definition, the expression on the right-hand side of the last equality is non-positive. Hence, function (24) is concave.

4 Conclusion

Our paper contributes to the deepening of the knowledge on the recent application of the inventory control theory in the logistics networks analyses and optimal planning. It focuses on a problem of optimization of logistics practices by simultaneously considering the supply, production and transportation plans of two types of inventories: raw materials and finished products to arrive at a cost total logistics minimum. For this purpose, the approach has been developed to modeling and optimization of logistic network on the basis of inventory control theory. The classical Wagner-Whitin model is generalized for the case of final set of suppliers, manufacturers, and points of destination. Our approach allows to realize the optimal synergism in result of better coordination between all participants of multi-echelon supply chain.

The further research in this direction may be focused on the following topics:

* taking into account the influence of stochastic fluctuations of demand at destinations in each period if time;
* consideration of competition among different supply chains forming the logistic network (Postan et al., 2017).

References

Zipkin, P.H.: Foundations of Inventory Management, 1st edn. McGrow-Hill/Irwin, Boston (2000)

Bramel, J., Simchi-Levi, D.: The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management. Springer, New York (1997)

Brandimarte, P., Zotteri, G.: Introduction to Distribution Logistics. Wiley, New York (2007)

Christopher, M.: Logistics & Supply Chain Management (Creating Value-Adding Networks), 4th edn. Prentice Hall, Harlow (2011)

Dachkovski S, Wirth, F., Jagalski, T.: Autonomous control in shop floor logistics: analytic models. In: Chryssolouris G, Mourtzis D (eds.) Manufacturing, Modelling, Management and Control 2004. Elsevier Science Ltd, Amsterdam (2005)

Smith, J.M., Tan, B. (eds.): Handbook of Stochastic Models and Analysis of Manufacturing System Operations, vol. XXVIII. Springer, New York (2013)

Morozova, I.V., et al.: Dynamic optimization model for planning of integrated logistical system functioning. In: Kreowski, H.-J., et al. (eds.) Dynamics in Logistics. Lecture Notes in Logistics, pp. 291–300. Springer, Heidelberg (2013)

Postan, M.Y.: Economic-Mathematical Models of Multimodal Transport. Astroprint, Odessa (2006). (in Russian)

Postan, M.Y., et al.: Dynamic model for optimization of production and finished products delivery plans in supply chain. Logistyka **4**, 2345–2352 (2014)

Shapiro, J.F.: Modeling the Supply Chain. Duxbury/Thomson Learning, Pacific Grove (2001)

Williams, A.C.: A stochastic transportation problem. Oper. Res. **11**(5), 759–770 (1963)

Postan, M.Y., et al.: Method of finding equilibrium solutions for duopoly of supply chains taking into account the innovative activity of enterprises. East. Euro. J. Enterp. Technol. **3**(87), 25–30 (2017). https://doi.org/10.15587/1729-4061.2017.103989