



Ambiguity Preference and Context Learning in Uncertain Signaling

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Abstract. Lexical ambiguity is present in many natural languages, but ambiguous words and phrases do not seem to be advantageous. Therefore, the presence of ambiguous words in natural language warrants explanation. We justify the existence of ambiguity from the perspective of the context dependence. The main contribution of the paper is that we constructed a context learning process such that the interlocutors can infer opponent's private belief from the conversation. A sufficient condition is proved to show if the learning can be successful. Furthermore, we investigate when the learning fails, how the interlocutors choose among degrees of ambiguous expressions through an adaptive learning.

Keywords: Ambiguity · Context learning · Uncertain signaling · Reinforcement learning

1 Introduction

Natural language involves various kinds of uncertainties such as vagueness, synonymy and ambiguity. Among those uncertainties, lexical ambiguity is one of the most common features in language. Lexical ambiguity lies in the fact that a word could have more than one interpretations. For example, the word “mole” in English can be used to refer to “a dark spot on the skin”, to “a burrowing mammal”, to “a spy”. In terms of information transaction, ambiguity does not seem an optimal choice. It is because the use of ambiguous expressions may cause the failure of information transaction and misunderstandings. We have not run out of possible words, why not invent a new word for any one of the meanings for ambiguous words? Therefore, the existence of ambiguous words needs an explanation.

In linguistics and game theory, many people have discussed this problem, and in most works, it is argued that being precise is expensive and unnecessary. Language, therefore, optimizes the balance of the benefits of precision with the costs of lexicon size (see Piantadosi et al. 2012; O'Connor 2014a; Santana 2014). The core of this argument relies on the fact that the context of the conversation can fill in information gaps left by ambiguity. According to Grice's cooperative principle (see Grice 1968) the conversational inference is based on the notion

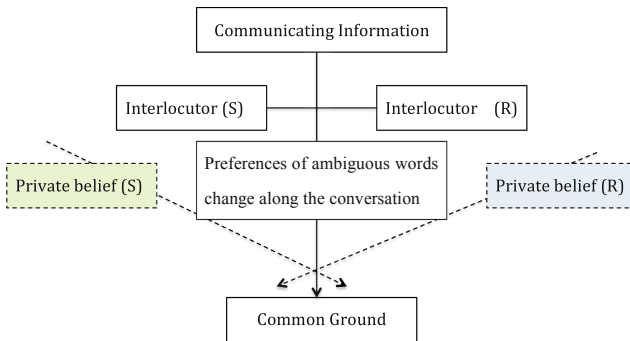
of common ground. From Stalnaker (2002), common ground is defined as the mutually recognized shared information in a situation where an act of trying to communicate takes place. In a conversation, common ground is treated as the conversational context, which plays an essential role in the pragmatic understanding of the language.

Furthermore, the influence of the context on the use of language depends on how much mutual information the interlocutors share and what kind of vocabularies is available. As the context gets clear, the more ambiguous word may be sufficient for transferring the information. However, it has not been fully investigated that where the context comes from and how the context affects the interlocutors' choice of words with different degrees of ambiguity.

The goal of this paper is to justify the existence of ambiguity from the perspective of the context dependence. We construct a context learning process in a signaling game for building the common ground of the conversation along the interactions. After that, the interlocutors' preferences of ambiguous words can be tracked as the context varies.

More specifically, we consider two interlocutors, a sender (S) and a receiver (R), are conducting a conversation for transferring information. They both have some personal beliefs about the communicating information. As the communication goes on, the interlocutors gradually infer the other's private belief from the result of each interaction. After repeated interactions, players are able to form a common ground, which serves as the context for the conversation. In addition, during the learning process, interlocutors' choices of ambiguous words may change as their beliefs about the context vary.

The following graph summarises the discussion above.



For implementing the idea above, we use Lewis's signaling game as our base model for the learning process (see Lewis 1969). Lewis's signaling game describes a very general communication scenario where a sender observes the situation (a state of the world) and then sends a signal to a receiver. The receiver takes an action based on the signal he receives. The payoff in the game depends on the state of the world and the action the receiver takes. The uncertainty of the signaling game comes from the receiver's ignorance of the true state. He can only obtain the information from the signals that the sender sends. The model has been widely used in exploring language communication (see O'Connor 2014b; Huttegger et al. 2010; Huttegger 2007; Zollman 2005; Jäger 2014).

Based on the Lewis’s signaling game, we made two main changes to the model. The first change is to add players’ private beliefs about the communicating information into the game. The second change is to expand the set of signals such that we can discuss ambiguous words with different degrees. With these two changes, we construct a context learning process through which the players are learning each other’s beliefs during the repeated interactions. A sufficient condition is provided under which the learning is successful. After the players learned each other’s private belief, they can form a mutual belief that serves as the common ground of the conversation. Furthermore, when the learning fails, though there is uncertainty about opponent’s belief, we show that players still have a strong tendency to choose ambiguous expressions. We explore an uncertain signaling through the reinforcement learning mechanism.

The structure of the paper is the following. In Sect. 2, we first extend Lewis’s signaling game such that each player has private belief about the communicating information. Then we discuss how players can learn opponent’s private belief from the repeated communication. In Sect. 3, we establish a sufficient condition on the game under which the learning is successful. Under this condition, we discuss how the common ground of the conversation can be formed. In Sect. 4, we explore, when the learning fails, players’ preferences of different ambiguous expressions through the reinforcement learning signaling. The paper ends with a comparison between our model and existing models on discussing uncertain signaling and pragmatic reasoning in conversation.

2 Signaling Game with Private Belief (SPB)

As the conversation goes, the interlocutors’ beliefs about the communicating information change. For studying this dynamic process, we develop a new signaling game called signaling game with private belief (SPB). Based on Lewis’s classical model, we assume that each player has private belief about the communicating information before the communication starts. Formally, we define players’ private beliefs as partitions on the set of the states as follows.

Definition 1. *Given a set of finite states T , a private belief of player i indicated as B_i is defined as a partition on T . Each component of the belief partition is called an element of the partition that is indicated as $B_{ik}, k \in N$.*

When the game begins, nature reveals the information to the sender. At the same time, since the receiver has private information about the state, he is also aware of relative information about the true state¹. The sender then sends a signal that carries information about the true state. By combining the signal information and his private information, the receiver takes an action that decides both players’ benefits. Moreover, We argue that by observing the outcome of the game, the sender can possibly infer the receiver’s private belief.

States and actions can have a broad interpretation. The most obvious way to understand them is that the state is some important fact about the outside

¹ Similar idea appeared in Santana (2014)’s work.

world and the action is some response to that fact. The state might be whether it is raining and the action might be to take an umbrella. But, the state might be something more internal, like the desire for the receiver to hold a certain belief. Similarly the action might be private, like coming to believe something.

Formally, SPB model is defined as follows.

Definition 2. *A signaling game with private belief (SPB) consists of the following parts:*

- two players: player 1 and player 2;
- two roles: a sender and a receiver;
- a set of states $T : \{1, 2, \dots, n\}$, each state is assumed to occur with the same probability.
- a set of possibly ambiguous signals S with conventional meanings;
- each player i has private belief B_i that is unknown to their opponents;
- a set of actions $\mathcal{A} : \{a_1, a_2, \dots, a_n\}$;
- Sender's strategy is a function $f : T \rightarrow S$, the receiver's strategy is a function $g : S \times B_i \rightarrow \mathcal{P}(\mathcal{A}), i \in \{1, 2\}$, where $\mathcal{P}(\mathcal{A})$ is the power set on \mathcal{A} .
- A payoff function u for both players,

$$u(t, A, B_{i \in \{1,2\}}) = \begin{cases} \frac{1}{|A|} & \text{if } a_t \in A, |A| \text{ represents the size of the set } A, \\ 0 & \text{Otherwise.} \end{cases}$$

where $t \in T$ and $A \in \mathcal{P}(\mathcal{A})$;

An example (Example 1) is provided below for illustrating the concepts of the game.

Example 1. The game consists of the following parts.

- Two players: player 1, player 2
- Two roles: a sender, a receiver
- A set of states $\{1, 2, 3, 4\}$ occurring with equal probability;
- A set of signals s_1, s_2 with commonly known conventional meanings (s_1 indicates the states $\{1, 2\}$, s_2 indicates the states $\{3, 4\}$).

$$\begin{array}{c} 1, 2 || 3, 4 \\ s_1 || s_2 \end{array}$$

Since each signal can represent two different states, we say both signals are ambiguous. We use “||” to indicate partitions on the meaning of signals and “|” to indicate the partitions of the private belief.

- Each player has a private belief about the states and does not know the opponent's belief. We assume that player 1's private belief is

$$\begin{array}{c} 1, 2 | 3, 4 \\ B_{11} | B_{12} \end{array}$$

which means that player 1 can distinguish $\{1, 2\}$ from $\{3, 4\}$ but not further. And player 2's private belief is

$$1|2, 3, 4 \\ B_{21}|B_{22}$$

- A set of actions $\{a_1, a_2, a_3, a_4\}$

Firstly, we suppose that player 1 is the sender and player 2 is the receiver. Then, for each possible state, the game is played as follows.

Table 1. Play of the game in Example 1 (1)

State	Signal	Revealed information	Receiver's reasoning	Payoff
1	s_1	B_{21}	$B_{21} \cap s_1$	1
2	s_1	B_{22}	$B_{22} \cap s_1$	1
3	s_2	B_{22}	$B_{22} \cap s_2$	1/2
4	s_2	B_{22}	$B_{22} \cap s_2$	1/2

Table 1 shows for each state, what signal the sender sends, how the receiver reasons and what the outcome of the game is. For instance, the first row indicates that when state 1 occurs, the sender sends signal s_1 . The receiver reads the information from the signal that is $\{1, 2\}$, then combines it with his private belief B_{21} that is $\{1\}$, which yields the information $\{1\}$. Because $\{1\}$ is precise, the receiver is able to take the correct action which guarantees the best payoff for both players. In this example, the result holds trivially because the receiver's private belief already reveals the precise state information even without the information from the sender. Since the sender does not know the receiver's private belief, he sends the ambiguous signal anyway. But in row 3, the outcome information is $\{3, 4\}$, which yields the finer information than the receiver's private belief. Hence, combining personal belief and signal information is essential in this case.

After communicating repeatedly, player 1 (sender) is able to infer player 2's (receiver) private belief by comparing the differences between the outcomes resulting from his inference and the actual outcome of the game. Player 1's inference about player 2's private belief with respect to Table 1 can be described as follows. For simplicity, we assume that both players know that players' belief partition contains two elements². At each stage of the inference, from the sender's point of view, all the possible configurations of the receiver's private belief are listed.

At the initial stage, since only partitions with two elements are considered, the receiver's private belief has only three possibilities. They are

$$1|2, 3, 4 \quad 1, 2|3, 4 \quad 1, 2, 3|4$$

² This assumption is just for simplifying the illustration.

After state 1 occurs, the sender does some counterfactual reasoning. Specifically, the sender reasons that what the outcome of the game will be if he plays the game with the receiver who holds one of the three possible beliefs. It is easy to observe that only the first one yields payoff 1, which is consistent with the actual outcome in Table 1. Therefore, the other two possibilities are eliminated. Therefore, player 1 has learned that player 2’s private belief is $1|2, 3, 4$.

The order of the occurrence of the states affect the learning process. If state 3 or state 4 occurs first, then the learning procedure may be different. The following analysis shows player 1’s reasoning process when state 3 occurs first.

The initial stage is the same, there are three possibilities

$$1|2, 3, 4 \quad 1, 2|3, 4 \quad 1, 2, 3|4$$

After state 3 is communicated, the third possibility yields payoff 1 that is different from the actual payoff in Table 1. Therefore, the third possibility can be eliminated. Hence, two possibilities still remain.

$$1|2, 3, 4 \quad 1, 2|3, 4$$

After state 1 is communicated, applying the similar reasoning, player 1 infers player 2’s private belief precisely as follows.

$$1|2, 3, 4$$

Therefore, in this simple toy game, in at most two steps, namely, after communicating state 3 or state 4 and state 1 or state 2, player 1 (sender) is able to infer player 2’s (receiver) private belief correctly. In addition, if player 1 is lucky enough that state 1 or state 2 occurs earlier than state 3 or state 4, then player 1 is able to learn player 2’s private belief quickly.

Similarly, player 2 is also able to infer player 1’s private belief by playing the role of the sender. The following part shows the reasoning process where player 2 is the sender and player 1 is the receiver.

Table 2. Play of the game in Example 1 (2)

State	Signal	Revealed information	Receiver’s reasoning	Payoff
1	s_1	B_{11}	$B_{11} \cap s_1$	1/2
2	s_1	B_{11}	$B_{11} \cap s_1$	1/2
3	s_2	B_{12}	$B_{12} \cap s_2$	1/2
4	s_2	B_{12}	$B_{12} \cap s_2$	1/2

Table 2 shows the outcomes of the game for different states. Player 2’s inference about player 1’s private belief with respect to Table 2 is simply described as follows.

At the initial stage, there are three possible beliefs.

$$1|2, 3, 4 \quad 1, 2|3, 4 \quad 1, 2, 3|4$$

After state 1, the first one can be eliminated.

$$1, 2|3, 4 \quad 1, 2, 3|4$$

After state 2, it keeps the same.

$$1, 2|3, 4 \quad 1, 2, 3|4$$

After state 3, only one belief remains. That is

$$1, 2|3, 4$$

In this process, state 1 (or state 2) and state 3 (or state 4) are important for player 2 to learn player 1's private belief.

Therefore, after a few rounds of signaling communication with role switching, both players can learn each other's private belief. After the opponents' beliefs are learned, both players can combine their beliefs. Then a common belief $1|2|3, 4$ can be induced, which is obtained by taking the coarsest common refinement of the two belief partitions.

This common belief is important for further communication. It serves as the knowledge base for processing ambiguous expressions. For example, under this common belief, a two signal language s_1 and s_2 indicating the information $\{1, 2, 3\}$ and $\{4\}$ is sufficient to communicate precisely all the information in Example 1. Nevertheless, this kind of successful communication can not be achieved before the common belief is formed.

Moreover, if another set of signals $\{s_1, s_2, s_3, \}$ (with meanings $1, 2|3|4$) is available as well, then the more ambiguous signal set $\{s_1, s_2\}$ (with meanings $1, 2, 3|4$) is preferred given that each signal is costly.

As a result, we have built a dynamic learning process between interlocutors through the repeated SPB model. After the learning is accomplished, even by using the ambiguous language, players might be able to communicate all the information precisely.

However, the learning may not be successful in the sense that some player's private belief can not be singled out from all possible belief partitions. Therefore, it is natural to ask under what conditions players' private belief is learnable. We answer this question in details in the next section.

3 When Is Private Belief Learnable?

In the previous section, we developed a learning procedure for players to learn each other's private belief in a conversation. However, there are situations in which the learning fails. The following is an example to demonstrate that players may sometimes fail to learn their opponent's private belief.

Example 2. Suppose there are six states $\{1, 2, 3, 4, 5, 6\}$ occurring with equal probability, the signal structure is the following.

$$12||34||56$$

$$s_1||s_2||s_3$$

Assume player 1’s private belief and play 2’s private belie are the followings:

$$1, 2, 3|4, 5, 6 \quad 1, 2|3, 4, 5, 6$$

$$B_{11}|B_{12} \quad B_{21}|B_{22}$$

Firstly, we assume that player 1 is the sender and player 2 is the receiver. Then from the sender’s point of view, his inference is as follows.

Table 3. Play of the game in Example 2

State	Signal	Revealed information	Receiver’s reasoning	Payoff
1	s_1	B_{21}	$B_{21} \cap s_1$	1/2
2	s_1	B_{21}	$B_{21} \cap s_1$	1/2
3	s_2	B_{22}	$B_{22} \cap s_2$	1/2
4	s_2	B_{22}	$B_{22} \cap s_2$	1/2
3	s_3	B_{22}	$B_{22} \cap s_3$	1/2
4	s_3	B_{22}	$B_{22} \cap s_3$	1/2

Following Table 3, we examine player 1’s learning process for player 2’s private belief. Similarly, we assume that player 1 knows that player 2’s belief takes the form of a two-element partition on the set of the states. Therefore, player 1’s learning process can be constructed as follows. Without lose of generality, we list player 1’s learning process by communicating from state 1 to state 6.

At the initial stage, there are five possibilities.

$$1|2, 3, 4, 5, 6 \quad 1, 2|3, 4, 5, 6 \quad 1, 2, 3|4, 5, 6 \quad 1, 2, 3, 4, |5, 6 \quad 1, 2, 3, 4, 5|6$$

After state 1, the first possible belief can be eliminated.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3|4, 5, 6 \quad 1, 2, 3, 4, |5, 6 \quad 1, 2, 3, 4, 5|6$$

After state 2, it stays the same.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3|4, 5, 6 \quad 1, 2, 3, 4, |5, 6 \quad 1, 2, 3, 4, 5|6$$

After state 3, the second possible belief above can be eliminated.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3, 4, |5, 6 \quad 1, 2, 3, 4, 5|6$$

After state 4, it keeps the same.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3, 4, |5, 6 \quad 1, 2, 3, 4, 5|6$$

After state 5, one more possibility is eliminated.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3, 4, |5, 6$$

After state 6, two possibilities still remain.

$$1, 2|3, 4, 5, 6 \quad 1, 2, 3, 4, |5, 6$$

From this learning process, it is obvious that even though every state is communicated, player 1 still can not distinguish player 2’s private belief from $1, 2|3, 4, 5, 6$ to $1, 2, 3, 4, |5, 6$. In other words, player 1 knows that player 2’s private belief is one of these two, but there is no means for player 1 to figure out which one it is. Therefore, it is an example where players’ private belief is not learnable. Thus, it arises a natural question that under what conditions players are able to learn each other’s private belief. For answering this question, we examine more about the reason of the failure in Example 2.

We calculate the payoffs in the remaining two possible private belief under each state in Example 2.

Table 4. Payoff under each state

Private belief	State 1	State 2	State 3	State 4	State 5	State 6
$1, 2 3, 4, 5, 6$	1/2	1/2	1/2	1/2	1/2	1/2
$1, 2, 3, 4, 5, 6$	1/2	1/2	1/2	1/2	1/2	1/2

From Table 4, both possible beliefs yield the same payoffs under all the states. Recall that, in the learning process, the trigger for the sender to eliminate some private belief is the payoff differences. For example, in Example 2, from the initial stage to stage one, the private belief $1|2, 3, 4, 5, 6$ is eliminated from the possible set. It is because $1|2, 3, 4, 5, 6$ yields the payoff 1 for state 1, whereas all other possibilities yield 1/2 for state 1. Since the true payoff is 1/2, therefore, $1|2, 3, 4, 5, 6$ should be eliminated. The essential feature here is that the payoff differences resulting from different possible private beliefs provide the sender the opportunities to learn about the receiver’s private belief. If all the possible private beliefs yield the same payoff, then the sender has no chance to learn anything.

Inspired by this phenomena, we established a sufficient condition under which the player’s private belief is learnable. Before stating the condition, we first define formally what it means that two possible private belief are distinguishable.

Definition 3. *Given a SPB game with the set of states T , a set of actions \mathcal{A} and a set of signals, we say that from the player i 's point of view, two possible private beliefs B_{-i}^1, B_{-i}^2 are distinguishable, if there exists a state j , such that $u(j, A_j | s_j \cap B_{-i}^{1j}) \neq u(j, A_j | s_j \cap B_{-i}^{2j})$, where $j \in T$, s_j is the corresponding signal, B_{-i}^{1j} and B_{-i}^{2j} are partition elements containing state j , and $A \in \mathcal{P}(\mathcal{A})$.*

Where B_{-i} indicates player i 's opponent's possible private belief. $u(j, A_j | s_j \cap B_{-i}^{1j})$ is read as the payoff with respect to the receiver's action set A_j (indicating that $a_j \in A$) given signal s_j and the private information B_{-i}^{1j} .

The intuition behind this definition is just saying that if there exists a state under which two belief partitions yield different payoffs, then they are distinguishable. For example, in Example 2, the belief partitions $1|2, 3, 4, 5, 6$ and $1, 2|3, 4, 5, 6$ are distinguishable under state 1. Moreover, the structure of the game guarantees that under state 1, at least one of the two beliefs yields the wrong payoff and can be eliminated.

Now we can present the sufficient condition under which a private belief is learnable by using Definition 3.

Theorem 1. *Given the SPB game, if any two of receiver's possible private belief partitions from the sender's point of view are distinguishable, then the receiver's private belief is learnable.*

Proof: See Appendix.

Theorem 1 tells us under what conditions receiver's private belief is learnable. The sufficient condition meets our intuition that the payoff differences provide the sender an indication of distinguishing possible private beliefs from impossible ones. An example can illustrate the intuition behind the theorem.

Conversation A:

Ann: Hi, morning! Are you going to the bank?

Bob: Yes, I go there every day.

Conversation B

Ann: Hi, morning! Are you going to the bank?

Bob: Yes, I have an appointment with the financial manager.

In the two conversations above, because of the ambiguity of the word "bank", there maybe uncertainty in the conversation. In conversation A, Ann does not know which meaning of the bank that Bob is using. It is because from Bob's response, Ann can not distinguish the financial bank from the river bank. On the contrary, in conversation B, Ann can easily infer from Bob's response that Bob is using the word "bank" for the financial institute.

4 Uncertain Signaling and Ambiguity Preference

In the previous section, we provide a sufficient condition under which the players can learn opponents' private beliefs through the repeated signaling game. However, there are many situations such as Example 2 in which the learning fails. We want to ask if the players are uncertain about each other's belief, whether ambiguous signals can be chosen. In this section, we conduct simulation studies for exploring players' preferences of ambiguous expressions when the opponent's private belief is not fully learned.

The idea is to model a communicative scenario where given a set of signaling systems with different ambiguities, players are learning which signaling system is more optimal. A signaling system is a set of signals with conventional meanings with respect to the given set of the states. In the classical signaling game and our previous discussions, we consider a game with single signaling system only. In this section, we consider multiple signaling systems simultaneously.

In Lewis's signaling game, each equilibrium can be represented as a partition on a set of states, we call it a signaling system. For example, given a set of states $T : \{1, 2, 3, 4, 5, 6\}$, the separating equilibrium can be induced from the partition $\{1||2||3||4||5||6\}$, where we simply use $||$ to indicate the elements in the partition. One of the possible meaningful set of signals for the partition is that signal s_i carries the meaning of state i . Apparently, in the separating equilibrium, each signal precisely represents each state information.

One the other hand, the partial pooling equilibrium involves uncertainties for the meaning of the signals. For instance, the partition $\{1234||56\}$ can produce a partial pooling equilibrium, in which two signals are used and each signal carries the meanings of multiple states. Therefore, ambiguous signals can appear in the partial pooling equilibrium. We say that signaling system $\{1||2||3||4||5||6\}$ containing more signals is more precise than the signaling system $\{1234||56\}$. In general, for different partitions on the same set of states, the partition contains more signals is considered less ambiguity than the one with fewer signals. We assume that each signal in a partition have a cost c , then the partition with more signals is more precise but more expensive.

Assuming the existence of multiple signaling systems, we assign each signaling system a weight m by considering three factors. Firstly, the signaling system gains credences from two simultaneously occurring process: one is from the conversational information transaction, the other is from providing partial information about opponent's personal beliefs. In addition, we take into account of the cost of signals. Through keeping track of the weight of each signaling system in a reinforcement learning process, we show that ambiguous signaling systems have advantages in this uncertain signaling process.

The reinforcement learning has been widely applied in the studies of language evolution (see O'Connor 2014b; Skyrms 2010; Wagner 2009; Zollman 2005; Franke and van Rooij 2011). Reinforcement learning can be described by a simple urn model with two colored balls. Every time a ball is drawn from the urn randomly. Then, the same ball and another same colored ball are returned to the urn. As a result, the probability of the ball with the same color being drawn

next time is increased. When reinforcement learning is applied in the signaling game, players' strategies can be imagined as drawing the colored balls from the urns of signals and acts.

We define the reinforcement learning for only sender's choice among the signaling systems. Since we assume the signaling systems are common knowledge, once the sender's strategy is fixed, the receiver's action is also fixed. Hence, it is sufficient to consider only the sender's strategy.

The updating rule is the following.

$$w_{P_i}(t + 1) = w_{P_i}(t) + u_j + u_l - kc$$

where w_{P_i} is the weight for the signaling system P_i , u_j is the payoff for the result of communicating the state information j , u_l is the credence from learning the opponent's private beliefs while P_i is used. Formally, kc is the cost of the signaling system P_i in which k number of signals are contained. u_l is decided by counting how many possible belief partitions can be eliminated when P_i is used. $u_l = l$, if one possibility is eliminated. $u_l = 2l$, if two possibilities are eliminated. u_l is understood as how much impossibilities can be eliminated by using certain signals. The more impossible belief can be eliminated the more learning credences the signal can obtain. The learning here is an epistemic learning process which is also a process of reducing uncertainties.

By calculating the weight of each signaling system, we can define a response rule for the learning system to capture how frequently certain signals are used. The response rule for the reinforcement learning is defined as follows.

$$p_{P_i}(t) = \frac{w_{P_i}(t)}{\sum_j w_{P_j}(t)}$$

in which the probability of P_i being chosen is calculated by the proportion of the weight of P_i among the weight of all the possible partitions. The higher the probability of certain signaling system is, the more frequently this signaling system is used and hence more advantages this signaling system obtains in the evolutionary system.

We use Example 2 for the simulation study. The signaling systems under the consideration are

$$P_1 : 12||34||56 \quad P_2 : 1234||56 \quad P_3 : 123456$$

The costs of the signaling systems are $3c$, $2c$ and c . The ambiguity increases from P_1 to P_3 .

The reinforcement learning process is the following. Firstly, the occurring state and who plays the role of the sender are decided randomly with equal probability. Then, the sender chooses the signaling system by the response rule. The results of communication and learning are reflected on the weight w . We assume the original weights for all the signaling systems are the same.

If we assume the response probability $p_{P_i} = \frac{1}{3}, i = 1, 2, 3$ at time $t = 0$, we can calculate the expected weights according to the following equation.

$$E_{w_{P_i}} = \frac{1}{2} \sum_j \frac{1}{6} * \frac{1}{3} (u_j + u_l) + \frac{1}{2} \sum_j \frac{1}{6} * \frac{1}{3} (u_j + u_l) - kc$$

Therefore,

$$E_{w_{P_1}} = \frac{7}{12} + \frac{3}{2}l \approx 0.58 + 1.5l - 3c,$$

$$E_{w_{P_2}} = \frac{4}{9} + 3l \approx 0.44 + 3l - 2c,$$

$$E_{w_{P_3}} = \frac{1}{3} + \frac{7}{2}l \approx 0.33 + 3.5l - c$$

Apparently, for comparing $E_{w_{P_i}}$, we have to specify the particular values of l and c . As the proportion of P_i changes along the learning process, it becomes impossible to calculate manually. Hence we conduct simulations to explore the dynamic of this learning process.

By changing the values of l and c , we got the simulation results by conducting each trial for 2000 generations.

Figure 1 presents one instance of simulation results for different values of l and c . The X axis shows the repeated times of communication. We repeated 2000 times for each trial. The Y axis shows the probability of each signaling system being chosen. It is obvious that in a short time of communication, the most precise signaling system P_1 still has some advantages. However, in the long run, the precise signaling system is dominated by the more ambiguous signaling systems P_2 and P_3 .

For examining the stability of the result, we also conduct simulations for 200 trials for each case. During the 200 trials, we record the frequency of each signaling system being the best among the three with respect to the best average probability in each trial. For example, when $l = 0.2, c = 0.15$ (case (f)), in the 200 trials, 77% times, P_3 has the best average performance, P_2 takes 22.5% of the time while P_1 takes only 0.5% of the time. When $l = 0.1, c = 0.05$ (case (a)), among the 200 trials, P_3 has the best average performance 45% of the time, P_2 takes 41% of the time while P_1 takes 14% of the time. Overall, the most ambiguous signaling system has the best average performance among the given three signaling systems.

The explanation of the simulation results relies on two facts. Firstly, on balancing the information transaction and the costs of signals, the ambiguous signaling systems turn out to be more optimal. Secondly, the ambiguous signaling systems have advantages in the learning process in our models. When the communication is successful through using the ambiguous expression, which means the receiver's private belief plays an important role in the information transaction, as a result, the sender can infer the receiver's private belief through the outcome of this play. On the contrary, the precise signals do not have this merit.

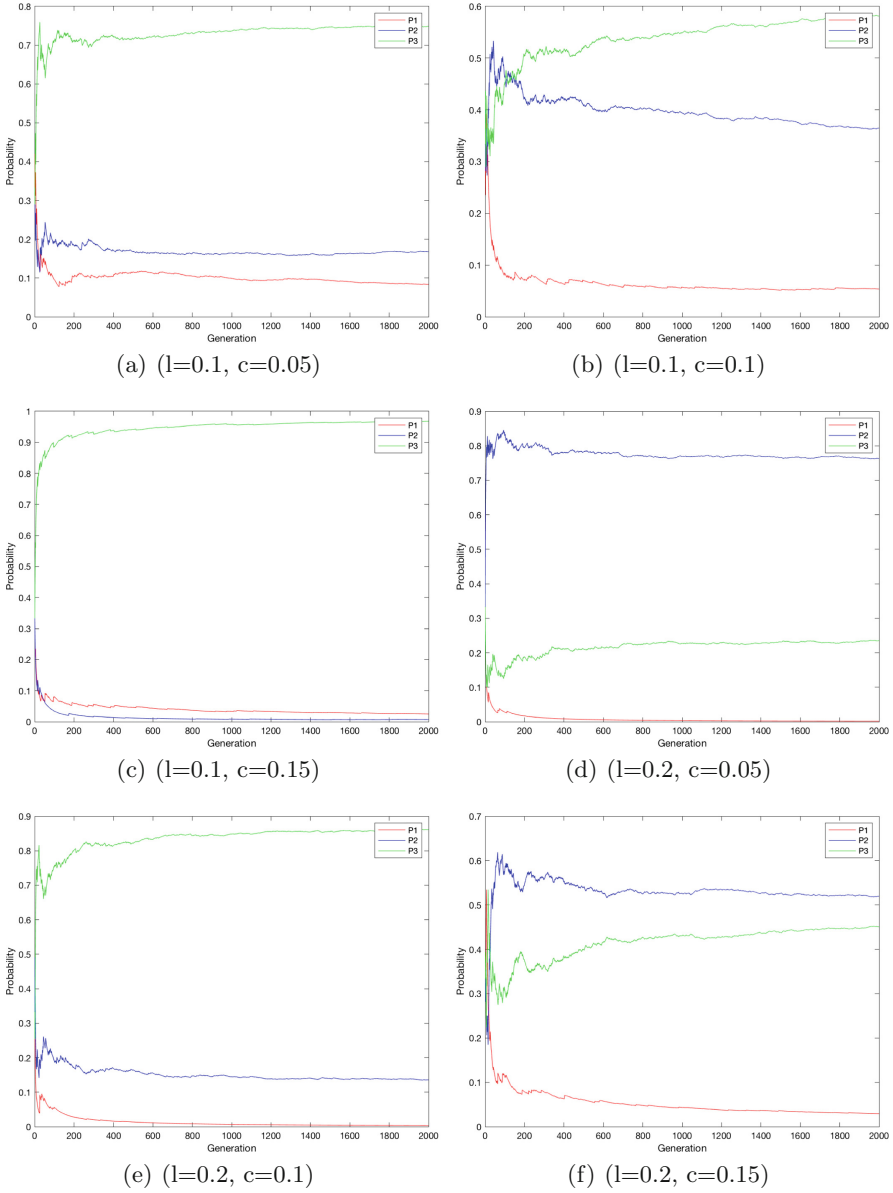


Fig. 1. The probability of each signaling system being chosen

To conclude, in this section, we use simulation studies to examine player's preference on ambiguous signals when the opponent's private belief is uncertain. Three factors are considered in the simulations: the benefits from information transactions, the partial information of opponents' belief through the learning

and the cost of the signals. The simulation results show that more ambiguous signals are preferred in most of the situations.

5 Discussion and Conclusion

In the literature, there are many discussions about uncertain signaling and its communication features. We discuss the similarities and differences between our model and the established models in the literature. The models we concern are Santana's signaling model with belief context (see Santana 2014), the rational speech act model from Frank and Goodman (2012), the iterated response model by Franke and Jäger (2014) and the uncertain signaling model from Thomas (2017).

We proposed a dynamic learning procedure of private belief for communication, which differs from the models in which a common prior of beliefs is assumed. Santana's signaling model is a typical signaling game with a given context background. It studies the emergence of ambiguity in a cooperation signaling game. Based on Lewis's signaling game, a context is added to the model. Players combine both the signal information and the independent context information for making decisions. The paper argues that the evolution favors the ambiguous signaling. Our model has the similar motivation and structure as Santana's model. The major difference is that in Santana's model, the context is given as the common knowledge independent of the communication. One of the main contributions of our paper is building a learning process of the context belief during the communication. A dynamic perspective is taken in our model for both the context formation and the preference of ambiguities.

Our model is also different from the models built on the probabilistic (Bayesian) iterated learnings (see Frank and Goodman 2012; Goodman and Frank 2016; Franke and Jäger 2014). Rational language use is captured by a hierarchy over reasoning types in Franke and Jäger's hierarchy model. An iterated rationality reasoning is constructed on the strategy types in the model, which captures the back and forth pragmatics reasoning. Rational speech act game (in Frank and Goodman 2012) uses a Bayesian reasoning to predicate interlocutors' language use. Franke and Jäger's model and the Rational speech act model focus on the rationality and pragmatics in use of the language. The interlocutors' context belief is coded into the strategy types and the context information is not fully explored.

Thomas (2017)'s uncertain signaling model generates an adaptive dynamic to predicate ambiguous communication under which the players are lacking a common prior. Brochhagen's model focuses on the learning of the context belief, but only the adaptive dynamics is explored. It lacks a full analysis of whether and when a common context is possible to be established.

The investigation in this paper is built on the notion of information partition that is more basic than the concept of probability. The model follows Aumann (1976)'s tradition of "Agree to Disagree". The difference is that Aumann's theorem has a common prior assumption and is based on only one random variable. In

our model, we works on multiple random variables (all possible private beliefs) and different priors. Another advantage of using information partition is that we can discuss the content of the information from the signals as well as from the context beliefs instead of just posterior probabilities. From the discussion in Geanakoplos and Polemarchakis (1982), the learning of posterior probability does not equal to learning the information itself. Hence, a model built on the notion of information partition has more potential for exploring belief updating and learning. Moreover, the discussion based on information partition can be easily extended to other related fields such as other extensive games and possible world semantics in Modal Logic.

To conclude, the paper tries to justify the existence of language ambiguity from the perspective of context dependence. When the context about the conversation is commonly known, the ambiguous expression is possible to communicate all the information. Furthermore, as the interlocutors' beliefs change during the repeated conversations, the interlocutors' preferences of degrees of ambiguity may change as well. The main contribution of the paper is that we construct a learning process for the players to update beliefs from the result of each conversation. We also establish a testing condition under which whether the learning process is successful. In addition, we discussed players's choice of ambiguous language when the opponent's private belief is not fully known. A reinforcement learning signaling game is developed for the uncertain signaling situations.

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Appendix

Theorem 1. *Given the SPB game, if any two of receiver's possible private belief partitions from the sender's point of view are distinguishable, then the receiver's private belief is learnable.*

Proof: This theorem can be proved from the players' reasoning process on inferring opponent's private belief by playing the SPB game repeatedly. The algorithm of this learning can be described as follows. For convenience, we eliminate the subscribe indicating the players in B in the proof.

- Step 1 Since T is finite, we can list all the possible private belief partitions as a sequence $B : B^1, \dots, B^m$;
- Step 2 Calculate all the expected payoffs yielded by each $B^j, j \in \{1, 2, \dots, m\}$ for each state $i, i \in \{1, 2, \dots, n\}$;

Step 3 Pick the first two partitions in the sequence B , B^1 and B^2 , since any belief partition are distinguishable, then there exists a state k such that $u(k, A_k | s_k \cap B^{1k}) \neq u(k, A_k | s_k \cap B^{2k})$. Therefore, once state k happens (the occurrence of state k can be guaranteed because players are playing this game repeatedly and every state is possible to occur.), by comparing the true payoff with the payoffs given by B^1 and B^2 , There are two situations:

- One of the beliefs yields the true payoff, then the sender just return the correct partition back to the sequence B .
- Neither belief yields the true payoff, then both beliefs should be eliminated.

Step 4 Update the sequence B , then repeat from step 1.

Since for any two private belief partitions, they are distinguishable, and at least one of them is wrong. Hence, the list B can be eliminated to only one element in finite steps. The remaining belief partition is receiver's true private belief. Therefore, receiver's private belief is learnable. \square

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