Chapter 25
Lateral Thinking in Learning Computational Fluid Dynamic Methods

Priya C. Sande

Abstract Although a traditional engineering education system is good in imparting the intended know-how, it does not focus much on the delivery system. Applying lateral thinking techniques can provide a value addition to delivery as illustrated in this work, for the course of Computational Fluid Dynamics (CFD). CFD is a powerful engineering simulation tool and a trending elective course taught at several universities. CFD methods build on numerical methods to resolve flows and design flow related equipment. Here several methods form the core of the course, making teaching and learning them all a rigorous mathematical exercise. Hence lateral thinking techniques were applied to sustain and enhance student interest. A basic CFD method (Lax-Wendroff) was first taught. It was then illustrated how *suspended judgment*, *generation of alternatives* and *fractionation* were applied to further elucidate closely related CFD methods, namely those of Maccormack and Richtmyre. Hence lateral thinking was successfully applied for value addition to delivery in the class room, and a framework was thereby suggested to explore other CFD methods in this way. The work demonstrates the possibilities in amalgamating lateral thinking with analytical content in engineering education.

Keywords Lateral thinking techniques · CFD methods · Creative engineering pedagogy

25.1 Introduction

Traditionally, it is assumed that creativity is more associated, understood and more frequently spoken about in the Arts (Heilmann and Korte 2010), thus the term 'creative arts'. But William J. J. Gordon affirms that the intellectual processes that characterize creativity are the same whether applied in arts, science or even engineering (Gordon 1961). Edward de Bono the pioneer in creativity, propagated the

P. C. Sande (✉)
Department of Chemical Engineering, Birla Institute of Technology and Science, Pilani-Campus, RJ 333031 Pilani, India
e-mail: priya@pilani.bits-pilani.ac.in

term 'lateral thinking' instead of 'creative thinking' (De Bono 1970), and these terms will hence be used interchangeably in this work. His work describes the advantages and means of applying lateral thinking to the process of design, which is at the very heart of engineering. He affirms that while vertical thinking is excellent, it is not sufficient. Lateral thinking provides enhanced value. Values can be equated with benefits, which when propagated can arguably 'complete' the process of knowledge dissemination which we have come to call 'education'. If vertical thinking is concerned with selecting and using standard patters (or solutions) for problem solving, then lateral thinking is all about reconstructing and transforming information into new patterns. Even though the importance of creative thinking in academic settings has been stressed for a long time (Senra and Fogler 2014), pedagogy in engineering courses has largely revolved around vertical thinking (Tekic et al. 2015). This is not surprising, considering the analytical nature of the courses. With little agreement on the definition of creativity or how to teach it, firsthand information about creativity and the creative process, as experienced by working engineers was studied (Klukken et al. 1997). Creativity in a general sense has been researched in engineering education, with one study concluding that creativity in engineering curricula is not appropriately taught or rewarded, and that students who view themselves as highly creative were actually less likely to graduate in engineering (Atwood and Pretz 2016). Another work which aimed to document opportunities for creative growth, found that convergent thinking was well represented in the engineering courses, but generation of ideas, divergent thinking and idea exploration was largely lacking (Daly et al. 2014). However in one case study on mechanical engineering project work, special attention was paid to the nature of creativity, with exercises introduced to facilitate this historically neglected aspect of engineering education (Conwell et al. 1993).

Whereas creativity is difficult to define and too often only the description of a result, lateral thinking is the description of a process (De Bono 1970) and described in the next section. In this work the lateral thinking techniques proposed by de Bono were adapted for the first time in pedagogy for the course 'Computational Fluid Dynamics' (CFD). The work is a novel attempts at integrating creative thinking with the mathematical methods of CFD course, thereby opening up new possibilities for extending such pedagogy in engineering education. The benefit to the student is two fold in terms of easier and stronger grasp of the complex and varied CFD methods, and familiarity with lateral thinking techniques, which have broad application in problem solving.

25.2 Some Lateral Thinking Techniques

Lateral thinking as the name suggests, can be best viewed as complementary or even opposite in some sense to vertical thinking. The action of vertical thinking is mainly related to selection, rejection and development of arranged bits of information called patterns, whereas lateral thinking acts to restructure the information bits into completely different patterns. Lateral thinking involves escape from the

old, and provocation of new patterns, so it is 'provocative' while vertical thinking is 'analytical'. With regard to the way the mind works: vertical thinking results from familiar pattern reinforcement and stocking. These patterns become difficult to change once established, so a deliberate application of lateral thinking techniques aims to restructure patterns and put information together in new ways. The following three techniques of de Bono's several lateral thinking techniques (De Bono 1970) were applied in this work.

25.2.1 Suspended Judgment

This is a lateral thinking effort, in which one does not try to size-up an idea quickly, but rather lets it live on with the intention of converting it into a better idea. This runs contrary to vertical thinking, where ideas which are deemed unwise, impractical, fanciful or fantastic are snuffed out at their inception and are not allowed to advance in the design process. This is because correct ideas are valued in vertical thinking over a diversity of ideas, as in creative thinking.

25.2.2 Generation of Alternatives

Of all techniques, the generation of alternatives is the one which forms the core of creative thinking. Here there is a deliberate search and exploration of alternatives, which goes beyond a mere natural inclination which usually halts when a promising approach presents itself. Instead a wide ranging search is engaged in, to gather a diversity of ideas. Outcomes could be: (a) one of the generated ideas leads to the best idea; (b) the original idea is still chosen, but for the reason that it is indeed the best among many; (c) generation of ideas may simply loosen up rigid patterns, open-up new information or challenge assumptions which provoke further idea generation.

25.2.3 Fractionation

Fractionation enables new views of a situation for better and easier generation of solutions. Usually the commonly used standard patterns were originally developed from smaller patters that are no longer referred to. To restructure and bring about insight one may have to go back and break-up patterns into fractions, which can be further restructured into new options or solutions.

25.3   Illustration of CFD Methods Using Lateral Thinking

CFD is an approach to solving flow problems by numerically resolving the governing (Navier-Stokes) partial differential equations, over discrete grid points, rather than analytically over a continuum. In an introductory CFD course, all aspects of the governing equations and grid generation are dealt with in the first half, followed by the all important computational methods. Although commercial CFD software platforms use variants of the Pressure Correction method, understanding of several precursor methods is paramount; not only to understand evolution of the methods, but also to grasp mature methods like Pressure Correction. The Lax-Wendroff (Lax-W) method is conceptually the simplest (only two main steps) and probably the easiest for a novice to grasp. It is also usually taught as the first of many methods in a CFD course, and several methods evolved as improvements. Teaching and learning all these methods can be laborious due to their complexities and similarities. By introducing lateral thinking, it is intended not only that the learning process would thereby become less arduous and more interesting, but also that lateral thinking techniques would be conveyed as an added benefit. To illustrate, two improvement methods are detailed as derived from the fundamental Lax-W method, using the said lateral thinking techniques. Standard CFD textbook, Anderson (Anderson 2012) covers these methods in detail.

25.3.1   *Maccormack Method Illustrated from Lax-Wendroff Method*

The Lax-W method, is conceptually the simplest method, in the case that flow variable (density (ρ), x-component of velocity (u), y-component of velocity (v) and specific internal energy (e)) are know at initial time t. In this method the standard continuity, x-momentum, y-momentum and energy equations for compressible fluid are modified and used in the form of Eqs. (25.1), (25.2), (25.3), and (25.4) respectively:

$$\frac{\partial \rho}{\partial t} = -\left(\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial v}{\partial y} + v \frac{\partial \rho}{\partial y} \right) \tag{25.1}$$

$$\frac{\partial u}{\partial t} = -\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \tag{25.2}$$

$$\frac{\partial v}{\partial t} = -\left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} \right) \tag{25.3}$$

$$\frac{\partial e}{\partial t} = -\left(u \frac{\partial e}{\partial x} + v \frac{\partial e}{\partial y} + \frac{p}{\rho} \frac{\partial u}{\partial x} + \frac{p}{\rho} \frac{\partial v}{\partial y} \right) \tag{25.4}$$

To derive the flow variables (for example for a ρ) at the next time step '$t + \Delta t$' (called marching solution), and for all grid points (i, j), the Taylor series expansion including the second derivative (to maintain second-order solution accuracy) is used:

$$\rho_{i,j}^{t+\Delta t} = \rho_{i,j}^{t} + \left(\frac{\partial \rho}{\partial t}\right)_{i,j}^{t} \Delta t + \left(\frac{\partial^2 \rho}{\partial t^2}\right)_{i,j}^{t} \frac{(\Delta t)^2}{2} + \cdots \qquad (25.5)$$

Similar equations are written for all flow variables. The first derivative is substituted directly from governing Eqs. (25.1)–(25.4), and a second application of differentiation operator on the same equations provides the second derivatives, as for example Eq. (25.6).

$$\begin{aligned}
\frac{\partial^2 \rho}{\partial t^2} = &-\rho \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial u}{\partial x}\frac{\partial \rho}{\partial t} + u\frac{\partial^2 \rho}{\partial x \partial t} + \frac{\partial \rho}{\partial x}\frac{\partial u}{\partial t} \\
&+ \rho \frac{\partial^2 v}{\partial y \partial t} + \frac{\partial v}{\partial y}\frac{\partial \rho}{\partial t} + v\frac{\partial^2 \rho}{\partial y \partial t} + \frac{\partial \rho}{\partial y}\frac{\partial v}{\partial t}
\end{aligned} \qquad (25.6)$$

The mixed derivatives on the right-hand-side of Eq. (25.6) can similarly be derived from governing equations. Subsequently, the determined first and second derivatives are written using their second-order central difference forms and substituted in Eq. (25.5) (not shown for brevity) to provide the required solution at next time step.

Hence the essence of Lax-W method is that: with only current time data inputs, a mathematical device (Taylor series expansion) advances the solution to the next time step, with the physics injected in by the time derivates, which are appropriately derived and substituted from governing equations with spatial derivatives.

The Maccormack (Mac) method can be viewed and easily remembered as a lateral thinking variant of Lax-W method. First, an effort at *suspended judgment* offers a simplified Taylor series expansion that includes only the first derivative, as in Eq. (25.7). Such an approximation assumes straight line relationship of a variable between time steps, which would otherwise be rejected because it offers only first-order solution accuracy. On the other hand omitting the second derivative drastically cuts computational. From here it can easily be seen that a *better* time derivative (designated with subscript 'av' in Eq. (25.8)) would enhance utility of Eq. (25.7); so thinking to *generate alternative* derivatives unfolds as an aim, which leads to the possibility of constructing this '*better* time derivative' from '*better* predicted variables' (designated with bar on top, as given by Eq. (25.9) for example).

$$\rho_{i,j}^{t+\Delta t} = \rho_{i,j}^{t} + \left(\frac{\partial \rho}{\partial t}\right)\Delta t \qquad (25.7)$$

$$\rho_{i,j}^{t+\Delta t} = \rho_{i,j}^{t} + \left(\frac{\partial \rho}{\partial t}\right)_{av}\Delta \qquad (25.8)$$

$$(\bar{\rho})_{i,j}^{t+\Delta t} = \rho_{i,j}^t + \left(\frac{\partial \rho}{\partial t}\right)_{i,j}^t \Delta t \qquad (25.9)$$

When these predicted variables for next time step, obtained from Taylor series simplification (only first two terms), are substituted for example in the difference equation form of continuity equation, a 'better alternative time derivative' for ρ is obtained:

$$\left(\overline{\frac{\partial \rho}{\partial t}}\right)_{i,j}^{t+\Delta t} = -\left[(\bar{\rho})_{i,j}^{t+\Delta t} \frac{(\bar{u})_{i,j}^{t+\Delta t} - (\bar{u})_{i-1,j}^{t+\Delta t}}{\Delta x} + (\bar{u})_{i,j}^{t+\Delta t} \frac{(\bar{\rho})_{i,j}^{t+\Delta t} - (\bar{\rho})_{i-1,j}^{t+\Delta t}}{\Delta x} \right.$$
$$\left. + (\bar{\rho})_{i,j}^{t+\Delta t} \frac{(\bar{v})_{i,j}^{t+\Delta t} - (\bar{v})_{i,j-1}^{t+\Delta t}}{\Delta y} + (\bar{v})_{i,j}^{t+\Delta t} \frac{(\bar{\rho})_{i,j}^{t+\Delta t} - (\bar{\rho})_{i,j-1}^{t+\Delta t}}{\Delta y} \right] \quad (25.10)$$

Since average of the derivative at t (using forward differencing) and derivative using predicted variables at $t + \Delta t$ (using backward differencing) is taken (Eq. (25.11)), information from both grid points lying on the two sides of grid point i, j is included in the same way as in the central differencing employed in Lax-W method. Hence both methods are second order accurate.

$$\left(\frac{\partial \rho}{\partial t}\right)_{av} = \frac{1}{2}\left[\left(\frac{\partial \rho}{\partial t}\right)_{i,j}^t + \left(\overline{\frac{\partial \rho}{\partial t}}\right)_{i,j}^{t+\Delta t} \right] \qquad (25.11)$$

25.3.2 Richtmyer Method Illustrated from Lax-Wendroff Method

The Richtmyer method, also called two-step Lax-Wendroff method, was proposed for the case of non-linear equations where the substitution of temporal with spatial derivatives no longer remains straightforward or unique. This method can also be viewed and easily remembered as a lateral thinking variant of Lax-W method. First, the same line of lateral thinking as in Mac method is followed, until thinking to an *generate alternative* derivative leads to the new idea of *fractionation* to predict variables at only *half* the time step for *half* grid points $(i, j + 1/2)$ and $(i, j-1/2)$ using Eqs. (25.12–25.13), and similarly for full grid point i, j using Eq. (25.14). Here time derivative at time t is substituted as before from Eq. (25.1) to inject the physics, but with forward differencing proposed for the time derivative in Eq. (25.13), likewise backward differencing for Eq. (25.14) and central differencing for Eq. (25.14).

$$\rho_{i,j+\frac{1}{2}}^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(\rho_{i,j+1}^t + \rho_{i,j}^t\right) + \frac{\Delta t}{2}\left(\frac{\partial \rho}{\partial t}\right)_{i,j}^t \qquad (25.12)$$

$$\rho_{i,j-\frac{1}{2}}^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(\rho_{i,j}^t + \rho_{i,j-1}^t\right) + \frac{\Delta t}{2}\left(\frac{\partial \rho}{\partial t}\right)_{i,j}^t \tag{25.13}$$

$$\rho_{i,j}^{t+\frac{\Delta t}{2}} = \rho_{i,j}^t + \frac{\Delta t}{2}\left(\frac{\partial \rho}{\partial t}\right)_{i,j}^t \tag{25.14}$$

Substituting these *better* variables again in Eq. (25.1) (similar to substituting variables with bar on top as in Mac method) leads to the *alternative derivative* sought (represented with a double bar on top), which is used in the final prediction (Eq. 25.16).

$$\left(\overline{\overline{\frac{\partial \rho}{\partial t}}}\right)_{i,j} = -\left(\rho_{i,j}^{t+\frac{\Delta t}{2}}\left[\frac{u_{i,j+\frac{1}{2}}^{t+\frac{\Delta t}{2}} - u_{i,j-\frac{1}{2}}^{t+\frac{\Delta t}{2}}}{\Delta x}\right] + u_{i,j}^{t+\Delta t}\left[\frac{\rho_{i,j+\frac{1}{2}}^{t+\frac{\Delta t}{2}} - \rho_{i,j-\frac{1}{2}}^{t+\frac{\Delta t}{2}}}{\Delta x}\right]\right.$$
$$\left. + \rho_{i,j}^{t+\frac{\Delta t}{2}}\left[\frac{v_{i,j+\frac{1}{2}}^{t+\frac{\Delta t}{2}} - v_{i,j-\frac{1}{2}}^{t+\frac{\Delta t}{2}}}{\Delta y}\right]\right) \tag{25.15}$$

$$\rho_{i,j}^{t+\Delta t} = \rho_{i,j}^t + \Delta t\left(\overline{\overline{\frac{\partial \rho}{\partial t}}}\right)_{i,j} \tag{25.16}$$

Hence lateral thinking gives insight into the genesis of the two methods described.

25.4   Student Feedback

A pilot class covering 'CFD methods with creative thinking' as detailed in this work, was conducted during CFD course 'Computational Fluid Dynamics', taught at Birla Institute of Technology and Science, Pilani, Pilani campus. An anonymous paper based survey gauged the response of the 16 students (Fig. 25.1) registered for the course (January–May 2019). The class comprised eight undergraduate students and seven master's level students which included two female students. The questionnaire appropriated the 5 point Likert scale (Joshi et al. 2015) with six statements (see Table 25.1).

The first three questions were related to the potential of applying lateral thinking specifically to learning CFD, and the next three questions were related to engineering courses in general. It is encouraging to note the positive responses for both sets of questions with slightly more number strongly agreeing with the second set of questions. 85.42% of the responses were either 'agree' or 'strongly agree', 14.58% were 'neutral' and 0% responded with either 'disagree' or 'strongly agree'. The overall-average of all responses was approx. 4, which corresponds to 'agree'.
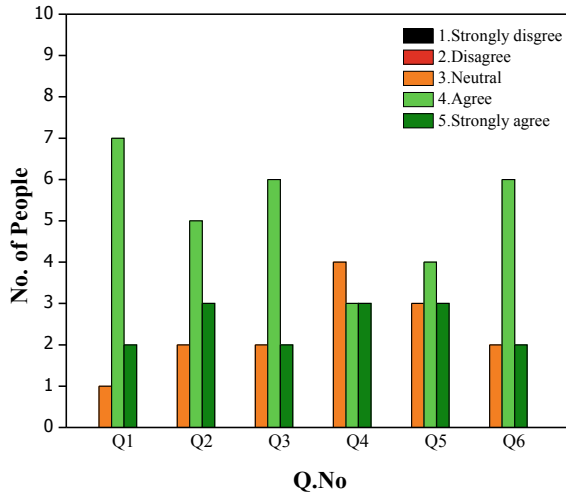
Fig. 25.1 Survey responses of engineering students to the pilot class conducted using lateral thinking to teach CFD methods

Table 25.1 Survey of student feedback

Statements in questionnaire
Q1. I found the techniques of lateral thinking (LT) interesting
Q2. Applying LT techniques to CFD methods provided more insight into the method
Q3. Applying the LT techniques to CFD methods has potential to help me remember them
Q4. Applying the LT techniques has potential to simplify learning for me
Q5. Applying LT may become stress reliving in engineering course, especially when there is so much emphasis on equation based problem solving
Q6. LT techniques have the potential to enhance teaching and learning in other complex and analytical engineering courses as well
Student comments: It may be useful also in optimization and machine leaning type courses; LT techniques helps to remember things which are otherwise difficult; It is a relatively new idea and have the potential to eventually simplify complex concepts for me

25.5   Conclusions

This work explores using lateral thinking in a popular engineering elective such as CFD. Application of Lateral thinking techniques evolved two related CFD methods, from the framework of a basic method. This novel treatment was implemented in a pilot class, as an aid in teaching, learning, simplifying and remembering these complex methods. Survey of student feedback was positive, with all agreeing with the benefits of lateral thinking, not only for CFD, but also potentially for other engineering courses as well.

References

Anderson JD (2012) Computational fluid dynamics: the basics with applications, Indian edn. McGraw Hill, New York

Atwood SA, Pretz JE (2016) Creativity as a factor in persistence and academic achievement of engineering undergraduates. J Eng Educ 105(4):540–559. https://doi.org/10.1002/jee.20130

Conwell JC, Catalano GD, Beard JE (1993) A case study in creative problem solving in engineering design. J Eng Educ 82(4):227–231. https://doi.org/10.1002/j.2168-9830.1993.tb01079.x

Daly SR, Mosyjowski EA, Seifert CM (2014) Teaching creativity in engineering courses. J Eng Educ 103(3):417–449. https://doi.org/10.1002/jee.20048

De Bono E (1970) Lateral thinking: a textbook of creativity. Ward Lock Educational, London

Gordon WJJ (1961) Synectics. Harper & Row, New York

Heilmann G, Korte WB (2010) The role of creativity and innovation in school curricula in the EU27: a content analysis of curricula documents. Joint Research Centre, Luxembourg: European Commission

Joshi A, Kale S, Chandel S, Pal DK (2015) Likert scale: explored and explained. Br J Appl Sci Tech 7(4):396–403. https://doi.org/10.9734/bjast/2015/14975

Klukken PG, Parsons JR, Columbus PJ (1997) The creative experience in engineering practice: implications for engineering education. J Eng Educ 86(2):133–138. https://doi.org/10.1002/j.2168-9830.1997.tb00276.x

Senra M, Fogler HS (2014) Teaching creative thinking and transitioning students to the workplace in an academic setting. Chem En Educ 48(1):9–16

Tekic Z, Tekic A, Todorovic V (2015) Modelling a laboratory for ideas as a new tool for fostering engineering creativity. Procedia Eng 100:400–407. https://doi.org/10.1016/j.proeng.2015.01.384