

A Preliminary Study for the Comparison of Different Pacejka Formulations Towards Vehicle Dynamics Behaviour

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Abstract. The implementation of a tire model in a simulation environment is fundamental to characterize the vehicles and to predict the dynamic behaviour during the design phase, e.g. to test automotive control systems like ADAS [\[1\]](#page-8-0) or different parameters or working conditions like tire compound, pressure, and speed. Moreover, the output of a tire model can be employed also to predict its temperature distribution [[2](#page-8-0)].

This paper deals with the comparison between different Pacejka formulations, differing for the sensitivity to physical factors, like inflation pressure and slight analytical variations.

Since the discussed tire models are different version of the same formulation, the microparameters concerning specific physical effects have been zeroed, in order to make the comparison more reliable. In particular, a Pacejka's MF 5.2 has been compared towards the MF 6.1 tire model employing a tire vehicle in specific dynamic manoeuvres.

Some longitudinal (braking and acceleration) and lateral manoeuvres (spiral, steering pad, fishhook, line change, constant speed curve) have been adopted to compare the results of the implemented tire model influence on the overall vehicle dynamics.

Finally, to evaluate the effect of different tire configurations, a sensitivity test was carried out.

Keywords: Tire – road interaction \cdot Magic formula \cdot Real-time simulation

1 Introduction

The prediction of vehicle dynamics behaviour can be profitably obtained thanks to the development of a proper simulation environment. Several and different simulation environments, each one characterized by peculiar solvers, features and physical subsystems like tire models (depending on the output required) are on the market. Indeed, literature is rich of tire models, the most used are the physical brush model [\[3](#page-8-0), [4\]](#page-8-0), Magic Formula [[5\]](#page-8-0), an empirical model characterized by low computational cost, TMeasy [\[6](#page-8-0)], characterized by a formulation simpler than the MF, however by a less accuracy, FEA models [\[7](#page-8-0)], adopted to evaluate static characteristics due to significant computational load, multibody models [[8,](#page-8-0) [9](#page-8-0)], etc. Moreover, the need for higher model accuracy in certain conditions led the researchers to the development of other models like Bratch et al., that have developed a tire model for vehicle dynamics and accident reconstruction [\[10](#page-8-0)], R. Lot, who developed a motorcycle tire model [[11\]](#page-8-0), and Mancuson et al., that developed a model that considers also the comfort characteristics of the tire [\[12](#page-8-0)]. Most of the tire models have been developed for high speed, however due to the increase of the focus on autonomous driving in the last years, e.g. the automatic parking system [\[13](#page-8-0), [14](#page-8-0)], and observing that the automobiles usually spend long times at speed under 60 km/h during their entire life, also due to urban traffic [\[15](#page-8-0)], new different formulations to model tire behaviour at low speed have been presented. One of the main causes requiring the development of a low speed tire models is charged in the calculation of sideslip angle and longitudinal slip at low longitudinal velocities which leads to numerical problems [[16\]](#page-8-0). In fact, Garcia-Pozuelo et al., have been taken a great interest on the contact between tire and road during low speed applications [\[15](#page-8-0)].

Moreover, other researchers have been developed a tire models that taking into account also the thermodynamic effect [[17\]](#page-8-0) and tire wear [\[18](#page-8-0)] which are fundamental phenomena in tire-road interaction.

Tire models characterized by low computational load, able to perform quickly a large number of calculations are often used in real-time vehicle simulation environments. For this reason, the most used tire models in real-time simulations are often semi-empirical models, characterized by analytical formulations.

Commercial simulation softwares need to be validated to be used with awareness, because the software companies protect the implemented function by means of compiled algorithm, therefore it is impossible to get fundamental information on the effective tire model present in the simulation software.

Furthermore, it can be useful to substitute the tire model implemented by the software house because an open tire model gives the possibility of customization and makes the performance vary physically under different conditions or different compounds.

The paper's aim is to implement and preliminary analyse different Pacejka formulations, with the eventual target to highlight not disclosed subformulations employed in commercial environments; in particular, a native and compiled MF 6.1 [\[19](#page-8-0)] software formulation) and a MF 5.2 [[6\]](#page-8-0) implemented in parallel and called MF UniNa will be compared. As it is known [[19\]](#page-8-0), the two formulations diverge meanly for the introduction of pressure inflation microparameters.

Since the discussed tire models are different versions of the same formulation, the microparameters concerning specific physical effects have been zeroed, in order to make the comparison more reliable.

Some longitudinal (braking and acceleration) and lateral (spiral, steering pad, fishhook, line change, constant speed curve) manoeuvres have been adopted to compare the results of the implemented tire model and the influence on the overall vehicle dynamics.

Finally, Key Performance Indices (KPIs) characteristics of behaviour in longitudinal and lateral interaction have been determined and a sensitivity test on the models has been considered in order to evaluate the influence on the KPIs.

2 Simulation Environment

The commercial simulation software selected for the activity uses the Matlab-Simulink design environment to simulate different subsystems and submodels. These are the implementation of Engine, Drivetrain and Vehicle Dynamics equations; moreover there are the basic simulation of the ECUs that are present in the real vehicle, the so called "Soft ECUs", and the simulation of the so called "Environment" in which the vehicle works, i.e. a driver able to drive the vehicle itself, the Roads and the Traffic.

Inside the Vehicle Dynamics system there is the Tire sub-system where the four vehicle tires are managed. The Tire subsystem can be configured in order to use two different tire modelling: TMeasy [[7\]](#page-8-0) and the Magic Formula [\[19](#page-8-0)]. The vehicle dynamics is modelled as a nonlinear multibody system with 13 degrees of freedom and the suspensions are characterized by a nonlinear table-based model with kinematics and compliance.

The vehicle considered in this work has the characteristics of a MidSize Car of 1880 kg, the Drivetrain Type is a Front Wheel Drive and is equipped with a manual transmission.

3 Manoeuvers

As introduced in the last paragraph, some manoeuvres have been made in order to provide a preliminary comparison of the two Pacejka formulations output.

Among all the manoeuvres studied (braking and acceleration, spiral, steering pad, fishhook, line change, constant speed curve), have been chosen two representative manoeuvres, for the longitudinal and lateral interaction respectively.

As regards the longitudinal tire-road interaction analysis, a braking has been carried out. In the first phase of the breaking manoeuvre, the vehicle starts lunched at 100 km/h and keeps constant speed for two seconds. In the final phase, the vehicle brakes until it stops (0 km/h). Such braking has been performed in the absence of ABS.

The lateral manoeuvre is Slow Ramp Steer (SRS) [[20\]](#page-8-0). In the initial phase of the SRS manoeuvre, the vehicle starts at 100 km/h and keeps the speed constant for some seconds before starting to steer with a ramp steering law. Therefore, achieved the requested steering angle, it is maintained constant.

4 Implementation and Results

The MF UniNa has been implemented in Matlab-Simulink design environment, which is the basis of commercial vehicle dynamics software operation. In the first development phases of the MF UniNa, prefixed inputs were used and, including all the microparameters, loaded by a Matlab code. In the second step the MF UniNa was inserted in parallel to the one present in the commercial vehicle dynamics software. The final step was to compare, through the manoeuvres described in the dedicated paragraph of this paper, the outputs of the MF UniNa with the same ones obtained by default formulation.

The following results are focused on the characteristics manoeuvres described above, one of it highlighting the longitudinal behaviour of the tire (therefore of the vehicle) and the other considering the lateral behaviour.

All the results shown are referred to the same tire, in particular the front left tire.

The Fig. 1 shows the longitudinal tire-road interaction forces and slip during a braking manoeuvre from 100 km/h to 0 km/h for the UniNa model and for the commercial software model.

Fig. 1. Longitudinal forces and slip ratio in braking manoeuvre

Once reached the constant speed of 100 km/h, the results of the analysed models are similar. At the time of 2 s, the braking system is activated, at 3.2 s the wheel is blocked, and the tire starts to slide. In fact, it is possible to observe in the Fig. 1, the absolute value of the longitudinal slip calculated by the UniNa model is equal to 1.00, while the amount of slip calculated through the commercial software formulation is equal to 0.80 therefore, starting from this phenomenon the outputs of such models are different.

The outputs of such longitudinal manoeuvres display a first difference among the models under investigation caused by a different calculation of one of the Pacejka MF inputs that is the slip ratio [[6\]](#page-8-0).

The difference arises in terms of both maximum value and time required to achieve such a maximum value. In fact, the maximum value of the slip ratio (k) calculated in the implemented submodel following the standard formulation:

$$
k=-\frac{V_x-\Omega\bigwedge R}{V_x}
$$

is equal to 1.00 in according to Pacejka [\[6](#page-8-0)]. Differently, the maximum value slip amount calculated using the commercial software formulation is 1.06.

In order to investigate on the lateral forces during the braking manoeuvre, the F_v vs Time is shown in the Fig. 2 in which is possible to observe a perfect overlap of the two models.

Fig. 2. Lateral forces and slip angle in braking manoeuvre

Moreover, the Fig. 2 shows that the braking manoeuvre implemented is mainly a pure longitudinal tire-road interaction; for the first seconds of the analysis, lateral forces are present but they are not significant (they are an order of magnitude lower than the F_x) and are compensated by the corresponding forces on the opposite tire, moreover when the tire is blocked F_y is zero. Slip angle is zero during the entire manoeuvre. Although, observing the Fig. 2 it is not possible to understand if both models use the same formulation to calculate the slip angle. For this reason, a SRS manoeuvre has been analysed. Finally, the Fig. [3,](#page-5-0) shows the overturning torque, the rolling resistance and the self-aligning moment. As regards the overturning torque (M_x) , slight differences in the models are shown. Indeed, the results obtained for the rolling resistance (M_v) are the same for both models. Significative differences are present in the self-aligning moment (M_z) .

These results show that there is a different evaluation for the F_x , therefore in the longitudinal slip among the UniNa (and consequently the standard Pacejka's model) and commercial software tire models.

Fig. 3. Torques in braking manoeuvre

As already said, to analyse the lateral behaviour of the tire models, a SRS manoeuvre has been considered.

Fig. 4. Lateral forces in SRS manoeuvre

The Fig. [4](#page-5-0) shows the results of the models in a SRS manoeuvre. The lateral forces of the models differ significantly when the vehicle is about to close the steering ramp. To understand such differences, as for the longitudinal manoeuvre, is needed to investigate on the inputs of the models, in particular on the slip angle. In fact, as it is possible to observe in the Fig. [4](#page-5-0), the differences between the forces calculated with the UniNa formulation became significant starting from the same time in which is possible to observe differences in the calculation of the slip angle. Therefore, having observed such phenomena, it could be preliminary stated that the commercial software evaluates differently both slip ratio and slip angle with respect to the Pacejka formulation [\[6](#page-8-0)] with the potential intention to limit numerical issues, but at the same time, creating substantial modifications in vehicle dynamics.

5 KPI and Sensitivity Test

Focusing on Figs. [1](#page-3-0) and [4](#page-5-0), a Key Performance Index (KPI) linked to the behaviour in longitudinal and lateral interaction have been determined. Such KPIs are representative index of the behaviour during each manoeuvre and they make easy and immediate the comparison between different models.

In the longitudinal manoeuvre the KPI is defined as the ratio between the maximum F_x and the F_x stabilized value at $5th$ second:

$$
KPI_{braking} = \frac{Fx_{max}}{Fx_{(5s)}}
$$

In the lateral manoeuvre, the KPI_{SRS} is defined as the slope of the curve obtained by the interpolation of the points on the F_y curve, in the range identified by the first two seconds of steering.

The Table 1 below shows the KPIs for both manoeuvre and for both the models:

Finally, the KPIs can play an important role in the development of a sensitivity analysis. In fact, the Fig. [5](#page-7-0) below shows the variation of the slope for both models varying the steering angle (160°, 180°, 200°) in the lateral manoeuvre. The further summarization of the KPIs values can help to get a fast overview of the effects due to the different models in the same manoeuvre (Table [2\)](#page-7-0).

Fig. 5. Sensitivity analysis on lateral forces during the SRS manoeuvre

KPI_{SRS} [N/s]		
		Unina Commercial SW
	160° -1705 -1536	
	180° -1780 -1696	
	200° -2012 -1774	

Table 2. KPIs and sensitivity test for SRS manoeuvre for both the models

6 Conclusions

In this work a MF 5.2 tire model has been implemented in a vehicle dynamics simulation software. Subsequently, several manoeuvres have been analysed to compare the implemented magic formula tire model with the MF 6.1 tire model present in the cited commercial software.

The analysis of the manoeuvres underlines differences between the models, due to a different calculation of the slip ratio, and to objectivize these differences a KPI for each manoeuvre has been defined.

Finally, a sensitivity test on the models has been considered in order to evaluate the influence on the KPIs.

The limits of this work are related to the limits of the MF, as the presence of well known analytical indeterminacies when the vehicle speed is close to zero. Therefore, future developments could involve the implementation of a low speed tire model able to extend the domain of investigation and to focus on the numerical issues linked with such dynamics.

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